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Private Provision of Public Goods between Families*

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Abstract

We consider a two-stage voluntary provision model where individuals in a family contribute to inter-family public goods, and, at the same time, the parent makes private transfers to her child within the same family. We show not only that Warr's neutrality holds regardless of the different timings of parent-to-child transfers, but also that there is a continuum of Nash equilibria in the sense that individuals' contributions and parental transfers are indeterminate, although the allocation of each's private consumption and total public good provision is uniquely determined. We further show that, in the presence of impure altruism, neutrality and uniqueness of the equilibrium allocation persist.

Key Words: private provision, public good, Nash equilibrium, subgame perfect equilibrium, family
JEL Classification: C72, D64, H41

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1 Introduction

The standard framework for the analysis of private provision of public goods is the Nash-Cournot model in which agents choose their contributions simultaneously and independently. A celebrated theoretical result is that, within certain bounds, the distribution of incomes among contributors is irrelevant for the equilibrium allocation of goods. Consequently, governmental lump-sum income redistribution policy may have no real effect on either the allocation of economic resources between private and public consumption or the well-being of individuals. A characteristic feature of these models is that a contributor is a single individual or a private organization which behaves as a single player in provision games. In this paper we relax this assumption, and consider a model in which contributing agents are family members who make voluntary income transfers within a family, in addition to contributing to public goods.

The literature on family economics has not viewed a family as a single decision unit, since it consists of several heterogeneous agents characterized by different preferences as well as different income sources such as a husband, a wife, children or a grandmother, etc. Furthermore, those family members are interdependent in two respects (see, e.g., Chen and Woolley, 2001; and Ermisch, 2003). First, there are family or household public goods whose benefits are enjoyed only by members of the same family, such as a housework, a beautiful garden, a clean house, care for sick family members, the well-being of elderly parents and children, and so on. Second, either since family members care about each other or for other motivations such as self-interested exchange (e.g., Bernheim et al., 1985), they make private income transfers within the family.

In reality, households also voluntarily contribute to many public goods whose benefits spill over to members of other families. Such contributions include donations to charities, blood, and community orchestras, various volunteer activities and so on. As a result, several families will be interdependent through such interfamily public goods. Therefore, it is very natural to tie together two different stands in the literature; voluntary provision to public goods and the economics of family. Aside from reality, such a hybrid model would provide new theoretical implications to both fields.

Our analysis reveals three major findings. First, even if the distribution of income among individuals is fixed, an infinite number of combinations of private contributions to a public good and intrafamily transfers may be compatible with a unique profile of individual private consumptions and the total supply of public good. In other words, there is a continuum of (subgame perfect) Nash equilibria in the strategy space of individual voluntary contri-

butions. This non-uniqueness property contrasts sharply with the result of Bergstrom et al. (1986) in which, when agents have convex preferences defined over a private good and a normal public good, there exists a unique Nash equilibrium, with a unique set of individual contributions and the total provision of public good.

The source of this non-uniqueness result appears to be very simple. Suppose a parent simultaneously makes two types of voluntary contributions; private donations to interfamily public goods and parental altruistically motivated transfers to their child. From the viewpoint of the parent, both contributions would be regarded as perfect substitutes for provisions to the same public good, although such private transfers would be indirect contributions to that public good via the well-being of the child. Yet, we shall show that this intuition is not sufficient to account for the non-uniqueness of the Nash equilibrium.

The main focus of our model lies on the timing of parental transfers to children in the presence of voluntary contributions to interfamily public goods. There are two possible timings of parent-to-child transfers; more precisely, the parent makes private transfers to a selfish child after or before observing the child's action (i.e., ex-post transfers or pre-committed transfers, respectively). Assuming further that the parent simultaneously contributes to public goods as well as making parent-to-child transfers (which is the most plausible assumption), we demonstrate the robustness of Warr's (1983) neutrality theorem by considering these different sequential orders of actions chosen by the parent and child. Put differently, the neutrality result is independent of the details of the environment in which strategic behavior associated with the timing of parent's transfers.

Bernheim and Bagwell (1988) have argued that the existence of interfamily linkages through a common child produced by originally unrelated individuals makes any government policy, such as public transfers, distortionary taxation and so on, neutral in the sense that those policies have no real effect. The neutrality property they found, which is called 'cross sectional neutrality', is much stronger than Barro's neutrality which works through altruistically motivated intergenerational transfers. Bernheim and Bagwell's cross sectional neutrality operates through interfamily transfers based on marital connections.[1] In addition to such links provided by altruistically motivated transfers, we introduce an interfamily public good which provides another possible link connecting individuals in different families as well as within the same family. We show not only that the latter link acts as

[1]However, Laitner (1991) points out that when material connections are explicitly modeled via a market for marriage, this neutrality is not robust.

a perfect substitute for the link of operative interfamily transfers assumed in the model of Bernheim and Bagwell (1988), but also that the introduction of the strategic motives for transfers (i.e., different sequential orders of transfers) does not affect the likelihood of Bernheim and Bagwell's cross sectional neutrality. Indeed, the presence of voluntarily supplied public goods will enhance Bernheim and Bagwell's cross sectional neutrality in the sense that even if interfamily transfers are not operative, the redistribution neutrality holds as long as private donations to a public good are operative.

Cornes and Itaya (2003) show that in a one-shot, Nash provision game with many public goods, any income redistribution has no effect on the original equilibrium allocation, as long as that redistribution occurs within a set of *linked individuals* who are eventually connected each other through effective private contributions to many interfamily public goods. They call it 'partial neutrality'. We shall show that the concept of a 'link' they suggest plays a key role in generating the neutrality as well as non-uniqueness of an equilibrium allocation in our two-stage provision game. In our model we define a 'link' as either positive parent-to-child transfers or private donations to interfamily public goods, and then show that when a redistribution of income is carried over among the *linked individuals* who are eventually connected each other through the latter link, neutrality arises. Moreover, if the number of links is larger than the minimum number of links which connect individuals, the indeterminacy in terms of choice variables at the node where extra links emerge arises.

The third finding is that even if parents or children stop contributing to a public good, the neutrality with respect to a redistribution of income involving those non-contributing agents may remain valid, as long as the parental transfers are operative. In other words, even if we take '*non-contributors*' at face value; namely, the individuals who do not contribute a positive amount to the public good, the redistribution policy either between non-contributors or between contributors and non-contributors may not destroy neutrality, as long as they are connected through operative private transfers. Accordingly, our finding has not been addressed in the literature on private provision of public goods (see, e.g., Bergstrom et al., 1986) in which a redistribution of income among individuals involving non-contributors usually negates neutrality.

Abel and Bernheim (1991) point out the possibility that if there are some frictions such as impure altruism, incomplete information about others' preferences and egalitarian social norms that constrain parents to divide their transfers evenly among children, Bernheim and Bagwell's cross sectional neutrality fails to hold. Nevertheless, we show that even if there are such frictions, neutrality and uniqueness of the Nash equilibrium persist. On the other

hand, Andreoni (1990) finds that introducing a "warm glow" (i.e., impure altruism) destroys Warr's neutrality result, while in the presence of effective intrafamily private transfers neutrality again recovers. This neutrality reflects the fact that the number of links matters in generating neutrality rather than the mere on 2 presents the basic model. Section 3 considers the two-stage provision game with ex-post transfers to children. Section 4 considers the two-stage provision game with pre-committed transfers to children. Section 5 considers the two-stage provision game where different families adopt the different modes of transfers; one of the families adopt ex-post transfers, while another family adopts ex-post transfers. Section 6 performs the same analysis under Cobb-Douglas preferences. Section 7 concludes the paper with a discussion of some possible extensions of the model.

2 The Model

In this paper, without loss of generality, we consider an economy of two families, each consisting of a single altruistic parent and a single selfish child. The utility function of the parent, who is altruistic towards her child, is given by

$$U_p^i(c_p^i, G; c_k^i) \equiv u_p^i(c_p^i, G) + \alpha^i u_k^i(c_k^i, G),\ i = 1, 2, \tag{1}$$

where c_p^i and c_k^i are the parent's and child's consumptions of family i, respectively, G is an interfamily public good, α^i is the parameter which measures the strength of parent i's altruism towards her child, and u_p^i and u_k^i, respectively, represent the parent's and child's utility functions of family i which are twice-continuously differentiable, strictly quasi-concave, strictly increasing in each argument and $\partial u_j^i/\partial c_j^i \to \infty$ as $c_j^i \to 0$ for $i = 1, 2$ and $j = p, k$.[2] It is assumed that c_p^i (and c_k^i) and G are normal goods. We further assume that $\alpha^i \in [0, 1]$, which implies that the parent neither cares about her child more than herself nor hates her child.

The public good G is an interfamily public good whose benefits spill over to members of the other family. Moreover, the public good is entirely supplied by voluntary contributions made by the parent and child, g_p^i and g_k^i of family $i = 1, 2$, respectively. The public good is thus produced according to the following summation technology:

[2] Although the form of the utility function given by (1) has been commonly used in the literature on family economics, it is easy to show that the results obtained in the present paper remain valid in the more general utility function such as $U_p^i(c_p^i, G, u_k^i(c_k^i, G))$.

$$G = \sum_{j=1}^{2} \left(g_p^j + g_k^j \right). \tag{2}$$

3 Nash Equilibrium with Ex-post Transfers

In this section we consider a two-stage contribution game in which the child and parent play sequentially, and characterize the resulting subgame perfect Nash equilibrium. We shall consider two different timings that the parent and child play. We first investigate the following timing of actions. In stage 1, the child of family i chooses her own consumption, c_k^i, and contribution to the public good, g_k^i. In stage 2, after having observed the contributions made by the children of both families, (g_k^1, g_k^2), the parent of family i chooses c_p^i and g_p^i (or equivalently, π^i and g_p^i) so as to maximize her utility function (1) subject to

$$c_p^i + \pi^i + g_p^i = y_p^i, \tag{3}$$

$$c_k^i + g_k^i = y_k^i + \pi^i, \tag{4}$$

where π^i represents the transfer from the parent of family i to her child and y_p^i is the fixed income of the parent of family i.[3]

We use backward induction to solve the parent's optimization problem first. After substitution of the (3) and (4) into (1), the parent's problem is:

$$\max_{\{g_p^i, \pi^i\}} U_p^i = u_p^i(y_p^i - \pi^i - g_p^i, G) + \alpha^i u_k^i(y_k^i + \pi^i - g_k^i, G). \tag{5}$$

[3]Since one may alternatively view the present model as one in which one spouse makes an income transfer to the other – perhaps motivated by altruistic concerns – the present analysis may well be applied to those cases.

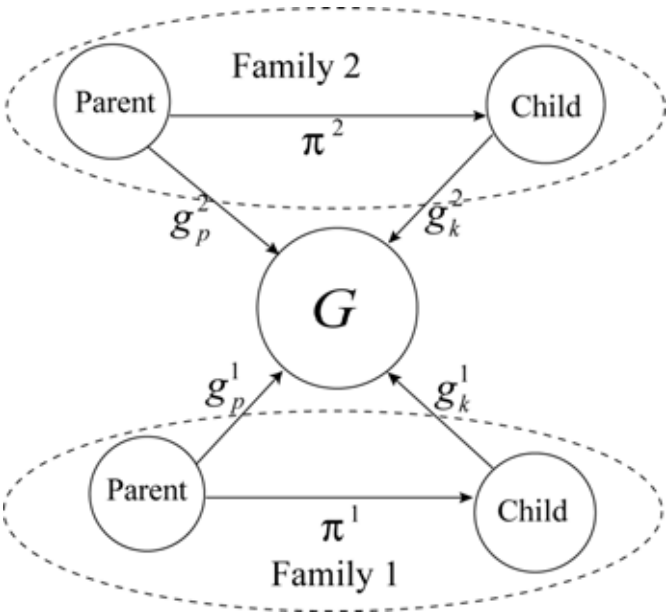

Figure 1. Both parent and child contribute to the public good G.

The first-order conditions characterizing an interior solution are

$$\frac{\partial U_p^i}{\partial g_p^i} = -\frac{\partial u_p^i(y_p^i - \pi^i - g_p^i, G)}{\partial c_p^i} + \frac{\partial u_p^i(y_p^i - \pi^i - g_p^i, G)}{\partial G} + \alpha^i \frac{\partial u_k^i(y_k^i + \pi^i - g_k^i, G)}{\partial G} = 0,\ i = 1, 2, \tag{6}$$

$$\frac{\partial U_p^i}{\partial \pi^i} = -\frac{\partial u_p^i(y_p^i - \pi^i - g_p^i, G)}{\partial c_p^i} + \alpha^i \frac{\partial u_k^i(y_k^i + \pi^i - g_k^i, G)}{\partial c_k^i} = 0,\ i = 1, 2. \tag{7}$$

According to (6), the parent's optimal contribution equates the marginal utility of one unit increase in her own consumption to the sum of the parent's and child's marginal utilities resulting from one unit increase in the total provision of public good. Condition (7) implies that the parent's optimal transfer equates the marginal utility of her own consumption to the discounted marginal utility of the child's consumption. Since (6) and (7) form the system of equations in g_p^i and π^i, $i = 1, 2$, given g_k^i and g_k^j, we can solve it for $\hat{g}_p^i(g_k^1, g_k^2)$ and $\hat{\pi}^i(g_k^1, g_k^2)$, $i = 1, 2$.

Given the optimal transfer and contribution functions $\hat{\pi}^i(g_k^1, g_k^2)$ and $\hat{g}_p^i(g_k^1, g_k^2)$ for $i = 1, 2$, the child's maximizing problem of family i at stage 1 is given by

$$\max_{\{g_k^i\}} U_k^i = u_k^i(y_k^i + \hat{\pi}^i\left(g_k^i, g_k^j\right) - g_k^i, \hat{G}),$$

where $\hat{G} \equiv \hat{g}_p^1\left(g_k^1, g_k^2\right) + \hat{g}_p^2\left(g_k^1, g_k^2\right) + g_k^1 + g_k^2$. The first-order conditions for an interior solution are given by

$$\frac{\partial U_k^i}{\partial g_k^i} = \frac{\partial u_k^i(y_k^i + \hat{\pi}^i - g_k^i, \hat{G})}{\partial c_k^i}\left[\frac{\partial \hat{\pi}^i}{\partial g_k^i} - 1\right] + \frac{\partial u_k^i(y_k^i + \hat{\pi}^i - g_k^i, \hat{G})}{\partial G}\left[\frac{\partial \hat{g}_p^i}{\partial g_k^i} + \frac{\partial \hat{g}_p^j}{\partial g_k^i} + 1\right] = 0,\ i = 1, 2. \tag{8}$$

As shown in Appendix A, it follows from (A2), (A3) and (A4) that

$$\frac{\partial \hat{\pi}^i}{\partial g_k^i} = 1 \text{ and } \frac{\partial \hat{g}_p^i}{\partial g_k^i} + \frac{\partial \hat{g}_p^j}{\partial g_k^i} + 1 = 0,\ i = 1, 2, \tag{9}$$

which automatically leads to that $\partial U_k^i / \partial g_k^i = 0$ for $i = 1, 2$. This implies that the first-order conditions (8) are satisfied for any value of g_k^i, $i = 1, 2$, so that indeterminacy in terms of g_k^i arises.

Next, in order to examine the validity of neutrality, we suppose that $dy_p^1 + dy_p^2 + dy_k^1 + dy_k^2 = 0$. We further assume that

$$dy_p^i = d\pi^i + dg_p^i,\ i = 1,2, \tag{10}$$

$$dy_k^i + d\pi^i = dg_k^i,\ i = 1,2, \tag{11}$$

which implies that the *constant* private consumption of every family member will satisfy the budget constraints (3) and (4). Moreover, it is shown that

$$\begin{aligned} dG &= dg_p^1 + dg_p^2 + dg_k^1 + dg_k^2, \\ &= dy_p^1 - d\pi^1 + dy_p^2 - d\pi^2 + dy_k^1 + d\pi^1 + dy_k^2 + d\pi^2, \\ &= dy_p^1 + dy_p^2 + dy_k^1 + dy_k^2 = 0, \end{aligned} \tag{12}$$

where the second equality in (12) follows from (10) and (11), and the last equality follows from the assumed scheme of income redistribution. Since the private consumption of every individual as well as total public good provision both remain invariant, the first-order conditions (6) and (7) are also unaffected. Since (8) is always fulfilled at equality, neutrality obtains. In words, when lump-sum taxes are imposed on, say, the parent of family 1, she can undo their effect by either withdrawing the transfer to her child,

reducing her public good contribution, or both, in anticipation that the child of family 1 or/and the members of family 2 will receive a lump-sum transfer of the same amount.

Furthermore, it follows from the budget constraints (3) and (4), in conjunction with the invariance of c_k^i and c_p^i, that the resulting indeterminacy of g_k^i entails the indeterminacy of π^i and g_p^i as well, although the values of $\pi^i + g_p^i$ and thus $g_p^i + g_k^i$ are uniquely determined for each i. In other words, there are an infinite number of combinations of g_k^i and π^i (or equivalently, combinations of g_p^i and π^i) which are consistent with a unique profile of private consumptions and the total provision of public good in a (subgame perfect) Nash equilibrium.[4]

As for non-interior solutions, there are several patterns as illustrated in Figures 2-4. We first consider the case where only the children of both families are contributors, which corresponds to Figure 2. In this case we consider the following hypothetical changes in the contributions of children and the parental transfers in both families in response to an income redistribution that satisfies $dy_p^1 + dy_p^2 + dy_k^1 + dy_k^2 = 0$:

$$dy_p^i = d\pi^i,\ i = 1,2, \tag{13}$$

$$dy_k^i + d\pi^i = dg_k^i,\ i = 1,2. \tag{14}$$

These hypothetical responses, in conjunction with the budget constraints (3) and (4), result in the constant private consumption of every individual and

$$\begin{aligned} dG &= dg_k^1 + dg_k^2, \\ &= dy_k^1 + d\pi^1 + dy_k^2 + d\pi^2, \\ &= dy_k^1 + dy_p^1 + dy_k^2 + dy_p^2 = 0, \end{aligned}$$

where the second equality follows from (14), while the third equality follows from (13). In addition, the constancy of the variables c_p^i, c_k^i and G leaves

[4] When the child has another choice variable such as a labor supply (i.e., 'a lazy kid' in the sense of Bergstrom; 1989), the optimizing problem of the child becomes

$$\max_{\{g_k^i\}} U_k^i = u_k^i(y_k^i(a^i) + \hat{\pi}^i(g_k^i, g_k^j, a^i) - g_k^i, \hat{G}, a^i),$$

where $y_k^i(a^i)$ represents the labor income earned by the child which depends positively on the child's labor supply a^i, and is a C^2-class function. The optimal transfer function $\hat{\pi}^i(g_k^i, g_k^j, a^i)$ has been obtained by solving the first-order condition for the parent's optimizing problem given by (6) and (7), given that g_k^i, g_k^j and a^i have been determined at stage 1. Assuming (10) and (11), we can show that neutrality as well as indeterminacy hold in a similar way.

the first-order conditions (7) and (8) intact as well. Taken together, the hypothetical responses prescribed by (13) and (14) are fulfilled at equilibrium, thus leading to the neutrality of income redistribution policy. Both π^i and thus g_k^i are uniquely determined from (3) and (4) owing to the neutrality property, respectively, for π^i is a sole decision variable confronted by the parent.[5]

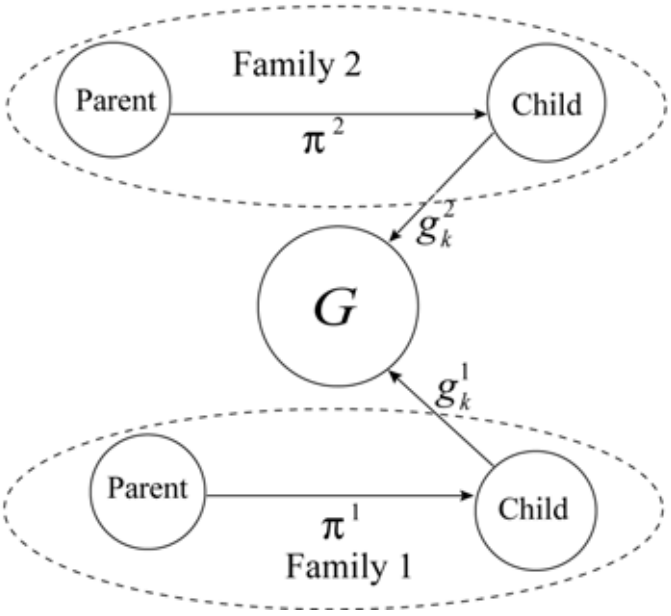

Figure 2. Only the child contributes to the public good G.

Next, consider the case where only the parents of both families are contributors, as illustrated in Figure 3. As before, we consider the following hypothetical changes in the children's contributions and the parental transfers in response to the income redistribution:

$$dy_p^i = d\pi^i + dg_p^i,\ i = 1,2, \tag{15}$$

$$dy_k^i + d\pi^i = 0,\ i = 1,2. \tag{16}$$

In an analogous manner, we can confirm that the hypothetical responses are fulfilled thus resulting in the validity of neutrality, and that π^i and g_p^i are uniquely determined from the budget constraint (3) and (4), respectively.

Finally, consider the case depicted in Figure 4. In this case we assume the following responses of private contributions as well as parental transfers to the income redistribution:

[5] When all individuals make positive contributions to the public good but the parents stop the transfers to their children, the resulting case precisely corresponds to the standard Nash provision game (see, e.g., Bergstrom et al., 1986).

$$dy_p^1 = d\pi^1 + dg_p^1 \text{ and } dy_k^1 + d\pi^1 = 0, \tag{17}$$

$$dy_p^2 = d\pi^2 \text{ and } dy_k^2 + d\pi^2 = dg_k^2, \tag{18}$$

which ensures Warr's neutrality theorem as well as the uniqueness of π^1, π^2, g_p^1 and g_k^2 as before.

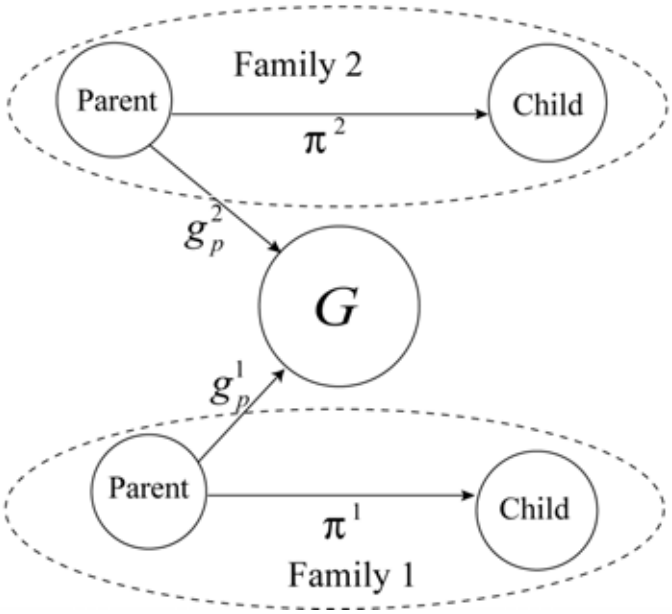

Figure 3. Only the parent contributes to the public good G.

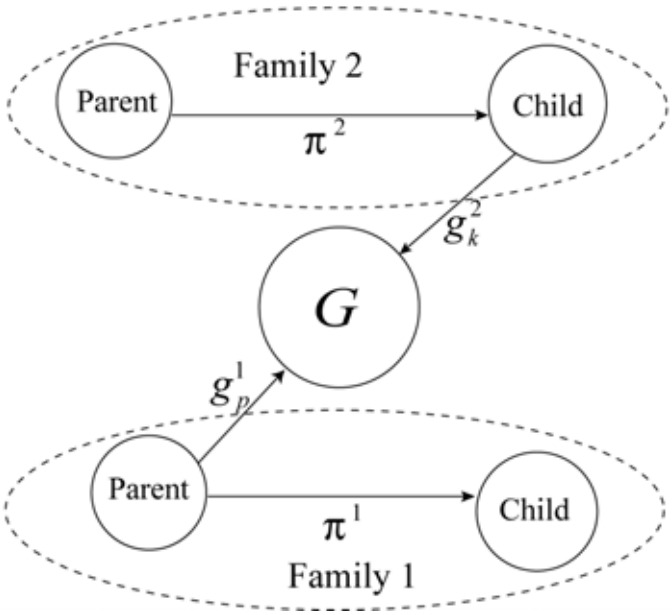

Figure 4. Only the parent contributes to the public good G in Family 1, while only the child contributes to the public good G in Family 2.

To sum up:

Proposition 1 *Consider a two-family, two-stage contribution game with the preferences in (1), where the parents decide how much to make transfers to the children after having observed the contributions of all children. Then, we have*
(*i*) *Neutrality in terms of a redistribution of income as well as indeterminacy in terms of the private contributions of all family members as well as the transfers made by the parents hold, provided the contribution of every individual and transfers made by the parents are positive;*
(*ii*) *If only the parents (only children) of both families are contributors, then the Nash equilibrium is unique, but neutrality remains valid, provided that the parents of both families make positive transfers to their children, and;*
(*iii*) *If only the child is a contributor in one family while only the parent is a contributor in another family, then the Nash equilibrium is unique, but neutrality remains valid, provided that the parents of both families make positive transfers to their children.*

Varian (1994) investigates a private provision model in which two agents sequentially choose voluntary contributions to a public good, and finds that in a Stackelberg equilibrium where both agents contribute a positive amount

to the public good the neutrality result holds, which is consistent with our result. Nevertheless, his model does not entail a continuum of Nash equilibria. This implies that the concept of a Stackelberg equilibrium itself will cause redistributional income neutrality but does not suffice to generate indeterminacy.

Statements (ii) and (iii) in Proposition 1 have not been addressed in any of the literature on private provision of public goods. These properties appear to be similar to Proposition 3 in the multiple-public good model of Cornes and Itaya (2003) in which, even if two individuals do not contribute to the same public good, a redistribution of income that is restricted to a set of '*linked individuals*', who are eventually connected each other through effective private contributions to voluntarily provided public goods, has no effect on the original equilibrium allocation (they call it 'partial neutrality'). However, their analysis is limited to a one-shot, simultaneous Nash provision game. In light of Proposition 1, it is easy to show that 'partial neutrality' holds true in our model with the sequential order of actions associated with the Stackelberg equilibrium concept.

In order to clarify the relationship between our current treatment and the 'partial neutrality' of Cornes and Itaya, we have to slightly modify their definition of *linked individuals* as follows:

Definition 1 *Individuals h and k are* **linked** *at an equilibrium if at least one of the following conditions holds: (i) there are positive transfers between h and k: (ii) they contribute to the same public good.*

Clearly, if individuals h and k are linked, regardless of whether they belong to the same family or not, so too are any two individuals belonging to the chain, which may consist of operative private transfers within a family and/or positive private contributions to public goods whose benefits spill over to members of other families, that links them.[6] Using this definition, Proposition 1 can be restated in the following corollary:

Corollary 1 *Consider a two-family, two-stage contribution game with the preferences in (1), where the parents decide how much to make transfers to the children after having observed the contributions of all children. An income redistribution that is restricted to a set of linked individuals, that maintains the links between them, has no effect on the original equilibrium allocation.*

[6] As evident in Definition 1, the concept of linked individuals is not restricted in the case involving a single public good and two families, each consisting of a single parent and a single child. It is straightforward to demonstrate the validity of partial neutrality in the context of many public goods, many families and many children.

Moreover, if every individual is linked through at least *one link supported by positive transfers to her child or positive private contributions to the public good, neutrality holds. In particular, if every individual is linked through* only *one link (we call such links "the minimum set of links"), the uniqueness of the associated equilibrium profile emerges.*

Stated differently, if there are *double* links at some node associated with a particular individual; for instance, the parents simultaneously make both positive transfers to their children and private contributions to the public good, indeterminacy as to the associated choice variables emerges.[7] In Figure 1 the double links occur at the nodes labeled by *Parent* in both families and thus indeterminacy between the parents' choice variables such as g_p^i and π^i arises, while in Figures 2-4 there is no double link so that indeterminacy does not arise. Since in all diagrams all nodes are connected, neutrality arises.

Nevertheless, it is still unclear whether or not the property of indeterminacy is robust with respect to the different timings of the sequential ordering of actions. To verify this, we need to further investigate the model with alternative timing of parental transfers and private donations, and the hybrid model where different families adopt different timings of transfers and private donations in the following sections, respectively.[8]

[7] The concept of "double link" can more carefully be defined using the terminology of graph theory as follows. Since all nodes including the node labeled G in Figures 1-4 are tied by several links (each of which is called "edge"), all graphs depicted in Figures 1-4 are termed as "the graph is connected". Hence, if the graph is connected, neutrality emerges as shown in Proposition 1. Moreover, if one of those edges is to be dropped in Figures 2-4, the graph becomes "disconnected". Then the original graph is called a "tree" (we also call these links "the minimum set of links"). Therefore, if the graph is a tree (or if the links consist of the minimum set of links), indeterminacy never arises. Thus the concept of "double link" means that if there are more links than the number of the minimum set of links, there exist extra links at some node and we call them "double links".

[8] We can address the efficiency problem. To do this, combining (6) and (7) and rearranging yields

$$\frac{\partial u_{\mathsf{p}}^{\mathsf{i}}(c_{\mathsf{p}}^{\mathsf{i}}, G)/\partial G}{\partial u_{\mathsf{p}}^{\mathsf{i}}(c_{\mathsf{p}}^{\mathsf{i}}, G)/\partial c_{\mathsf{p}}^{\mathsf{i}}} + \frac{\partial u_{\mathsf{k}}^{\mathsf{i}}(c_{\mathsf{k}}^{\mathsf{i}}, G)/\partial G}{\partial u_{\mathsf{k}}^{\mathsf{i}}(c_{\mathsf{k}}^{\mathsf{i}}, G)/\partial c_{\mathsf{k}}^{\mathsf{i}}} = 1,\ i = 1, 2.$$

This implies that the Samuelson's efficiency condition for provision holds within a family. Stated differently, the parent of each family is willing to internalize the externality of the public good so that the Rotten-kid theorem in terms of private provision of public goods holds true. However, it should be noted that it is not fully Pareto-efficient in the whole society.

4 Nash Equilibrium with Pre-committed Transfers

In this section we suppose that the parent pre-commits to a fixed transfer before the child chooses her public good contribution. Given the pre-committed transfer π^i, at stage 2 the child chooses her contribution to maximize the utility function. More formally, given the transfers (π^1, π^2) and the contributions made by the parents (g_p^1, g_p^2), after substituting the budget constraint (4), the child's decision problem in family i at stage 2 is to maximize her own utility function:

$$\max_{\{g_k^i\}} U_k^i = u_k^i(y_k^i + \pi^i - g_k^i, G).$$

Assuming an interior solution, the first-order conditions for the respective families are given by

$$\frac{\partial u_k^i\left(y_k^i + \pi^i - g_k^i, G\right)}{\partial c_k^i} = \frac{\partial u_k^i\left(y_k^i + \pi^i - g_k^i, G\right)}{\partial G}, \; i = 1, 2. \tag{19}$$

Since the first-order conditions (19) constitute the system of equations in g_k^i for $i = 1, 2$, we can solve it for $\hat{g}_k^i\left(\pi^1, \pi^2, g_p^1, g_p^2\right)$ for $i = 1,2$.

After substitution of the above optimal consumption and contribution functions into the parent's utility function, the parent's decision problem in family i at stage 1 is to maximize her utility function:

$$\max_{\{g_p^i, \pi^i\}} U_p^i = u_p^i(y_p^i - \pi^i - g_p^i, \hat{G}) + \alpha^i u_k^i(y_k^i + \pi^i - \hat{g}_k^i\left(\pi^1, \pi^2, g_p^1, g_p^2\right), \hat{G}),$$

where $\hat{G} \equiv g_p^i + g_p^j + \hat{g}_k^i\left(\pi^1, \pi^2, g_p^1, g_p^2\right) + \hat{g}_k^j\left(\pi^1, \pi^2, g_p^1, g_p^2\right)$.

The first-order conditions characterizing an interior solution are

$$\frac{\partial U_p^i}{\partial g_p^i} = -\frac{\partial u_p^i}{\partial c_p^i} - \alpha^i \frac{\partial u_k^i}{\partial c_k^i}\frac{\partial \hat{g}_k^i}{\partial g_p^i} + \left[\frac{\partial u_p^i}{\partial G} + \alpha^i \frac{\partial u_k^i}{\partial G}\right]\left[\frac{\partial \hat{g}_k^i}{\partial g_p^i} + \frac{\partial \hat{g}_k^j}{\partial g_p^i} + 1\right] = 0, \; i = 1, 2, \tag{20}$$

$$\frac{\partial U_p^i}{\partial \pi^i} = -\frac{\partial u_p^i}{\partial c_p^i} + \alpha^i \frac{\partial u_k^i}{\partial c_k^i}\left[1 - \frac{\partial \hat{g}_k^i}{\partial \pi^i}\right] + \left[\frac{\partial u_p^i}{\partial G} + \alpha^i \frac{\partial u_k^i}{\partial G}\right]\left[\frac{\partial \hat{g}_k^i}{\partial \pi^i} + \frac{\partial \hat{g}_k^j}{\partial \pi^i}\right] = 0, \; i = 1, 2. \tag{21}$$

We first show that the neutrality property holds. We once again consider hypothetical changes in family member's private contributions and parental transfers prescribed by (10) and (11) in response to the redistribution of income. As before, it is straightforward to show that *constant* private consumptions of parents and children, as well as *constant* total provision of

public good, are consistent with the budget constraints (3) and (4), as well as the first-order conditions (19), (20) and (21). This entails redistributional income neutrality.

Furthermore, substituting (B2), (B6), (B7), and (B8) in Appendix B, into the first-order conditions (20) and (21), these conditions boil down to a single equation so that there are too few equations to determine the unknown variables π^i and g_p^i, i = 1,2; consequently, indeterminacy arises, although the values of $g_p^i + \pi^i$ and thus $g_p^i + g_k^i$ are uniquely determined.

As for non-interior solutions, we have to consider the three cases associated with Figures 2-4. Applying the hypothetical changes in individual contributions and parental private transfers prescribed by either (14) and (13), (16) and (15) or (18) and (17) to the respective cases leads to both the validity of neutrality and also the failure of indeterminacy as before.

In summary;

Proposition 2 *Consider a two-family, two-stage contribution game with the preferences in (1), where the children decide how much to make their contributions to the public good after having observed the transfers made by the parents to the children as well as contributions of the parents. Then, statements (i), (ii) and (iii) in Proposition 1 hold true.*

Hence, Corollary 1 holds as well.

Cornes and Silva (1999) obtain a result similar to ours in that within a single family framework the voluntary contributions to a family public good and the transfers to children are indeterminate, when contributions to the family public good are perfect substitutes. Consequently, there is a continuum of Nash equilibria. Unfortunately, according to Chiappori and Werning (2002), Cornes and Silva's finding is not robust in the sense that an interior Nash equilibrium where every child makes a positive contribution is non-generic.

In contrast, our indeterminacy result holds regardless of the forms of utility functions which satisfy the standard properties. The key reason underlying our non-uniqueness result is the presence of an additional choice confronted by the parents, in conjunction with the well-established neutrality property in the literature on voluntary provision of public goods independently of the form of utility functions. Suppose that, initially, there is a minimal set of links. In view of Corollary 1, this means that the original graph would become disconnected if any one edge – or link – were dropped. Then adding a link to the original graph (see footnote 6) generates an indeterminacy of the associated additional choice variable confronted by the parents.

5 Nash Equilibrium with the Mix of Ex-post and Pre-committed Transfers

So far we have investigated homogeneous economies in the sense that there is only one type of family: either all families make ex-post transfers or all families make pre-committed transfers. We now relax this setting to allow for the coexistence of those two types of families in the economy. Suppose, without loss of generality, that the parent of family 1 makes ex-post transfers, while that of family 2 makes pre-committed transfers, as illustrated in Figure 5.

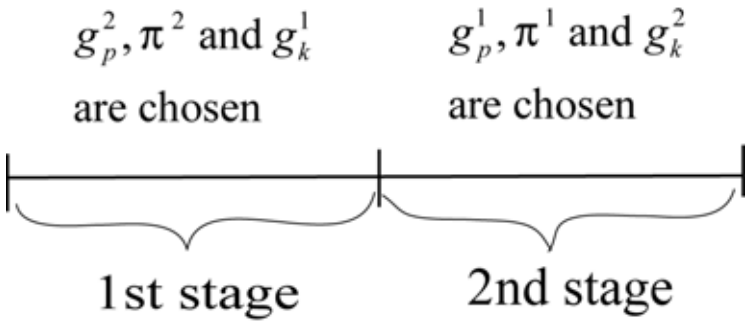

Figure 5. The timing of individual actions in the mixed case.

In stage 2, after having observed $(g_k^1,\ \pi^2,\ g_p^2)$, the parent of family 1 chooses π^1 and g_p^1, while the child of family 2 chooses g_k^2. Assuming an interior solution, the first-order conditions for the family 1's parent and the family 2's child are respectively given by

$$\frac{\partial U_p^1}{\partial g_p^1} = -\frac{\partial u_p^1(y_p^1 - \pi^1 - g_p^1, G)}{\partial c_p^1} + \frac{\partial u_p^1(y_p^1 - \pi^1 - g_p^1, G)}{\partial G} + \alpha^1 \frac{\partial u_k^1(y_k^1 + \pi^1 - g_k^1, G)}{\partial G} = 0, \tag{22}$$

$$\frac{\partial U_p^1}{\partial \pi^1} = -\frac{\partial u_p^1(y_p^1 - \pi^1 - g_p^1, G)}{\partial c_p^1} + \alpha^1 \frac{\partial u_k^1(y_k^1 + \pi^1 - g_k^1, G)}{\partial c_k^1} = 0, \tag{23}$$

$$\frac{\partial U_k^2}{\partial g_k^2} = -\frac{\partial u_k^2\,(y_k^2 + \pi^2 - g_k^2, G)}{\partial c_k^2} + \frac{\partial u_k^2\,(y_k^2 + \pi^2 - g_k^2, G)}{\partial G} = 0, \tag{24}$$

which form the system of equations in g_p^1, g_k^2 and π^1, given g_k^1, π^2 and g_p^2. Solving this, we can get $\hat{g}_p^1\left(g_k^1,\ \pi^2,\ g_p^2\right)$, $\hat{g}_k^2\left(g_k^1,\ \pi^2,\ g_p^2\right)$ and $\hat{\pi}^1\left(g_k^1,\ \pi^2,\ g_p^2\right)$.

Given those optimal functions, the child's maximizing problem in family 1 and the parent's decision problem in family 2 at stage 1 are respectively given by

$$\max_{\{g_k^1\}} U_k^1 = u_k^1(y_k^1 + \hat{\pi}^1\left(g_k^1,\ \pi^2,\ g_p^2\right) - g_k^1, \hat{G}),$$

and

$$\max_{\{g_p^2,\pi^2\}} U_p^2 = u_p^2(y_p^2 - \pi^2 - g_p^2, \hat{G}) + \alpha^2 u_k^2(y_k^2 + \pi^2 - \hat{g}_k^2\left(g_k^1,\ \pi^2,\ g_p^2\right), \hat{G}),$$

where $\hat{G} \equiv \hat{g}_p^1\left(g_k^1,\ \pi^2,\ g_p^2\right) + g_p^2 + g_k^1 + \hat{g}_k^2\left(g_k^1,\ \pi^2,\ g_p^2\right)$. The first-order conditions for an interior solution are respectively given by

$$\frac{\partial U_k^1}{\partial g_k^1} = \frac{\partial u_k^1}{\partial c_k^1}\left[\frac{\partial \hat{\pi}^1}{\partial g_k^1} - 1\right] + \frac{\partial u_k^1}{\partial G}\left[\frac{\partial \hat{g}_p^1}{\partial g_k^1} + \frac{\partial \hat{g}_k^2}{\partial g_k^1} + 1\right] = 0, \tag{25}$$

$$\frac{\partial U_p^2}{\partial g_p^2} = -\frac{\partial u_p^2}{\partial c_p^2} - \alpha^2 \frac{\partial u_k^2}{\partial c_k^2}\frac{\partial \hat{g}_k^2}{\partial g_p^2} + \left[\frac{\partial u_p^2}{\partial G} + \alpha^2 \frac{\partial u_k^2}{\partial G}\right]\left[\frac{\partial \hat{g}_p^1}{\partial g_p^2} + \frac{\partial \hat{g}_k^2}{\partial g_p^2} + 1\right] = 0, \tag{26}$$

$$\frac{\partial U_p^2}{\partial \pi^2} = -\frac{\partial u_p^2}{\partial c_p^2} + \alpha^2 \frac{\partial u_k^2}{\partial c_k^2}\left[1 - \frac{\partial \hat{g}_k^2}{\partial \pi^2}\right] + \left[\frac{\partial u_p^2}{\partial G} + \alpha^2 \frac{\partial u_k^2}{\partial G}\right]\left[\frac{\partial \hat{g}_p^1}{\partial \pi^2} + \frac{\partial \hat{g}_k^2}{\partial \pi^2}\right] = 0. \tag{27}$$

As shown in Appendix C, it follows from (C2), (C3) and (C3) that[9]

$$\frac{\partial \hat{\pi}^1}{\partial g_k^1} = 1 \text{ and } \frac{\partial \hat{g}_p^1}{\partial g_k^1} + \frac{\partial \hat{g}_k^2}{\partial g_k^1} + 1 = 0, \tag{28}$$

$$\frac{\partial \hat{g}_k^2}{\partial g_p^2} = -1 + \frac{\partial \hat{g}_k^2}{\partial \pi^2} \text{ and } \frac{\partial \hat{g}_p^1}{\partial g_p^2} + \frac{\partial \hat{g}_k^2}{\partial g_p^2} + 1 = \frac{\partial \hat{g}_p^1}{\partial \pi^2} + \frac{\partial \hat{g}_k^2}{\partial \pi^2}. \tag{29}$$

Property (28) ensures that $\partial U_k^1/\partial g_k^1 = 0$ in (25) always holds, which implies that values of g_k^1 are indeterminate. Moreover, owing to property (29), both (26) and (27) boil down to a single equation, implying that the division between g_p^2 and π^2 is also indeterminate. Furthermore, assuming the hypothetical responses prescribed by (10) and (11), we can easily verify that neutrality holds.

As for non-interior solutions, we have to consider the following four cases; the first three cases correspond to the ones depicted in Figures 1-3, and the last case to the one opposed to Figure 4, where the only child in family 1 is a contributor while the only parent of family 2 is a contributor. Assuming (13) and (14), (15) and (16), as well as (17) and (18) to the first three cases and the following hypothetical changes:

[9] Note that the two expressions in (29) simplify to $\partial \hat{g}_p^1/\partial g_p^2 = \partial \hat{g}_p^1/\partial \pi^2$.

$$dy_p^1 = d\pi^1 \text{ and } dy_k^1 + d\pi^1 = dg_k^1,$$
$$dy_p^2 = d\pi^2 + dg_p^2 \text{ and } dy_k^2 + d\pi^2 = 0,$$

to the last case, respectively, we can establish the validity of neutrality as well as the uniqueness of the equilibrium profile of private contributions and parental transfers in either case. Since the links appearing in Figures 2-4 consist of the *minimum set of links*, these results are compatible with Corollary 1. In summary:

Proposition 3 *Consider a two-family, two-stage contribution game with the preferences in (1), where the parent of one family makes ex-post transfers to the child and the parent of the other family makes pre-committed transfers to the child. Then, statements (i), (ii) and (iii) in Proposition 1 hold true.*

6 Cobb-Douglas Preferences

This section presents examples of the foregoing arguments. A special example may help to show algebraically that Warr's neutrality theorem holds, and, in particular, enables us to identify the range of income distributions that are consistent with the existence of an interior equilibrium. To this end, let us assume the following particular utility functions for the parent and child, respectively:

$$u_p^i(c_p^i, G) \equiv \ln c_p^i + \gamma_p \ln G,\ i = 1, 2, \tag{30}$$
$$u_k^i(c_k^i, G) \equiv \ln c_k^i + \gamma_k \ln G,\ i = 1, 2, \tag{31}$$

while the rest of the model is the same as before. Without loss of generality, we further assume that $\alpha = \alpha^i$ for $i = 1, 2$.

6.1 Ex-post Transfers

We first consider the two-stage problem associated with ex-post transfers under the utility functions (30) and (31). Assuming an interior solution and given g_k^i for i = 1,2, at stage 2 the parent's first-order conditions of family i (6) and (7) are respectively expressed by

$$\frac{\partial U_p^i}{\partial \pi^i} = \frac{-1}{y_p^i - \pi^i - g_p^i} + \frac{\alpha}{y_k^i + \pi^i - g_k^i} = 0,\ i = 1, 2, \tag{32}$$
$$\frac{\partial U_p^i}{\partial g_p^i} = \frac{-1}{y_p^i - \pi^i - g_p^i} + \frac{\gamma_p + \alpha\gamma_k}{G} = 0,\ i = 1, 2. \tag{33}$$

Solving (32) for π^i yields

$$\hat{\pi}^i = \frac{1}{1+\alpha}\left[\alpha\left(y_p^i - g_p^i\right) - y_k^i + g_k^i\right],\ i = 1,2. \tag{34}$$

Substituting (34) into π^i in (33) for each i and forming the system of equations in g_p^1 and g_p^2, we solve for:

$$\hat{g}_p^1 + \hat{g}_k^1 = \frac{\left(1+\alpha+\gamma_p+\alpha\gamma_k\right)\left(y_p^1+y_k^1\right) - (1+\alpha)\left(y_p^2+y_k^2\right)}{2(1+\alpha)+\gamma_p+\alpha\gamma_k}, \tag{35}$$

$$\hat{g}_p^2 + \hat{g}_k^2 = \frac{\left(1+\alpha+\gamma_p+\alpha\gamma_k\right)\left(y_p^2+y_k^2\right) - (1+\alpha)\left(y_p^1+y_k^1\right)}{2(1+\alpha)+\gamma_p+\alpha\gamma_k}, \tag{36}$$

which implies that there exists a certain range of income distributions among individuals for which an interior solution occurs. Summing (35) and (36) to get:

$$\hat{G} \equiv \hat{g}_p^1 + \hat{g}_p^2 + \hat{g}_k^1 + \hat{g}_k^2 = \frac{\left(\gamma_p+\alpha\gamma_k\right)Y}{2(1+\alpha)+\gamma_p+\alpha\gamma_k}, \tag{37}$$

where $Y \equiv y_p^1 + y_k^1 + y_p^2 + y_k^2$.

Given (37), the child's first-order condition at stage 1 is given by

$$\frac{\partial U_k^i}{\partial g_k^i} = \frac{1}{y_k^i + \hat{\pi}^i - g_k^i}\left(\frac{\partial \hat{\pi}^i}{\partial g_k^i} - 1\right) = 0,\ i = 1,2. \tag{38}$$

Differentiating (34) with respect to g_k^i gives rise to

$$\frac{\partial \hat{\pi}^i}{\partial g_k^i} = \frac{1}{1+\alpha}\left(-\alpha\frac{\partial \hat{g}_p^i}{\partial g_k^i} + 1\right) = 1,\ i = 1,2, \tag{39}$$

since it follows from (35) and (36) that $\partial \hat{g}_p^i / \partial g_k^i = -1$ for $i = 1,2$. Substituting (39) into (38) results in that $\partial U_k^i / \partial g_k^i = 0$ for any value of g_k^i, which implies that the optimal values of g_k^i are indeterminate. Moreover, since the values of $\hat{g}_p^i + \hat{g}_k^i$, $i = 1,2$, are uniquely determined from (35) and (36), g_p^i and thus π^i turn out to be indeterminate.

Furthermore, combining (34) and (35), (36) with (3) and (4) yields

$$\hat{c}_p^i = \frac{Y}{2(1+\alpha)+\gamma_p+\alpha\gamma_k} \text{ and } \hat{c}_k^i = \frac{\alpha Y}{2(1+\alpha)+\gamma_p+\alpha\gamma_k}, i = 1,2, \tag{40}$$

which, together with (37), imply neutrality.

When the only parents of both families stop contributing, i.e., $g_p^i = 0$, $i = 1, 2$, g_k^i is uniquely determined from (35) and (36), so does π^i. Yet, neutrality continues to hold, which is confirmed by observing the fact that

$$\begin{aligned} \hat{G} &\equiv \hat{g}_k^1 + \hat{g}_k^2 = \frac{\gamma_k Y}{2 + \gamma_k}, \\ \hat{c}_p^i &= \frac{Y}{(1+\alpha)(2+\gamma_k)} \text{ and } \hat{c}_k^i = \frac{\alpha Y}{(1+\alpha)(2+\gamma_k)}, \ i = 1, 2. \end{aligned}$$

Conversely, when the only children of both families stop contributing, i.e., $g_k^i = 0$, $i = 1, 2$, we obtain

$$\begin{aligned} \hat{G} &\equiv \hat{g}_p^1 + \hat{g}_p^2 = \frac{\gamma_k Y}{2 + \gamma_k}, \\ \hat{c}_p^i &= \frac{Y}{2(1+\alpha) + \gamma_p + \alpha\gamma_k} \text{ and } \hat{c}_k^i = \frac{\alpha Y}{2(1+\alpha) + \gamma_p + \alpha\gamma_k}, \ i = 1, 2, \end{aligned}$$

so that neutrality follows.

Finally, we may consider the case depicted in Figure 4. In this case we have:

$$\begin{aligned} \hat{G} \equiv \hat{g}_p^1 + \hat{g}_k^2 &= \frac{\gamma_k Y}{2 + \gamma_k}, \\ \hat{c}_p^1 = \frac{Y}{(1+\alpha)(1+\gamma_k)} &\text{ and } \hat{c}_k^1 = \frac{\alpha Y}{(1+\alpha)(1+\gamma_k)}, \\ \hat{c}_p^2 = \frac{\gamma_k Y}{(1+\alpha+\gamma_p+\alpha\gamma_k)(1+\gamma_k)} &\text{ and } \hat{c}_k^2 = \frac{\alpha\gamma_k Y}{(1+\alpha+\gamma_p+\alpha\gamma_k)(1+\gamma_k)}, \end{aligned}$$

which clearly implies neutrality.

6.2 Pre-committed Transfers

In this subsection we consider the pre-committed case. Assuming that every family member makes a positive contribution to the public good, the child's first-order conditions of the respective families (19) at stage 2 are given by

$$\frac{\partial U_k^i}{\partial g_k^i} = \frac{-1}{y_k^i + \pi^i - g_k^i} + \frac{\gamma_k}{G} = 0, \ i = 1, 2,$$

which are rearranged as follows:

$$\begin{aligned} (1+\gamma_k) g_k^1 + g_k^2 &= \gamma_k \left(y_k^1 + \pi^1\right) - g_p^1 - g_p^2, \\ g_k^1 + (1+\gamma_k) g_k^2 &= \gamma_k \left(y_k^2 + \pi^2\right) - g_p^1 - g_p^2. \end{aligned}$$

Solution of this pair of equations yields

$$\hat{g}_k^1 = \frac{(1+\gamma_k)(y_k^1+\pi^1)-\left(g_p^1+g_p^2+y_k^2+\pi^2\right)}{2+\gamma_k}, \tag{41}$$

$$\hat{g}_k^2 = \frac{(1+\gamma_k)(y_k^2+\pi^2)-\left(g_p^1+g_p^2+y_k^1+\pi^1\right)}{2+\gamma_k}. \tag{42}$$

Summing these contributions over all individuals yields

$$\hat{G} \equiv g_p^1+g_p^2+\hat{g}_k^1+\hat{g}_k^2 = \frac{\gamma_k\left(g_p^1+g_p^2+y_k^1+\pi^1+y_k^2+\pi^2\right)}{2+\gamma_k}. \tag{43}$$

Given (41), (42) and (43), the parent's first-order conditions of the respective families at stage 1, (20) and (21) are, for $i = 1, 2$,

$$\frac{\partial U_p^i}{\partial g_p^i} = -\frac{1}{y_p^i-\pi^i-g_p^i}-\frac{\alpha}{y_k^i+\pi^i-\hat{g}_k^i}\frac{\partial \hat{g}_k^i}{\partial g_p^i}+\frac{\gamma_p+\alpha\gamma_k}{\hat{G}} = 0, \tag{44}$$

$$\frac{\partial U_p^i}{\partial \pi^i} = -\frac{1}{y_p^i-\pi^i-g_p^i}+\frac{\alpha}{y_k^i+\pi^i-\hat{g}_k^i}\left(1-\frac{\partial \hat{g}_k^i}{\partial \pi^i}\right)+\frac{\gamma_p+\alpha\gamma_k}{\hat{G}} = 0. \tag{45}$$

Differentiating (41) and (42) with respect to g_p^i and π^i, respectively, yields

$$\frac{\partial \hat{g}_k^i}{\partial g_p^i} = -\frac{1}{2+\gamma_k} \text{ and } \frac{\partial \hat{g}_k^i}{\partial \pi^i} = \frac{1+\gamma_k}{2+\gamma_k},\ i = 1, 2, \tag{46}$$

which together implies that $\partial \hat{g}_k^i/\partial g_p^i = 1-(\partial \hat{g}_k^i/\partial \pi^i)$. Using this, we can confirm that (44) and (45) coincide, and thus the division between $\hat{g}_p^i$ and $\hat{\pi}^i$ is indeterminate.

Substituting (46) into (44) and (45), and rearranging yields

$$\begin{aligned}\left[1+\gamma_p+(1+\gamma_k)\alpha\right]S_1+S_2 &= B_1,\\ S_1+\left[1+\gamma_p+(1+\gamma_k)\alpha\right]S_2 &= B_2,\end{aligned} \tag{47}$$

where $S_i \equiv \pi^i+g_p^i$ and $B_i \equiv \left[\gamma_p+(1+\gamma_k)\alpha\right]y_p^i-y_k^i-y_k^j, i, j = 1, 2, i \neq j$.

Solving (47) for S_1 and S_1 yields

$$S_1 \equiv \hat{\pi}^1+g_p^1 = \frac{\left[1+\gamma_p+(1+\gamma_k)\alpha\right]B_1-B_2}{\left[2+\gamma_p+(1+\gamma_k)\alpha\right]\left[\gamma_p+(1+\gamma_k)\alpha\right]}, \tag{48}$$

$$S_2 \equiv \hat{\pi}^2+\hat{g}_p^2 = \frac{\left[1+\gamma_p+(1+\gamma_k)\alpha\right]B_2-B_1}{\left[2+\gamma_p+(1+\gamma_k)\alpha\right]\left[\gamma_p+(1+\gamma_k)\alpha\right]}. \tag{49}$$

Substituting S_1 and S_2 into $\pi^1 + g_p^1$ and $\pi^2 + g_p^2$ in (43), respectively, results in

$$\hat{G} = \frac{\gamma_k(\alpha + \gamma_p + \alpha\gamma_k)Y}{(2 + \gamma_k)\left[2 + \gamma_p + (1 + \gamma_k)\,\alpha\right]}, \tag{50}$$

while the private consumptions of the parent and child are given by

$$\hat{c}_p^i = \frac{Y}{2 + \gamma_p + (1 + \gamma_k)\,\alpha} \text{ and } \hat{c}_k^i = \frac{(\alpha + \gamma_p + \alpha\gamma_k)Y}{(2 + \gamma_k)\left[2 + \gamma_p + (1 + \gamma_k)\,\alpha\right]},\ i = 1, 2, \tag{51}$$

where $Y \equiv y_p^1 + y_k^1 + y_p^2 + y_k^2$. Neutrality follows from (50) and (51).

For brevity, we omit non-interior cases.

6.3 Mix of Ex-post and Pre-committed Transfers

Given $(g_k^1,\ \pi^2,\ g_p^2)$ and assuming (30) and (31), we can solve (22), (23) and (24) to get

$$\hat{g}_p^1 = \frac{(1 + \gamma_k)\,(\gamma_p + \alpha\gamma_k)(y_k^1 + y_p^1) - (1 + \alpha)\gamma_k(y_k^2 + g_p^2 + \pi^2)}{\gamma_k(1 + \alpha) + (1 + \gamma_k)\,(\gamma_p + \alpha\gamma_k)} - g_k^1, \tag{52}$$

$$\hat{\pi}^1 = \frac{\alpha\gamma_k(y_p^1 + y_k^2 + g_p^2 + \pi^2) - [\gamma_k + (1 + \gamma_k)(\gamma_p + \alpha\gamma_k)]y_k^1}{\gamma_k(1 + \alpha) + (1 + \gamma_k)\,(\gamma_p + \alpha\gamma_k)} + g_k^1, \tag{53}$$

$$\hat{g}_k^2 = \frac{\gamma_k(1 + \alpha + \gamma_p + \alpha\gamma_k)(y_k^2 + \pi^2) - (\gamma_p + \alpha\gamma_k)(y_p^1 + y_k^1 + g_p^2)}{\gamma_k(1 + \alpha) + (1 + \gamma_k)\,(\gamma_p + \alpha\gamma_k)}. \tag{54}$$

Manipulating (52) and (54), and adding g_p^2 yields

$$\hat{G} = \hat{g}_p^1 + g_k^1 + g_p^2 + \hat{g}_k^2 = \frac{\gamma_k(\gamma_p + \alpha\gamma_k)(y_p^1 + y_k^1 + y_k^2 + \pi^2 + g_p^2)}{\gamma_k(1 + \alpha) + (1 + \gamma_k)\,(\gamma_p + \alpha\gamma_k)}. \tag{55}$$

Given the optimal contribution functions $\hat{g}_p^1$, $\hat{\pi}^1$, $\hat{g}_k^2$ in (52)-(54) and the total provision of $\hat{G}$ in (55), the parent's decision problem of family 2 at stage 1 gives the following first-order conditions:

$$\frac{\partial U_p^2}{\partial g_p^2} = -\frac{1}{y_p^2 - \pi^2 - g_p^2} - \frac{\alpha}{y_k^2 + \pi^2 - \hat{g}_k^2}\frac{\partial \hat{g}_k^2}{\partial g_p^2} + \frac{\gamma_p + \alpha\gamma_k}{\hat{G}} = 0, \tag{56}$$

$$\frac{\partial U_p^2}{\partial \pi^2} = -\frac{1}{y_p^2 - \pi^2 - g_p^2} + \frac{\alpha}{y_k^2 + \pi^2 - \hat{g}_k^2}\left(1 - \frac{\partial \hat{g}_k^2}{\partial \pi^2}\right) + \frac{\gamma_p + \alpha\gamma_k}{\hat{G}} = 0 \tag{57}$$

Differentiating $\hat{g}_k^2$ given by (54) with respect to g_p^2 and π^2, respectively, yields

$$-\frac{\partial \hat{g}_k^2}{\partial g_p^2} = \frac{\gamma_p + \alpha\gamma_k}{\gamma_k(1+\alpha) + (1+\gamma_k)(\gamma_p + \alpha\gamma_k)} = 1 - \frac{\partial \hat{g}_k^2}{\partial \pi^2},$$

which implies that the parent's first-order conditions (56) and (57) become identical. As a result, the division between $\hat{\pi}^2$ and $\hat{g}_p^2$ is indeterminate, although $\hat{\pi}^2 + \hat{g}_p^2$ is given by:

$$\hat{\pi}^2 + \hat{g}_p^2 = \frac{(\alpha + \gamma_p + \alpha\gamma_k)y_p^2 - y_p^1 - y_k^1 - y_k^2}{1 + \alpha + \gamma_p + \alpha\gamma_k}. \tag{58}$$

Further substitution of (58) into (55) yields

$$\hat{G} = \frac{(\alpha + \gamma_p + \alpha\gamma_k)Y}{[\gamma_k(1+\alpha) + (1+\gamma_k)(\gamma_p + \alpha\gamma_k)](1 + \alpha + \gamma_p + \alpha\gamma_k)}, \tag{59}$$

while the private consumptions of the parent and child in the respective families are given by

$$\hat{c}_p^1 = \frac{\gamma_k(\alpha + \gamma_p + \alpha\gamma_k)Y}{(1+\alpha)(1+\gamma_k)} \text{ and } \hat{c}_k^1 = \frac{\alpha\gamma_k(\alpha + \gamma_p + \alpha\gamma_k)Y}{(1+\alpha)(1+\gamma_k)},$$

$$\hat{c}_p^2 = \frac{Y}{1 + \alpha + \gamma_p + \alpha\gamma_k} \text{ and } \hat{c}_k^2 = \frac{(\gamma_p + \alpha\gamma_k)(\alpha + \gamma_p + \alpha\gamma_k)Y}{[\gamma_k(1+\alpha) + (1+\gamma_k)(\gamma_p + \alpha\gamma_k)](1 + \alpha + \gamma_p + \alpha\gamma_k)}.$$

These results establish neutrality.

7 Impure Altruism

In this section we consider the following parent's utility functions:

$$U_p^i(c_p^i, G, g_p^i; c_k^i) \equiv u_p^i(c_p^i, G, g_p^i) + \alpha^i u_k^i(c_k^i, G),\ i = 1, 2, \tag{60}$$

or

$$U_p^i(c_p^i, G, \pi^i; c_k^i) \equiv u_p^i(c_p^i, G, \pi^i) + \alpha^i u_k^i(c_k^i, G),\ i = 1, 2, \tag{61}$$

while the rest of the model is the same as before.[10]

Abel and Bernheim (1991) show that, under the utility function (61) without the preference toward the total provision of the public good, Bernheim

[10] Alternatively, we may assume that the utility function of the child is of the form $u_k^i(c_k^i, G, g_k^i)$. In a similar way, since we can obtain the same implication as in the utility function (60) in the text, we omit this investigation.

and Bagwell's cross sectional neutrality fails to hold, while Andreoni (1990) shows that under the utility function (60), without the preference toward the welfare of the child, Warr's neutrality theorem fails. We first examine the neutrality issue under the utility function (61). Assuming the ex-post transfers made by the parents of both families, the first-order conditions for an interior solution are given by (6), which is modified by adding the variable π^i, and

$$\frac{\partial U_p^i}{\partial \pi^i} = -\frac{\partial u_p^i(y_p^i - \pi^i - g_p^i, \pi^i, G)}{\partial c_p^i} - \frac{\partial u_p^i(y_p^i - \pi^i - g_p^i, \pi^i, G)}{\partial \pi^i} + \alpha^i \frac{\partial u_k^i(y_k^i + \pi^i - g_k^i, \pi^i, G)}{\partial c_k^i} = 0, \; i = 1, 2. \tag{62}$$

Under the hypothetical changes prescribed by (10) and (11), the above first-order conditions (6) and (62) are not satisfied at the original values of c_p^i, c_k^i and G, because π^i is free to change. Instead, assuming π^i to be fixed at the original value, the hypothetical changes are amended as follows; $dy_p^i = dg_p^i$ and $dy_k^i = dg_k^i$, $i = 1,2$. It is easy to see that under these hypothetical changes the allocation of resource allocation remains unchanged and thus neutrality holds, yet the indeterminacy between g_p^i and π^i clearly disappears. Similarly, as long as an interior equilibrium is assumed, the neutrality and uniqueness of an equilibrium allocation are robust regardless of the different timings of parental transfers.

Under the utility function (60), on the other hand, we consider the hypothetical changes given by $dy_p^i = d\pi^i$ and $dy_k^i + d\pi^i = dg_k^i$, $i = 1,2$, keeping g_p^i unchanged. In a similar way, we can show that the neutrality and uniqueness of the equilibrium allocation hold.

In either case, since one of the parent's choice variables such as g_p^i or π^i does not appear in the utility function and thus the first-order condition, that variable is able to freely change so as to neutralize the effect of income redistribution policy. Moreover, the presence of imperfect altruism will eliminate the non-uniqueness of a Nash equilibrium allocation even in an interior equilibrium, because one of the variables g_p^i and π^i should be kept at the original value in order to realize neutrality; consequently, it should be uniquely determined so as to satisfy the relevant first-order condition. Nevertheless, when both types of impure altruism are simultaneously present; that is, the parent's (or child's) utility function depends directly both on her own transfers as well as on her own private contribution, neutrality no longer holds true.

8 Concluding Remarks

In this paper we have shown that when parents voluntarily and simultaneously make both private donations to public goods and transfers to children in an interior equilibrium, the contributions made by parents are indeterminate. In other words, in a two-stage Nash provision game an infinite number of combinations of individual contributions and transfers are sustained as a (subgame perfect) Nash equilibrium with a unique profile of every individual's consumption as well as the total supply of public good, regardless of whether the parents decide to make parent-to-child transfers before or after having observed the contributions made by the children. However, the introduction of some friction such as impure altruism with respect to either her own private donations or her own intrafamily transfers eliminates such indeterminacy, although neutrality may still remain valid.

There is one important policy implication. Bergstrom et al. (1986) point out that if the redistribution of income from non-contributors to contributors takes place, not only neutrality fails, but also the total provision of public good will increase. In contrast to their claim, in our multi-family setting there is the possibility that neutrality continues to hold under such a redistribution, as long as intrafamily transfers are operative. On the other hand, even if most of interfamily links through transfers are not operative, Bernheim and Bagwell's cross sectional neutrality may survive because the private donations to interfamily public goods play a perfect substitutable role for interfamily private transfers.

The present framework can be applied to other different contexts. Examples in an international context are transboundary pollution and international public goods, such as reducing the risk of infectious deceases and preserving valuable cultural heritage (such as the World Heritage) and rare (perhaps potentially threatened) species. Suppose that developed (richer) countries make (conditional) foreign aid to developing (poorer) countries to curve pollution emissions of developing countries, which may in part be motivated by altruistic concern towards the welfare of those poor countries, and that all countries share an environmental good. The results of our paper indicate that even if the donor countries increases *conditional* foreign aid which aims at reductions in their pollution emissions, the quality of environment and thus the welfare of donor and receipt countries remains unchanged.

There are extensions in several directions. First, a natural extension is to introduce an intrafamily public good to the present model in addition to an interfamily public good. In this case the parents have three choices such as voluntary contributions to non-family and to family public goods, and voluntary transfers to children. As found by Cornes and Itaya (2003), in a

model with many public goods, the existence of an interior Nash equilibrium in which all individuals make positive contributions to all public goods is not a generic property. In addition, we have to investigate many variations in terms of the profile of individual contributions as well as total public supply. Second, it is also interesting to investigate the case of several children, possibly coupled with the endogenous fertility decision of the parent. Although the analysis will be very complicated, we expect that our basic results would be still valid. Third, our non-uniqueness result relies crucially on identical marginal costs between contributing to public goods and transfers made by the parents. Relaxation of this assumption to one that allows either for different marginal costs of contributing to a public good and making parental transfers, or for more general technologies of supplying a public good will undermine both neutrality and indeterminacy.

Appendix A

In order to prove that (9) holds true, we totally differentiate (6) and (7) to yield

$$\begin{bmatrix} u_{cc}^{1p}-u_{Gc}^{1p}+\alpha^1 u_{Gc}^{1k} & 0 \\ 0 & u_{cc}^{2p}-u_{Gc}^{2p}+\alpha^2 u_{Gc}^{2k} \\ u_{cc}^{1p}+\alpha^1 u_{cc}^{1k} & 0 \\ 0 & u_{cc}^{2p}+\alpha^2 u_{cc}^{2k} \end{bmatrix}$$

$$\begin{bmatrix} u_{cc}^{1p}-2u_{Gc}^{1p}+u_{GG}^{1p}+\alpha^1 u_{GG}^{1k} & -u_{cG}^{1p}+u_{GG}^{1p}+\alpha^1 u_{GG}^{1k} \\ -u_{cG}^{2p}+u_{GG}^{2p}+\alpha^2 u_{GG}^{2k} & u_{cc}^{2p}-2u_{Gc}^{2p}+u_{GG}^{2p}+\alpha^2 u_{GG}^{2k} \\ u_{cc}^{1p}-u_{cG}^{1p}+\alpha^1 u_{cG}^{1k} & -u_{cG}^{1p}+\alpha^1 u_{cG}^{1k} \\ -u_{cG}^{2p}+\alpha^2 u_{cG}^{2k} & u_{cc}^{2p}+u_{cG}^{2p}-\alpha^2 u_{cG}^{2k} \end{bmatrix} \begin{bmatrix} d\pi^1 \\ d\pi^2 \\ dg_p^1 \\ dg_p^2 \end{bmatrix} =$$

$$\begin{bmatrix} \left(\alpha^1 u_{Gc}^{1k}+u_{cG}^{1p}-u_{GG}^{1p}-\alpha^1 u_{GG}^{1k}\right) dg_k^1+\left(u_{cG}^{1p}-u_{GG}^{1p}-\alpha^1 u_{GG}^{1k}\right) dg_k^2 \\ \left(\alpha u_{Gc}^{2k}+u_{cG}^{2p}-u_{GG}^{2p}-\alpha^2 u_{GG}^{2k}\right) dg_k^2+\left(u_{cG}^{2p}-u_{GG}^{2p}-\alpha^2 u_{GG}^{2k}\right) dg_k^1 \\ \left(\alpha^1 u_{cc}^{1k}+u_{cG}^{1p}-\alpha^1 u_{cG}^{1k}\right) dg_k^1+\left(u_{cG}^{1p}-\alpha^1 u_{cG}^{1k}\right) dg_k^2 \\ \left(u_{cG}^{2p}-\alpha^2 u_{cG}^{2k}\right) dg_k^1+\left(\alpha^2 u_{cc}^{2k}+u_{cG}^{2p}-\alpha^2 u_{cG}^{2k}\right) dg_k^2 \end{bmatrix} \quad \text{(A1)}$$

Applying Cramer's rule, we can get the following comparative statics results:

$$\frac{d\hat{\pi}^i}{dg_k^i} = 1,\ i = 1,2, \tag{A2}$$

$$\frac{d\hat{g}_p^i}{dg_k^i} = -1,\ i = 1,2, \tag{A3}$$

$$\frac{d\hat{g}_p^j}{dg_k^i} = 0,\ i,j = 1,2,\ i \neq j. \tag{A4}$$

Appendix B

In order to show that the first-order conditions (20) and (21) reduce to a single equation, we totally differentiate (19) to yield

$$\begin{bmatrix} -u_{cc}^{1k} + 2u_{Gc}^{1k} - u_{GG}^{1k} & u_{cG}^{1k} - u_{GG}^{1k} \\ u_{cG}^{2k} - u_{GG}^{2k} & -u_{cc}^{2k} + 2u_{Gc}^{2k} - u_{GG}^{2k} \end{bmatrix} \begin{bmatrix} dg_k^1 \\ dg_k^2 \end{bmatrix} = \begin{bmatrix} \left(u_{Gc}^{1k} - u_{cc}^{1k}\right) d\pi^1 + \left(u_{GG}^{1k} - u_{cG}^{1k}\right) dg_p^1 + \left(u_{GG}^{1k} - u_{cG}^{1k}\right) dg_p^2 \\ \left(u_{Gc}^{2k} - u_{cc}^{2k}\right) d\pi^2 + \left(u_{GG}^{2k} - u_{cG}^{2k}\right) dg_p^2 + \left(u_{GG}^{2k} - u_{cG}^{2k}\right) dg_p^1 \end{bmatrix}. \tag{B1}$$

Application of Cramer's rule yields

$$\frac{d\hat{g}_k^i}{dg_p^i} = |B|^{-1} \left(u_{GG}^{ik} - u_{cG}^{ik}\right)(-u_{cc}^{jk} + u_{Gc}^{jk}), \tag{B2}$$

$$\frac{d\hat{g}_k^j}{dg_p^i} = |B|^{-1} (u_{cG}^{jk} - u_{GG}^{jk})(-u_{cc}^{ik} + u_{Gc}^{ik}), \tag{B3}$$

$$\frac{d\hat{g}_k^i}{d\pi^i} = |B|^{-1} \left(u_{Gc}^{ik} - u_{cc}^{ik}\right)(-u_{cc}^{jk} + u_{Gc}^{jk} + u_{cG}^{jk} - u_{GG}^{jk}), \tag{B4}$$

$$\frac{d\hat{g}_k^j}{d\pi^i} = |B|^{-1} \left(u_{Gc}^{ik} - u_{cc}^{ik}\right)(u_{cG}^{jk} - u_{GG}^{jk}), \tag{B5}$$

where $|B|$ represents the determinant of the matrix appearing on the left-hand side of (B1).

Furthermore, by straightforward calculation, we have

$$1 - \frac{\partial \hat{g}_k^i}{\partial \pi^i} = |B|^{-1} \left(u_{cG}^{ik} - u_{GG}^{ik}\right) \left(-u_{cc}^{jk} + u_{Gc}^{jk}\right), \; i = 1, 2, \tag{B6}$$

$$\frac{\partial \hat{g}_k^i}{\partial g_p^i} + \frac{\partial \hat{g}_k^j}{\partial g_p^i} + 1 = |B|^{-1} \left(u_{Gc}^{ik} - u_{cc}^{ik}\right) \left(3u_{cG}^{jk} - 2u_{GG}^{jk} - u_{cc}^{jk}\right), \; i, j = 1, 2, \; i \neq j, \tag{B7}$$

$$\frac{\partial \hat{g}_k^i}{\partial \pi^i} + \frac{\partial \hat{g}_k^j}{\partial \pi^i} = |B|^{-1} \left(u_{Gc}^{ik} - u_{cc}^{ik}\right) \left(-u_{cc}^{jk} + 3u_{Gc}^{jk} - 2u_{GG}^{jk}\right), \; i, j = 1, 2, \; i \neq j. \tag{B8}$$

Comparison of (B6) with (B2) and (B7) with (B8) reveals that

$$\begin{aligned} 1 - \frac{\partial \hat{g}_k^i}{\partial \pi^i} &= -\frac{d\hat{g}_k^i}{dg_p^i}, \; i = 1, 2, \\ \frac{\partial \hat{g}_k^i}{\partial g_p^i} + \frac{\partial \hat{g}_k^j}{\partial g_p^i} + 1 &= \frac{\partial \hat{g}_k^i}{\partial \pi^i} + \frac{\partial \hat{g}_k^j}{\partial \pi^i}, \; i, j = 1, 2, \; i \neq j, \end{aligned}$$

under which the first-order conditions (20) and (21) become identical.

Appendix C

In order to prove that (28) and (29) holds true, we totally differentiate (22), (23) and (24) to yield

$$\begin{bmatrix} u_{cc}^{1p} - u_{Gc}^{1p} + \alpha^1 u_{Gc}^{1k} \\ u_{cc}^{1p} + \alpha^1 u_{cc}^{1k} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} u_{cc}^{1p} - 2u_{Gc}^{1p} + u_{GG}^{1p} + \alpha^1 u_{GG}^{1k} & -u_{cG}^{1p} + u_{GG}^{1p} + \alpha^1 u_{GG}^{1k} \\ u_{cc}^{1p} - u_{cG}^{1p} + \alpha^1 u_{cG}^{1k} & -u_{cG}^{1p} + \alpha^1 u_{cG}^{1k} \\ u_{cG}^{2k} - u_{GG}^{2k} & -u_{cc}^{2k} + u_{cG}^{2k} + \alpha^2 u_{Gc}^{2k} - u_{GG}^{2k} \end{bmatrix} \begin{bmatrix} d\pi^1 \\ dg_p^1 \\ dg_k^2 \end{bmatrix} =$$

$$\begin{bmatrix} \left(u_{cG}^{1p} - u_{GG}^{1p} + \alpha^1 u_{Gc}^{1k} - \alpha^1 u_{GG}^{1k}\right) dg_k^1 + \left(u_{cG}^{1p} - u_{GG}^{1p} - \alpha^1 u_{GG}^{1k}\right) dg_p^2 \\ \left(u_{cG}^{1p} + \alpha^1 u_{cc}^{1k} - \alpha^1 u_{cG}^{1k}\right) dg_k^1 + \left(u_{cG}^{1p} - \alpha^1 u_{cG}^{1k}\right) dg_p^2 \\ \left(u_{cG}^{2k} + u_{GG}^{2k}\right) dg_k^1 + \left(-u_{cc}^{2k} + u_{Gc}^{2k}\right) d\pi^2 + \left(-u_{cG}^{2k} + u_{GG}^{2k}\right) dg_p^2 \end{bmatrix}. \tag{C1}$$

Application of Cramer's rule yields

$$\frac{\partial \hat{\pi}^1}{\partial g_k^1} = 1, \tag{C2}$$

$$\frac{\partial \hat{g}_p^1}{\partial g_k^1} = -1 \text{ and } \frac{\partial \hat{g}_k^2}{\partial g_k^1} = 0, \tag{C3}$$

$$\frac{\partial \hat{g}_k^2}{\partial g_p^2} = -1 + \frac{\partial \hat{g}_k^2}{\partial \pi^2}, \text{ and} \tag{C4}$$

$$\frac{\partial \hat{g}_p^1}{\partial g_p^2} = \frac{\partial \hat{g}_p^1}{\partial \pi^2}. \tag{C5}$$

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