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Symmetric Equilibrium of Monopolistic Competition and Comparative Statics

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The traditional theory of monopolistic competition concentrates on the market form of a specific industry and states its implications from the standpoint of partial equilibrium analysis (see F.M. Scherer (1980)). Recent progress in this field, however, has enabled us to consider the theory of monopolistic competition by the general equilibrium framework. There are at least three different approaches to this problem. First, the neo-Hotelling approach yields the circular solution, combined with the two sector model and avoids the end-firm problem originated by H. Hotelling (see H. Hotelling (1929) and E. Helpman (1981)). Second, the characteristics approach, with the aid of specification in individual preference, can be used to analyse the theory of monopolistic competition, and to distinguish between group goods and outside goods (see K. Lancaster (1979)). Third, the traditional microeconomic analysis, based on individual optimizing behavior, could be applied to the problem of choice between differentiated goods and homogeneous goods (see A.K. Dixit and J.E. Stiglitz (1977)).

However, most of the above contributions have been directed only to the equilibrium state and have never seriously discussed the stability problem, except for Lancaster's work. It should be noted that scale economies are a dominant factor in the monopolistic industry (see Scherer (1980, *ibid.*) and that in this case stability of the system is not necessarily trivial.*

Since our main concern is with the symmetric equilibrium, let us first state the reason why consideration of stability is essential at the beginning. Symmetric equilibrium typically characterises firms in the differentiated goods sector as facing the same demand curve and charging the same price with identical technology. Around this equilibrium we expect that entrants as well as incumbent firms could maintain the above state of competition. According to Lancaster's terminology, under perfect monopolistic competition, no explicit distinction is necessary between incumbents and entrants. If any firm makes a profit (a loss), there would occur

* When there exists industries with increasing returns to scale, the production possibility frontier does not necessarily maintain the nice property of being concave toward the origin, which would generate the possibility of instability (see M. Kemp (1959)).

entrance into (exit from) the market. Since the price of differentiated goods depends on the demand/supply relation and the latter is to a large extent influenced by the number of firms in the differentiated goods sector, it is of great importance to answer how entry or exit will be initiated. Lancaster answered this question as follows:

"If the profit per firm at this equilibrium is positive, there will always be new potential entrants. They will, however, enter one (or very few) at a time, ... If profits per firm are negative, firms will leave the group (one or a few at a time) until profit per firm is zero ... (p.187)"

This description on dynamic adjustment of the system seems far from complete. When any firm in the differentiated goods sector suffers a loss, exit will occur. However, since all the incumbents there are identical, who will leave the market is stochastic in nature, except in an unrealistic case where all firms simultaneously leave the differentiated goods sector, which suddenly raises the price and induces entrants into the equilibrium with repeating oscillatory convergence. Therefore, we shall provide a plausible explanation on dynamic adjustment in the differentiated goods sector by using the idea of stochastic processes. In fact, our dynamic adjustment would correspond to the Galton-Watson branching process in biology (see T.E. Harris (1963) and D.R. Cox and H.D. Miller (1965)).

In this paper, we basically employ the closed economy version of the Lawrence-Spiller model (1983). It has an advantage in that production technology is non-homothetic. In particular, there are two factors of production, one variable and the other fixed. It is known that in the case of a monopolistic situation, the fixed factor plays a critical role in explaining strategic aspect of entry barriers by incumbent(s) (see A.M. Spence (1976)). While we do not explicitly treat the strategic aspect, it seems of great significance to distinguish different roles played by disparate factors of production. Especially, this is crucial in considering the problem of innovation. Usually, innovative activity is defined as a firm's unit cost reducing effort (see P. Dasgupta and J.E. Stiglitz (1980) and M.T. Flaherty (1980)). However, possible effects derived from it might be quite different, according to whether unit variable cost is reduced (this may correspond to technical progress) or unit fixed cost is reduced.

This paper consists of the following five sections. Section 1 will describe basic features of our model, which are based on Lawrence-Spiller (1983, *ibid.*) and Dixit-Stiglitz (1977, *ibid.*). Section 2 will provide a dynamic view of the model. Explicit treatment will be made between short-run adjustment and long-run adjustment. Section 3 will examine stability conditions. It will be shown that short-run equilibrium is always stable. However, in the long-run the possibility of instability cannot be eliminated. Section 4 will give comparative static analysis. Our main

interest is on effects of productivity improvement and of a change in the fixed cost. Section 5 will briefly touch upon main results in Section 4 and relate them to topics on innovation.

1. the Model

Consider an economy with two sectors: one producing homogeneous goods with perfect competition, and the other producing differentiated goods. Labor and capital are requisite for production of both commodities. In order to dramatise the role of increasing returns to scale in the monopolistically competitive goods sector, we employ the Lawrence-Spiller model, where each local monopolist must hold a fixed amount of physical capital stock to stay in the industry. Its variable cost comes solely from labor employment. That is, we consider the following non-homothetic technology.

$$(1) \quad x_i = \begin{cases} \frac{1}{\beta} L_i & \text{for } k_i \geq \gamma \\ 0 & \text{for } k_i < \gamma \end{cases} ,$$

where X_i , L_i , K_i and γ respectively stand for output, labor, capital, and fixed amount of physical capital. $1/\beta$ obviously measures the marginal product of labor. Under the above technological assumption and perfectly competitive factor markets, equation (2) will show total cost incurred to monopolistic firm i .

$$(2) \quad TC_i = \gamma r + w\beta x_i ,$$

where r and w respectively denote rental on capital and wage rate.

In the case of the homogeneous goods sector, production technology is given by equation (3).

$$(3) \quad Y = K_y^\epsilon L_y^{1-\epsilon} ,$$

where K_y and L_y respectively denote capital and labor employed there. Since the homogeneous goods market is perfectly competitive, marginal cost pricing, described by equations (4) and (5), prevails.

$$(4) \quad WL_y = (1 - \epsilon) Y ,$$

$$(5) \quad rk_y = \epsilon Y ,$$

Next, let us consider the consumer side. We assume the average consumer having the following homothetic preference:

$$(6) \quad u = U(Y, \left\{ \sum_{i=1}^n X_i^\rho \right\}^{1/\rho}) . \quad 0 < \rho < 1$$

The budget constraint is given by equation (7).

$$(7) \quad Y + \sum_{i=1}^n p_i X_i = I \quad .$$

where p_i are prices of the differentiated goods being produced, and I is income in terms of the numeraire.

Following Dixit-Stiglitz, we use a two-stage budgeting procedure. Define dual quantity and price indexes.

$$(8) \quad \bar{X} = \left\{ \sum_{i=1}^n X_i^\rho \right\}^{1/\rho} \quad , \quad q = \left\{ \sum_{i=1}^n p_i^{-\frac{1}{1-\rho}} \right\}^{-\frac{1-\rho}{\rho}} \quad .$$

Then in the first stage

$$(9) \quad \bar{X} = Is(q)/q \quad , \quad Y = I(1 - s(q))$$

for function s which depends on the form of u . The elasticity of the function s is related to the elasticity of substitution between Y and $\bar{X}$.

$$(10) \quad \theta(q) = \{1 - \sigma(q)\} \{1 - s(q)\} < 1 \quad ,$$

where $\theta(q) = qs'(q)/s(q)$ and $\sigma = d \ln(Y/\bar{X})/d \ln q$. Next, from the second stage of the problem,

$$(11) \quad x_i = \bar{X} \left(\frac{q}{p_i} \right)^{\frac{1}{1-\rho}} \quad \text{for each } i \quad .$$

when n is reasonably large,

$$(12) \quad \frac{\partial \ln X_i}{\partial \ln p_i} = - \frac{1}{1-\rho} \quad ,$$

which shows the elasticity of the Chamberlinian dd curve. In the so-called symmetric equilibrium, $p_i = p$ and $X_i = X$ for all i . Therefore, the symmetric equilibrium equations (8), (9), and (11) yield the following:

$$(13) \quad p = \frac{s(q)}{1 - s(q)} \frac{Y}{n\bar{X}} \quad .$$

The elasticity of the DD curve in Chamberlinian terminology can be measured by equation (14).

$$(14) \quad \frac{\partial \ln X}{\partial \ln p} = - (1 - (q)) \quad .$$

Dixit and Stiglitz assume the dd curve be more elastic than the DD curve, that is,

$$(CI) \quad \frac{\rho}{1-\rho} + \theta(q) > 0 \quad .$$

It is easily seen that condition (CI) is automatically satisfied if the utility function is of Cobb-Douglas type (which corresponds to the Lawrence-Spiller case). Dixit and Stiglitz derived the implication of (CI) from uniqueness of the equilibrium and suggested dynamic motion of entry (p.300). However, they did not provide a dynamic mechanism by which entry or exit would be generated in the system. Therefore, condition (CI) does not exactly correspond to our stability conditions. But as far as

elasticity of substitution is equal to or less than unity, both (C1) and our stability conditions will be satisfied, which is discussed in Section 3.

2. Equilibrium Conditions

Let us focus our attention on the differentiated goods market, because it is assumed that equilibrium in the homogeneous goods market is realized through changes in commodity prices and factor prices, resulting in equality of its demand to supply and full employment of production factors. In the Dixit-Stiglitz-Lawrence-Spiller model (DSLS, for short) they examined only the long-run steady state where marginal cost always equals marginal revenue and the zero profit condition prevails. However, they did not distinguish the difference between short-run adjustment and long-run adjustment. In the short-run, each firm behaves monopolistically. Under perfect monopolistic competition, each firm shares identical technology (or cost function), faces the same demand curve, and has equal opportunity for gaining profit. Furthermore, each firm behaves independently. Therefore, we can choose any incumbent firm from the differentiated goods sector and investigate the dynamic motion around the equilibrium. Under these circumstances, each firm will make an attempt for marginal revenue to exceed or at least equal marginal cost. In our case, since $MR = \rho p$ and $MC = \beta w$, the monopolist wants the following condition to hold.

$$\rho p \geq \beta w \quad .$$

When $\rho p > \beta w$, then the monopolist will increase employment of the variable factor, labor. Since marginal productivity of labor evaluated by the monopolist is equal to $p/\rho\beta$ and the monopolist can hire any amount of labor at current wage level w , adjustment in labor employment could be expressed by equation (15).

$$(15) \quad \dot{L}_x = \lambda_1 \left(\frac{p}{\rho\beta} - w \right) \quad .$$

The short-run (Nash) equilibrium is defined as the state of $\dot{L}_x = 0$. That is, inflow and/or outflow of labor into the differentiated goods sector force marginal revenue to equal marginal cost. *Figure 1* shows the short-run equilibrium situation. The monopolist enjoys a positive profit shown by the shaded area.

In the previous papers, it is always assumed that marginal revenue will equal marginal cost. There has been no explicit dynamic process developed before. This implies that we cannot answer how the quality of marginal revenue to marginal cost be sustained and whether the situation here be stable. We have supplied a tentative answer to these questions. In order to study long-run equilibrium, consider the implication of perfect monopolistic competition. Each firm, which can enter the differentiated goods market, has to hold the same amount of set-up cost (γ) and to share the same technology. After short-run equilibrium is realized, every endogenous variable would become a function of the number of firms in the differentiated goods

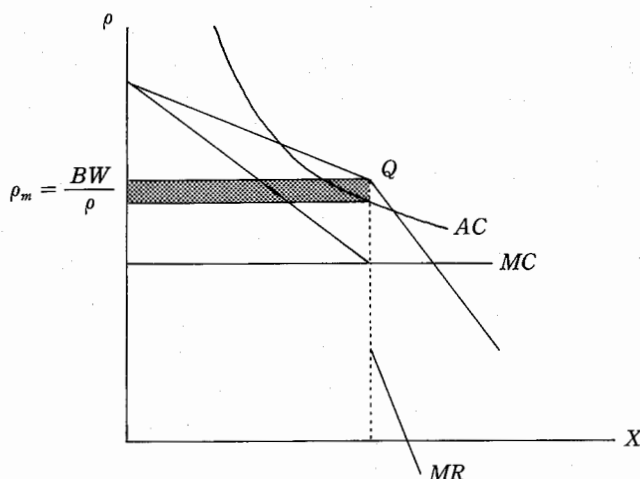


Figure 1

sector (n). Therefore, in the long-run our concern is on the equilibrium number n . However, because of perfect monopolistic competition, we can concentrate on the behavior of only one incumbent firm. If it has an incentive to reproduce itself, this will naturally imply inducement to entry. However, since every firm has equal opportunity with the same technology, who will enter or leave the market would be considered stochastic in nature. This stochastic process is known as the Galton-Watson branching process, or especially as the birth-death process in biology (see T.E. Harris (1963) and S. Karlin and H.M. Taylor (1975)) Consider one firm (particle in biological terminology) at time zero. The number of firms at time t is stochastic. Let $X(t)$ be the number of firms at time t , given $X(0) = 1$. Then whether one firm at time zero turns out to be zero or more than one would depend on birth rate and death rate in biological terminology. We can translate these respectively as entry rate (ξ) and exit rate (μ). Then it will be shown in Appendix B that the rate of change in the expected value of $X(t)$ is a function of the difference between entry rate and death rate. That is,

$$\dot{m}(t) = (\xi - \mu) m(t) ,$$

where $m(t) = EX(T)$. Under Nash equilibrium, each firm behaves independently. Therefore, the above relationship exactly holds in terms of the expected value of the total number of firms in the differentiated goods sector. That is,

$$\dot{M}(t) = (\xi - \mu) M(t) ,$$

where $M(t) = EX(t; n_0)$ and n_0 is the initial given number of firms.

It is natural to assume that entry and/or exit would be generated by a positive profit level of the average firm, which is a function of n . Therefore, we can rewrite the entry-generating equation as follows.

$$\frac{\dot{M}(t)}{M(t)} = \lambda \Pi(n) ,$$

where λ is the coefficient of speed of adjustment. It is clear in Appendix B that this stochastic process is a Poisson process and that the corresponding deterministic version of an entry-generating function will be expressed as follows.

$$\frac{\dot{n}(t)}{n(t)} = \lambda_2 \Pi(n(t)) .$$

Since we are only interested in the neighborhood of the equilibrium point and $\Pi(n^*) = 0$, we can specify the entry-generating equation as equation (16).

$$(16) \quad \dot{n} = \lambda_2 \Pi(n) .$$

The implication of equation (16) exactly corresponds to Lancaster's idea on entry-exit equilibrium (p.187)*. For convenience of later analysis, we employ the following equivalent expression.

$$(16)' \quad \dot{n} = \lambda_2 (p - AC) \\ = \lambda_2 \left(p - \left(w\beta + \frac{r\gamma}{X} \right) \right) .$$

Long-run equilibrium will be realized once $\dot{n} = 0$, in addition to $\dot{L}_x = 0$.

3. Stability Analysis

Now let us restate the key equations of our system.

$$\left. \begin{array}{l} (17) \quad wL_y = (1 - \varepsilon)Y \\ (18) \quad rK_y = \varepsilon Y \end{array} \right\} \text{profit maximization in the homogeneous goods sector.}$$

$$\left. \begin{array}{l} (19) \quad L = L_y + nL_x \\ (20) \quad K = K_y + n \end{array} \right\} \text{full employment in the factor market.}$$

$$\left. \begin{array}{l} (21) \quad Y = K_y^\varepsilon L_y^{1-\varepsilon} \\ (22) \quad X = \frac{1}{\beta} L_x \end{array} \right\} \text{production function.}$$

$$(23) \quad p = \frac{s(q)}{1 - s(q)} \frac{Y}{nX} \quad \text{symmetric equilibrium.}$$

* As cited in the introduction, Lancaster states that entry occurs by one or very few entrants at a time. In our case, this means the value of $(\xi - \mu)$ is finite.

$$(q = n^{-\frac{\rho}{1-\rho}} p)$$

There are seven equations and nine variables ($p, w, r, X, Y, L_x, L_y, K_y$, and n). In the short-run equilibrium, for given n , $\dot{L}_x = 0$. In the long-run, n itself will be endogenously determined by setting $\dot{n} = 0$.

For later analytical convenience, let us express seven variables (p, w, r, X, Y, L_y and K_y) in terms of L_x and n .

$$(17)' \quad w = (1 - \varepsilon) k_y^\varepsilon L_y^{1-\varepsilon} .$$

$$= (1 - \varepsilon) [\bar{K} - n\gamma]^{1-\varepsilon} [L - nL_x]^{1-\varepsilon} .$$

$$(18)' \quad r = \varepsilon [\bar{K} - n\gamma]^{-1} [L - nL_x]^{1-\varepsilon} .$$

$$(19)' \quad L_y = \bar{L} - nL_x .$$

$$(20)' \quad k_y = \bar{k} - n\gamma .$$

$$(21)' \quad Y = [\bar{k} - n\gamma]^\varepsilon [\bar{L} - nL_x]^{1-\varepsilon} .$$

$$(22)' \quad X = L_x/\beta .$$

$$(23)' \quad p = \frac{s(q)}{1-s(q)} \frac{\beta}{nL_x} [\bar{k} - n\gamma]^\varepsilon [\bar{L} - nL_x]^{1-\varepsilon} .$$

$$(q = n^{-\frac{1-\rho}{\rho}} p) .$$

1. Short-Run Stability

The short-run equilibrium is stable if and only if

$$\frac{\partial L_x}{\partial L_x} = \lambda_1 \frac{\partial f}{\partial L_x} < 0 .$$

where $f = \frac{\rho}{\beta} p - w$.

Note that $\frac{\partial f}{\partial L_x} = \frac{\rho}{\beta} \frac{\partial p}{\partial L_x} - \frac{\partial w}{\partial L_x}$.

Using equations (10) and (23)', we have:

$$\frac{1}{p} \frac{\partial p}{\partial L_x} = - \frac{L - \varepsilon n L_x}{\sigma L_x L_y} < 0 .$$

That is, labor inflow from the differentiated goods sector to the homogeneous goods sector raises the former's output and reduces the latter's. This change in a relative ratio of output between the sectors basically decreases the price of differentiated

goods.

On the other hand, using equation (17)', we have:

$$\frac{1}{w} \frac{\partial w}{\partial L_x} = \xi \frac{n}{L_y} .$$

That is, the above labor movement will induce a wage increase. Using these results, we obtain the following:

$$\frac{\partial f}{\partial L_x} = -w \frac{L + (\sigma - 1) \varepsilon n L_x}{\sigma L_x L_y} .$$

Since $\sigma > 0$ and $0 < \varepsilon < 1$,

$$L + (\sigma - 1) \varepsilon n L_x > L - \varepsilon n L_x > 0 .$$

Therefore,

lemma 1

The system is stable in the short-run. As long as each firm pursues the behavior of profit maximization with hiring a variable factor, the economy can stay at point Q as in *Figure 1*.

2. Long-Run Stability

Long-run equilibrium is stable if and only if

$$\frac{d\dot{n}}{dn} < 0 .$$

$$\text{Let } g = p - \left(w\beta + \frac{\gamma}{x} \right) = g(L_x(n, \alpha), n, \alpha),$$

where α is an arbitrary parameter. In the above, we use the fact that L_x is a function of n in the short-run. Then the long-run stability condition can be written by

$$Z \equiv \frac{dg}{dn} = g_{L_x} \frac{\partial L_x}{\partial n} + g_n < 0 ,$$

$$\text{where } \frac{\partial L_x}{\partial n} = -\frac{f_n}{f_{L_x}} \text{ and } f_\beta = \frac{\partial f}{\partial \beta} (\beta = L_x, n).$$

We may compute the value of Z as follows:

$$Z = -A\tilde{Z} ,$$

$$\text{where } A \equiv \frac{1-p}{p} \frac{\beta w}{n k_y 1_y [1 + (\sigma - 1) \varepsilon 1_x]} \text{ and}$$

$$\tilde{Z} = k[1 + (\sigma - 1) \varepsilon 1_x] - (\sigma - 1) \left[\varepsilon k_x 1_x + \frac{1-p}{p} k_y 1_y \right] .$$

$\tilde{Z}$ can alternatively be expressed as follows:

$$\tilde{Z} = (k - k_x)(\sigma - 1) \varepsilon 1_x + k_x 1_x + k_y 1_y \left[1 - (\sigma - 1) \frac{1-p}{p} \right] .$$

Note that $1 + (\sigma - 1) \varepsilon_{1x} > 0$.

Theorem 1

The long-run equilibrium is stable if

$$(1) \quad 0 < \sigma < 1 \quad , \quad \text{or}$$

$$(2) \quad k_y > k_x \quad \text{and} \quad 1 - (\sigma - 1) \frac{1-p}{p} > 0 \quad .$$

It should be noted that sufficient conditions given in *Theorem 1* have several similarities with condition (CI) suggested by Dixit and Stiglitz. Note (CI) is rewritten as follows:

$$(CI)' \quad 1 - (\sigma - 1)(1-s) \frac{1-p}{p} > 0 \quad ,$$

which guarantees the Chamberlinian dd curve be more elastic than the DD curve. Similarities could be stated as follows: On the one hand, when $0 < \sigma < 1$, then (CI)' is clearly satisfied. On the other hand, when $\sigma > 1$ and (CI)' is satisfied, then $1 - (\sigma - 1) \frac{1-p}{p} > 0$. The difference between our conditions and (CI)' is that we have derived our conditions from the full dynamic system, while Dixit and Stiglitz obtained condition (CI) from the static framework.*

4. Comparative Statics

First, let us show general treatment on comparative statics. The short-run equilibrium will be stated as follows:

$$(24) \quad f = \frac{p\beta}{\beta} - w = f(L_x; n, \alpha) \quad .$$

Using Equation (24), we obtain comparative static results.

$$(25) \quad \frac{\partial L_x}{\partial n} = - \frac{f_n}{f_{L_x}}$$

$$(26) \quad \frac{\partial L_x}{\partial \alpha} = - \frac{f_\alpha}{f_{L_x}}$$

From the stability condition, $f_{L_x} < 0$.

The long-run equilibrium will be expressed as follows:

$$g \equiv p - \left(w\beta + \frac{r\gamma}{x} \right) = g(L_x(n, \alpha), n, \alpha) \quad .$$

Therefore, using the comparative static method, we could compute the following:

* K. Lancaster suggests that 1 is necessary for stability (p.200).

However, what he claims as stability conditions are the second-order conditions for profit-maximization, which is satisfied in our case (see Figure 1).

$$\begin{aligned}
 (27) \quad Z \frac{dn}{d\alpha} &= -g_{L_x} \frac{\partial L_x}{\partial \alpha} - g_\alpha \\
 &= g_{L_x} \frac{f_\alpha}{f_{L_x}} - g_\alpha .
 \end{aligned}$$

Therefore, by specifying a parameter and computing necessary partial derivatives, we obtain both short-run and long-run comparative static results. We will consider two cases: (1) reduction in variable cost (decline in β), and (2) reduction in fixed cost (decrease in γ). Note that both cases result in unit cost reduction. However, as seen below, there are several conflicting effects, depending on which cost should be reduced.

(1) decline in β

As already shown in the previous section, short-run equilibrium is stable ($f_L < 0$). A decrease in β , in our case, implies improvement in productivity of labor. Therefore, the effect of a change in β on labor demand (L_x) will be known once f_β is computed (see equation (26)). Since cost-reducing effort implies improvement in productivity in the diversified goods sector, there are two elements which affect demand for labor. (1) Improvement in productivity would, *ceteris paribus*, increase the supply of differentiated goods, which tends to decrease relative commodity price (p) and the motivation for hiring labor. However, (2) since unit variable cost is decreased, each firm has a tendency for employing more labor. Therefore, the result of these conflicting forces would depend on the substitutability between differentiated goods and homogeneous goods. In the case where elasticity of substitution be unity, commodity share between differentiated goods and homogeneous goods is not disturbed by a change in composite price q . Therefore, an improvement in productivity simply reduces the relative commodity price at the same rate. That is,

$$\frac{\partial p}{\partial \beta} = \frac{p}{\beta} .$$

In this case,

$$f_\beta = \frac{\rho}{\beta} \left(\frac{\partial p}{\partial \beta} - \frac{p}{\beta} \right) = 0$$

In general, however,

$$\frac{\partial p}{\partial \beta} = \frac{p}{\sigma \beta} .$$

Then

$$f_\beta = \frac{\rho p}{\beta} \frac{1 - \sigma}{\sigma} .$$

Once the effect of a change in β on L_x is computed, we can calculate effects on the other remaining variables such as X , L_y , Y , w , r , and p . Table 1 summarizes short-run effects by productivity improvement (or variable cost reduction) in the

Table 1 Short-run Effects of Productivity Improvement

	L_x	X	L_y	K_y	Y	w	r	p
$\sigma > 1$	+	+	-	0	-	+	-	-
$\sigma < 1$	-	+	+	0	+	-	+	-
$\sigma = 1$	0	+	0	0	0	0	0	-

Table 2 Long-run Effects of Productivity Improvement

		n	L_x	X	L_y	K_y	Y	w	r	p
$\sigma > 1$	$k_x > k_y$	+	+	+	-	-	-	-	+	?
	$k_x < k_y$	+	-	+	-	-	-	+	-	?
$\sigma < 1$	$k_x > k_y$	-	-	+	+	+	+	+	-	?
	$k_x < k_y$	-	+	+	+	+	+	-	+	-
$\sigma = 1$		0	0	+	0	0	0	0	0	-

differentiated goods sector.

Our analysis has shown that process innovation embodied by reducing variable cost definitely increases production of the differentiated commodity in each firm and decreases the relative commodity price. However, since labor employment in each sector is influenced by substitutability between commodities, production of homogeneous goods also depends on the elasticity of substitution. Similarly, long-run effects will be summarised as in Table 2. In the long-run, an equilibrium number of firms in the differentiated goods sector critically depends on the elasticity of substitution. Further complication will arise because the order of factor intensity of the two sectors affects the employment of labor in the differentiated goods sector and factor prices in addition. As a result, productivity improvement does not always reduce commodity price p . Suppose that the case of $0 < \sigma \leq 1$ is normal and that the case of $\sigma > 1$ is less than normal. Then we may summarise our results as follows:

Theorem 2

In the normal case ($0 < \sigma \leq 1$), productivity improvement in the differentiated goods sector increases output of each firm both in the short-run and in the long-run. Production in the homogeneous goods sector is benefited by this improvement. Furthermore, we expect in the long-run that productivity improvement will result in higher concentration, that is, fewer firms and greater output level in each firm.

Similarly,

lemma 2

In a less than normal case ($\sigma > 1$), productivity improvement in the differentiated goods sector increases only its own output and decreases production of homogeneous goods. In the long-run, we have a lower concentration, that is, a larger number of firms in the differentiated goods sector and greater output level in each firm.

(2) decline in γ

Suppose that process innovation, without changing productivity, is realised as reduction in the fixed cost in each symmetric firm of the differentiated goods sector. Then when the elasticity of substitution is smaller than unity, the share of differentiated goods will be increased, due to a rise in composite commodity price q (see equation (10)), and so will the demand for labor and production of differentiated goods. While labor employment in the homogeneous goods sector depends on elasticity of substitution, production in that sector always increases, because process innovation enables it to use abundant capital stock. Evidently, rental on capital is lowered by process innovation. Since wage rate moves in opposite direction against change in rental on capital, it will be increased. Process innovation results in an increase in commodity price, because increase in production of the homogeneous goods sector must lower its price. Table 3 summarises the short-run effects by process innovation embodied in fixed cost reduction.

In the long-run, an equilibrium number of firms in the differentiated goods sector will be expected to increase. However, in the less than normal case where

Table 3 Short-run Effects by Fixed Cost Reduction

	L_x	X	L_y	K_y	Y	w	r	p
$\sigma > 1$	-	-	+	+	+	+	-	+
$\sigma < 1$	+	+	-	+	+	+	-	+
$\sigma = 1$	0	0	0	+	+	+	-	+

Table 4 Long-run Effects by Fixed Cost Reduction

		n	L_x	X	L_y	K_y	Y	w	r	p
$\sigma > 1$	$k_x > k_y$	?	?	?	-	-	-	-	+	-
	$k_x < k_y$	+	?	?	-	-	-	+	-	+
$\sigma < 1$	$k_x > k_y$	+	-	-	+	+	+	+	-	+
	$k_x < k_y$	+	-	-	+	+	+	-	+	-
$\sigma = 1$		+	-	-	0	0	0	0	0	0

substitutability between commodities is elastic ($\sigma > 1$) and the differentiated goods sector is more capital intensive, strong incentive for entry does not necessarily follow. In a less elastic case where process innovation creates more entry, each firm in the differentiated goods sector is engaged in smaller production. Production of the homogeneous goods sector critically depends on values of the elasticity of substitution. Both factor prices and commodity price depend on the order of factor intensities as well as on the elasticity of substitution (See *Table 4*). We can summarise our results as follows:

Theorem 3

In normal cases ($\sigma \leq 1$), symmetric firms in the differentiated goods sector, as a result of reduction in fixed cost, increase output in the short-run. However, in the long-run each firm produces less because of entry, which lowers concentration in the differentiated goods sector. The homogeneous goods sector always increases its production.

Similarly,

lemma 3

In a less than normal case ($\sigma > 1$), the effect of process innovation on the differentiated goods sector is generally not known. However, when the second sufficient condition for stability (see *Theorem 1*) is satisfied, we have lower concentration (an increase in n), lower labor employment (L_x), and lower output of each firm. However, production of the homogeneous goods sector is always decreased.

4. Concluding Remarks

In this paper, we extended the DSLS model so as to explain a dynamic aspect of entry and/or exit in the monopolistically competitive market from the general equilibrium framework. We have derived the entry-generating equation, whose background idea is quite similar to Lancaster's. However, entry and/or exit in the symmetric equilibrium seems stochastic in nature. We have provided rigorous justification for Lancaster's formulation. Furthermore, our model will clarify some issues related to process innovation, resulting in unit cost reduction. We have shown that process innovation must be carefully investigated, according to whether it reduces variable cost or fixed cost. When process innovation improves productivity in the differentiated goods sector, it would raise production, irrespective of substitutability between the differentiated goods and the homogeneous goods. However, in the case of reducing fixed cost, a distinct difference emerges whether we consider short-run equilibrium or long-run equilibrium. In the short-run, production of differentiated

goods will be increased in normal cases ($\sigma \leq 1$), because a rise in the composite commodity price would raise the share of differentiated goods. In a less than normal case ($\sigma > 1$), the opposite will prevail. These short-run results, however, may be drastically modified. Now, let us state our main results:

- (1) When the elasticity of substitution is inelastic ($\sigma \leq 1$), long-run equilibrium is stable, as has been anticipated by the Dixit-Stiglitz model. Instability would be caused when σ is quite high from unity.
- (2) It is anticipated that when a degree of concentration in industries is small, industry-wide R & D effort is positively correlated with concentration (see Dasgupta and Stiglitz (1980, *ibid.*)). However, once outside repercussions are taken into account, this does not necessarily hold. In order for this to be true, (1) process innovation should be realised by technological progress, and (2) substitutability between commodities must be inelastic ($\sigma \leq 1$). If reducing unit cost implies a decrease in fixed cost, it is likely that the market would become competitive.
- (3) Careful attention must be paid to the case of unitary elasticity of substitution. In this case, technical progress in the differentiated goods sector increases production, resulting in a lowering of its commodity price. However, there is no impact on the homogeneous goods sector, and the rate of concentration remains unchanged. On the other hand, reduction in the fixed cost has quite disparate effects on production and on commodity price, according to whether we consider short-run equilibrium or long-run equilibrium. In the short-run, fixed cost reduction does increase only production of the homogeneous goods sector and raises relative commodity price p . However, in the long-run, it amplifies competitiveness in the differentiated goods sector and reduces production of each firm there. There is no impact either on the homogeneous goods sector or on relative commodity price. These clear-cut results, however, cannot be generalised to the extent that elasticity of substitution is not necessarily unity.

Appendix A.

(I) Short-run Results

(1) productivity improvement (a change in β)

(a) effect on L_x

$$\frac{\partial L_x}{\partial \beta} = - \frac{f_\beta}{f_{L_x}}$$

where

$$f_{L_x} = -w \frac{1 + (\sigma - 1) \epsilon 1_x}{\sigma L_x 1_y}, \quad 1_x = \frac{n L_x}{L} \quad \text{and} \quad 1_y = \frac{L_y}{L}$$

Note that $f_{L_x} < 0$.

$$f_\beta = \frac{\rho}{\beta} \frac{\partial p}{\partial \beta} - \frac{\rho p}{\beta^2} - \frac{\partial w}{\partial \beta}$$

Simple computation will yield:

$$\frac{1}{p} \frac{\partial p}{\partial \beta} = \frac{1}{1 - s(q)} \frac{s'(q) q}{s(q)} \left(\frac{1}{q} \frac{\partial q}{\partial \beta} \right) + \frac{1}{y} \frac{\partial y}{\partial \beta} - \frac{1}{x} \frac{\partial x}{\partial \beta}.$$

Note that

$$(i) \quad \frac{1}{q} \frac{\partial q}{\partial \beta} = \frac{1}{p} \frac{\partial p}{\partial \beta}$$

$$(ii) \quad \frac{\partial y}{\partial \beta} = 0$$

$$(iii) \quad \frac{\partial x}{\partial \beta} = - \frac{x}{\beta}, \quad \text{and}$$

$$(iv) \quad \theta = \frac{s'(q) q}{s(q)} = (1 - \rho)(1 - s)$$

Therefore

$$\frac{\partial p}{\partial \beta} = \frac{p}{\rho \beta}$$

Since $\frac{\partial w}{\partial \beta} = 0$,

$$f_\beta = \frac{\rho p}{\beta^2} \frac{1 - \sigma}{\sigma}.$$

(b) effect on X

Since $X = L_x / \beta$,

$$\frac{\beta}{x} \frac{\partial x}{\partial \beta} = \frac{\beta}{L_x} \frac{\partial L_x}{\partial \beta} - 1.$$

Therefore, substituting the value of $\frac{\partial L_x}{\partial \beta}$ into the above equation, we obtain

$$\frac{\beta}{x} \frac{\partial x}{\partial \beta} = \frac{1 + (\sigma - 1)[1_y + \epsilon 1_x]}{1 + (\sigma - 1) \epsilon 1_x} < 0.$$

(c) effect on L_y

$$\frac{\partial L_y}{\partial \beta} = -n \frac{\partial L_x}{\partial \beta}.$$

(d) effect on K_y

$$\frac{\partial K_y}{\partial \beta} = 0.$$

(e) effect on Y

$$\frac{1}{Y} \frac{\partial Y}{\partial \beta} = (1 - \epsilon) \frac{1}{L_y} \frac{\partial L_y}{\partial \beta}$$

(f) effect on w

$$\frac{1}{w} \frac{\partial w}{\partial \beta} = (-\epsilon) \frac{(-n)}{L_y} \frac{\partial L_x}{\partial \beta} = \frac{n\epsilon}{L_y} \frac{\partial L_x}{\partial \beta}$$

(g) effect on r

$$\frac{1}{r} \frac{\partial r}{\partial \beta} = -\frac{n(1 - \epsilon)}{L_y} \frac{\partial L_x}{\partial \beta}$$

(h) effect on p

$$\text{Since } p = \frac{s(q)}{1 - s(q)} \frac{Y}{nX},$$

$$\begin{aligned} \frac{1}{p} \frac{\partial p}{\partial \beta} &= \frac{s'q}{s} \left(\frac{1}{q} \frac{\partial q}{\partial \beta} \right) + \frac{s'q}{1 - s} \left(\frac{1}{q} \frac{\partial q}{\partial \beta} \right) + \frac{1}{Y} \frac{\partial Y}{\partial \beta} - \frac{1}{X} \frac{\partial X}{\partial \beta} \\ &= \frac{1}{\beta [1 + (\sigma - 1) \epsilon 1_x]} \end{aligned}$$

(2) fixed cost reduction (a change in γ)

(a) effect on L_x

$$\text{Now that } f_r = \frac{p}{\beta} \frac{\partial p}{\partial \gamma} - \frac{\partial w}{\partial \gamma}$$

using equation (13), we have the following:

$$\frac{1}{p} \frac{\partial p}{\partial \gamma} = -\frac{n\epsilon}{\partial K_y}$$

Similarly,

$$\frac{1}{w} \frac{\partial w}{\partial \gamma} = -\frac{n\epsilon}{K_y}$$

Therefore,

$$\begin{aligned} f_r &= \frac{n\epsilon w}{K_y} \left(\frac{\sigma - 1}{\sigma} \right) \\ \frac{\partial L_x}{\partial \gamma} &= \frac{(\sigma - 1) \epsilon 1_x}{K_y [1 + (\sigma - 1) \epsilon 1_x]} \end{aligned}$$

(b) effect on X

$$\frac{1}{x} \frac{\partial x}{\partial \gamma} = \frac{1}{L_x} \frac{\partial L_x}{\partial \gamma}$$

(c) effect on L_y

$$\frac{\partial L_y}{\partial \gamma} = -n \frac{\partial L_x}{\partial \gamma} .$$

(d) effect on Y

$$\begin{aligned} \frac{1}{Y} \frac{\partial Y}{\partial \gamma} &= \varepsilon \frac{1}{K_y} \frac{\partial K_y}{\partial \gamma} + (1 - \varepsilon) \frac{1}{L_y} \frac{\partial L_y}{\partial \gamma} . \\ &= -\frac{n\varepsilon}{K_y} \frac{1 + (\sigma - 1) l_x}{1 + (\sigma - 1) \varepsilon l_x} < 0 . \end{aligned}$$

(f) effect on w

$$\begin{aligned} \frac{1}{w} \frac{\partial w}{\partial \gamma} &= \varepsilon \left[\frac{1}{K_y} \frac{\partial K_y}{\partial \gamma} - \frac{1}{L_y} \frac{\partial L_y}{\partial \gamma} \right] . \\ &= -\frac{n\varepsilon}{K_y [1 + (\sigma - 1) \varepsilon l_x]} < 0 . \end{aligned}$$

(g) effect on r

$$\begin{aligned} \frac{1}{r} \frac{\partial r}{\partial \gamma} &= -(1 - \varepsilon) \left[\frac{1}{K_y} \frac{\partial K_y}{\partial \gamma} - \frac{1}{L_y} \frac{\partial L_y}{\partial \gamma} \right] . \\ &= \frac{n(1 - \varepsilon)}{K_y [1 + (\sigma - 1) \varepsilon l_x]} > 0 . \end{aligned}$$

(h) effect on p

$$\frac{1}{p} \frac{\partial p}{\partial \gamma} = -\frac{n\varepsilon}{K_y [1 + (\sigma - 1) \varepsilon l_x]} < 0 .$$

(II) Long-run Results

(1) productivity improvement

(a) effect on n

Note:

$$(A1) \quad Z \frac{dn}{d\beta} = g_{L_x} \frac{f_\beta}{f_{L_x}} - g_\beta$$

$$(i) \quad \frac{f_\beta}{f_{L_x}} = \frac{L_x l_y (\sigma - 1)}{\beta [1 + (\sigma - 1) \varepsilon l_x]}$$

$$(ii) \quad g_{L_x} = -\frac{w\beta}{\sigma L_x l_y} \left\{ 1 - (\sigma - 1) \left[\frac{1 - p}{p} (1 - \varepsilon l_x) - \varepsilon l_x \right] \right\}$$

$$(iii) \quad g_\beta = -\frac{w(\sigma - 1)}{p\sigma}$$

Substituting the above results into (A1), we have:

$$g_{L_x} \frac{f_\beta}{f_{L_x}} - g_\beta = \frac{1 - p}{p} \frac{(\sigma - 1) w}{[1 + (\sigma - 1) \varepsilon l_x]}$$

Since Z can be arranged to $-A\bar{Z}$,

$$\frac{dn}{d\beta} = -\frac{1}{Q} \frac{(\sigma - 1) w}{[1 + (\sigma - 1) \varepsilon l_x]}$$

where

$$Q = \frac{\beta}{nK_y 1_y [1 + (\sigma - 1) \epsilon 1_x]} \bar{Z}, \quad K_x = \frac{\gamma}{L_x} \quad \text{and} \quad K_y = \frac{K_y}{L_y}$$

If the system is stable, $z < 0$, and $\bar{Z}, Q > 0$.

(b) effect on L_x

$$L_x = L_x(n, \beta)$$

$$\frac{dL_x}{d\beta} = \frac{\partial L_x}{\partial n} \frac{dn}{d\beta} + \frac{\partial L_x}{\partial \beta}$$

Note that $\frac{\partial L_x}{\partial n} = -\frac{f_n}{f_{L_x}}$ and $\frac{\partial L_x}{\partial \beta} = -\frac{f_\beta}{f_{L_x}}$

Final result will be:

$$\frac{dL_x}{d\beta} = -\frac{1}{Q} \frac{(\sigma - 1) L_x}{n [1 + (\sigma - 1) \epsilon 1_x]} \left(\frac{K}{K_y} - 1 \right)$$

(c) effect on X

$$\begin{aligned} \frac{1}{x} \frac{dx}{d\beta} &= \frac{1}{L_x} \frac{dL_x}{d\beta} - \frac{1}{\beta} \\ &= -\frac{1}{Q} \frac{K[\sigma 1_y(1 - \epsilon) + 1_x + \epsilon 1_y] + (1 - \sigma)(1 - \epsilon)K_y 1_y + (1 - \sigma)\left(\frac{1 - \beta}{\beta}\right)K_y 1_y}{nK_y 1_y [1 + (\sigma - 1) \epsilon 1_x]} \end{aligned}$$

(d) effect on L_y

$$\begin{aligned} \frac{dL_y}{d\beta} &= -L_x \frac{dn}{d\beta} - n \frac{dL_x}{d\beta} \\ \frac{dL_y}{d\beta} &= \frac{L_x}{Q} \frac{\sigma - 1}{[1 + (\sigma - 1) \epsilon 1_x]} \frac{K}{K_y} \end{aligned}$$

(e) effect on K_y

$$\frac{dK_y}{d\beta} = -\gamma \frac{dn}{d\beta} = \frac{\gamma}{Q} \frac{(\sigma - 1)}{[1 + (\sigma - 1) \epsilon 1_x]}$$

(f) effect on Y

$$\begin{aligned} \frac{1}{Y} \frac{dY}{d\beta} &= \frac{\epsilon}{K_y} \frac{dK_y}{d\beta} + (1 - \epsilon) \frac{1}{L_y} \frac{dL_y}{d\beta} \\ &= -\frac{1}{Q} \frac{(\sigma - 1) [\epsilon \gamma + (1 - \epsilon) K L_x]}{K_y [1 + (\sigma - 1) \epsilon 1_x]} \end{aligned}$$

(g) effect on w

$$\begin{aligned} \frac{1}{w} \frac{dw}{d\beta} &= \epsilon \frac{1}{K_y} \left\{ \frac{dK_y}{d\beta} - K_y \frac{dL_x}{d\beta} \right\} \\ &= -\frac{1}{Q} \frac{\epsilon}{K_y} \frac{(\sigma - 1) (K_x - K)}{1 + (\sigma - 1) \epsilon 1_x} \end{aligned}$$

(h) effect on r

$$\frac{1}{r} \frac{dr}{d\beta} = -(1 - \epsilon) \frac{1}{K_y} \left\{ \frac{dK_y}{d\beta} - K_y \frac{dL_y}{d\beta} \right\}$$

$$= -\frac{1}{Q} \frac{1-\varepsilon}{K_y} \frac{(\sigma-1)(K_x-K)}{1+(\sigma-1)\varepsilon 1_x}$$

(i) effect on p

$$\begin{aligned} \frac{1}{p} \frac{dp}{d\beta} &= \frac{1}{\beta} + \frac{1}{w} \frac{dw}{d\beta} \\ &= \frac{1}{Q} \frac{K - (\sigma-1) \frac{1-p}{p} 1_y K_y + (\sigma-1)(1-\varepsilon)(K_x - K_y) 1_x 1_y}{n 1_K K_y [1 + (\sigma-1)\varepsilon 1_x]} \end{aligned}$$

(2) fixed cost reduction

(a) effect on n

$$(A2) \quad Z \frac{dn}{d\gamma} = g_{L_x} \frac{f_\gamma}{f_{L_x}} - g_\gamma$$

Note

$$\begin{aligned} (i) \quad \frac{f_\gamma}{f_{L_x}} &= -\frac{(\sigma-1)\varepsilon 1_x}{K_y [1 + (\sigma-1)\varepsilon 1_x]} \\ (ii) \quad g_{L_x} &= -\frac{w\beta}{\sigma L_x 1_y} \left\{ 1 - (\sigma-1) \left[\frac{1-p}{p} (1 - \varepsilon 1_x) - 1_x \right] \right\} \\ (iii) \quad g_\gamma &= \beta w \frac{n}{K_y} \left\{ \frac{\varepsilon}{p} \frac{\sigma-1}{\sigma} - \frac{1-p}{p} \frac{K}{1_x K_x} \right\} \end{aligned}$$

Substituting these results into (A2), we obtain:

$$\frac{dn}{d\gamma} = -\frac{1}{Q} \frac{\beta n}{K_y} \frac{K + (K - K_x)(\sigma-1)\varepsilon 1_x}{K_x 1_x [1 + (\sigma-1)\varepsilon 1_x]}$$

or

$$= -\frac{1}{Q} \frac{\beta n}{K_y} \frac{K[1 + (\sigma-1)\varepsilon 1_x] - (\sigma-1)\varepsilon K_x 1_x}{K_x 1_x [1 + (\sigma-1)\varepsilon 1_x]}$$

(b) effect on L_x

$$\frac{dL_x}{d\gamma} = \frac{\partial L_x}{\partial n} \frac{dn}{d\gamma} + \frac{\partial L_x}{\partial \gamma}$$

where

$$\frac{\partial L_x}{\partial n} = -\frac{f_n}{f_{L_x}} = -\frac{L_x}{n} \left\{ 1 - \frac{(\sigma-1)1_y \left[\frac{1-\rho}{\rho} + \varepsilon \frac{n\gamma}{K_y} \right]}{1 + (\sigma-1)\varepsilon 1_x} \right\}$$

and

$$\frac{\partial L_x}{\partial \gamma} = \frac{n L_x 1_y \varepsilon (\sigma-1)}{K_y [1 + (\sigma-1)\varepsilon 1_x]}$$

Therefore,

$$\begin{aligned} \frac{dL_x}{d\gamma} &= \frac{K[1 + (\sigma-1)\varepsilon 1_x] - (\sigma-1) \left[\frac{1-p}{p} K 1_y + \varepsilon 1_x K_x \right]}{K_x \tilde{Z}} \\ &= \frac{K \left[1 - (\sigma-1) \frac{1-p}{p} 1_y \right] + \varepsilon 1_x (\sigma-1)(K - K_x)}{K_x \tilde{Z}} \end{aligned}$$

(c) effect on X

$$\frac{1}{X} \frac{dx}{d\gamma} = \frac{1}{L_x} \frac{dL_x}{d\gamma}$$

(d) effect on L_y

$$\begin{aligned} \frac{dL_y}{d\gamma} &= -n \left(\frac{dL_x}{d\gamma} + \frac{L_x}{n} \frac{dn}{d\gamma} \right) \\ &= \frac{n(\sigma - 1) \frac{1-p}{p} K l_y}{K \bar{Z}} \end{aligned}$$

(e) effect on K_y

$$\frac{dK_y}{d\gamma} = -n - \gamma \frac{dn}{d\gamma} = \frac{n}{\bar{Z}} \frac{1-p}{p} (\sigma - 1) K_y l_y$$

(f) effect on Y

$$\begin{aligned} \frac{1}{Y} \frac{dY}{d\gamma} &= \frac{1}{K_y} \left\{ \epsilon \frac{dK_y}{d\gamma} + (1 - \epsilon) K_y \frac{dL_y}{d\gamma} \right\} \\ &= \frac{\sigma - 1}{K_x} \frac{1-p}{p} \frac{n K_y l_y [\epsilon K_x + (1 - \epsilon) K]}{\bar{Z}} \end{aligned}$$

(g) effect on w

$$\begin{aligned} \frac{1}{w} \frac{dw}{d\gamma} &= \frac{\epsilon}{K_y} \left\{ \frac{dK_y}{d\gamma} - K_y \frac{dL_y}{d\gamma} \right\} \\ &= \frac{\epsilon}{K_y} \frac{n K_y l_y (\sigma - 1) (K_x - K)}{K_x \bar{Z}} \frac{1-p}{p} \end{aligned}$$

(h) effect on r

$$\begin{aligned} \frac{1}{r} \frac{dr}{d\gamma} &= - (1 - \epsilon) \frac{1}{K_y} \left\{ \frac{dK_y}{d\gamma} - K_y \frac{dL_y}{d\gamma} \right\} \\ &= - \frac{1 - \epsilon}{K_y} \frac{1-p}{p} \frac{n K_y l_y (\sigma - 1) (K_x - K)}{K_x \bar{Z}} \end{aligned}$$

(i) effect on p

$$\frac{1}{p} \frac{dp}{d\gamma} = - \frac{\epsilon l_x}{r} \frac{1-p}{p} \frac{(\sigma - 1) (K - K_x)}{\bar{Z}}$$

Appendix B Note on Entry-Generating Equation

As already shown in the paper, every endogenous variable is determined in the short-run, once the number of firms in the differentiated goods sector is known. The number of firms there is expected to increase for a positive profit and to decrease for a loss, and it would in the long-run converge to an equilibrium value with zero profit. In the DSLS model, however, a dynamic adjustment process of entry or exit is not self-evident. In the DSLS model, since every firm is supposed to be identical, entrants as well as incumbents would have equal incentive to enter and remain in the differentiated goods sector if one incumbent (in fact, they are identical) enjoys a positive profit, and every incumbent would have equal motivation to leave the sector if it faces a loss. But a change in the number of firms would be continuously approximated in the economy as a whole. (In particular, there is no uncertainty explicitly taken into account as in the DSLS model.) In order to explain an approximate dynamic adjustment process of change in the number of firms, it would be best to focus first on the firm's incentive to enter or leave from the microeconomic decision formula, and then derive an entry-generating equation from the macroeconomic standpoint.

First we described the concept of perfect monopolistic competition by Lancaster. Perfect monopolistic competition implies the state of competition where entrants and incumbents are treated equally. That is, there is no entry barrier for entrants to share the same technology as incumbents, and they have equal opportunity for obtaining profits. Furthermore, they behave independently and yet are influenced by the number of firms in the differentiated goods sector. In deriving an entry-generating equation in perfect monopolistic competition, we do not need to distinguish who is an entrant. We can arbitrarily choose any one incumbent and examine whether he has a motive to expand. Thus the future evolution of firms in a sector will be stochastically determined once the present number of firms in the sector is known exactly. Knowledge on the past record of the number of firms does not play any significant role at all. This situation is analogous to the situation where a single particle splits producing k (identical) particles. This process of evolution is known in the field of biology as the Galton-Watson branching process (see T.E. Harris (1963)). For $k > 1$, there is entry into the differentiated goods sector, and for $k < 1$, exit from the sector. This situation of entry or exit would be solved by using the method of continuous Markov branching process (see Karlin and Taylor (1975, chapters 4 and 8) and Cox and Miller (1978, chapter 4)).

Let $X(t) = X_1(t) + X_2(t) + \dots + X_k(t)$ and $P_{ij}(t)$ respectively denote the number of firms in the differentiated goods sector at time t and a probability that the population of size i at time s will be of size j at time t . That is,

$$P_{ij}(t) = \Pr \{X(t+s) = j | X(s) = i\} \quad .$$

In addition, we assume that the $P_{ij}(t)$ satisfy:

1. $P_{i,i+1}(h) = \xi_i h + o(h)$ as $h \downarrow 0$ $i > 0$,
2. $P_{i,i-1}(h) = \mu_i h + o(h)$ as $h \downarrow 0$ $i > 1$,
3. $P_{i,i}(h) = 1 - (\xi_i + \mu_i) h + o(h)$ as $h \downarrow 0$ $i > 0$,
4. $P_{i,j}(0) = \delta_{ij}$, and
5. $\xi_0 = 0$, $\mu_0 > 0$, $\xi_i, \mu_i > 0$,

where δ_{ij} denotes Kronecker's delta symbol. Parameters ξ_i and μ_i are respectively the infinitesimal birth and death rates. Because of the assumption of perfect monopolistic competition, we can choose any incumbent firm in the differentiated goods sector and concentrate on its evolution over a period of time. Therefore, we determine a continuous time Markov branching process with state variable $X(t) = \{\text{number of firms at time } t, \text{ given } X(0) = 1\}$ by specifying the infinitesimal probabilities of the process. Let

$$(1) \quad \delta_{1k} + a_k h + o(h) \quad , \quad k = 0, 1, 2 \quad ,$$

represent the probability that an average firm will produce k (identical) firms during a small time interval $(t, t+h)$ of length h . Then for simplicity, assume $a_0 = \xi$, $a_1 = -(\mu + \xi)$ and $a_2 = \mu$, and $a_k = 0$ otherwise. $(\mu + \xi)^{-1}$ can be interpreted as the probability of an entry or exit event: $\xi/(\mu + \xi)$ and $\mu/(\mu + \xi)$, respectively, are the probability of an entry and exit under condition that an event has happened. a_k ($k = 0, 1, 2$) are not functions of time, because μ and ξ would depend on whether a firm earns a positive (or negative) profit, which in turn depends on the number of firms in the sector. Therefore, we can assume time homogeneity for transition probabilities. We introduce generating function

$$(2) \quad \phi(t; s) = \sum_{j=0}^{\infty} P_{1j}(t) s^j$$

Since all firms act independently, we have the following functional relation:

$$(3) \quad \sum_{j=0}^{\infty} P_{ij}(t) s^j = [\phi(t; s)]^i$$

Equation (3) implies that the population $X(t; i)$ evolving in time t from i initial firms is the same, probabilistically, as the combined sum of i population, each with one firm.

In view of the time homogeneity, the Chapman-Kolmogorov equations take the form

$$(4) \quad P_{ij}(t + \tau) = \sum_{k=0}^{\infty} P_{ik}(t) P_{kj}(\tau)$$

With the aid of equations (2) and (3), we have

$$[\phi(t + \tau; s)]^i = [\phi(t; \phi(\tau; s))]^i .$$

In particular,

$$(5) \quad \phi(t + \tau; s) = \phi(t; \phi(\tau; s)) .$$

Next, we introduce the generating function of the infinitesimal probabilities defined in equation (1). Specifically, let

$$u(s) = \sum_{j=0}^{\infty} a_j s^j .$$

Consider

$$(6) \quad \phi(h; s) = \sum_{j=0}^{\infty} p_{ij}(h) s^j = s + hu(s) + o(h) .$$

We substitute $t = h$ in equation (5). Then

$$\phi(h + \tau; s) = \phi(h; \phi(\tau; s)) = \phi(\tau; s) + hu(\phi(\tau; s)) + o(h) ,$$

by using equation (6). Note:

$$\frac{\phi(\tau + h; s) - \phi(\tau; s)}{h} = u(\phi(\tau; s)) + \frac{o(h)}{h}$$

Letting $h \rightarrow 0^+$ and replacing τ by t , we have

$$(7) \quad \frac{\partial \phi(t; s)}{\partial t} = u(\phi(t; s)) .$$

Differentiating equation (7) with respect to s , we interchange the order of differentiation on the left side. Then

$$(8) \quad \frac{\partial}{\partial t} \frac{\partial \phi(t; s)}{\partial s} = \frac{\partial^2 \phi(t; s)}{\partial s^2} u(s) + \frac{\partial \phi(t; s)}{\partial s} u'(s)$$

Set $s = 1$. Since $u(1) = \sum_{j=0}^{\infty} a_j = 0$, we have

$$(9) \quad \frac{\partial m(t)}{\partial t} = u'(1) m(t) .$$

where $m(t) = EX(t) = \frac{\partial \phi(t; s)}{\partial s} \Big|_{s=1}$. Note that $u'(1) = \xi - \mu$. It is natural to assume that the net entry rate would depend on profit of the average firm, which is counted once the number of firms in the sector is known. That is, $\xi - \mu = \lambda \Pi(n)$ (The Malthusian parameter now reflects the density effect.) Equation (9) will be rewritten as equation (10).

$$(10) \quad \frac{\dot{m}(t)}{m(t)} = \lambda \Pi(n) .$$

Referring to the implication derived from equation (3), we may analogously write an entry-generating equation in the differentiated goods sector as a whole:

$$\frac{\dot{M}(t)}{M(t)} = \lambda \Pi(n) ,$$

where $M(t) = EX(t; n_0)$ and n_0 is the initially given number of firms in the differentiated goods sector at time zero. The corresponding deterministic version of an entry-generating equation will be expressed as equation (11).

$$(11) \quad \dot{n}(t) = \lambda \Pi(n) .$$

If the above Poisson process is a good approximation, we can use equation (11) as the one showing dynamic motion of entry in the system. In stability analysis, furthermore, we are only interested in the neighborhood of equilibrium and $\Pi(n^*) = 0$, where n^* denotes the equilibrium number of firms in the differentiated goods sector. Therefore, we can safely specify entry-generating equation (12)

$$(12) \quad \dot{n} = \lambda \Pi(n) .$$

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