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Quota cum Quality Control in Vertical Product Differentiation*
—Welfare Aspect of A Simple General Equilibrium Model—

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As a practical trade policy, quota is as important as tariff. Furthermore, in order to avoid trade frictions among trading countries, the voluntary export restraint (hereafter VER, for short), which is understood as a variant on the import quota, is taken, more often than not, as a realistic policy. In addition, commodities concerned are sometimes not homogeneous as traditional analysis assumes, but graded or differentiated according to, for instance, quality. We supply a general equilibrium model to explain various quota policies in vertical product differentiation.

Suppose that there are two countries, called the North and the South, both of which can produce differentiated products and other composite goods, called homogeneous goods. The former products are differentiated by quality. The North has comparative cost advantage on production of high quality goods, while the South possesses comparative cost advantage on production of low quality goods. Under plausible assumptions, which will be shown below, H.Flam and E.Helpman (1987a) (hereafter FH, for short) have demonstrated that the North exported high quality products and imported both low quality products and homogeneous products. While they succeeded in explaining a phenomenon of product cycles in the above case, the welfare aspect of trade policies is not fully investigated yet.(1) This paper is concerned with quota cum quality control, because it brings out several interesting issues. First, this setting of our model does not hold the equivalence between tariff and quota.(2) Because of cost differences between the North and the South, quotas create forced consumption of homogeneous goods by some marginal consumers who are indifferent to consuming either low or high quality products before quotas. This never occurs in case of tariff, in which each consumer optimally chooses a pair of both a differentiated item and homogeneous goods. Second, there are at least two different kinds of quota policies. Either country, the North or the South, can import a fixed number of differentiated products, whose qualities are either under or over a specific level. Suppose that the North fixes a maximum level of quality on imported low quality products. This policy may be promoted to protect marginal

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firms in the North. Conversely, the North fixes the minimum level of quality importables. For instance, since qualities of importables are relatively unknown, the North may make an attempt to maintain their minimum level of qualities if they are related to health or securities.(See R.E.Falvey (1989).) As to their application, consider VER on automobiles between the U.S. and Japan. Both R.E.Falvey (1989) and R.C.Feenstra (1988) report upward quality shifts of exportables by Japan after VER. Why does Japan voluntarily restrict the quantity of exports and export higher quality products? How does that policy affect both the U.S and Japan? These questions themselves are interesting questions to be answered. Alternatively, the South has similar choices for quality in case of restricting the quantity of importables. Which policy is more beneficial for the South-restricting the maximum quality or the minimum quality? This paper supplies a synthetic model to answer these questions. Third, in treating commercial policies, traditional analysis assumes the existence of a representative consumer, whose utility level exhibits social welfare.[3] (See M.Kemp (1969).) In our case, there is a continuum of consumers whose utility levels are dependent upon their income levels. We here assume the Bergson-Samuelson type social welfare function for many consumers and derive meaningful policy implications from optimization procedures.

The purpose of this paper is to examine welfare changes by quota cum quality control policy on the basis of the FH framework. Since individuals possess different skills represented by means of income classes, we cannot find a representative individual, even if they have the same type of utility function. In this paper, we simplify the form of utility functions so as to satisfy the Gorman polar form. Let us regard a change in the (relative) wage rate as a change in the terms of trade, because our model is the Ricardian type (see R. Dornbusch, S. Fisher, and P.A. Samuelson (1977)). Then we shall show that the terms of trade always change favorably for a quota-imposing country and against a quota-imposed country. However, those marginal individuals who are forced to purchase differentiated goods of a certain level of quality at the sacrifice of other goods may suffer welfare losses. Suppose the South voluntarily restricts its exportables and only exports relatively higher quality goods among the low.[4] This southern action may be justified in our paper that while the South may suffer welfare losses voluntarily taken from VER, its losses are smaller than those in the case where the North restricts the maximum level of quality of differentiated products.

Section 1 demonstrates our model under free trade. We simplify some of the functional forms, while essential features are the same as the FH model. Section 2 then discusses effects on several endogenous variables such as the wage rate when quotas are exercised. Section 3 sheds light on welfare changes both in the North and the South, when one of them takes a quota policy. VER is considered as an application of our quota cum quality control policy. In case of VER, since a major

concern of an exporter is to avoid trade frictions against trade partners, even if such a policy is not the best choice from the standpoint of its own interest, it is important to assess the welfare effects to its trade partners as well as to its own. Section 4 compares welfare changes in both countries by available quota cum quality control policies. Section 5 gives brief concluding remarks.[5]

1. Preliminary Remarks

This section briefly shows the basic framework of our model based upon free trade, which owes much to the FH model.[6] There are two countries —the North and the South. They can produce both homogeneous goods and differentiated goods, the latter of which are classified by a continuum of qualities. While the same technology is shared between the North and the South in producing homogeneous goods, the North has comparative cost advantage in producing high quality differentiated goods, and the South in producing low quality differentiated goods. Since labor is the only factor of production and the wage rate in the North is assumed higher than that in the South, free trade implies only that the South is capable of producing homogeneous goods. Individuals in both countries possess different skills endowed from the beginning, which are reflected by means of income classes.

Now let us specify our model. We basically assume Ricardian technologies. In the production of homogeneous goods, for the sake of simplicity, one unit of labor produces one unit of products. The price of homogeneous goods is taken as a numeraire. Since labor is the only factor of production and we assume a competitive labor market, the wage rate in the South becomes unity. As for the case of differentiated goods, input coefficients are constant, so far as quality z remains unchanged. Equation (1. 1) gives a specific form in our model.[7]

$$(1.1) \qquad a_{LX}(z) = az + b.$$

Equation (1. 2) presents the unit cost of differentiated goods with quality z in the North.

$$(1.2) \qquad c(z) = awz + bw.$$

The constant term bw in equation (1. 2) may be regarded as either an entry fee or setting-up cost (refer to R.E. Falvey and H. Kierzkowski (1987) for its explanation). Under competitive markets, the commodity price of quality z, $p(z)$, is equal to the unit cost, c (z).

$$(1.3) \qquad p(z) = awz + bw.$$

We express variables of the South by attaching an asterisk notation to corresponding variables in the North. Therefore, equations (1. 4) and (1. 5) respectively show input coefficients and the commodity price of quality z in the South.

$$(1.4) \qquad a^*_{LX}(z) = a^* z + b^*.$$

$$(1.5) \qquad p^*(z) = a^* z + b^*.$$

Note that $w^*=1$. The relation in inequality, (C1), shows technological differences between the North and the South.

(C1) $\quad a^* > aw$ and $bw > b^*$.

Under condition (C1), the South has comparative cost advantage in producing low quality goods and the North, high quality goods.

Next, consider the behavior of consumers. We assume a simple Cobb-Douglas type utility function.[8]

$$(1.6) \qquad u_h(y_h, z_h) = y_h^{\alpha} z_h^{\beta}, \ \alpha + \beta = 1.$$

where y_h denotes the amount of homogeneous goods demanded. Subscript h indicates an income class to which individuals belong. Let $f(h)$ and $n(h)$ respectively stand for the distribution of effective labor units across income classes and the distribution of the population over income classes. Then the income of individuals in income class h equals $I_h = wLf(h)/Nn(h)$, where L and N respectively show the quantity of labor available to the North, and its population size. We usually drop subscript h in case of no confusion. Take an individual of income class h. Because of the Cobb-Douglas type of utility functions, we can easily derive the forms of indirect utility function. Equations (1.7) respectively show utility levels when the individual purchases one unit of differentiated goods, produced either in the South or in the North.

$$(1.7) \qquad \begin{aligned} V_L(I) &= Co\,(a^*)^{-\beta}(I_d - b^*). \\ V_H(I, w) &= Co\,(aw)^{-\beta}(I_d - bw), \end{aligned}$$

Where $Co = \alpha^{\alpha}\beta^{\beta}$.

Figure 1 shows individual utility levels originated from consuming either domestic goods (line aa in the Figure) or imported goods (line a^*a^*). Individuals who belong to income class h_d and earn income I_d are indifferent to purchasing a pair of commodities either from foreign firms or from domestic firms. That is, equation (1.8) holds.

$$(1.8) \qquad c_0\,(a^*)^{-\beta}(I_d - b^*) = c_0\,(aw)^{-\beta}(I_d - bw),$$

We can derive a unique relation between critical income level and the wage rate; $I_d = I_d(w)$. It should be noted that the elasticity of income with respect to the wage rate, ε_{wd}, is greater than unity.[9] It is clear that proportional increases in both

income and wage rate do disturb the equality relation stated in equation (1. 8). Suppose both income and wage rate are doubled. Under this situation, in case where individuals demand high quality goods, consumption of homogeneous goods is doubled, while that of differentiated goods remains unchanged. In contrast to this, when importables are purchased, consumption of both homogeneous goods and low quality goods is more than doubled. Since the price of quality is cheaper in the North than that in the South, the only way to maintain equality is to raise income at a higher rate than the wage rate. Now that the level of critical income is determined, so is the corresponding income class, h_n.

$$(1.9) \qquad I_d(w) = wLf(h_d)/Nn(h_d).$$

Without any difficulty, we apply our analysis to the South and obtain equations (1. 10) and (1. 11).

$$(1.10) \qquad \begin{aligned} V_L^*(I^*) &= Co(a^*)^{-\beta}(I^* - b^*). \\ V_H^*(I^*, w) &= Co(aw)^{-\beta}(I^* - bw). \end{aligned}$$

$$(1.11) \qquad I_d^*(w) = L^* f^*(h_d^*)/N^* n^*(h_d^*).$$

We know from the above that $\varepsilon_{wd}^* = d \text{ low } I_d^*/d \log w > 0$.[10]

2. The Model of Quota as Quality Control

First, consider a quota policy taken by the North. Since each individual demands one unit of differentiated goods and his income level determines the degree of quality in differentiated goods, quota in fact means quality control in our case. Let X_F denote the number of imported low quality goods under free trade. Then

$$X_F = \int_0^{hd} Nn(h)\, dh$$

When quota is imposed by the North, there are two distinct ways under which restricted amount X_Q is imported.

$$(Q1) \qquad X_Q = \int_0^{hQ} Nn(h)\, dh < X_F.$$

$$(Q2) \qquad X_Q = \int_{hQ}^{hd} Nn(h)\, dh < X_F.$$

Policy (Q1) may be realised under the situation where the North sets an allowable maximum level of quality and prohibits imports which exceed this level. This policy may be taken as a means for expanding domestic shares of differentiated goods. We call (Q1) a share-expanding quota (S-quota, for short). On the contrary, if the North only fixes the total volume of imports and does not care about the content of quality, the South may export relatively higher quality goods among

exportables. This pattern of trade is predicted in case of VER (see R.E. Falvey (1979) and R.C. Feenstra (1988)). We call (Q2) a quality-sifting quota policy (E-quota, for short).

2.1 S-Quota Policy

Consider an individual in income class h_s', whose income is $I_s'(=wLf(h_s')/Nn(h_s'))$ less than I_d. He prefers to purchase an imported good (refer to Figure 1). When imported goods are permitted only below quality $z_s'(=\beta[I_s'-b^*]/a^*)$, those individuals whose incomes are a little higher than I_s' do not immediately switch to purchasing domestically produced differentiated goods. They rather spend a fixed amount of money for imports of quality z_s' and use up the rest for homogeneous goods. With rising income levels, we find a critical income level I_s, showing that individuals are indifferent to continue purchasing imports of quality z_s'. Income level I_s is determined so as to satisfy equality given in equation (2.2).

$$(2.2) \qquad [I_s-(a^*z_s'+b^*)]^{\alpha}(z_s')^{\beta}=[\alpha(I_s-bw)]^{\alpha}[\beta(I_s-bw)/aw]^{\beta}.$$

Income level I_s could be a function of both w and h_s'.

$$(2.3) \qquad I_s=I_s(w, h_s').$$

Define the following elasticities.

$$\varepsilon_{ws}=\partial \log I_s/\partial \log w \quad \text{and} \quad \varepsilon_{hs}'=\partial \log I_s/\partial \log h_s'.$$

It is easy to verify that $\varepsilon_{ws}>1$ and $\varepsilon_{hs}'>0$. Now we classify income groups into three categories.

(1) $\quad h\in[0, h_s']$

Individuals optimally choose both homogeneous goods and one unit of imported low quality goods.

(2) $\quad h\in[h_s', h_s]$

Individuals spend the amount of $p(z_s')=a^*z_s'+b^*$ for imported low quality goods and allocate the rest into purchase of homogeneous goods.

(3) $\quad h\in[h_s, 1]$

Individuals optimally choose a pair of homogeneous goods and one unit of high quality goods domestically produced.

Since the equilibria in labor markets for both countries guarantee the equilibria in commodity markets as well as the equilibrium in the balance of trade, we supply the minimum expression of equilibrium conditions in our model.[11]

$$(2.7) \qquad I_s(w, h_s') = wLf(h_s)/Nn(h_s)$$

$$(2.8) \qquad X_s = \int_0^{hs} Nn(h)\,dh$$

$$(2.9) \qquad I_d^*(w) = L^*f^*(h_d^*)/N^*n^*(h_d^*)$$

$$(2.10) \qquad L = \int_{hs}^{1} (az + b)\,Nn(h)\,dh + \int_{hd}^{1*} (az^* + b)\,N^*n^*(h)\,dh.$$

Equation (2.7) defines the level of critical income, I_s. Individuals with income less than I_s consume imported low quality goods. On the contrary, individuals with income greater than I_s purchase high quality goods. Equation (2.8) expresses the amount of quota imposed by the North. For simplicity, we assume that the integrated number of individuals between income classes 0 and h_s is exactly equal to X_s. Equation (2.9) states the critical income level I_d^*, with which individuals are indifferent to buying either imported high quality goods or low quality goods domestically produced. Equation (2.10) demostrates equilibrium in the labor market of the North.

Now, how does the imposition of quota affect endogenous variables? A decline in X_s directly reduces the number of individuals who purchase imported low quality goods. Therefore, from equation (2.8), h_s is decreased. Now that $\varepsilon_{ws} > 1$[12] and h_s is decreased, equation (2.7) in fact reveals that the number of individuals optimally purchasing importables is decreased. Since the imposition of quota turns a part of the demand for importables towards domestic products, labor demand in the North is increased, which raises the wage rate. Since the wage rate is increased, so is the critical income level, h_d^*, by equation (2.9).

2.2 E-Quota Policy

In this case quota means that importables with quality less than a certain level, say z_E', are prohibited. Then those individuals whose incomes are a little lower than I_E' can still enjoy higher satisfaction from consuming importables with quality z_E' and sacrificing a part of the optimal amount of homogeneous goods, rather than optimally choosing a pair of homogeneous goods and low quality goods domestically produced. Individuals with income I_E become indifferent to consuming either importables or domestic products. Therefore, we can classify income groups into four categories.

(1) $\quad h \in [0, h_E]$

Individuals optimally choose a pair of homogeneous goods and one unit of differentiated goods, the latter of which are domestically produced.

(2) $h \in [h_E, h_E']$

Individuals stick to demanding importables with quality z_E'.

(3) $h \in [h_E', h_d]$

Individuals consume a pair of importables and homogeneous goods.

(4) $h \in [h_d, 1]$

Individuals optimally purchase high quality goods domestically produced.

Equilibrium conditions are summarised by five equations.

(2. 11) $I_d(w) = wLf(h_d)/Nn(h_d).$

(2. 12) $X_E = \int_{hE}^{hd} Nn(h)\, dh.$

(2. 13) $I_E(w, h_E') = wLf(h_E)/Nn(h_E).$

(2. 14) $I_d^*(w) = L^* f^*(h_d^*)/N^* n^*(h_d^*).$

(2. 15)
$$L = \int_0^{hE} (az + b) Nn(h)\, dh + \int_{hd}^1 (az + b) Nn(h)\, dh + \int_{hd*}^1 (az^* + b) N^* n^*(h)\, dh.$$

It should be noted that $\varepsilon_{wE} = \partial \log I_E / \partial \log w < 1$ and $\varepsilon = \partial \log I_E / \partial \log h_E' > 0$, because $I_E' > I_E$.[13] When quota is imposed, individuals with income less than I_E turn their demands for low quality goods domestically supplied. Labor demand is increased in the North and the wage rate is raised. Then since I_d and I_d^* both increase at a higher rate than a wage increase, both h_d and h_d^* increase. Now that some of those individuals who have purchased high quality goods now shift their demands for imported low quality goods, quota from equation (2. 12) implies an increase in h_E. Since $\varepsilon_{wE} < 1$ and h_E increases, h_E' must increase.

2. 3 E*-Quota Policy

In this case, the South does not allow the importation of high quality goods, whose qualities are higher than z_E^*. This policy may be exercised in order to protect domestic firms producing high quality goods. Individuals with income $I_E'^*$ have the option of choosing either point D or E. We can divide income classes into four categories.

(1) $h \in [0, h_d^*]$

Individuals optimally purchase low quality goods domestically produced.

(2) $h \in [h_d^*, h_E'^*]$

Individuals optimally consume imported high quality goods.

(3) $h \in [h_E'^*, h_E^*]$

Individuals continue to purchase importables with quality $z_E'^*$ and spend the rest to purchase homogeneous goods.

(4) $h \in [h_E^*, 1]$

Individuals switch their demand for high quality goods domestically produced. Equilibrium conditions will be stated by the following five equations.

(2. 16) $I_d(w) = wLf(h_d)/Nn(h_d).$

(2. 17) $I_d^*(w) = L^*f^*(h_d^*)/N^*n^*(h_d^*).$

(2. 18) $I_E^*(w, h_E'^*) = L^*f^*(h_E^*)/N^*n^*(h_E^*).$

(2. 19) $X_E^* = \int_{hd*}^{hE*} N^*n^*(h)\,dh.$

(2. 20) $$L = \int_{hd}^{1} (az + b) Nn(h)\,dh + \int_{hd*}^{hE'*} (az^* + b) N^*n^*(h)\,dh + \int_{hE'*}^{hE*} (az_E'^* + b) N^*n^*(h)\,dh...$$

Since a rise in the wage rate means only an increase in the price of differentiated goods, the critical income level in the South is decreased. Let us define the following elasticities.

$$\varepsilon_{wE} = -\partial \log I_E^*/\partial \log w > 0 \text{ and } \varepsilon_{hE}'^* = \partial \log I_E^*/\partial \log h_E'^* > 0.$$

Since quota policy determined by the South turns a part of the demand for imported high quality goods to domestic ones, labor demand in the North will be reduced and the wage rate decreased.[14] Then $\varepsilon_{wd} > 0$ implies from equation (2. 16) that a decrease in the wage rate results in a decrease in h_d. Similarly, h_d^* also declines. Since marginal consumers can now purchase imported differentiated goods because of declines in the price of exportables from the North, quota necessarily means a decrease in h_E^*. These changes in w and h_E^* necessarily bring about a decrease in $h_E'^*$.

2.4 S*-Quota Policy

The South allows imports of high quality goods with qualities beyond a certain level, say $z_s'^*$. This policy intends to expand the share among domestic firms. We group income classes by the following three.

(1) $h \in [0, h_s^*]$.

Individuals purchase an optimal choice of homogeneous goods and low quality goods domestically produced.

(2) $h \in [h_s^*, h_s'^*]$

Individuals continue purchasing one unit of high quality goods with quality $z_s'^*$ and spend the rest for homogeneous goods.

(3) $h \in [h_s'^*, 1]$

Individuals optimally choose one unit of imported high quality goods.

Equilibrium conditions will be stated by equations from (2.21) to (2.24).

(2.21) $$I_d(w) = wLf(h_d)/Nn(h_d).$$

(2.22) $$I_s^*(w, h_s'^*) = L^*f^*(h_s^*)/N^*n^*(h_s^*).$$

(2.23) $$X_s^* = \int_{hs*}^{1} N^*n^*(h)\,dh.$$

(2.24) $$L = \int_{hd}^{1} (az + b) Nn(h)\,dh + \int_{hs*}^{hs'^*} (az_s'^* + b) N^*n^*(h)\,dh + \int_{hs*}^{1} (az^* + b) N^*n^*(h)\,dh.$$

The imposition of quota means a decrease in the number of invididuals who consume high quality goods. Equation (2.23) reveals that only higher income classes can afford to do this. Furthermore, since quota reduces the demand for importables, there will be an excess supply in the North's labor market, which decreases both the wage rate and the critical income class, h_d. These changes also imply an increase in $h_s'^*$.

3. Effects on Social Welfare

In this section, we assume the Bergson-Samuelson type social welfare function, where individual utilities are contents of the social welfare function. Evaluating them is always problematic in public choice. We simply assume here that social contribution of each individual in a particular income class is evaluated by the inverse of the marginal utility income of that income class, which reveals the state

of Pareto efficiency (see H.Varian (1978)).

When the policy of quota is exercised, its effects on social welfare generally consist of four parts.

(1) A change in the (relative) wage rate

Since our model is basically Ricardian, a change in the wage rate is regarded as a change in the terms of trade. We call this variation the terms of trade effect (expressed as E_1). (See R. Dornbusch, et al. (1977).)

(2) A change in either h_d or h_d^*

This change implies that marginal consumers optimally substitute high quality goods for low and vice versa. We call this the substitution effect and express it as E_2.

(3) A change in either h_Q or h_Q^* ($Q = E$ or S)

This change means that marginal consumers in this income class switch under constraints their demands from imported goods to domestic goods and vice versa. We call this the diversion effect and express it as E_3.

(4) A change in either h_Q' or $h_Q'^*$ ($Q = E$ or S)

Even if individuals of all income classes except h_d or h_d^* optimally purchase either importables or domestic goods, there are clear differences in utility levels. These differences force some of them to stick to purchasing restrained quality goods. We call this change the adjustment effect and express it as E_4.

Tables 1 and 2 respectively show signs of these effects on both northern and southern social welfare. Positive (negative) signs indicate that a quota policy increases (correspondingly decreases) welfare. Two tables show that the terms of trade effects play an anticipated role. On the one hand, a quota-enforcing country always increases its social welfare. On the other hand, a partner country always loses its social welfare. As for the adjustment effect, it always reduces social welfare, because imposing quota means that more individuals abandon their optimal choice and are forced to over-consume or under-consume homogeneous goods. In case of the diversion effect, there is a clear difference whether a country will exercise its quota policy or not. When the North does it, it suffers welfare losses. In case of the South, it enjoys welfare gains. Finally, both the terms of trade and the substitution effect move in the same direction in the North, but opposite in the South.[15]

Table 1. Changes in Northern Social Welfare

	S-Quota	E-Quota	E*-Quota	S*-Quota
E_1	+	+	−	−
E_2	0	+	−	−
E_3	−	−	0	0
E_4	−	−	0	0

Table 2. Changes in Southern Social Welfare

	S-Quota	E-Quota	E*-Quota	S*-Quota
E_1	−	−	+	+
E_2	+	+	−	0
E_3	0	0	+	+
E_4	0	0	−	−

4. Comparison of Policy Effects

In the previous sections, we have investigated several qualitative aspects of different quota policies. In this section, we want to answer the following quantitative question. Suppose the North can restrict the number of importables. Which is better from a welfare point of view — choosing an S-quota policy or an E-quota policy? First, consider the terms of trade effect. Even if the North imports the same number of low quality products from the South, taking an S-policy rather than an E-policy, and promotes the production of relatively higher quality products among low quality products in the North, which requires relatively larger demand for labor, an increase in the wage rate will be greater in S-quota than in E-quota. Since a change in the wage rate causes the terms of trade effect, this has an influence over trade activities, E_X.[16] Second, consider the behavior of marginal consumers, who are indifferent either to purchasing low or high quality products (see equation (1. 8) and (2. 2)). Suppose their income is I_d. (We can also choose either I_E' or I_S'.) Even if the level of their utility, whether to purchase low or high quality products, is the same, the level of their disposable income is different. When they switch their demand for differentiated products from low quality to high (S-quota), a change in disposable income is equal to $(I_d-bw)-(I_d-b^*)=b^*-bw$, which is negative. Contrary to this, when we take the E-quota policy, marginal consumers who belong to both income classes h_d and h_E' shift their demand from high to low. The signs of these effects crucially depends upon technological differences between the North and the South. Finally, the diversion effect always creates forced consumption for homogeneous goods in either case, which negatively affects social welfare. However, this effect will be negligible when we evaluate changes at the free trade level. It is seen that either policy has the same qualitative effect on southern social welfare. An increase in the wage rate deteriorates social welfare through the terms of trade effect. However, marginal consumers whose income is I_d^* switch their purchases from high to low quality products, which results in the South's positive welfare gain. Since the terms of trade effect affects the volume of trade ($E_X=I_M^*$), we would generally expect a welfare loss in the South. If the terms of trade effect is a dominant factor, the South suffers more when the North takes the S-quota policy rather than the E-policy. In this sense we understand the reason why the South voluntarily restricts its exports with relatively higher quality. Next, consider that the South imposes quota on importables. When it takes the E*-quota policy

rather than the S*-quota policy, its effect on the (northern) wage rate is larger than that in the S*-quota case. However, because of a decline in the price of exportables from the North, marginal consumers at income classes h_d^*, $h_s'^*$, and $h_E'^*$ suffer welfare losses. The diversion effect works favorably for social welfare in both cases. As for northern welfare, either policy taken by the South definitely deteriorates it. It is seen that terms of the trade effect negatively work against northern welfare. In addition, marginal consumers at income class h_d now are forced to consume low quality products domestically produced, which creates the other type of welfare losses. Since wage changes are larger in the E*-quota case than in the S*-quota case, the North prefers the South to perform the S*-quota policy. That is, the North has a strong incentive to protect domestic sectors in order to produce higher quality products.(17)

5. Concluding Remarks

We have presented several quota cum quality control policies and shown that impacts of quotas are not uniform over income classes. Generally speaking, there are many conflicts of interest among individuals as to which income class they belong. While the above problem is assumed away in some sense under the state of Pareto efficiency, we have derived several interesting results.

(1) The terms of trade effect always raises welfare in a quota- imposing country and deteriorates the welfare of a trade partner.

(2) Since the terms of trade effect is closely related to trade activities (E_X or I_M^* in our notation), we expect this effect to play a dominant role for explaining changes in social welfare. The same reasoning is essentially held by traditional analysis.

(3) When the South exercises a quota policy irrespective of either E*-quota or S*-quota, the North unambiguously suffers welfare losses.

(4) From the standpoint of the South, the incentive is to voluntarily take a VER policy, which has the same effect as an E-quota policy by the North.(18)

(5) From the standpoint of the North, it is preferable for the South to take an S*-quota policy, rather than an E*-quota policy. The North has a strong motivation in blaming the South for protecting domestic sectors that produce high quality products, which have comparative cost disadvantages. In this case, it should be noted that protected sectors are not infant industries.

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Footnotes

(1) In case of the so-called horizontal product differentiation, there are several cases of

research on record. See A.J. Venables (1987) and H. Flam and E. Helpman (1987b).

(2) It is argued that the equivalence between tariff and quota breaks down when monopoly, for instance, is introduced into the traditional framework. See J. Bhagwati (1968).

(3) Since a representative consumer is safely assumed in case of horizontal product differentiation, there are several trade policies complement to traditional analysis. See, for instance, H. Flam and E. Helpman (1987b) and A.J. Venables (1987).

(4) As for general guidance on the analysis of VER, see J. Bhagwati (1977). For empirical analysis, see B.Y. Aw. and M.J. Roberts (1986).

(5) We leave most of our computations in Mathematical Notes. While we assume every individual consumes one unit of differentiated products (the Hotelling's assumption), we relax this in Mathematical Note D and touch upon the strong possibility that quota may hurt a quota-imposing country. Mathematical Notes are available on reader's request.

(6) R.E. Falvey and H. Kierzkowski (1987) also presented a model of vertical product differentiation under the same spirit of the FH model. However, their model assumes two factors of production, labor and capital.

(7) In case of FH, they assume the exponential form, while ours is linear.

(8) The Cobb-Douglas type of utility functions satisfies the Gorman form and is used in avoiding complications in aggregation. See also the Falvey-Kierzkowski model, in which they assume the Stone-Geary form.

(9) Since FH (1987a) also assumes an entry fee, $\varepsilon_{wd}>1$ in their model. (In their notation, entry fees are equal to $1/A$ and $1/A^*$.) In fact, $\varepsilon_{wd}>1$ implies signs of change in both w, h_d, and h_d^* are the same.

(10) In case of both free trade and optimum tariff, we have shown that we could employ two types of indirect utility function for representative consumers by using a typical aggregation procedure (see H. Ono (1990)).

(11) We can add the equilibrium condition in the South's labor market.

$$L^* = \int_0^{hs'} (a^*z + b^*)\, Nn(h)\, dh + \int_{hs'}^{hs} (a^*z_s' + b^*)\, Nn(h)\, dh$$

$$\int_0^{hd*} (a^*z^* + b^*)\, N^*n^*(h)\, dh + L_y^*.$$

Since both z_s, z_s', and z^* are functions of w and/or h_s', adding one equation supplies one more variable, L_y^*. Therefore, we omit this equation. Furthermore, the equilibria in e labor markets both in the North and the South imply equilibria in both the commodity markets and in trade balance. Refer to FH (1987a).

(12) We can compute ε_{ws} as B/A,

where

$$A = I_s/(I_s - bw) - \alpha I_s/[I_s - (a^* z_s' + b^*)] > 0 \text{ and}$$

$$B = \beta - [\alpha a^* z_s'/(I_s - (a^* z_s' + b^*))(I_s' - b^*)] + (\beta I_s + \alpha bw)/(I_s - bw)$$

Then

$$\varepsilon_{ws} - 1 = (B - A)/A = \{[\beta I_s'(I_s - I_s')/(I_s - (a^* z_s' + b^*))(I_s' - B^*)] + \alpha(a^* Z_s' + b^*)/(I_s - (a^* z_s' + b^*))\}/A > 0.$$

(13) We calculate $\varepsilon'_{ws} = B/A$ and $\varepsilon'_{hE} = C/A$,

where

$$A = \alpha I_E/(I_E - (a^* z_E' + b^*)) - I_E/(I_E - bw),$$

$$B = [\alpha a^* z_E'/(I_E - (a^* z_E' + b^*)) - \beta] I_E'/(I_E' - b^*) - [\beta + bw/(I_E - bw)],$$

and

$$C = [\alpha a^* z_E'/(I_E - (a^* z_E' + b^*))] - \beta.$$

Now we respectively compute A and C as follows:

$$A = I_E[p(z_E') - p(z_E)]/(I_E - (a^* z_E' + b^*))(I_E - bw) > 0, \text{ and}$$

$$C = \beta(I_E' - I_E)/(I_E - (a^* z_E' + b^*)) > 0.$$

Note also that

$$\varepsilon'_{wE} - 1 = -\{\beta I_E'(I_E - \alpha b^*) + \alpha^2 b^*(I_E' - b^*)\}/A(I_E - (a^* z_E' + b^*))(I_E' - b^*) < 0.$$

(14) We assume stability in the labor market. Using equations (2.16) to (2.19), we express h_d, h_d^*, h_E^*, and $h_E'^*$, as functions of w. Then substituting these relations into equation (2.20), we derive the stability condition in thelabor market.

(15) Computational results are obtained on request.

(16) For detailed analysis, see Mathematical Note C.

(17) Our argument is not directly related to the infant industry argument. In our case, protected sectors have comparative cost disadvantage.

(18) We suppose the North and the South respectively to be the U.S. and Japan. Then Japanese VER in automobiles may be explained by using our model. R.C. Feenstra (1989) pointed out quality shifts by Japanese exports. See also R.M. Crandall (1987) and A. Dixit (1987).

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Mathematical Note A. Comparative Static Results

We use the following expression on elasticities below.

$$\varepsilon_Q = d \log [f(h_Q)/n(h_Q)]/d \log h_Q, \qquad Q = d, E, S.$$

$$\varepsilon_Q = d \log [f^*(h_Q^*)/n^*(h_Q^*)]/d \log h_Q^*, \qquad Q = d, E, S.$$

$$\varepsilon_{wQ} = \partial \log I_Q/\partial \log w, \qquad Q = d, E, S.$$

$$\varepsilon_{wQ}^* = -\partial \log I_Q^*/\partial \log w, \qquad Q = E, S.$$

$$\varepsilon_{wd}^* = \partial \log I_d^*/\partial \log w.$$

We have stated equilibrium conditions in the paper. Performing the method of comparative statics, we derive the following computational results. "Caret" notation below shows proportional changes in variables.

(A-1) Case of S-quota

$$\hat{h}_s/\hat{X}_s = 1/a_1 > 0.$$

$$\hat{h}_s'/X_s = \{\varepsilon_s \Delta_s + b_1 \varepsilon_d^* (\varepsilon_{ws} - 1)\}/a_1 \varepsilon_{hs}' \Delta_s > 0.$$

$$\hat{h}_d^*/\hat{X}_s = (\varepsilon_{wd}^*/\varepsilon_d^*)(w/X_s) < 0.$$

$$\hat{w}/\hat{X}_s = -(b_1/a_1)(\varepsilon_d^*/\Delta_s) < 0,$$

where $\Delta_s = b_3 \varepsilon_d^* + b_2 \varepsilon_{wd}^*$, $a_1 = Nn(h_s) h_s/X_s$,

$$b_1 = [\beta I_s + \alpha b w] Nn(h_s) h_s/X_s = p(z_s) Nn(h_s) h_s/w,$$

$$b_2 = p(z_d^*) N^* n^*(h_d^*) h_d^*/w, \quad b_3 = \int_{hd*}^{1} [\beta L^* f^*(h)/w]\, dh.$$

Note that $b_1/a_1 = p(z_s) X_s/w$.

(A-2) Case of E-quota

$$\hat{h}_d/\hat{X}_E = [(\varepsilon_{wd} - 1)/\varepsilon_d](w/X_E)/a_2' \varepsilon_d \Delta_E.$$

$$\hat{h}_E/\hat{X}_E = -[b_4' \varepsilon_d \varepsilon_d^* + b_3' \varepsilon_d \varepsilon_{wd}^* + b_1' \varepsilon_d^* (\varepsilon_{wd} - 1)]/a_2' \varepsilon_d \Delta_E.$$

$$\hat{h}_E'/\hat{X}_E = -\{\varepsilon_d [b_4' \varepsilon_E \varepsilon_d^* + b_2' \varepsilon_d^* (1 - \varepsilon_{wE}) + b_3' \varepsilon_E \varepsilon_{wd}^*] + b_1' \varepsilon_E \varepsilon_d^* (\varepsilon_{wd} - 1)\}/a_2' \varepsilon_d \Delta_E.$$

$$\hat{h}_d^*/\hat{X}_E = (\varepsilon_{wd}^*/\varepsilon_d^*)(w/X_E).$$

$$\hat{w}/\hat{X}_E = -(b_2'/a_2')(\varepsilon_d^*/\Delta_E),$$

where

$$\Delta_E = b_4'\varepsilon_d^* + b_3'\varepsilon_{wd}^* + (\varepsilon_d^*/\varepsilon_d)(\varepsilon_{wd} - 1)[b_1' - (a_1'/a_2')b_2'],$$

$$a_1' = Nn(h_d)\, h_d/X_E,$$

$$a_2' = Nn(h_E)\, h_E/X_E,$$

$$b_1' = p(z_d)\, Nn(h_d)\, h_d/w,$$

$$b_2' = p(z_E)\, Nn(h_E)\, h_E/w,$$

$$b_3' = p(z_d^*)\, N^* n^*(h_d^*)\, h_d^*/w,$$

$$b_4' = \int_{hd}^{1^*} [\beta L^* f^*(h)/w]\, dh.$$

Note that $b_1' - (a_1'/a_2')\, b_2' = \{p(z_d) - p(z_E)\}\, Nn(h_d)\, h_d/w > 0$.
Stability in the labor market quarantees $\Delta_E > 0$.
Therefore,

$$\hat{w}/\hat{X}_E < 0, \quad \hat{h}_d/\hat{X}_E > 0, \quad \hat{h}_E/\hat{X}_E > 0, \quad \hat{h}_E'/\hat{X}_E > 0, \quad \hat{h}_d^*/\hat{X}_E > 0.$$

(A–3) Case of E*-quota

$$\hat{h}_d/\hat{X}_E^* = [(\varepsilon_{wd} - 1)/\varepsilon_d]\,(w/X_E^*).$$

$$\hat{h}_d^*/\hat{X}_E^* = \varepsilon_{wd}^*\varepsilon_d[b_4^*\varepsilon_E^* + b_3^*\varepsilon_{hE}'^*]/\Delta_E^*.$$

$$\hat{h}_E^*/\hat{X}_E^* = \{\varepsilon_d[b_5^*\varepsilon_d^*\varepsilon_{hE}'^* + b_2^*\varepsilon_{hE}'^*\varepsilon_{wd}^* - b_4^*\varepsilon_d^*\varepsilon_{wE}^*] + b_1^*\varepsilon_d^*\varepsilon_{hE}'^*(\varepsilon_{wd} - 1)\}/\Delta_E^*.$$

$$\hat{h}_E'^*/\hat{X}_E^* = \{b_5^*\varepsilon_d^*\varepsilon_E^* + b_2^*\varepsilon_E^*\varepsilon_{wd}^* + b_3^*\varepsilon_d^*\varepsilon_{wE}^* + b_1^*(\varepsilon_{wd} - 1)\,\varepsilon_d^*\varepsilon_E^*\}/\Delta_E^*.$$

$$\hat{w}/\hat{X}_E^* = \varepsilon_d\varepsilon_d^*[b_4^*\varepsilon_E^* + b_3^*\varepsilon_{hE}'^*]/\Delta_E^*,$$

where

$$\Delta_E^* = \varepsilon_d\{\varepsilon_d^*[a_1^*b_5^*\varepsilon_{hE}'^* - a_1^*b_4^*\varepsilon_{wE}^* + \varepsilon_{wd}^*[a_1^*b_2^*\varepsilon_{hE}'^* - a_2^*b_3^*\varepsilon_{hE}'^* - a_1^*b_2^*\varepsilon_{hE}'^*] + (1 - \varepsilon_{wd})\, a_1^*b_1^*\varepsilon_d^*\varepsilon_{hE}'^*,$$

$$a_1^* = N^* n^* (h_E^*)\, h_E^*/h_E^*,$$

$$a_2^* = N^* n^* (h_d^*)\, h_d^*/w,$$

$$b_1^* = p (z_d)\, Nn (h_d)\, h_d/w,$$

$$b_2^* = p (z_d^*)\, N^* n^* (^*_{hd})\, h_d^*/w,$$

$$b_3^* = p (z_E'^*)\, N^* n^* (h_E^*)\, h_E^*/w,$$

$$b_4^* = (\beta \varepsilon_E'^* I_E'^*/w) \int_{hE}^{hE*'*} N^* n^* (h)\, dh,$$

$$b_5^* = (\beta/w) \{\int_{hd}^{hE'*} I^* N^* n^* (h)\, dh + \int_{hE}^{hE*'*} I_E' N^* n^* (h)\, dh\}.$$

Stability in the labor market implies $\Delta_E^* > 0$. That is,

$$\hat{h}_d/\hat{X}_E^* > 0,\ \hat{h}_d^*/\hat{X}_E^* > 0,\ \hat{h}_E^*/\hat{X}_E^* > 0,\ \hat{h}_E'^*/\hat{X}_E^* > 0,\ \hat{w}/\hat{X}_E^* > 0.$$

(A-4) Case of S*-quota

$$\hat{h}_d/\hat{X}_s^* = [(\varepsilon_{wd} - 1)/\varepsilon_d]\, (\hat{w}/\hat{X}_s^*)$$

$$\hat{h}_s^*/\hat{X}_s^* = -1/a_1'^*.$$

$$\hat{h}_s'^*/\hat{X}_s^* = -\{b_4'^* \varepsilon_d \varepsilon_s^* + b_1'^* \varepsilon_s^* (\varepsilon_{wd} - 1) + b_2'^* \varepsilon_{ws}^* \varepsilon_d\}/\Delta_s^*.$$

$$\hat{w}/\hat{X}_s^* = \varepsilon_d [b_2'^* \varepsilon_{hs}'^* - b_3'^* \varepsilon_s^*]/\Delta_s^*,$$

where

$$\Delta_s^* = a_1'^* \{b_4'^* \varepsilon_d \varepsilon_{hs}^* + b_1'^* \varepsilon_{hs}'^* (\varepsilon_{wd} - 1) + b_3'^* \varepsilon_d \varepsilon_{wd}^*\},$$

$$a_{17}^* = N^* n^* (h_s^*)\, h_s^*/X_s^*,$$

$$b_1'^* = p (z_d)\, Nn (h_d)\, h_d/w,$$

$$b_2'^* = p (z_s'^*)\, N^* n^* (h_s^*)\, h_s^*/w,$$

$$b_3'^* = (\beta \varepsilon_s'^* I_s'^*/w) \int_{hs*}^{hs'*} N^* n^* (h)\, dh,$$

$$b_4'^* = (\beta/w) \int_{hs*}^{hs'*} I_s'^* N^* n^* (h)\, dh + \int_{hs'*}^{1} I^* N^* n^* (h)\, dh\}.$$

Assuming stability in the labor market, we have $\Delta_s^* > 0$. This implies;

$$\hat{w}/\hat{X}_s^* > 0, \quad \hat{h}_d/\hat{X}_s^* > 0, \quad \hat{h}_s^*/\hat{X}_s^* < 0, \quad \hat{h}_s'^*/\hat{X}_s^* < 0.$$

Mathematical Note B. Changes in Social Welfares

(B-1) Case of S-quota

According to the division of income classes in section 2, the social welfare function in the North consists of the following three parts.

$$W = W_1 + W_2 + W_3,$$

where $w_1 = \int_0^{hs'} a_h u_h Nn(h)\, dh$, $w_2 = \int_{hs'}^{hs} a_h u_h Nn(h)\, dh$, and $w_3 = \int_{hs}^{1} a_h u_h Nn(h)\, dh$. a_h shows the social contribution of an individual in income class h, which should be measured, under the state of Pareto efficiency, by the inverse of the marginal utility of income. Let us derive the marginal utility of income in each income class.

(1) $h \in [0, h_s']$

Individual optimization is to choose y and z so as to maximize the following Lagrangian.

$$L_h = (y_h)^\alpha (z_h)^\beta + \lambda_h [I_h - y_h - (a^* z_h + b^*)].$$

The first order condition supplies λ_h.

$$\lambda_h = 1/a_h = \alpha (u_h/y_h).$$

Using the envelope theorem, we have the following relation.

$$du_h/dX_s = \partial L/\partial X_s = \lambda_h [\partial I_h/\partial X_s] = (\lambda_h I_h/X_s)(\hat{w}/\hat{X}_s).$$

(2) $h \in [H_s', h_s]$

In this case the Lagrangian is given by the following equation.

$$L_h = (y_h)^\alpha (z_s')^\beta + \lambda_h [I_h - y_h - (a^* z_s' + b^*)].$$

Since the form of λ_h is the same as above, we only compute the results from applying the envelope theorem below.

$$du_h/dX_s = (\lambda_h/X_s)\{[I_h + ((\beta y_h/\alpha z_s') - a^*)(\beta I_s'/a^*)](w/X_s) + [(\beta y_h/\alpha z_s') - a^*](\beta I_s'/a^*)\varepsilon_s'(\hat{h}_s'/\hat{X}_s).$$

In deriving this, we have used the following relations.

$$z_s' = \beta (I_s' - b^*)/a^*, \text{ where } I_s' = wLf(h_s')/Nn(h_s').$$

Note that $(\beta y_h/az_s') - a^* > (\beta y_s'/az_s') - a^* = 0$.

(3) $h \in [h_s, 1]$

The Lagrangian is given as follows.

$$L_h = (y_h)^{\alpha}(z_h)^{\beta} + \lambda_h [I_h - y_h - (awz_h + bw)].$$

We similarly compute the following.

$$du_h/dX_s = \lambda_h \{(\partial I/\partial X_s) - (az_h + b)(\partial w/\partial X_s)\}$$
$$= (\lambda_h/X_s)\{I_h - (awz_h + bw)\}(\hat{w}/\hat{X}_s)$$

Employing the above results, we derive the effects of social welfare due to S-quota as follows.

$$X_s(dW/dX_s) = A_1(\hat{w}/\hat{X}_s) + A_2(\hat{h}_s/\hat{X}_s) + A_3(\hat{h}_s'/\hat{X}_s),$$

where

$$A_1 = E_X + \int_{hs'}^{hs} [(\beta y_h/\alpha z_s') - a^*](\beta I_s'/a^*) Nn(h)\, dh > E_X,$$

$$A_2 = [(1-\alpha)(I_s - I_s') + \alpha(bw - b^*)/\alpha > 0, \text{ and}$$

$$A_3 = \varepsilon_s'(\beta I_s'/a^*)\{\int_{hs'}^{hs} [\beta y_h/\alpha z_s') - a^*] Nn(h)\, dh\}.$$

As for the social welfare of the South, we divide two parts.

$$W^* = W_1^* + W_2^*,$$

where

$$W_1^* = \int_0^{hd^*} a_h^* u_h^* N^* n^*(h)\, dh, \text{ and } W_2^* = \int_{hd^*}^1 a_h^* u_h^* N^* n^*(h)\, dh.$$

By using the same procedures taken in the above, we derive the following:

$$\lambda_h = u_h^*/(I_h^* - b^*).$$

When $h \epsilon [0, h_d^*]$, then $du_h^*/dX_s = 0$. In case of $h \epsilon [h_d^*, 1]$,

$$du_h^*/dX_s = -(\lambda_h^*/X_s)(awz_h^* + bw)(\hat{w}/\hat{X}_s).$$

The following equation gives welfare changes in the South.

$$X_s(dw^*/dX_s) = A_1^*(\hat{w}/\hat{X}_s) + A_2^*(\hat{h}_d^*/\hat{X}_s),$$

where

$$A_1^* = -I_M^*, \text{ and } A_2^* = (bw - b^*) N^* n^*(h_d^*) h_d^* > 0, \text{ because of (C1).}$$

(B-2) Case of E-quota

Social welfare in the North may be divided by three parts. In order to calculate social changes in each component, we should measure both the marginal utility of income and changes in individual utility levels by quota.

(1) $h\epsilon\ [0,\ h_E]^{\cup}\ [h_d,\ 1]$

In this case individuals consume differentiated products domestically produced. We form the following Lagrangian.

$$L_h = (y_h)^{\alpha}\,(Z_h)^{\beta} + \lambda_h\,[I_h - y_h - (awz_h + b)].$$

The first order condition yields the value of λ_h as follows.

$$\lambda_h = \alpha u_h / y_h.$$

The marginal utility of income, λ_h, takes the above form for individuals in any income classes. The enveope theorem gives a change in individual utility as follows.

$$X_E\,(du_h/dX_E) = \lambda_h\,\{I_h - (awz_h + bw)\}\,(\hat{w}/\hat{X}_E).$$

(2) $h\epsilon\ [h_E,\ h_E']$

In this case individuals stick to consume Z_E'. Therefore, the Lagrangian will be given by the following.

$$L_h = (y_h)^{\alpha}\,(z_E')^{\beta} + \lambda_h\,[I_h - y_h - (a^* z_E' + b^*)].$$

We compute a change in individual utility as follows.

$$X_E\,(du_h/dX_E) = (\beta u_h/z_E')\,(\partial z_E'/\partial X_s) + \lambda_h\,[I_h\,(w/X_E) - a^*\,(\partial z_E'/\partial X_E)].$$

Since $z_E' = \beta\,[I_E' - b^*]/a^*$, we finally obtain:

$$\begin{aligned} X_E\,(du_h/dX_E) = {} & \lambda_h\,\{[I_h + ((\beta y_h/\alpha z_E') - a^*)\,(z_E' I_E')/(I_E' - b^*)]\,(\hat{w}/\hat{X}_E) \\ & + [(\beta y_h/\alpha z_E') - a^*]\,[(z_E' I_E')/(I_E' - b^*)]\,\epsilon_E'\,(\hat{h}_E'/\hat{X}_E). \end{aligned}$$

It should be noted that $(\beta y_h/\alpha z_E') - a^* < (\beta y_E'/\alpha z_E') - a^* = 0$.

(3) $h\epsilon\ [h_E',\ h_d]$

We form the following Lagrangian.

$$L = (y_h)^{\alpha}\,(z_h)^{\beta} + \lambda_h\,[I_h - y_h - (a^* z_h + b^*)].$$

We easily obtain the following.

$$X_E\,(du_h/dX_E) = \lambda_h I_h\,(\hat{w}/\hat{X}_E).$$

The welfare changes in the North will be computed as follows.

$$X_E\,(dW/dX_E) = B_1\,(\hat{w}/\hat{X}_E) + B_2\,(\hat{h}_d/\hat{X}_E) + B_3\,(\hat{h}_E/\hat{X}_E) + B_4\,(\hat{h}_E'/\hat{X}_E),$$

where

$$B_1 = E_X - [\int_{hE}^{hE'} (I_E' - I_h)\,Nn\,(h)\,dh]\,(\beta I_E'/(I_E' - b^*)) < E_X,$$

$$B_2 = (bw - b^*)\, Nn\,(h_d)\, h_d,$$

$$B_3 = \{(I_E - bw) - [I_E - (a^* z_E' + b^*)]/\alpha\}\, Nn\,(h_E)\, h_E,$$

and $B_4 = \varepsilon_E' \{\int_{hE}^{hE'} [(\beta y_h/\alpha z_E') - a^*]\, Nn\,(h)\, dh\}\, (z_E' I_E')/(I_E' - b^*)$.

It should be noted that

$$\alpha\,(I_E - bw) - [I_E - (a^* z_E' + b^*)] = p\,(z_E') - p\,(z_E) > 0.$$

As for the case of welfare changes in the South, we basically have the same form as in case of S-quota. That is,

$$X_E\,(dW^*/dX_E) = A_1^*\,(\hat{w}/\hat{X}_E) + A_2^*\,(\hat{h}_d^*/\hat{X}_E).$$

(B-3) Case of E*-quota

The social welfare in the North is given by $W = W_1 + W_2$,
where

$$W_1 = \int_0^{hd} a_h u_h Nn\,(h)\, dh, \text{ and } W_2 = \int_{hd}^1 a_h u_h Nn\,(h)\, dh.$$

We derive the following.

$$X_E^*\,(dW/dX_E^*) = C_1\,(\hat{w}/\hat{X}_E^*) + C_2\,(\hat{h}_d/\hat{X}_E^*),$$

where

$$C_1 = E_X \text{ and } C_2 = (bw - b^*)\, Nn\,(h_d)\, h_d > 0.$$

In case of social welfare in the South, we have the following.

$$W^* = W_1^* + W_2^* + W_3^* + W_4^*,$$

$$W_1^* = \int_0^{hd*} a_h^* u_h^* N^* n^*\,(h)\, dh,$$

$$W_2^* = \int_{h*d}^{h*E'} a_h^* u_h^* N^* n^*\,(h)\, dh,$$

$$W_3^* = \int_{h*E}^{h*E} a_h^* u_h^* N^* n^*\,(h)\, dh, \qquad \text{and}$$

$$W_4^* = \int_{h*E}^1 a_h^* u_h^* N^* n^*\,(h)\, dh,$$

$$X_E^*\,(dW^*/dX_E^*) = C_1^*\,(\hat{w}/\hat{X}_E^*) + C_2^*\,(\hat{h}_d/\hat{X}_E^*) + C_3^*\,(\hat{h}_E^*/\hat{X}_E^*) + C_4^*\,(\hat{h}_E'^*/\hat{X}_E^*),$$

where

$$C_1^* = -\{I_M^* + [\int_{hE'^*}^{hE^*} [\beta(I_h^* - (awz_E'^* + bw)/(\alpha z_E'^*) - aw] N^* n^*(h)\, dh] (z_E'^* I_E'^*/(I_E'^* - bw),$$

$$C_2^* = (bw - b^*) N^* n^*(h_d^*) h_d^*,$$

$$C_3^* = \{[I_E^* - (awz_E'^* + bw)]/\alpha - (I_E^* - b^*)\} N^* n^*(h_E^*) h_E^*,$$

and $$C_4^* = \{\int_{hE'^*}^{hE^*} [\beta(I_h^* - (awz_E'^* + bw))/(\alpha z_E'^*) - aw] N^* n^*(h)\, dh\} z_E'^* I_E'^* \varepsilon_E'^*/(I_E'^* - bw).$$

It should be noted that $\beta[I_h^* - (awz_E'^* + bw]/(\alpha z_E'^*) - aw > (\beta y_E'^*/\alpha z_E'^*) - aw = 0$.

(B-4) Case of S*-quota

In case of welfare changes in the North, we have the same form as in case of E*-quota.

As for the case of southern welfare changes, we may have the following:

$$X_s^*(dW^*/dX_s^*) = D_1^*(\hat{w}/\hat{X}_s^*) + D_2^*(\hat{h}_s^*/\hat{X}_s^*) + D_3^*(\hat{h}_s'^*/\hat{X}_s^*),$$

where

$$D_1^* = -\{I_M^* + [\int_{hs^*}^{hs'^*} [(\beta y_h^*/\alpha z_s'^*) - aw] N^* n^*(h)\, dh] [z_s'^* I_s'^*/(I_s'^* - bw)],$$

$$D_2^* = \{(I_s^* - b^*) - [I_s^* - (awz_s'^* + bw)/\alpha]\} N^* n^*(h_s^*) h_s^*, \text{ and}$$

$$D_3^* = \{\int_{hs^*}^{hs'^*} [(\beta y_h^*/\alpha z_s'^*) - aw] N^* n^*(h)\, dh\} [\varepsilon_s'^* z_s'^* I_s'^*/[\varepsilon_s'^* z_s'^*/(I_s'^* - bw)].$$

It should be noted that

$$(\beta y_h^*/\alpha z_s'^*) - aw < (\beta y_s'^*/\alpha z_s'^*) - aw = 0.$$

Mathematical Note C. Comparison of Quota Policies

Suppose that the North is free to choose either a S-quota policy or a E-quota policy, except that it restricts importables at the same quantity ($X_E = X_S$). First note that $b_4' = b_3$ and $b_3' = b_2$. Therefore,

$$\Delta_E - \Delta_S = (\varepsilon_d^*/\varepsilon_d)(\varepsilon_{wd} - 1)[b_1' - (a_1'/a_2') b_2'].$$

Since $\varepsilon_{wd} - 1 > 0$ and $b_1' - (a_1'/a_2') b_2' > 0$, $\Delta_E > \Delta_S$, which implies $|\hat{w}/\hat{X}_S| > |\hat{w}/\hat{X}_E|$ and $|\hat{h}_d^*/\hat{X}_S| > |\hat{h}_d^*/X_E^*|$. If we evaluate welfare changes at initial free trade ($h_E = h_E' = 0$ and $h_S = h_d = 0$), then

$$X_s\,(dW/dX_s) = E_X\,(\hat{w}/\hat{X}_s) + (bw - b^*)\,Nn\,(h_s)\,h_s\,(\hat{h}_s/\hat{X}_s)\,|_{hs=hd}.$$

and

$$X_E\,(dW/dX_E) = E_X\,(\hat{w}/\hat{x}_E) + (bw - b^*)\,\{Nn\,(h_d)\,h_d\,(h_d/X_E) - Nn\,(h_E)\,(dh_E/dX_E)\,|_{hE=0}\}.$$

As for southern welfare, a northern quota policy affects as follows.

$$X_Q\,(dW^*/dX_Q) = A_1^*\,(\hat{w}/\hat{X}_Q) + A_2^*\,(\hat{h}_d^*/\hat{X}_Q^*) \qquad (Q=S,\ E)$$
$$= [A_1^* + (\varepsilon_{wd}^*/\varepsilon_d^*)\,A_2^*]\,(\hat{w}/\hat{X}_Q).$$

Since $A_1^*+(\varepsilon_{wd}^*/\varepsilon_d^*)\,A_2^*>0$ probably, $dW/dX_s > dW/dX_E$.
A E-quota policy is preferable from the southern standpoint of view.

Next, consider that the South exercises either a S*-quota policy or a E*-quota policy. When we evaluate changes in variables at initial free trade, $h_E^*=h_E'^*=1$ and $h_s^*=h_s'^*=h_d^*$. That is, $b_4^*=0$ and $b_3'^*=0$. We have the following relations among variables.

$$b_2'^* = b_2^* \text{ and } b_4'^* = b_5^*.$$

Then

$$\Delta_2^* = a_1^*\varepsilon_{hE}'^*\,\{b_5^*\varepsilon_d + (\varepsilon_{wd} - 1)\,b_1^*\} > 0, \qquad \text{and}$$

$$\Delta_E^* = (\varepsilon h_E'^*/\varepsilon_{hs}'^*)\Delta_s^* - (\varepsilon_{hE}'^*\varepsilon_d\varepsilon_{wd}^*/\varepsilon_d^*) - [a_2^*b_3^* - a_1^*b_2^*] > 0.$$

Note that $a_2^*b_3^*-a_1^*b_2^*>0$. Therefore, $\Delta_s^*>\Delta_E^*$. Note also $p(z_E'^*)>p(z_s'^*)$. Since $(\hat{w}/\hat{X}_E^*)=[\varepsilon_d p\,(z_E'^*)\,X_E^*/w\Delta_E^*]$ and $(\hat{w}/\hat{X}_s^*)=[\varepsilon_d p\,(z_s'^*)\,X_s^*/w\Delta_s^*]$,

$$\hat{w}/\hat{X}_E^* > \hat{w}/\hat{X}_s^*.$$

This implies $X_E^*\,(dW/dX_E^*)>X_s^*\,(dW/dX_s^*)$

Mathematical Note D. Case Where Individuals Do Not Consume Differentiated Goods At All

In the paper we implicitly assumed that each individual consumes one unit of differentiated products. However, we may face a situation where individuals in some income class prefer to purchase only homogeneous goods. In particular, this may happen in case of E-quota. The imposition of E-quota forces individuals in some class to switch from importables to domestic products. In the transition, they stick to purchase differentiated goods of excessive quality. However, if the price of domestic goods is expensive to consume, then they may be forced to consume only homogeneous goods. Our formulation of utility functions implicitly assumes away

this possibility, because in this case individuals utility becomes zero. If we assume a Stone-Geary type of utility functions, we can avoid this difficulty. That is,

$$u_h = (y_h)^{\alpha} (z_h + C_0)^{\beta}.$$

This modifications will be carried out by similar procedures made in the paper. We respectively use terms $bw - awc_0$ and $b^* - a^* c_0$ instead of bw and b^*.

We may classify income classes as follows.

(1) $h \in [0, h_E]$

Individuals purchase only homogeneous goods.

(2) $h \in [h_E, h_E']$

Individuals purchase one unit of differentiated goods with quality z_E', together with homogeneous goods.

(3) $h \in [h_E', h_d]$

Individuals optimally choose a pair of homogeneous goods and one unit of imported differentiated goods.

(4) $h \in [h_d, 1]$

Individuals consume both one unit of differentiated products domestically produced and homogeneous goods.

In Figure I, points A and B are indifferent to individuals with income I_E. We expreses equilibrium conditions as following five equations.

(D1) $$I_d(w) = wLf(h_d)/Nn(h_d).$$

(D2) $$I_d^*(w) = L^* f^*(h_d^*)/N^* n^*(h_d^*).$$

(D3) $$X_E = \int_{hE}^{hd} Nn(h)\, dh.$$

(D4) $$I_E(w, h_E') = wLf(h_E)/Nn(h_E).$$

(D5) $$L = \int_{hd}^{1} (az + b) Nn(h)\, dh + \int_{hd*}^{1} (az^* + b) N^* n^*(h)\, dh.$$

Functional relation stated in the left-hand-side of equation (D4) are derived by the following equality.

$$[I_E - (a^* z_E' + b^*)]^{\alpha} (z_E' + C_0)^{\beta} = (I_E)^{\alpha} C_0.$$

We immediately find that equations (D1), (D2), (D3), and (D5) determine w, h_d, h_d^*, and h_E. Then h_E' will be obtained from equation (D4). While the imposition of quota by the North forces individuals there to consume (imported) homogeneous goods instead of imported differentiated goods, there is no change in the wage rate (no

terms of trade effect), and so are both h_d and h_d^*. The number of individuals who consume only homogeneous goods increases, which implies increases in both h_E and h_E'. The effect of E-quota on social welfare in the North is the same as that in the paper except there are no terms of trade effect and substitution effect. We have a negative adjustment effect, because quota imposition forces individuals in $[h_E, h_E']$ to keep differentiated goods with quality z_E', which are not optimal choice. On the contrary, we have a positive diversion effect, because a switch from low quality goods to homogeneous goods, even if that change is evaluated at the same utility level, brings more homogeneous goods and the social welfare is measured by income terms.

Finally, as for the social welfare in the South, there is no change at all.

Figure 1. Indirect Utility Functions

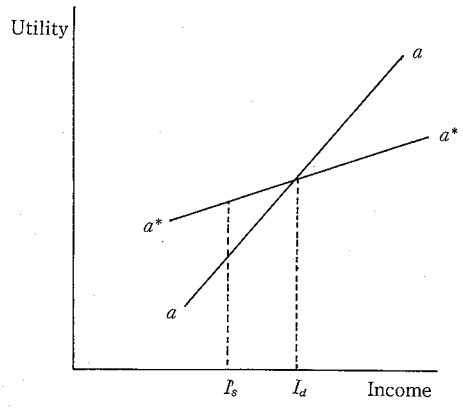

Figure 2. Free Trade Case

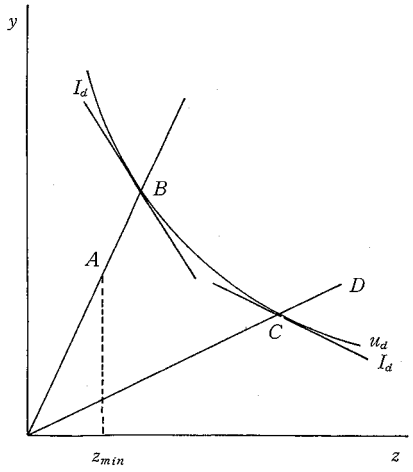

Figure 3. S-quota Case

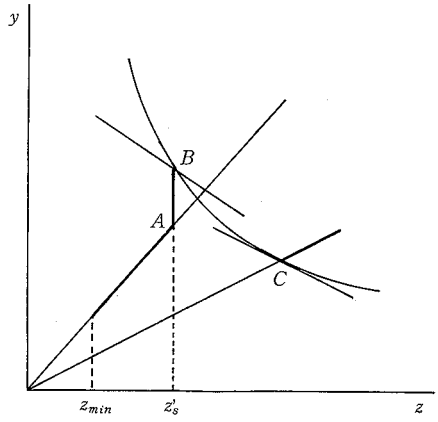

Figure 4. E-quota Case

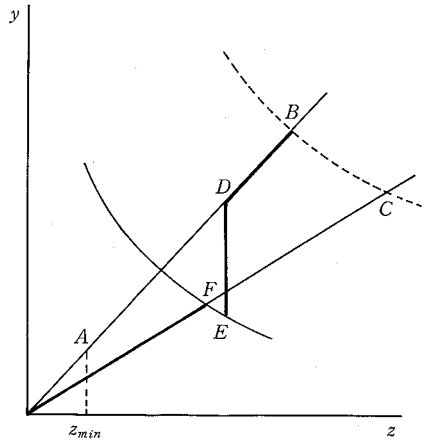

Figure 5. E*-quota Case

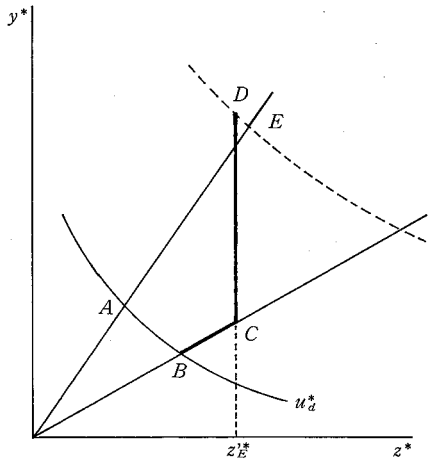

Figure 6. S*-quota Case

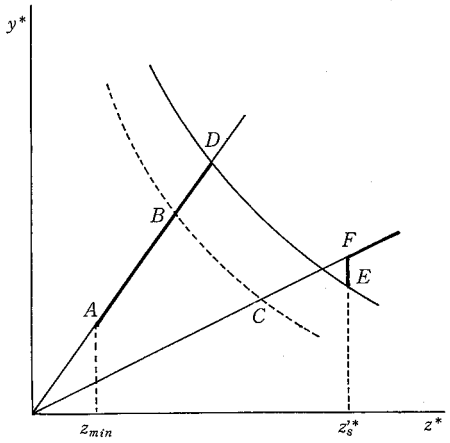
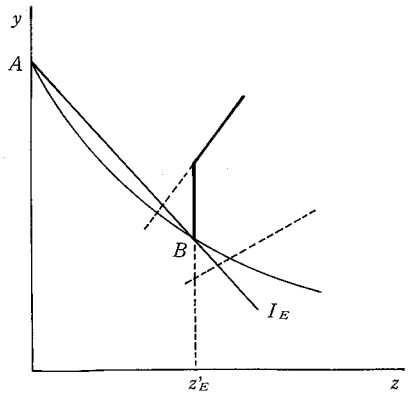
Figure I