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## A Bibliography of Research on Heavy Traffic Limit Theorems for Queues

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This paper provides a classified list of research on heavy traffic limit theorems for queues. Heavy traffic limit theorems provide not only rigorous descriptions of unstable queues but also useful justifications of diffusion-process approximations of stable queues. Our main focus is on the former aspect of the heavy traffic limit theorems.

This paper provides a classified list of research on heavy traffic limit theorems for queues. This bibliography is supplementary in the sense that it does *not* include rather classical work in the previous review papers of Whitt[8] and Lemoine [6].

Heavy traffic limit theorems provide not only rigorous descriptions of unstable queues but also useful justifications of diffusion-process (or Brownian) approximations of stable queues. Because our main focus is on the former aspect of the heavy traffic limit theorems, research on the latter aspect, e.g., by Gordon F. Newell, Donald P. Gaver, John P. Lehoczky, J. Michael Harrison, Martin I. Reiman, Lawrence M. Wein and many others, are excluded from the list. A survey work on the diffusion-process approximations for queues is in progress and will be reported elsewhere.

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