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## Durable Goods and Tariff Protection

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This paper investigates the effect tariff protection has on welfare when the commodities concerned are durable. Using a model based on Bulow-Mann-Waldman's two-period monopoly model, we show that tariff protection of non-durable goods causes a transfer of monopoly rents to the tariff imposing country and improves its welfare. This result, however, may not hold when goods are durable. When the market is imperfect, a foreign monopolist can increase current production, while at the same time reducing future production and creating shortage of future used goods ; thereby raising current, future and future used durable goods' prices. We thus conclude that tariff protection of durable goods may result in a reduction of the welfare of the tariff-imposing country.

**Keywords :** Durable goods, Tariff protection, Imperfect competition, Used goods market, Two-period model, Subgame perfect

JEL Classification: F 1

### 1. Introduction

In a seminal paper, Brander and Spencer(1984) established that a trade policy, such as tariff, may be used as a strategic tool to improve the welfare of the imposing country. While the introduction of the policy results in Pareto inefficiency in the case of imperfect competition, income transfer from the imposed country to the imposing country makes the latter better off and the former worse off. However, their argument implicitly assumes that the commodities concerned be non-durable. We extend their analyses to the case of durable goods. The results obtained have significant implications, both from a theoretical and empirical point of view. A number of contributions have been made to the theoretical literature on durable goods. One of the most celebrated results is the "Coase conjecture," which claims that even a monopolist can not sustain his monopoly price if he produces durable goods (see Coase (1972) and Stokey (1986)) . Bulow (1982) first used the two-period model, which was later extended by Mann (1992) and Waldman (1996). To investigate whether or not a monopolist will practice planned obsolescence, Bulow (1986) also extends his

original model to a symmetric oligopoly model. While this paper is based on the Bulow-Mann-Waldman's (BMW, for short) two-period model, there are several complications when we apply the BMW model to an open economy. In particular, we can not maintain symmetry in a trade model. Theoretically, this is not trivial. Furthermore, durable goods are traded heavily and play a critical role in real trade policies. In particular, recently, interest in the trade of automobiles between US and Japan has result in both theoretical and empirical research (see Dean (1991) , Feenstra (1985) and Yano (1989)) . These studies, however, neglect that automobiles are durable goods. The purpose of this paper is to provide a simple two-period model of international trade, based on BMW. In order to make our analysis as simple as possible, we neglect the problem of planned obsolescence in this paper. We assume that the quality of commodities supplied in the two periods is the same. However, the quality of commodities supplied in period 1 deteriorate in period 2 and some of them will be traded in the competitive used durable goods market. We believe that the introduction of used durable goods is natural in dealing with the problem of durable goods (see Waldman and Mann). We are particularly interested in how the announcement of introducing a tariff in period 2 will affect production decisions in period 1 and period 2 and commodity prices. We find that all commodity prices rise. The reason is that the imposition of tariff reduces the demand for durable goods in period 2 , which in turn decreases the supply of used durable goods and raises their price. The rise in the price of used durable good also implies the rise in the imputed price of durable goods supplied in period 1 . These price changes certainly will have strong influence on the welfare of the home country.

Section 2 briefly examines the demand side of our analysis, which is basically the same as BMW. Section 3 examines the simple case of a foreign exporter who monopolizes home country's durable goods market. It will be shown that the tariff always worsens the home country's welfare, irrespective of the degree of durability. Section 4 extends our model to the duopoly case, of a home firm and foreign firm engaging in Cournot's type competition. It will be shown that the tariff may worsen the home country's welfare if the degree of durability are relatively high. Brief concluding remarks are given in Section 5.

## 2. Demand Side Analysis

Since our model is based on the BMW two-period model, we will only briefly explain our model. In this section, we demonstrate the price formation by consumers.

We employ the following notation.

$P_i$  : price of a durable good in period  $i$  ( $i = 1, 2$ )

- $P_u$  : price of a used durable good  
 $x_i$  : quantity demanded in period  $i$  ( $i = 1, 2$ )<sup>1</sup>  
 $\delta$  : rate of quality deterioration ( $0 < \delta < 1$ )  
 $q$  : discount rate

We will make the following assumptions.

- (A 1) The commodities concerned last for two periods.  
 (A 2) Consumers are specified by their preference (utility)  $R_i$ , which is uniformly distributed over one unit interval  $[0, 1]$ .  
 (A 3) Consumers purchase, if any, one unit of the durable good.  
 (A 4) Both consumers and producers have perfect foresight.  
 (A 5) The used durable goods market is competitive.

Now we demonstrate the price formation by consumers. First, we will consider the second period problem. There are two markets: (i) the market for newly supplied durable goods and (ii) the market for used durable goods. We will consider the used durable goods market, first. Those consumers who satisfy inequality (1) will purchase the used durable goods.

$$(1 - \delta) R_i - P_u \geq 0 \quad (1)$$

The equality in equation (1) will be satisfied by marginal consumers  $R_i = 1 - x_1 - x_2$ . Therefore,

$$\begin{aligned}
 P_u &= (1 - \delta) (1 - x_1 - x_2) \\
 &= a_3 (1 - x_1 - x_2),
 \end{aligned} \quad (2)$$

where  $a_3 \equiv 1 - \delta > 0$ . In period 2, some consumers may purchase the newly supplied durable goods. We will assume that

$$(A 6) \quad x_1 > x_2.$$

Assumption (A6) implies that those consumers who purchase durable goods in period 2 are at the same time suppliers of used durable goods.<sup>2</sup> Those consumers who satisfy inequality (3) will purchase durable goods in period 2.

$$R_i - P_2 + P_u \geq a_3 R_i \quad (3)$$

Since marginal consumers are located at  $R_i = 1 - x_2$  on the preference line, (A4) implies equation (4);

$$\begin{aligned}
 P_2 &= \delta (1 - x_2) + P_u \\
 &= 1 - a_3 x_1 - x_2 \\
 &= G(x_1, x_2)
 \end{aligned} \quad (4)$$

Define:

$$G_1 \equiv \frac{\partial G}{\partial x_1} = -a_3 < 0 \quad \text{and} \quad G_2 \equiv \frac{\partial G}{\partial x_2} = -1 < 0. \quad (D1)$$

Finally, we will consider period 1. Those consumers who purchase durable goods in period 1 evaluate their utility over two periods and find inequality (5) is satisfied.

$$R_i - P_1 + q a_3 R_i \geq q [a_3 R_i - P_u] \quad (5)$$

Since marginal consumes are located at  $R_i = 1 - x_1$  over the preference line, (A 4) implies equation (6);

$$\begin{aligned} P_1 &= 1 - x_1 + qP_u \\ &= (1 + a_3q)(1 - x_1) - a_3qx_1 - x_2 \\ &= H(x_1, x_2) \end{aligned} \quad (6)$$

Define:

$$H_1 \equiv \frac{\partial H}{\partial x_1} = -(1 + a_3q) < 0 \quad \text{and} \quad H_2 \equiv \frac{\partial H}{\partial x_2} = -a_3q < 0. \quad (D2)$$

Since the demand side is fully analyzed, we will next consider the supply side, which consists of two cases; (1) the monopoly case and (2) the duopoly case.

### 3. The Monopoly Exporter Case

In this section, we will consider the simplest case where a foreign monopolist, who is solely engaged in production for exports, is the only supplier of durable goods. For simplicity, we assume:

(A 7) Both marginal and fixed costs are zero.<sup>3</sup>

As a trade policy, we will consider tariff imposition.

(A 8) Home country announces in period 1 that it will impose a tariff on durable goods in period 2.

Under (A7) and (A8), the foreign monopolist's profit in period 2 will be expressed by equation (7).

$$\pi_2 = [G(x_1, x_2) - t]x_2, \quad (7)$$

where  $t$  denotes for the tariff rate. A necessarily condition for profit maximization can be stated as follows:

$$\frac{\partial \pi_2}{\partial x_2} = G(x_1, x_2) - t + G_2x_2 = 0, \quad (8)$$

which yields the following supply function in period 2.

$$\begin{aligned} x_2 &= \frac{1}{2} [1 - t - a_3x_1] \\ &= g(x_1, t) \end{aligned} \quad (9)$$

Define:

$$g_1 \equiv \frac{\partial g}{\partial x_1} = -\frac{a_3}{2} < 0 \quad \text{and} \quad g_t \equiv \frac{\partial g}{\partial t} = -\frac{1}{2} < 0. \quad (D3)$$

Using equation (8), we can derive the profit function in period 2.

$$\pi_2 = \{g(x_1, t)\}^2. \quad (10)$$

We return to period 1. The foreign monopolist, who knows that a tariff will be introduced in period 2 and calculates his profits in period 2, determines  $x_1$  so as to maximize overall profits.

$$\begin{aligned} \Pi &= \pi_1 + q\pi_2 \\ &= H(x_1, g(x_1, t))x_1 + q\{g(x_1, t)\}^2 \end{aligned}$$

A necessary condition for profit maximization will be given by equation (11).

$$\frac{\partial \Pi}{\partial x_1} = H(x_1, g(x_1, t)) + [H_1 + H_2 g_1]x_1 + 2qg(x_1, t)g_t = 0 \quad (11)$$

We can solve equation (11) as follows.

$$\begin{aligned} x_1 &= \frac{1 + a_3 q t}{2 + \frac{1 + 3\delta}{2} a_3 q} \\ &= h(t) \end{aligned} \quad (12)$$

Evaluating the above function at  $t=0$  yields

$$h(0) = \frac{1}{2 + \frac{1 + 3\delta}{2} a_3 q} \quad \text{and} \quad h'(0) = a_3 q h(0). \quad (D4)$$

Substituting equation (12) into (9) yields

$$x_2 = g(h(t), t). \quad (13)$$

We summarize our result as lemma 1.

**Lemma 1.**

$$\frac{dx_1}{dt} > 0 \quad \text{and} \quad \frac{dx_2}{dt} < 0.$$

Proof:

From equation (12),

$$\frac{dx_1}{dt} = h'(0) = \frac{a_3 q}{2 + \frac{1 + 3\delta}{2} a_3 q} > 0.$$

Using equation (12) and (13), we derive:

$$\frac{dx_2}{dt} = g_1 \frac{dx_1}{dt} + g_t < 0,$$

Since  $g_1 < 0$ ,  $\frac{dx_1}{dt} > 0$  and  $g_t < 0$ . (q.e.d)

Lemma 1 shows that the announcement of tariff imposition in period 1 makes the monopolist shift his production from period 2 to period 1.

Lemma 2 states the effect of durability on production in each period.<sup>4</sup>

**Lemma 2.**

- (i)  $\frac{\partial h(0)}{\partial a_3} \gtrless 0$ , depending on if  $a_3 \lesseqgtr \frac{2}{3}$
- (ii)  $\frac{\partial g(h(0), 0)}{\partial a_3} < 0$ .

Proof:

To prove (i) is trivial. To prove (ii) it is sufficient to show that  $\frac{\partial [a_3 h(0)]}{\partial a_3} > 0$ . That is,

$$\frac{\partial}{\partial a_3} [a_3 h(0)] = \{2 + \frac{3}{2} a_3^2 q\} h(0) > 0 \quad (\text{q.e.d.})$$

Lemma 2 shows that an increase in durability encourages the monopolist to reduce production in period 2. However, when durable goods are not very durable ( $a_3 < \frac{2}{3}$ ), then this means a low price for used durable goods. Since those consumers who purchase durable goods in period 1 have already imputed the durable goods' value in period 2, reflected by the price of used goods, the price of durable goods in period 1 also becomes lower, which decreases the incentive to produce in period 1. On the other hand, if durable goods are sufficiently durable ( $a_3 > \frac{2}{3}$ ), then the monopolist, anticipating a high price in period 1, produces more.

This completes the formal analysis of tariff imposition. In the following subsections, we will now consider the effect of tariff imposition on (i) monopoly profits, (ii) durable goods prices, and (iii) the welfare of the home country.

### (i) The Effect of Tariff Imposition on Monopoly Profits

The profit function of the foreign monopolist is stated as follows:

$$\begin{aligned} \Pi &= \Pi_1 + q\Pi_2 \\ &= H(h(t), g(h(t), t))h(t) + q\{g(h(t), t)\}^2 \\ &= \Pi(t) \end{aligned} \quad (13)$$

Totally differentiating equation (13) with respect to  $t$  yields equation (14).

$$\frac{d\Pi}{dt} = \{H_2 h(t) + 2qg(h(t), t)g_t\} \quad (14)$$

In deriving equation (14), we used the Envelope Theorem. Using (D2) and (D3) and evaluating equation (14) at  $t=0$ , we can derive equation (15).

$$\frac{d\Pi}{dt} = Q(a_3)g_t, \quad (15)$$

$$\text{Where } Q(a_3) = \frac{2\delta + \frac{1+3\delta}{2}a_3q}{2 + \frac{1+3\delta}{2}a_3q} > 0; \text{ given } g_t < 0, \frac{d\Pi}{dt} < 0.$$

### Lemma 3.

Tariff imposition unanimously reduces the foreign monopolist's profit.

It should be noted, however, that  $Q'(a_3) < 0$ .<sup>5</sup> That is, the impact of tariff imposition on the foreign monopolist profits will be weakened if commodities' dura-

bility increases (i.e. an increase in the value of  $a_3$ ). Lemma 3 also indicates that an income transfer from the foreign monopolist to home country occurs.

### (ii) The Effect of Tariff Imposition on Commodity Prices

The effect of tariff imposition on commodity prices seems unanimous. Since the tariff clearly reduces their demand in period 1, the supply of used durable goods is also decreased, which raises the price of used durable goods and the imputed value of durable goods in period 1. This anticipation will be showed in lemma 4.

#### Lemma 4.

$$\frac{dP_u}{dt} > 0, \quad \frac{dP_1}{dt} > 0 \quad \text{and} \quad \frac{dP_2}{dt} > 0$$

Proof:

Note that

$$P_u = a_3 [1 - h(t) - g(h(t), t)] = P_u(t).$$

Totally differentiating  $P_u$  with respect to  $t$ , yields

$$\frac{dP_u}{dt} = a_3 [- (1 + g_1) h'(0) - g_t].$$

Substituting (D2) and the result in lemma 1, we obtain

$$\frac{dP_u}{dt} = \frac{1}{2} a_3 \{ 2 + \frac{4 - 5 a_3}{2} a_3 q \} h(0) > 0.$$

Similarly, note that

$$P_1 = H[h(t), g(h(t), t)] = P_1(t).$$

Totally differentiating  $P_1(t)$  with respect to  $t$  yields the following.

$$\frac{dP_1}{dt} = \{H_1 + H_2 g_1\} h'(0) + H_2 g_t.$$

Substituting (D1) and (D2) and using Lemma 1, we obtain

$$\frac{dP_1}{dt} = \frac{1}{2} a_3 q \{ 1 + (1 - a_3) a_3 q \} h(0) > 0.$$

Finally, since

$$P_2 = G[h(t), g(h(t), t)] = P_2(t),$$

Total differentiation of the above with respect to  $t$  yields;

$$\frac{dP_2}{dt} = \{G_1 + G_2 g_1\} h'(0) + G_2 g_t.$$

Therefore,

$$\frac{dP_2}{dt} = \frac{1}{2} \{1 - a_3^2 q\} h(0) > 0. \quad (\text{q.e.d.})$$

Lemma 4 shows that the tariff induces a rise in the price of durable goods. While lemma 3 suggests that an income transfer from the foreign exporter to

the home country occurs, in the case of commodities being durable, the foreign monopolist can decrease their production over time and push up their prices, which decreases consumers surplus. This problem will be discussed in a next subsection.

### (iii) Welfare changes by tariff imposition

In this subsection, we prove Theorem 1.

Theorem 1.

Tariff imposition unambiguously increases the welfare of the home country.

The proof of Theorem 1 consists of two parts: (a) changes in welfare in period 1 and (b) changes in welfare in period 2.

#### (a) Changes in Welfare in Period 1

The welfare in period 1 is stated as equation (15).

$$W_1 = \int_{1-h(t)}^1 [R - P_1(t)] dR \quad (15)$$

A change in welfare in period 1 now is computed as follows:<sup>6</sup>

$$\frac{dW_1}{dt} = [1 - h(0)]h'(0) - h(0)\frac{dP_1}{dt} - P_1(0)h'(0).$$

Using equation (6) and lemma 1, we can derive the following.

$$\frac{dW_1}{dt} = -qP_u(0) - h(0)\frac{dP_1}{dt}; \quad (16)$$

Using lemma 4,  $\frac{dW_1}{dt} < 0$ . Welfare in period 1, due to tariff imposition, definitely is reduced, partly because of an imputed price increase ( $P_u$ ) and partly because of a rise in  $P_1$ .

#### (b) Change in Welfare in Period 2

Welfare in period 2 consists of four parts: (1) consumers surplus derived from purchasing newly produced durable goods ( $CS_2$ ), (2) consumers surplus derived from purchasing used durable goods ( $CS_u$ ), (3) consumers surplus derived from durable goods bought in period 1 ( $CS_{21}$ ), and tariff revenue ( $GS$ ). They are defined as follows:

$$CS_2 = \int_{1-g(h(t),t)}^1 [R - P_2(t) + P_u(t)] dR$$

$$CS_u = \int_{1-h(t)-g(h(t),t)}^{1-h(t)} [a_3R - P_u(t)] dR$$

$$CS_{21} = \int_{1-h(t)}^{1-g(h(t),t)} a_3R dR$$

$$GS = t g(h(t), t)$$

Summing these surpluses together yields equation (17):

$$W_2 = \int_{1-g(h(t),t)}^1 R dR + \int_{1-h(t)-g(h(t),t)}^{1-g(h(t),t)} a_3 R dR - (P_2(t) - t)g(h(t),t). \quad (17)$$

Totally differentiating equation (17) with respect to  $t$ , we obtain the change in welfare in period 2.

$$\begin{aligned} \frac{dW_2}{dt} &= [1 - g(0)][g_1 h'(0) + g_t] - a_3 [1 - g(0)][g_1 h'(0) + g_t] \\ &\quad + a_3 [1 - h(0) - g(0)][h'(0) + g_1 h'(0) + g_t] \\ &\quad - g(0) \frac{dP_2}{dt} - P_2 [g_1 h'(0) + g_t] + g(0), \end{aligned}$$

where  $g(0) \equiv g(h(0), 0)$ . Substituting (D2) into the above equation and rearranging, we can derive equation (16).

$$\frac{dW_2}{dt} = P_u(0)h'(0) - g(0) \frac{\partial P_2}{\partial t} + g(0) \quad (18)$$

Now, combining equations (16) and (18), we can derive equation (19).

$$\begin{aligned} \frac{dW}{dt} &= \frac{dW_1}{dt} + q \frac{dW_2}{dt} \\ &= qg(0) \left\{ 1 - \frac{dP_2}{dt} \right\} - h(0) \frac{dP_1}{dt}. \end{aligned}$$

Giving lemma 4,

$$\frac{dW}{dt} = \frac{1}{2} \{ 1 - a_3(1 - a_3q)h(0) - a_3(1 + a_3q)h(0)^2 \}.$$

Note that  $0 < h(0) < \frac{1}{2}$ ,  $0 < a_3 < 1$  and  $0 < q < 1$ , which implies  $\frac{dW}{dt} > 0$ . Theo-

rem 1 states, under the special situation where only foreign monopolist supplies the durable goods, a tariff causes an increase in the welfare of the home country.

#### 4. The Duopoly Case

In this section we allow for activities of a home firm. There are two firms in the home country, a home firm, denoted firm  $i$ , and a foreign firm, denoted firm  $j$ . Both firms have the same cost function and we assume (A7). Assumption (A8) also holds add (A9).

(A9) Both firms play a Cournot game.

Before we investigate the subgame perfect solution, we should note that the demand functions given by equations (2), (4) and (6) in section 2 are still valid.

First, consider the optimization problem of each firm in period 2. Firm  $i$ 's profit will be given by equation (19).

$$\Pi_{2i} = G(x_1, x_2) x_{2i} \quad (19)$$

Differentiating equation (19) with respect to  $x_{2i}$  yields the following necessary condition for maximizing profit.

$$\frac{\partial \Pi_{2i}}{\partial x_{2i}} = G(x_1, x_2) + G_{2i} x_{2i} = 0 \quad (20)$$

Using (D1), equation (20) provides the reaction function of firm  $i$  in period 2.

$$x_{2i} = \frac{1 - a_3 x_1 - x_{2i}}{2} \quad (21)$$

Similarly, firm  $j$ 's profit is given by equation (22).

$$\Pi_{2j} = [G(x_1, x_2) - t] x_{2j}, \quad (22)$$

Where  $t$  denotes the tariff. Firm  $j$ 's optimization procedure also yields its reaction function.

$$x_{2j} = \frac{1 - t - a_3 x_1 - x_{2i}}{2} \quad (23)$$

Solving equations (22) and (23) together, we can obtain Cournot-Nash solutions of  $x_{2i}$  and  $x_{2j}$  as follows:

$$\begin{aligned} x_{2i} &= \frac{1}{3} [1 - a_3 x_1 + t] \equiv g_i(x_1, t) \\ x_{2j} &= \frac{1}{3} [1 - a_3 x_1 - 2t] \equiv g_j(x_1, t) \end{aligned} \quad (24)$$

Since  $x_2 = x_{2i} + x_{2j}$ ,

$$x = \frac{2(1 - a_3 x_1) - t}{3} = g(x_1, t).$$

Now, define:

$$\begin{aligned} g_{i1} &\equiv \frac{\partial g_i}{\partial x_1} = -\frac{a_3}{3} < 0, \quad g_{it} \equiv \frac{\partial g_i}{\partial t} = \frac{1}{3} > 0 \\ g_{j1} &\equiv \frac{\partial g_j}{\partial x_1} = -\frac{a_3}{3} < 0, \quad g_{jt} \equiv \frac{\partial g_j}{\partial t} = -\frac{2}{3} < 0 \\ g_1 &\equiv \frac{\partial g}{\partial x_1} = -\frac{2a_3}{3}, \quad g_t \equiv -\frac{1}{3} \end{aligned} \quad (D5)$$

By employing the necessary conditions, the profit functions of both firms can be expressed by a similar functional representation.

$$\pi_{2k} = \{g_k(x_1, t)\}^2, \quad k=i, j \quad (25)$$

Next we consider the optimization problem of both firms in period 1. By using equation (25), over time (discounted) profits are given by equation (26).

$$\begin{aligned} \Pi_k &= \pi_{1k} + q \pi_{2k} \\ &= H(x_1, g(x_1, t)) x_{1k} + q \{g_k(x_1, t)\}^2, \quad k=i, j \end{aligned} \quad (26)$$

The necessary conditions for maximizing profits can be expressed as follows.

$$\frac{\partial \Pi_k}{\partial x_{1k}} = H(x_1, g(x_1, t)) + \{H_1 + H_2 g_1\} x_{1k} + 2q g_k(x_1, t) g_{k1} = 0 \quad (27)$$

We can derive the reaction function of each firm and solve subgame perfect Nash equilibrium solutions. Here we rather solve equations (27) simultaneously. Equations (27) may be presented by the following matrix representation.

$$\begin{bmatrix} 2\alpha_1 - \alpha_2 & \alpha_1 - \alpha_2 \\ \alpha_1 - \alpha_2 & 2\alpha_1 - \alpha_2 \end{bmatrix} \begin{bmatrix} x_{1i} \\ x_{1j} \end{bmatrix} = \begin{bmatrix} \alpha_0 + \beta_1 t \\ \alpha_0 + \beta_2 t \end{bmatrix},$$

where  $\alpha_0 = 1 + \frac{1}{9}a_3q$ ,  $\alpha_1 = 1 + \frac{3-2a_3}{3}a_3q$ ,  $\alpha_2 = \frac{2}{9}a_3^2q$ ,  $\beta_1 = \frac{1}{9}a_3q$ ,  $\beta_2 = \frac{7}{9}a_3q$ . Simple calculation yields the following solutions.

$$\begin{aligned} x_{1i} &= \frac{1}{\alpha_1 Y} \{ \alpha_0 \alpha_1 - [(\alpha_1 - \alpha_2)(\beta_2 - \beta_1) - \alpha_1 \beta_1] t \} \\ &= h_i(t) \\ x_{1j} &= \frac{1}{\alpha_1 Y} \{ \alpha_0 \alpha_1 + [(\alpha_1 - \alpha_2)(\beta_2 - \beta_1) + \alpha_1 \beta_2] t \} \\ &= h_j(t) \end{aligned} \quad (28)$$

By using equation (28), we can derive  $x_1$  as follows.

$$\begin{aligned} x_1 &= x_{1i} + x_{1j} = \frac{1}{Y} \{ 2\alpha_0 + (\beta_1 + \beta_2) t \} \\ &= h(t) \end{aligned} \quad (29)$$

The effect of tariff imposition on the supply side is summarized by lemma 5.

**Lemma 5.**

$$\begin{aligned} \frac{dh_i(0)}{dt} &= h_i' < 0, \quad \frac{dh_j(0)}{dt} = h_j'(0) > 0, \quad \frac{dh(0)}{dt} = h'(0) > 0, \\ \frac{dg_i(0)}{dt} &> 0, \quad \frac{dg_i(0)}{dt} < 0, \quad \frac{dg(0)}{dt} < 0 \end{aligned}$$

**Proof:**

In order to prove  $h_i'(0) < 0$ , it is sufficient to show that  $(\alpha_1 - \alpha_2)(\beta_2 - \beta_1) - \alpha_1 \beta_1 > 0$ , which is equal to  $(5\alpha_1 - 6\alpha_2)\beta_1 = 5 + \frac{15-14a_3}{3}a_3q > 0$ .  $h_j'(0) > 0$  and  $h'(0) > 0$  are trivial to show. In order to show  $\frac{dg_i(0)}{dt} < 0$ , we note that  $\frac{dg_i(0)}{dt} = g_{i1}h'(0) + g_{i2}$ . Substituting (D5) into the above equation yields

$$\frac{dg_i(0)}{dt} = \frac{1}{3Y} \{ 3 + [27 - 23\alpha_3]\beta_1 \} > 0.$$

Similarly,

$$\frac{dg_i}{dt} = g_{i1}h'(0) + g_{i2} = -\frac{1}{3} [2 + a_3h'(0)] < 0,$$

and

$$\frac{dg}{dt} = \frac{dg_i}{dt} + \frac{dg_j}{dt} = -\frac{1}{3} [1 + 2a_3h'(0)] < 0. \quad (\text{q.e.d.})$$

Corresponding to lemma 2, we can establish lemma 6.

**Lemma 6.**

$$(1) \quad \frac{\partial h(0)}{\partial a_3} \geq 0, \text{ depending on as } a_3 \geq a_0.$$

$$(2) \quad \frac{\partial g(0)}{\partial a_3} < 0,$$

$$\text{where } a_0 = \frac{9}{q} \left\{ -1 + \sqrt{1 + \frac{2}{99}q} \right\}.$$

Proof:

We note that  $h_i(0) = h_j(0) = \frac{1}{2}h(0) = \frac{\alpha_0}{Y}$ . In order to prove (1), it is sufficient to show:

$$\frac{\partial}{\partial a_3} \left( \frac{\alpha_0}{Y} \right) = \frac{q}{Y^2} \left\{ \frac{22}{81} a_3^2 q + \frac{44}{9} a_3 - \frac{3}{8} \right\}.$$

In order to prove (2), it is sufficient to show that  $\frac{\partial}{\partial a_3}(a_3 h(0)) > 0$ . (q.e.d)

Lemma 6 indicates that tariff imposition reduces total production in period 2, while its effect on the total production in period 1 crucially depends upon the durability of the commodities concerned. In the following subsections we use the same analytical procedure undertaken in section 3.

**(i) The Effect of Tariff on Firms's Profit**

The overall (discounted) profits are given by equation (26):

$$\begin{aligned} \Pi_k &= H(h(t), g(h(t)), t) h_k + q |g_k(h(t), t)|^2 \\ &\equiv \Pi_k(h_i(t), h_j(t), t) \quad k=i, j \end{aligned} \quad (26)'$$

The imposition of tariff affects firms' profit as follows.

$$\frac{d\Pi_k}{dt} = A_k h'_1(0) + B_k g_{kt}, \quad 1 \neq k, \quad (30)$$

where  $A_i = A_j = A = \{H_1 + H_2 g_1\} h_i(0) + 2q g_i(0) g_{i1}$ ,  $B_i = 2q g_i(0) - H_2 h_i(0)$  and  $B_j = 2q g_j(0) + \frac{1}{2} H_2 h_j(0)$ . Equation (30) was derived using the Envelope Theorem. Note that  $h_i(0) = h_j(0)$ ,  $g_i(0) = g_j(0)$ ,  $g_t = -g_{it} = \frac{1}{2} g_{jt}$ . Since  $A < 0$ ,  $B_i > 0$ ,  $B_j > 0$ ,  $h'_1(0) < 0$ ,  $h'_j(0) > 0$ ,  $g_{it} > 0$ , and  $g_{jt} < 0$ , we can summarize our result as lemma 7.

**Lemma 7.**

The effect of tariff imposition in period 2 on firms's profits consists of two parts: (1) direct effect through production changes in period 2 and (2) indirect effect through production changes in period 1. These effects are counteractive. However, when durability is insignificant ( $a_3 \rightarrow 0$ ), the imposition of tariff always raises the home firm's profit and lowers the foreign firm's profit.

Proof:

It should be noted that as  $a_m \rightarrow 0$ ,  $h_i'(0) \rightarrow 0$  and  $h_j'(0) \rightarrow 0$ . (q.e.d)

We have already shown in lemma 3 that tariff imposition always decreases the foreign monopolist's profits, irrespective of the degree of durability. We also expect in the duopoly case that  $\frac{d\Pi_i}{dt} > 0$  and  $\frac{d\Pi_j}{dt} < 0$  for all  $a_3$  or  $\delta$ . However except for the case  $a_3 = 0$ , it is not clear.<sup>7</sup>

### (ii) The Effect of Tariff Imposition on Commodity Prices

Corresponding to lemma 4, we propose lemma 8.

**Lemma 8.**

$$\frac{dP_1}{dt} > 0, \quad \frac{dP_2}{dt} > 0 \quad \text{and} \quad \frac{dP_u}{dt} > 0.$$

Proof:

We know that the commodity price in period 1 will be specified as follows.

$$\begin{aligned} P_1 &= H[h(t), g(h(t), t)] \\ &= P_1(t) \end{aligned} \tag{31}$$

Totally differentiating equation (31) with respect to  $t$ , we obtain

$$\frac{dP_1}{dt} = [H_1 + H_2 g_1] h'(0) + H_2 g_t \tag{32}$$

Substituting (D1), (D2) and equations (28) yields

$$\frac{dP_1}{dt} = \frac{1}{qY} \{ 1 - a_3^2 g + (1 - a_3) a_3 q \} > 0.$$

Similarly,

$$\begin{aligned} P_2 &= G[h(t), g(h(t), t)] \\ &= P_2(t) \end{aligned}$$

Therefore,

$$\frac{dP_2}{dt} = \frac{1}{3} \left\{ 1 - \frac{1}{Y} \frac{8}{9} a_3^2 q \right\} > 0.$$

Finally, it is easy to show that

$$\frac{dP_u}{dt} = \frac{a_3}{3} \{ 1 - (3 - 2a_3) a_3 h'(0) \} > 0. \tag{q.e.d}$$

Lemma 8 indicates the robustness of lemma 4 and suggests the importance of the existence of durable goods in considering welfare changes.

### (iii) Welfare Changes of Tariff Imposition

In this section, we prove Theorem 2.

Theorem 2 .

The effect of tariff imposition on welfare of the home country is given by equation (33).

$$\frac{dW}{dt} = P_1(0)h_i'(0) - h_j(0)\frac{dP_1}{dt} + \underline{qg_i(0)}. \quad (33)$$

In order to prove Theorem 2, we prove lemma 9 and lemma 10.

**Lemma 9.**

The welfare changes in period 1 due to tariff imposition are given by equation (34).

$$\frac{dW_1}{dt} = [1 - h(0)]h_j'(0) - qP_u(0)h_j'(0) - h_j(0)\frac{dP_1}{dt} \quad (34)$$

It is clear that  $\frac{dW_1}{dt} < 0$ .

Proof:

Since consumers' surplus in period 1 is given by equation (15), we add the home firm's profit in period 1,  $\pi_{1t}$ , to it. Therefore,

$$W_1 = \int_{1-h(t)}^1 R dR - P_1(t)h_j(t).$$

Totally differentiating the above equation with respect to  $t$  and using equation (6), we obtain equation (34). We have already derived that  $h_i'(0) < 0$  and  $h_j'(0) > 0$  from lemma 5 and  $\frac{dP_1}{dt} > 0$  from lemma 8. (q.e.d)

**Lemma 10.**

$$\frac{dW_2}{dt} = P_u(0)h'(0) + \underline{g_i(0)} > 0 \quad (35)$$

Proof:

The expressions of consumers surplus are the same as in section 3. Government revenue will be  $GS = tP_2(t)g_i(h(t), t)$ . We also must add the home firm's profit in period 2. Therefore,

$$W_2 = \int_{1-g(h(t), t)}^1 R dR + \int_{1-h(t)-g(h(t), t)}^{1-g(h(t), t)} R dR - \underline{(P_2(t) - t)g_i(h(t), t)}$$

Totally differentiating the above equation with respect to  $t$  yields

$$\begin{aligned} \frac{dW_2}{dt} = & P_2(0)[g_i h'(0) + g_i] - P_u(0)h'(0) - g_j(0)\frac{dP_2}{dt} \\ & - P_2[g_j h'(0) + g_j] + \underline{g_i(0)}. \end{aligned}$$

Note that  $g_1 = g_{i1} + g_{j1}$  and  $g_i = g_{i1} + g_{i2}$  and that  $P_2(t) = g_i(h(t), t)$  and  $g_i(0) = g_i(0)$ . We obtain

$$P_2(0)[g_i h'(0) + g_i] - g_j(0)\frac{dP_2}{dt} - P_2[g_j h'(0) + g_j] = 0,$$

Which yields equation (35). (q.e.d)

Combining equations (34) and (35), we can derive equation (33). That is,

$$\begin{aligned}\frac{dW}{dt} &= \frac{dW_1}{dt} + q \frac{dW_2}{dt} \\ &= [1 - h(0) + qP_u(0)]h_i'(0) - h_j(0)\frac{dP_1}{dt} + qg_j(0).\end{aligned}$$

From equation (6), the bracketed first term is equal to  $P_1(0)$ . This completes the proof. Theorem 2 suggests interesting characteristics of the effect of tariff on welfare in the case of durable goods. When durability is not significant ( $a_3 \rightarrow 0$ ),  $h_i' \rightarrow 0$  and  $h_j' \rightarrow 0$ . Therefore,  $\frac{dW}{dt} > 0$  as expected in the case of non-durable goods. However, if the degree of durability plays an important role, indirect effects, which are given by the terms,  $h_i'(0)$  and  $\frac{dP_1}{dt}$ , negatively affect welfare.<sup>8</sup> That is,

Corollary.

$$\lim_{a_3 \rightarrow 1} \frac{dW}{dt} < 0 \quad \text{for } q \leq 0.9$$

Proof:

By inserting  $a_3 = 1$ , we can compute the following for  $q = 0.9$ :

$$P_1(0) = \frac{1}{Y} \left\{ 1 + \frac{8}{9}q + \frac{5}{27}q^2 \right\} = 0.5571,$$

$$h_i'(0) = -\frac{q}{Y} \left\{ \frac{5}{9} + \frac{1}{27}q \right\} = -0.1514,$$

$$\frac{dP_1}{dt} = q(1 - q) = 0.09,$$

$$h_j(0) = \frac{1}{Y} \left\{ 1 + \frac{1}{9}q \right\} = 0.3142,$$

$$qg_j(0) = q \frac{1 - h(0)}{3} = 0.1114.$$

Substituting the above results into equation (33) proves the corollary. (q.e.d)

The corollary implies that in the duopoly case when the degree of durability becomes significant, tariff protection may decrease the welfare of home country.<sup>9</sup> It also highlight the importance of taking account of durable goods in trade policy.

## 5. Concluding Remarks

In this paper, we have shown that a trade policy such as tariff, should be carefully exercised. While it is well-known that a tariff may be an effective

welfare enhancing tool when foreign firm(s) have monopoly power in the home market, that belief should be modified when we consider the case of durable goods. When the foreign monopoly exporter is the sole supplier of durable goods, the imposition of tariff improves the home country's welfare, similar to the case of non-durable goods. This result does not hold if we allow for a home firm and consider a duopoly game. When the degree of durability is strong, the imposition of tariff makes foreign firms switch production from period 2 to period 1 which increases production in period 1. However, since a tariff reduces the supply of used durable goods in period 2 and raises the price of used goods, it pushes up the prices in period 1 and period 2. This kind of spillover effect never occurs in the case of non-durable goods case. Our analysis yields the important research that an element of "durability" may play a much significant role in trade policies.

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## Notes

1.  $\delta$  can be considered as related to either quality deterioration or physical deterioration. In the former case, the case where  $\delta < 0$ , corresponds to planned obsolescence. In the latter case, there will be related problems like maintenance (see Schmalensee (1974)).
2. (A2) is also assumed Waldman (1996) and Mann (1992). While this is a normal case, we can also assume the opposite case where  $x_1 < x_2$ . In this case marginal consumers are new entrants.
3. Since the level of fixed cost does not affect the necessary conditions for maximizing profit, the assumption that fixed costs are zero does not affect our result. Similarly, the assumption that marginal costs are zero implies that we deal with net price.
4. Since an increase in durability means an increase in  $a_3$ , we can measure a parametric change in durability as a change in  $a_3$ .
5.  $Q(a_3)$  can be rewritten as follows:

$$Q(a_3) = 1 - a_3 + \frac{s(a_3)a_3}{2 + s(a_3)},$$

Where  $s(a_3) = \frac{4 - 3a_3}{2} a_3 q$ . Then

$$Q'(a_3) = \frac{4 + 2s(a_3) - 2a_3s'(a_3)}{[2 + s(a_3)]^2} < 0.$$

6. We have used the following assumption.

$$\frac{d}{dt} \int_{g(t)}^{f(t)} R dR = f(t)f'(t) - g(t)g'(t).$$

7. We have already known that

$$\left. \frac{d\Pi_i}{dt} \right|_{a_3=0} > 0 \quad \text{and} \quad \left. \frac{d\Pi_i}{dt} \right|_{a_3=1} < 0.$$

We can also check that

$$\left. \frac{d\Pi_i}{dt} \right|_{a_3=1} = \frac{D}{qY} \quad \text{and} \quad \left. \frac{d\Pi_i}{dt} \right|_{a_3=1} = -\frac{D}{qY^2} E,$$

Where  $Y = 3 + \frac{5}{9}q$ ,  $D = (3 + \frac{7}{9}q)Y - (1 + \frac{1}{3}q)(3 + \frac{7}{3}q + \frac{2}{3}q^2)(\frac{7}{9} + \frac{1}{9}q)$  and

$E = (1 + q)Y - (1 + \frac{1}{3}q)(3 + \frac{7}{3}q + \frac{2}{3}q^2)(\frac{5}{9} + \frac{1}{27}q)$ . Note that

$D > 9 - (1 + \frac{1}{3})(3 + \frac{7}{3}q + \frac{2}{3}q^2)(\frac{7}{9} + \frac{1}{9}q) > 9 - \frac{4}{3} \times 6 \times \frac{8}{9} > 0$  and

$E > 3 - (1 + \frac{1}{3})(3 + \frac{7}{3}q + \frac{2}{3}q^2)(\frac{5}{9} + \frac{1}{27}q) > 0$ . Therefore,  $\left. \frac{d\Pi_i}{dt} \right|_{a_3=1} > 0$   $\left. \frac{d\Pi_i}{dt} \right|_{a_3=0} < 0$ . If

$\frac{\partial}{\partial a_3} \left( \left. \frac{d\Pi_i}{dt} \right| \right) < 0$  and  $\frac{\partial}{\partial a_3} \left( \left. \frac{d\Pi_i}{dt} \right| \right) > 0$  for  $a_3 \in [0, 1]$ , which are not proven yet.

8. While we expect  $\frac{\partial}{\partial a_3} \left( \left. \frac{dW}{dt} \right| \right) < 0$ , it is not proven yet, because of (1) in lemma 6.
9. Since the value of  $\frac{dW}{dt}$  not only depends on  $a_3$  but also on  $q$ , it is impossible generally to prove that  $\frac{\partial}{\partial a_3} \left( \left. \frac{dW}{dt} \right| \right) < 0$ . However, for a relatively large value of  $a_3$ ,  $\frac{dW}{dt}$  becomes negative.

$$\text{Note that } P_1(0)h'_1(0) = -\frac{a_3q}{9Y^2} \left[ 1 + \frac{2-10a_3}{9}a_3q + \frac{7-6a_3}{9}a_3^2q^2 \right] \left[ 5 + \frac{15-14a_3}{3}a_3q \right],$$

$$h_j(0)\frac{dP_1}{dt}=\frac{a_3q}{Y^2}\left[1+(1-a_3)a_3q\left[1+\frac{1}{9}a_3q\right]\right]\text{ and }qP_2(0)g_j(0)=\frac{q}{9Y^2}\left[3-2a_3+(\frac{9-8a_3}{3})a_3q\right]^2.$$

Therefore,  $\frac{dW}{dt}=\frac{q}{9Y^2}B$ . It may numerically be shown that there exist several combinations of  $a_3$  and  $q$  which

yield  $\frac{dW}{dt}<0$ . We expect that larger values of  $a_3$  and smaller values of  $q$  will satisfy  $\frac{dW}{dt}<0$ .

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