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BEHAVIOR IN SHEAR OF REINFORCED CONCRETE BEAMS
UNDER FATIGUE LOADING *

(疲労荷重下の鉄筋コンクリートはりのせん断挙動)

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ABSTRACT

Fatigue tests of eleven T-beams with stirrups and sixteen rectangular beams without web reinforcement were carried out. Based on the test results, a procedure is derived to calculate stirrup strain under general variable loading whose maximum and minimum loads change at random as well as repeated loading with constant maximum and minimum load. Besides, an equation for the prediction of the fatigue strength in shear of beam without web reinforcement is proposed in the process of derivation of the procedure, and it is made clear that the fatigue strength of beam failing due to fatigue fracture of stirrups can be evaluated from the stress range calculated by the procedure. Lastly a design method for stirrup under fatigue loading is presented tentatively.

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1. INTRODUCTION

Recent design codes for shear which were revised referring to the static test results tend to require less web reinforcement than in the previous codes. This tendency demands the further study on fatigue, because a reinforced concrete beam sometimes fails in shear under fatigue loading due to the fracture of web reinforcement even if the applied maximum shear force is much smaller than the ultimate static strength [1][2].

Since the fatigue fracture of web reinforcement depends on the stress range, the characteristics of stress under fatigue loading are firstly to be investigated. Fatigue fracture of web reinforcement is considered to be affected by the local stresses caused by the diagonal crossing of shear cracks. Bending operation of a steel bar makes the fatigue strength smaller. Therefore, fatigue strength of web reinforcement should be investigated in consideration of these effects.

1.1 Previous Studies

Some previous reports pointed out that (1) stirrup strains increased during fatigue loading [2][3][4][5], (2) fatigue fracture of stirrup occurred at bend [2][6][7] and (3) fatigue strength of stirrup was smaller than that of bar itself [2][7]. Some of the reports proposed the equation for the calculation of stirrup strain under fatigue loading according to the observed relationship between applied shear forces and stirrup strains. Hawkins [2] proposed Eq.(1.1) derived from the assumption of classical truss analogy without considering any contribution of concrete (see Fig.1.1).

$$\epsilon_w = \frac{V}{A_w E_w z/s} \quad (1.1)$$

where ϵ_w : stirrup strain under fatigue loading
 V : applied shear force
 A_w : cross-sectional area of a pair of stirrup
 E_w : Young's modulus of stirrup
 z : arm length of truss ($=d/1.15$)
 d : effective depth
 s : spacing of stirrups

Ruhnau [3] proposed Eq.(1.2), considering residual stirrup strain under fatigue loading (see Fig.1.2).

$$\epsilon_w = k_1 + k_2 \frac{V}{A_w E_w z/s} \quad (1.2)$$

where k_1 : an empirical coefficient showing residual stirrup strain
 k_2 : an empirical coefficient (0.45 to 0.60)

Higai [5] proposed temporarily Eq.(1.3) (see Fig.1.3).

$$\epsilon_w = \frac{\epsilon_{wy}}{0.55 V_{co} + A_w E_w \epsilon_{wy} z/s} V \quad (1.3)$$

where ϵ_{wy} : yield strain of stirrup

V_{co} : shear force carried by concrete at the initial loading

Although proposed equations had different forms, the assumed $V-\epsilon_w$ curves were linear, and the calculated strains were larger than those derived from Eq.(1.4) under static loading.

$$\epsilon_w = \frac{V - V_{co}}{A_w E_w z/s} \quad (1.4)$$

Farghaly [1] carried out fatigue tests of seventeen rectangular beams with detail observation of stirrup strains, considering the previous reports. The outlines of this study, which was the base of the author's study, were as shown in Sec.1.2.

1.2 Farghaly's Study

A constant maximum and minimum load were applied to each beam. The minimum load was the same for all the beams and the maximum load was changed as a parameter. Detail of specimens, material constants for concrete and reinforcement and detail of loading were as shown in Appendix A. Following conclusions were derived from the test results.

(1) Stirrup strains increased with loading cycles. Although the rates of the increase were different among the stirrups, the average strain in a shear span increased at an almost constant rate against the logarithm of loading cycles. The rates were not influenced by the magnitude of the applied maximum shear force.

(2) It was assumed that the increase of strain was caused by the decrease of shear force carried by concrete, and this decrease was the same as that of the fatigue strength in shear of beam without web reinforcement. From the assumption, Eq.(1.5) was derived to calculate the stirrup strain at the applied maximum shear force after 10^6 cycles of loading.

$$\epsilon_{wmax} = \frac{V_{max} - 0.6 V_{co}}{A_w E_w z/s} \quad (1.5)$$

where ϵ_{wmax} : stirrup strain at V_{max} after 10^6 cycles of loading

V_{max} : applied maximum shear force

0.6 : value derived from the fact showing that the fatigue strength in shear of beam without web reinforcement was 60% of the static strength at 10^6 cycle

(3) The observed relationship between applied shear forces and stirrup strains was idealized as shown in Fig.1.4, so that Eq.(1.6) was obtained to calculate the strain after subjected to repeated loading.

$$\epsilon_w = \frac{V}{V_{\max}} \epsilon_{w\max} \quad (1.6)$$

where $\epsilon_{w\max}$: stirrup strain at the applied maximum shear force calculated by Eq.(1.5)

This equation was transposed to Eq.(1.7) for the calculation of strain range.

$$\epsilon_{wr} = \frac{V_{\max} - V_{\min}}{V_{\max}} \epsilon_{w\max} \quad (1.7)$$

where ϵ_{wr} : strain range of stirrup
 $V_{\min}$: applied minimum shear force

(4) Fatigue fracture of stirrups occurred in all the specimens. Forty one legs were fractured at the bends around longitudinal bars, and one leg was fractured at hook. First fracture usually occurred in the leg that developed the largest strain in the specimen. After fracture of one leg, the strain of another leg or the leg of the adjacent stirrup increased exceedingly, and the next fracture usually occurred in either of the legs after some thousand cycles. The beam failed when it became unable to sustain the applied maximum shear force with the remaining legs.

(5) The fatigue strength of stirrup was 40 % in the cases where the stirrups were supported partially by longitudinal bars because of the large radius of bend, and 66 % in the cases where the stirrups were supported fully because of the small radius of bend.

1.3 Objects of the Author's Study

Although a design method for stirrups under fatigue loading was proposed by Farghaly, the following problems still remained. The author's study was carried out to solve the problems.

(1) Evaluation of the decrease of the fatigue strength in shear of beam without web reinforcement or the decrease of shear force carried by concrete ----- The influence of load range on the fatigue strength and the size effect had not yet been cleared. Any S-N curve which included these factors had never existed.

(2) Applicability of the assumption on which Eq.(1.5) was based ----- The tests by Farghaly were limited, so that 1) the applied minimum shear force was made constant for all the specimens, 2) the shape of cross section was rectangular only, 3) the ratio of shear span to effective depth was 2.5 only, 4) the spacings of stirrups were relatively large compared with the beam depth.

(3) Evaluation of stirrup strain of the beam where the applied maximum shear force is less than the shear capacity of concrete (or shear force carried by concrete under the initial loading)

(4) Evaluation of stirrup strain at the applied minimum shear force -----

The residual strains were neglected.

(5) Evaluation of scatter of strain in each stirrup from the calculated one

(6) Evaluation of stirrup strain under fatigue loading with varied load range ——— This evaluation is necessary for design because the actual loading is generally the fatigue loading with varied load range.

(7) Influence of radius of bend of stirrup on the fatigue strength

(8) Evaluation of fatigue strength of beam failing in shear due to fatigue fracture of stirrup

The fatigue tests of eleven T-beams with stirrups were carried out to make clear the problems (2)-(8), and the fatigue tests of sixteen rectangular beams without web reinforcement were carried out to make clear the problem (1).

2. OUTLINES OF TESTS

In this section the fatigue tests of T-beams with stirrups are referred to. The fatigue tests of rectangular beams are referred to in Sec.3.3.

All the specimens had the same T cross section as shown in Fig.2.1. Loading points were determined to make shear span-depth ratio 2.0 for the right span and 4.0 for the left. There were seven pairs of stirrups in the right span and nine pairs in the left, so that the spacing of stirrups were relatively small compared with the effective depth. The specimens were designed so that simultaneous yielding of main bars and all the stirrups except for the one nearest to the loading point should occur in both the shear spans. The shear force at yielding of stirrups was evaluated according to Eq.(2.1) (see Table 2.1).

$$V_y = V_{co} + A_w f_{wy} (z/s) / \beta_x \quad (2.1)$$

where V_y : shear force at yielding of stirrups

f_{wy} : yield strength of stirrup

β_x : a coefficient for each stirrup to cover the influence of support and loading point reducing stirrup strains (see Sec.4.1)

Position of stirrups were determined by using Eq.(2.1) to produce the same stress in each stirrup under static loading except for the stirrup nearest to the loading point. The latter stirrup was located to make its strain much smaller than the strains in other stirrups. For analyzing test results the values of V_{co} and β_x were corrected to fit the results well (see Table 2.1 and Fig.2.2). The values of V_{co} were approximately equal to calculated values of the static shear strength of beam without web reinforcement, whose ratio of shear span to effective depth was 5.6 [12] (see Eq.(3.1)).

All the specimens were loaded statically during the first hundred cycles, and after that, loaded dynamically 210 cycles per minute with sine loading curve. A hydraulic jack was used for cyclic and static loading. Details of loading history of each specimen were as shown in Fig.2.3. Two specimens, FS1 and FL2, were subjected to a larger load than the maximum of the repeated loading before they were subjected to fatigue loading. All the specimens except for the specimen FS1 were subjected to fatigue loading of multi-level. Load ranges were changed with constant minimum load for two specimens, FS7 and FL8, and with constant maximum load for two specimens, FS9 and FL10. The minimum load was changed for one specimen, FS11, with constant load range. For seven specimens, FS3, FL4, FS5, FL6, FS7, FL8 and FS11, the maximum shear force of the first repeated loading was less than the shear capacity of concrete.

All the bars used in the tests were deformed bars having two longitudinal ribs and parallel transverse lugs perpendicular to bar axis. Their material constants were as shown in Table 2.2. The stirrups were bent around the longitudinal bars and all the specimens were divided into two groups, called as FS series and FL series respectively, according to the radius of bend. The radius was 1.25 times as much as the diameter of the stirrup in FS series and 2.5 times in FL series. Three batches of ready mixed concrete were used. Four specimens were made from each batch, and compressive strengths at the ages of testing which were between two and

twelve months after casting were as presented in Table 2.2. Electrical resistance strain gauges of 5 mm in length were used for measuring stirrup strains. These gauges were attached to all the stirrups at the position of 50 mm above the center of the longitudinal bars (see Fig.2.1).

The pulsator was stopped after appropriate loading cycles, and stirrup strains were measured under static loading and the propagation of diagonal cracks was recorded. Concrete cover was removed to confirm the fatigue fracture of stirrup after the tests. Widths of diagonal cracks were measured for one specimen, FS11, by using the contact type gauges. The contact points were put on the surface of concrete in order to make triangles (see Fig.4.10).

3. FATIGUE STRENGTH IN SHEAR OF BEAM WITHOUT WEB REINFORCEMENT

It is observed that stirrup strains in beams subjected to repeated loading increase with loading cycles (see Sec.1). It is observed that stirrup strains increase after diagonal cracking due to repeated loading, even if no stirrup strain is developed at the initial loading (see Sec.4.1). These phenomena are thought to be related closely to behavior in shear of beam without web reinforcement. Therefore, it is significant to clarify this behavior for designing web reinforcement in beams subjected to fatigue loading.

3.1 Previous Studies and Their Problems

Some previous studies were carried out to investigate behavior in shear of beam without web reinforcement under repeated loading. A summary of the studies was as follows.

- (1) S-N curves of beams were expressed as shown in Figs.3.1, 3.2 and 3.3. The co-ordinate of S generally represented the ratio of the applied maximum shear force to the static shear strength. The fatigue strength of beam at 10^6 cycle was about 60 % of the static strength. However, the shapes of the S-N curves and the methods of the evaluation of the static shear strength varied. Chang - Kesler [8] reported that the S-N curve was a curved line not to cross the co-ordinate of S at the point representing S equal to one and a fatigue limit existed (Fig.3.1). Higai [5] and Farghaly [1] reported that the S-N curve was a straight line and a fatigue limit did not exist (Figs.3.2 and 3.3).
- (2) Farghaly [1] pointed out that the S-N curve was not unique, and that the reduction of the fatigue strength for the beam with small ratio of shear span to effective depth (a/d) was smaller than that for the beam with large a/d (Fig.3.4(a)).
- (3) Taylor [9] predicted that the fatigue strength was influenced by not only the applied maximum load, but also load range and size of specimen, and with the prediction he explained the reason for the difference between the test results by Chang - Kesler [11] and those by Stelson - Cernica [10] and himself [9].
- (4) When a beam with a relatively large a/d was subjected to repeated loading, the beams enduring for a while after diagonal cracking were observed as well as the beams failing soon after diagonal cracking. Although Chang - Kesler and Higai pointed out that these beams had the different failure mechanisms, they considered that two S-N curves for these beams were identical.
- (5) When beam had a relatively small a/d , it was observed [1][5] that the beam often failed due to fatigue fracture of tensile bar at the portion crossed by a diagonal crack, although the beam would not fail soon after diagonal cracking. Farghaly [1] reported the S-N curve for the beam failing due to the fatigue fracture of tensile bar as shown in Fig.3.4(b). This curve was different from that for the beam failing due to the propagation of diagonal crack as shown in Fig.3.4(a).
- (6) On the other hand the fatigue fracture of tensile bar tended to occur at the maximum moment region in the beam with a relatively large a/d . Chang - Kesler [11] pointed out that the fatigue fracture of tensile bar at the maximum moment region occurred in the beam subjected to smaller maximum load, but the shear failure due to diagonal crack occurred in the beam subjected to larger maximum load.

(7) This test result showed that the beam, which would fail in flexure under static loading, could fail in shear under fatigue loading.

From the summary, the following problems could be pointed out.

- (1) The influences of a/d , load range and size of specimen on the fatigue strength of beam without web reinforcement failing in shear have never been cleared. Therefore, no S-N curve including these factors has been known.
- (2) The method of the evaluation of the static shear strength has never been standardized.
- (3) It could not be predicted accurately whether a beam failed due to fatigue of concrete or tensile bar.

The objects of the author's study were to make clear the above problems for the beam with a relatively large a/d . For the objects fatigue tests of rectangular beams with a/d equal to 3.5 were carried out, considering load range as a main parameter.

The beam with a relatively small a/d was not dealt with in the author's study. Because it was difficult to evaluate the static shear strength of the beam and the fatigue strength of tensile bar at the portion crossed by a diagonal crack, and there were few experimental data.

3.2 Fatigue Strength in Shear of Beam without Any Consideration of Influence of Load Range

The evaluation of static strength is important, when fatigue strength is represented by the ratio of the applied maximum shear force to the static strength. It is an inefficient work to obtain the average ultimate strength of the identical specimens by static tests. Therefore, Eq.(3.1), from which the most accurate static strength is thought to be evaluated [12], is used for the calculation of the static strength of beam without web reinforcement.

$$V_{cu} = 0.20 f_c'^{1/3} (0.75 + 1.40 d/a) (1 + \beta_p + \beta_d) b_w d \quad (3.1)$$

where $\beta_p = \sqrt{100 p_w} - 1$
 $\beta_d = (1000 / d)^{1/4} - 1$
 $p_w = A_s / (b_w d)$: reinforcement ratio
 V_{cu} : static strength in shear of beam without web reinforcement
 f_c' : cylinder strength (MPa)
 a : shear span
 d : effective depth (mm)
 b_w : web width
 A_s : cross-sectional area of tensile bar

The S-N curve in Fig.3.5 expresses the relationship between the ratios of the previous tested values of fatigue strength in shear to the calculated values by Eq.(3.1) and the tested values of fatigue lives of beams (see Appendix B). Since the calculated values overestimate the tested ones in the cases of beams with light weight concrete, the calculated ones are multiplied by 0.8. The value of 0.8 is determined boldly, but the average ratio of tested fatigue strengths to calculated ones by Eq.(3.2) is 1.02. It is observed in Fig.3.5 that the fatigue strength decreases linearly with logarithm of loading cycles and the rate of decrease is somewhat smaller

when the logarithm of loading cycles is larger than six. Although there is no sure information about the fatigue limit because of lack of data in the cases where logarithm of loading cycles is larger than seven, the fatigue limit is supposed not to appear within the range of the tests. From these facts a following equation which is represented by solid line in Fig.3.5 is derived for the prediction of the fatigue strength.

$$\log\left(\frac{V_{\max}}{V_{cu}}\right) = m' \log N_f \quad (3.2)$$

where $m' = -0.035$
 N_f : fatigue life

The average ratio of tested values to calculated ones ($V_{\max\text{test}} / V_{\max\text{cal}}$) is 1.00 and the coefficient of variation is 7.4 %. The tested values include both the cases of the beam failing soon after diagonal cracking and the beam enduring for a while after diagonal cracking. Therefore it is appropriate to consider that the two S-N curves for the two types of beams are identical.

3.3 Fatigue Strength in Shear of Beam with Consideration of Influence of Load Range

Fatigue tests of sixteen rectangular beams without web reinforcement were carried out, considering load range as a main parameter. All the specimens which consisted of eight kinds were made as shown in Fig.3.6 and Table 3.1. For eight specimens effective depth was 440 mm which was much larger than that of 108 to 220 mm in the previous tests [1][5][8][9][10][11]. Concentrated load was applied at two points to make a/d equal to 3.5. The static flexure strength was larger than the static shear strength in the specimens with reinforcement ratio of 1.67 %, while the static shear strength was larger in the specimens with reinforcement ratio of 0.68 %.

Two batches of ready mixed concrete with maximum aggregate size of 25 mm were used. The cylinder strengths were obtained as shown in Table 3.1. All the bars were SD35 and had two longitudinal ribs and parallel transverse lugs perpendicular to bar axis. The diameters were 25 mm for reinforcement ratio of 1.67 % and 16 mm for reinforcement ratio of 0.68 %, and the cross-sectional areas were 1,470 and 600 mm² respectively. The yield strength of the bar with the diameter of 25 mm was 370 MPa and that of the bar with the diameter of 16 mm was 400 MPa.

The constant maximum and minimum load were applied to each specimen. The applied maximum shear force was 61 to 92 % of the static shear strength calculated by Eq.(3.1). The applied minimum shear force was changed widely between 10 and 90 % of the maximum to investigate the influence of load range. The apparatus for loading consisted of a steel frame and a hydraulic jack connected to a pulsator. All the specimens were loaded statically during the first hundred cycles and after that loaded dynamically 210 cycles per minute with sine loading curve until the specimens failed. Four specimens which did not fail under repeated loading were made to fail due to static loading or changing level of repeated loading.

All the specimens were placed in the laboratory more than one month before the tests. The fatigue tests were carried out between sixty and hundred fifty days after casting.

Load level and fatigue life for each specimen are shown in Table 3.1. Twelve of the sixteen specimens fail in shear due to propagation of main diagonal crack. Two of the other specimens fail in flexure due to fatigue fracture of tensile bar at the maximum moment region and two fail under static loading after the fatigue tests.

There are three pairs of the specimens, 1a-1b, 2a-2b and 7a-7b, which are identical except for the magnitude of the applied minimum load, among the specimens failing in shear under repeated loading. The fatigue lives of two specimens for each pair are different. That is, the fatigue life of one specimen whose ratio of the minimum load to the maximum is larger is smaller than that of another. And it is observed that the specimen 8b does not fail under the first repeated loading, but fails under the second repeated loading whose ratio of the minimum load to the maximum becomes larger than that of the first one without the change of the maximum load. The test results excluding those of the beams failing in flexure under repeated loading are rearranged in Table 3.2 to confirm the influence of load range once more. The tested values of fatigue lives are classified according to the value of the ratio, V_{max} / V_{cu} , of the applied maximum shear force to the static shear strength. The table shows that the smaller the ratio, V_{min} / V_{max} (called as 'r' hereafter), of the applied minimum shear force to the maximum is, the shorter is the fatigue life among the specimens whose ratios, V_{max} / V_{cu} , are the same. Consequently it can be said that the larger the load range is, the shorter is the fatigue life.

The relationship between the ratio, r , and the ratio, $V_{maxtest} / V_{maxcal}$, of the tested values of fatigue strength to the values calculated by Eq.(3.2) which was proposed without any consideration of the influence of load range is shown in Fig.3.7. The calculated values have a tendency to become smaller than the tested ones with increase of r . Although the fatigue strength can be evaluated approximately from Eq.(3.2) in the case of r smaller than 0.5, the fatigue strength cannot be evaluated in the case of r larger than 0.6. From this fact it can be supposed that the influence of load range appears not so clearly from the previous tests where most of r 's were smaller than 0.5. Finally, a following equation for the prediction of the fatigue strength in shear of beam is proposed with consideration of the influence of load range.

$$\log\left(\frac{V_{max}}{V_{cu}}\right) = m (1 - r|r|) \log N_f \quad (3.3)$$

where $m = -0.036$

The absolute value of r is used in Eq.(3.3) to consider alternating cyclic loading as well, but the applicability of Eq.(3.3) in this case has never been verified experimentally. The relationship between r and the ratio of the tested value of the fatigue strength to that calculated by Eq.(3.3) is shown in Fig.3.8. The average of the ratios, $V_{maxtest} / V_{maxcal}$, is 0.97 and the coefficient of variation is 7.5 %. The correlation cannot be seen in spite of the clear correlation seen in Fig.3.7. Fig.3.9 shows the ratio of the previously tested value of the fatigue strength [1][5][8][9][10][11]

(see Appendix B) to the calculated one. The average is 0.99 and the coefficient of variation is 7.4 %. The correlation can be seen neither. Therefore, the calculated values agree the tested ones nicely.

The relationship between the ratio, $V_{maxtest} / V_{maxcal}$, and cylinder strength, f_c' , ratio of shear span to effective depth, a/d , reinforcement ratio, ρ_w , and effective depth, d , which are parameters in Eq.(3.1) for the calculation of the static shear strength are given in Fig.3.10. The relationship between the ratio, $V_{maxtest} / V_{maxcal}$, and the tested value of fatigue life, N_f , is given in Fig.3.11. Because the ratio, $V_{maxtest} / V_{maxcal}$, is not correlated to any parameter, the applicability of Eq.(3.3) is confirmed from the figures.

The tested values lie scatteringly still around the values calculated by Eq.(3.3), which is proposed newly to predict the fatigue strength more accurately. One of the reasons for the scatter can be thought to be related to the location of diagonal crack in the vicinity of loading points. For example, the location of diagonal cracks in the specimens 3a and 3b are shown in Fig.3.12. In the specimen 3a the compressive concrete portion above the diagonal crack has depth of 70 mm right under the outer edge of loading plate. The specimen 3a does not fail under repeated loading, although the diagonal crack arrives at the inside of loading plates. The specimen fails due to plastic hinge formed between loading plates under static loading after the fatigue test. On the other hand the specimen 3b fails due to the penetration of the diagonal crack at the outside of loading plate under repeated loading. The specimen 3a can endure the repeated load after the diagonal crack propagates fully, because the compressive concrete above the diagonal crack is deep. While the specimen 3b cannot endure the repeated load before the diagonal crack propagates fully, because the compressive concrete is shallow. It can be predicted that this difference of the location of diagonal crack makes the tested values of the fatigue strength for the specimen 3a larger than the calculated one and makes the tested values for the specimen 3b smaller than the calculated one.

Another example shows that the difference of the location of diagonal crack influences the speed of the propagation of diagonal crack. The specimen 4b fails at 2,300 cycle, while the specimen 8b fails at 2,700 cycle. Although the fatigue lives are almost same, the speeds of the propagation of diagonal cracks are quite different. The specimen 4b endures three hundred cycles after the recognition of the diagonal crack, but the specimen 8b endures more than two thousand cycles after the recognition of the diagonal crack and about seventy cycles after the full propagation of the diagonal crack. The compressive concrete has depth of 30 mm and width of 200 mm in the specimen 4b with effective depth of 440 mm, although it has depth of 30 mm and width of 400 mm in the specimen 8b with effective depth of 220 mm. Therefore, the area of the compressive concrete in the specimen 8b is substantially four times as large as that in the specimen 4b. The tested value of the fatigue strength for the specimen 8b with a relatively large compressive portion is larger than the calculated one, but for the specimen 4b both the values are reversed.

Consequently it can be said that the deeper the compressive concrete above diagonal crack at the vicinity of loading points is, the larger is the tested value of the fatigue strength in shear of beam. Because this phenomenon is thought to be observed under static loading as well, it is

necessary to investigate the magnitude of the scatter and the reason why the location of diagonal crack changes.

3.4 Prediction of Failure Modes between Shear and Flexure Failure under Fatigue Loading

Chang - Kesler test results [11] show that the beam with a relatively large a/d , which will fail in flexure under static loading, has two failure modes under fatigue loading, which are a shear mode and a flexure mode due to fatigue fracture of tensile bar. Because the static flexure strength of a reinforced concrete member is designed to be smaller than the static shear strength, it is important to examine whether the member will fail in shear or in flexure under fatigue loading.

In the author's tests the eight specimens with reinforcement ratio of 0.68 % will fail in flexure under static loading. Five of the eight specimens, 1a, 1b, 2a, 5b and 6b, fail in shear and the other two specimens, 5a and 6a, fail in flexure under fatigue loading. The shear fatigue lives of the eight specimens calculated by Eq.(3.3) are compared with the flexure ones in Table 3.3. When the flexure fatigue lives are to be calculated, Eq.(3.4) for the prediction of the S-N curve of bar is derived from the test results and Eq.(3.5) for the calculation of stress range in bar is used.

$$\log \sigma_{sr} = -0.106 \log N_f + 3.10 \quad (3.4)$$

$$\sigma_{sr} = \frac{3 a (V_{max} - V_{min})}{A_s d \{ 3 + p n - \sqrt{(p n)^2 + 2 p n} \}} \quad (3.5)$$

where $p = A_s / (b d)$: reinforcement ratio

$n = E_s / E_c$: ratio of Young's modulus of bar to that of concrete

σ_{sr} : stress range of bar

b : flange width

The failure modes predicted from the comparison between the calculated flexure and shear fatigue lives coincide with the actual modes for all the specimens except for the specimen 5a in which the actual fatigue strength in shear is much larger than the calculated one. All the specimens in Chang - Kesler's tests [11] will have failed in flexure under static loading. The predicted modes coincide with the actual modes for twenty one of the twenty five specimens as shown in Table 3.4. Consequently, it is quite practicable to predict the failure mode of the beam which will fail in flexure under static loading, using Eq.(3.3) for the calculation of the fatigue strength in shear of beam.

It is predicted from Eqs.(3.3) and (3.5) that the extension of the flexure fatigue life becomes much larger than that of the shear fatigue life when load range becomes small, in other words, the ratio, r , becomes large. Therefore, a larger ratio of r tends to cause shear failure under fatigue loading. The S-N curves of beams failing in shear under repeated loading and those of beams failing in flexure under repeated loading are shown in Fig.3.13. These S-N curves are for the pairs of specimens, 5a-5b and 6a-6b, which are tested under the same condition except for the value of r . The figures clearly indicate that the larger the value of r is, the

wider is the range where the shear failure can occur. The predicted failure modes for the specimens 5a and 5b are shear modes. However, the specimen 5a fails in flexure, because it is probable that the scatter of the fatigue strength in shear of beam causes the fatigue strength in shear to become smaller than the fatigue strength in flexure. The predicted failure modes for the specimens 6a and 6b are flexure and shear modes respectively. The test result for the specimen 6a coincides with the prediction. While it can be considered that the specimen 6b does not fail under the first repeated loading because the cyclic number of the first repeated loading is smaller than the predicted fatigue life (Table 3.3).

It is predicted from Eqs.(3.3) (3.4) and (3.5) that the other factors influencing failure mode than the ratio, r , are the fatigue strength of bar, cylinder strength, f_c' , ratio of shear span to effective depth, a/d , reinforcement ratio, p or p_w , and effective depth, d .

4. STIRRUP STRAIN UNDER FATIGUE LOADING WITH CONSTANT MAXIMUM AND MINIMUM LOAD

Stirrup strains under fatigue loading increase due to the decrease of shear force carried by concrete as mentioned in Sec.1. The decrease is assumed to be essentially the same as that of the fatigue strength in shear of beams without web reinforcement, which is evaluated by Eq.(3.3). From this assumption the equation for the calculation of stirrup strain under fatigue loading with constant maximum and minimum load is obtained according to the procedure mentioned in this section.

4.1 Stirrup Strain at the Applied Maximum Shear Force Larger than the Shear Capacity of Concrete

The applied maximum shear force, V_{max} , is carried by two components, V_s and V_c , where V_s is the shear force carried by the assumed truss with 45° diagonals and V_c is the one carried by concrete. Thus Eq.(4.1) is obtained.

$$V_s = V_{max} - V_c \quad (4.1)$$

However Eq.(4.1) is for the part of a beam where the influences of supports or loading points are negligibly small. For the part where these influences exist, V_s is lightened [13].

$$V_s = \beta_x (V_{max} - V_c) \quad (4.2)$$

where β_x : a coefficient for each stirrup to cover the influence of support and loading point on reducing stirrup strains as shown in Fig.2.2

The V_c' expressed as in Eq.(4.3) is the shear force transferred to the support or the loading point directly. That is, V_c' is transferred without a chord of truss mechanism, and this should be distinguished from the V_c .

$$V_c' = (1 - \beta_x) (V_{max} - V_c) \quad (4.3)$$

Then the following equation is derived.

$$V_{max} = V_s + V_c + V_c' \quad (4.4)$$

where $V_s = A_w E_w \epsilon_w z/s$

The decrease of V_c is assumed to be the same as that of the fatigue strength in shear of beam without web reinforcement, so that Eq.(4.5) is derived from Eq.(3.3).

$$V_c = V_{co} 10^{-0.036 (1 - r |r|) \log N} \quad (4.5)$$

where $r = V_{min} / V_{max}$
 N : loading cycles

When V_{max} is constant, the value of V_s should increase with the decrease of V_c as indicated in Eq.(4.2). Finally, the average of stirrup strains at

the applied maximum shear force is expressed by Eq.(4.6).

$$\bar{\epsilon}_{swmax} = \frac{\bar{\beta}_x \{ V_{max} - V_{co} 10^{-0.036 (1 - r|x|) \log N} \}}{A_w E_w z/s} \quad (4.6)$$

where $\bar{\epsilon}_{swmax}$: average of stirrup strains at applied maximum shear force
 $\bar{\beta}_x$: average of β_x 's as shown in Table 2.1

The average strains calculated by Eq.(4.6) are compared with the tested ones in Figs.4.1, 4.2, 4.3, 4.4 and 4.5. Eq.(4.6) is confirmed to be applicable to all the T-beam tests with changing minimum load (Figs.4.1, 4.2, 4.3 and 4.4) as well as the rectangular beam tests [1] with constant minimum load (Fig.4.5). Fig.4.2 shows the cases in which the maximum load is changed for one beam with constant minimum load. Fig.4.3 shows the case in which the minimum load is changed for one beam with constant load range producing shear force range, V_r . Fig.4.4 shows the case in which the load range is changed for one beam as well as the maximum and minimum loads. The solid lines in these figures derived from Eq.(4.6) agree nicely with tested values. The range of V_{max} / V_{min} (1.23 to 11.1) and V_{max} / V_{co} (1.42 to 2.94) covers all the practical cases. Eq.(4.6) is confirmed to be applicable also to the cases in which the influence of support or loading point is either small ($a/d = 4.0$) or large ($a/d = 2.0$).

4.2 Influence of Load Range on Stirrup Strain

It is shown by Eq.(4.6) that the larger the ratio of r is, the smaller is the increase of stirrup strain, and that the increase does not exist, when the r is equal to one. Actually stirrup strain increases even under sustained loading (see Sec.5.2), because the duration of the loading causes stirrup strain to increase as well as the loading cycles. Eq.(4.6) is an equation for the prediction of the increase mainly due to the loading cycles, so that the calculated increase becomes zero when the r is one. However, Eq.(4.6) is applicable to the practical case, because it is made clear by the test results that the increase due to the duration of the loading is smaller than the one due to the loading cycles in usual cases.

The increase of average stirrup strains in FS9(I), whose value of V_{min} is very close to that of V_{max} , is smaller than those in other cases as shown in Fig.4.1(e). The calculated line represents the small increase nicely. It may be said that Fig.4.1(e) shows the influence of load range on the decrease of V_c clearly.

4.3 Stirrup Strain at the Applied Maximum Shear Force Less than the Shear Capacity of Concrete

It was observed from the Farghaly's tests that stirrup strains hardly increased in the early stage of loading, and stirrup strains began to increase noticeably after some cycles (Fig.4.6(g)(h)(i)), when the applied maximum shear force was less than the shear capacity of concrete ($V_{max} < V_{co}$). In the author's tests it is observed that the average of strains in stirrups increases little until two or three diagonal cracks occur in the same shear span and one of them is so long to cross some

stirrups. The gradual propagation of diagonal cracks during fatigue loading is something like a slow-motion of the propagation under static loading.

The idea on which Eq.(4.6) is based can be extended to estimate not only the number of cycles, N_c , when stirrup strains begin to increase but also the increase of the strains thereafter. The value of the shear capacity of concrete is the same as that of shear force, V_{co} , carried by concrete at the initial loading, and decreases like the fatigue strength in shear of beam without web reinforcement until N_c cycles. After N_c cycles the applied maximum shear force, V_{max} , is carried by two components V_s and V_c , and the decrease of V_c can be evaluated from Eq.(4.5) in this case as well. Eq.(4.7) is then obtained.

$$V_{max} = V_{co} 10^{-0.036 (1 - r |r|) \log N_c} \quad (4.7)$$

where N_c : number of cycles when stirrup strains begin to increase

Eq.(4.7) is transposed to Eq.(4.8).

$$\log N_c = - \frac{1}{0.036 (1 - r |r|)} \log \frac{V_{max}}{V_{co}} \quad (4.8)$$

The stirrup strain at the applied maximum shear force can be calculated by Eq.(4.6), substituting for N the total loading cycles from the start of fatigue loading. The calculated values, which are compared with the experimental values in Fig.4.6, can be said to represent the actual phenomena nicely.

4.4 Stirrup Strain at the Applied Minimum Shear Force with Consideration of Residual Strain (Strain Range of Stirrup)

In order to make the fatigue strength of stirrup clear, the stress range in stirrup under fatigue loading need be determined. The equation for the calculation of the stress range was temporarily proposed as a result of observation which showed that stirrup stress changed linearly with the change of load [1]. But the equation was incomplete due to the neglected residual strain. In the author's study the stirrup strain at the applied minimum load is examined in detail, and an improved equation is derived.

Fig.4.7(a) shows the typical relationship between applied shear force, V , and average strain, $\bar{\epsilon}_w$. Although $V-\bar{\epsilon}_w$ curve of the first cycle is a straight line at unloading, the one of the second cycle is a folded line at reloading. The both curves of the 10^4 cycle at unloading and reloading approach each other and are folded lines. The shear force at the folded point becomes larger with loading cycles. In another specimen, however, no folded point is observed, when the applied minimum shear force is larger than the shear force at the folded point (Fig.4.7(b)).

The applied minimum shear force is generally thought of to be larger than the shear force at the folded point, so that the $V-\bar{\epsilon}_w$ curve can be roughly said to be a straight line. The inclination of the straight line

becomes smaller with increase in number of cycles of fatigue loading, so that the strain range corresponding to the same range of shear force becomes larger. And a significant increase of the residual strain is observed during fatigue loading.

Fig.4.8 shows the relationship between applied shear force, V , and width of diagonal crack, w , in the specimen FS11. Fig.4.9 shows the relationship between shear force and stirrup strain, $\bar{\epsilon}_w$, recorded very near to the point where the crack width is measured. Both the measurement points are shown in Fig.4.10 together with all the diagonal cracks in the same shear span. From Figs.4.8 and 4.9 it can be seen that the relationship V - w is very similar to that of V - $\bar{\epsilon}_w$. The width is closely proportional to the strain as shown in Fig.4.11. The strain seems to increase during fatigue loading due to increase of crack width, and the residual strain is a result of unclosed diagonal crack. When the shear force is smaller than a certain level, there is little change in the strain with load, which corresponds to crack width. The points D-C-E in Fig.4.8 represent the increase of crack width with the increase of maximum shear force between two stages of fatigue loading. It seems that the rate of increase of crack width changes at the point C. The rate of increase between the points C and E is larger than between the points D and C. The rate of increase of stirrup strain is illustrated by the corresponding points D'-C'-E' in Fig.4.9. It is observed that between the points C and E both crack widths and lengths increase, although only crack widths increase, but crack lengths does not increase between the points D and C.

The observed V - $\bar{\epsilon}_w$ relationship in Fig.4.7 can be explained by assuming that V - $\bar{\epsilon}_w$ curve is always on the line between the point ($V_{\max}$, $\bar{\epsilon}_{w\max}$) representing the strain at the applied maximum shear force and the fixed point ($-V_{co}$, 0). The strain range, $\bar{\epsilon}_{wr}$, and residual strain, $\bar{\epsilon}_{wo}$, are assumed to increase during fatigue loading, because the maximum strain, $\bar{\epsilon}_{w\max}$, increases proportional to the logarithm of loading cycles according to Eq.(4.6). The assumed lines agree nicely with the actual V - $\bar{\epsilon}_w$ curve not only in the author's tests (Fig.4.7) but also in a previous test [14] (Fig.4.12). In this test T-beams with shear span of 1,800 mm and a/d equal to 2.5 were made. The specimens were tested under cyclic loading in which the maximum shear force was gradually increased to a certain load level after which the shear force was decreased to zero and was again increased to a load level higher than the previous maximum shear force.

From the assumption, the following equations are derived.

$$\bar{\epsilon}_{wr} = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{co}} \bar{\epsilon}_{w\max} \quad (4.9)$$

$$\bar{\epsilon}_{wo} = \frac{V_{co}}{V_{\max} + V_{co}} \bar{\epsilon}_{w\max} \quad (4.10)$$

where $\bar{\epsilon}_{wr}$: average of stirrup strain ranges
 $\bar{\epsilon}_{wo}$: average of residual stirrup strains
 $\bar{\epsilon}_{w\max}$ is given by Eq.(4.6), and the influence of the folded point in the V - $\bar{\epsilon}_w$ curve is neglected.

The calculated values are compared with the tested values in Figs.4.13, 4.14 and 4.15. The author's tested values are shown in Figs.4.13 and 4.15, and Farghaly's ones are in Fig.4.14. The equation for the calculation of strain range of stirrup nicely expresses the average behavior of stirrup in the author's tests. While the calculated values are generally smaller than the measured values in Farghaly's tests. In general it can be said that the calculated values tend to be smaller than the actual values in the cases where the applied maximum shear force is about the same as or less than the shear capacity of concrete or the shear reinforcement ratio is relatively small. This tendency causes the calculated values to be smaller in Farghaly's tests. However, the simple equation, Eq.(4.9) is considered to be applicable to practical cases.

4.5 Scatter of Measured Strain in Each Stirrup from Calculated One

Measured strain in each stirrup lies scatteringly around the calculated one in each stirrup. The observed scatter is influenced by propagation of diagonal crack. The fuller the propagation is, the smaller is the scatter. The scatter of stirrup strain range at the last measurement before the first finding of the damaged gauge is shown in Fig.4.16. The average ratios of tested value to calculated one are 1.09 ($a/d = 4.0$) and 1.00 ($a/d = 2.0$), and the coefficients of variation are 38 % ($a/d = 4.0$) and 28 % ($a/d = 2.0$). Because the ratios tend to be smaller than one in the cases of stirrups near to loading points and larger than one in the cases of stirrups near to supports, the assumed values of β_x may be necessary to be corrected.

In Farghaly's tests the scatter of measured strains is observed to be much larger than that in the author's tests. From this fact the small number of stirrups in a shear span is considered as one of the reasons for the large scatter.

5. STIRRUP STRAIN UNDER FATIGUE LOADING WITH VARIED LOAD RANGE (STIRRUP STRAIN UNDER GENERAL VARIABLE LOADING)

Since actual structures are not subjected to fatigue loading with constant maximum and minimum load but generally subjected to fatigue loading with varied load range and/or sustained loading, it is necessary to follow the behavior of web reinforcement under fatigue loading with varied load range. Therefore, each specimen was subjected to fatigue loading with varied load range. Especially the specimen FS11 was subjected to sustained loading as well.

5.1 Equivalent Cycles of Loading

When general fatigue loading is divided into some sets of repeated loading with constant maximum shear force, each set is named the first repeated loading, the second repeated loading and so on according to the sequence of loading. Although the stirrup strain after subjected to the first repeated loading can be calculated as mentioned in Sec.4, the strain during the second repeated loading may not be calculated. But, this strain can be estimated if it is assumed that loading history of the first repeated loading is equivalent to certain cycles (N_{eq}) of repeated loading whose maximum and minimum shear force are equal to those of the second repeated loading. By the similar way stirrup strain under the subsequent repeated loading can be estimated.

To evaluate the equivalent loading cycles a newly developed idea is proposed. When stirrup strains produced by applied shear forces whose magnitude is same are same in identical beams subjected to different loading historys, the strains during subsequent loading are essentially same in spite of the difference of the previous loading historys. In other words, the behavior of a stirrup after subjected to certain loading, static or fatigue or sustained loading, is only dependent on the strain corresponding to the shear force applied. Consequently any loading history can be substituted by equivalent fatigue loading with the constant maximum and minimum load.

5.2 Stirrup Strain under Fatigue Loading with Varied Load Range

The line(b) in Fig.5.1 is drawn between the fixed point ($-V_{co}, 0$) and the point representing the strain of stirrup after subjected to N_1 cycles of the first repeated loading whose maximum and minimum shear forces are V_{max1} and V_{min1} . When the maximum shear force, V_{max2} , of the second repeated loading is below the point A in Fig.5.1, the points representing the strains at the beginning of the second repeated loading are on the line(b), as mentioned in Sec.4.4. The strain, $\bar{\epsilon}_w$, at V_{max2} is calculated by the following equation.

$$\bar{\epsilon}_w = \frac{V_{max2} + V_{co}}{V_{max1} + V_{co}} \bar{\epsilon}_{wmax1} \quad (5.1)$$

where $\bar{\epsilon}_{wmax1}$ is the maximum stirrup strain calculated by Eq.(4.6) with $V_{max} = V_{max1}$, $V_{min} = V_{min1}$ and $N = N_1$.

Based on the newly developed idea, it is assumed that the state of strain after subjected to N_1 cycles of the first repeated loading is equivalent to the state of strain after subjected to the equivalent cycles, N_{eq} , of loading whose maximum and minimum shear forces are equal to those of the second repeated loading, V_{max2} and V_{min2} .

The assumption is expressed by Eq.(5.2).

$$\frac{\bar{\beta}_x \{ V_{max2} - V_{co} 10^{-0.036 (1 - r_2 |r_2|) \log N_{eq}} \}}{A_w E_w z/s} = \frac{V_{max2} + V_{co}}{V_{max1} + V_{co}} \frac{\bar{\beta}_x \{ V_{max1} - V_{co} 10^{-0.036 (1 - r_1 |r_1|) \log N_1} \}}{A_w E_w z/s} \quad (5.2)$$

The equivalent cycles, N_{eq} , is obtained by transposing Eq.(5.2).

$$\log N_{eq} = - \frac{1}{0.036 (1 - r_2 |r_2|)} \log \{ V_{max2} - V_{max1} + (V_{max2} + V_{co}) 10^{-0.036 (1 - r_1 |r_1|) \log N_1} \} \quad (5.3)$$

$$\text{where } r_1 = V_{min1} / V_{max1} \\ r_2 = V_{min2} / V_{max2}$$

When the maximum shear force, V_{max1} , of the first repeated loading is less than that of the second repeated loading, the equivalent cycles can be calculated by the similar way. After subjected to N_2 cycles of the second repeated loading, stirrup strain, $\bar{\epsilon}_{wmax2}$, at V_{max2} can be calculated by Eq.(4.6), substituting $N_{eq}+N_2$ for N . This means that the rate of increase in $\bar{\epsilon}_{wmax2}$ is very small if the order of N_2 is smaller than that of N_{eq} (Fig.5.2). Solid circles in Fig.5.3 show the observed relationship between $\bar{\epsilon}_{wmax2}$ and $\log N_2$ in the cases of beams, in which N_1 is one, and the solid lines in the figures are calculated by Eq.(4.6), substituting $N_{eq}+N_2$ for N . Fig.5.4 shows the cases in which V_{min2} is less than V_{min1} although V_{max2} is equal to V_{max1} . In the figures the small increase is recognized in the early stage of the second repeated loading, and the calculated lines express nicely the small increase. When the increase of the average strain at the applied maximum shear force is small, it is easy to predict the small increase of the strain range (see Fig.5.5).

When the maximum shear force, V_{max2} , of the second repeated loading is above the point A in Fig.5.1, the point representing the stirrup strain at V_{max2} is on the line(a). The line(a) represents the relationship between applied shear force and stirrup strain under static loading. Therefore, there is no influence of the previous fatigue loading on the stirrup strains under the second repeated loading, and Eq.(4.6) can be used without any modification for calculating $\bar{\epsilon}_{wmax2}$ (Fig.5.6). Measured and calculated strains in this case are shown in Figs.4.1(a)(b)(c)(d)(g), 4.2, 4.3 and 4.4.

The comparison between the calculated stresses and measured ones in Ruhnau's test [3] is shown in Fig.5.7. This case shows that the applied maximum shear forces of the subsequent repeated loading are larger than those of the previous ones, and are below the point A in Fig.5.1.

Stirrup strains increase clearly, even if sustained load is applied. It is observed in the specimen FS11 that the rate of increase is in proportion to logarithm of duration of the loading as shown in Fig.5.8. When a stirrup, whose strain has increased due to sustained loading, is subjected to fatigue loading, it is observed that the increase of strain is small in the early stage of the fatigue loading. When sustained loading is applied after some cycles of fatigue loading, the increase of stirrup strain is hardly recognized. Therefore, the increase of strain due to sustained loading is considered essentially same as the one due to fatigue loading. Considering this fact, loading speed may be one of the important factors. But the effect of sustained loading, which occurs during a usual fatigue test, is generally negligible.

5.3 Design Recommendation

Although the procedure is proposed to calculate stirrup strain under general variable loading, the procedure is too complicated for practical use in the cases with the changing maximum and minimum loads. Furthermore, it is difficult to modelize the general variable loading truly for design. However, a simple procedure for design is desirable actually.

The important characteristics for the design of stirrups under fatigue loading are considered that stirrup strain is not developed in proportion to the applied shear force under fatigue loading as well as static loading, and that stirrup strain is influenced greatly by the previous over loading as shown in Figs.5.2 and 5.3. These characteristics are quite different from those of longitudinal bar. Therefore, it is necessary to determine the magnitude of the design load for the fatigue limit state and its cyclic number separately for stirrup and longitudinal bar.

For example a following design procedure is thought out. A certain large load, V_{maxo} , which acts about once during a service lifetime of structure, is estimated from the actual distribution of the magnitude of load. Stirrup strain is then calculated, supposing that the large load has acted already. Because in this case the number of the equivalent loading cycles for the other relatively small loads acting frequently is generally very large, the increases of stirrup strain due to the small loads are negligible. Following simple equations are then obtained for design.

$$\sigma_{wrd} = \frac{\beta x (V_{maxo} - V_{co})}{A_w z/s} \cdot \frac{V_{rd}}{V_{maxo} + V_{co}} \quad (5.4)$$

$$\sigma_{wpd} = \frac{\beta x (V_{maxo} - V_{co})}{A_w z/s} \cdot \frac{V_{pd} + V_{co}}{V_{maxo} + V_{co}} \quad (5.5)$$

where V_{maxo} : design value of maximum of the applied maximum shear force
 V_{rd} : design value of shear force range
 V_{pd} : design value of permanent shear force

σ_{wrd} : design value of stirrup stress range produced by V_{rd}
 σ_{wpd} : design value of stirrup stress produced by V_{pd}

The above procedure is considered to be available in the cases where the distribution of the magnitude of load is wide like that for highway bridge. On the other hand, in the cases where the distribution is narrow like that for railroad bridge it is considered to be necessary to modelize the actual variable loading as some sets of repeated loading and to calculate stirrup strain as mentioned in Sec. 5.2.

6. BEAM FAILURE DUE TO FATIGUE FRACTURE OF STIRRUPS

The previous reports pointed out that a reinforced concrete beam sometimes failed in shear under fatigue loading due to the fracture of stirrup even if the applied maximum shear force was much smaller than the ultimate static strength. In the author's tests this type of beam failure is observed as well. In this section the characteristics of fractured stirrup and the fatigue strength of beam are mentioned.

6.1 Fatigue Fracture of Stirrup

Specifics of fatigue fractured stirrups in each specimen are given in Table 6.1. Fatigue life of the first fractured stirrup shown in this table is the loading cycle at the measurement previous to the recognition of the first fracture. However, the values of the fatigue lives are not so reliable, because the relatively small spacing of stirrups prevents the indication of the fractured stirrup from appearing and gauges are so damaged to be unavailable before the fracture.

Nine of the eleven beams fail in shear due to fatigue fracture of stirrups. The applied maximum shear forces are about 60 % of the calculated static flexure strength. Two beams fail under static loading after fatigue tests. The specimen FS1 fails in flexure and the ultimate strength is 103 % of the calculated ultimate strength. The specimen FS9 fails in shear and the ultimate strength is 97 % of the calculated ultimate strength.

Fatigue fractured stirrups are always found in the shear span where a/d is 2.0, and none of fracture occurs in the shear span where a/d is 4.0. Fatigue fractured stirrups are also found in the specimen FS9 which does not fail under fatigue loading. After the first fatigue fracture of stirrup, each specimen resists two hundred thousand to three million cycles. Following the first fracture, specimens fail due to the fracture of four to ten more legs of stirrups. Although the total number of fractured stirrups in each specimen does not seem to relate to the magnitude of fatigue loading or the fatigue lives of the beams, it is recognized that the shorter the fatigue life of beam is, the fewer is the loading cycles after the first fracture.

The fatigue fracture occurs not only at lower bent portion where stirrup is bent around longitudinal bars but also at middle straight portion and upper hook portion. The portion of fatigue fracture is generally along the main diagonal crack which causes the failure of beam (see Fig.6.1). Many fractured legs at the middle straight portion of stirrup are found in the center of shear span, and those at the upper hook are found in the vicinity of the loading point. This fact is different from Farghaly's tests [1] in which all the fatigue fracture except for one occurred at the lower bend. The following points can be considered as the reasons for the difference.

(1) The effective depth of the specimen in the author's tests is about two times as high as that in Farghaly's tests. A higher effective depth prevents the deterioration of bond from extending all over the stirrup. Therefore, the stress condition at the lower bend may be considerably lightened in the author's tests, if the point of the diagonal crack crossing is far from the lower bend.

(2) Seven pairs of stirrups are placed in one shear span with a/d of 2.0 in the author's tests, although only two or three pairs were placed in one shear span with a/d of 2.5 in Farghaly's tests. The ratio of spacing to effective depth influences on diagonal cracking a great deal. Many diagonal cracks occur uniformly over the shear span in the author's tests, while only a few diagonal cracks occurred in Farghaly's tests, aiming at the vicinity of the crossing points between stirrups and longitudinal bars. This fact seems to mean that the probability at which stirrup crossed diagonal crack in Farghaly's tests is higher than in the author's tests. The stress condition of stirrup in the vicinity of the lower bend in Farghaly's tests is possibly more severe.

(3) The ratios of the applied maximum shear force to the shear capacity of concrete, V_{co} , at the beam failure are 2.69 to 2.97 in the author's tests and 0.87 to 1.57 in Farghaly's tests. It can be supposed that concrete beams with the higher ratio are destroyed more extensively by diagonal cracks. Thus the portion of stirrup in the vicinity of diagonal crack may be damaged more severely in the author's tests.

(4) It is considered that the distance between the neutral axis of cross section and the upper hook of stirrup in the author's tests is shorter than in Farghaly's tests, and that the confinement surrounding the upper hook is inferior in the author's tests.

6.2 Fatigue Strength of Beam

It is considered that the fatigue strength of beam failing in shear due to stirrup fracture is related to the fatigue strength of stirrup. However, the measured strains show no more information except for that the fatigue strength of stirrup lies between the fatigue strength of the straight bar (100 %) and that of the bent bar (50 % , which is according to some previous reports [2][15]) as shown in Fig.6.2. Because it is difficult not only to recognize the fracture from the measured strains, but also to clear the relationship between the strain at the measured point and the strain at the fractured point.

On the other hand the closed relationship between calculated value of average stress range of stirrups at the the failure of beam and tested value of fatigue life of beam is found as shown in Fig.6.3. The average of the stress ranges is 70 % of the fatigue strength of the straight bar. The coefficient of variation is 0.96 %. This figure clearly indicates that the fatigue strength of beam can be evaluated from the average of stress range in stirrups calculated by Eq.(4.9).

The relationship between calculated value of average stress range of stirrups at the the failure of beam and tested value of fatigue life of beam in Farghaly's tests is shown in Fig.6.4. The average is 25 % which is much less than that in the author's tests. The coefficient of variation is 3.7 %, so that the scatter is much larger. One of the reasons for the difference of the averages can be considered that some factors, which cause the difference between the actual stirrup strain at the fractured point and the calculated average of stirrup strains to be larger than in the author's tests, exist in Farghaly's tests. The factors are (1) the location of stirrup which does not produce the same strain among all the stirrups, (2) the small number of stirrups which causes the scatter of stirrup strain to be large and (3) the lower effective depth which causes the stress condition at the lower bend to be more severe. The other reason can be

thought that the calculated strain ranges are generally smaller than the measured ones in Farghaly's tests (see Sec.4.4).

The difference of the fatigue strength of stirrup due to the difference of the radius of bend is not cleared, although Farghaly concluded that the fatigue strength of stirrup was 40 % in the cases where the stirrups were supported partially by longitudinal bars because of the large radius of bend, and 66 % in the cases where the stirrups were supported fully because of the small radius of bend. And no influence of the radius of bend on the fatigue strength of beam is observed.

6.3 Design Recommendation

Although it was mentioned in the previous section that the beam failure due to fatigue fracture of stirrups can be evaluated from the proposed equation, the S-N curve for the beam failure changes because of the change of the fatigue life of the first fractured stirrup when the detail of shear reinforcing changes. It is necessary to clarify the mechanism from the first fracture of stirrup to the beam failure, in order to evaluate more correctly the fatigue life of a general beam. On the other hand the applied stress of stirrups can be calculated by the author's proposed equations in the case of no fractured stirrup. The applied stress at the first fracture is thought to be able to be evaluated from the proposed equations in spite of the useless information from the strains measured with gauges. Considering these facts, it is thought to be proper that the first fracture of stirrup, which does not mean the beam failure but influence it greatly, is regarded as the fatigue limit state in design.

In design procedure the applied stress range calculated as in Sec.5.3 is checked by comparing with the fatigue strength of stirrup. There is not so many information on the fatigue strength. But the first fracture occurs at bent portion in most of the cases. Therefore, the fatigue strength is considered to be 50 % of that of the straight bar according to the fatigue strength of bar at the bent portion. In future it is necessary to estimate the fatigue strength more correctly and the scatter of the applied stress (see Sec. 4.5).

The strains of stirrups increase under fatigue loading. However, the most of increase occurs in the early stage of repeated loading, since the increase is proportional to logarithm of loading cycles. The applied stress can be assumed to be constant and calculated by the proposed equations, substituting for N the design value of loading cycles, which is one million cycles for example.

7. CONCLUSIONS

Fatigue tests of eleven T-beams with stirrups and sixteen rectangular beams without web reinforcement were carried out. In the T-beam tests the maximum and/or minimum load was changed for each specimen and all the stirrup strains were measured in detail to investigate the behavior of stirrup under general variable loading. The rectangular beam tests were carried out to support the assumption for the explanation of the behavior of stirrup under fatigue loading. In this tests load range was made a main parameter to investigate the influence of the load range on the fatigue strength in shear of beam without web reinforcement.

As test results, following conclusions are obtained.

- (1) In the rectangular beam tests twelve of the sixteen specimens fail in shear under repeated loading and two of the others fail in flexure due to fatigue fracture of tensile bar.

The tested values of the fatigue lives of beams show that load range influences the fatigue strength in shear of beam without web reinforcement in the case of the relatively large ratio of shear span to effective depth, a/d . The r , which represents the ratio of the applied minimum shear force to the maximum, the larger is, the longer is the fatigue life of beam without web reinforcement failing in shear. Especially when the r is larger than 0.6, the fatigue life becomes longer noticeably.

The fatigue strength in shear of beam without web reinforcement in the case of the relatively large a/d can be predicted by using Eq.(3.3) proposed with consideration of the influence of load range.

$$\log\left(\frac{V_{\max}}{V_{cu}}\right) = -0.036 (1 - r|r|) \log N_f \quad (3.3)$$

The values calculated by Eq.(3.3) coincide nicely with the previous tested values [1][5][8][9][10][11], so that the S-N curve of beam without web reinforcement failing in shear is said to be standardized by Eq.(3.3).

It can be thought that one of the reasons for the scatter of the tested values of the fatigue strength around the values calculated by Eq.(3.3) is to be related to the location of diagonal crack in the vicinity of loading points. The compressive concrete portion above the diagonal crack the deeper is, the larger is the ratio of the tested value to the calculated one. (see Sec.3.3)

Using Eq.(3.3), it is quite practicable to predict whether the failure mode of beam which will fail in flexure under static loading is a shear failure or a flexure one due to the fatigue fracture of tensile bar. It is predicted from Eq.(3.3) that the larger r tends to cause shear failure under fatigue loading, and this tendency is confirmed by the test results. (see Sec.3.4)

- (2) Considering Eq.(3.3) for the prediction of the fatigue strength in shear of beam without web reinforcement, Eq.(4.6) is newly proposed for the calculation of the average stirrup strain at the applied maximum shear

force under fatigue loading.

$$\bar{\epsilon}_{wmax} = \frac{\bar{\beta}_x \{ V_{max} - V_{co} 10^{-0.036 (1 - r|r|) \log N} \}}{A_w E_w z/s} \quad (4.6)$$

Eq.(4.6) is derived from the previous assumption [1] that the increase of stirrup strain was caused by the decrease of shear force carried by concrete, which was essentially the same as the decrease of the fatigue strength in shear of beam without web reinforcement.

It is proved experimentally that Eq.(4.6) does not depend on the shape of cross section nor on the shear span depth ratio, and is applicable not only to the cases of the constant minimum load but also to those of the constant maximum load or the constant load range. (see Sec.4.1)

(3) The influence of load range on the increase of stirrup strain is made clear by the proposed Eq.(4.6). The larger the ratio of r is, the smaller is the increase. This fact is confirmed by the test result of the beam in which the minimum load is very closed to the maximum. (see Sec.4.2)

(4) When the applied maximum shear force is smaller than the shear capacity of concrete, the stirrup strain does not increase at the early stage of fatigue loading but begins to increase after the specific cycles of fatigue loading. This phenomena can be explained by extending the assumption on which Eq.(4.6) is based. The stirrup strain can be calculated also by Eq.(4.6), where N is the total loading cycles from the start of the fatigue loading. (see Sec.4.3)

(5) Eq.(4.9) for the calculation of the average of strain ranges in stirrups is newly proposed to increase the accuracy of the previous one [1].

$$\bar{\epsilon}_{wr} = \frac{V_{max} - V_{min}}{V_{max} + V_{co}} \bar{\epsilon}_{wmax} \quad (4.9)$$

This equation is derived from such observation that the applied shear force - stirrup strain relationship can be considered practically linear at unloading and reloading and the line representing this relationship is assumed to cross the shear axis at the fixed point. (see Sec.4.4)

(6) A procedure to calculate the average of strains in stirrups under general variable loading is proposed. This procedure is based on the proposed Eqs.(4.6) and (4.9) and such a newly developed idea that the strains during the subsequent loading are essentially same in spite of the difference of the previous loading historys, when stirrup strains produced by the applied shear forces whose magnitude is same are same in identical beams subjected to different loading historys. Consequently any loading history can be substituted by equivalent fatigue loading with the constant maximum and minimum load. (see Sec.5.1)

The cyclic number of the equivalent fatigue loading is calculated by

Eq.(5.3).

$$\log N_{eq} = - \frac{1}{0.036 (1 - r_2 |r_2|)} \log \{ V_{max2} - V_{max1} - 0.036 (1 - r_1 |r_1|) \log N_1 + (V_{max2} + V_{co}) 10 \} \quad (5.3)$$

The applicability of the procedure is confirmed by the author's and the previous tests. (see Sec.5.2)

(7) Considering the characteristics of stirrup strain under fatigue loading, a temporary design procedure is recommended to calculate stirrup stress under general variable loading. (see Sec.5.3)

(8) In the T-beam tests nine of the eleven specimens fail in shear under fatigue loading whose maximum shear force is about 60 % of the static strength, although these specimens under static loading will have failed in flexure at the load when stirrups yield. In all the cases shear failure occurs due to fracture of several legs of stirrups. The number of fractured legs in each specimen does not seem to be related to the fatigue life of the beam. It is recognized that the shorter the fatigue life is, the fewer is the subsequent loading cycles after the first fracture of a stirrup. About half of the fractured legs are found at the middle straight portion while all the fractured legs were found at the bent portion in the previous tests [1][2][6][7]. It can be said that most of the fatigue fracture occurs along the main diagonal crack in the author's tests. (see Sec.6.1)

(9) There is large scatter of measured strain in each stirrup from the calculated one in each stirrup (see Sec.4.5). The measured strains show only that the fatigue strength of stirrup lies between the fatigue strength of the straight bar and that of the bent bar, because it is difficult to clear the relationship between the strain at the measured point and the strain at the fractured point.

However, it is made clear that the fatigue strength of beam can be evaluated from the calculated average of stress ranges in stirrups as shown in Fig.6.3.

Both the calculated and measured strains do not show information about the influence of the radius of bend of stirrup on the fatigue strength of stirrup nor beam. (see Sec.6.2)

(10) A design method for stirrup under fatigue loading is presented, considering that the fatigue limit state is the first fracture of stirrup. In the method the applied stress of stirrup is calculated as mentioned in Sec.5.3 and the fatigue strength of stirrup is considered tentatively to be 50 % of that of the straight bar. (see Sec.6.3)

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TABLES

Table 2.1 Properties of T-beams with stirrups

Specimens	V _{co}		V _y (3)		V _f (4)	
	Opt. (1) kN	Cal. (2) kN	a/d=4 kN	a/d=2 kN	a/d=4 kN	a/d=2 kN
FS1, FL2 FS3, FL4	96	99	229	428	229	459
FS5, FL6 FS7, FL8	97	101	230	429	231	462
FS9, FL10 FS11	106	106	239	438	235	470

(1) Tested values derived from applied shear force - stirrup strain curves

(2) $V_{co} = 0.20 f_c'^{1/3} (1 + \beta_p + \beta_d) bwd$ (f_c' : MPa)

where $\beta_p = \sqrt{100 A_s / (bwd)} - 1$,

$\beta_d = (1000/d)^{1/4} - 1$ (d: mm)

(3) $V_y = V_{co} + A_w f_{wy} (z/s) / \bar{\beta}_x$

where $\bar{\beta}_x$: average of β_x 's, $\bar{\beta}_{xs} = 155.5 \text{ mm}$ (a/d=4.0)

$\bar{\beta}_{xs} = 62.3 \text{ mm}$ (a/d=2.0)

(4) $V_f = A_s f_y (1 - 0.6 p f_y / f_c') d / a$

where bw : web width, a : shear span

Table 2.2 Properties of materials of T-beams with stirrups

Concrete		Stirrup				Longitudinal bar			
f_c' MPa	M.S. mm	Φ mm	A_w mm ²	f_{wy} MPa	E_w 10 ³ MPa	Φ mm	A_s mm ²	f_y MPa	p %
24.3 25.5 29.8	20	10	142.7	383.2	185.7	25	3040	342.2	1.40

f_c' : cylinder strength, M.S. : maximum size of coarse aggregate,

Φ : diameter, A_w , A_s : cross-sectional area,

f_{wy} , f_y : yield strength, E_w : Young's modulus,

$p = A_s / (bd)$: reinforcement ratio, b : flange width

Table 3.1 Test results of rectangular beams
without web reinforcement

Specimens	a mm	d mm	b mm	p _w %	f _c ' MPa	V _f (1) kN	V _{cu} kN	V _{max} kN	V _{min} kN	V _{min} V _{max}	V _{max} V _{cu}	N _f ×10 ⁴	Notes
1a	1540	440	200	0.68	33.4	65	69	49	4.9	0.1	0.72	0.05	
1b	1540	440	200	0.68	33.4	65	69	49	30	0.6	0.72	314.0	
2a	1540	440	200	0.68	45.5	66	76	46	4.6	0.1	0.61	1.86	
2b	1540	440	200	0.68	45.5	66	76	46	28	0.6	0.61	447.9<	(2)
								74			0.97		(3)
3a	1540	440	200	1.67	33.4	138	99	60	24	0.4	0.61	1032.0<	(2)
								82			0.83		(4)
3b	1540	440	200	1.67	33.4	138	99	71	36	0.5	0.72	0.07	
4a	1540	440	200	1.67	45.5	143	110	67	6.7	0.1	0.61	43.0	
4b	1540	440	200	1.67	45.5	143	110	79	32	0.4	0.72	0.23	
5a	770	220	400	0.68	34.2	65	85	58	5.8	0.1	0.69	[24.5]	(5)
5b	770	220	400	0.68	34.2	65	85	58	23	0.4	0.69	34.1	
6a	770	220	400	0.68	46.0	66	93	57	5.8	0.1	0.61	[31.1]	(5)
6b	770	220	400	0.68	46.0	66	93	57	23	0.4	0.61	312.1<	(2)
								62	25	0.3	0.61	23.15<	(2)
								67	34	0.5	0.72	36.9	
7a	770	220	400	1.67	34.2	139	115	98	59	0.6	0.85	0.049	
7b	770	220	400	1.67	34.2	139	115	98	9.8	0.1	0.85	0.024	
8a	770	220	400	1.67	46.0	143	127	99	79	0.8	0.78	70.6	
8b	770	220	400	1.67	46.0	143	127	108	97	0.9	0.85	398.4<	(2)
								108	86	0.8	0.85	387.0<	(2)
								117	94	0.8	0.92	123.5<	(2)
								117	70	0.6	0.92	0.267	

- (1) $V_f = A_s f_y (1 - 0.6 p f_y / f_c') d / a$
 where $p = A_s / (b d)$: reinforcement ratio
 b : flange width, V_f : static flexure strength
 f_y : yield strength of tensile bar
- (2) Non-failure
- (3) Flexure failure under static test after fatigue test
- (4) Shear failure under static test after fatigue test
- (5) Fatigue flexure failure due to fatigue fracture of tensile bar

Table 3.2 Fatigue lives of rectangular beams
without web reinforcement

V_{min}/V_{max} V_{max}/V_{cu}	0.1	0.4	0.5	0.6	0.8	0.9
92%				0.267 8b	(123.5) 8b	
85%	0.024 (1) 7b (2)			0.049 7a	(387.0) 8b	(398.4) 8b
78%					70.6 8a	
72%	0.05 1a	0.23 4b	0.07 36.9 3b 6b	314 1b		
69%	[24.5] (3) 5a	34.1 5b				
66%		(23.15) (4) 6b				
61%	1.86 43.0 [31.1] 2a 4a 6a	(1032) (312.1) 3b 6b		(447.9) 2b		

(1) Fatigue life of beam $\times 10^4$ cyc.

(2) Name of specimen

(3) [] means fatigue flexural failure due to fatigue fracture of tensile bar.

(4) () means non-failure.

Table 3.3 Failure modes of rectangular beams
without web reinforcement
in author's tests

Specimens	V_f kN	V_{cu} kN	$\log N_{fcal}$		$\log N_{ftest}$	Failure Modes	
			flexure (1)	shear		expected	tested
1a	65	69	6.05	4.04	2.70	shear	shear
1b	65	69	9.38	6.25	6.50	shear	shear
2a	66	76	6.33	6.06	4.27	shear	shear
2b	66	76	8.83	9.38	6.65<	non-failure	non-failure
5a	65	85	5.39	4.55	5.39	shear	flexure
5b	65	85	7.05	5.36	5.53	shear	shear
6a	66	93	5.49	6.06	5.49	flexure	flexure
6b	66	93	7.16	7.14	6.49<	non-failure	non-failure

(1) S-N curve for tensile bar :

$$\log \sigma_{sr} = -0.106 \log N_f + 3.10$$

Table 3.4 Failure modes of rectangular beams
without web reinforcement
in Chang - Kesler's tests [11]

Specimens	Vf kN	Vcu kN	logNfcal		logNftest	Failure Modes	
			flexure (1)	shear		expected	tested
1	11.2	17.2	7.93	9.64	6.84<	non-failure	non-failure
2	11.3	18.1	4.09	4.32	4.37	flexure	shear
3	11.3	17.9	4.49	4.82	3.48	flexure	shear
4	11.2	16.8	4.15	3.50	3.81	shear	shear
5	11.2	17.1	4.15	3.75	3.26	shear	shear
6	11.2	16.8	6.84	7.66	5.68	flexure	flexure
7	11.2	17.2	5.91	6.55	5.60	flexure	flexure
8	11.2	17.4	4.15	3.93	3.90	shear	flexure
9	11.2	17.2	4.16	3.84	3.41	shear	shear
10	11.1	16.2	5.93	5.82	4.67	shear	shear
11	11.2	17.6	9.14	11.73	7.31<	non-failure	non-failure
12	11.2	17.4	4.09	3.88	4.37	shear	shear
13	11.1	16.6	5.91	6.09	5.24	flexure	flexure
14	11.2	17.4	5.10	5.40	5.65	flexure	flexure
15	11.3	18.0	5.08	5.79	5.70	flexure	flexure
16	11.2	17.5	6.82	8.16	5.76<	non-failure	non-failure
17	11.1	16.3	6.84	7.32	5.85	flexure	flexure
18	11.2	17.1	5.10	5.19	5.62	flexure	flexure
19	11.3	17.9	5.93	7.03	5.67	flexure	flexure
20	11.3	18.2	4.08	4.37	3.51	flexure	flexure
21	11.3	18.7	4.06	4.66	4.41	flexure	flexure
22	11.3	18.3	4.21	4.65	4.48	flexure	shear
23	11.3	18.0	4.06	4.22	4.85	flexure	flexure
24	11.3	18.2	4.08	4.37	5.18	flexure	flexure
25	11.3	18.2	4.94	5.70	5.15	flexure	flexure

(1) S-N curve for tensile bar :
 $\log \sigma_{sr} = -0.0579 \log N_f + 2.79$

Table 6.1 Fractured stirrups and fatigue lives of T-beams

Specimens	Nu ×10 ⁴ (1)	First fractured stirrups		Other fractured stirrups No.	Number of frac- tured stirrups				Vmax Vf %
		Nf ×10 ⁴	No.		Position			Total	
					L (2)	M (3)	U (4)		
FS1	Static	—	—	—	0	0	0	0	103 (5)
FL2	224.7	33.9	30 L	26 27 28 29 31 U L M L L	4	1	1	6	62
FS3	235.0	32.0	30 L	24 26 27 28 29 31 M M M L L L	4	3	0	7	62
FL4	347.7	32.0	30 L	24 25 26 28 29 31 U U U L L L	4	0	3	7	62
FS5	41.9	10.0	30 L	20 22 23 24 25 26 27 28 29 31 U M M M M L L L L L	6	4	1	11	62
FL6	60.3	32.0	28 L	24 26 29 30 31 U U L L L	0	4	2	6	62
FS7	21.9	10.0	30 L	24 25 26 27 28 29 31 M L M L M L L	5	3	0	8	62
FL8	36.23	10.0	26 L	24 25 27 28 29 30 31 M M M L M L L	4	4	0	8	62
FS9	361.0	?	?	27 29 30 31 M M L L	2	2	0	4	61
FL10	45.6	10.1	27 M	22 24 25 26 28 29 30 31 M M M M M M M L	2	7	0	9	61
FS11	61.5	?	?	23 24 25 26 27 28 29 30 31 U L U M M M M L L	3	4	2	9	62
Total					34	32	9	75	

- (1) Nu : Fatigue life of beam (2) L : Lower bent portion
 (3) M : Middle straight portion (4) U : Upper hook portion
 (5) Ultimate shear force under static test after fatigue test

FIGURES

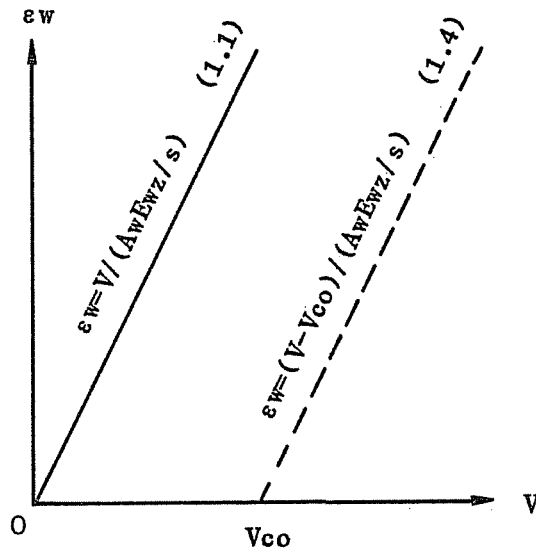


Fig.1.1 Relationship between applied shear force and stirrup strain under repeated loading reported by Hawkins [2]

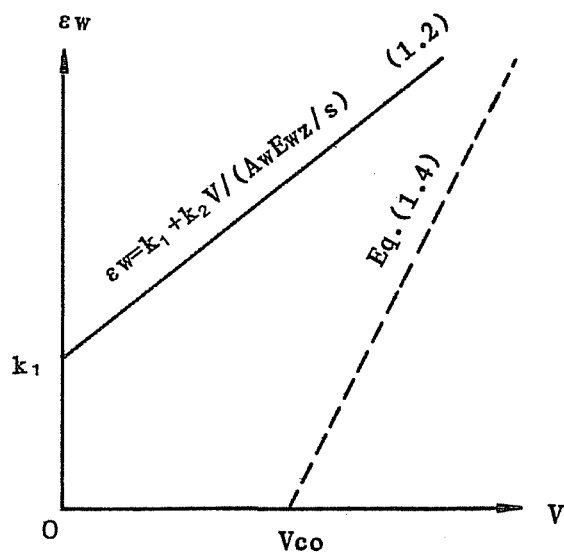


Fig.1.2 Relationship between applied shear force and stirrup strain under repeated loading reported by Ruhnau [3]

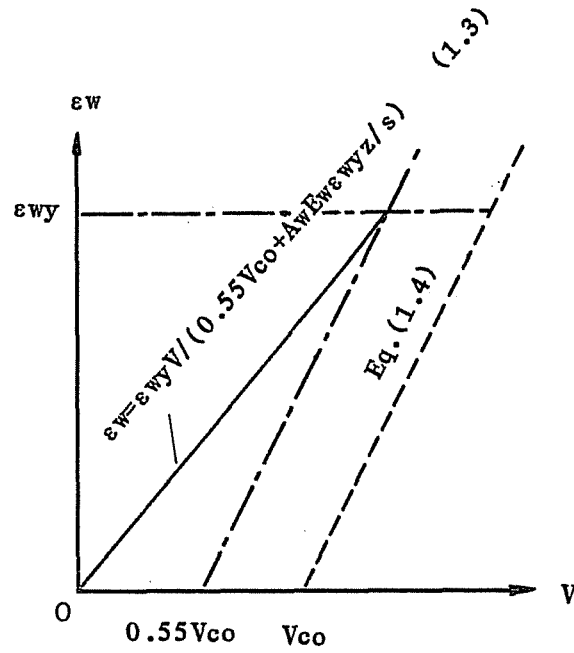


Fig.1.3 Relationship between applied shear force and stirrup strain under repeated loading reported by Higai [5]

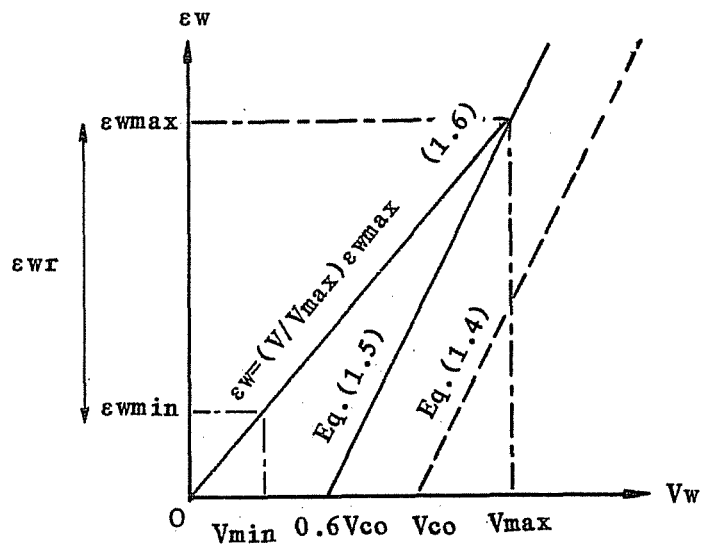


Fig.1.4 Relationship between applied shear force and stirrup strain under repeated loading reported by Farghaly [1]

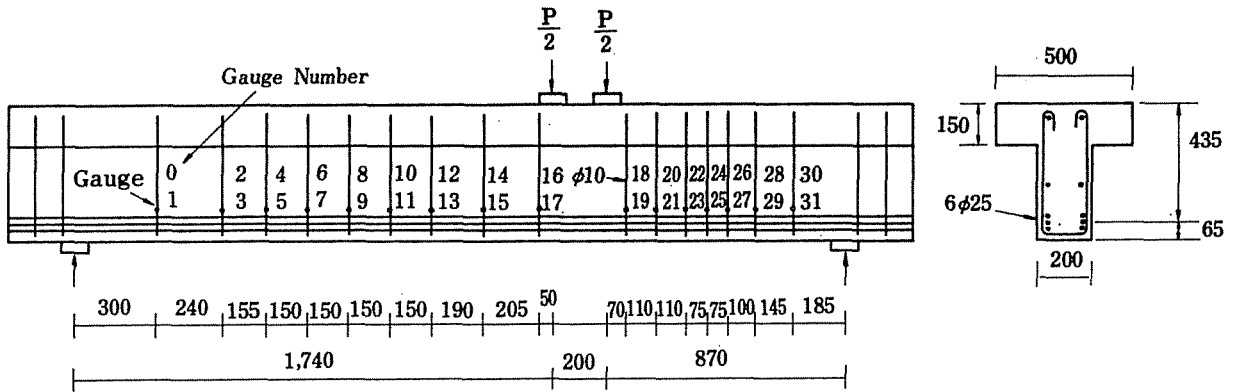


Fig.2.1 T-beam with stirrups

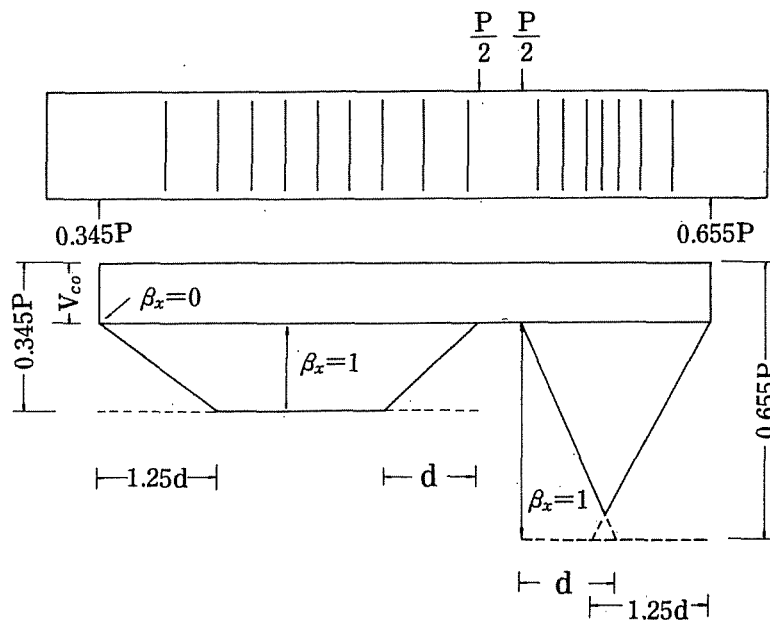


Fig.2.2 Shear diagram for T-beams

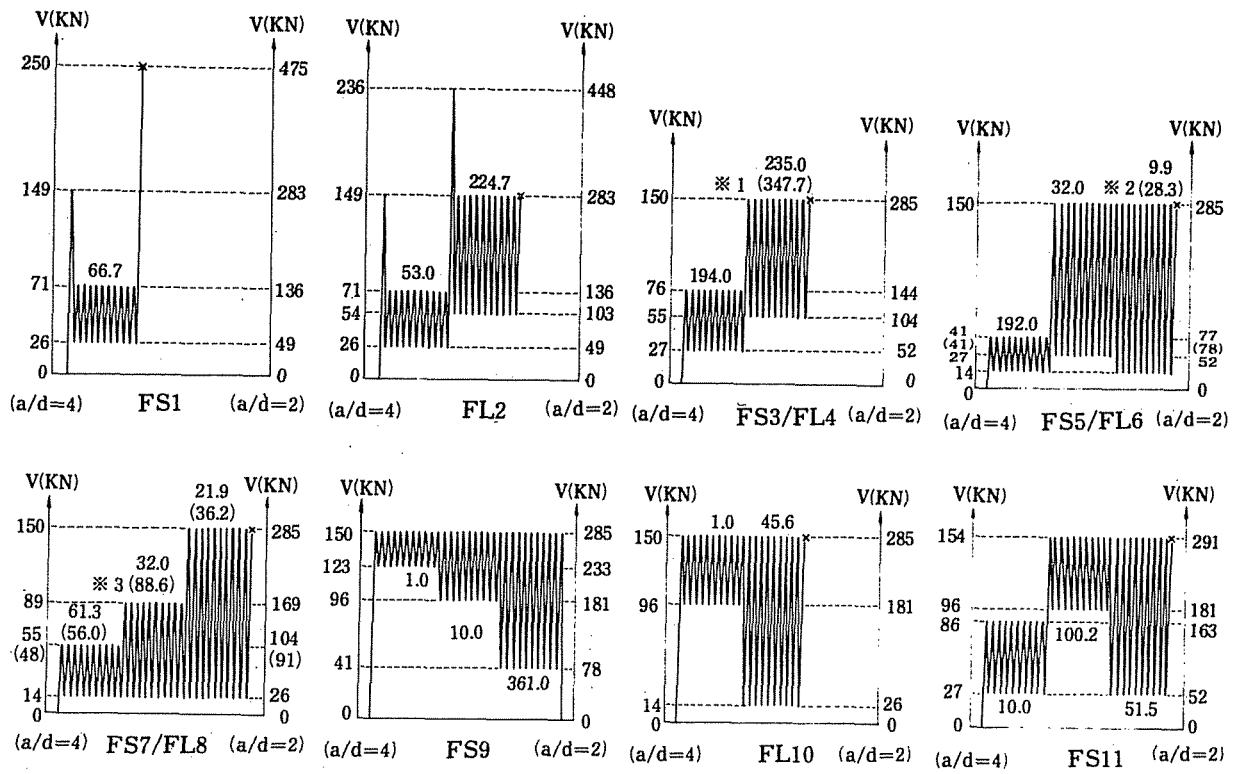


Fig.2.3 Loading history (*1 FL4, *2 FL6, *3 FL8 / Numerals in each figure indicate number of loading cycles $\times 10^4$. Static tests after fatigue tests were carried out for the specimen FS1 and FS9.)

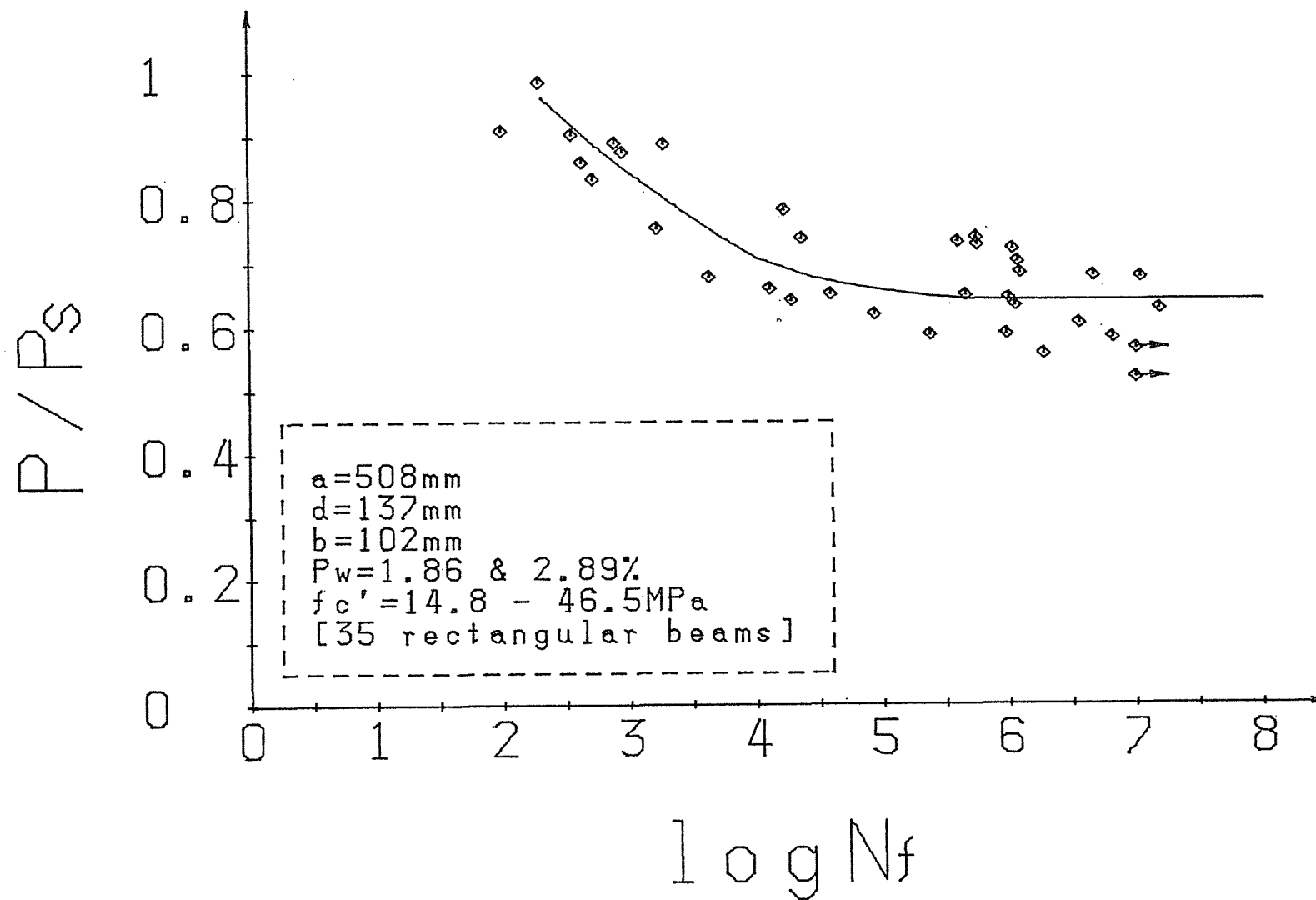


Fig.3.1 Fatigue strength in shear of beam without web reinforcement reported by Chang - Kesler [8] (P_s : static shear capacity calculated by Chang - Kesler's equation, P : tested maximum load)

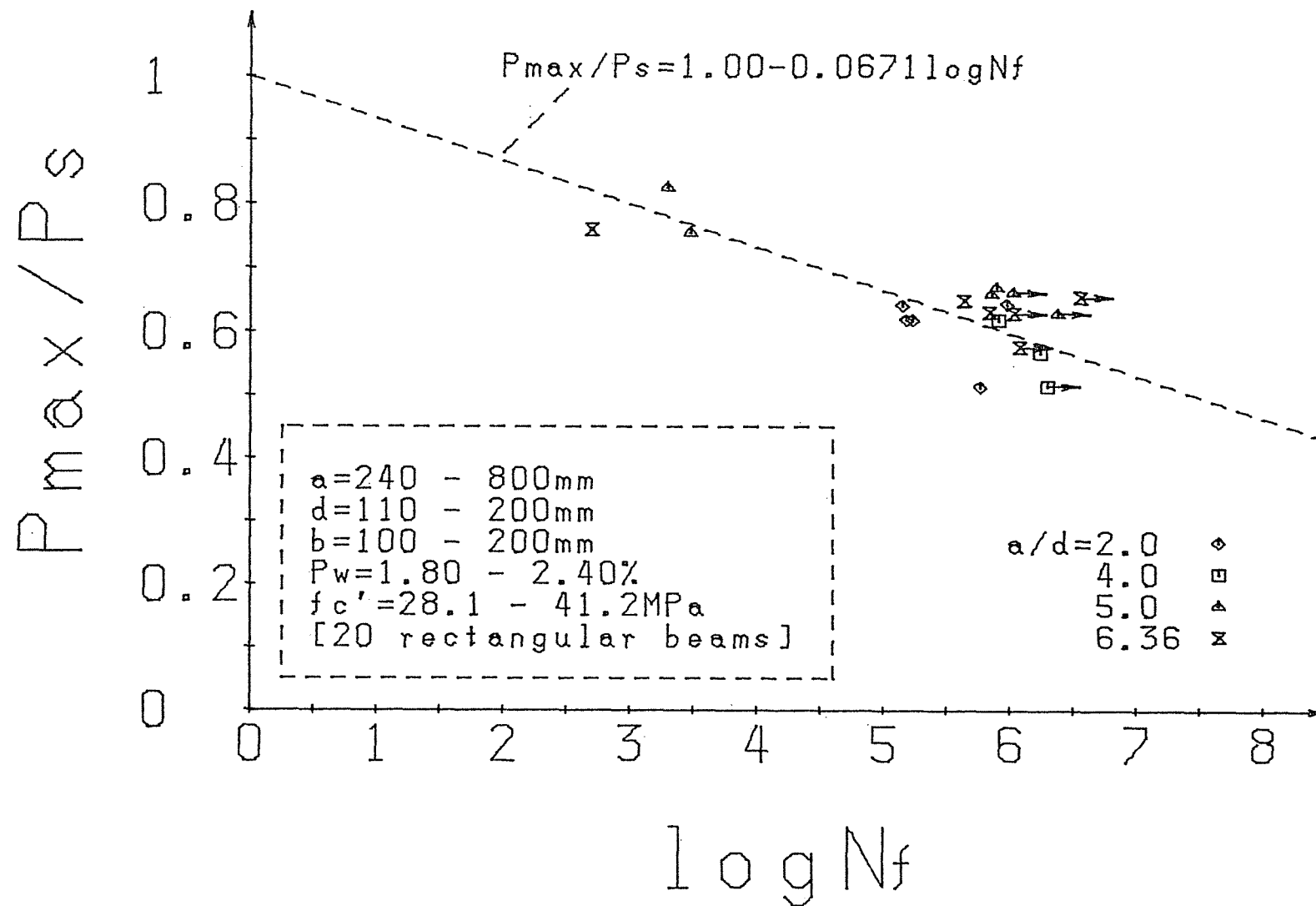


Fig.3.2 Fatigue strength in shear of beam without web reinforcement reported by Higai [5] ($P_s = 4.10/f_c'bd$ ($a/d=2.0$), $1.20/f_c'bd$ ($a/d \geq 4.0$) (f_c' : kgf/cm²) : static shear capacity, P_{max} : tested maximum load)

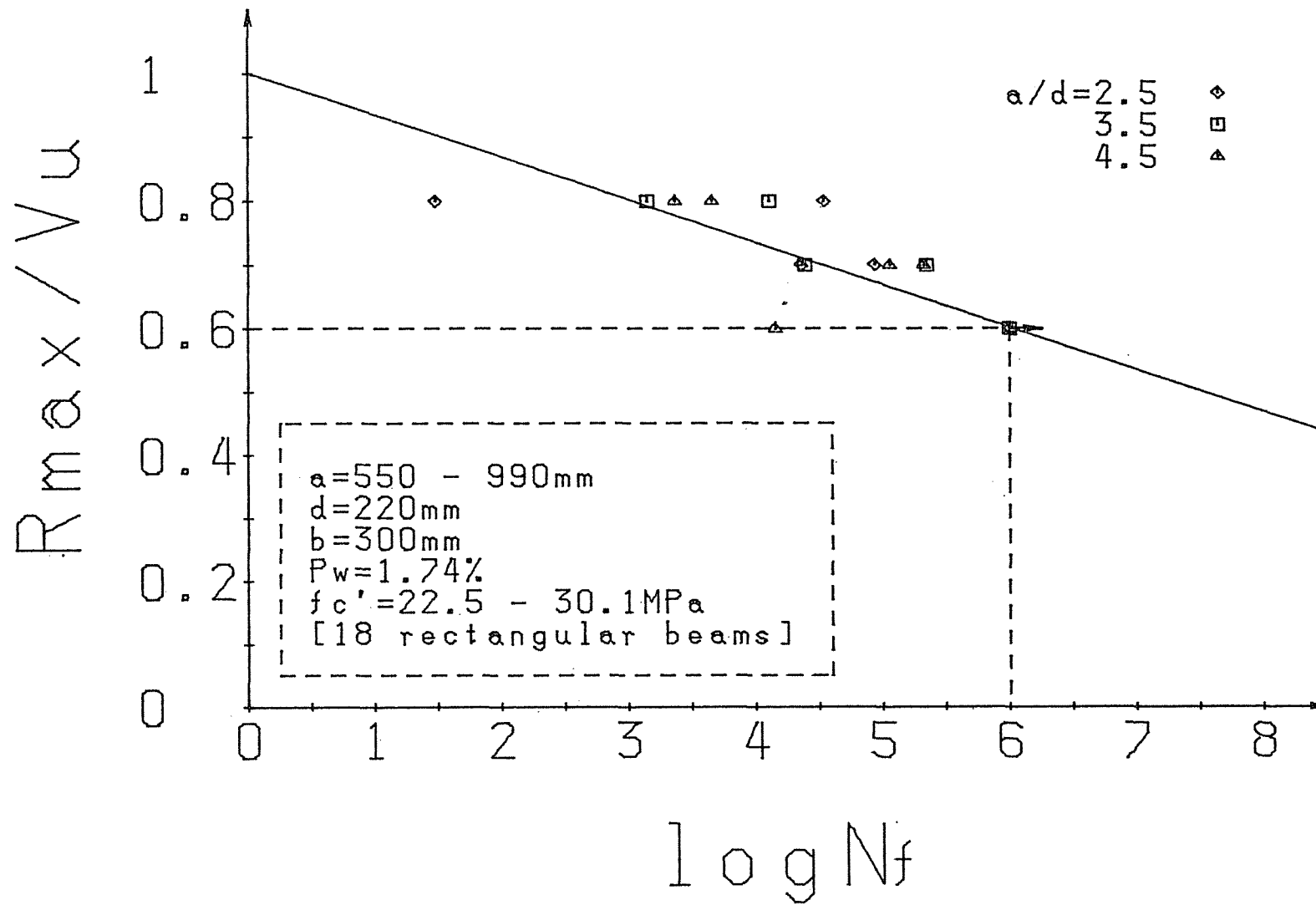


Fig.3.3 Fatigue strength in shear of beam without web reinforcement reported by Farghaly [1] (V_u : tested static strength in shear, R_{max} : tested maximum shear force)

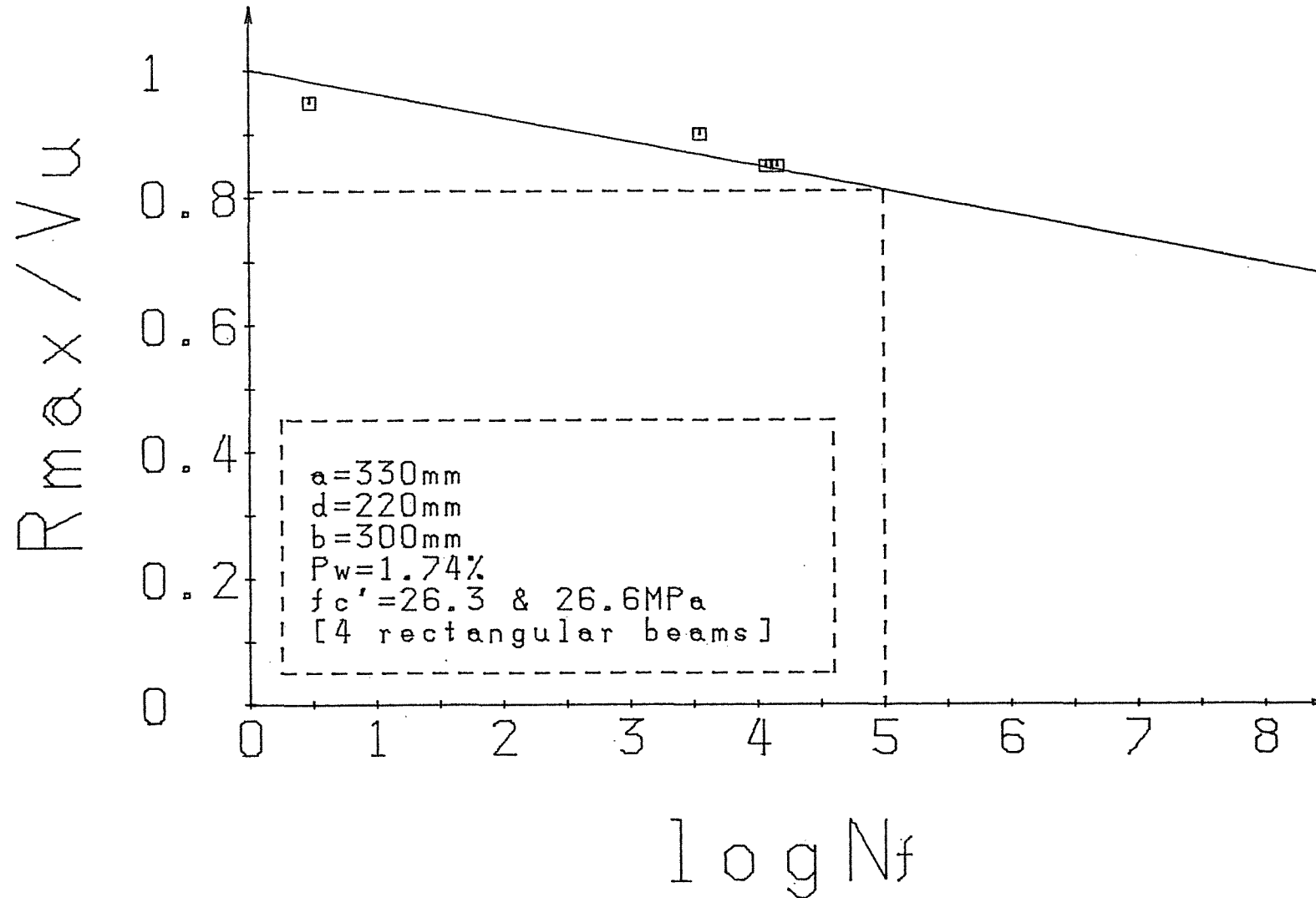


Fig.3.4(a)

Fig.3.4 Fatigue strength in shear of beam without web reinforcement in cases of beams in which ratio of shear span to effective depth is 1.5 [1]
 (a) failure due to propagation of diagonal crack, (b) failure due to fatigue fracture of tensile bar / V_u : tested static strength in shear, R_{max} : tested maximum shear force)

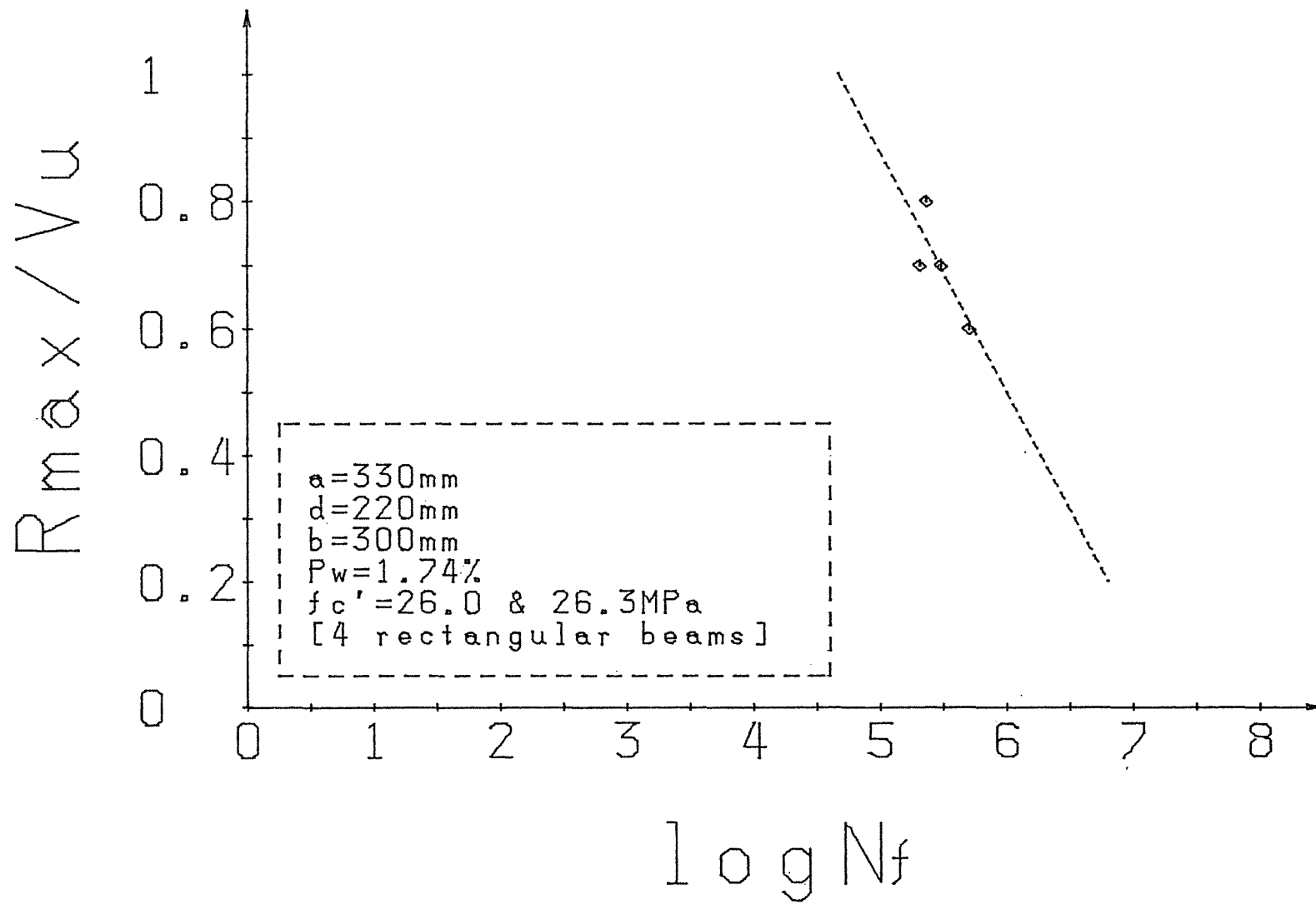


Fig.3.4(b)

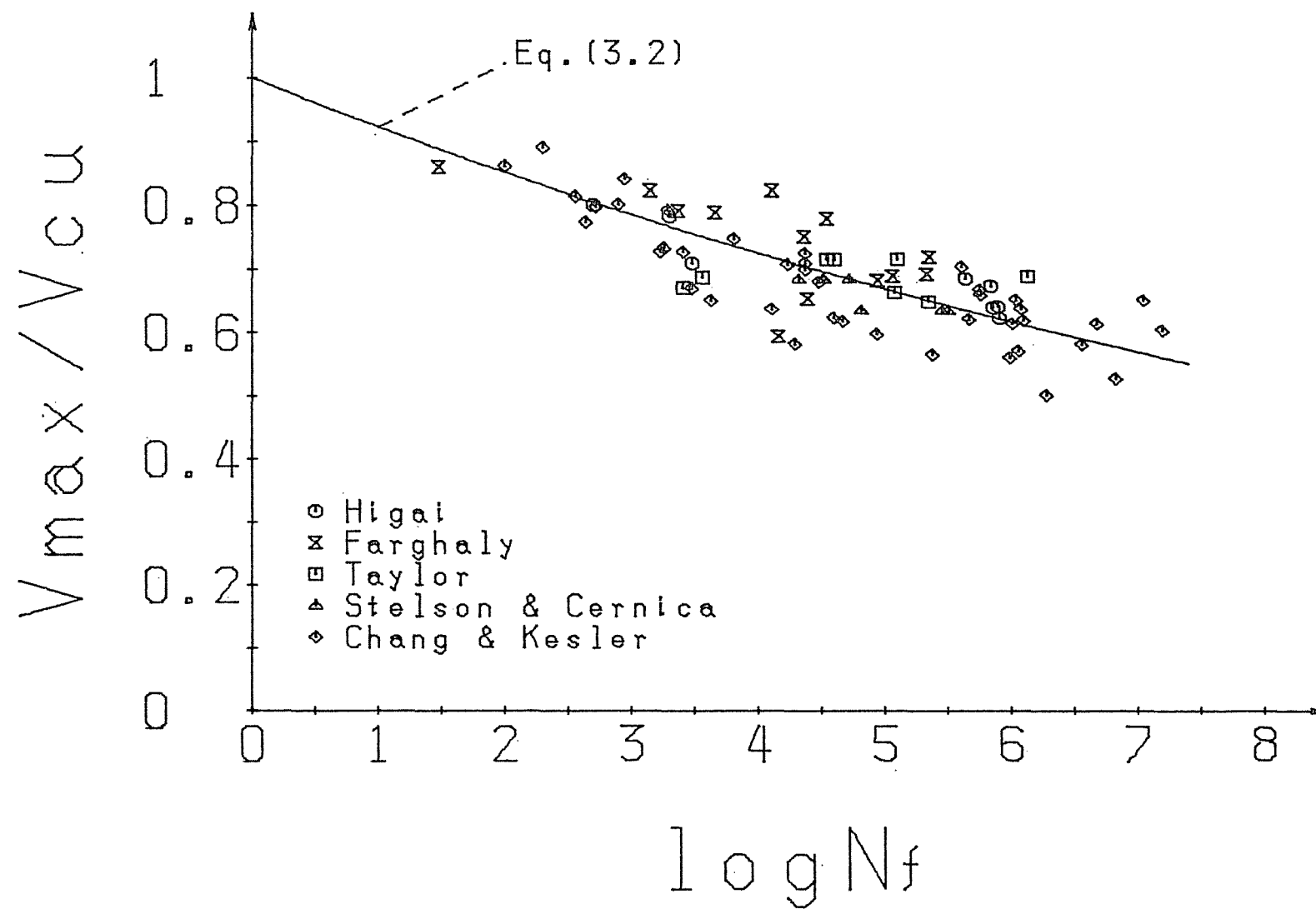


Fig.3.5 Fatigue strength in shear of beam without web reinforcement normalized with static strength calculated by Eq.(3.1)

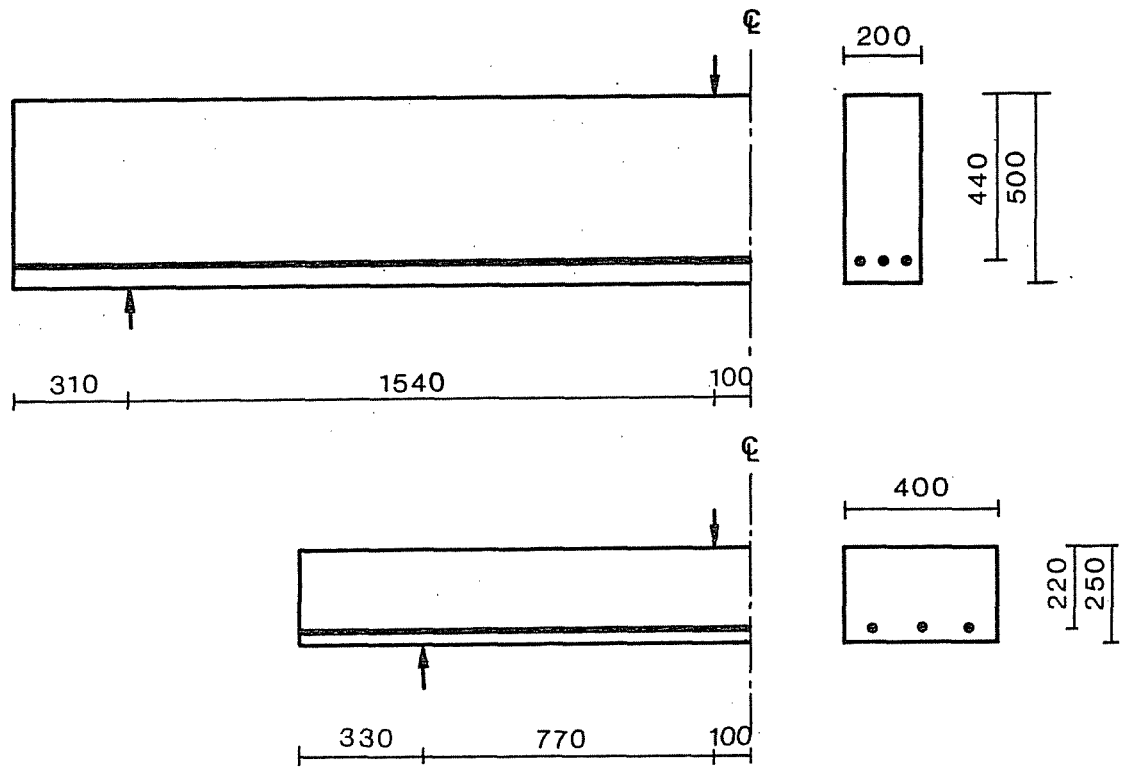


Fig.3.6 Rectangular beam without web reinforcement

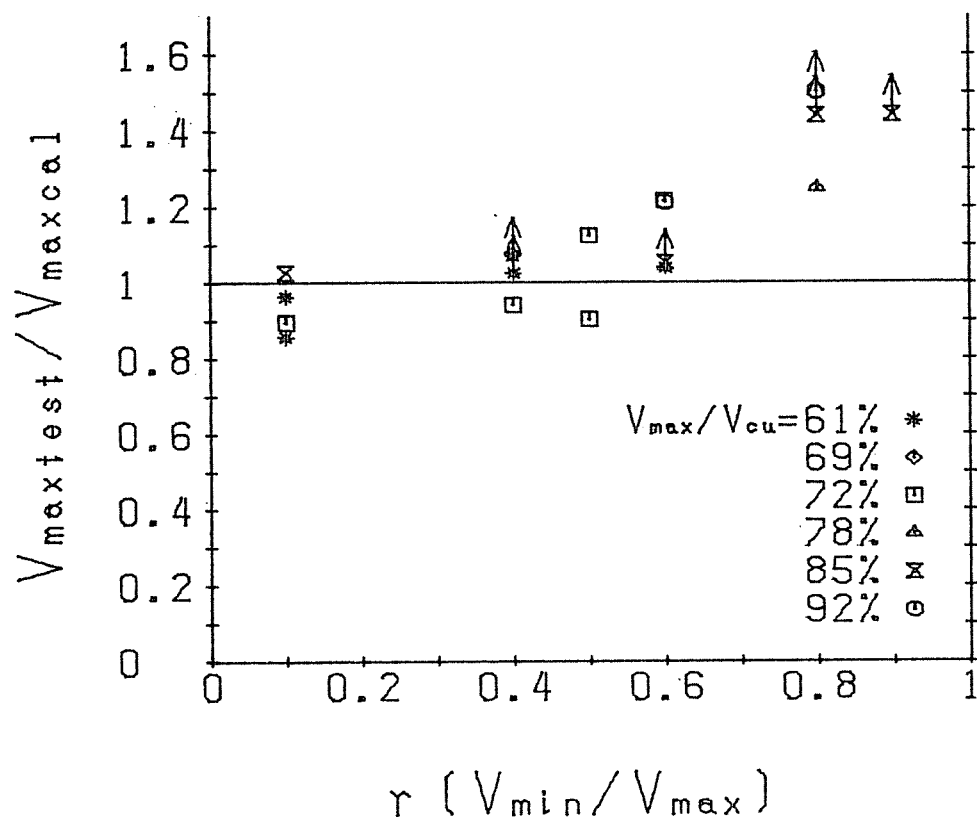


Fig.3.7 Influence of ratio, r , of minimum shear force to maximum one on fatigue strength in shear of beam without web reinforcement

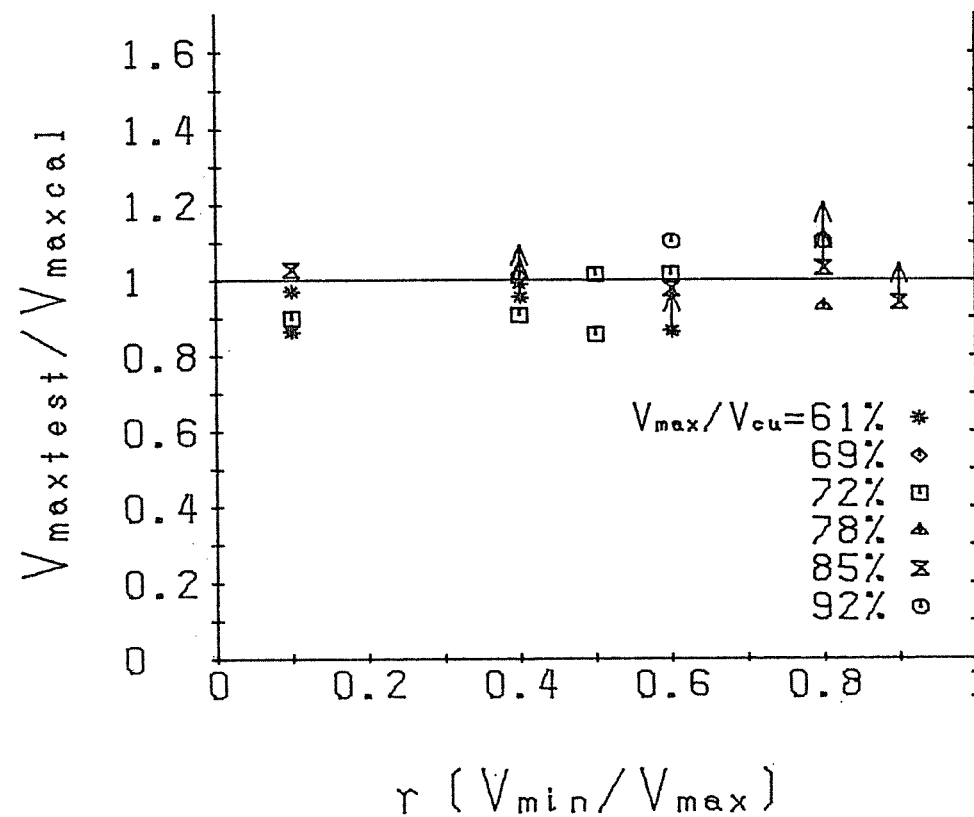


Fig.3.8 Relationship between ratio of tested value of fatigue strength in shear of beam without web reinforcement to that calculated by Eq.(3.3) and ratio, r , of minimum shear force to maximum one in author's tests

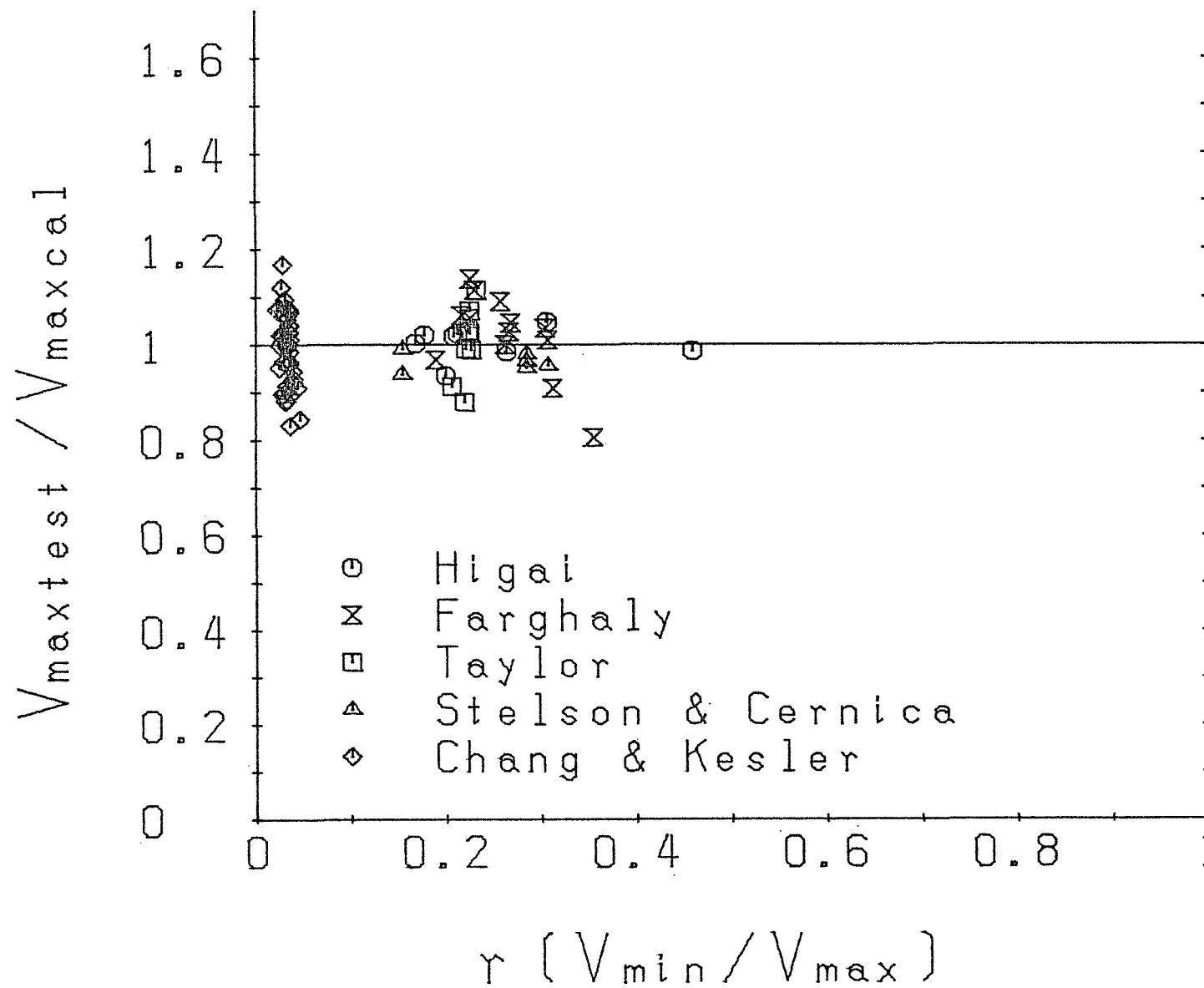


Fig.3.9 Relationship between ratio of tested value of fatigue strength in shear of beam without web reinforcement to that calculated by Eq.(3.3) and ratio, r , of minimum shear force to maximum one in previous tests [1][5][8][9][10][11]

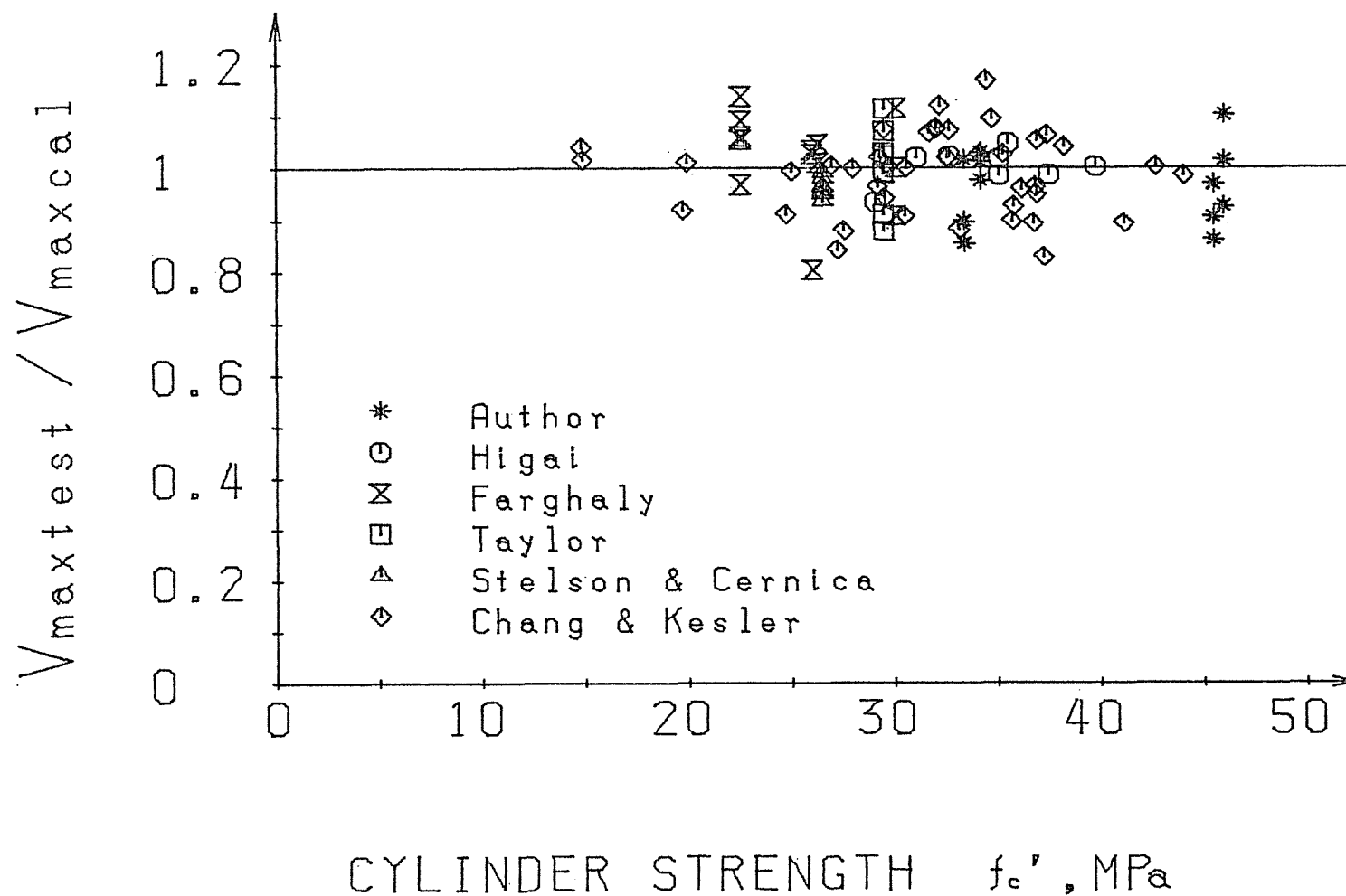


Fig.3.10(a)

Fig.3.10 Relationships between ratio of tested value of fatigue strength in shear of beam without web reinforcement to that calculated by Eq.(3.3) and such factors ----- cylinder strength, f_c' , shear span ratio, a/d , reinforcement ratio, ρ_w , and effective depth, d

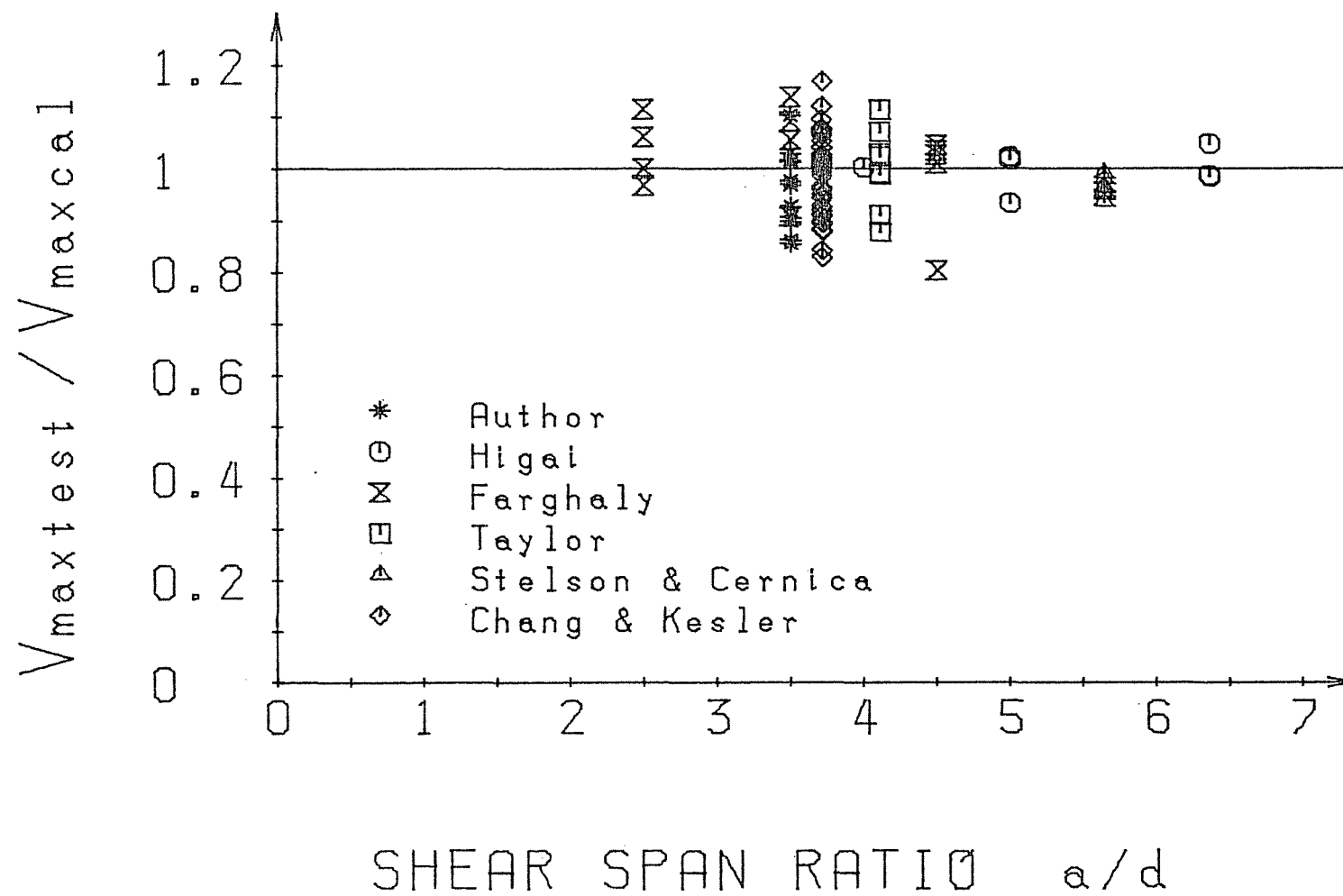


Fig.3.10(b)

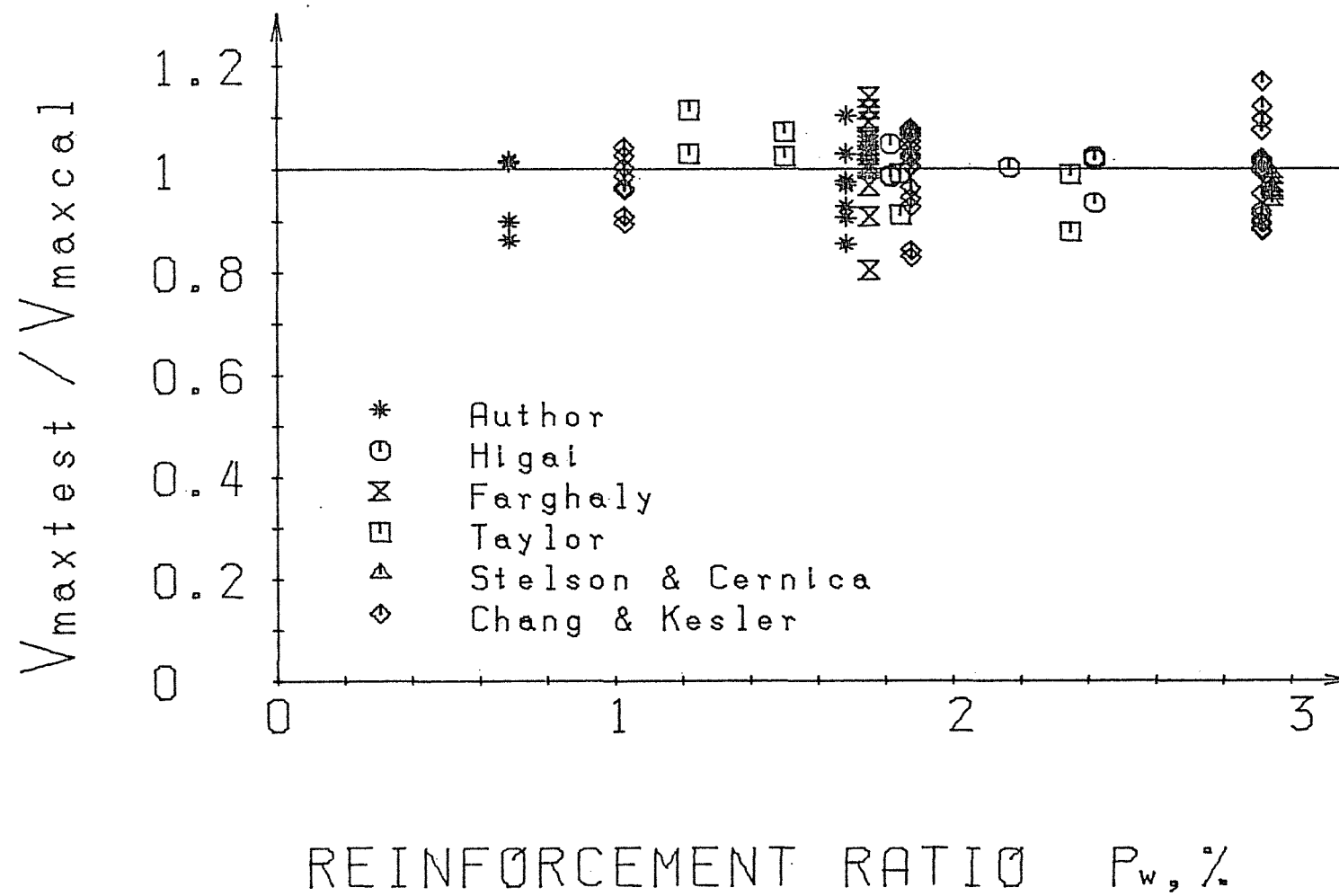


Fig.3.10(c)

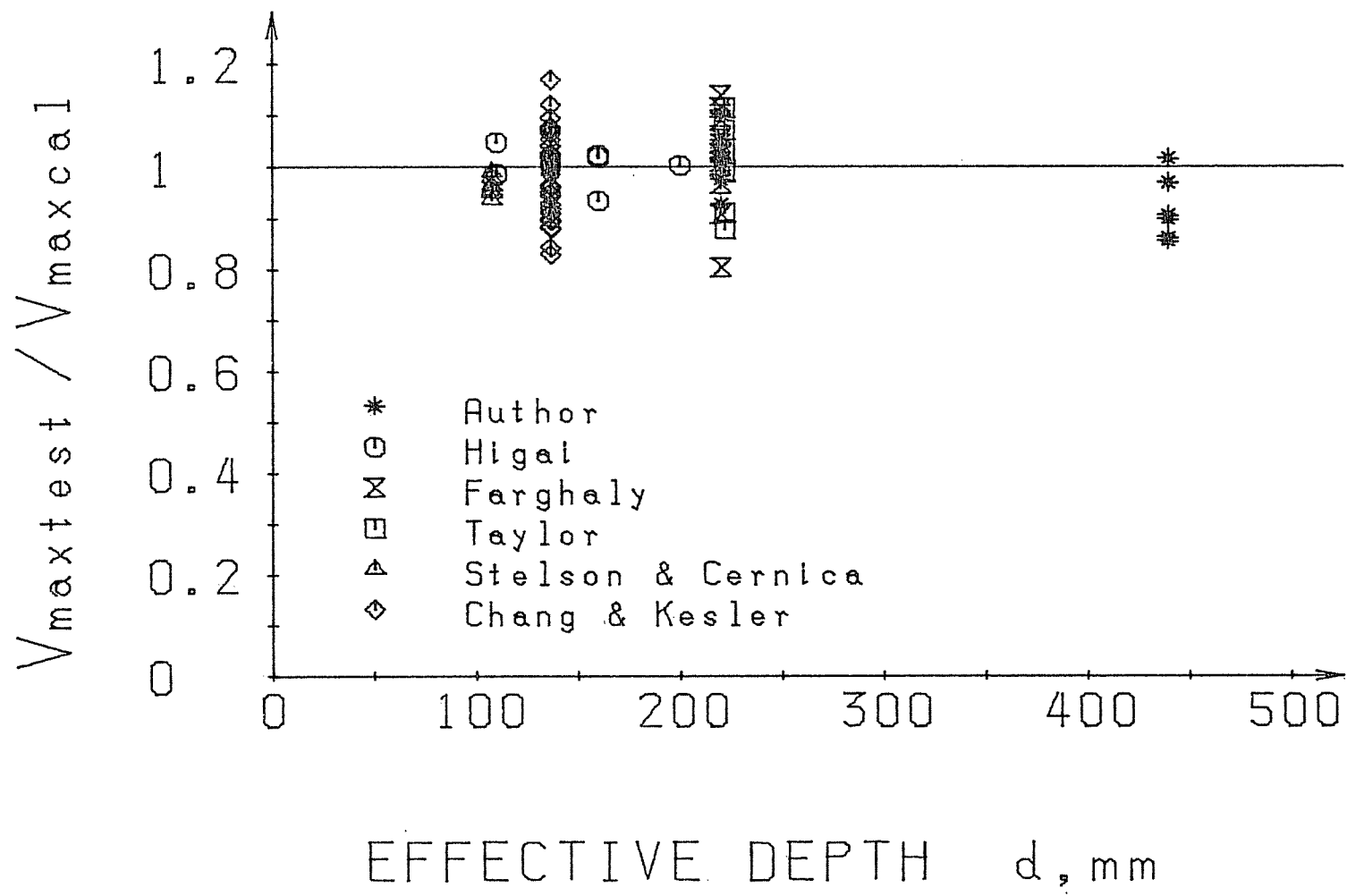


Fig.3.10(d)

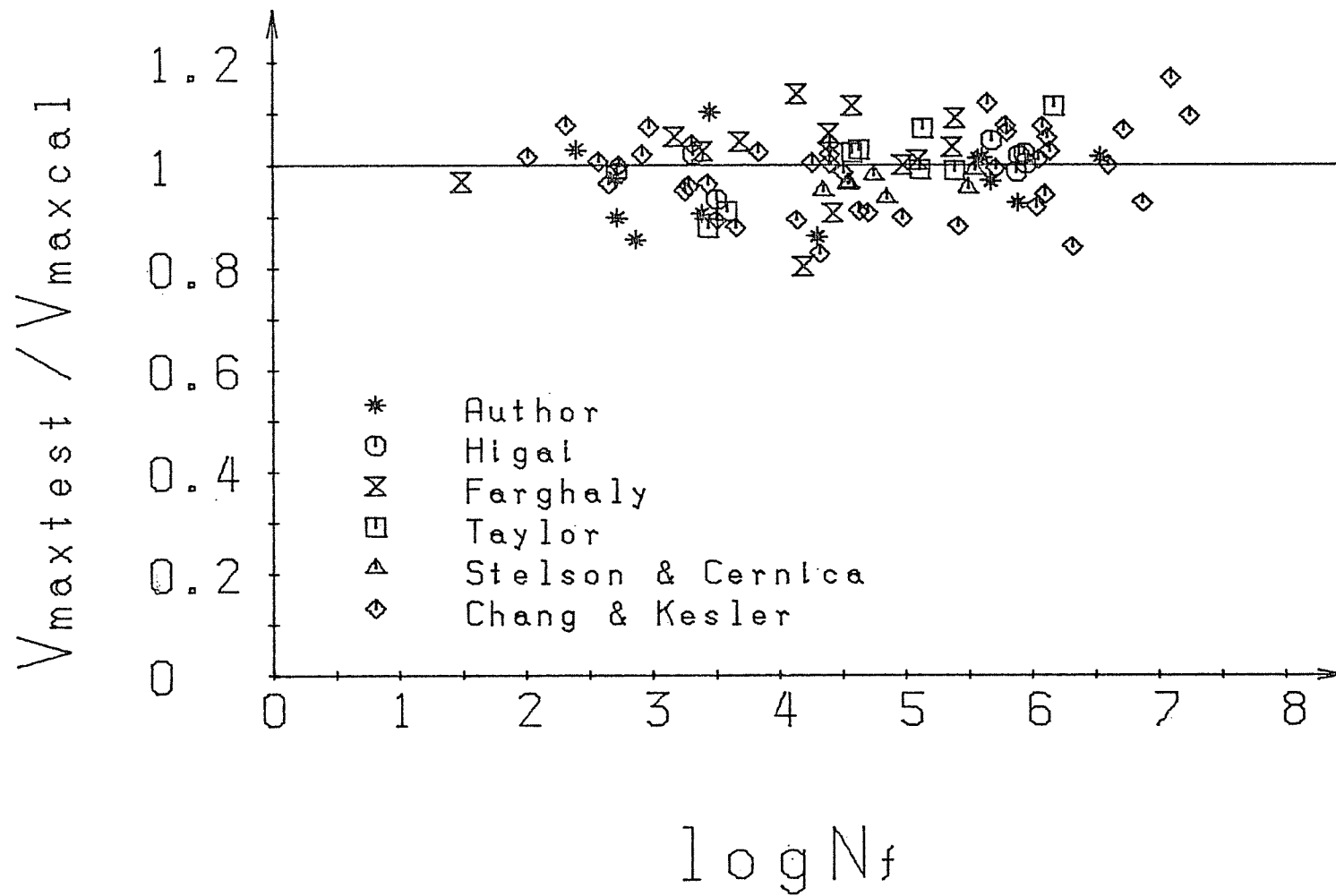


Fig.3.11 Relationship between ratio of tested value of fatigue strength in shear of beam without web reinforcement to that calculated by Eq.(3.3) and fatigue life of beam

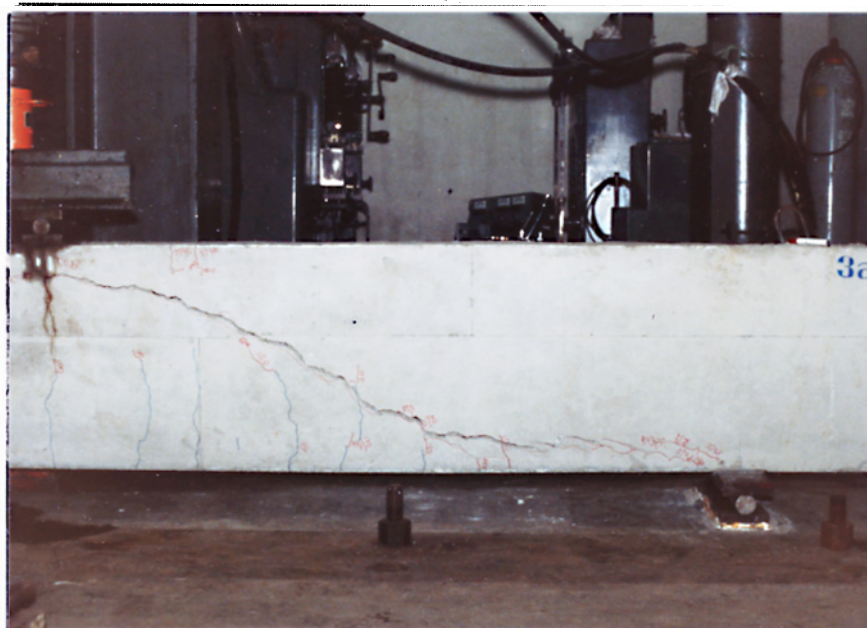


Fig.3.12(a)



Fig.3.12(b)

Fig.3.12 Location of diagonal crack in the vicinity of loading points
((a) Specimen 3a, (b) Specimen 3b)

Fig.3.13(a)

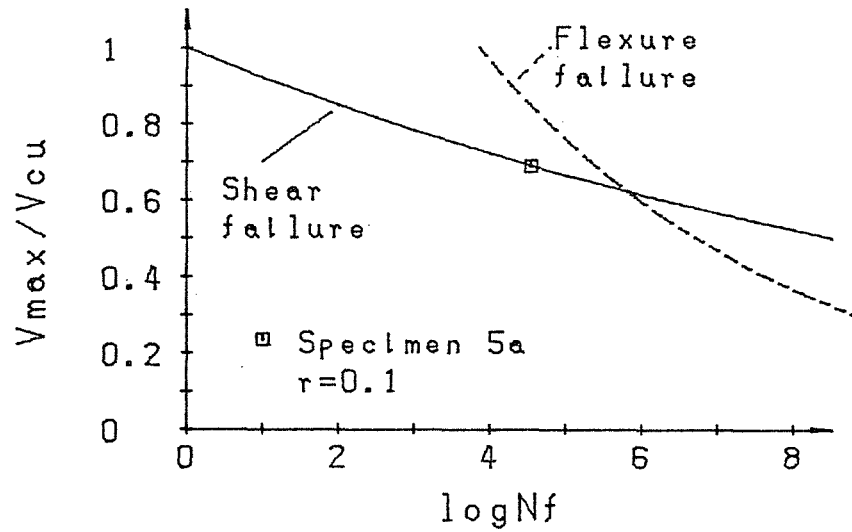


Fig.3.13(b)

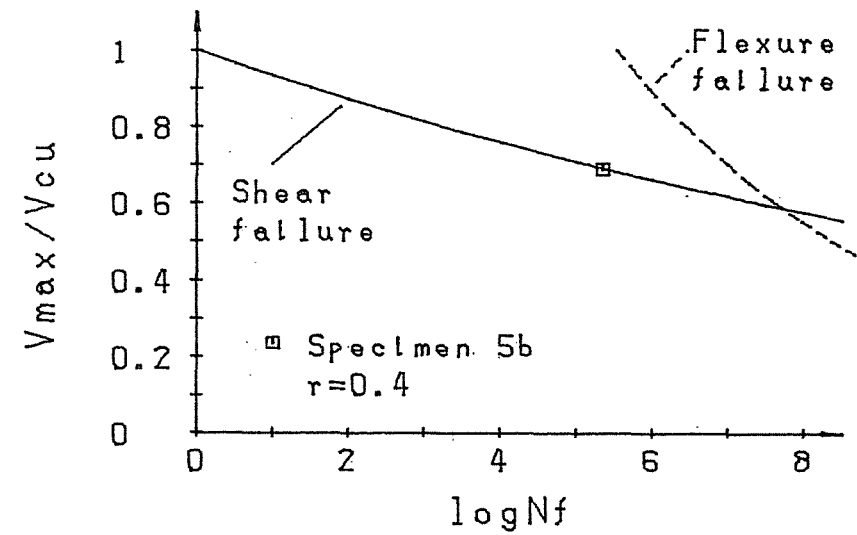


Fig.3.13(c)

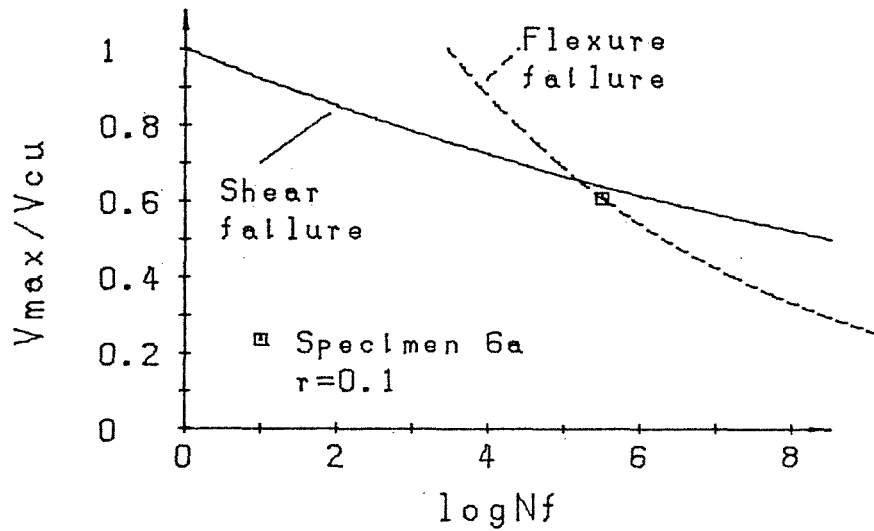


Fig.3.13(d)

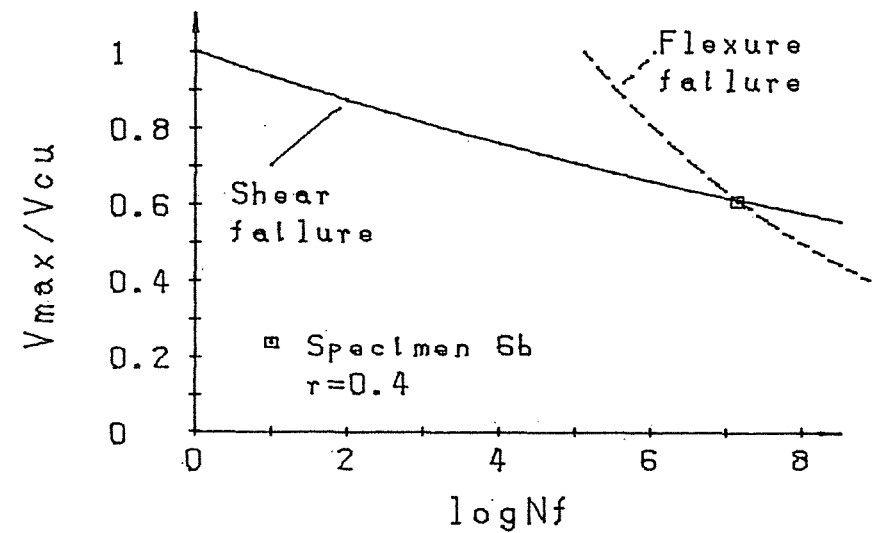


Fig.3.13 Influence of ratio, r , of minimum shear force to maximum one on failure mode of beam without web reinforcement under repeated loading

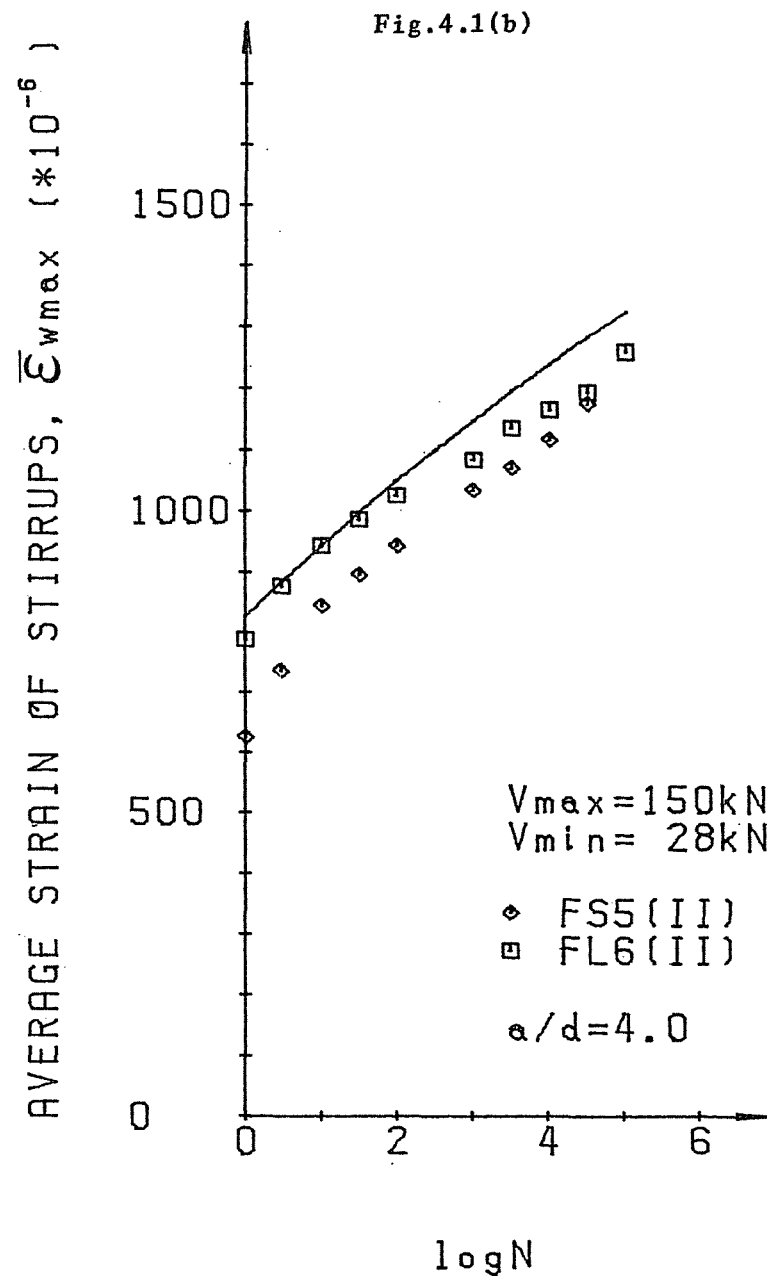
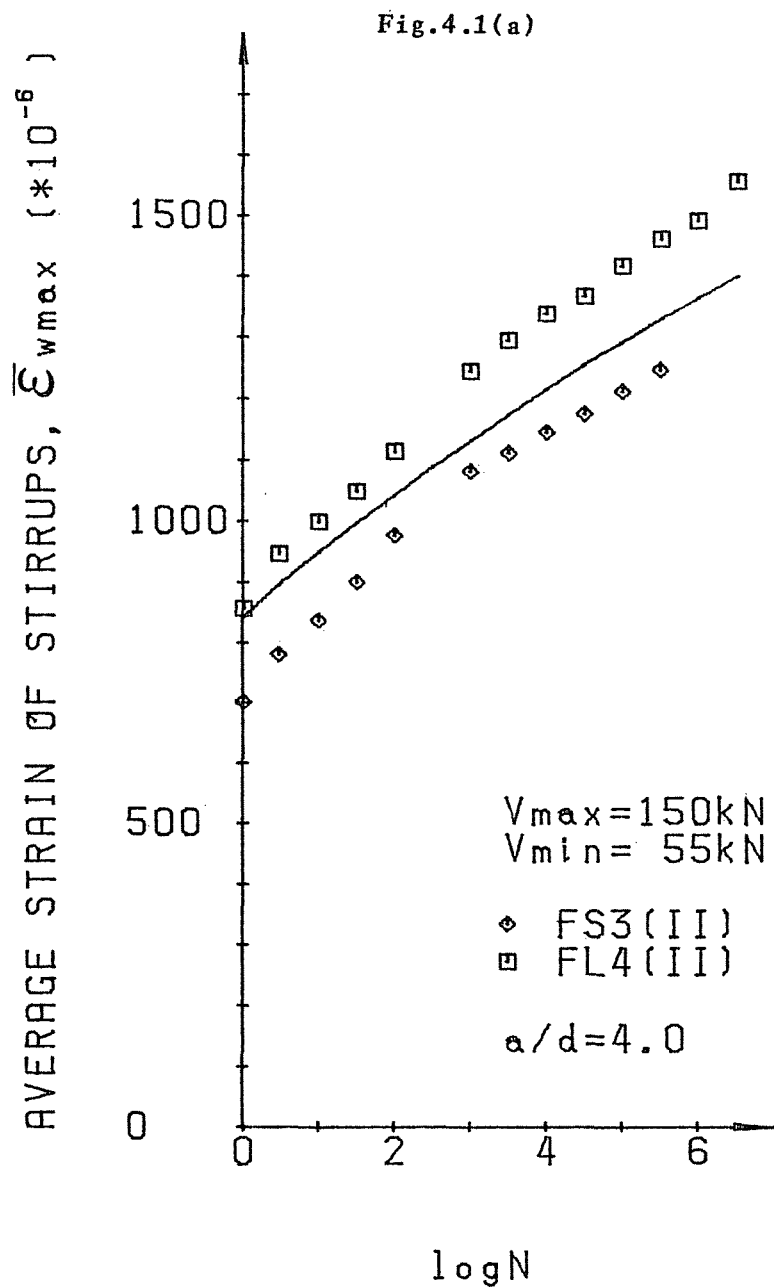


Fig.4.1 Average strain of stirrups at applied maximum shear force
 (T-beams in author's tests)

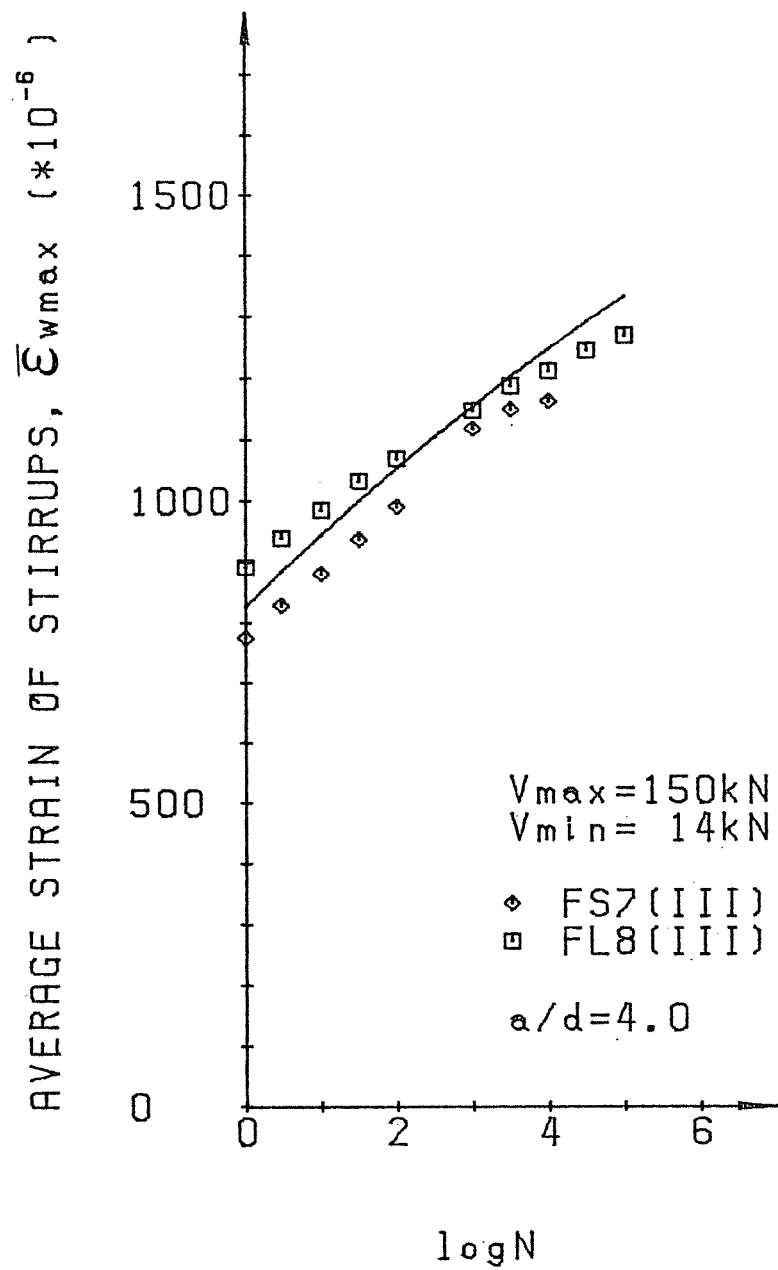


Fig.4.1(c)

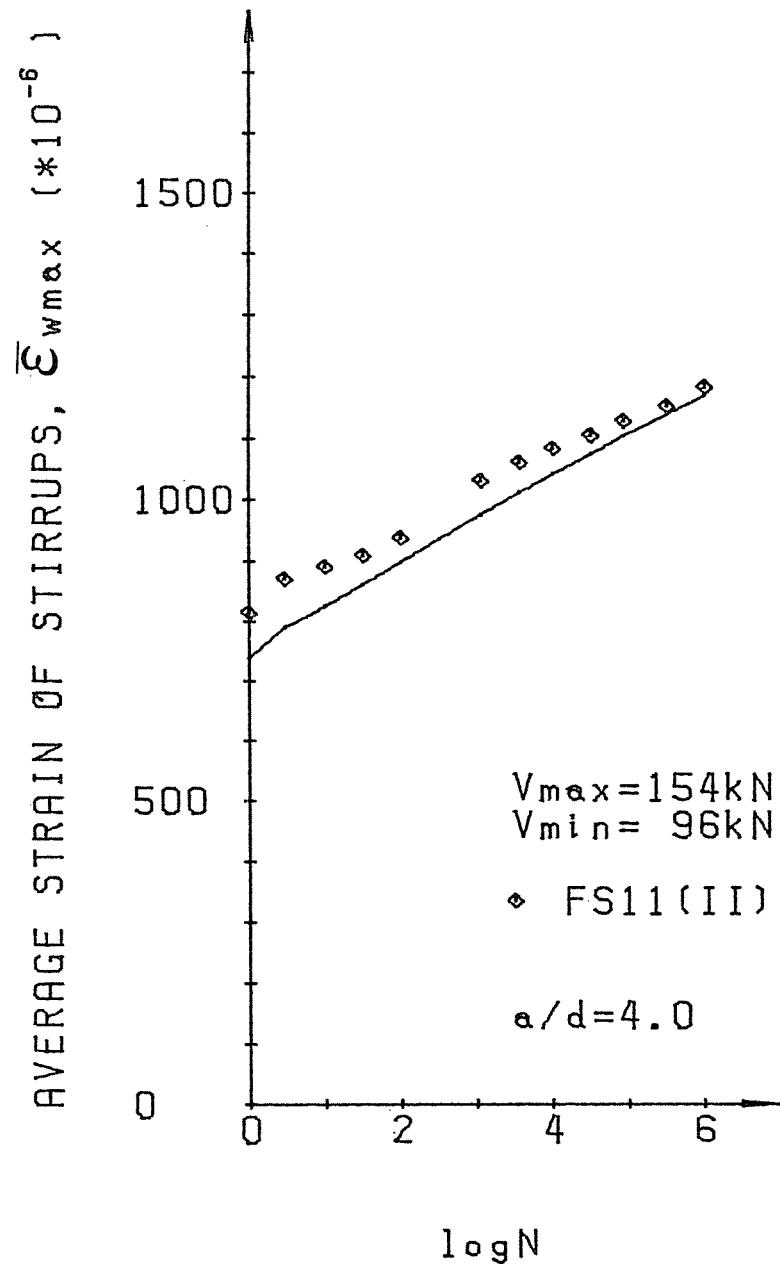


Fig.4.1(d)

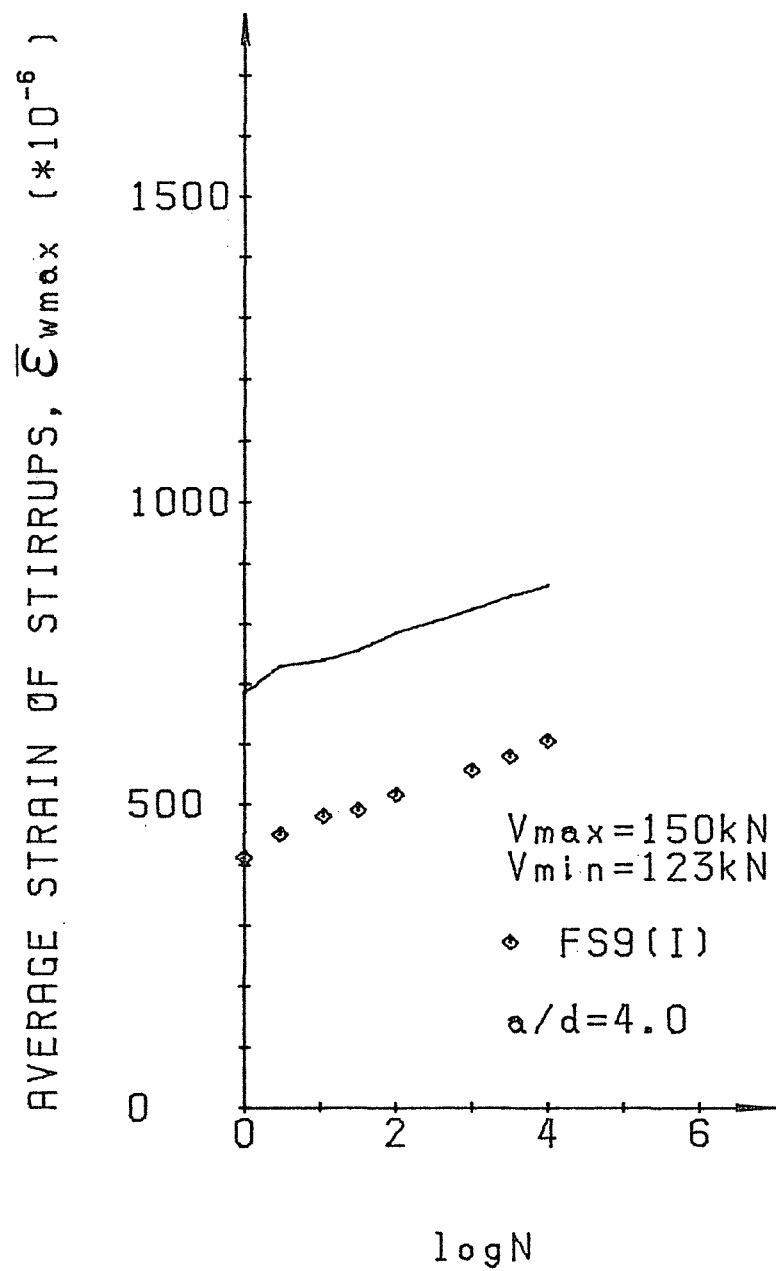


Fig.4.1(e)

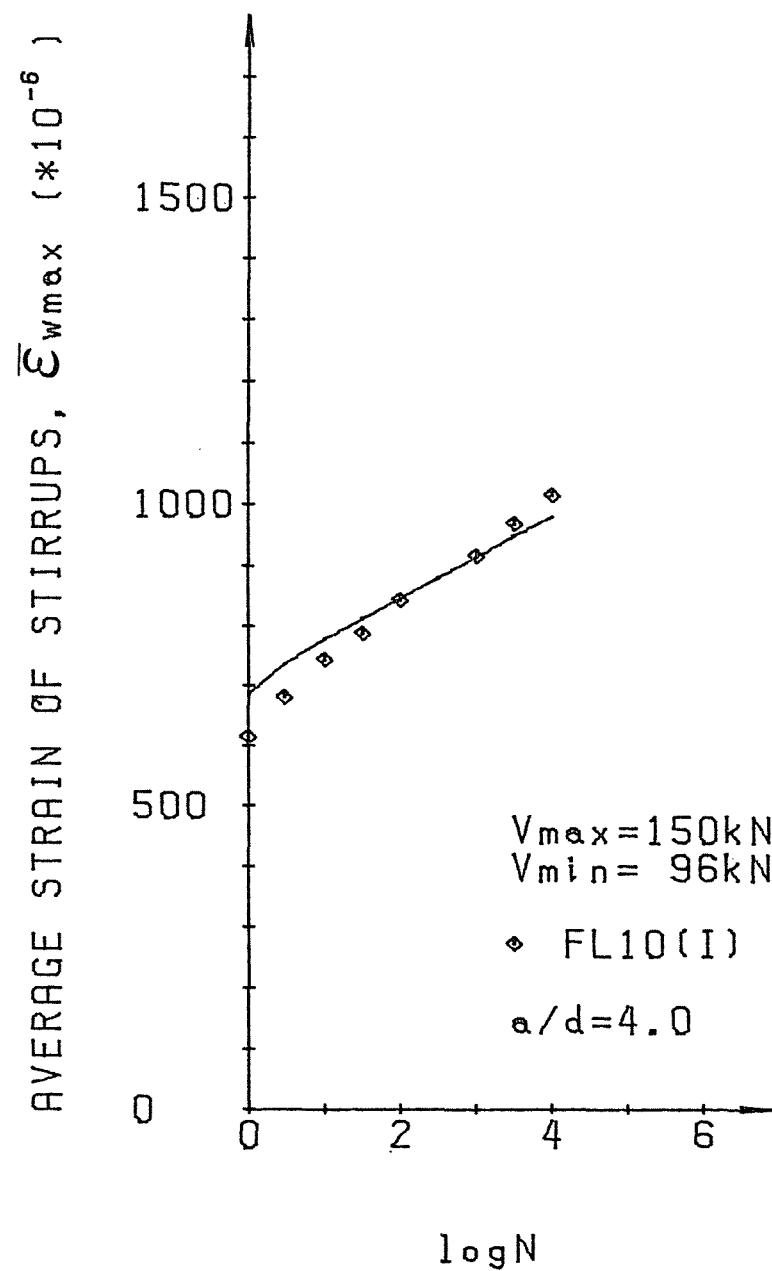


Fig.4.1(f)

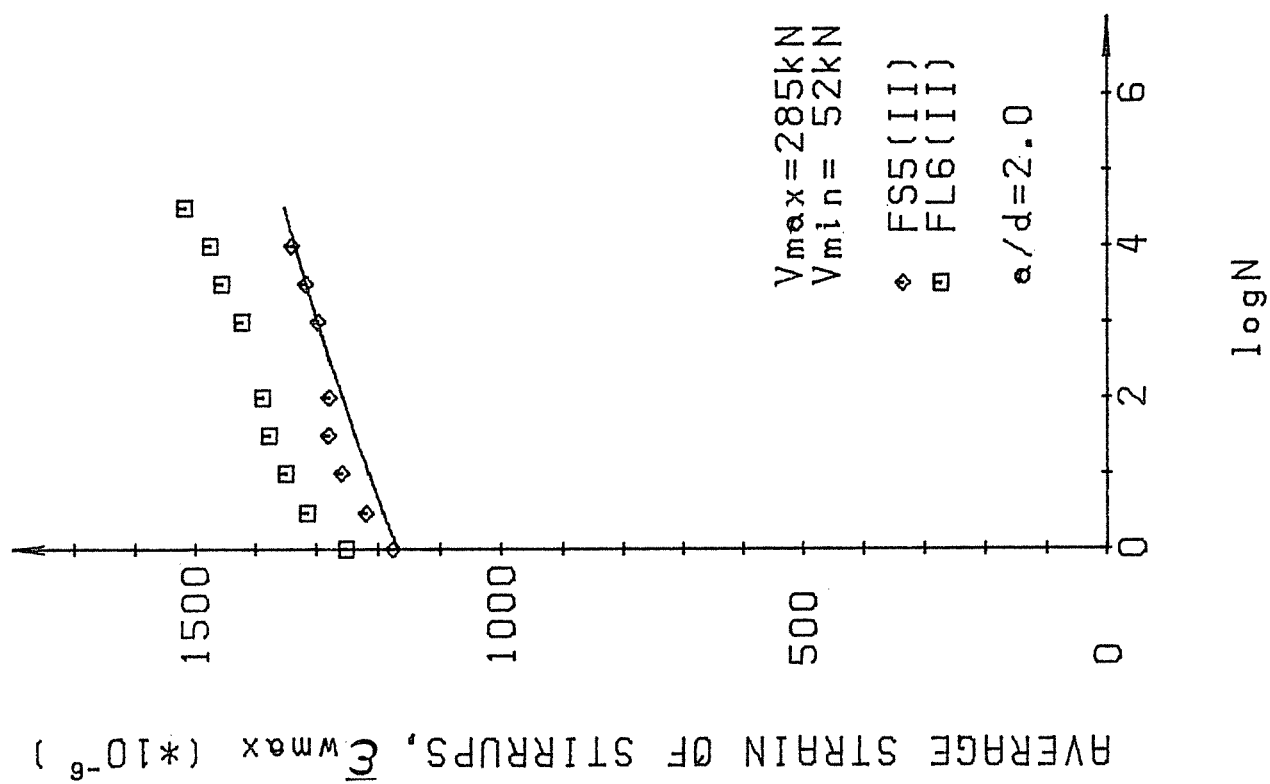


Fig.4.1(g)

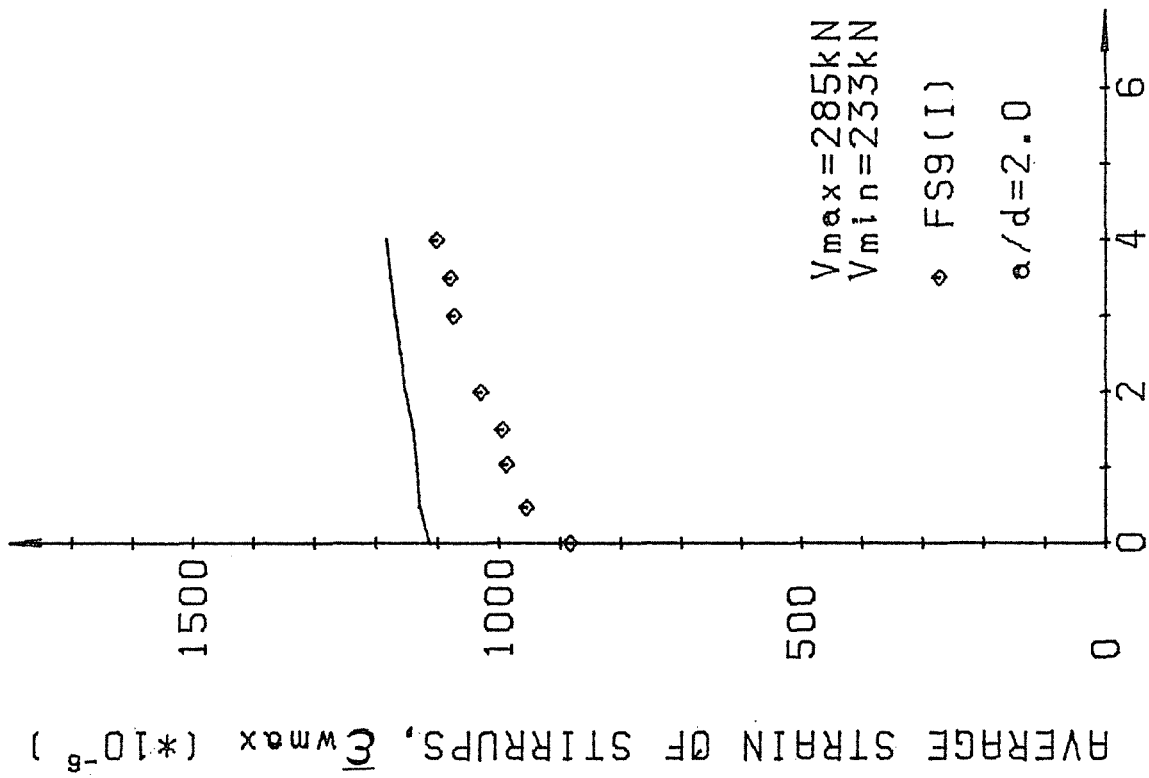
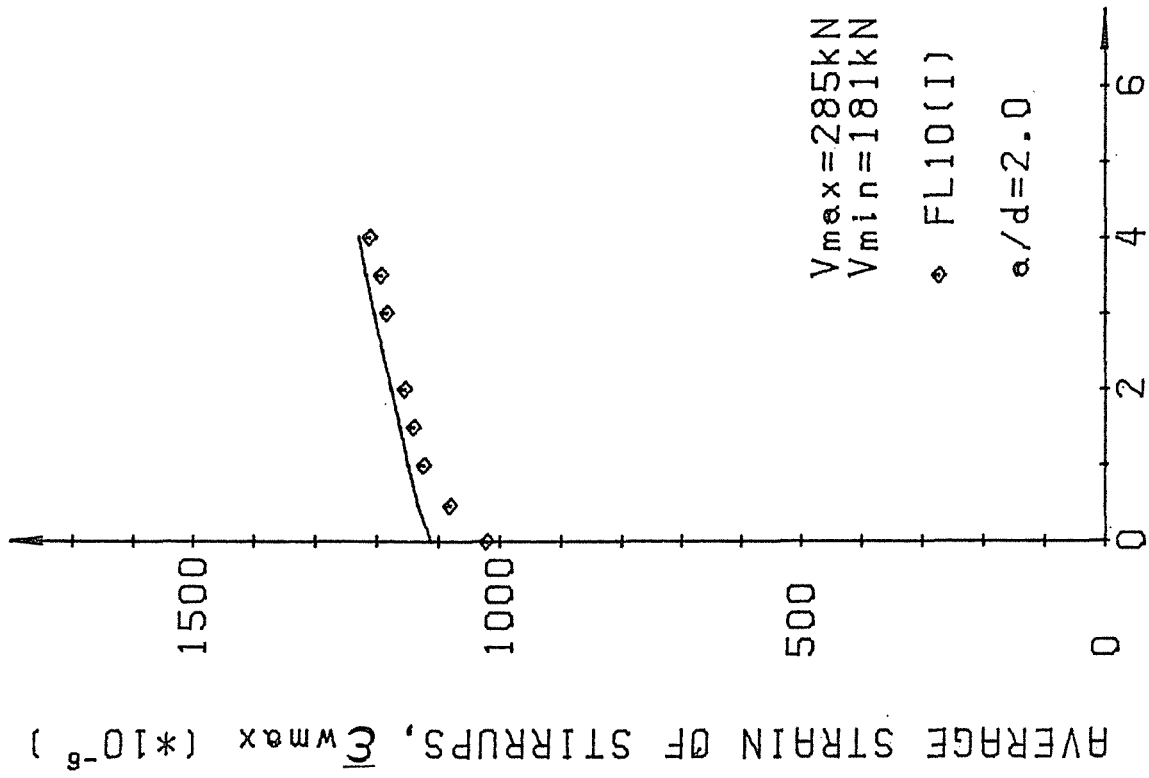


Fig. 4.1(h)

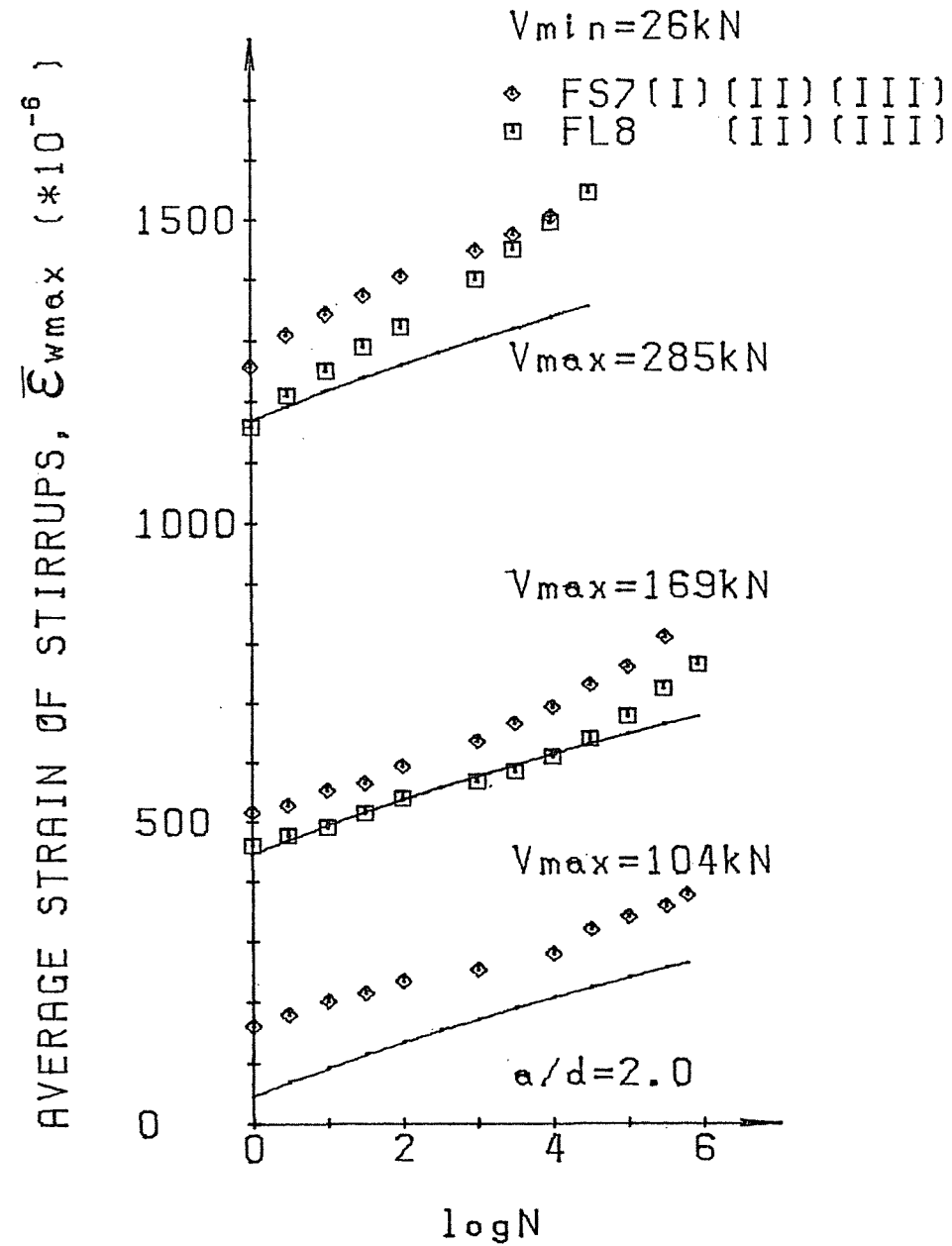


Fig.4.2 Average strain of stirrups at applied maximum shear force in caces where maximum load is changed for one beam with constant minimum load (T-beams in author's tests)

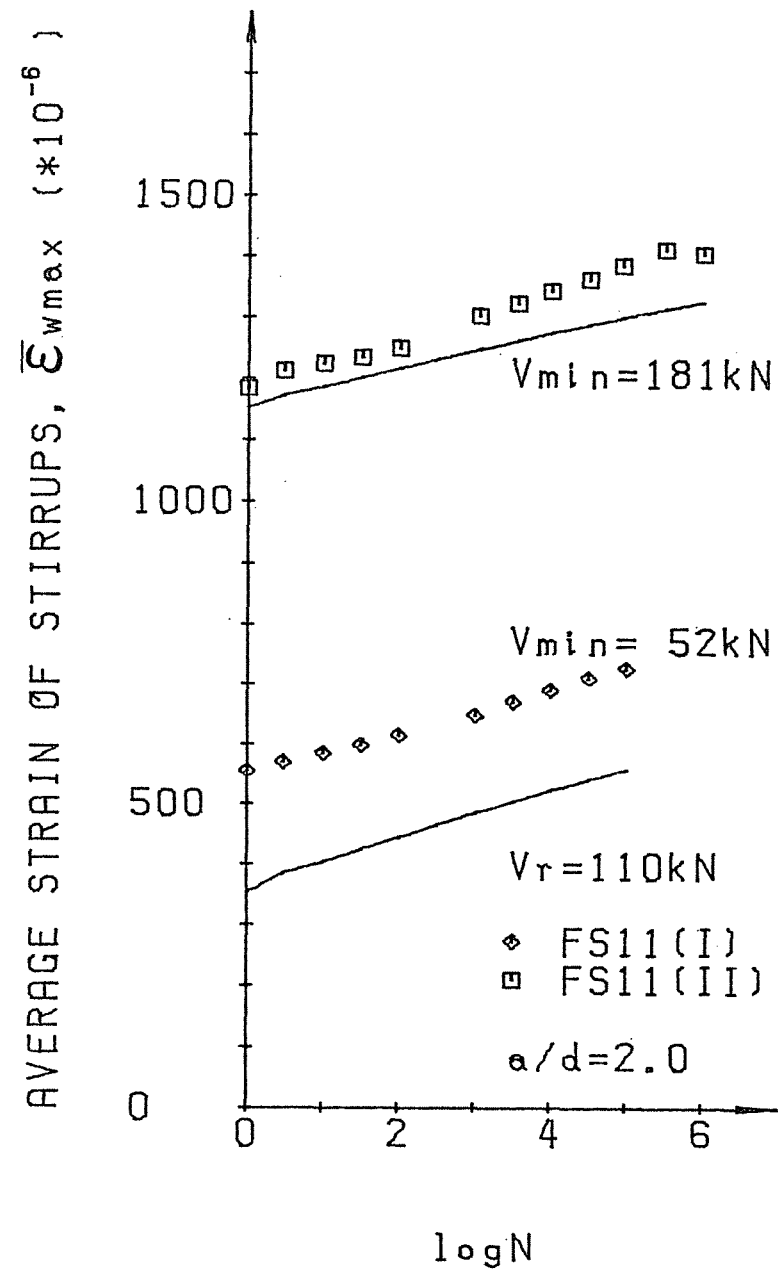


Fig.4.3 Average strain of stirrups at applied maximum shear force in a case where minimum load is changed for one beam with constant load range (T-beam in author's tests)

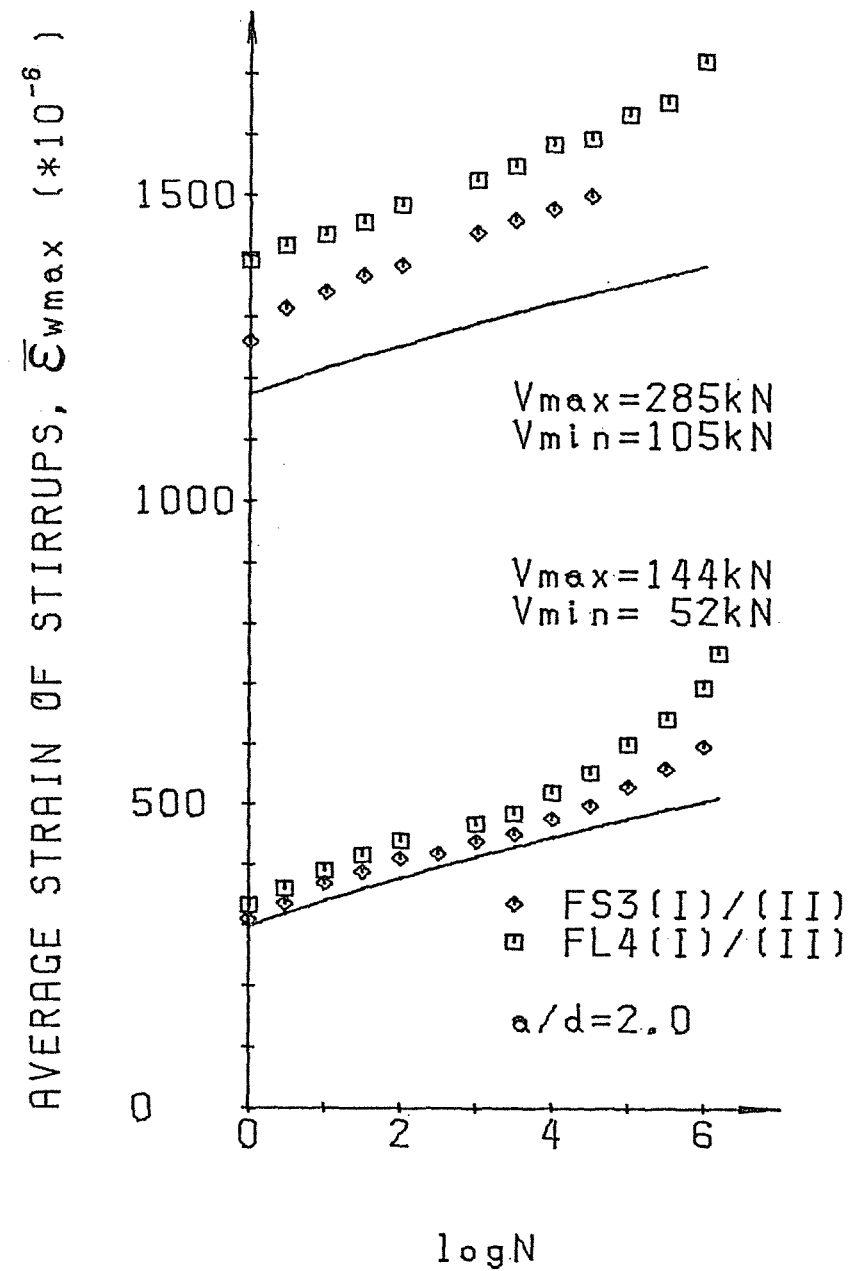


Fig.4.4 Average strain of stirrups at applied maximum shear force in cases where load range is changed for one beam as well as maximum and minimum loads (T-beams in author's tests)

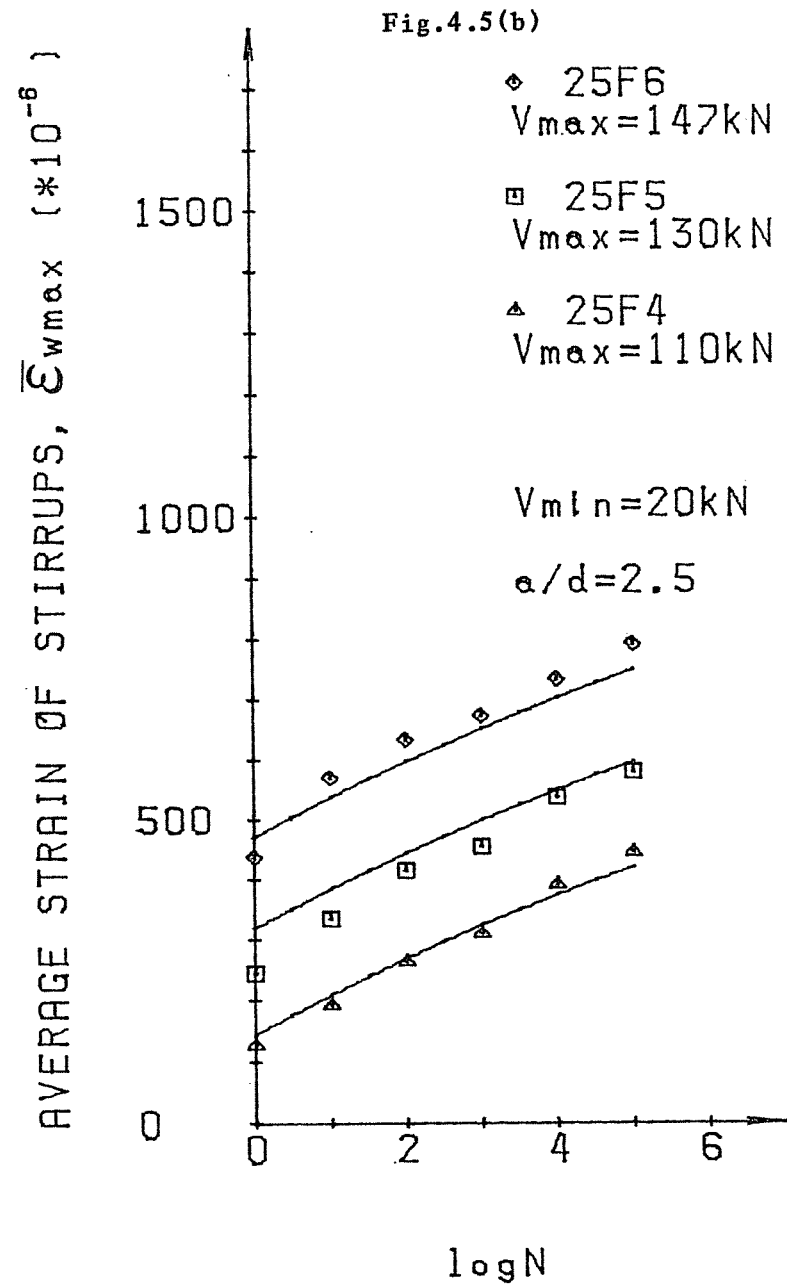
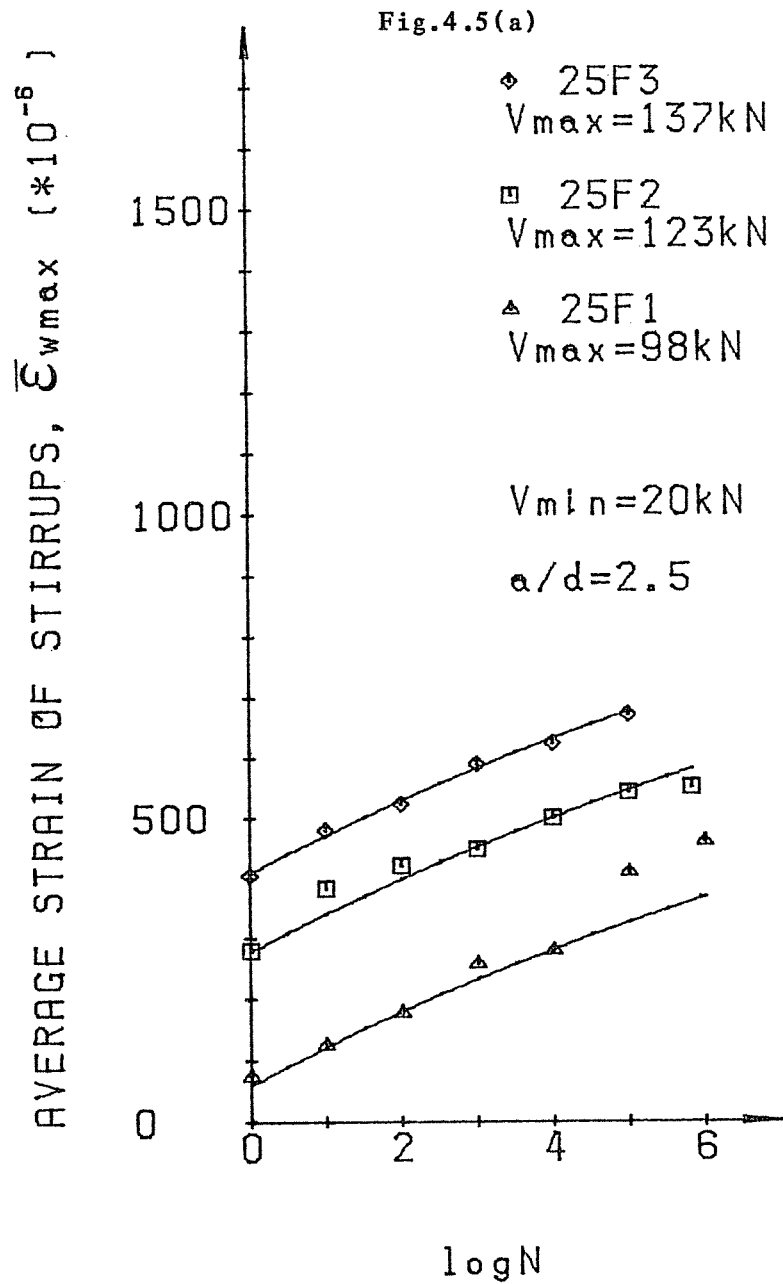


Fig.4.5 Average strain of stirrups at applied maximum shear force
 (rectangular beams in Farghaly's tests [1])

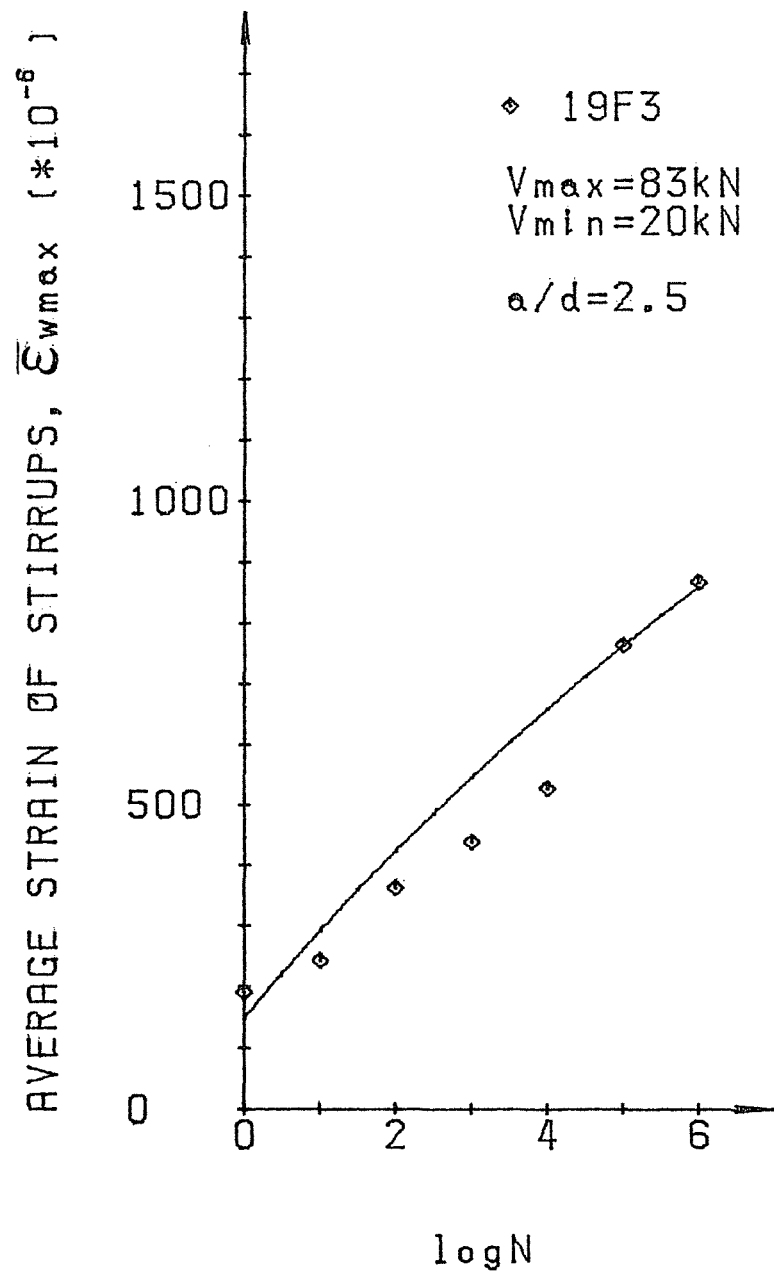


Fig.4.5(c)

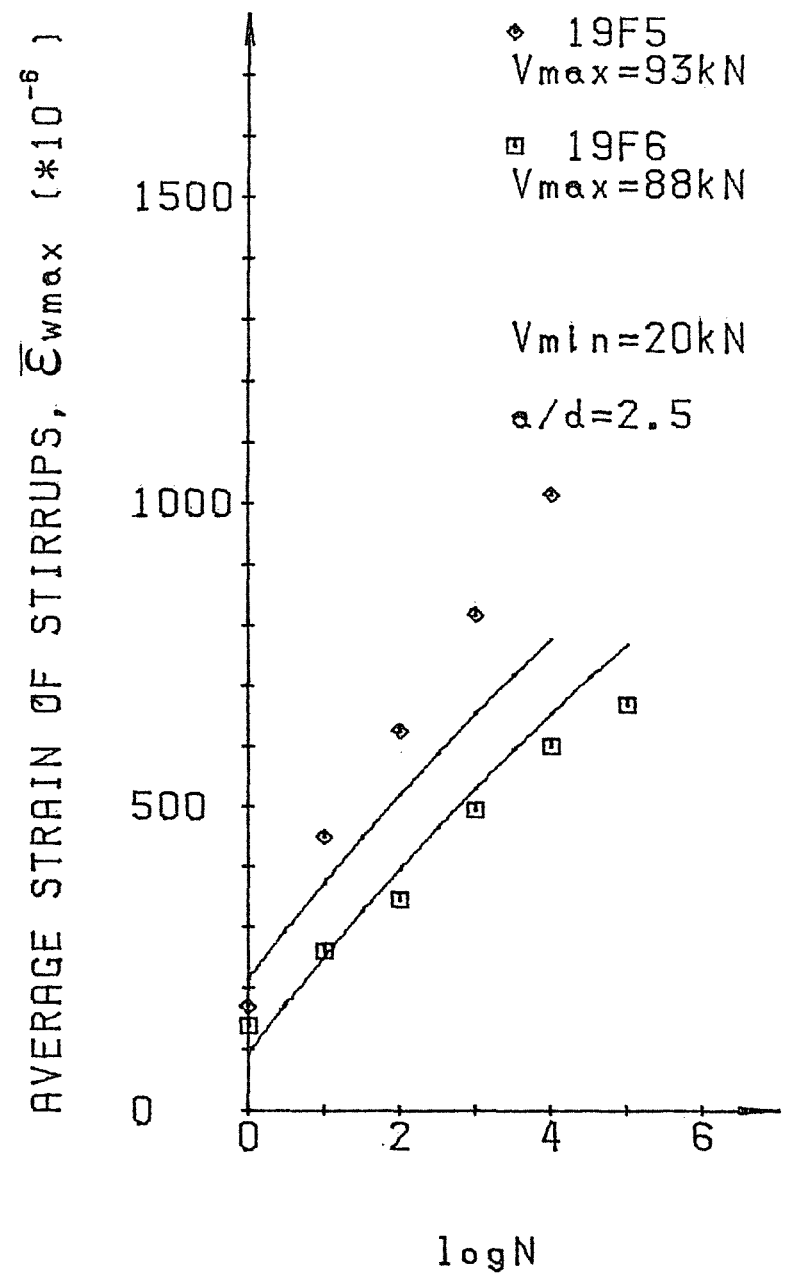


Fig.4.5(d)

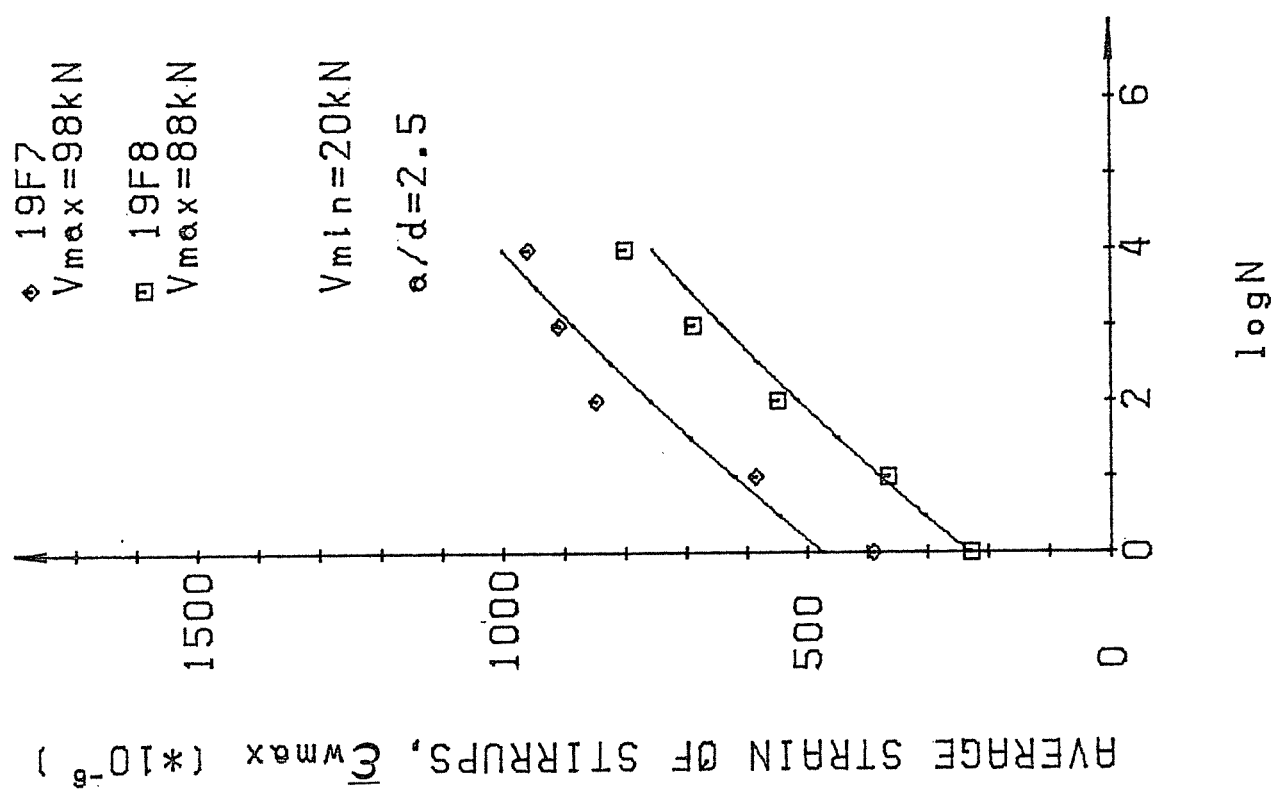


Fig.4.5(e)

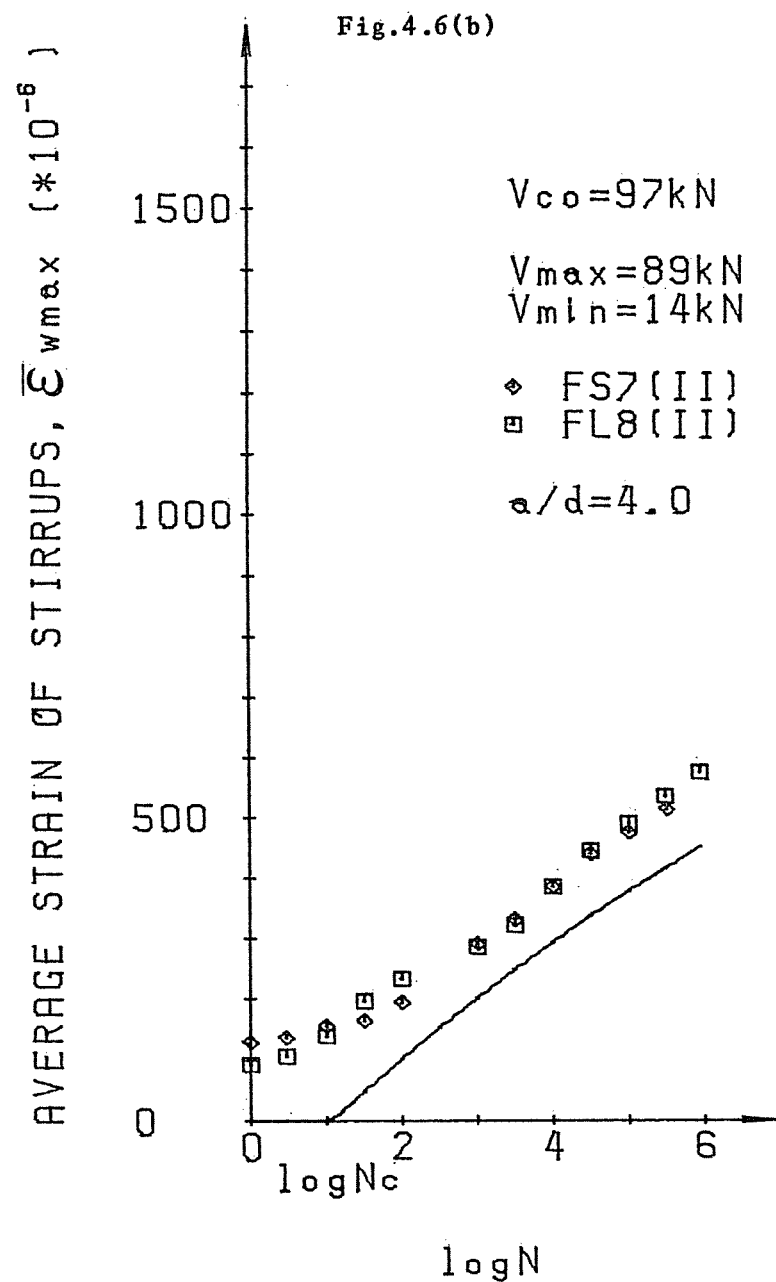
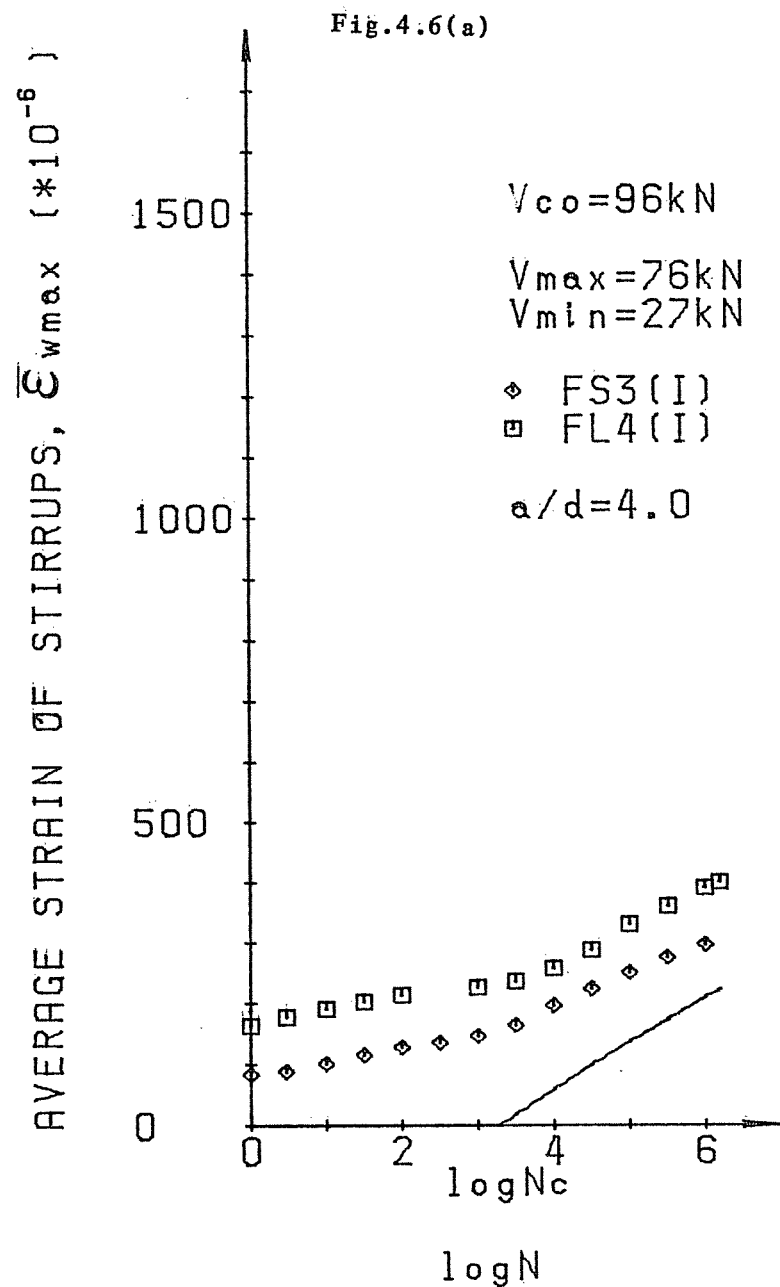


Fig.4.6 Average strain of stirrups at applied maximum shear force less than the shear capacity of concrete (T-beams in author's tests and rectangular beams in Farghaly's tests [1])

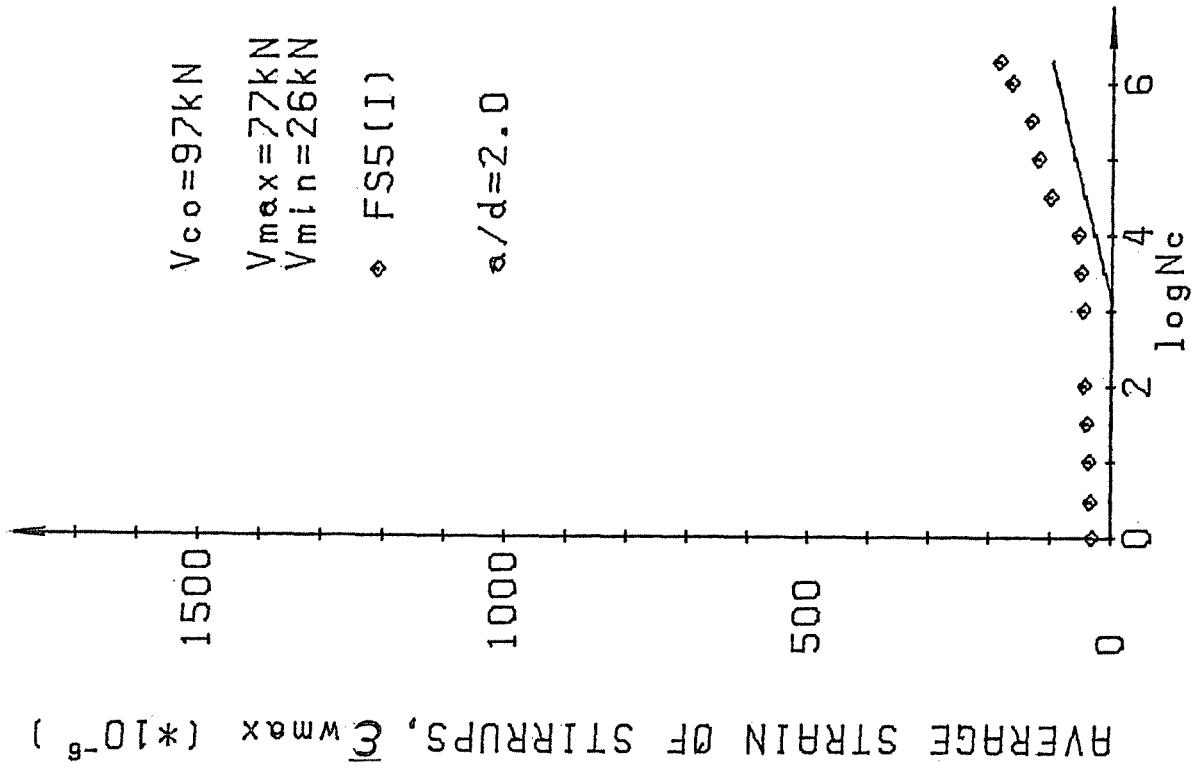


Fig.4.6(d)

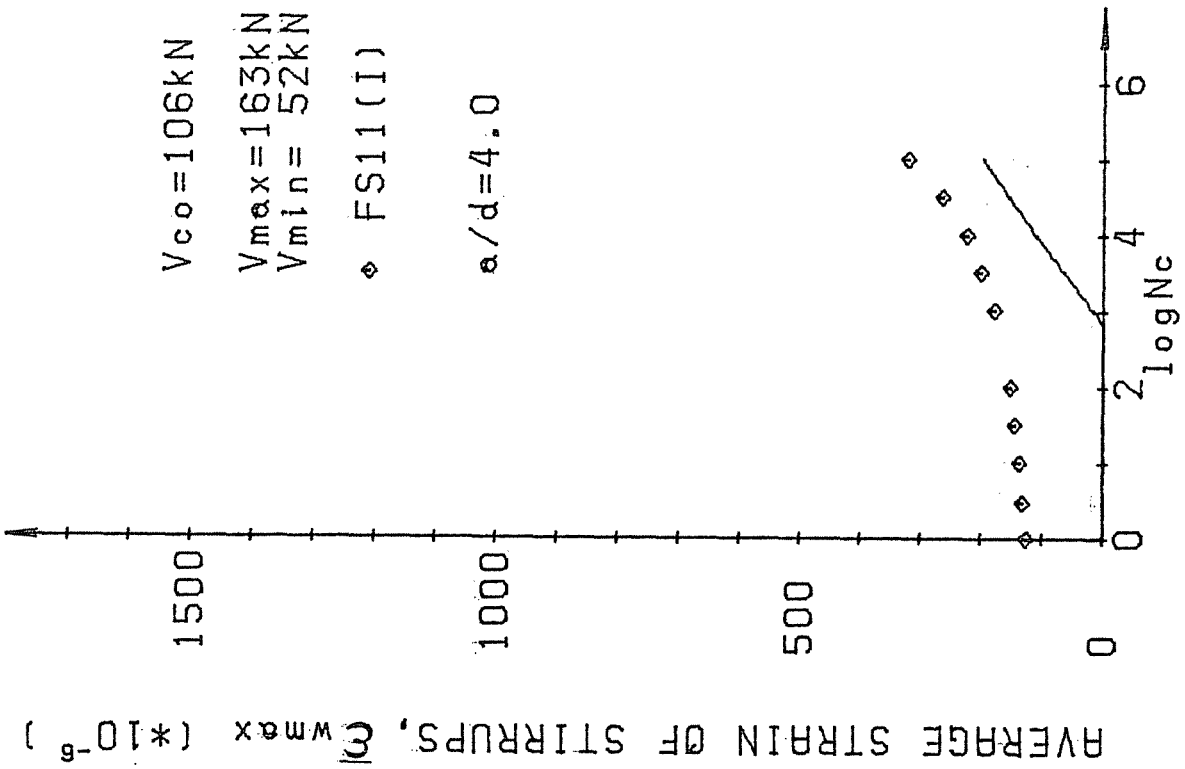


Fig.4.6(c)

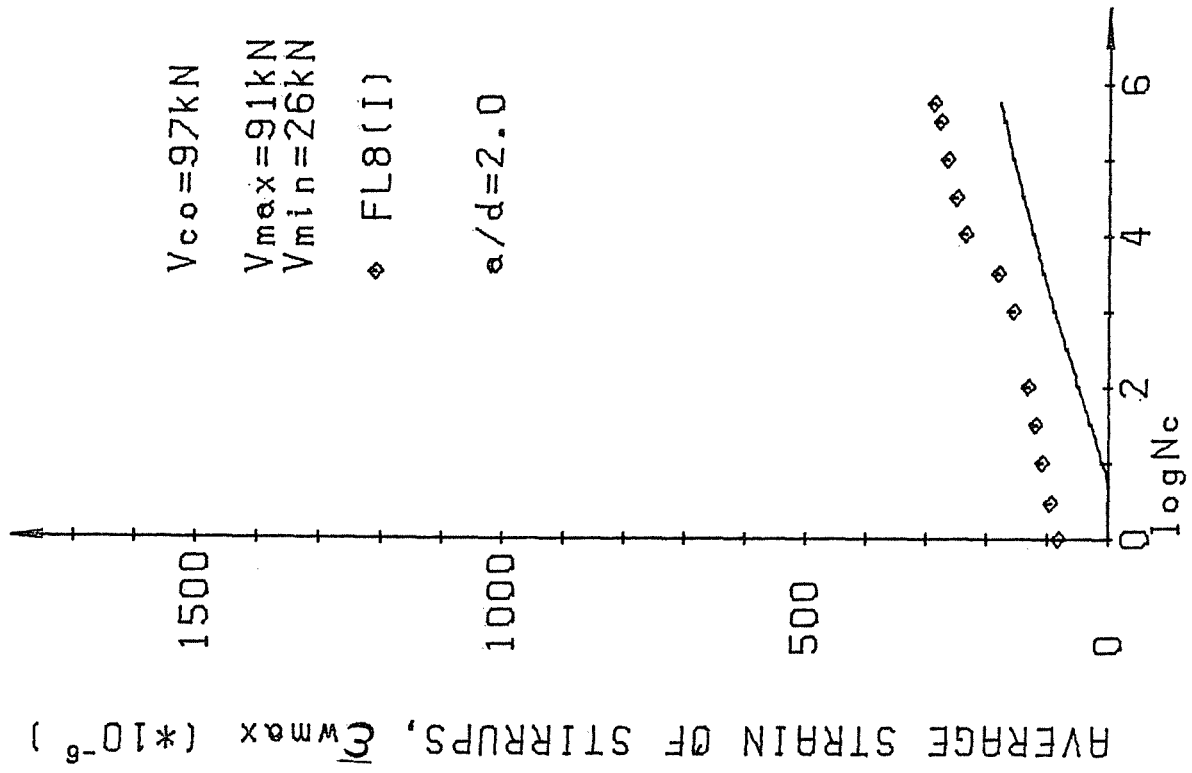


Fig. 4.6(f)

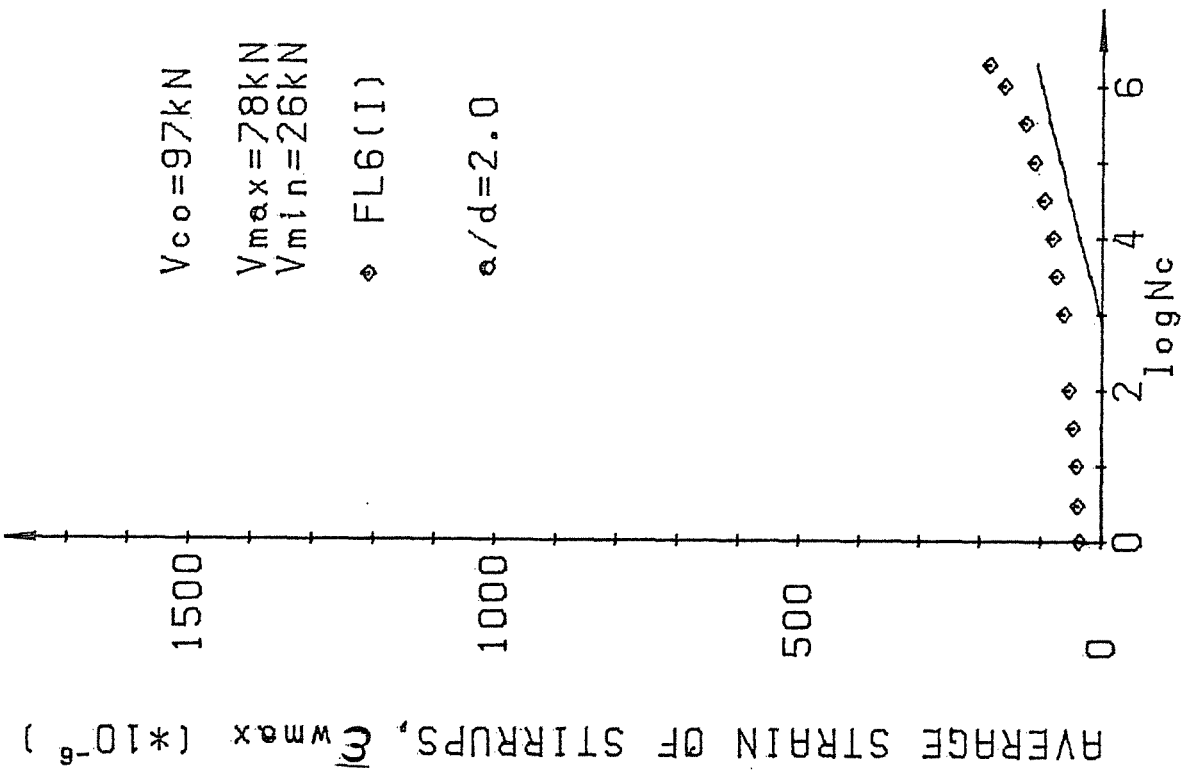


Fig. 4.6(e)

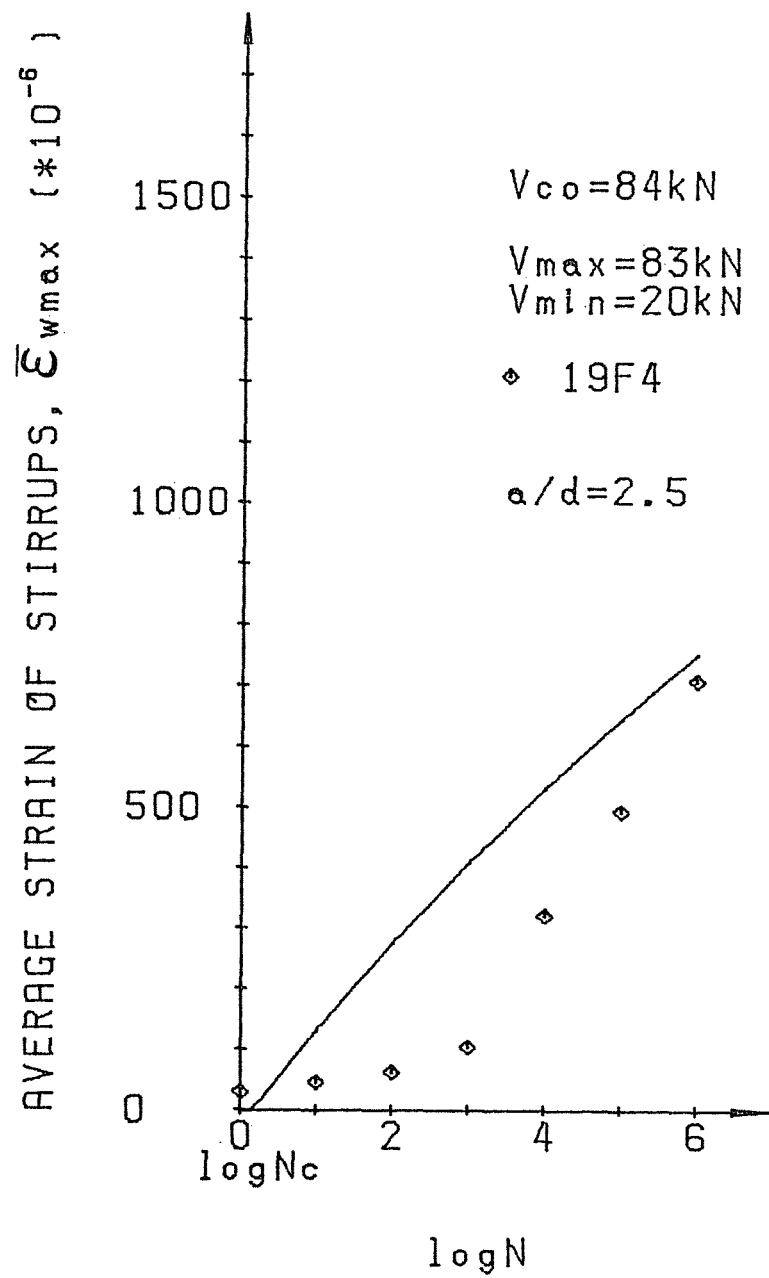


Fig.4.6(g)

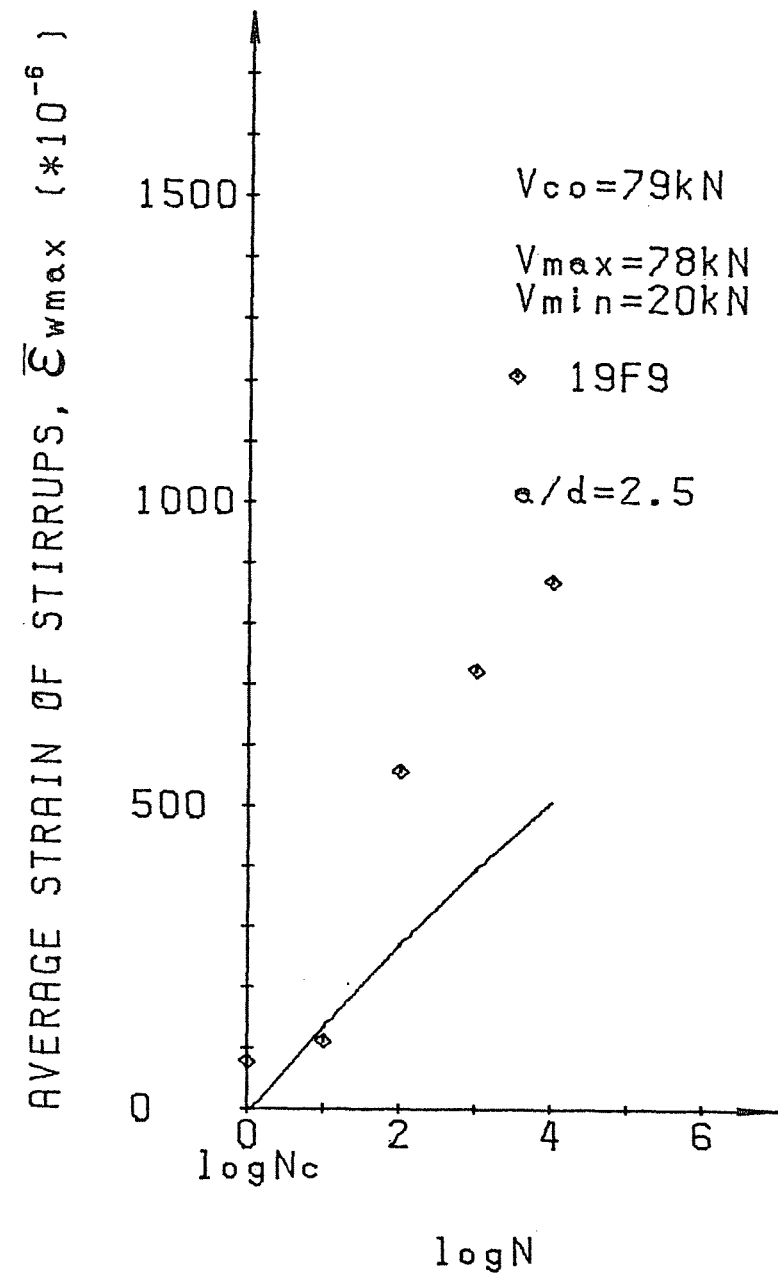


Fig.4.6(h)

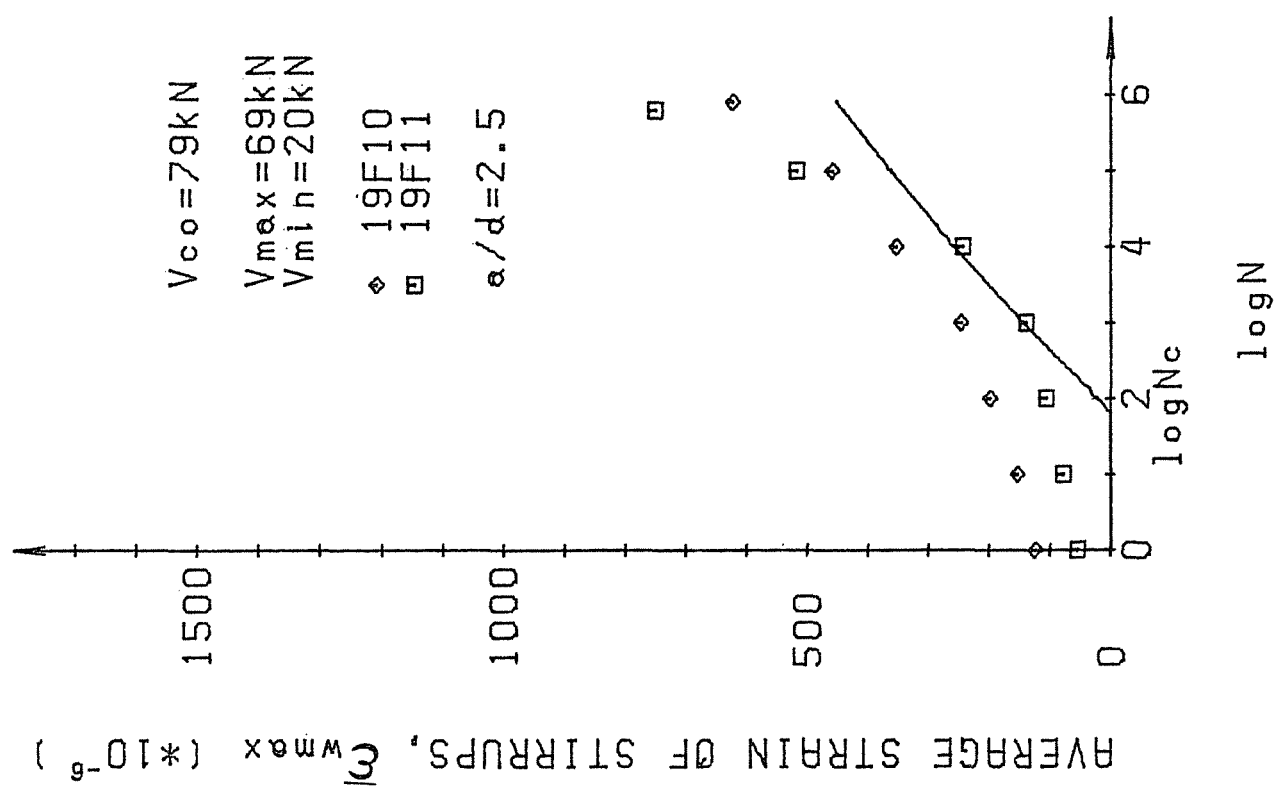


Fig.4.6(i)

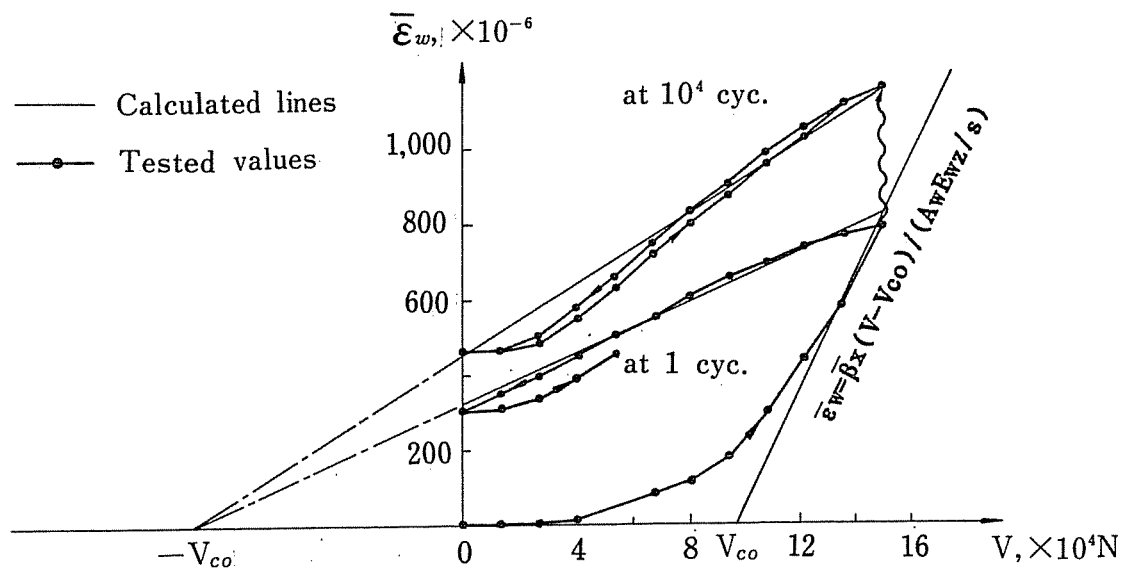


Fig.4.7(a)

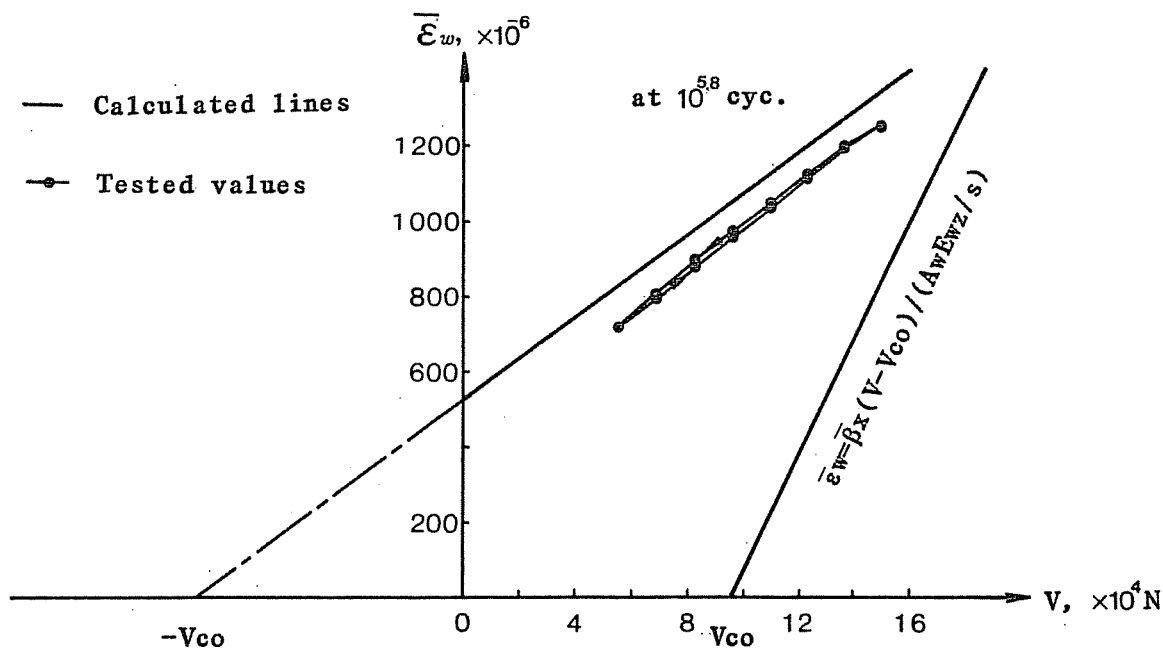


Fig.4.7(b)

Fig.4.7 Observed and assumed relationships between applied shear force and average of stirrup strains
 ((a) FL6(II) $a/d = 4.0$, (b) FS3(II) $a/d = 4.0$)

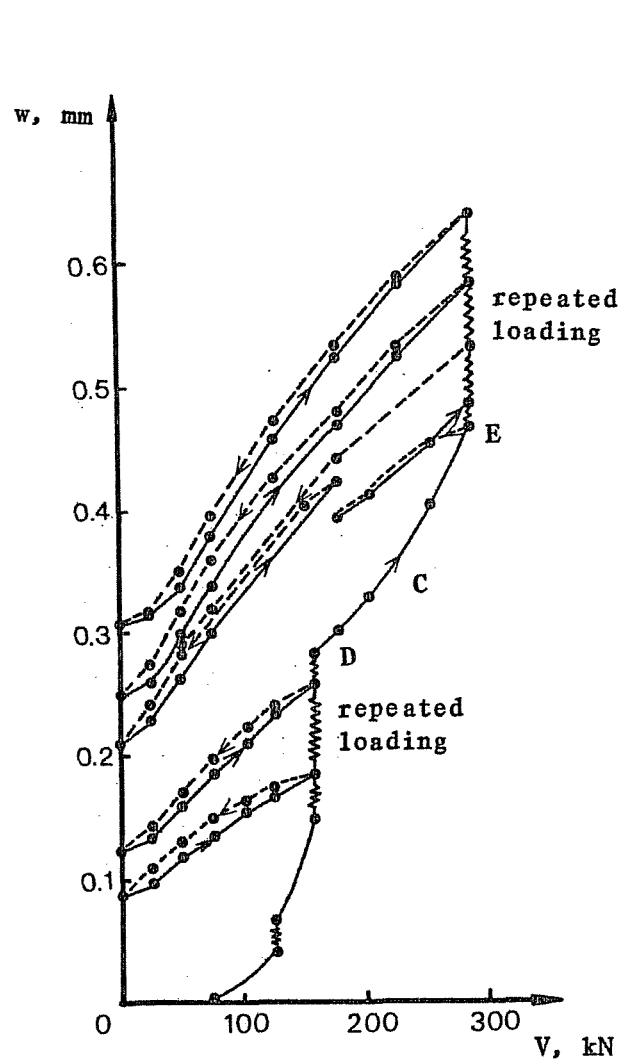


Fig.4.8 Relationship between applied shear force and width of diagonal crack (FS11 $a/d=2.0$)

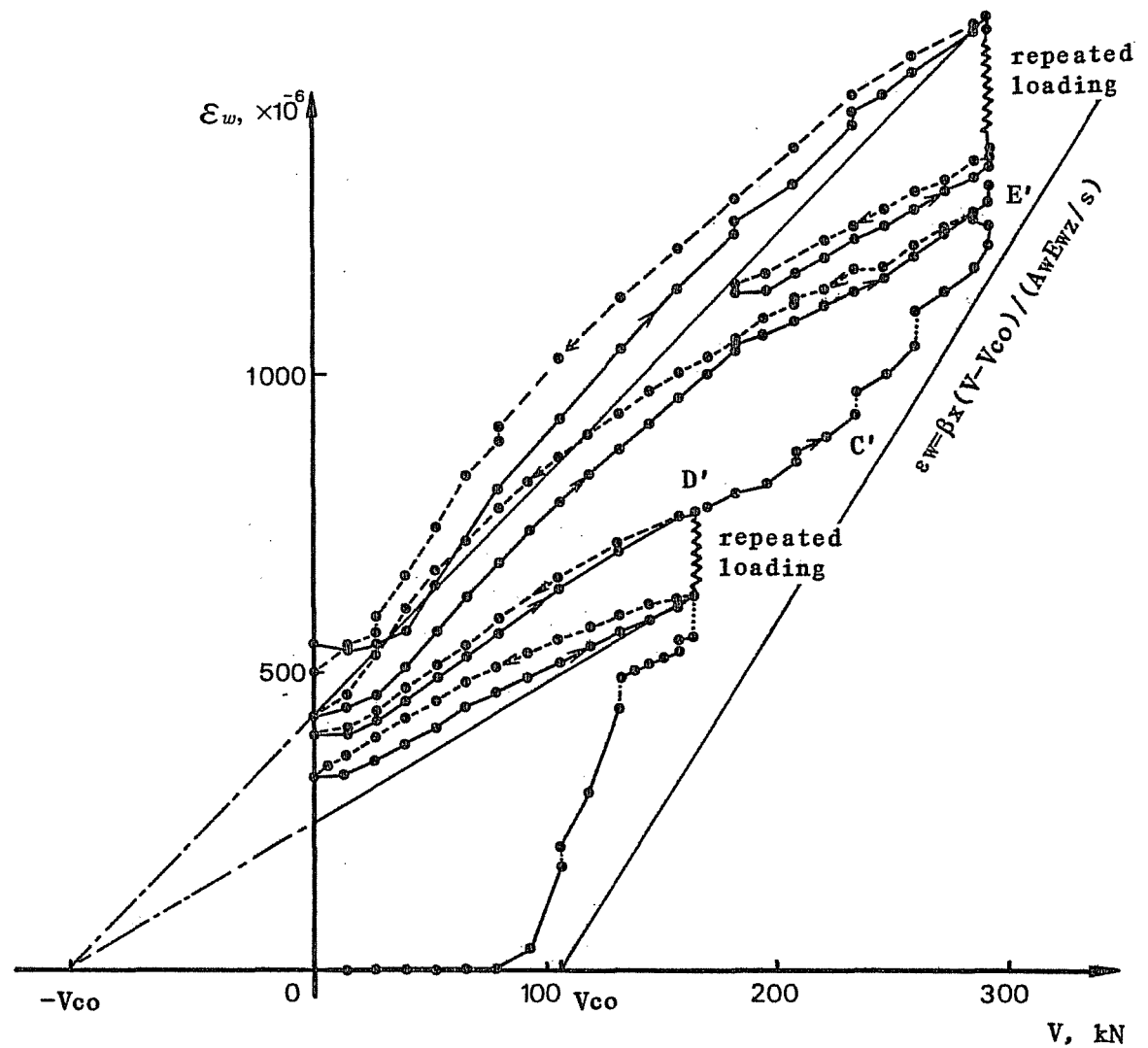


Fig.4.9 Relationship between applied shear force and stirrup strain (FS11 $a/d=2.0$, Gauge No.29)

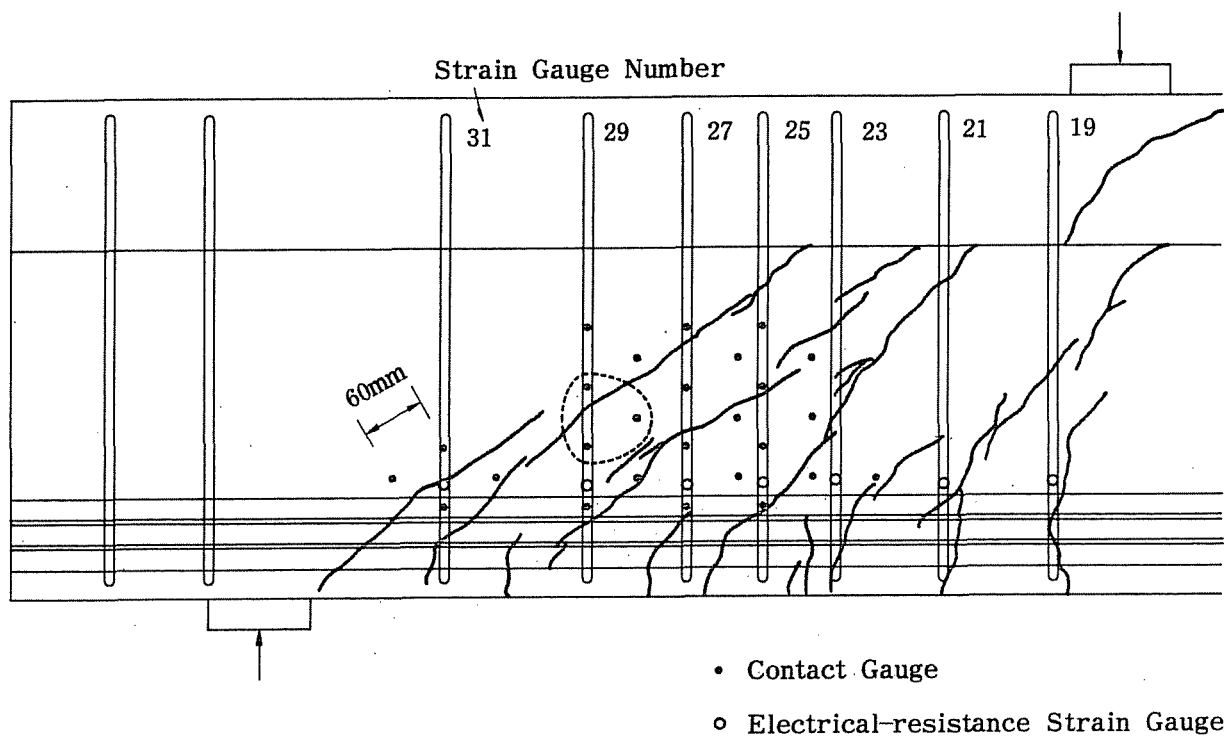


Fig.4.10 Positions of contact gauges and propagation of diagonal cracks
(FS11 $a/d=2.0$)

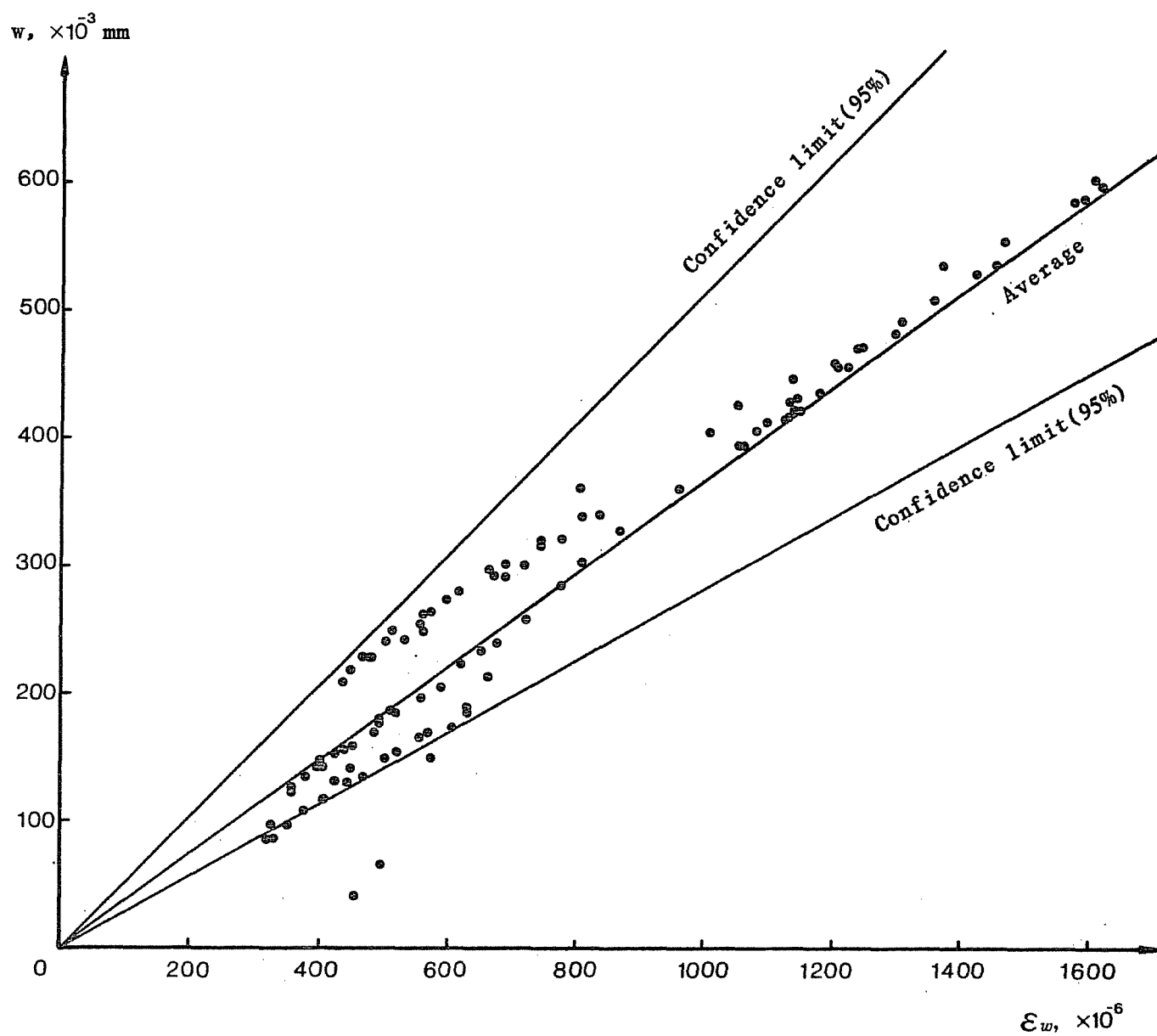


Fig.4.11 Relationship between stirrup strain shown in Fig.4.9 and crack width shown in Fig.4.8

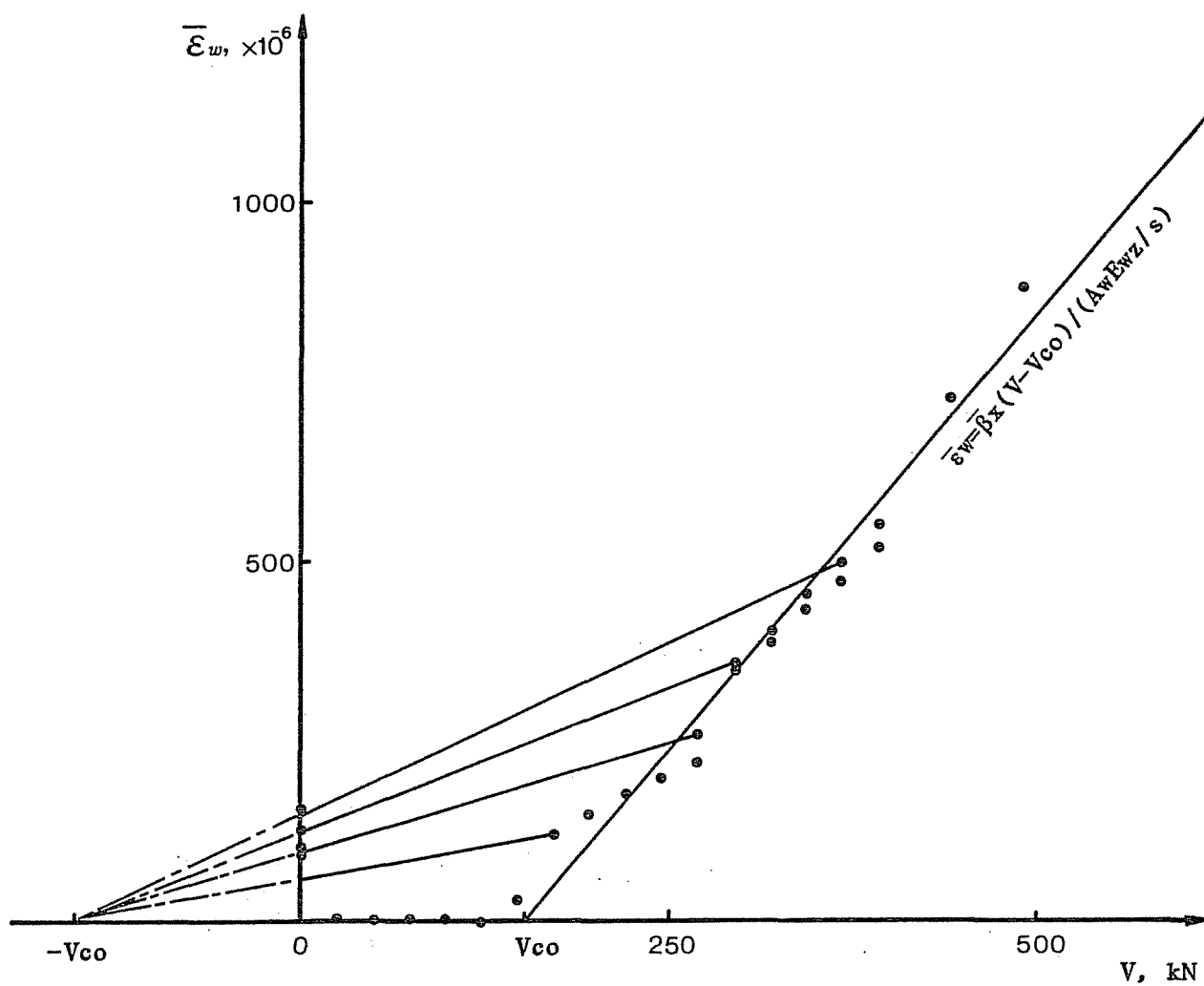


Fig.4.12 Relationship between applied shear force and average strain of stirrups in the previous test (Specimen No.3 [4])

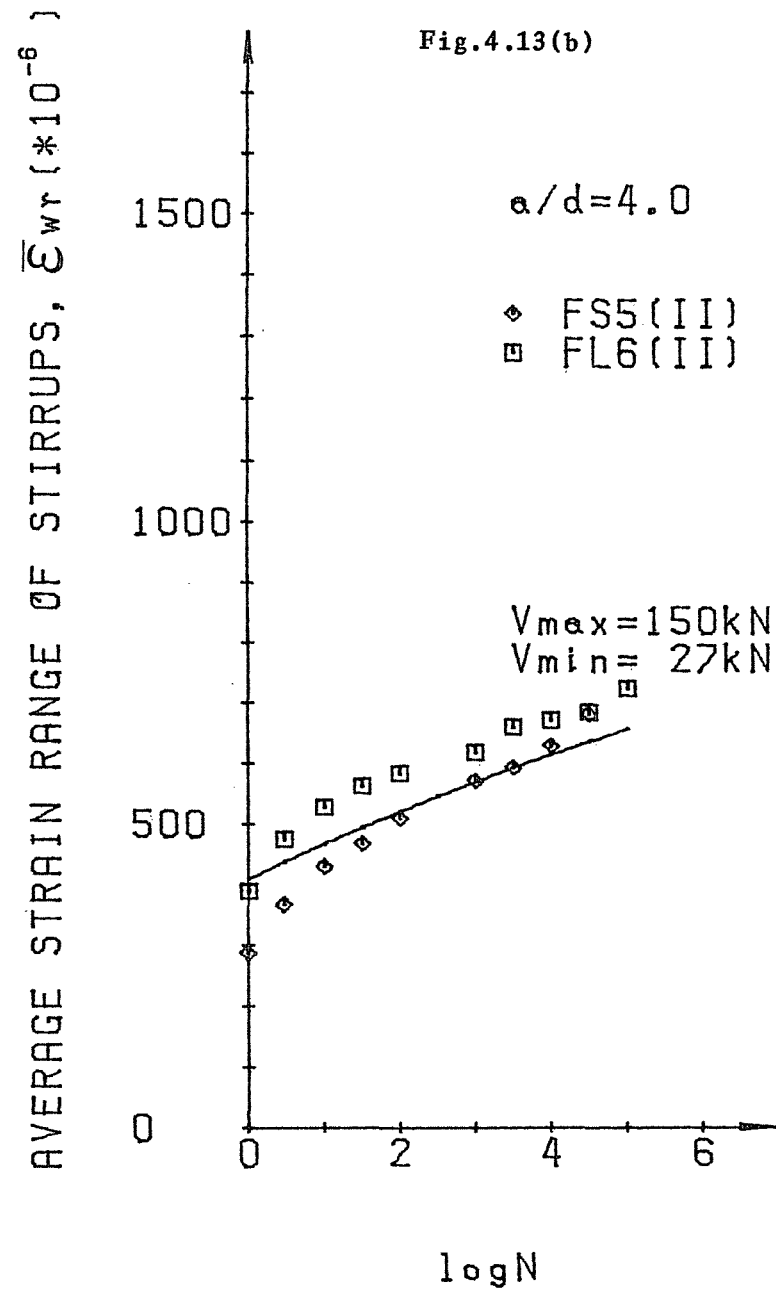
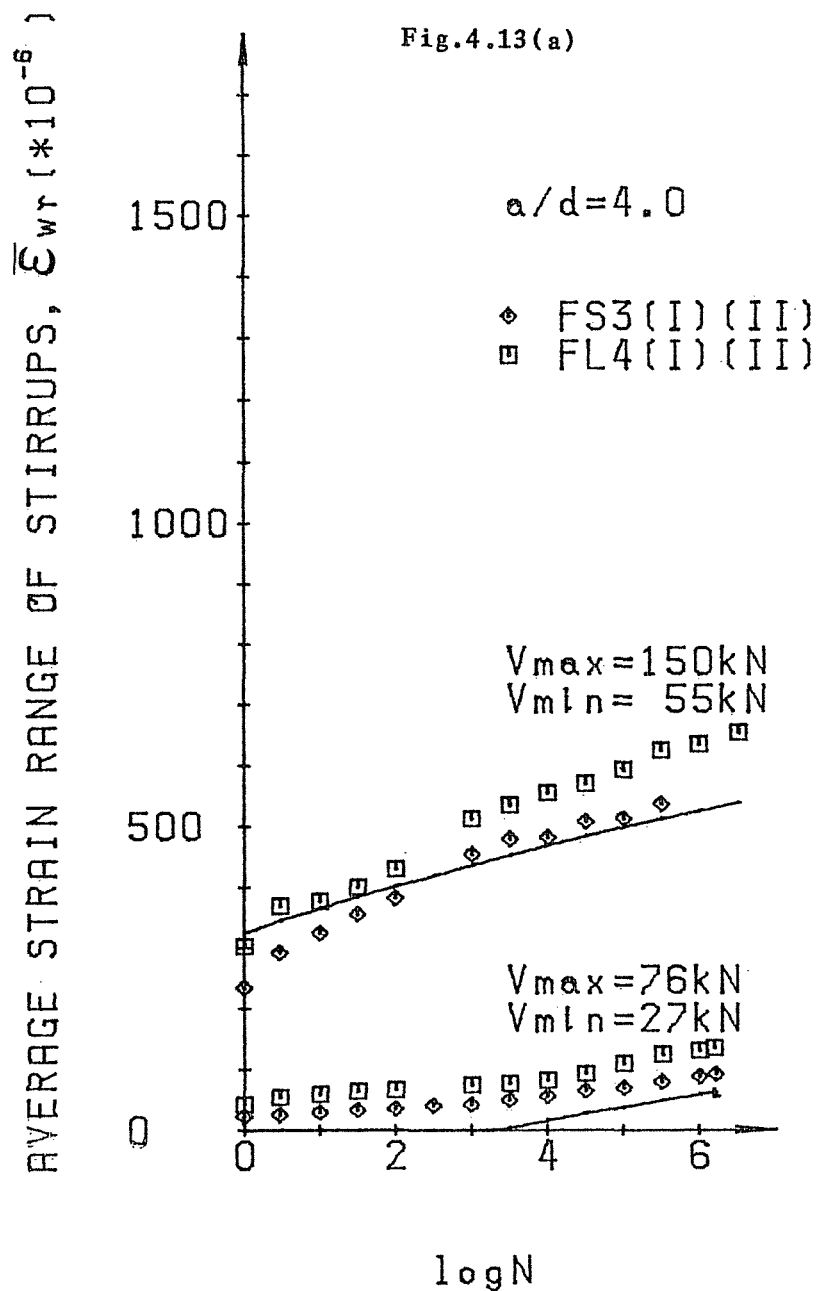


Fig.4.13 Average strain range of stirrups (T-beams in author's tests)

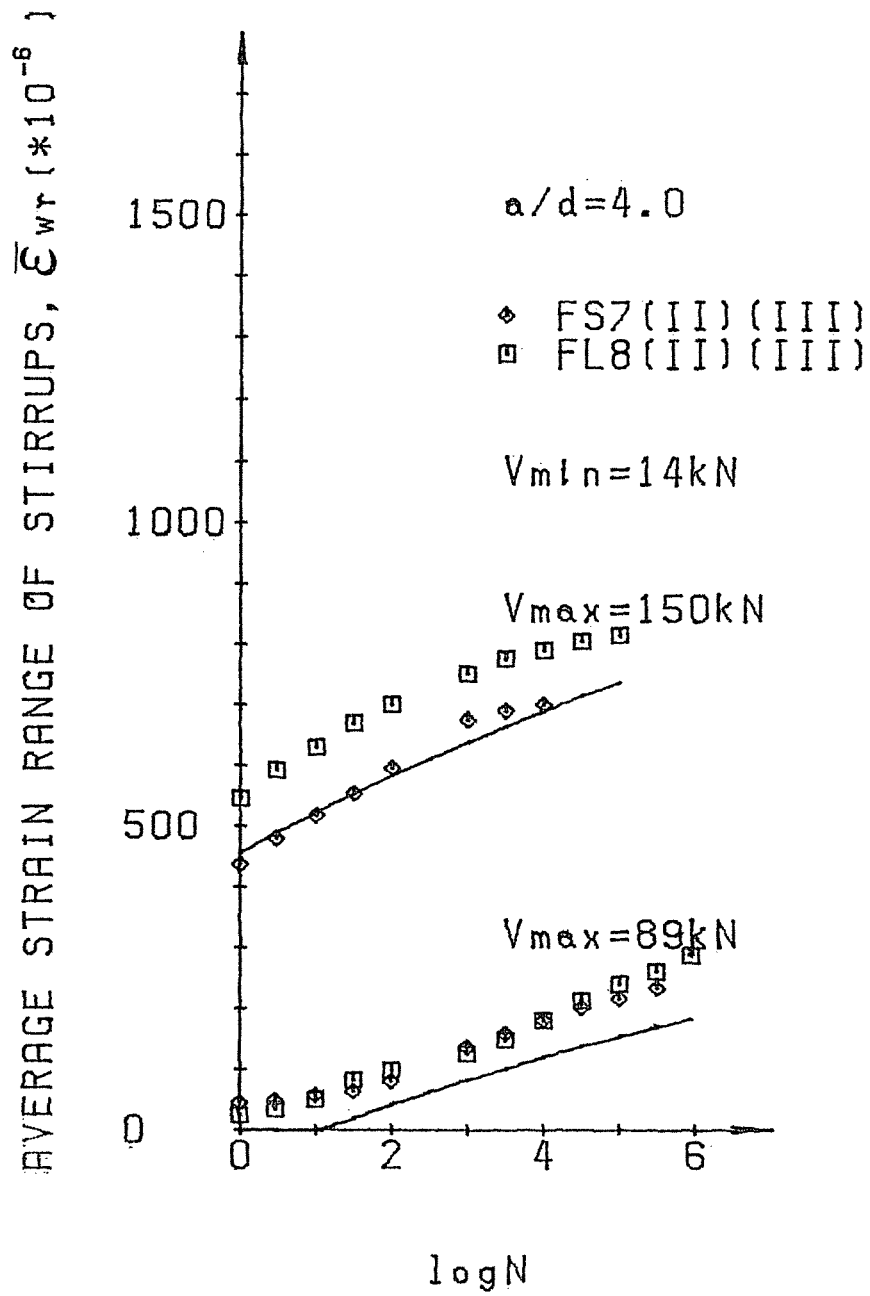


Fig.4.13(c)

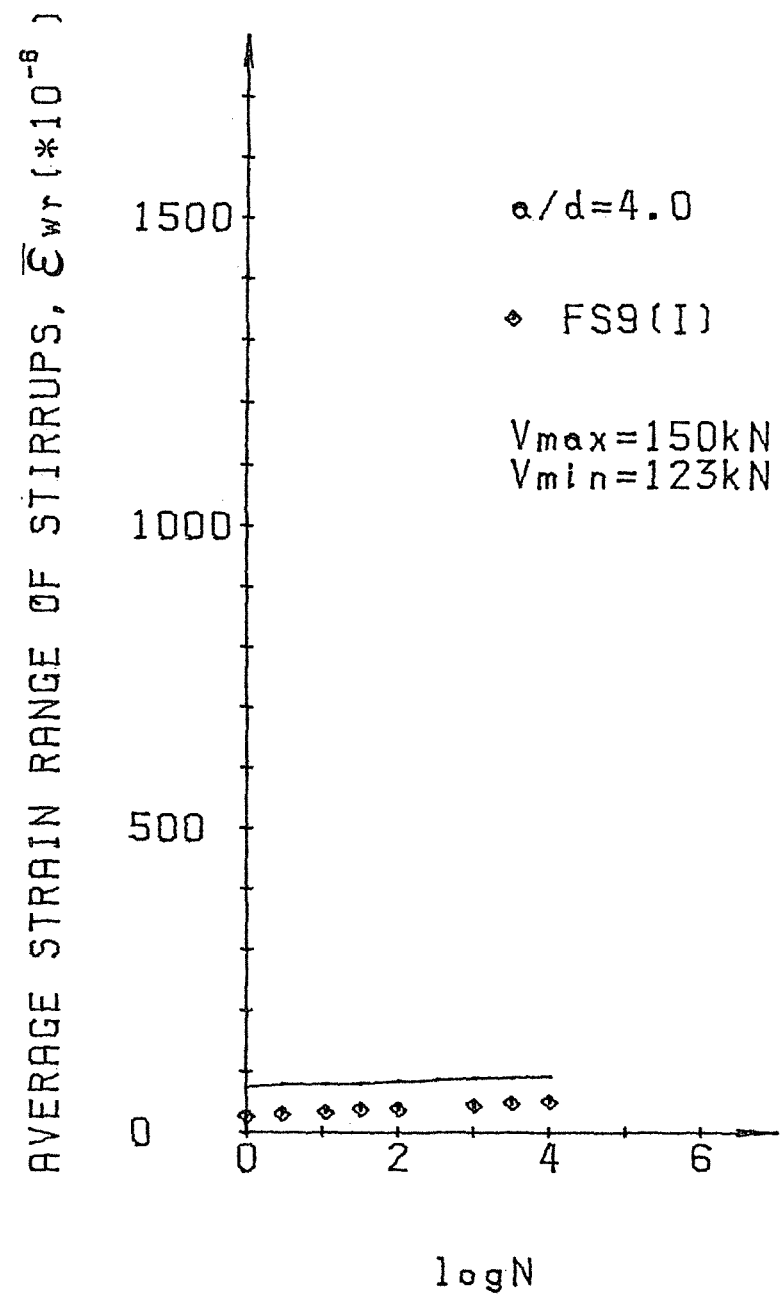


Fig.4.13(d)

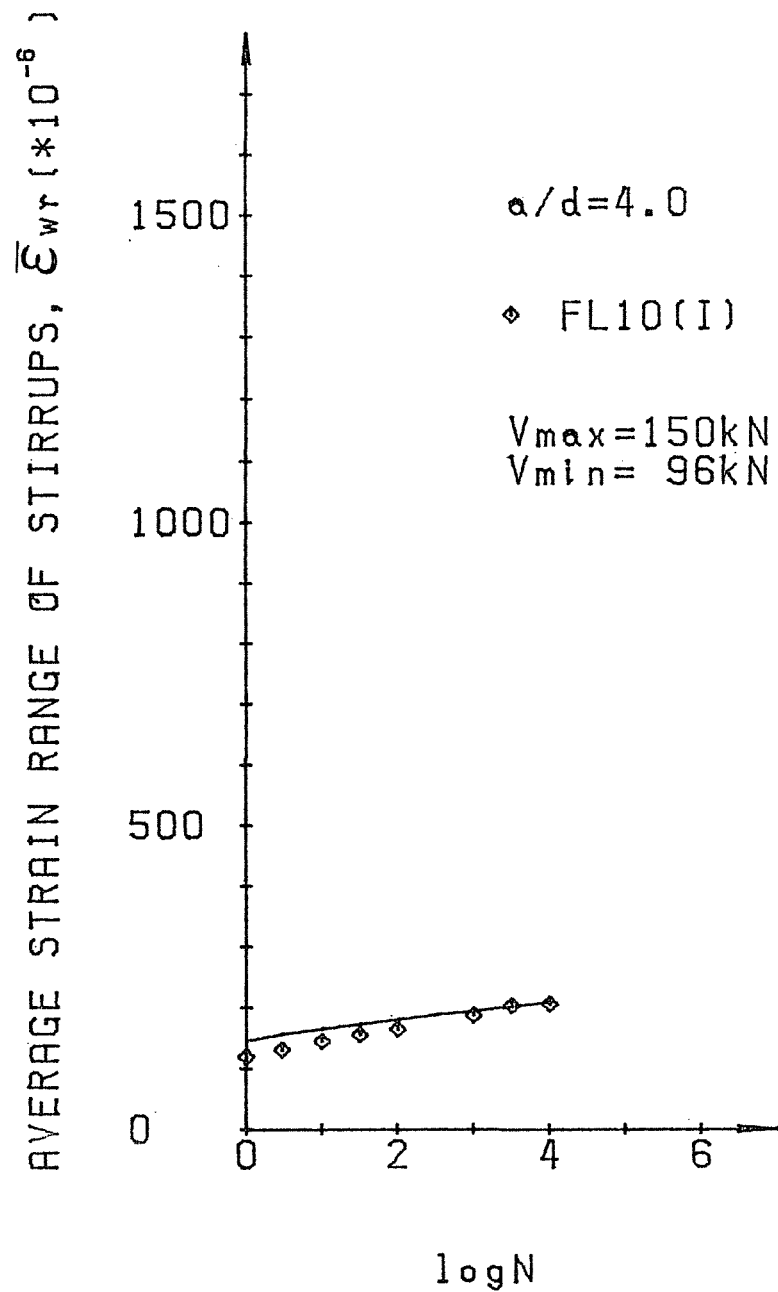


Fig.4.13(e)

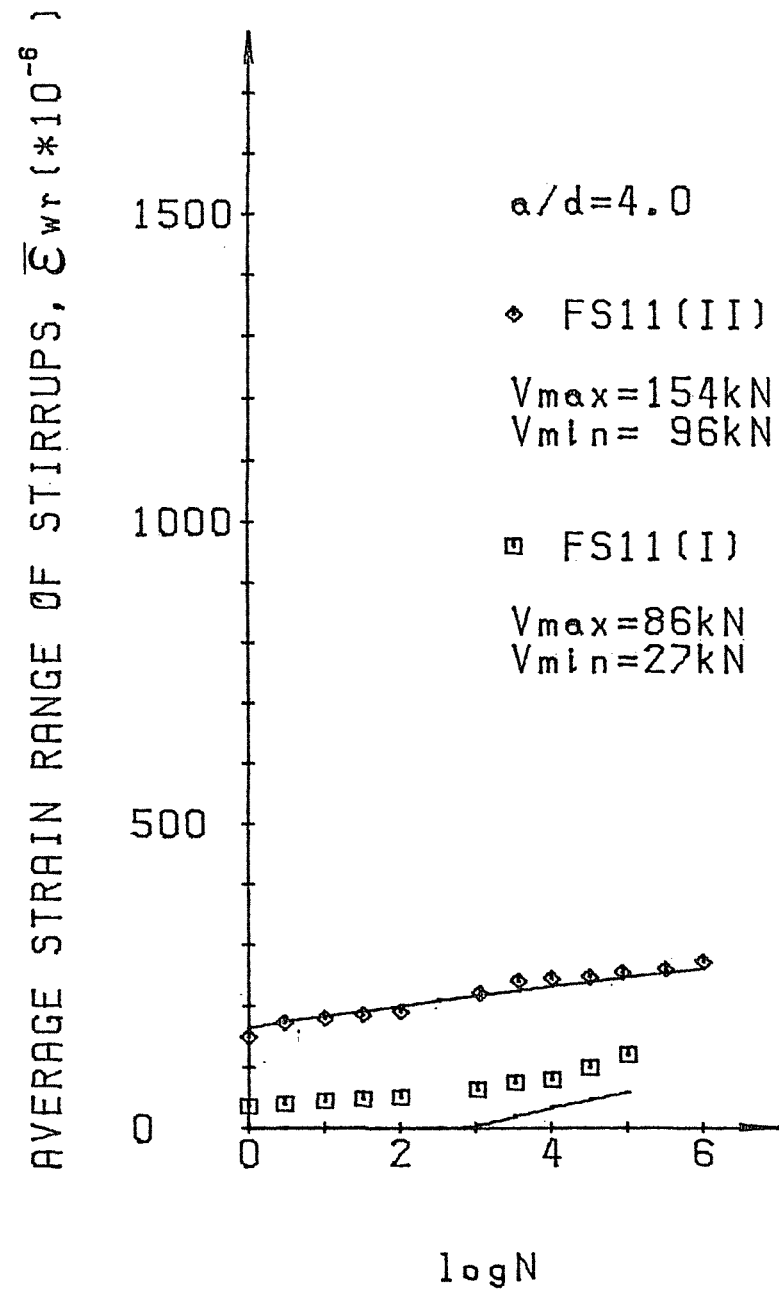


Fig.4.13(f)

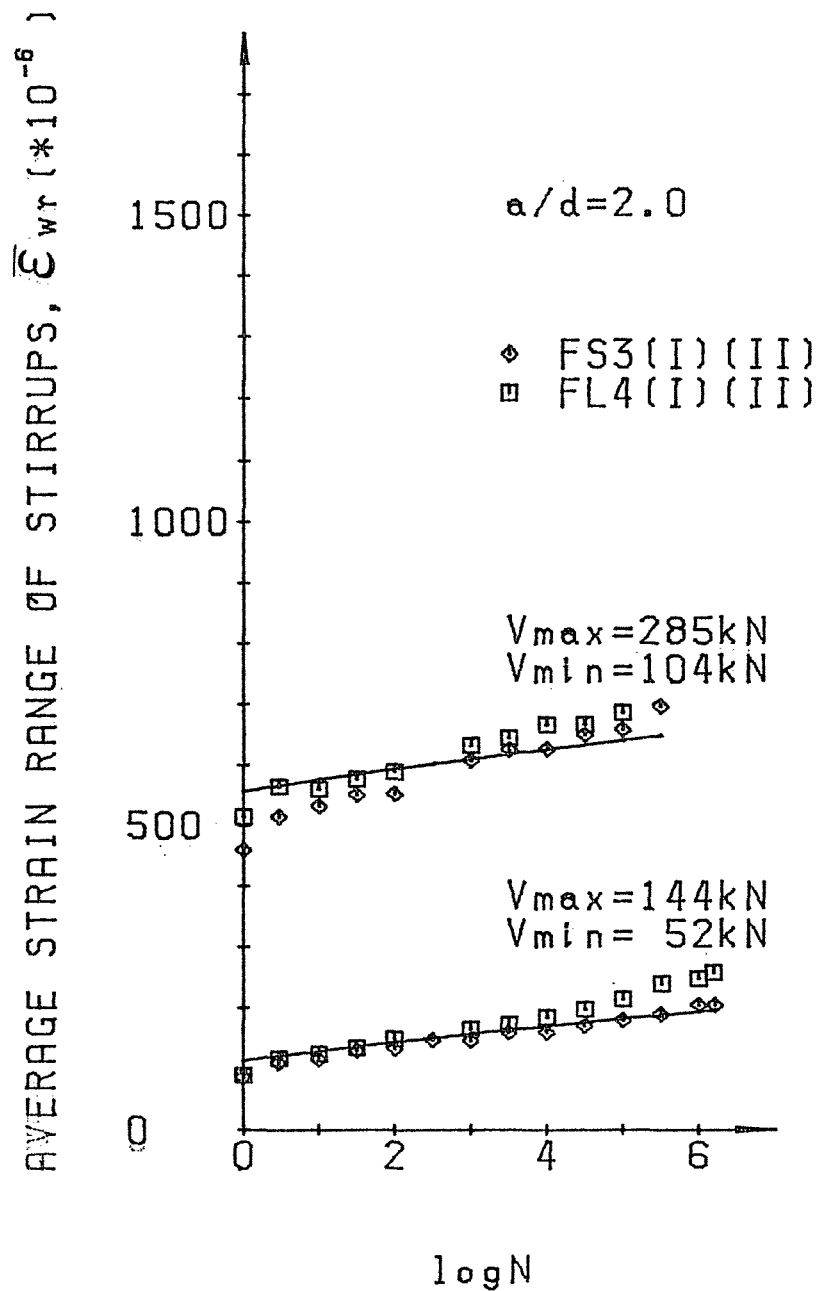


Fig.4.13(g)

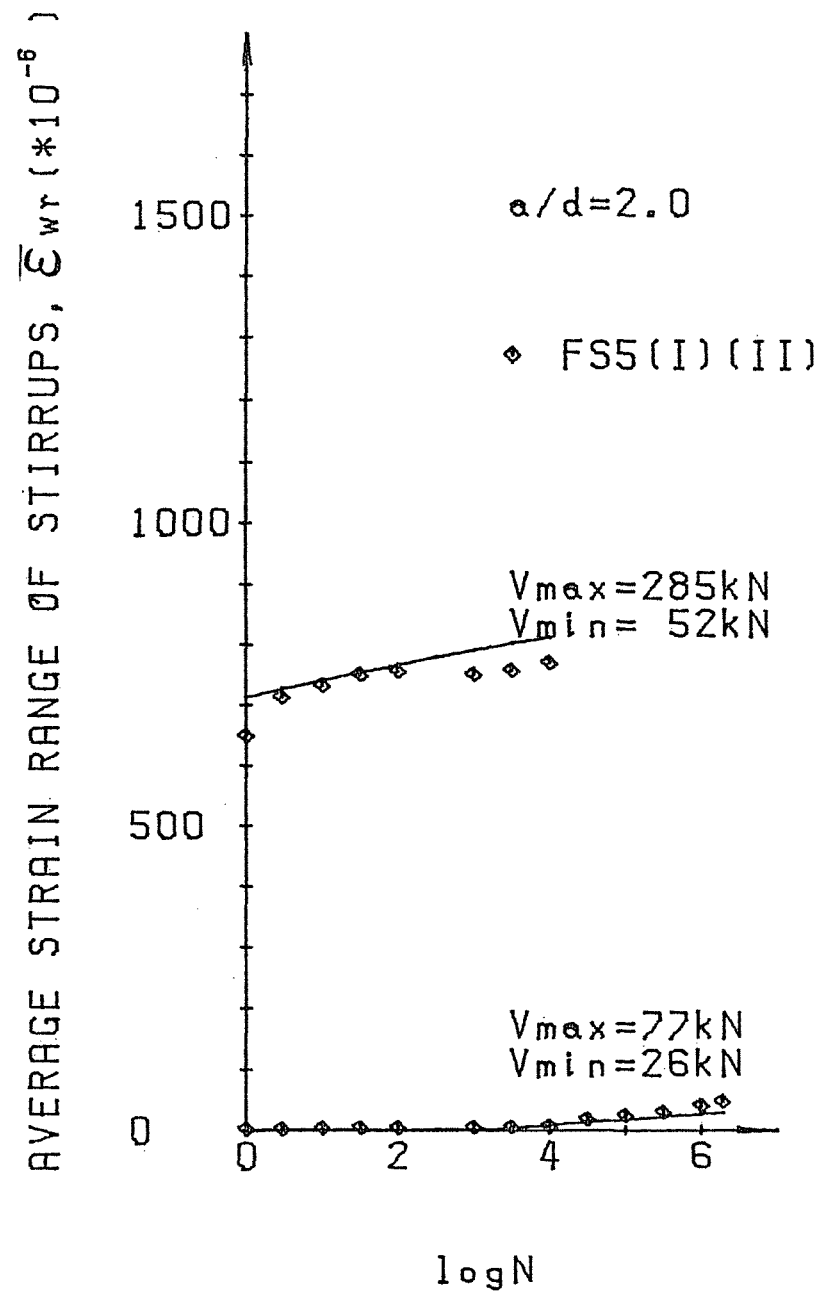


Fig.4.13(h)

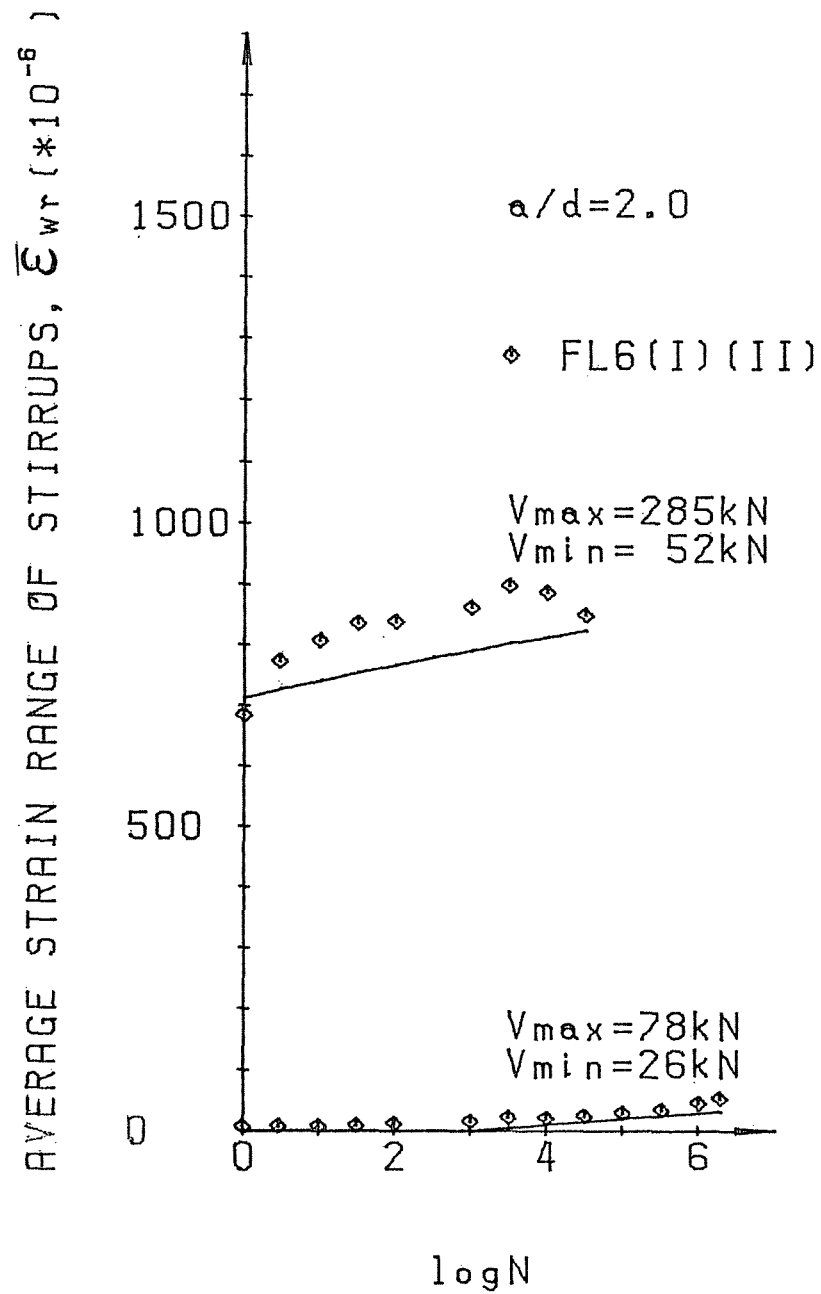


Fig.4.13(i)

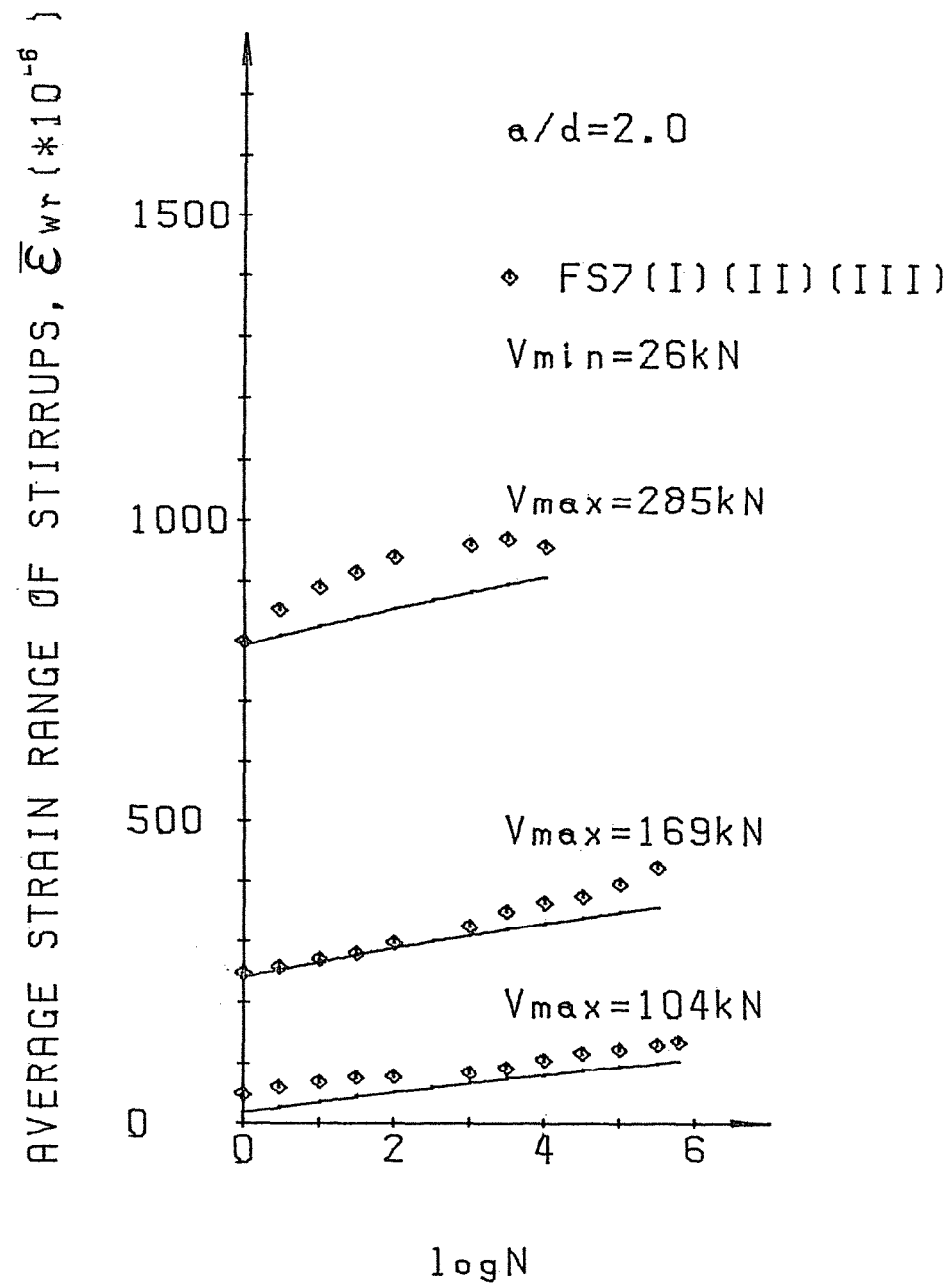


Fig.4.13(j)

AVERAGE STRAIN RANGE OF STIRRUPS, $\epsilon_w (*10^{-6})$

$a/d=2.0$

◆ FL8(I)(II)(III)

$V_{min}=26kN$

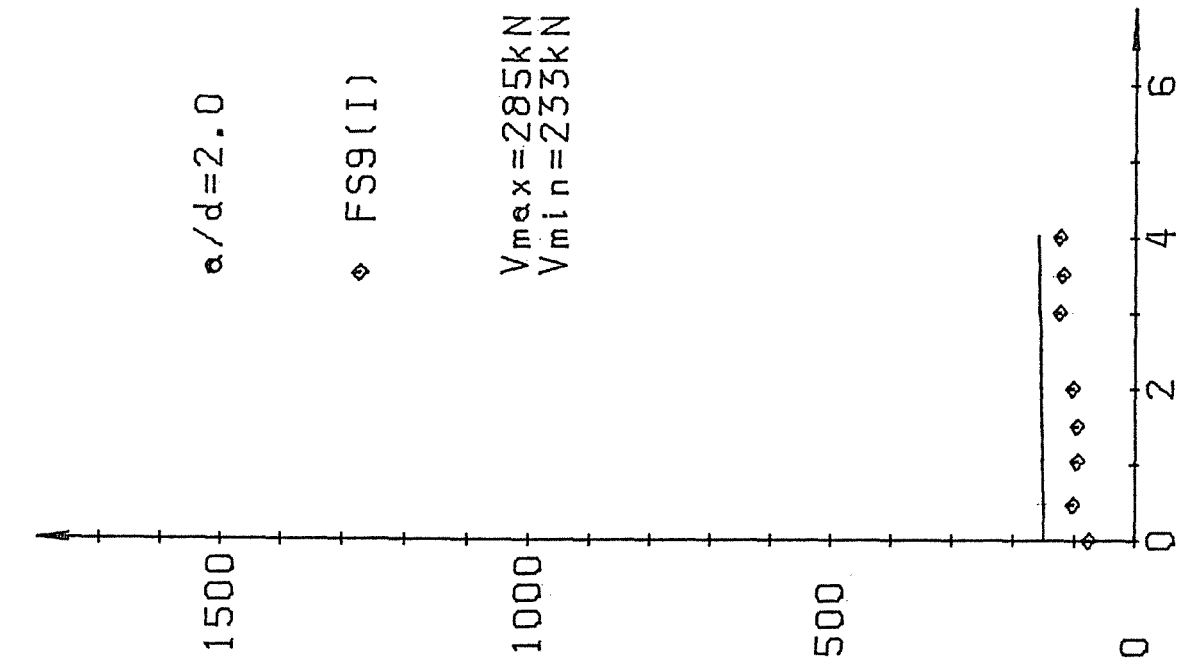
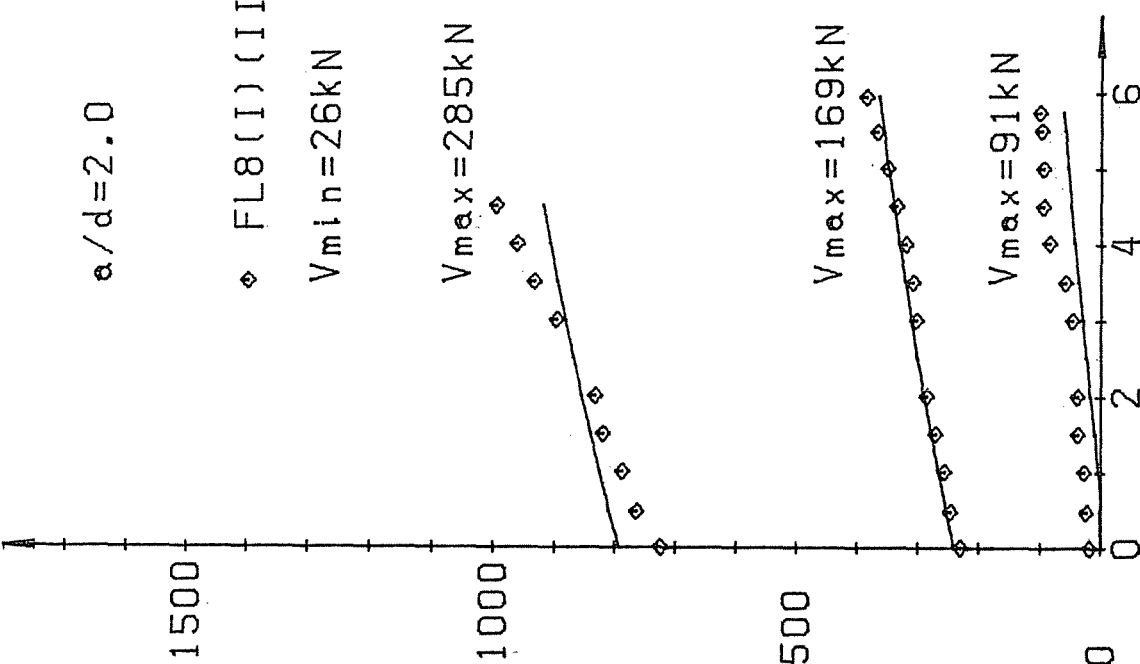
$V_{max}=285kN$

$V_{max}=169kN$

$V_{max}=91kN$

$\log N$

Fig.4.13(k)



$\log N$

Fig.4.13(l)

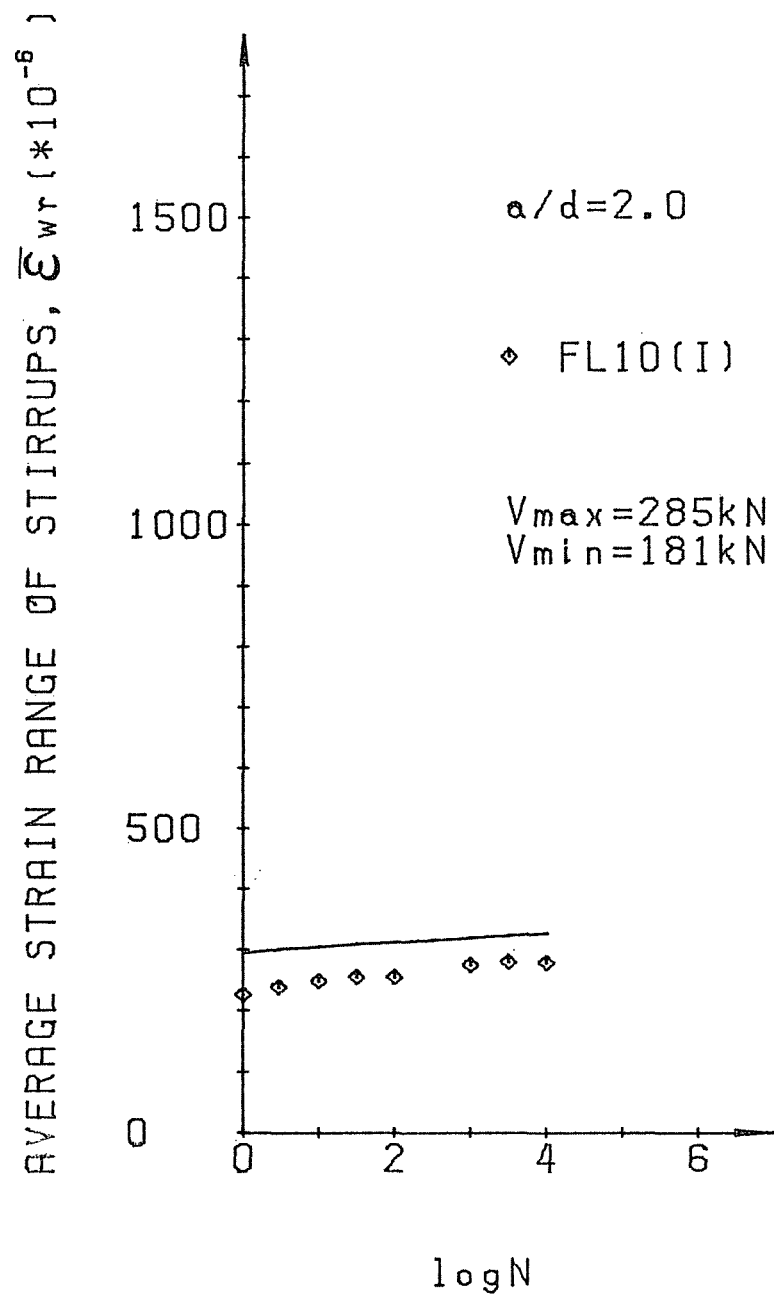


Fig.4.13(m)

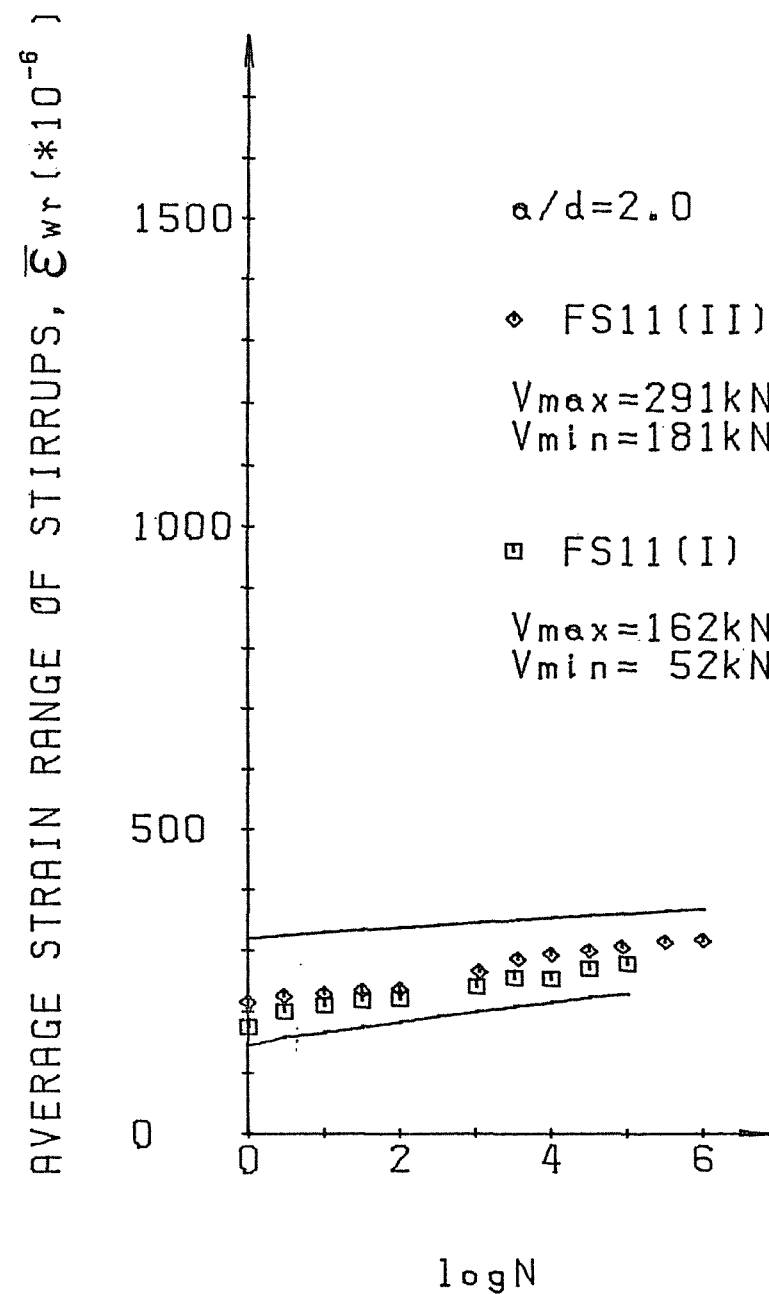


Fig.4.13(n)

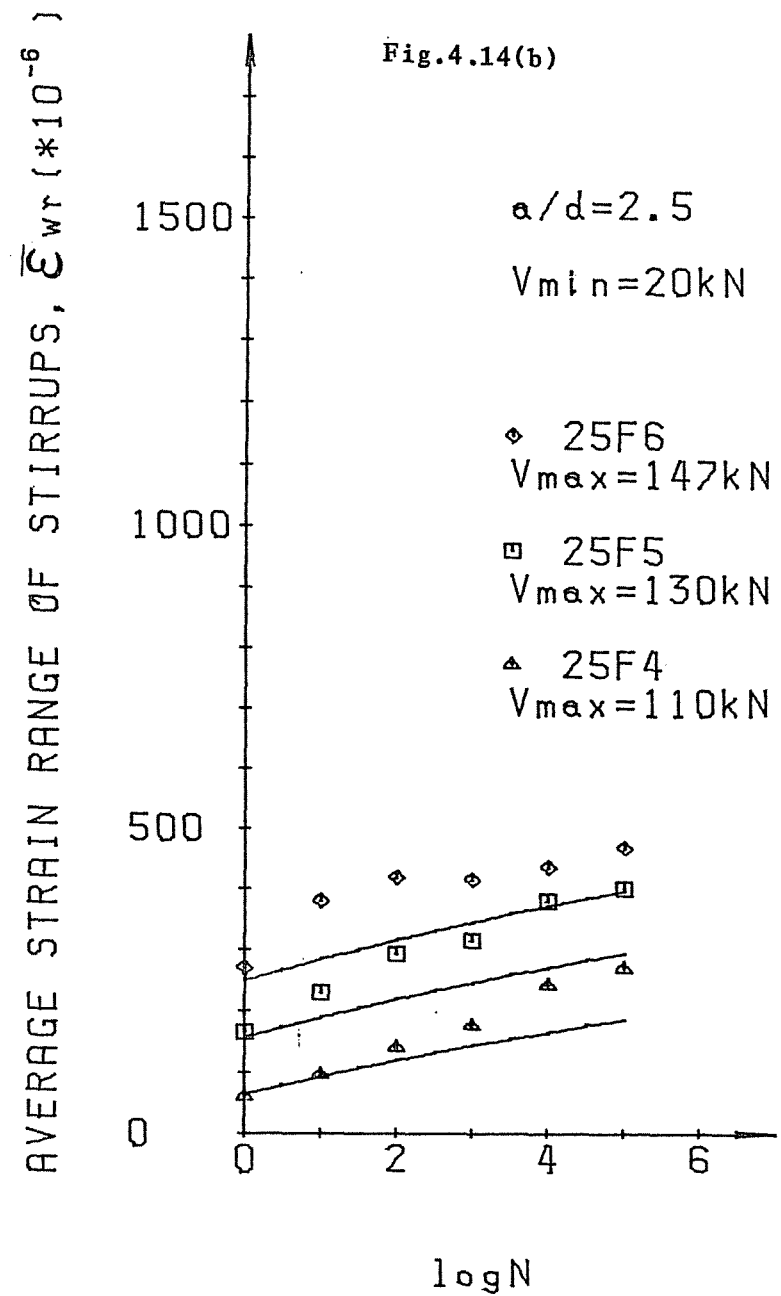
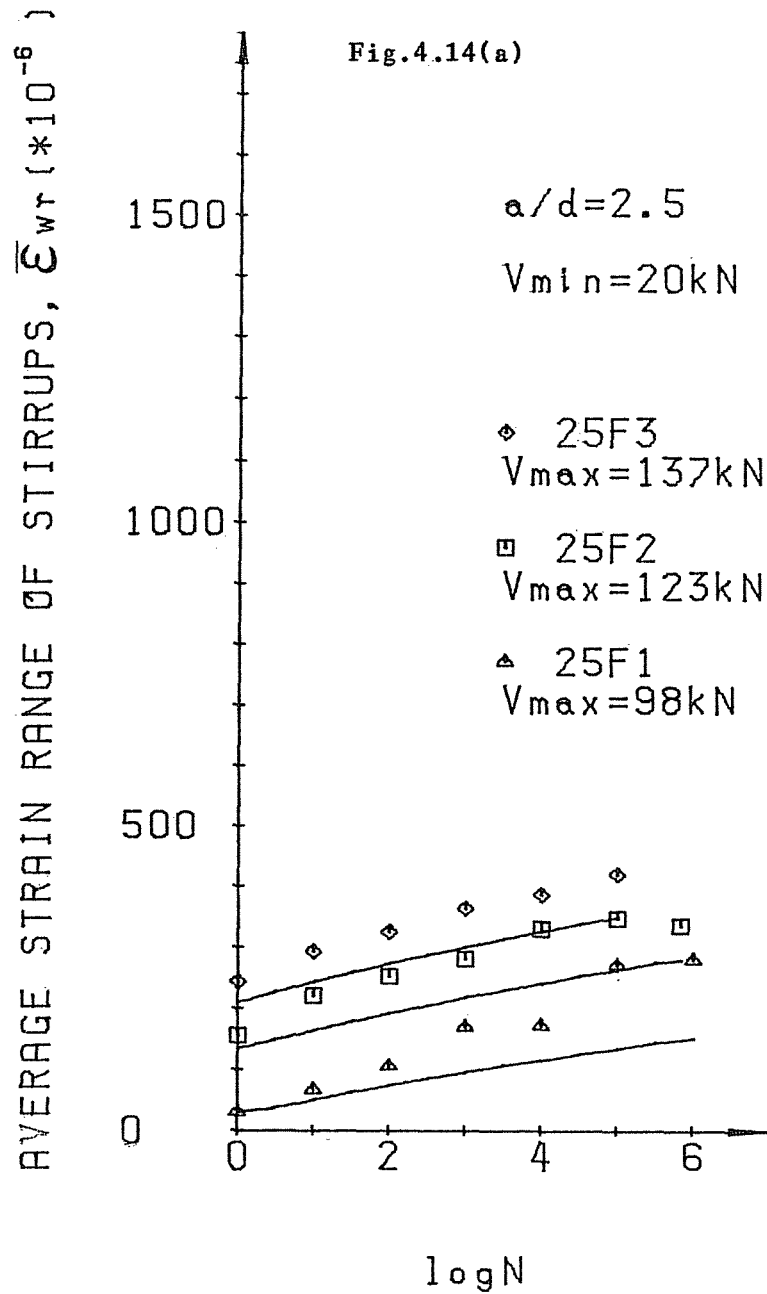


Fig.4.14 Average strain range of stirrups (rectangular beams in Farghaly's tests [1])

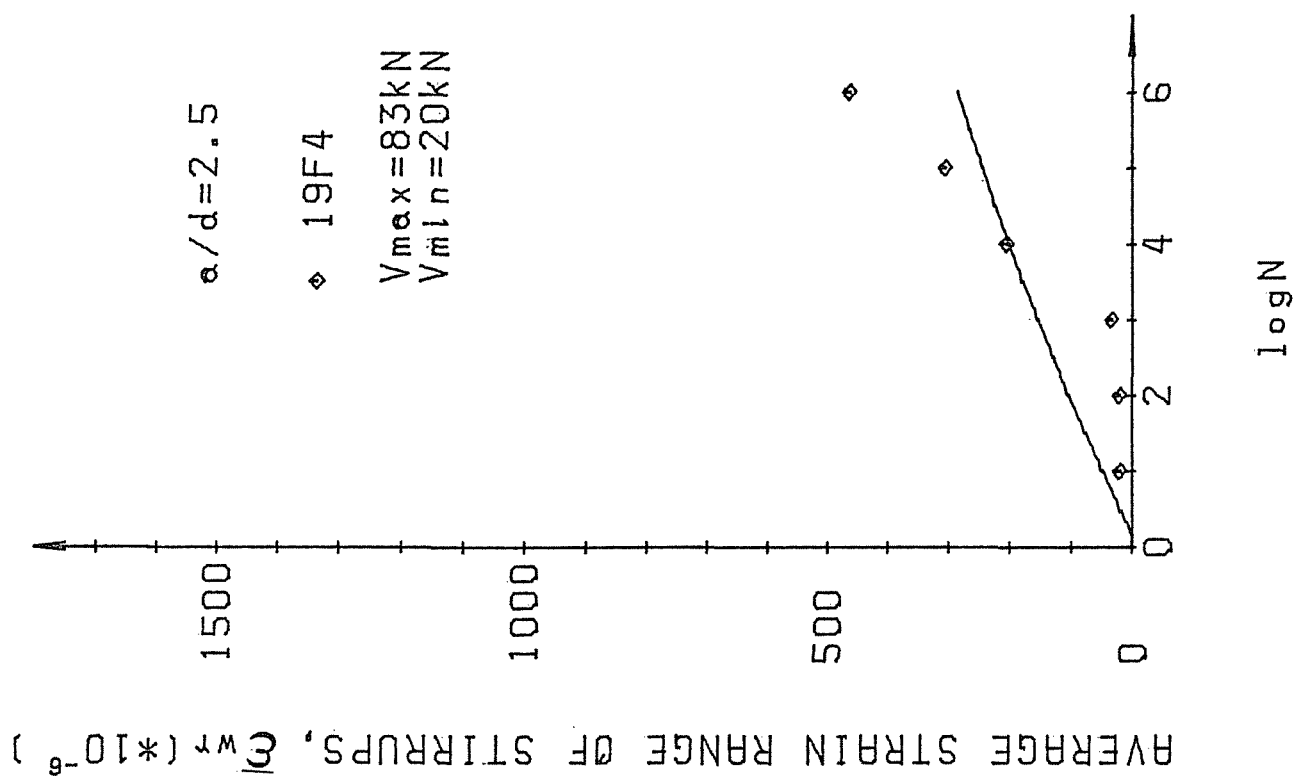


Fig. 4.14(d)

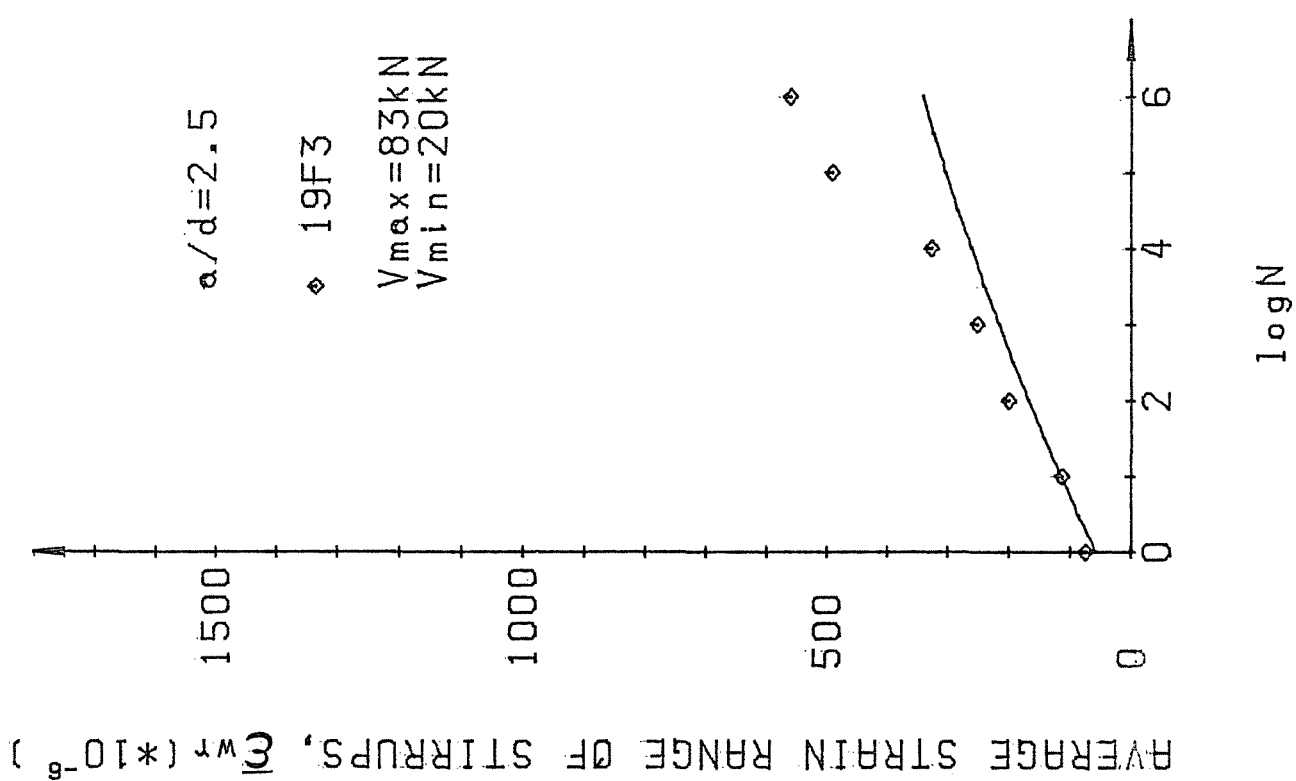


Fig. 4.14(c)

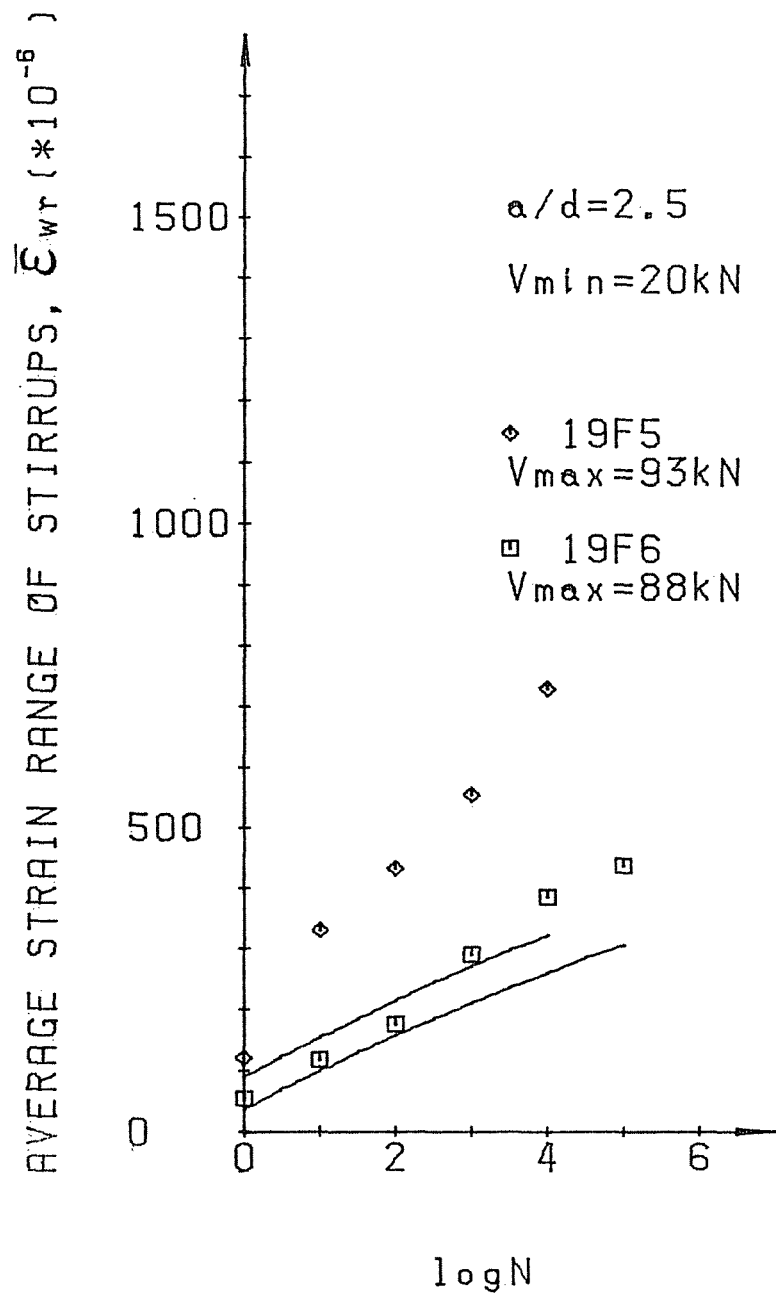


Fig.4.14(e)

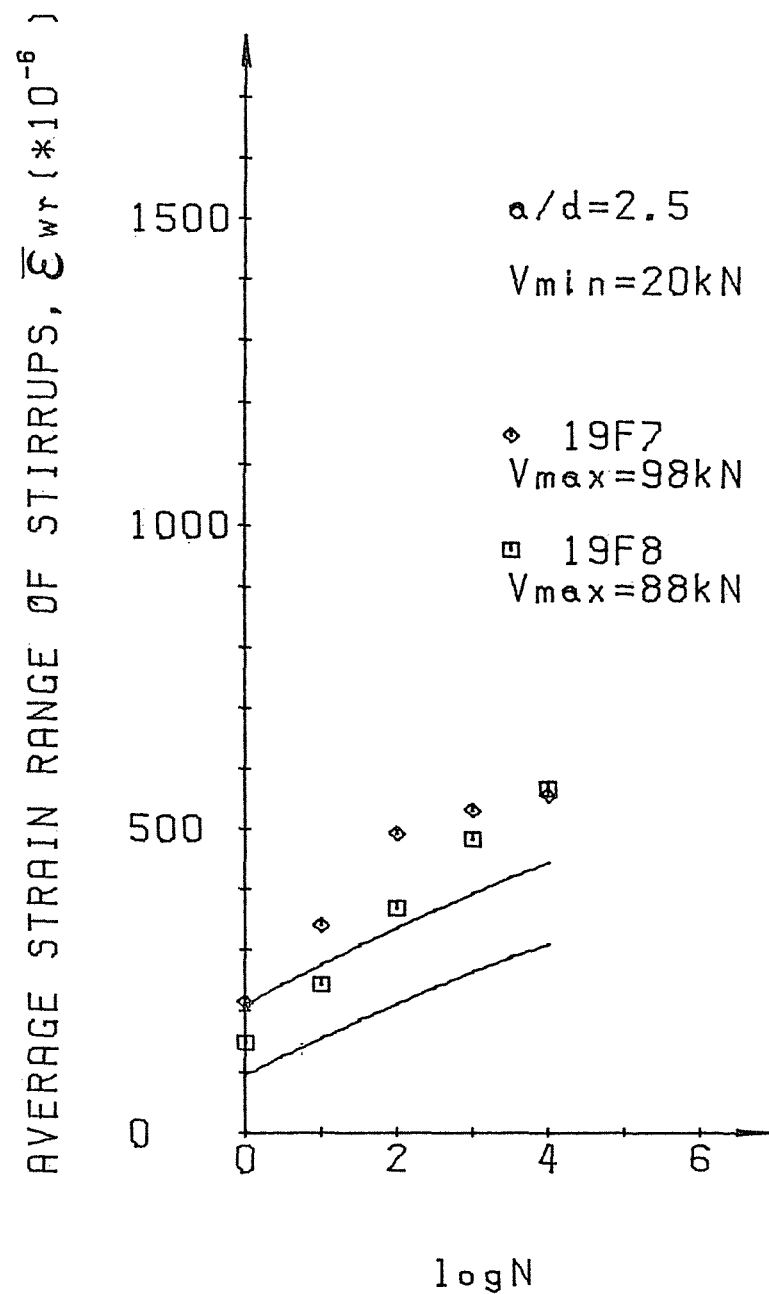


Fig.4.14(f)

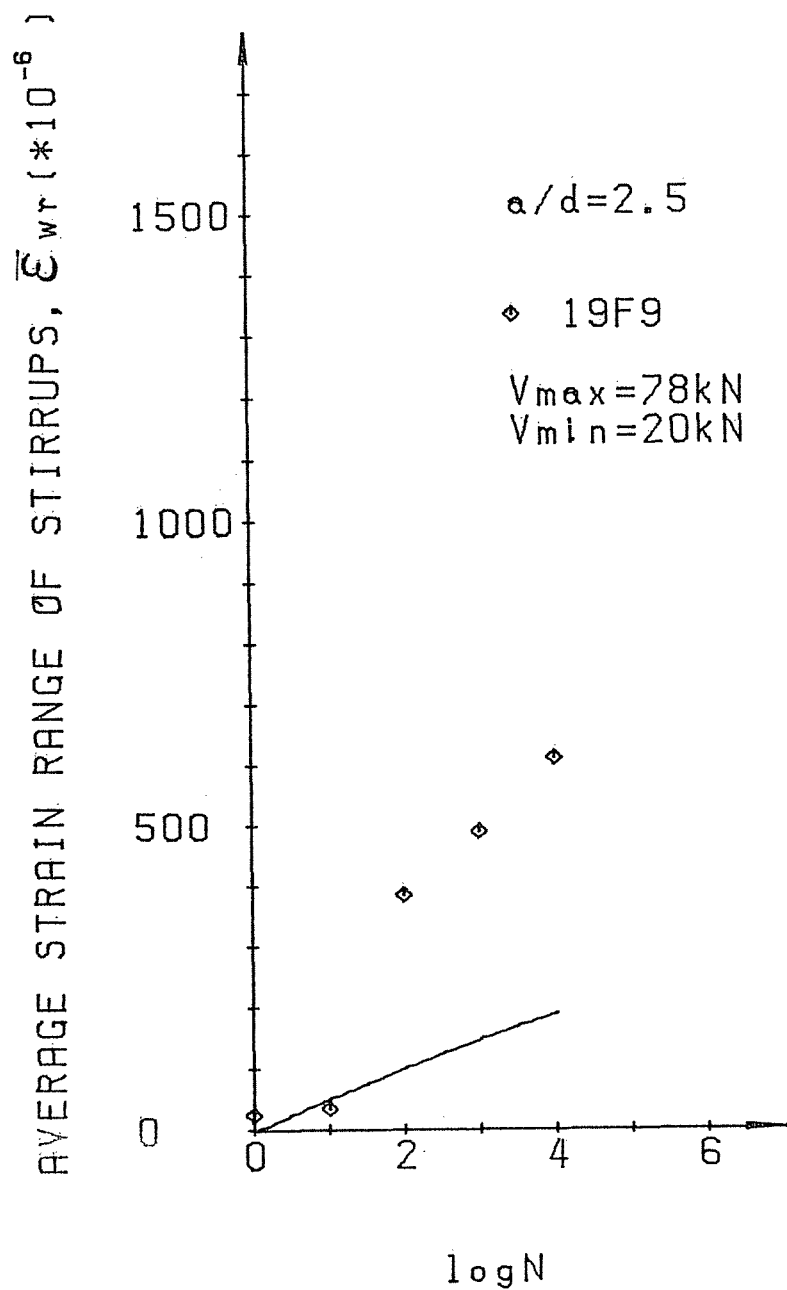


Fig.4.14(h)

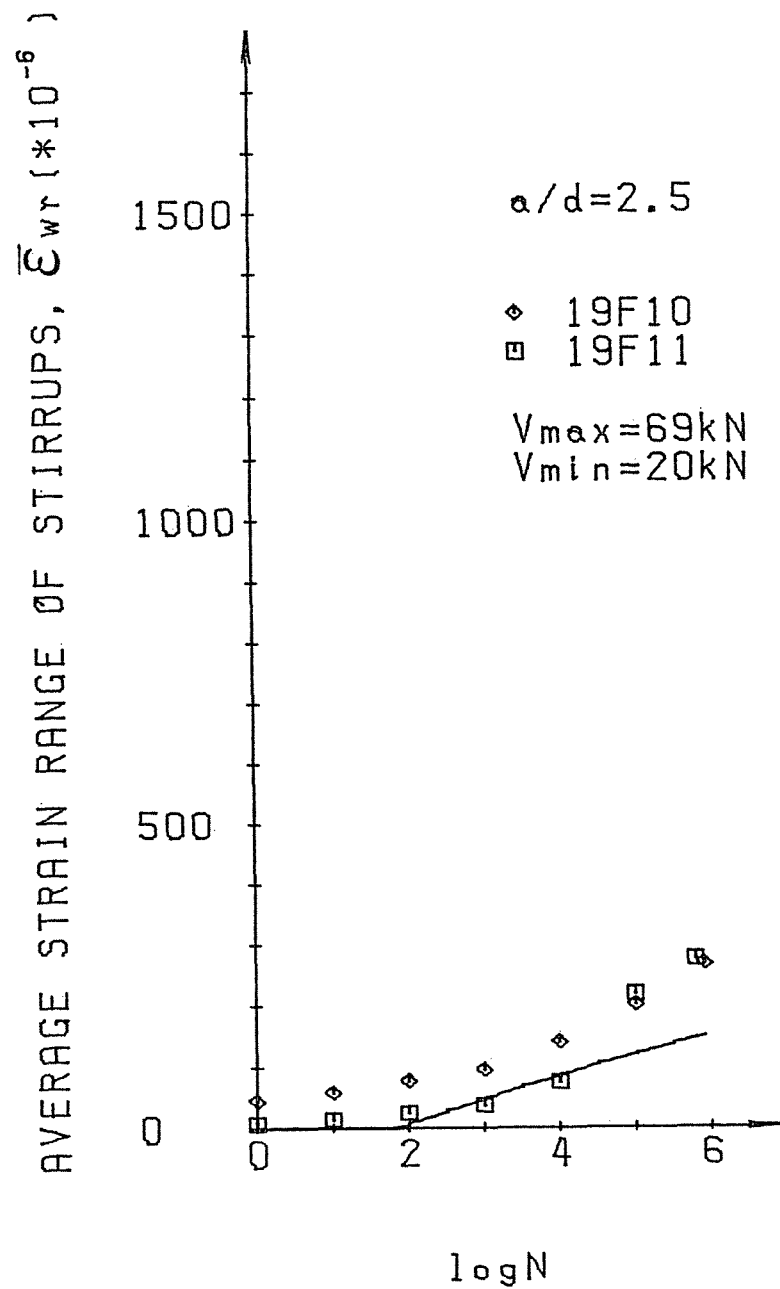


Fig.4.14(i)

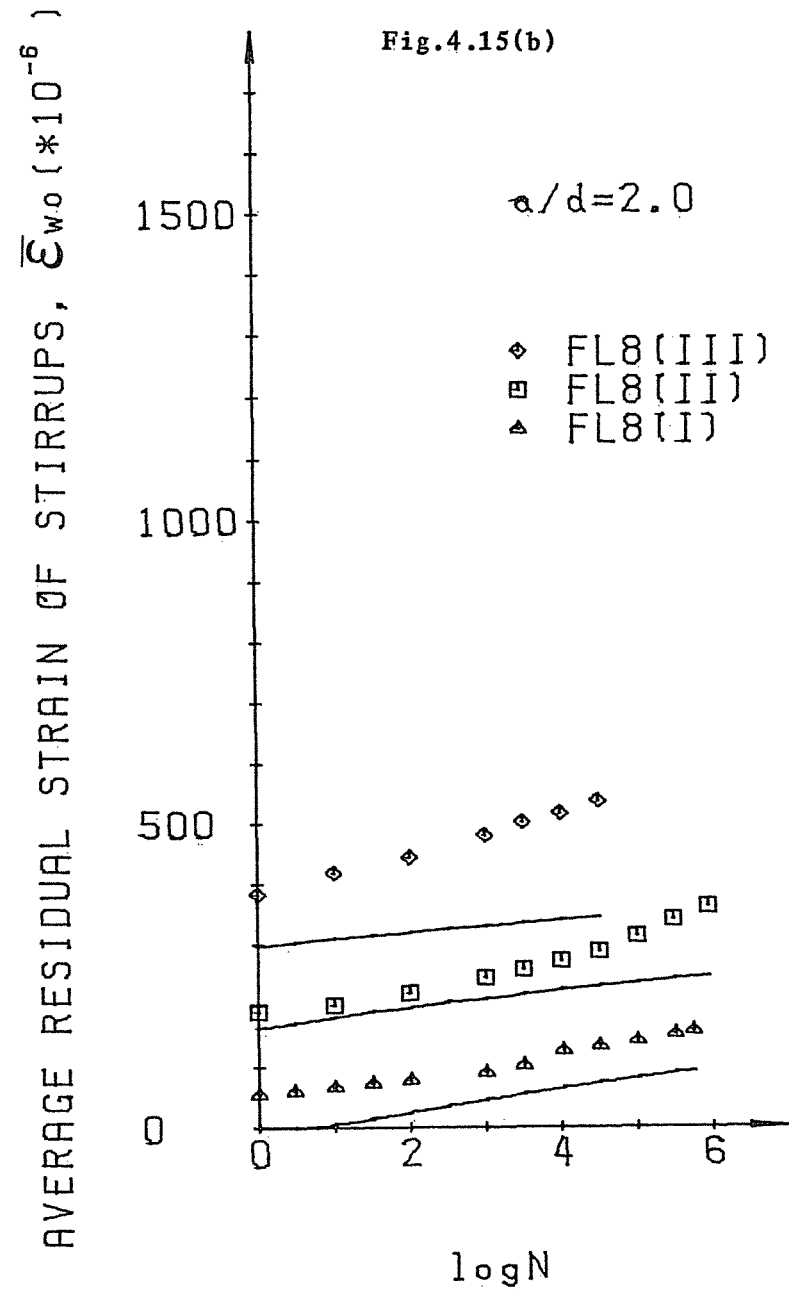
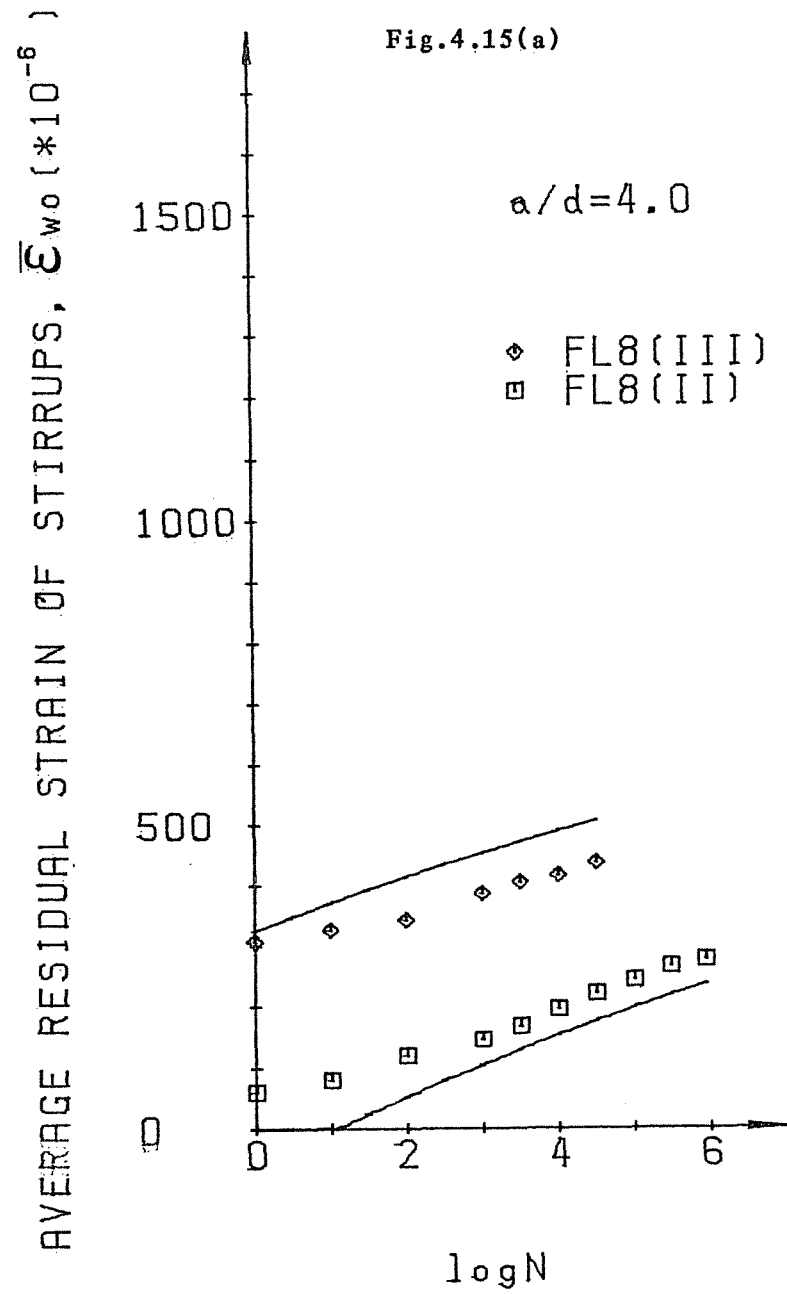


Fig.4.15 Average residual strain of stirrups (T-beam in author's tests)

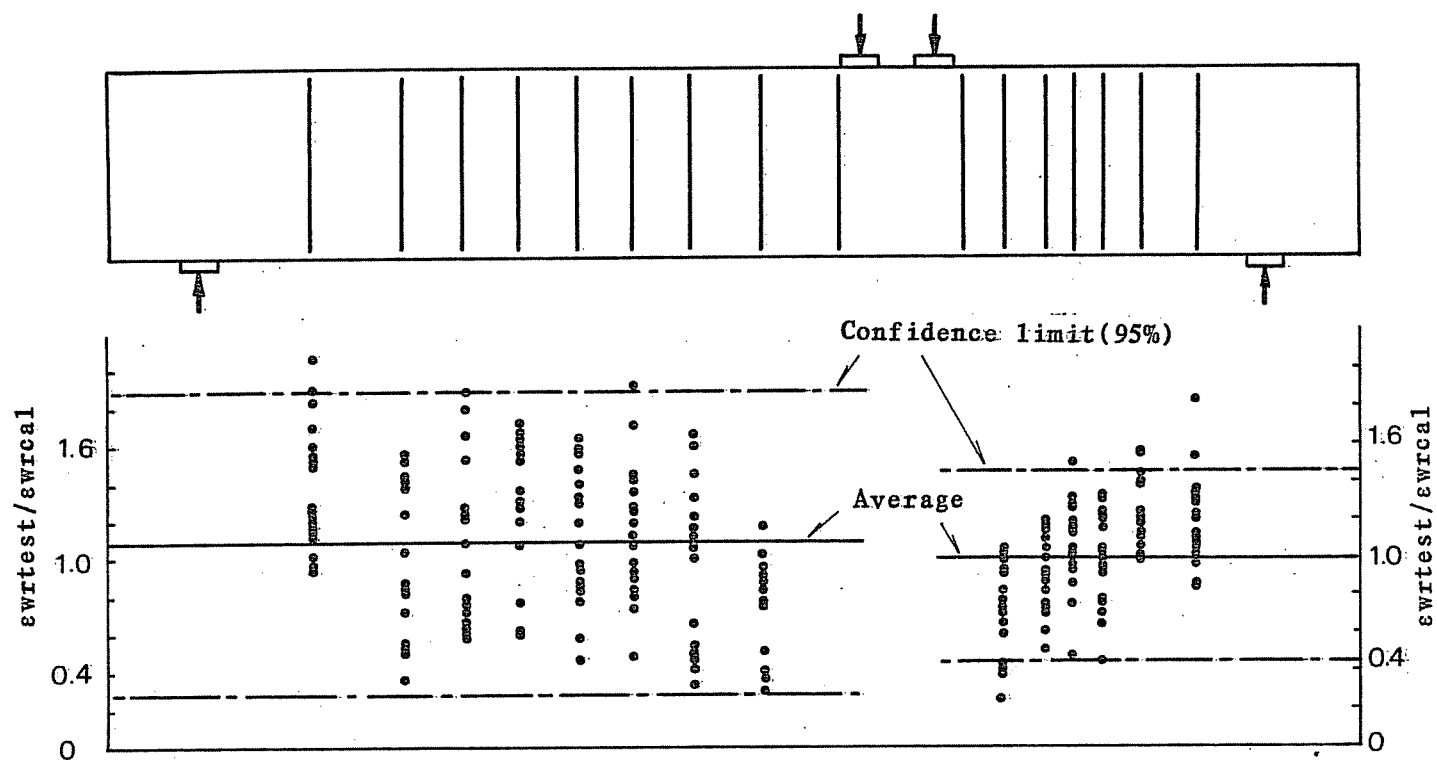


Fig.4.16 Scatter of measured strain of each stirrup around calculated one

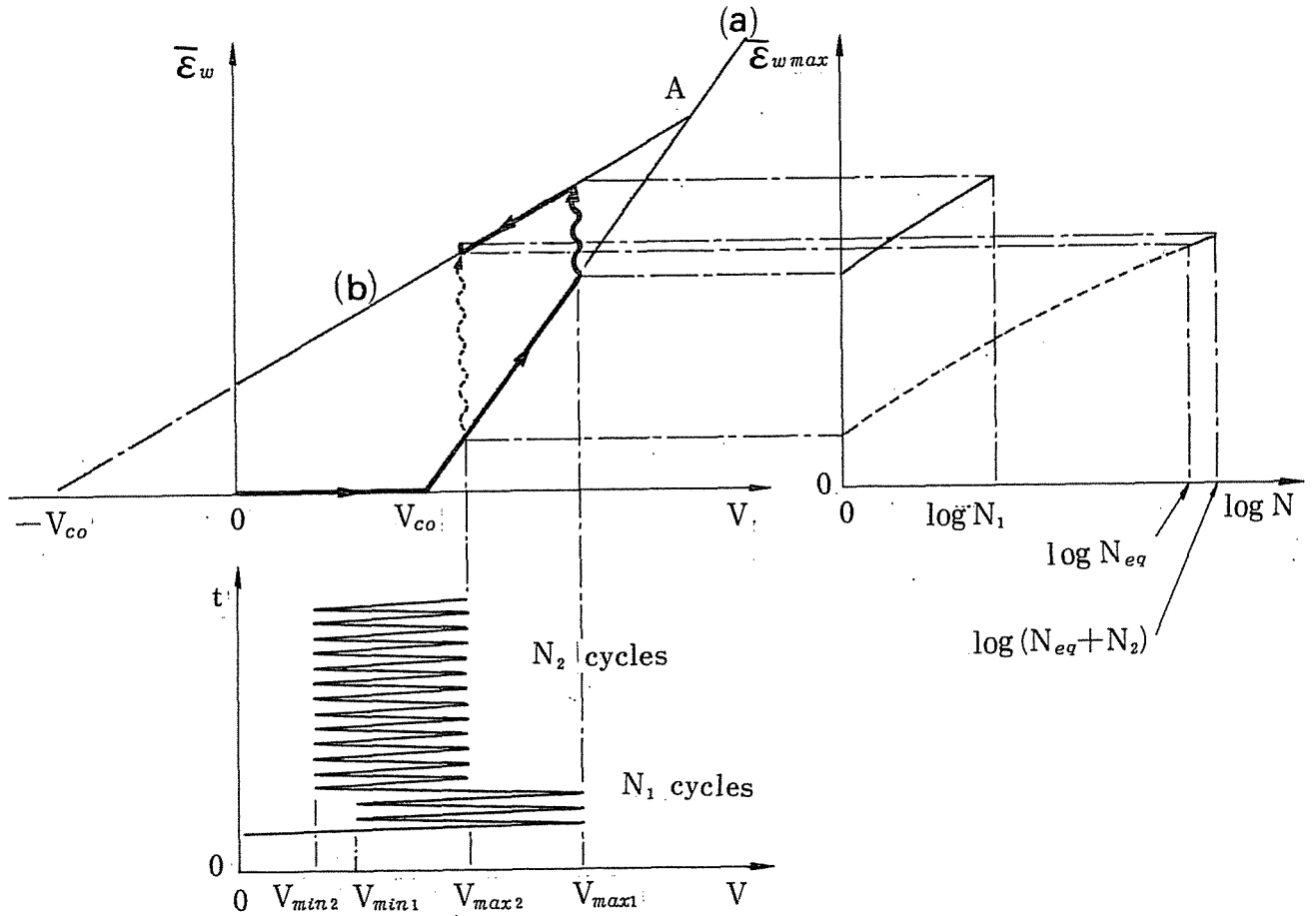


Fig.5.1 Idea of equivalent fatigue loading

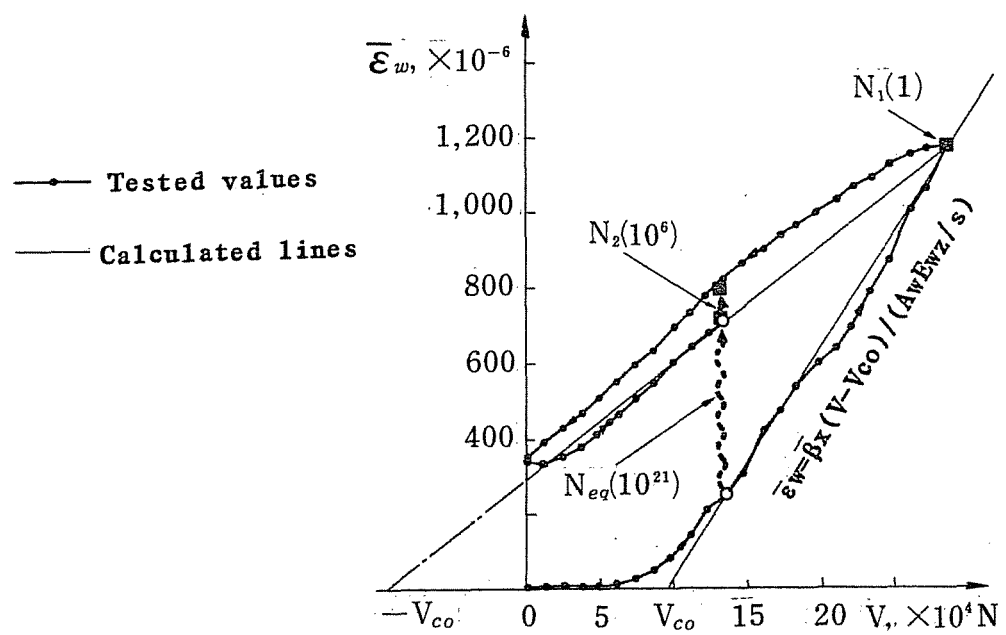


Fig.5.2 Average strain of stirrups after the first repeated loading a case of large influence of previous fatigue loading (FL2 a/d=2.0)

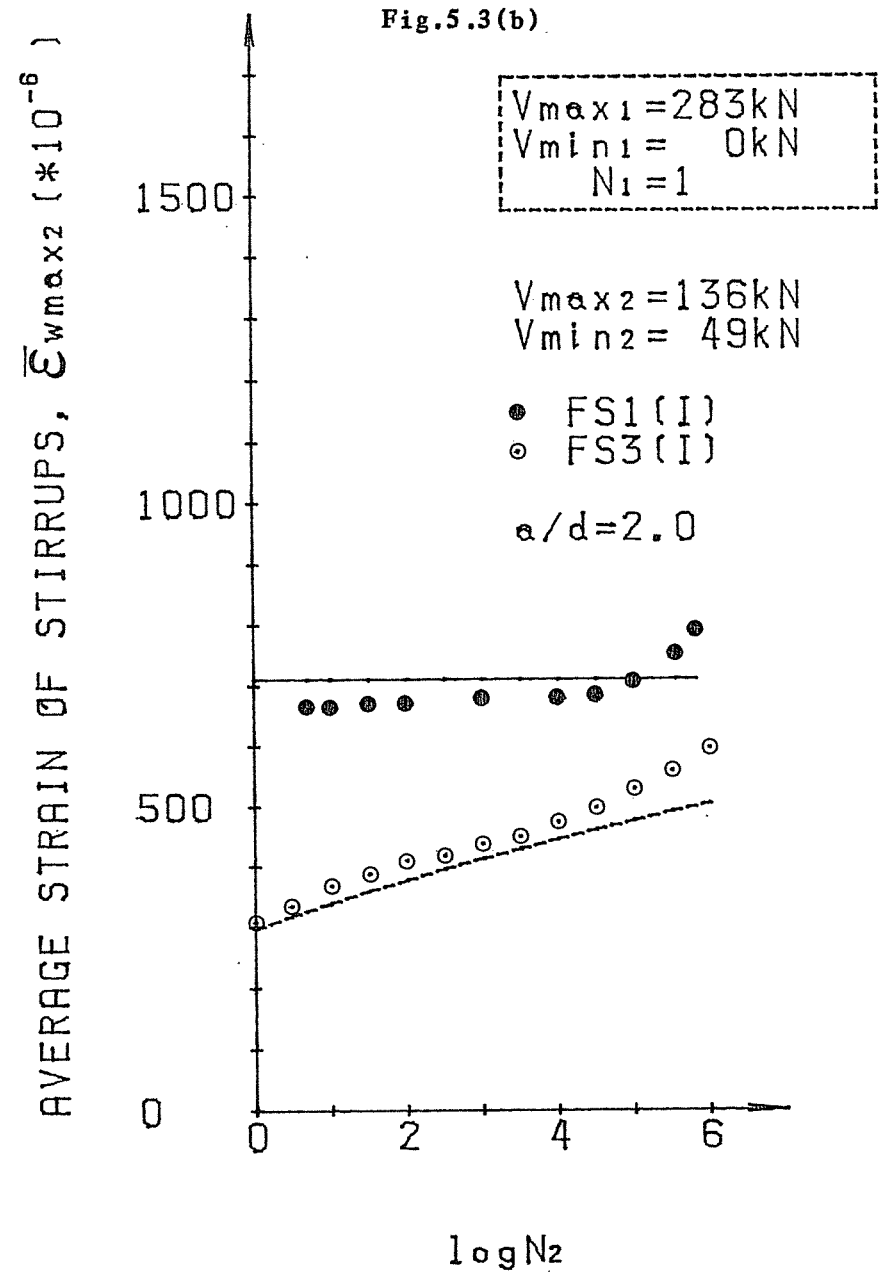
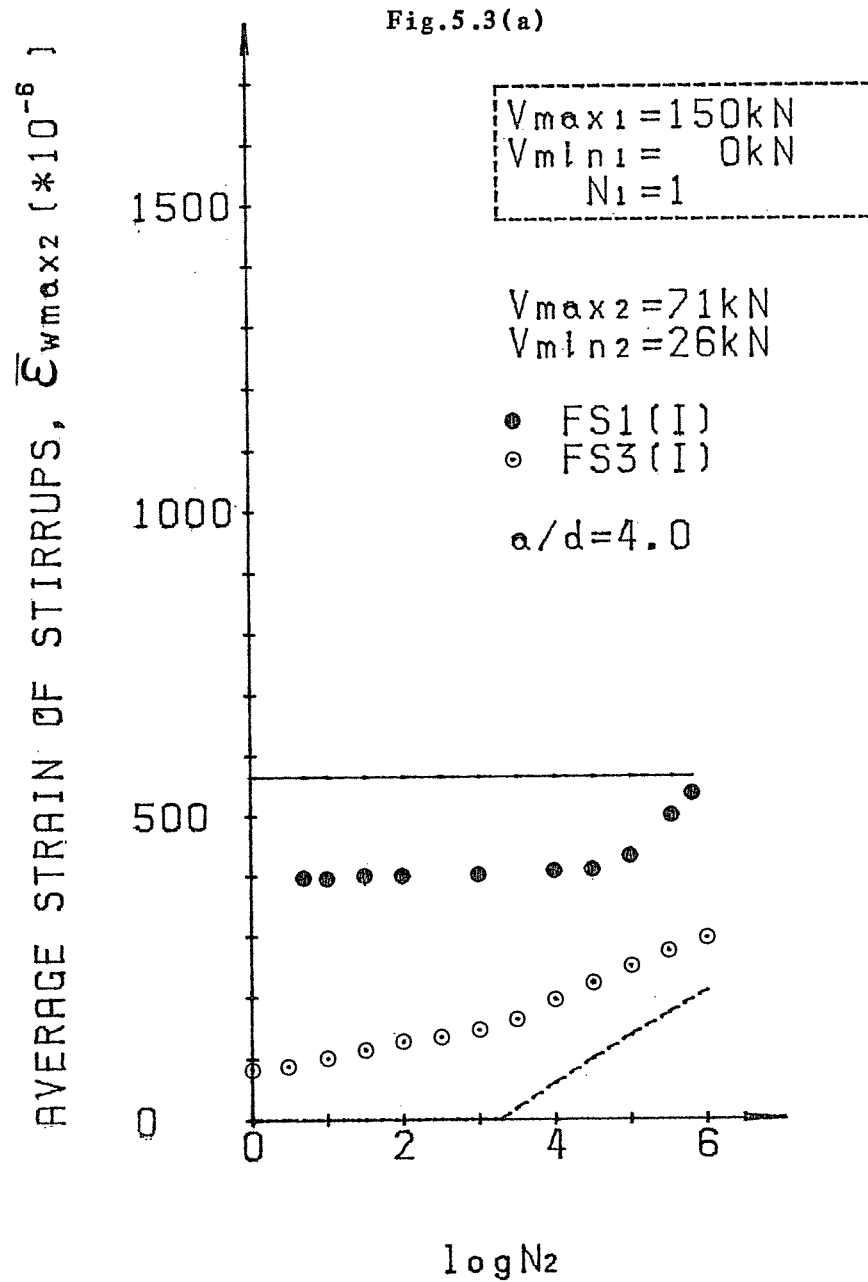


Fig.5.3 Influence of previous over-loading (Tested and calculated values in beams without over-loading are shown by open circles and dotted lines respectively.)

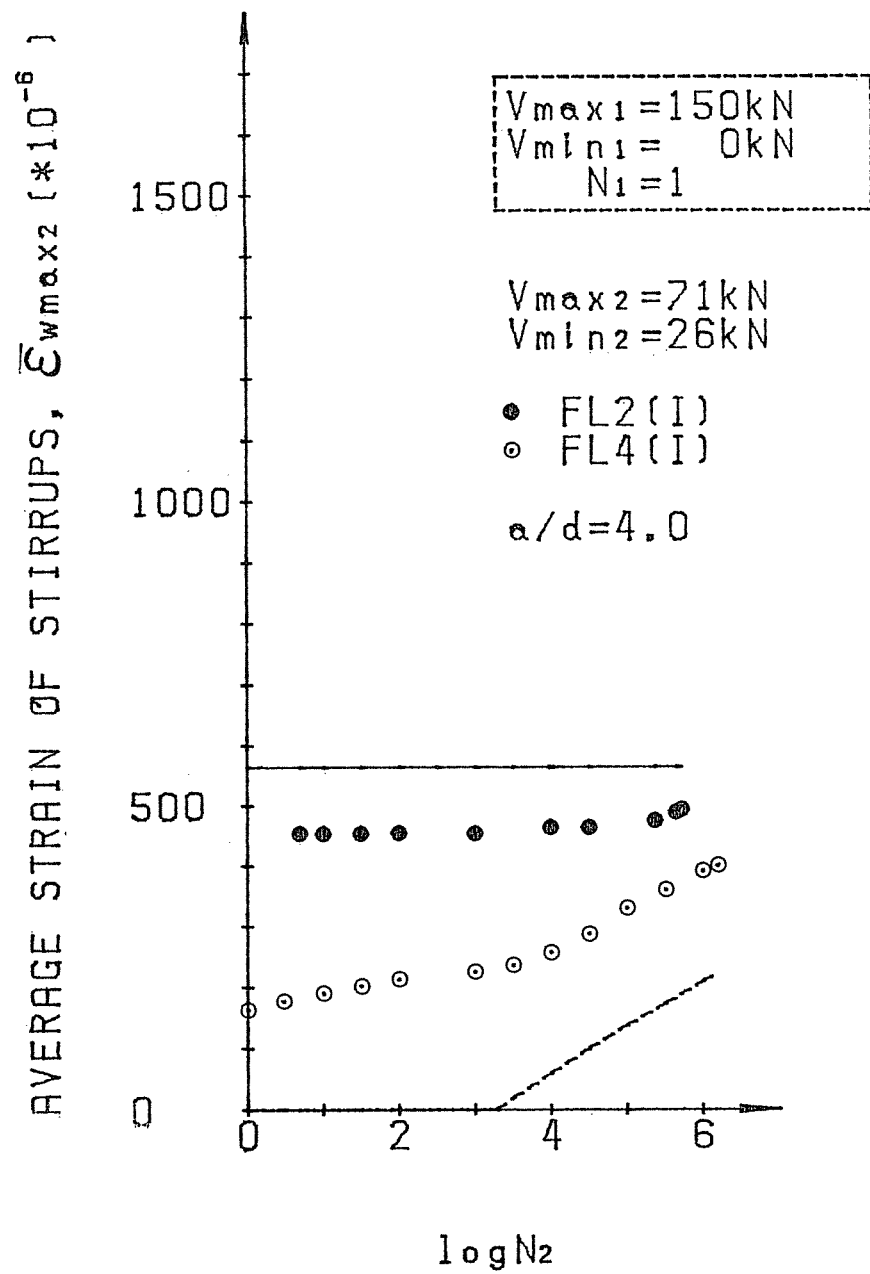


Fig.5.3(c)

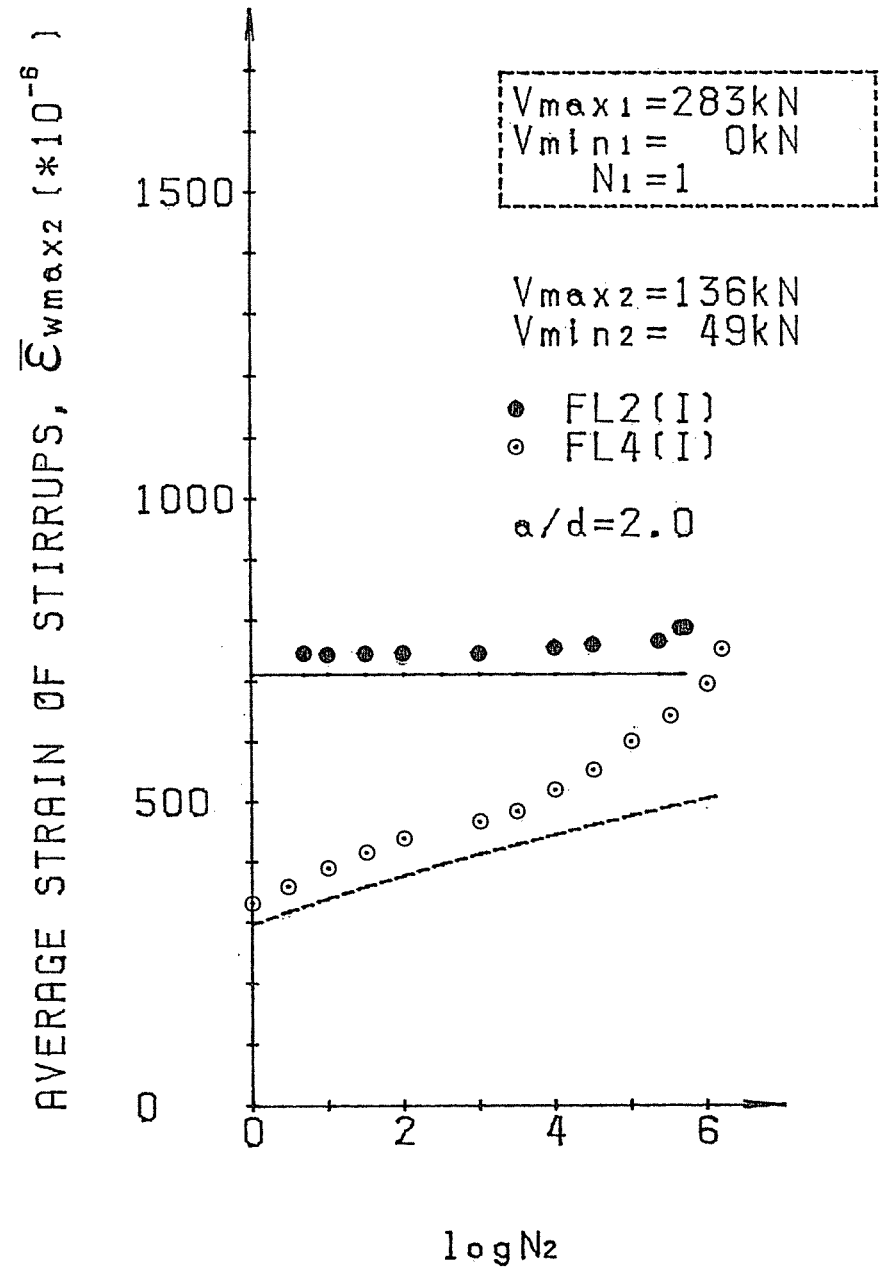


Fig.5.3(d)

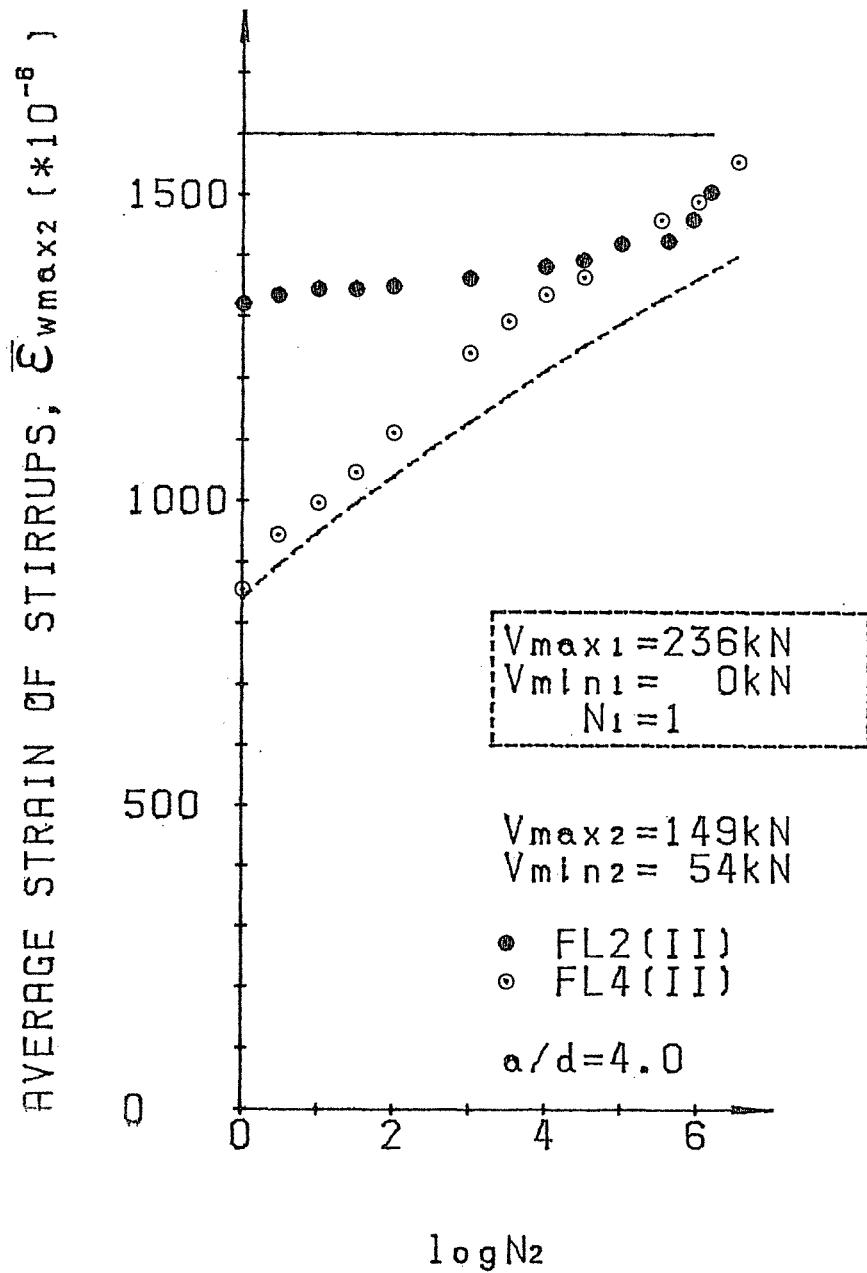


Fig.5.3(e)

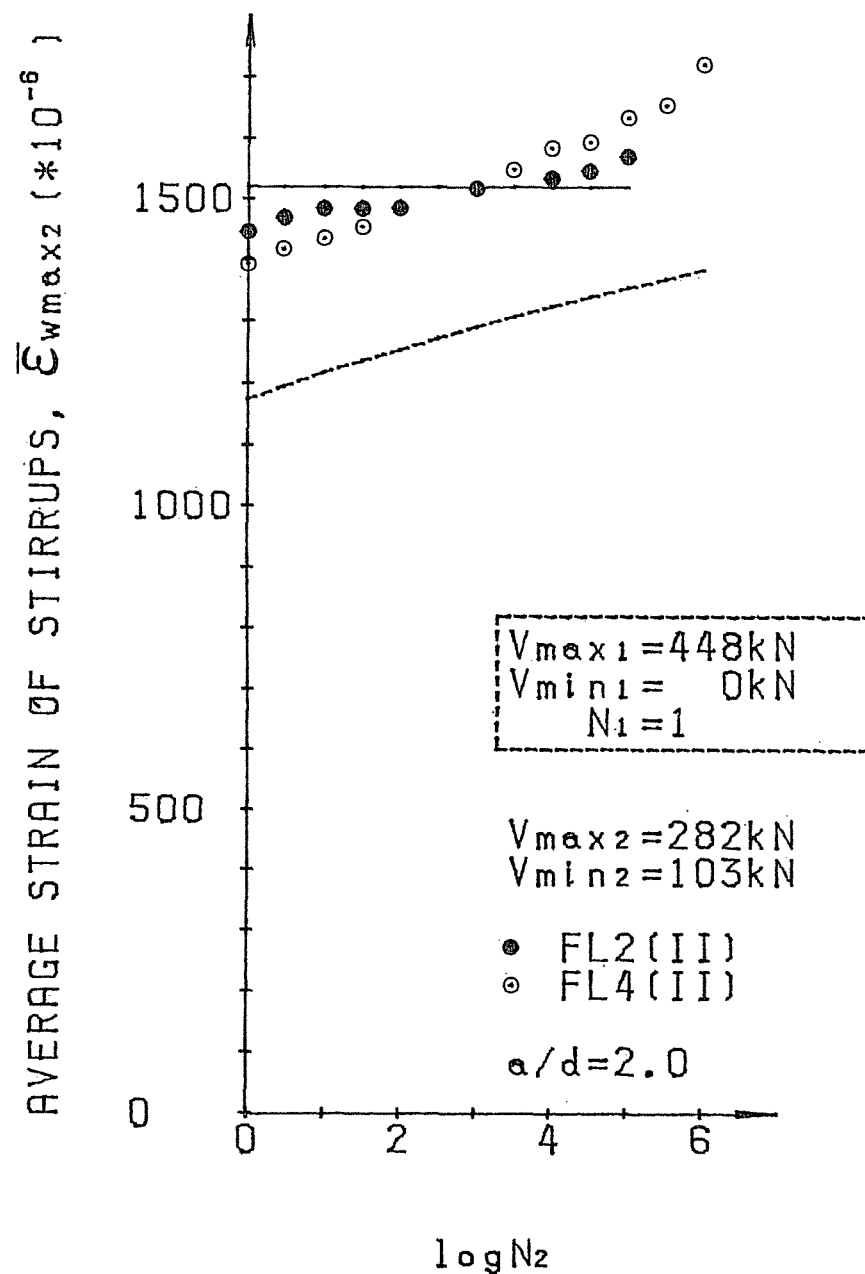


Fig.5.3(f)

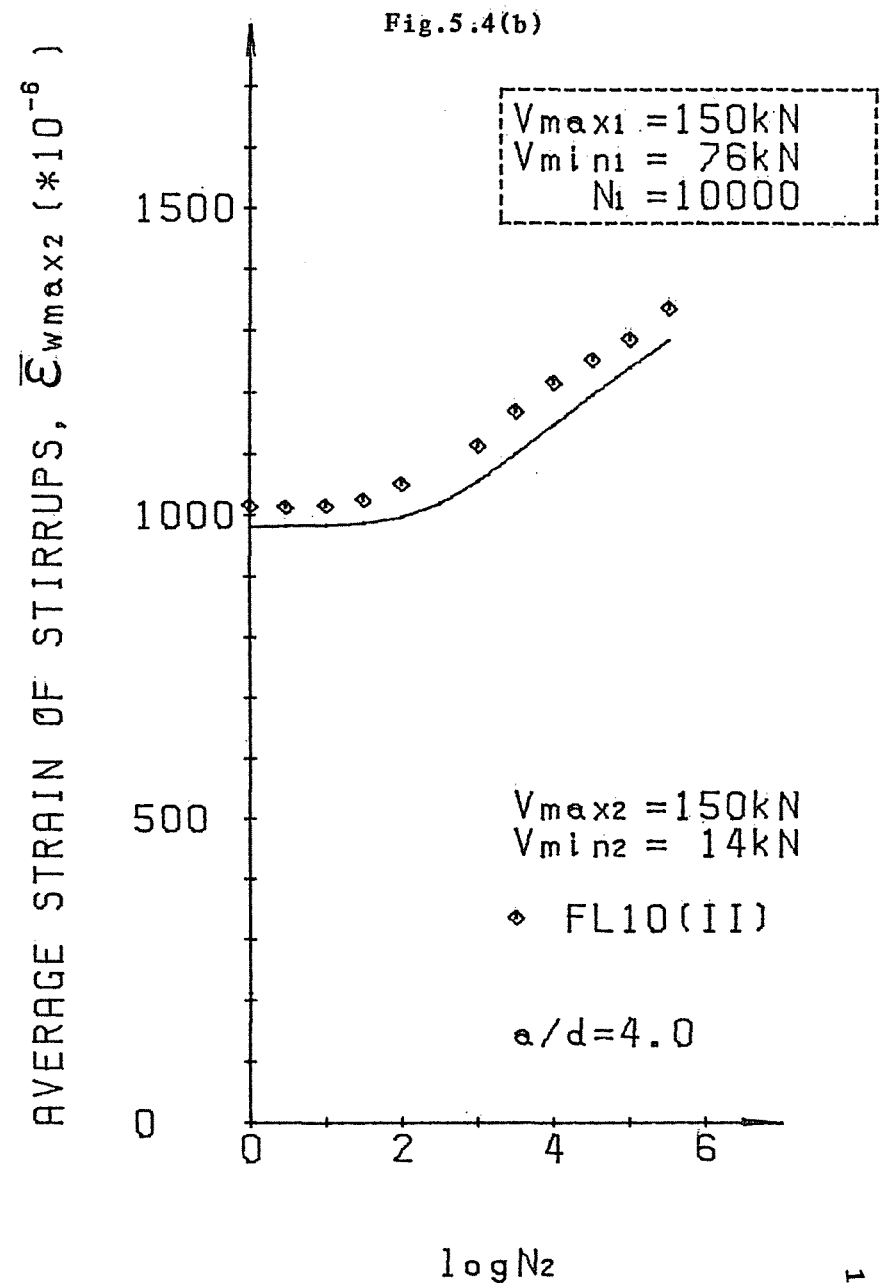
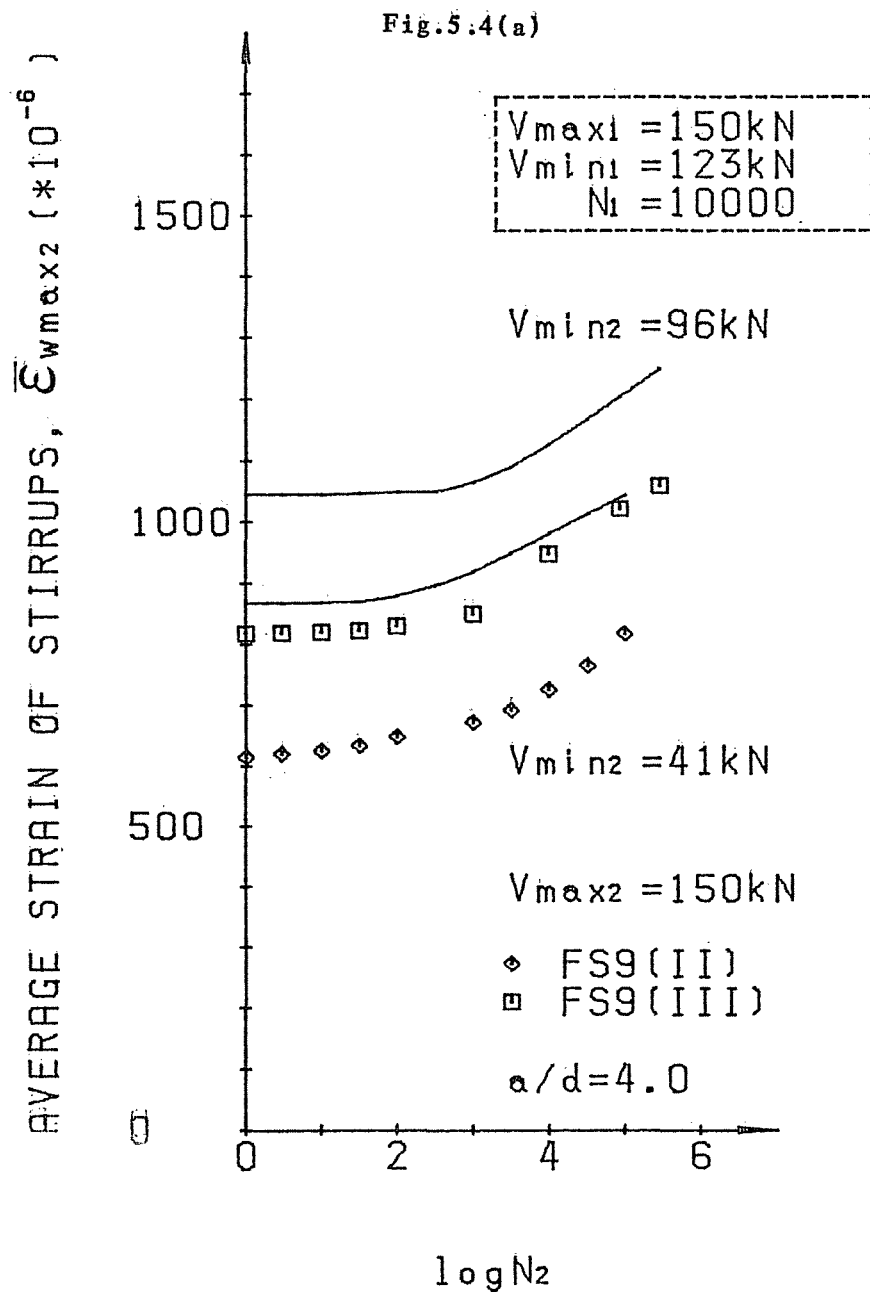


Fig.5.4 Average strain of stirrups after certain repeated loading in cases where V_{min2} is less than V_{min1} although V_{max2} is equal to V_{max1}

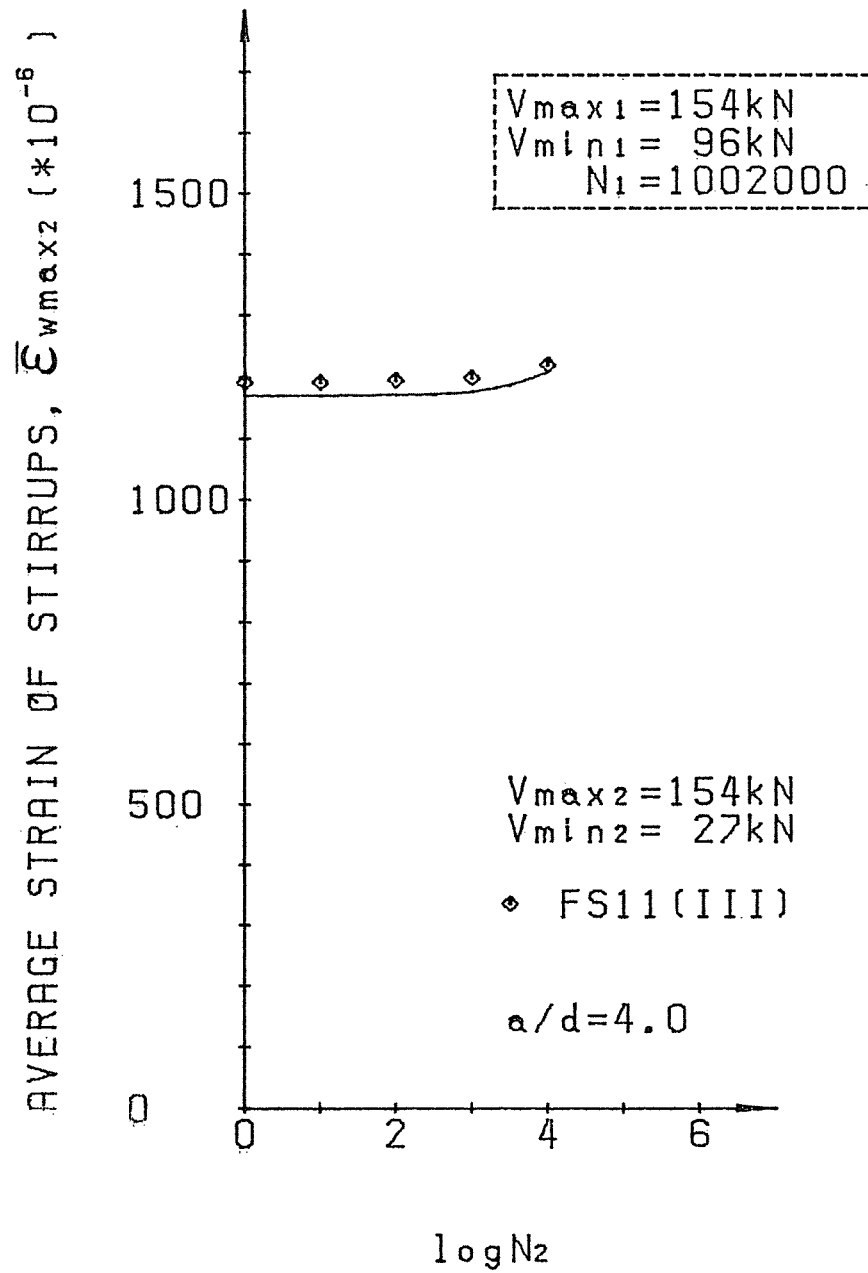


Fig.5.4(c)

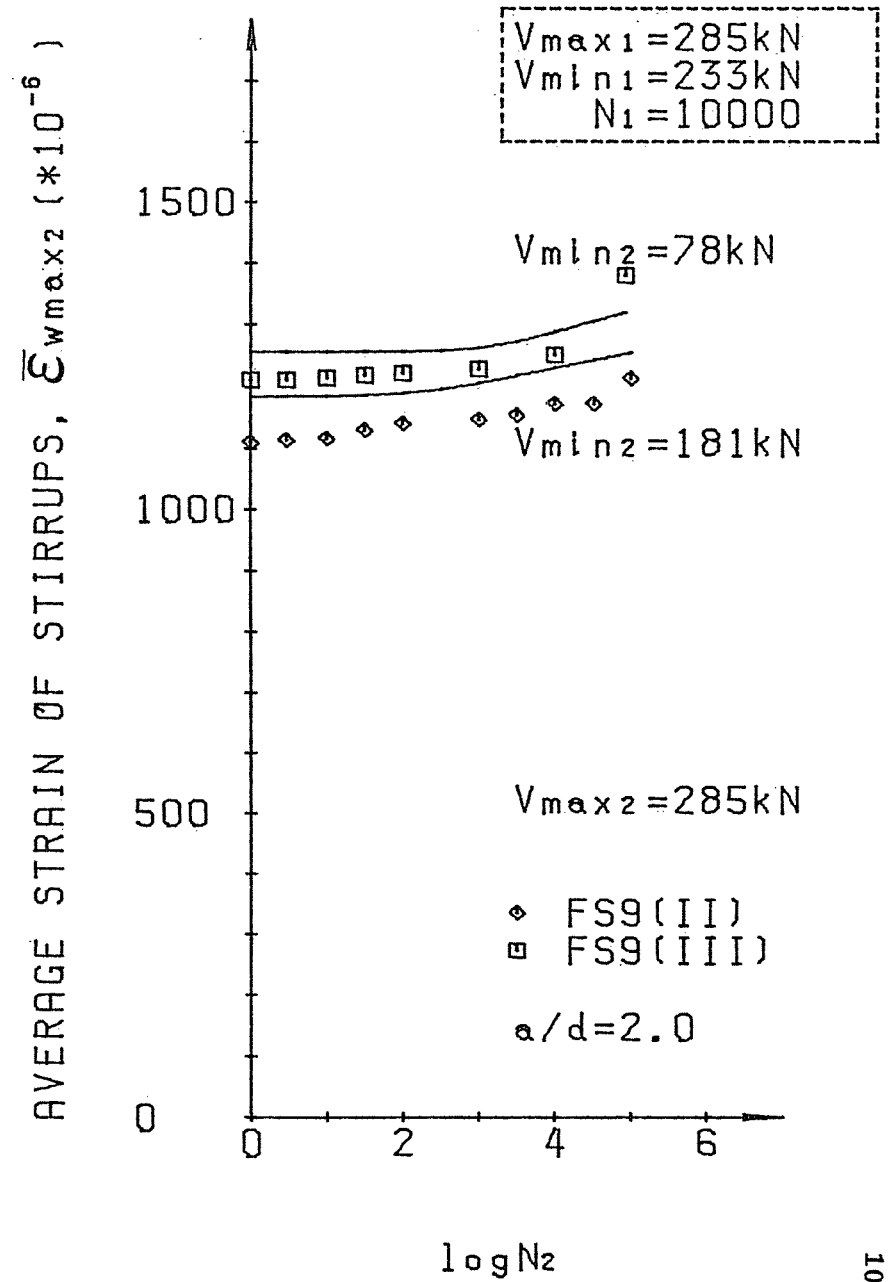


Fig.5.4(d)

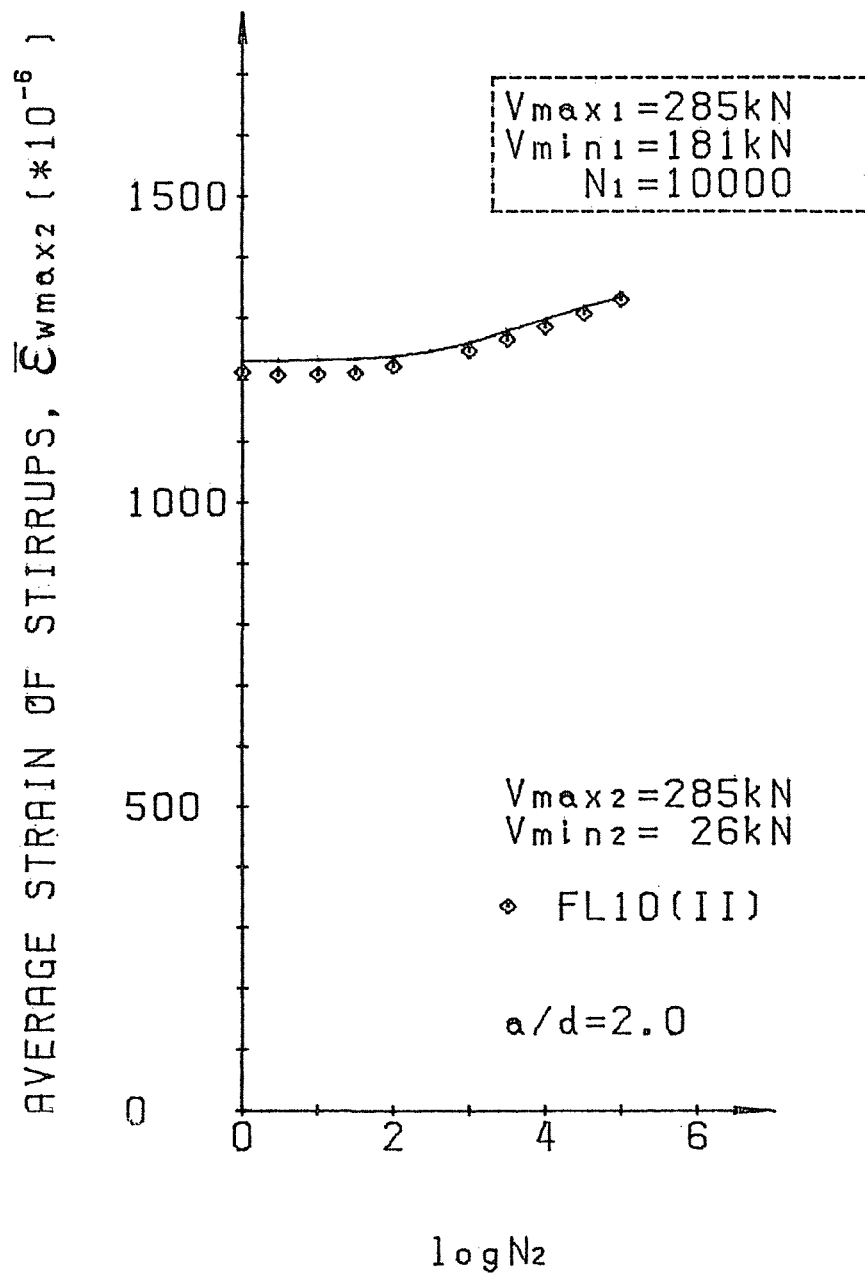


Fig.5.4(e)

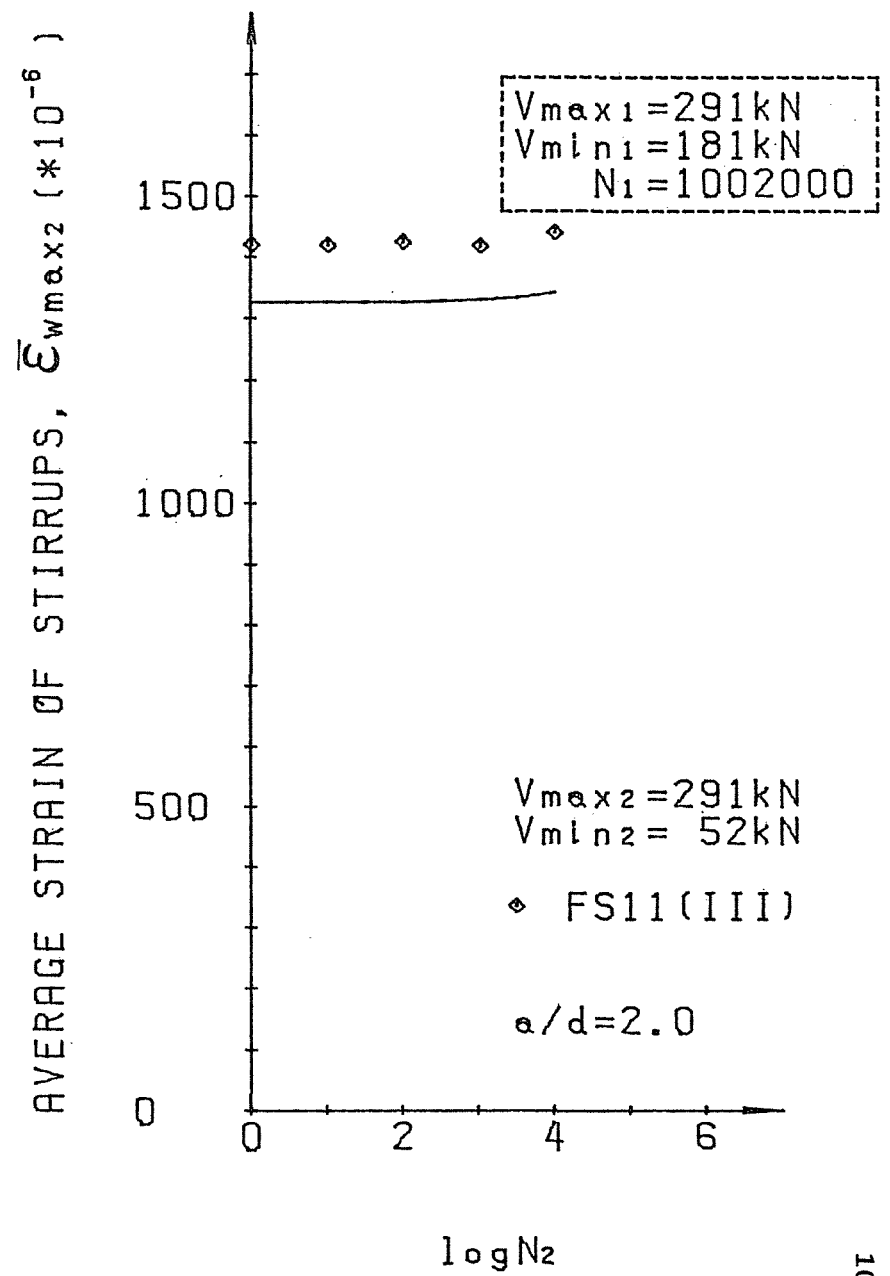


Fig.5.4(f)

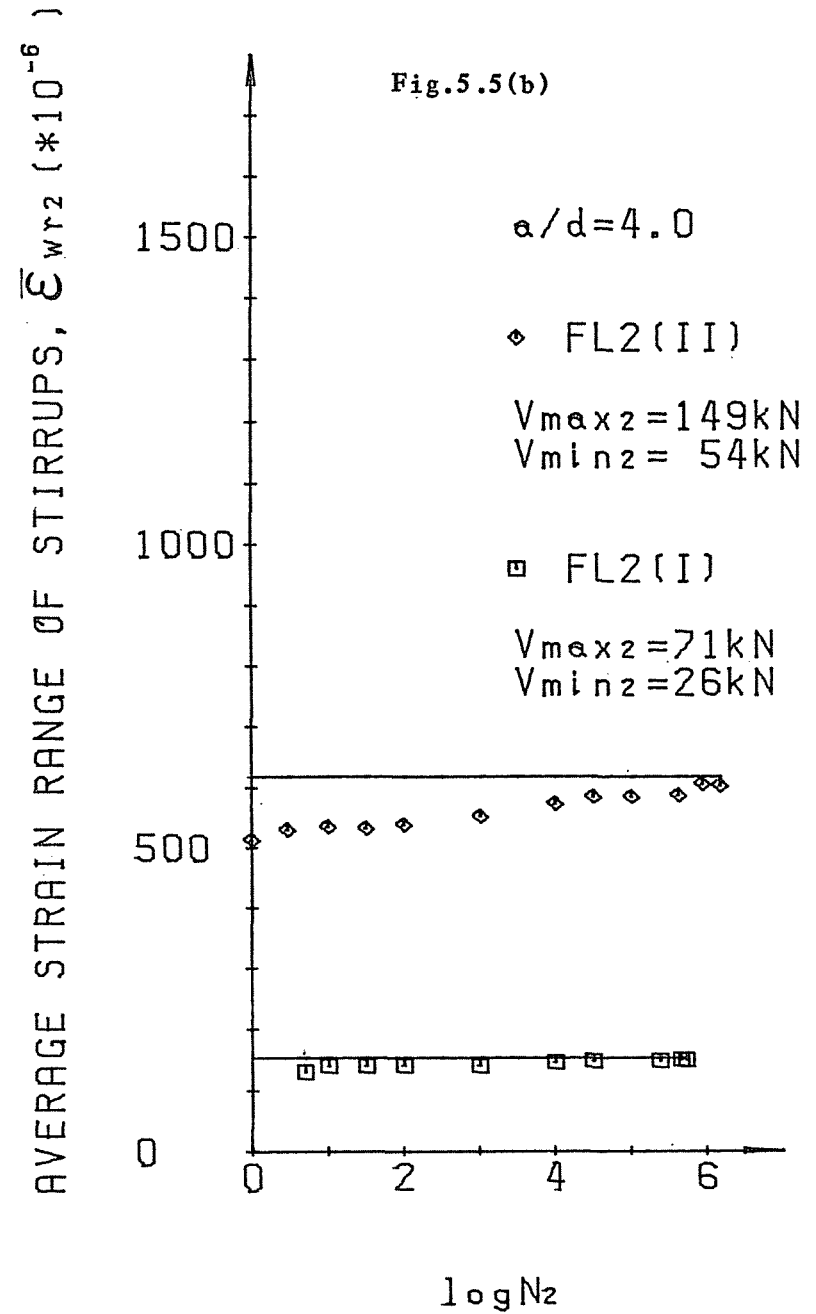
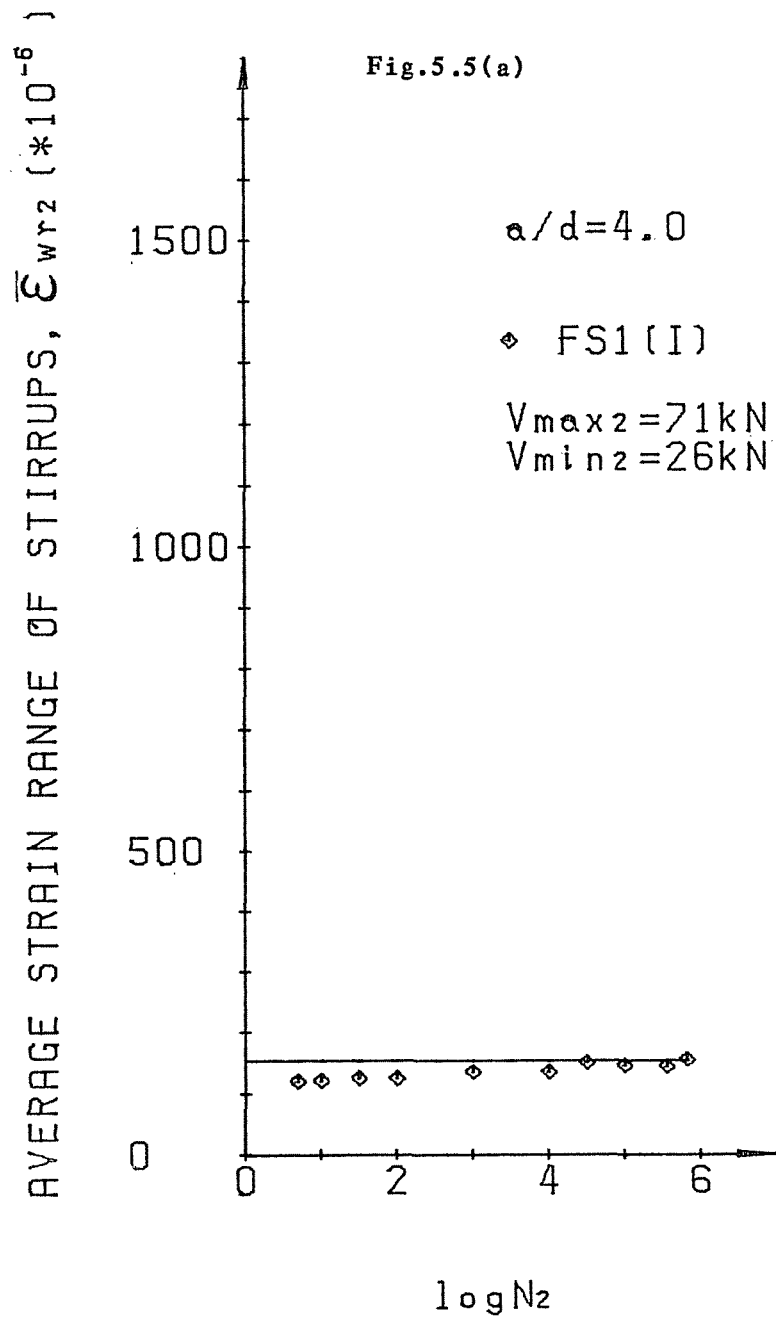
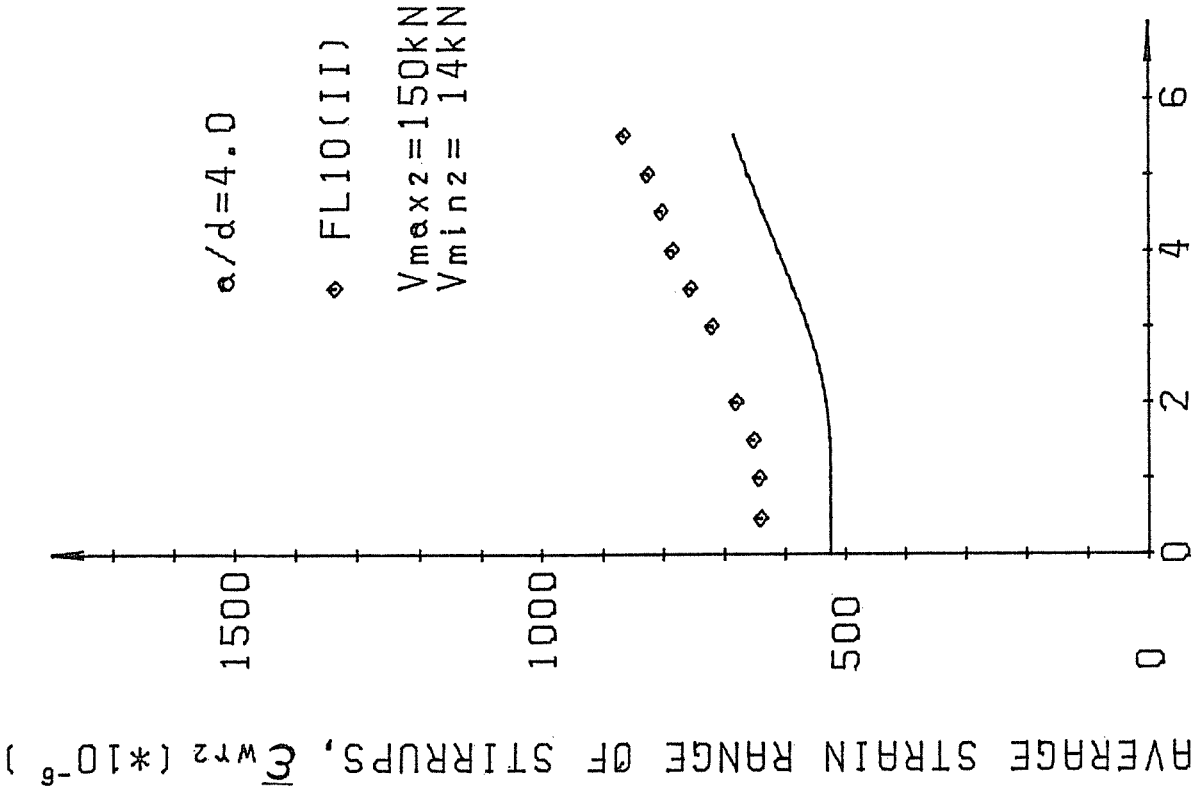
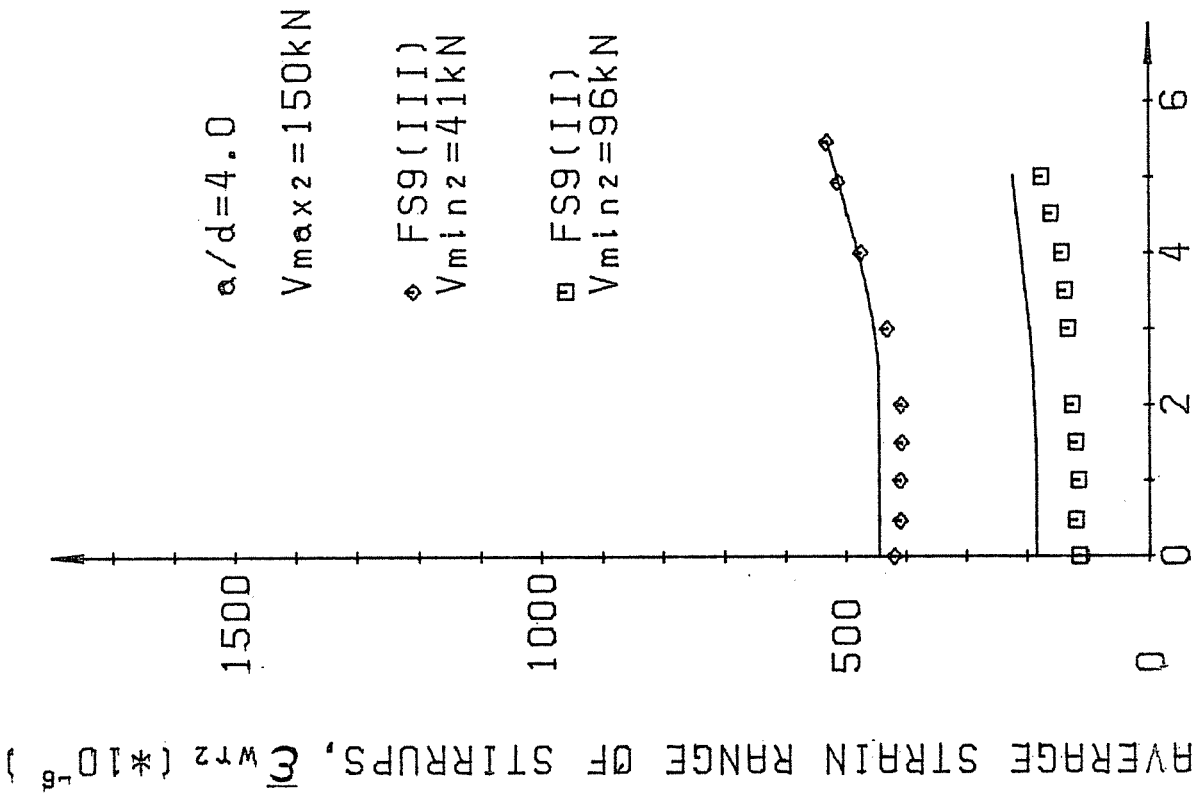


Fig.5.5 Average strain range of stirrups in cases of large influence of previous loading as shown in Figs.5.3 and 5.4



$\log N_2$

Fig.5.5(d)



$\log N_2$

Fig.5.5(c)

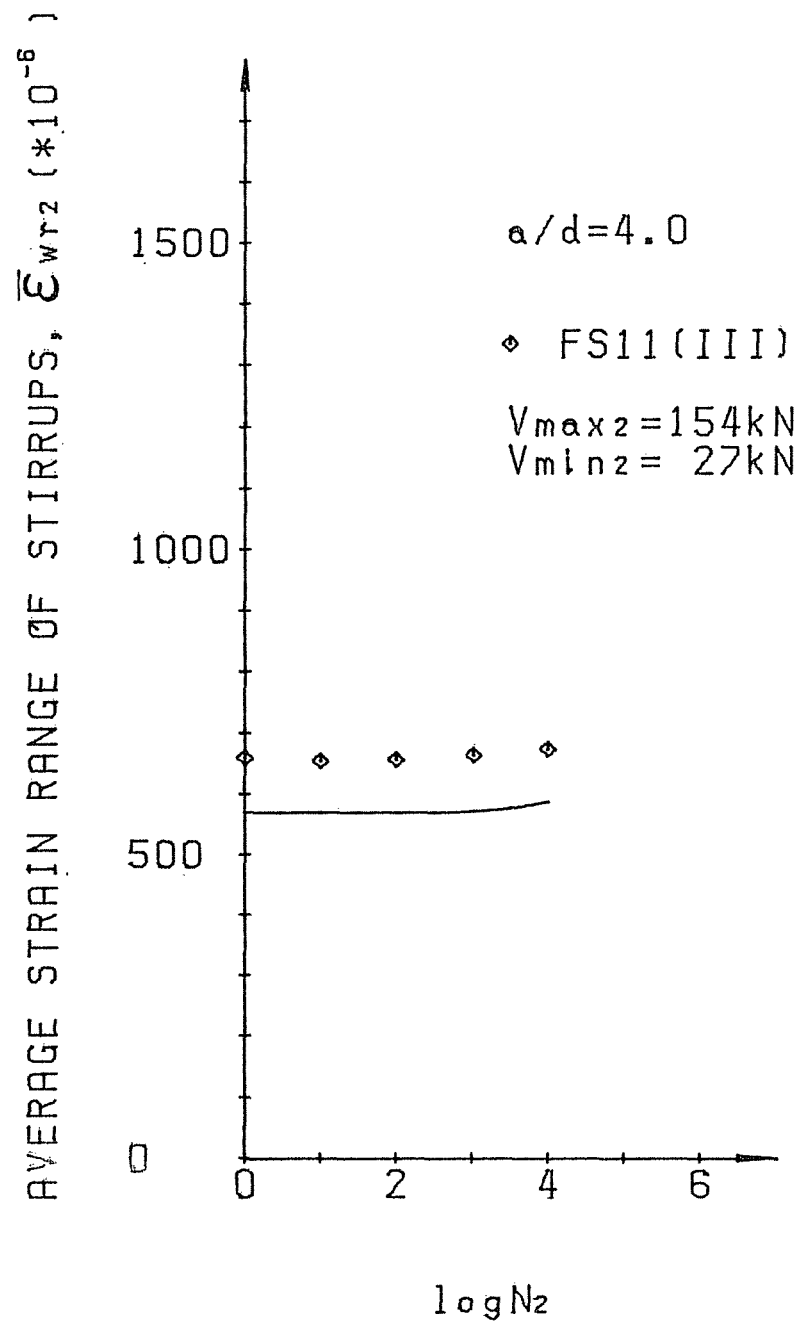


Fig.5.5(e)

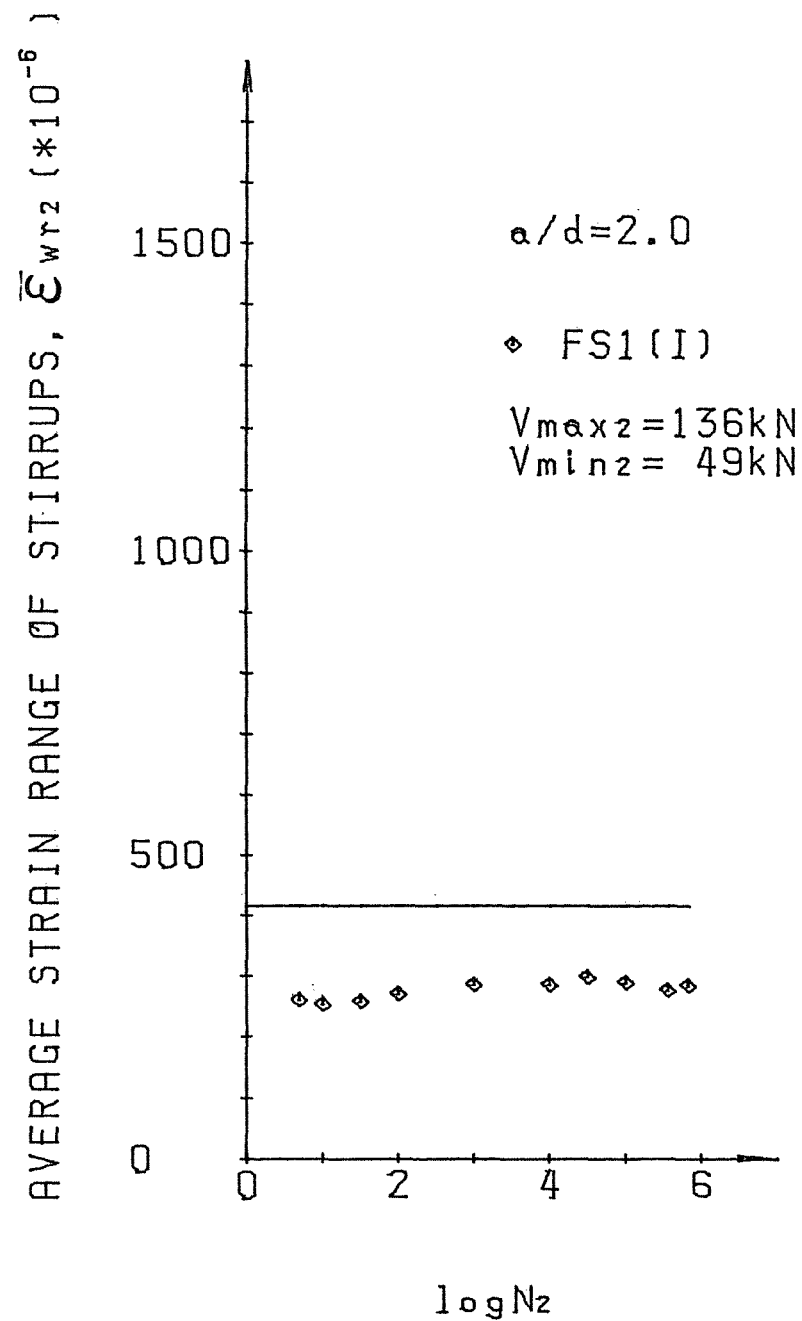


Fig.5.5(f)

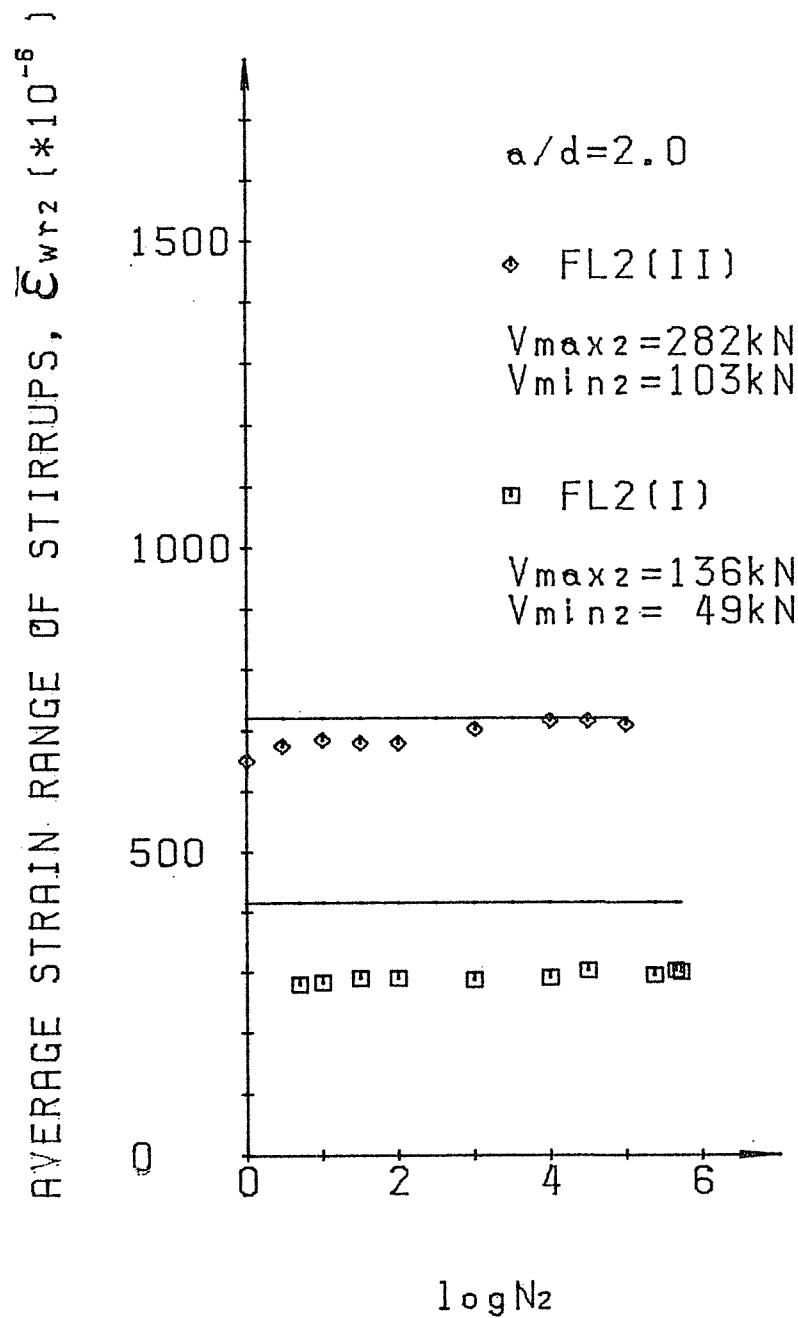


Fig.5.5(g)

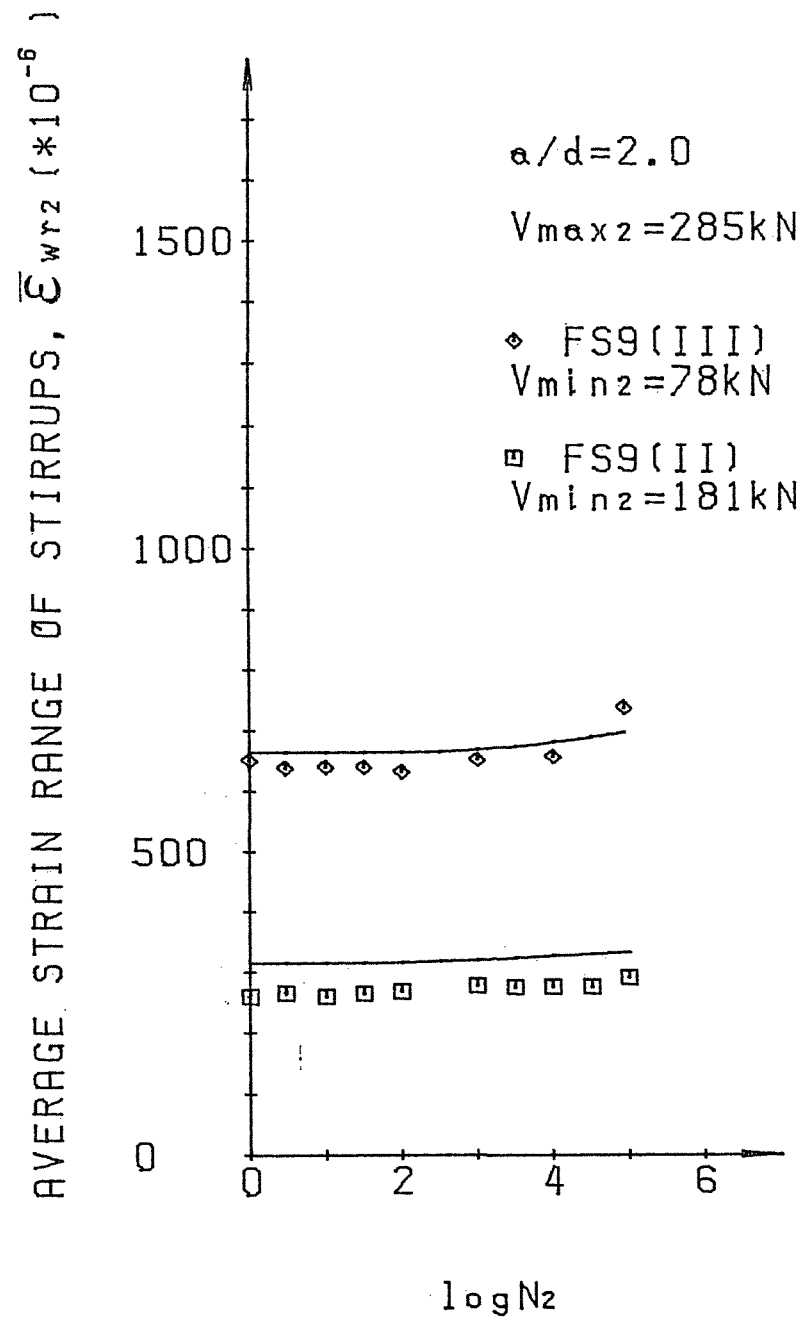


Fig.5.5(h)

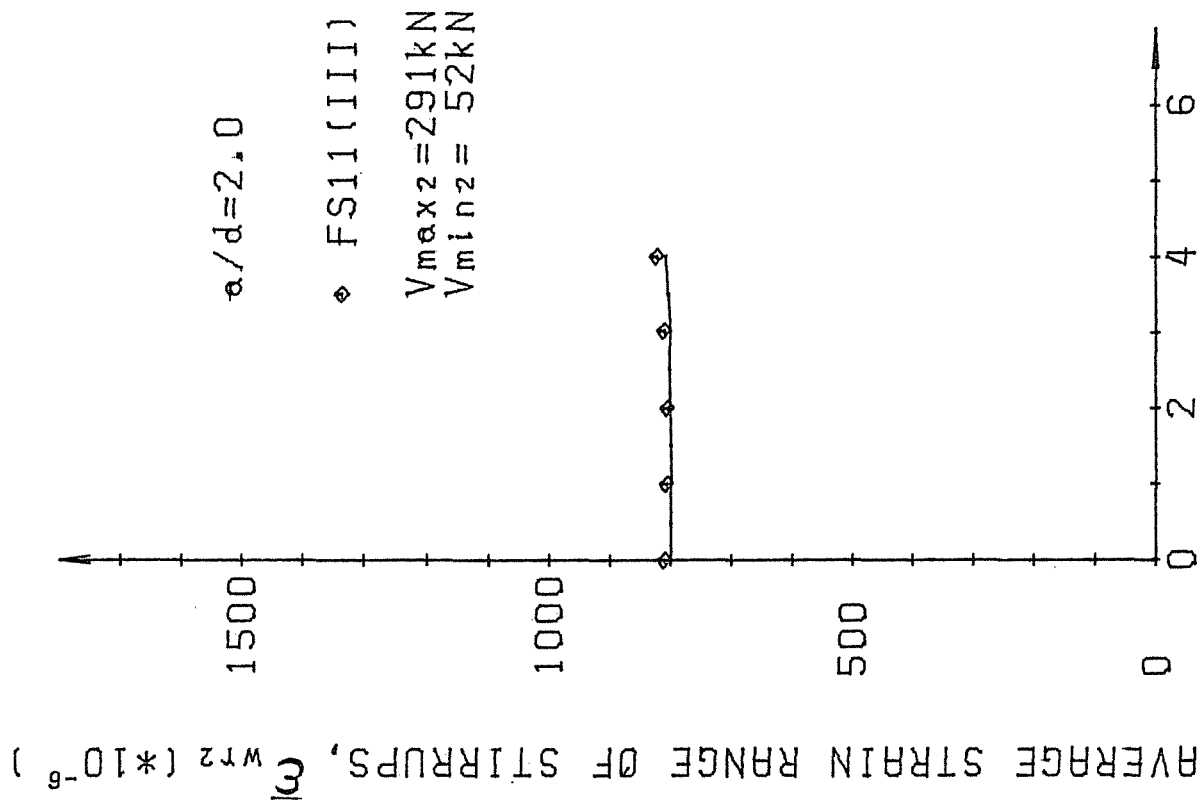


Fig.5.5(j)

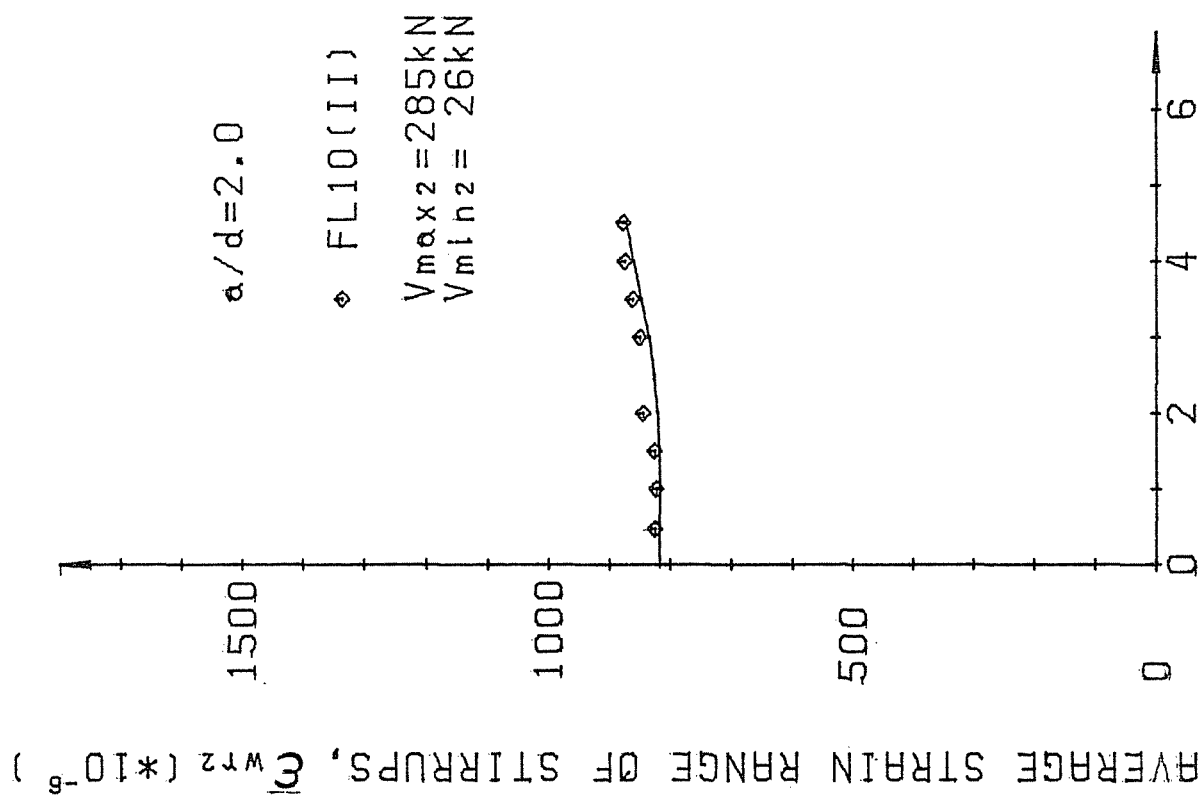


Fig.5.5(i)

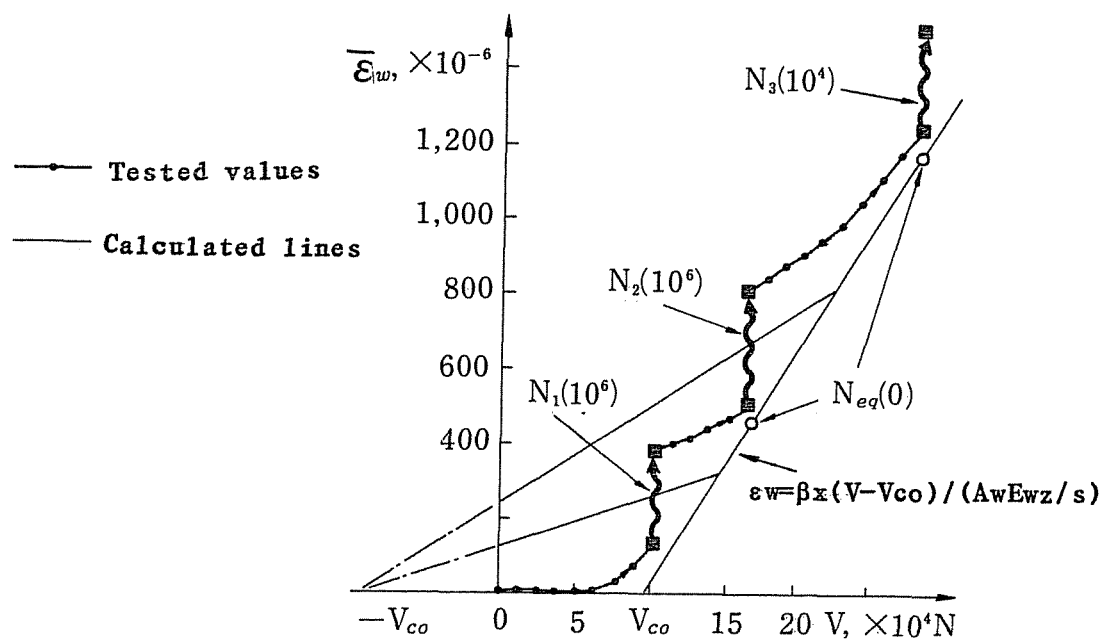


Fig.5.6 Average strain of stirrups after the first repeated loading ——— a case of no influence of previous fatigue loading (FS7 $a/d=2.0$)

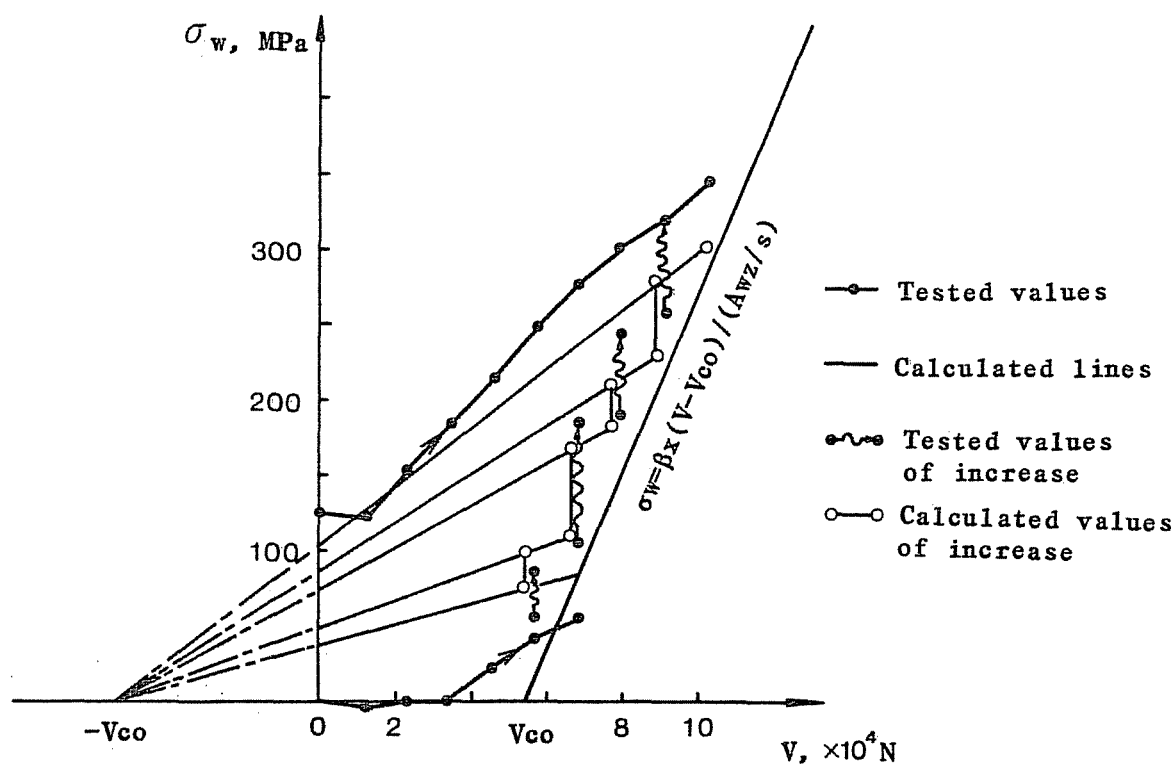


Fig.5.7 Comparison between calculated and tested stresses under fatigue loading with varied load range in Ruhnau's test [3]

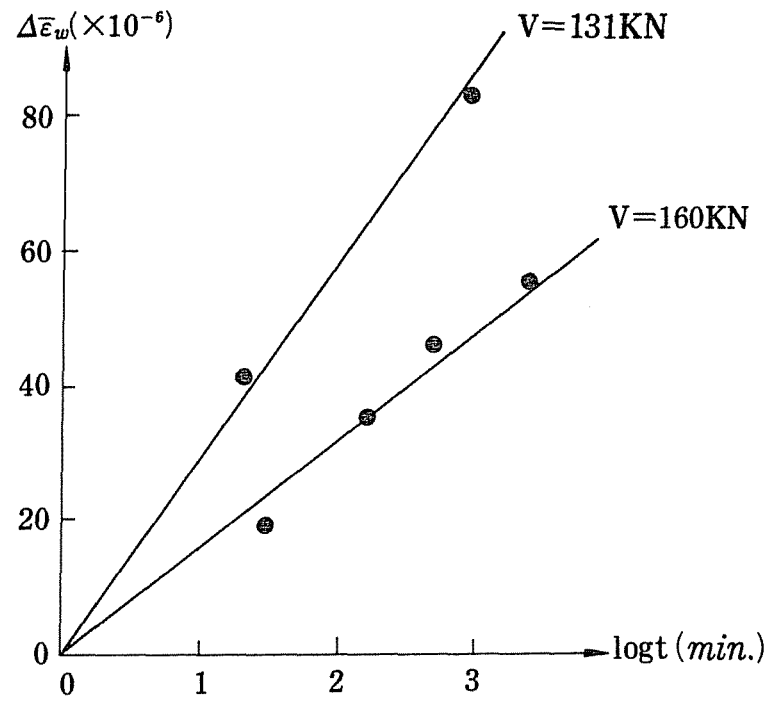


Fig.5.8 Increase of average stirrup strain under sustained loading (FS11
a/d=2.0)

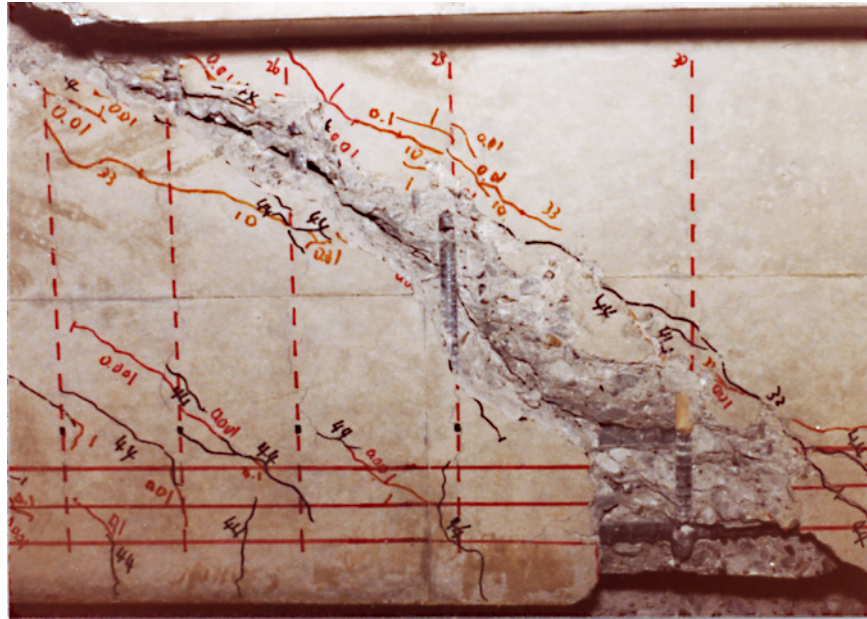


Fig.6.1 Examples of fatigue fractured stirrups (FL10 $a/d=2.0$)

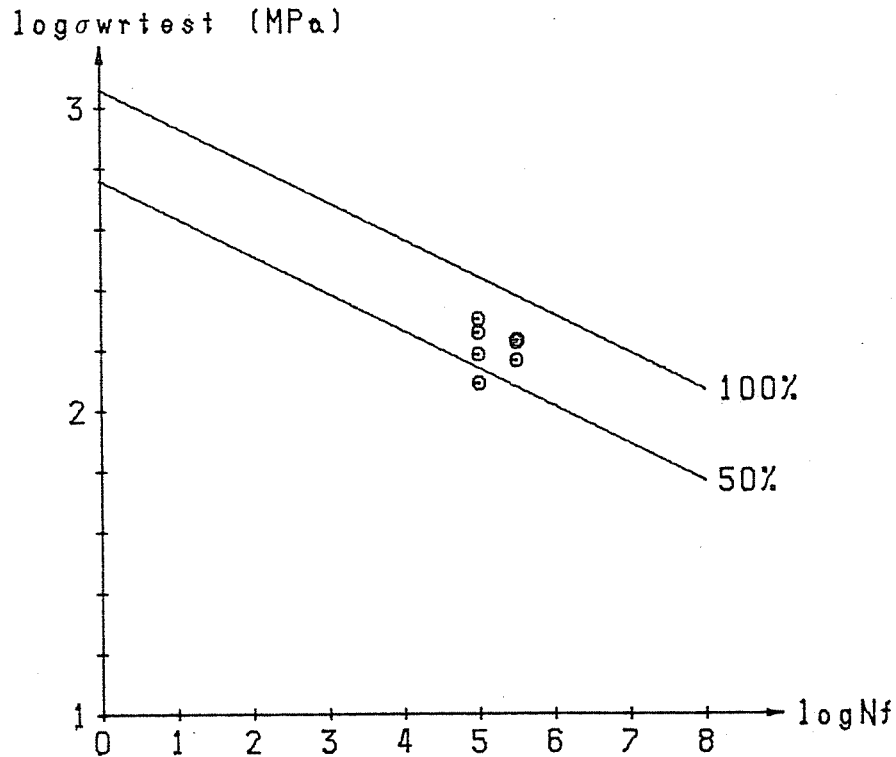


Fig.6.2 Relationship between measured stress range of first fractured stirrup and loading cycles at first stirrup fracture

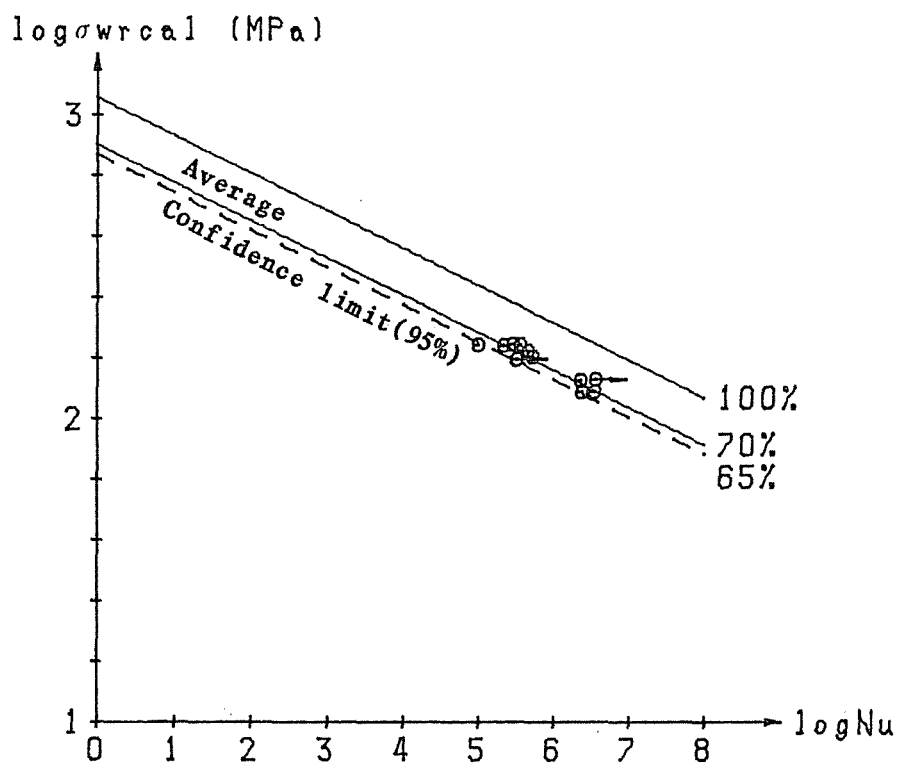


Fig.6.3 Relationship between calculated value of average stress range of stirrups and tested value of fatigue life of T-beam in author's tests

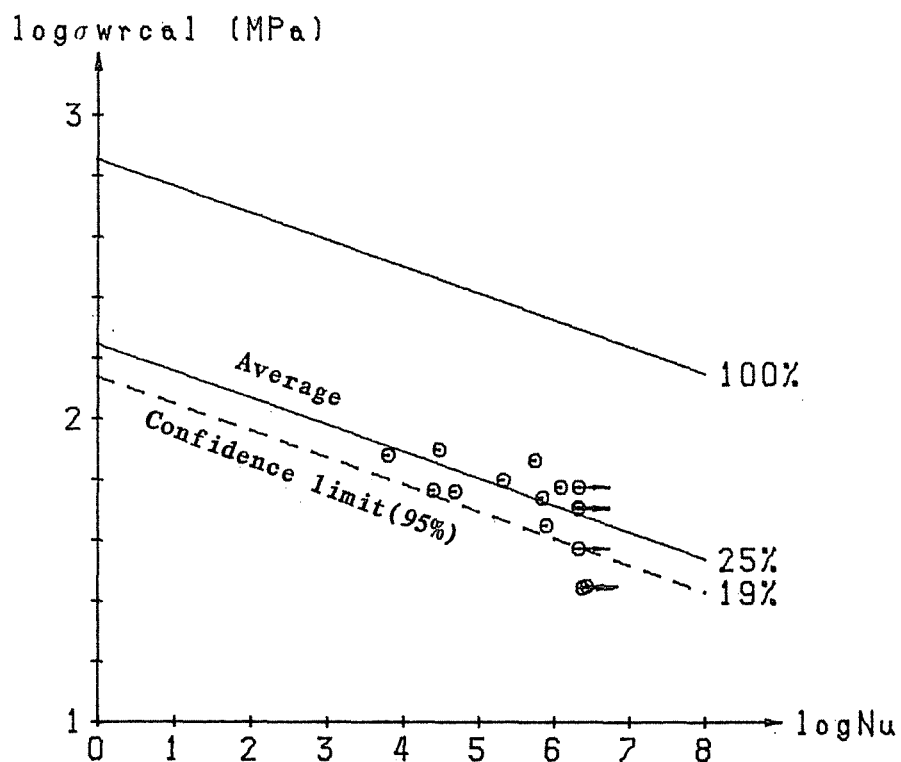


Fig.6.4 Relationship between calculated value of average stress range of stirrups and tested value of fatigue life of rectangular beam in Farghaly's tests [1]

APPENDIX A

Experimental Data in Farghaly's Tests

Table 1 Outline of test program

Series	Specimen		Concrete	Stirrup					Longitudinal Bar			
	Fatigue	Static	f_c' MPa	D mm	ϕ mm	A_w mm ²	f_{wy} MPa	E_w 10 ³ MPa	ϕ mm	A_s mm ²	p %	f_y MPa
I	25F1-3	25S1	18.6	65	13	253	324	176	25	2027	3.07	373
	25F4-6	25S2	35.2									
II	19F1-3	19S1	18.6	25	10	143	379	170	19	1146	1.73	377
	19F4-6	19S2	35.2									
III	19F7-11	—	30.5	50								

f_c' : cylinder strength, ϕ : diameter, A_w or A_s : cross-sectional area, f_{wy} or f_y : yield strength, E_w : Young's modulus, p : reinforcement ratio
D : pin diameter, Maximum size of coarse aggregate : 20 mm

Table 2 Properties of specimens

Series	Specimen		(1)	(2)	(3)	(4)	(5)
	Static	Fatigue	V_{co} kN	V_y kN	V_f kN	V_u kN	$\frac{V_u}{V_f}$
I	25S1	25F1-3	91.2	276	191	221	1.16
	25S2	25F4-6	93.9	278	243	252	1.04
II	19S1	19F1-3	77.3	159	136	162	1.19
	19S2	19F4-6	84.4	167	154	157	1.02
III	—	19F7-11	78.8	161	151	—	—

(1) V_{co} denotes tested value derived from V-sw relation of static specimen and fatigue specimens at the first cycle.

(2) $V_y = V_{co} + A_w f_{wy} (z/s)/\beta_x$

(3) $V_f = A_s f_y (1 - 0.6 p f_y / f_c') d/a$

(4) V_u denotes tested value.

Table 3 Results of fatigue tests

Series	Specimen	Maximum Load				First Fractured Stirrup														Others		
		Vmax kN	Vmax Vco	Vmax Vu	Vmax Vy	West Span					East Span									Stirrup (1)	Nf	
						Stirrup (1)	Nf (2) ×10 ⁴	(3) ×10 ⁴	σwr (4) MPa	(5) MPa	(4) (5)	Stirrup (1)	Nf (2) ×10 ⁴	(3) ×10 ⁴	(4) MPa	(5) MPa	(4) (5)	Nu (6) ×10 ⁴	Vu (7) kN		(2) ×10 ⁴	(3) ×10 ⁴
I	25F1	98.0	1.07	0.45	0.36	1S/1N	192.5	214.0	86.8	65.9	1.32							274.3	196.0			
	25F2	122.5	1.34	0.57	0.44	1N	124.0	133.0				6S	69.1	74.0	93.1	101.2	0.92	214.0	191.1	5S	124.0	133.0
	25F3	137.2	1.50	0.62	0.50							6S	10.0	21.12	128.6	117.3	1.10	21.12				
	25F4	110.3	1.17	0.44	0.40	2N	51.5	54.0	110.6	77.9	1.42	6S	10.0	33.0	114.1	71.1	1.61	214.0	186.2	5S(10)	114.0	135.0
	25F5	129.9	1.38	0.52	0.47	1N/2N	9.0	31.0	104.6	102.2	1.02	5N	31.0	39.0	124.7	107.5	1.16	69.5		1S/2S	39.0	61.0
	25F6	147.0	1.57	0.58	0.53	2S	20.0	31.7	93.8	133.9	0.70	5S	10.0	16.6	118.0	130.9	0.90	55.1		4S/4N/5N/6N	20.0	31.7
II	19F1	98.0	1.27	0.60	0.62	1N	0.1	0.63	(8)	136.2								0.63				
	19F2	83.3	1.08	0.51	0.52							4S/4N	10.0	31.0	161.9	(9)		46.0				
	19F3	83.3	1.08	0.51	0.52	1S	160.0	183.2	(8)	139.6		3N/4S/4N	183.2	213.2	130.7	140.7	0.93	213.2	151.9			
	19F4	83.3	0.99	0.53	0.50							3N/4S	155.8	210.7	135.6	129.1	1.05	210.7	161.7			
	19F5	93.1	1.10	0.59	0.56	1S	1.0	2.5	209.8	117.8	1.78							2.5				
	19F6	88.2	1.05	0.56	0.53	1N	20.5	49.0	47.0	126.2	0.37	3N	10.0	49.0	145.8	96.8	1.51	123.0		4S/4N	90.0	163.0
III	19F7	98.0	1.24	—	0.61	1S/1N	1.36	2.2	155.8	155.0	1.01							3.01		2N	2.2	2.85
	19F8	88.2	1.12	—	0.55	1S	3.86	4.65	156.8	124.5	1.26							4.81				
	19F9	78.4	0.99	—	0.49	1S	2.5	4.6	142.0	83.9	1.69	4S	4.6	10.0	124.1	89.0	1.39	78.1				
	19F10	68.6	0.87	—	0.43							4S	84.0	94.4	114.8	107.7	1.07	235.0	132.3			
	19F11	68.6	0.87	—	0.43							4S	64.8	85.4	87.6	105.6	0.83	205.0	129.9			

(0) V_{min} = 19.6 kN

(1) S and N denote south and north legs, respectively.

(2) Number of cycles at time of previous inspection

(3) Number of cycles at which fracture was found

(4) Measured value

(5) Calculated value

(6) Fatigue life of beam

(7) Final static test

(8) Unmeasurable case

(9) Uncalculated case

(10) Fracture at the start of its hook

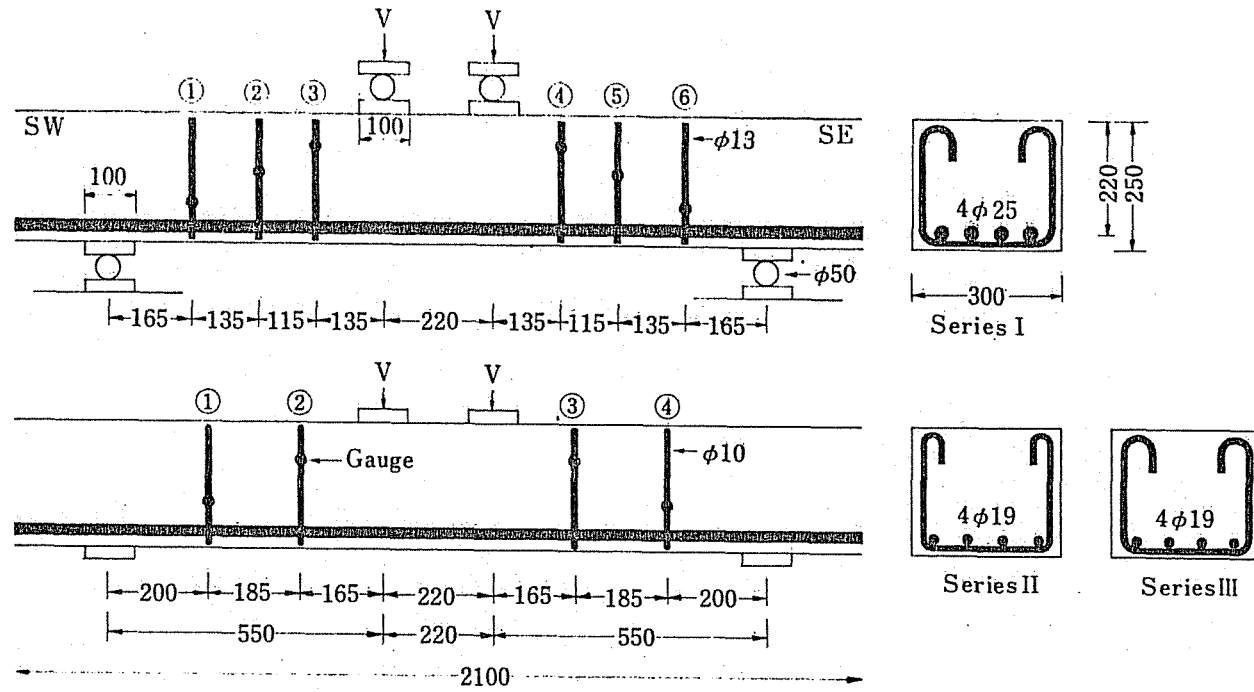


Fig.1 Test beams and gauge arrangement

APPENDIX B

Experimental Data in Previous Tests of Rectangular Beams without Web Reinforcement

Table 1 Experimental data in Farghaly's tests [1]

Specimens	a	d	bw	pw	fc'	Vf	Vcu	Vmax	Vmin	$\frac{Vmin}{Vmax}$	$\frac{Vmax}{Vcu}$	Nf	Notes
	mm	mm	mm	%	MPa	(1) kN	(2) kN	kN	kN			$\times 10^4$	
1.5-F-60-1	330	220	300	1.74	26.0	226	214	128	16	0.12	0.6	50.75	(4)
1.5-F-70-1	330	220	300	1.74	26.0	226	214	150	16	0.10	0.7	31.111	(4)
1.5-F-70-2	330	220	300	1.74	26.3	226	214	150	16	0.11	0.7	20.8	(4)
1.5-F-80-1	330	220	300	1.74	26.0	226	214	171	16	0.09	0.8	0.01	
1.5-F-80-2	330	220	300	1.74	26.3	226	214	171	16	0.09	0.8	23.4	(4)
1.5-F-85-1	330	220	300	1.74	26.3	226	214	182	16	0.09	0.85	1.48	
1.5-F-85-2	330	220	300	1.74	26.6	227	214	182	16	0.09	0.85	1.21	
1.5-F-90-1	330	220	300	1.74	26.6	227	214	193	16	0.09	0.9	0.36	
1.5-F-95-1	330	220	300	1.74	26.6	227	214	203	16	0.08	0.95	0.0003	
2.5-F-60-1	550	220	300	1.74	30.1	139	96	56	17	0.31	0.59	100.0<	(3)
2.5-F-60-2	550	220	300	1.74	22.5	132	87	56	14	0.25	0.65	100.0<	(3)
2.5-F-70-1	550	220	300	1.74	30.1	139	96	65	17	0.26	0.68	8.7	
2.5-F-70-2	550	220	300	1.74	22.5	132	87	65	14	0.22	0.75	2.3	
2.5-F-80-1	550	220	300	1.74	30.1	139	96	75	17	0.23	0.78	3.5	
2.5-F-80-2	550	220	300	1.74	22.5	132	87	75	14	0.19	0.86	0.003	
3.5-F-60-1	770	220	300	1.74	30.1	99	84	47	17	0.37	0.56	100.0<	(3)
3.5-F-60-2	770	220	300	1.74	22.5	94	76	47	14	0.30	0.62	100.0<	(3)
3.5-F-70-1	770	220	300	1.74	30.1	99	84	55	17	0.31	0.65	2.43	
3.5-F-70-2	770	220	300	1.74	22.5	94	76	55	14	0.26	0.72	22.35	
3.5-F-80-1	770	220	300	1.74	22.5	94	76	63	14	0.23	0.82	0.14	
3.5-F-80-2	770	220	300	1.74	22.5	94	76	63	14	0.23	0.82	1.3	
4.5-F-60-1	990	220	300	1.74	26.0	75	74	44	16	0.36	0.60	1.45	
4.5-F-60-2	990	220	300	1.74	26.3	75	74	44	16	0.36	0.59	100.0<	(3)
4.5-F-70-1	990	220	300	1.74	26.0	75	74	51	16	0.31	0.69	21.4	
4.5-F-70-2	990	220	300	1.74	26.3	75	74	51	16	0.31	0.69	11.37	
4.5-F-80-1	990	220	300	1.74	26.0	75	74	58	16	0.27	0.79	0.235	
4.5-F-80-2	990	220	300	1.74	26.3	75	74	58	16	0.27	0.79	0.46	

(1) $V_f = A_s f_y (1 - 0.6 p f_y / f_c') d / a$

where $p = A_s / (b d)$: reinforcement ratio

$A_s = 1146 \text{ mm}^2$: cross-sectional area of tensile bar

$f_y = 343 \text{ MPa}$: yield strength of tensile bar

b : flange width ($= b_w$), V_f : static strength in flexure

(2) $a/d \geq 2.5$

$V_{cu} = 0.20 f_c'^{1/3} (0.75 + 1.40 d / a) (1 + \beta_p + \beta_d) b_w d$

where $\beta_p = \sqrt{100 p w} - 1$, $\beta_d = (1000 / d)^{1/4} - 1$

$p w = A_s / (b_w d)$: reinforcement ratio

f_c' : cylinder strength (MPa), a : shear span

d : effective depth (mm), b_w : web width

V_{cu} : static strength in shear of beam without web reinforcement

$a/d < 2.5$

V_{cu} denotes tested value derived from static test of identical beam.

(3) Non-failure

(4) Fatigue shear failure due to fatigue fracture of tensile bar at portion crossed by a diagonal crack

Table 2 Experimental data in Higai's tests [5]

Specimens	a	d	bw	pw	fc'	Vf (1)	Vcu (2)	Vmax	Vmin	$\frac{Vmin}{Vmax}$	$\frac{Vmax}{Vcu}$	Nf $\times 10^4$	Notes
	mm	mm	mm	%	MPa	kN	kN	kN	kN				
FC4	240	120	100	2.12	35.1	47.6	45.6	19.6	4.9	0.25	0.43	200.0<	(3)
								24.5	4.9	0.20	0.54	200.0<	(3)
								29.4	4.9	0.17	0.64	95.0	(4)
FC5	240	120	100	2.12	31.9	46.7	43.5	27.0	4.9	0.18	0.62	17.0	(4)
FC6	240	120	100	2.12	31.9	46.7	43.5	27.0	4.9	0.18	0.62	15.0	(4)
FC7	240	120	100	2.12	38.2	48.4	47.6	24.5	4.9	0.20	0.51	59.0	(4)
FC8	240	120	100	2.12	35.3	47.7	45.7	29.4	4.9	0.17	0.64	14.0	
FT2	800	200	200	2.15	40.3	76.7	47.3	24.5	4.9	0.20	0.52	200.0<	(3)
FT3	800	200	200	2.15	39.8	76.6	47.2	27.0	4.9	0.18	0.57	174.0	(4)
FT4	800	200	200	2.15	39.8	76.6	47.2	29.4	4.9	0.17	0.62	82.0	
FT13	800	160	200	2.40	31.1	51.7	35.3	27.7	4.9	0.18	0.78	0.2	
FT14	800	160	200	2.40	29.1	50.9	34.6	24.5	4.9	0.20	0.71	0.3	
FT15	800	160	200	2.40	28.1	50.4	34.2	21.1	4.9	0.23	0.62	106.0<	(3)
FT16	800	160	200	2.40	32.7	52.3	35.9	23.0	4.9	0.21	0.64	79.0	
FT17	800	160	200	2.40	31.1	51.7	35.3	21.1	4.9	0.23	0.60	241.0<	(3)
FT19	800	160	200	2.40	35.1	53.1	36.8	23.5	4.9	0.21	0.64	73.0	
FT6	700	110	200	1.80	35.1	23.2	23.2	18.6	4.9	0.26	0.80	0.05	
FT7	700	110	200	1.80	35.5	23.3	23.3	16.0	4.9	0.31	0.69	44.0	
FT8	700	110	200	1.80	33.3	23.0	22.8	13.7	4.9	0.36	0.60	120.0<	(3)
FT9	700	110	200	1.80	32.0	22.9	22.5	14.7	4.9	0.33	0.65	106.0<	(3)
FT12	700	110	200	1.80	37.5	23.4	23.7	16.0	7.3	0.46	0.67	69.0	
FT10	700	110	200	1.80	35.0	23.2	23.2	16.0	9.8	0.61	0.69	366.0<	(3)

(1) $V_f = A_s f_y (1 - 0.6 p f_y / f_c') d / a$

where $p = A_s / (b d)$: reinforcement ratio

$f_y = 474 \text{ MPa}$ (if $p_w = 2.12 \%$), 411 MPa (if $p_w = 2.15 \%$)
 417 MPa (if $p_w = 2.40 \%$), 430 MPa (if $p_w = 1.80 \%$)

: yield strength of tensile bar

A_s : cross-sectional area of tensile bar

b : flange width (= b_w), V_f : static strength in flexure

(2) $a/d \geq 2.5$

$V_{cu} = 0.8 \times 0.20 f_c'^{1/3} (0.75 + 1.40 d / a) (1 + \beta_p + \beta_d) b_w d$

where $\beta_p = \sqrt{100 p_w} - 1$, $\beta_d = (1000 / d)^{1/4} - 1$

$p_w = A_s / (b_w d)$: reinforcement ratio

f_c' : cylinder strength (MPa), a : shear span

d : effective depth (mm), b_w : web width

V_{cu} : static strength in shear of beam without web reinforcement

$a/d < 2.5$

V_{cu} denotes tested value derived from static test of identical beam.

(3) Non-failure

(4) Fatigue shear failure due to fatigue fracture of tensile bar at portion crossed by a diagonal crack

Table 3 Experimental data in Chang - Kesler's tests [8]

Specimens	a	d	bw	pw	fc'	Vf (1)	Vcu (2)	Vmax	Vmin	$\frac{Vmin}{Vmax}$	$\frac{Vmax}{Vcu}$	Nf $\times 10^4$	Notes
mm	mm	mm	mm	%	MPa	kN	kN	kN	kN				
4-3	508	137	102	1.86	31.6	20.1	19.9	11.1	4.4	0.040	0.56	211.45	(4)
4-6	508	137	102	1.86	29.3	19.9	19.4	15.6	4.4	0.029	0.80	0.08	
4-7	508	137	102	1.86	32.0	20.1	20.0	17.8	4.4	0.025	0.89	0.02	
4-8	508	137	102	1.86	32.0	20.1	20.0	13.3	4.4	0.033	0.67	55.7	
4-9	508	137	102	1.86	26.9	19.7	18.8	13.3	4.4	0.033	0.71	1.68	
4-11	508	137	102	1.86	27.2	19.7	18.9	9.5	4.4	0.047	0.50	190.02	
4-12	508	137	102	1.86	29.6	19.9	19.4	11.1	4.4	0.040	0.57	114.21	
4-14	508	137	102	1.86	37.3	20.5	21.0	12.2	4.4	0.036	0.58	1.93	
4-15	508	137	102	1.86	14.8	17.1	15.4	12.2	4.4	0.036	0.79	0.194	
4-16	508	137	102	1.86	29.2	19.9	19.4	15.0	4.4	0.030	0.78	0.044	
4-17	508	137	102	1.86	29.6	19.9	19.5	15.8	4.4	0.028	0.82	0.036	
4-19	508	137	102	1.86	35.8	20.4	20.7	10.9	4.4	0.041	0.53	669.02	
4-20	508	137	102	1.86	38.8	20.6	21.3	10.0	4.4	0.044	0.47	—	(3)
4-24	508	137	102	1.86	31.7	20.1	19.9	12.2	4.4	0.036	0.62	482.24	
4-25	508	137	102	1.86	29.5	19.9	19.4	12.7	4.4	0.035	0.65	109.73	
4-26	508	137	102	1.86	32.3	20.2	20.0	12.2	4.4	0.036	0.61	149.36	(4)
4-27	508	137	102	1.86	35.3	20.4	20.6	12.8	4.4	0.035	0.62	125.04	
4-28	508	137	102	1.86	37.4	20.5	21.0	13.9	4.4	0.032	0.66	57.88	
4-29	508	137	102	1.86	36.9	20.5	20.9	13.3	4.4	0.033	0.64	120.76	
1-5	508	137	102	2.89	19.9	25.3	19.9	12.2	4.4	0.036	0.62	102.72	
2-5	508	137	102	2.89	19.7	25.2	19.8	11.1	4.4	0.040	0.56	97.61	
3-5	508	137	102	2.89	25.0	27.3	21.5	13.3	4.4	0.033	0.62	46.72	
5-1	508	137	102	2.89	46.5	31.0	26.4	14.4	4.4	0.031	0.55	—	(3)
5-3	508	137	102	2.89	32.6	29.2	23.4	16.7	4.4	0.027	0.71	2.32	
5-4	508	137	102	2.89	36.9	29.9	24.4	17.8	4.4	0.025	0.73	0.17	
5-5	508	137	102	2.89	32.2	29.1	23.3	16.4	4.4	0.027	0.70	40.29	
5-6	508	137	102	2.89	34.7	29.6	23.9	14.4	4.4	0.031	0.60	1587.17	
5-8	508	137	102	2.89	34.5	29.5	23.9	15.6	4.4	0.029	0.65	1121.77	
5-10	508	137	102	2.89	14.9	21.9	18.0	15.6	4.4	0.029	0.86	0.01	
5-11	508	137	102	2.89	24.7	27.2	21.4	13.3	4.4	0.033	0.62	3.98	
5-12	508	137	102	2.89	28.0	28.2	22.3	17.8	4.4	0.025	0.80	0.053	
5-13	508	137	102	2.89	30.6	28.8	22.9	13.3	4.4	0.033	0.58	366.65	
5-14	508	137	102	2.89	36.8	29.9	24.4	15.6	4.4	0.031	0.64	1.3	
5-15	508	137	102	2.89	33.1	29.3	23.6	13.3	4.4	0.033	0.57	23.9	
5-16	508	137	102	2.89	27.6	28.1	22.2	14.4	4.4	0.031	0.65	0.43	
5-17	508	137	102	2.89	35.8	29.7	24.2	14.4	4.4	0.031	0.60	8.78	
5-20	508	137	102	2.89	32.7	29.2	23.5	19.7	4.4	0.023	0.84	0.09	

$$(1) V_f = A_s f_y (1 - 0.6 p f_y / f_c') d / a$$

where $p = A_s / (b d)$: reinforcement ratio

$f_y = 327$ MPa : yield strength of tensile bar

A_s : cross-sectional area of tensile bar

b : flange width (= b_w), V_f : static strength in flexure

$$(2) V_{cu} = 0.20 f_c'^{1/3} (0.75 + 1.40 d / a) (1 + \beta_p + \beta_d) b_w d$$

where $\beta_p = \sqrt{100 p w} - 1$, $\beta_d = (1000 / d)^{1/4} - 1$

$p w = A_s / (b_w d)$: reinforcement ratio

f_c' : cylinder strength (MPa), a : shear span

d : effective depth (mm), b_w : web width

V_{cu} : static strength in shear of beam without web reinforcement

(3) Non-failure

(4) Fatigue shear failure due to fatigue fracture of tensile bar in pure moment region

Table 4 Experimental data in Taylor's tests [9]

Specimens	a	d	bw	pw	fc'	Vf (1)	Vcu (2)	Vmax	Vmin	$\frac{Vmin}{Vmax}$	$\frac{Vmax}{Vcu}$	Nf $\times 10^4$	Notes
	mm	mm	mm	%	MPa	kN	kN	kN	kN				
1	914	222	191	1.21	29.5	34.2	44.4	29.4	7.1	0.24	0.66	148.3	(3)
2	914	222	191	1.21	29.5	34.2	44.4	30.6	7.1	0.23	0.69	134.0	
3	914	222	191	1.21	29.5	34.2	44.4	31.8	7.1	0.22	0.72	4.0	
4	914	222	191	1.49	29.5	41.3	47.8	31.8	7.7	0.24	0.67	300.0	(3)
5	914	222	191	1.49	29.5	41.3	47.8	34.3	7.7	0.22	0.72	12.6	
6	914	222	191	1.49	29.5	41.3	47.8	34.3	7.7	0.22	0.72	3.5	
7	914	222	191	1.83	29.5	49.6	51.6	33.1	7.6	0.23	0.64	146.7	(3)
8	914	222	191	1.83	29.5	49.6	51.6	34.3	7.6	0.22	0.66	140.0	(3)
9	914	222	191	1.83	29.5	49.6	51.6	34.3	7.6	0.22	0.66	12.1	
10	914	222	191	1.83	29.5	49.6	51.6	35.5	7.6	0.21	0.69	0.36	
11	914	222	191	2.33	29.5	61.0	56.6	31.8	8.3	0.26	0.56	130.0	(3)
12	914	222	191	2.33	29.5	61.0	56.6	35.5	8.3	0.23	0.63	164.7	(3)
13	914	222	191	2.33	29.5	61.0	56.6	36.8	8.3	0.23	0.65	22.2	
14	914	222	191	2.33	29.5	61.0	56.6	38.0	8.3	0.22	0.67	0.26	

(1) $V_f = A_s f_y (1 - 0.6 p f_y / f_c') d / a$

where $p = A_s / (b d)$: reinforcement ratio

$f_y = 296$ MPa : yield strength of tensile bar

A_s : cross-sectional area of tensile bar

b : flange width ($= b_w$), V_f : static strength in flexure

(2) $V_{cu} = 0.20 f_c'^{1/3} (0.75 + 1.40 d / a) (1 + \beta_p + \beta_d) b_w d$

where $\beta_p = \sqrt{100 p w} - 1$, $\beta_d = (1000 / d)^{1/4} - 1$

$p w = A_s / (b_w d)$: reinforcement ratio

f_c' : cylinder strength (MPa), a : shear span

d : effective depth (mm), b_w : web width

V_{cu} : static strength in shear of beam without web reinforcement

(3) Non-failure

Table 5 Experimental data in Stelson - Cernica's tests [10]

Specimens	a	d	bw	pw	fc'	Vf (1)	Vcu (2)	Vmax	Vmin	$\frac{Vmin}{Vmax}$	$\frac{Vmax}{Vcu}$	Nf $\times 10^4$	Notes
	mm	mm	mm	%	MPa	kN	kN	kN	kN				
1	610	108	127	2.92	26.5	23.3	20.0	12.7	2.0	0.15	0.63	6.6	
2	610	108	127	2.92	26.5	23.3	20.0	12.7	2.0	0.15	0.63	32.3	
3	610	108	127	2.92	26.5	23.3	20.0	10.8	2.0	0.18	0.54	57.0	(3)
4	610	108	127	2.92	26.5	23.3	20.0	10.8	2.0	0.18	0.54	57.0	(3)
5	610	108	127	2.92	26.5	23.3	20.0	8.8	2.0	0.22	0.44	75.7	(3)
6	610	108	127	2.92	26.5	23.3	20.0	8.8	2.0	0.22	0.44	75.7	(3)
7	610	108	127	2.92	26.5	23.3	20.0	13.7	3.9	0.29	0.68	3.2	
8	610	108	127	2.92	26.5	23.3	20.0	13.7	3.9	0.29	0.68	2.08	
9	610	108	127	2.92	26.5	23.3	20.0	9.8	3.9	0.40	0.49	50.3	(3)
								11.7	3.9	0.33	0.59	52.0	(3)
								13.7	3.9	0.29	0.68	5.3	
10	610	108	127	2.92	26.5	23.3	20.0	11.7	3.9	0.33	0.59	50.3	(3)
								13.7	3.9	0.29	0.68	3.3	
11	610	108	127	2.92	26.5	23.3	20.0	7.8	3.9	0.50	0.39	54.2	(3)
								9.8	3.9	0.40	0.49	67.6	(3)
								12.7	3.9	0.31	0.63	28.6	

(1) $V_f = A_s f_y (1 - 0.6 p f_y / f_c') d / a$

where $p = A_s / (b d)$: reinforcement ratio

$f_y = 482$ MPa : yield strength of tensile bar

A_s : cross-sectional area of tensile bar

b : flange width (= b_w) , V_f : static strength in flexure

(2) $V_{cu} = 0.20 f_c'^{1/3} (0.75 + 1.40 d / a) (1 + \beta_p + \beta_d) b_w d$

where $\beta_p = \sqrt{100 p w} - 1$, $\beta_d = (1000 / d)^{1/4} - 1$

$p w = A_s / (b_w d)$: reinforcement ratio

f_c' : cylinder strength (MPa) , a : shear span

d : effective depth (mm) , b_w : web width

V_{cu} : static strength in shear of beam without web reinforcement

(3) Non-failure

Table 6 Experimental data in Chang - Kesler's tests [11]

Specimens	a	d	bw	pw	fc'	Vf (1)	Vcu (2)	Vmax	Vmin	$\frac{Vmin}{Vmax}$	$\frac{Vmax}{Vcu}$	Nf $\times 10^4$	Notes
	mm	mm	mm	%	MPa	kN	kN	kN	kN				
1	508	137	102	1.02	36.7	11.2	17.2	7.8	4.4	0.057	0.45	696.67<	(3)
2	508	137	102	1.02	42.7	11.3	18.1	12.7	4.4	0.035	0.70	2.35	
3	508	137	102	1.02	41.2	11.3	17.9	12.0	4.4	0.037	0.67	0.3	
4	508	137	102	1.02	34.1	11.2	16.8	12.6	4.4	0.035	0.75	0.65	
5	508	137	102	1.02	36.2	11.2	17.1	12.6	4.4	0.035	0.73	0.18	
6	508	137	102	1.02	34.0	11.2	16.8	8.9	4.4	0.050	0.53	48.3	(4)
7	508	137	102	1.02	36.8	11.2	17.2	10.0	4.4	0.044	0.58	39.56	(4)
8	508	137	102	1.02	37.9	11.2	17.4	12.6	4.4	0.035	0.72	0.8	(4)
9	508	137	102	1.02	36.9	11.2	17.2	12.5	4.4	0.035	0.73	0.26	
10	508	137	102	1.02	30.5	11.1	16.2	10.0	4.4	0.044	0.62	4.71	
11	508	137	102	1.02	39.0	11.2	17.6	6.7	4.4	0.067	0.38	2047.29<	(3)
12	508	137	102	1.02	38.2	11.2	17.4	12.7	4.4	0.035	0.73	2.32	
13	508	137	102	1.02	32.7	11.1	16.6	10.0	4.4	0.044	0.60	17.21	(4)
14	508	137	102	1.02	37.9	11.2	17.4	11.1	4.4	0.040	0.64	44.55	(4)
15	508	137	102	1.02	41.9	11.3	18.0	11.1	4.4	0.040	0.62	50.28	(4)
16	508	137	102	1.02	38.7	11.2	17.5	8.9	4.4	0.050	0.51	57.61<	(3)
17	508	137	102	1.02	31.2	11.1	16.3	8.9	4.4	0.050	0.55	71.35	(4)
18	508	137	102	1.02	35.8	11.2	17.1	11.1	4.4	0.040	0.65	42.02	(4)
19	508	137	102	1.02	41.2	11.3	17.9	10.0	4.4	0.044	0.56	46.35	(4)
20	508	137	102	1.02	43.4	11.3	18.2	12.7	4.4	0.035	0.70	0.32	(4)
21	508	137	102	1.02	46.9	11.3	18.7	12.7	4.4	0.035	0.68	2.58	(4)
22	508	137	102	1.02	44.1	11.3	18.3	12.4	4.4	0.036	0.68	3.0	
23	508	137	102	1.02	42.0	11.3	18.0	12.7	4.4	0.035	0.71	7.0	(4)
24	508	137	102	1.02	43.4	11.3	18.2	12.7	4.4	0.035	0.70	15.09	(4)
25	508	137	102	1.02	43.4	11.3	18.2	11.3	4.4	0.039	0.62	14.05	(4)

(1) $V_f = A_s f_y (1 - 0.6 f_y / f_c') d / a$

where $p = A_s / (b d)$: reinforcement ratio

$f_y = 310$ MPa : yield strength of tensile bar

A_s : cross-sectional area of tensile bar

b : flange width (= b_w) , V_f : static strength in flexure

(2) $V_{cu} = 0.20 f_c'^{1/3} (0.75 + 1.40 d / a) (1 + \beta_p + \beta_d) b_w d$

where $\beta_p = \sqrt{100 p w} - 1$, $\beta_d = (1000 / d)^{1/4} - 1$

$p w = A_s / (b_w d)$: reinforcement ratio

f_c' : cylinder strength (MPa) , a : shear span

d : effective depth (mm) , b_w : web width

V_{cu} : static strength in shear of beam without web reinforcement

(3) Non-failure

(4) Fatigue shear failure due to fatigue fracture of tensile bar in pure moment region