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Author(s)	Ueda, Kei-ichi; Teramoto, Takashi; Nishiura, Yasumasa
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SCATTERING OF PARTICLE-LIKE PATTERNS IN REACTION-DIFFUSION SYSTEMS

KEI-ICHI UEDA

Research Institute for Mathematical Sciences, Kyoto University, Kitashirakawa,
Sakyo-ku, Kyoto 606-8502, Japan
(ueda@kurims.kyoto-u.ac.jp)

TAKASHI TERAMOTO

Department of Photonics Material Science, Chitose Institute of Science and Technology,
758-65 Bibi, Chitose 066-8655, Japan
(teramoto@photon.chitose.ac.jp)

YASUMASA NISHIURA

Research Institute for Electronic Science, Hokkaido University, Kita 12 Nishi 6, Kita-ku,
Sapporo 060-0812, Japan
(nishiura@nsc.es.hokudai.ac.jp)

Abstract. Scattering of two 1-dimensional traveling pulses in reaction-diffusion systems is discussed. A variety of scattering dynamics can be observed in a three-component reaction-diffusion systems and the output changes depending on parameters in a sensitive way. It is found numerically that repulsion and annihilation dynamics can be observed near pitchfork and Hopf bifurcation (PH) points of single pulse for appropriate parameter regime. By using center manifold theory, bifurcation structure for single pulse near PH point is also investigated.

1 Introduction

Spatially localized moving patterns such as pulses and spots have been observed in reaction-diffusion systems. Such a phenomenon has been observed in experimentally and numerically, for instance, in gas-discharged system [1][3], CO-oxidization process [2], chemical reactions [4][8]. These findings introduce a variety of input-output relations for scattering process such as repulsion, fusion, annihilation and even switch to a chaotic regime depending on a parameter regime. A new viewpoint was introduced in [9] to shed a light on the mathematical mechanism controlling them, namely hidden saddles (called “scatter” in [10]) play a key role of traffic control for flows during scattering process.

A typical manner of collision is depicted as in Fig.3. A feature for this scattering is that there is a moment that solution profile right after the collision becomes almost symmetric and close to that of standing pulse as in Fig.3. Intuitively repulsion should persist for nearby parameters, however a tiny change of the parameter k_4 causes a sudden change to annihilation like Fig.3 (b-c), which does not look like a strong interaction. How do we understand this transition and what is the mathematical mechanism behind these scattering dynamics? It turns out that singularities of standing pulse of higher codimension are responsible for producing such dynamics.

In this article, we employ the following three-component system as an representative model for our purpose [11].

$$\begin{aligned} u_t &= D_u u_{xx} + k_2 u - u^3 - k_3 v - k_4 w + k_1, \\ \tau v_t &= D_v v_{xx} + u - \gamma v, \\ \theta w_t &= D_w w_{xx} + u - w. \end{aligned} \tag{1}$$

where $D_u = 5.0 \times 10^{-5}$, $D_v = 6.5 \times 10^{-3}$, $D_w = 1.0 \times 10^{-2}$, $\gamma = 8.0$, $k_1 = -3.0$, $k_2 = 2.0$, $k_3 = 2.0$, $\theta = 10.0$. We adopt $\Delta x = 5.0 \times 10^{-3}$ and $\Delta t = 1.0 \times 10^{-3}$.

2 Scattering dynamics near singularities

We consider the case in which the velocity of traveling pulses is small, namely two traveling pulses approach slowly. This typically occurs near pitchfork bifurcation from standing pulse. Such pitchfork bifurcation can be found for (1) when k_4 is increased with $\tau = 1200$ in which traveling pulses emanate from the standing pulse at $k_4 \approx 2.973$ supercritically. A collision of two slow pulses, for instance at $k_4 = 2.978$, is expected like Fig.2, in fact it was shown by [6] that pulses bifurcating supercritically from a stable standing pulse repel at collision. It turns out, however that this is not always the case, especially when the pitchfork bifurcation is combined with Hopf bifurcation.

A numerics reveals that the outputs after scattering are classified as in Fig.1. Solid and dashed lines represent pitchfork and Hopf instabilities respectively emanating from the branch of standing pulse. They intersect at one point PH forming a codim-2 point.

There is no traveling pulse in the left part of the pitchfork bifurcation line, in fact we only observe background state (region I) and standing pulse (region II) there. As shown in Fig.1, it is clear that PH is a kind of organizing center for the classification of outputs. When τ is below PH, standing pulses change into stable traveling ones as k_4 is

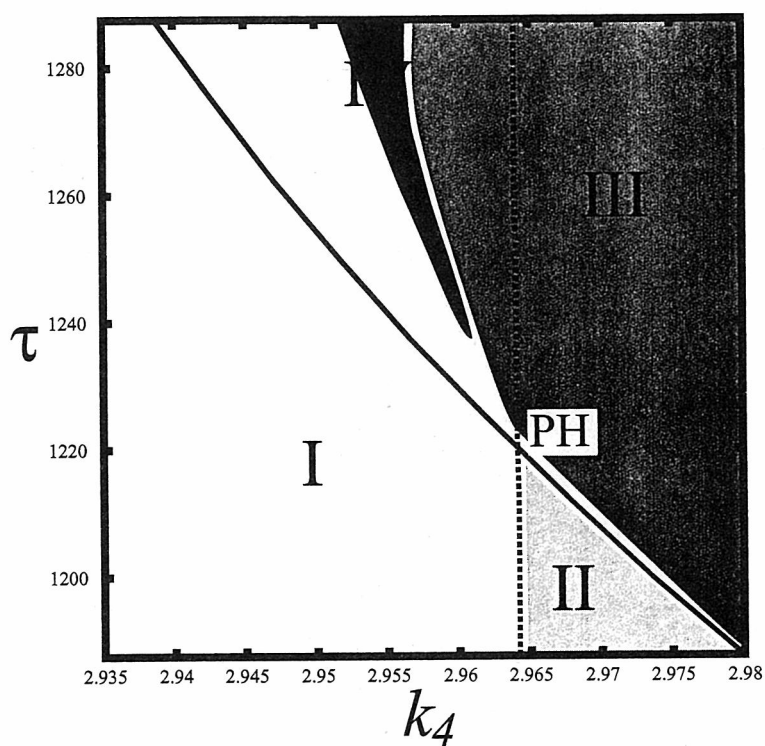


Figure 1: Phase diagram for (1). Solid (dashed) line indicates pitchfork (Hopf) bifurcation line for $D_v = 0.00065$. I: Background state. II: Existence region of stable standing pulse. III: Repulsion after collision. IV: Annihilation after collision.

increased, and repulsion of pulses is observed in region III. On the other hand, when τ is above PH, the same pitchfork bifurcation occurs as before, however there are no stable traveling pulses when k_4 is located near the bifurcation point. As k_4 is still increased, stable traveling pulses of finite speed appears, but the annihilate each other upon collision in region IV. Finally repulsion occurs for larger k_4 in region III. It should be noted that the speed of traveling pulse near PH of region IV is very slow.

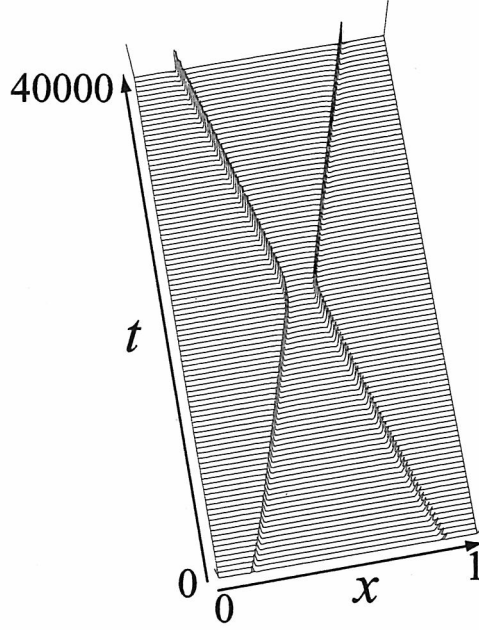


Figure 2: Repulsion for $\tau = 1200$, $k_4 = 2.978$.

3 Reduction to ODEs for a single pulse dynamics

In this section, we give a formal derivation of ODEs under some assumptions in order to describe the dynamics of single pulse near pitchfork and Hopf bifurcation points in a general setting as in [5].

Let us consider the following general form of an N -component reaction-diffusion equation.

$$\mathbf{u}_t = D\mathbf{u}_{xx} + F(\mathbf{u}, \mathbf{k})$$

where $\mathbf{u} = (u_1, u_2, \dots, u_N) \in \mathbf{R}^N$, $F : \mathbf{R}^N \rightarrow \mathbf{R}^N$ is a smooth function, D is a diagonal matrix with non-negative elements, and $\mathbf{k} = (k_1, k_2) \in \mathbf{R}^2$ are bifurcation parameters.

Let $\mathcal{L}(\mathbf{u}; \mathbf{k}) := D\mathbf{u}_{xx} + F(\mathbf{u}, \mathbf{k})$ and $\mathbf{k} = \tilde{\mathbf{k}} + \boldsymbol{\eta} = (\tilde{k}_1 + \eta_1, \tilde{k}_2 + \eta_2)$, then we have

$$\mathbf{u}_t = \mathcal{L}(\mathbf{u}) + \boldsymbol{\eta} \cdot g(\mathbf{u}) \quad (2)$$

where $\mathcal{L}(\mathbf{u}) = \mathcal{L}(\mathbf{u}; \tilde{\mathbf{k}})$, $g(\mathbf{u}) = g(\mathbf{u}; \boldsymbol{\eta})$, $\boldsymbol{\eta} \cdot g(\mathbf{u}; \boldsymbol{\eta}) = \mathcal{L}(\mathbf{u}; \tilde{\mathbf{k}} + \boldsymbol{\eta}) - \mathcal{L}(\mathbf{u})$, $\boldsymbol{\eta} = (\eta_1, \eta_2)$, $g(\mathbf{u}) = (g_1(\mathbf{u}), g_2(\mathbf{u}))$. We assume

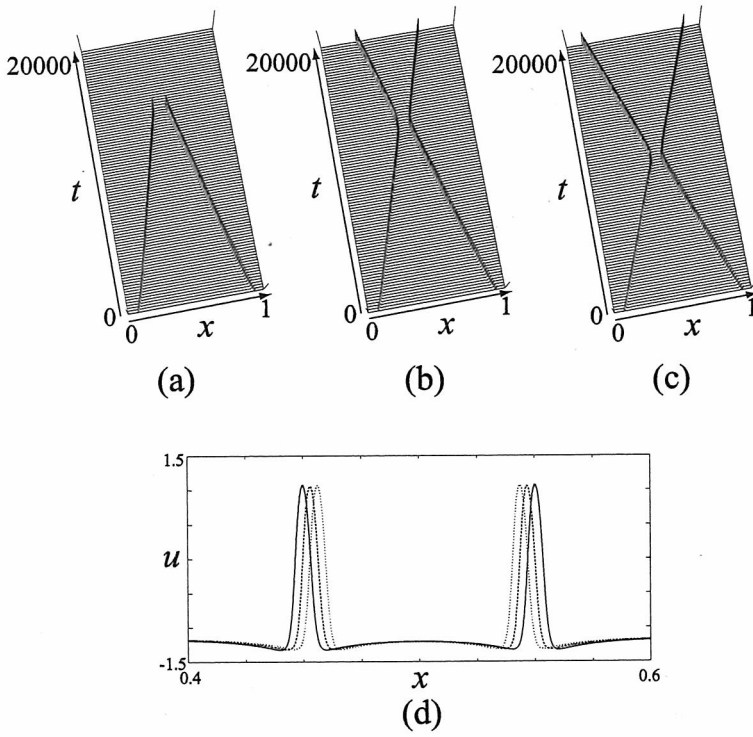


Figure 3: Scattering dynamics for $\tau = 1262.5$. (a) $k_4 = 2.956$. (b) $k_4 = 2.96$. (c) $k_4 = 2.97$. (d) Snapshots when the distance between two pulses attains minimum. Solid line: $t \approx 14700$ ((a)). Dashed line: $t \approx 13000$ ((b)). Dotted line: $t \approx 10480$ ((c)).

S1) There exist $\mathbf{k} = \tilde{\mathbf{k}} = (\tilde{k}_1, \tilde{k}_2)$ where pitchfork bifurcation and Hopf bifurcation occur, and a stationary pulse solution $S(x)$ of (2) such that $\mathcal{L}(S(x); \tilde{\mathbf{k}}) \equiv 0$ and $S(-x) = S(x)$.

Let $L = \mathcal{L}'(S(x); \tilde{\mathbf{k}})$ be the linearized operator of (2) with respect to $S(x)$. and Σ_c be the spectrum of L .

S2) Σ_c consists of $\Sigma_0 = \{0, i\omega_0, -i\omega_0\} (\omega_0 \in \mathbf{R}^+)$ and $\Sigma_1 \subset \{z \in \mathbf{C}; \text{Re} z < -\gamma_0\}$ for some positive constant γ_0 .

S3) ψ, ξ are eigenfuctions corresponding to pitchfork bifurcation and Hopf bifurcation respectively. $L\psi = -S_x$ and $L\xi = i\omega_0\xi$ are satisfied, and ξ is even while ψ is odd.

Let L^* be the adjoint operator of L . There exist ϕ^*, ψ^*, ξ^* such that $L^*\phi^* = 0$, $L^*\psi^* = -\phi^*$ and $L^*\xi^* = -i\omega_0\xi^*$.

By the normalizaion, $\psi, \xi, \phi^*, \psi^*, \xi^*$ are determined by

$$\langle \psi, S_x \rangle_{L^2} = 0, \langle S_x, \psi^* \rangle_{L^2} = 1, \langle \psi, \psi^* \rangle_{L^2} = 0, \langle \xi, \xi^* \rangle_{L^2} = 1.$$

We note that

$$\langle \psi, \phi^* \rangle_{L^2} = 1, \langle S_x, \phi^* \rangle_{L^2} = 0.$$

hold.

Let $E_1 = \text{span}\{P_x, \psi\}$, $E_2 = \text{span}\{\xi, \bar{\xi}\}$, and $E = E_1 \oplus E_2$. We introduce variables $q \in \mathbf{R}$ and $r \in \mathbf{C}$ and represent $\mathbf{u}$ for small parameters η_1 and η_2 as

$$\mathbf{u}(x, t) = S(x - p) + q\psi(x - p) + (r\xi(x - p) + c.c.) + \zeta(x - p) + \mathbf{w}. \quad (3)$$

where

$$\zeta = q^2\zeta_{2000} + (r^2\zeta_{0200} + qr\zeta_{1100} + c.c.) + |r|^2\zeta_{0110} + \eta_1\zeta_{0001} + \eta_2\zeta_{0002}$$

and c.c. means complex conjugate. ζ_{ijkl} are determined by

$$\begin{aligned} L\zeta_{2000} + \Pi_{2000} &= \alpha_{2000}\xi + \bar{\alpha}_{2000}\bar{\xi} \\ (L - 2i\omega_0 I)\zeta_{0200} + \Pi_{0200} &= \alpha_{0200}\xi + \bar{\alpha}_{0200}\bar{\xi} \\ (L - i\omega_0 I)\zeta_{1100} + \Pi_{1100} &= \alpha_{1100}\psi + \alpha'_{1100}\phi \\ L\zeta_{0110} + \Pi_{0110} &= \alpha_{0110}\xi + \bar{\alpha}_{0110}\bar{\xi} \\ L\zeta_{0001} + \Pi_{0001} &= \alpha_{0001}\xi + \bar{\alpha}_{0001}\bar{\xi} \\ L\zeta_{0002} + \Pi_{0002} &= \alpha_{0002}\xi + \bar{\alpha}_{0002}\bar{\xi} \end{aligned}$$

where

$$\begin{aligned} \Pi_{2000} &= \frac{1}{2}F'''(S)\psi^2 + \psi_x, \quad \Pi_{0200} = \frac{1}{2}F''(S)\xi^2, \quad \Pi_{1100} = F''(S)\psi \cdot \xi + \xi_x, \\ \Pi_{0110} &= F''(S)\xi \cdot \bar{\xi}, \quad \Pi_{0001} = g_1(S), \quad \Pi_{0002} = g_2(S). \end{aligned}$$

and α_{ijklm} satisfy the following equations:

$$\begin{aligned} \langle \Pi_{2000} - \alpha_{2000}\xi, \xi^* \rangle_{L^2} &= 0, \quad \langle \Pi_{0200} - \alpha_{0200}\xi, \xi^* \rangle_{L^2} = 0, \quad \langle \Pi_{1100} - \alpha_{1100}\psi, \phi^* \rangle_{L^2} = 0, \\ \langle \Pi_{1100} - \alpha'_{1100}\phi, \psi^* \rangle_{L^2} &= 0, \quad \langle \Pi_{0110} - \alpha_{0110}\xi, \xi^* \rangle_{L^2} = 0, \quad \langle \Pi_{0001} - \alpha_{0001}\xi, \xi^* \rangle_{L^2} = 0, \\ \langle \Pi_{0002} - \alpha_{0002}\xi, \xi^* \rangle_{L^2} &= 0, \quad \alpha_{0020} = \bar{\alpha}_{0200}, \quad \alpha_{1010} = \bar{\alpha}_{1010}, \quad \alpha_{1010} = \bar{\alpha}_{1100}. \end{aligned}$$

We assume the following property

S4) $w \in E^\perp$ with $\|w\|_{L^2}, \|w_x\|_{L^2} \leq O(q^3 + |r|^3 + |\eta|^{3/2})$.

Substituting (3) into (2), and taking the inner product of ψ^*, ϕ^*, ξ^* and both sides of (2), then we have

$$\begin{aligned}
 \dot{p} &= q + (m_{1100}qr + c.c.) + (m_{1200}qr^2 + c.c.) \\
 &\quad + m_{3000}q^3 + m_{0110}q|r|^2 + m_{1001}q\eta_1 + m_{1002}q\eta_2 + h.o.t. \\
 \dot{q} &= g_{1001}q\eta_1 + g_{1002}q\eta_2 + (g_{1100}qr + c.c.) \\
 &\quad + g_{3000}q^3 + (g_{1200}qr^2 + c.c.) + g_{1110}q|r|^2 + h.o.t. \\
 \dot{r} &= i\omega r + h_{0001}\eta_1 + h_{0002}\eta_2 + (h_{0101}r\eta_1 + c.c.) + (h_{0102}r\eta_2 + c.c.) \\
 &\quad + h_{2000}q^2 + (h_{0200}r^2 + c.c.) + h_{0110}|r|^2 + (h_{2100}q^2r + c.c.) \\
 &\quad + (h_{0300}r^3 + c.c.) + (h_{0210}r|r|^2 + c.c.) + h.o.t.
 \end{aligned} \tag{4}$$

where

$$\begin{aligned}
 m_{1100} &= -\alpha'_{1100}, \quad m_{1010} = -\alpha'_{1010}, \\
 m_{3000} &= -\langle F''(S)\psi \cdot \zeta_{2000} + \frac{1}{6}F'''(S)\psi^3 + \partial_x \zeta_{2000}, \psi^* \rangle_{L^2}, \\
 m_{1200} &= -\langle F''(S)\psi \cdot \zeta_{0200} + F''(S)\xi \cdot \zeta_{1100} + \frac{1}{2}F'''(S)\psi \cdot \xi^2 + \partial_x \zeta_{0200} - \alpha'_{1100}\xi_x, \psi^* \rangle_{L^2}, \\
 m_{1110} &= -\langle F''(S)\xi \cdot \zeta_{1010} + F''(S)\bar{\xi} \cdot \zeta_{1100} + F''(S)\psi \cdot \zeta_{0110} + F'''(S)\psi \cdot \xi \cdot \bar{\xi} \\
 &\quad + \partial_x \zeta_{0110} - \alpha'_{1100}\bar{\xi}_x - \alpha'_{1010}\xi_x, \psi^* \rangle_{L^2}, \\
 m_{1001} &= -\langle g'_1(S)\psi + \partial_x \zeta_{0001}, \psi^* \rangle_{L^2}, \\
 m_{1002} &= -\langle g'_2(S)\psi + \partial_x \zeta_{0002}, \psi^* \rangle_{L^2},
 \end{aligned}$$

$$\begin{aligned}
 g_{1100} &= \alpha_{1100}, \quad g_{1010} = \alpha_{1010}, \\
 g_{3000} &= \langle F''(S)\psi \cdot \zeta_{2000} + \frac{1}{6}F'''(S)\psi^3 + \partial_x \zeta_{2000}, \phi^* \rangle_{L^2}, \\
 g_{1200} &= \langle F''(S)\psi \cdot \zeta_{0200} + F''(S)\xi \cdot \zeta_{1100} + \frac{1}{2}F'''(S)\psi \cdot \xi^2 + \partial_x \zeta_{0200} - \alpha'_{1100}\xi_x, \phi^* \rangle_{L^2}, \\
 g_{1110} &= \langle F''(S)\xi \cdot \zeta_{1010} + F''(S)\bar{\xi} \cdot \zeta_{1100} + F''(S)\psi \cdot \zeta_{0110}, \phi^* \rangle_{L^2}, \\
 g_{1001} &= \langle g'_1(S)\psi + \partial_x \zeta_{0001}, \phi^* \rangle_{L^2}, \\
 g_{1002} &= \langle g'_2(S)\psi + \partial_x \zeta_{0002}, \phi^* \rangle_{L^2},
 \end{aligned}$$

$$\begin{aligned}
h_{2000} &= \alpha_{2000}, \quad h_{0200} = \alpha_{0200}, \quad h_{0020} = \alpha_{0020}, \\
h_{0110} &= \alpha_{0110}, \quad h_{0001} = \alpha_{0001}, \quad h_{0002} = \alpha_{0002}, \\
h_{2100} &= \langle F''(S)\psi \cdot \zeta_{1100} + F''(S)\xi \cdot \zeta_{2000} + \frac{1}{2}F'''(S)\psi^2 \cdot \xi + \partial_x \zeta_{1100} - \alpha'_{1100}\psi_x, \xi^* \rangle_{L^2}, \\
h_{0300} &= \langle F'''(S)\xi \cdot \zeta_{0200} + \frac{1}{6}F'''(S)\xi^3, \xi^* \rangle_{L^2}, \\
h_{0210} &= \langle F''(S)\xi \cdot \zeta_{0110} + F''(S)\bar{\xi} \cdot \zeta_{0200} + \frac{1}{2}F'''(S)\xi^2 \cdot \bar{\xi}, \xi^* \rangle_{L^2}, \\
h_{0101} &= \langle F''(S)\xi \cdot \zeta_{0001} + g'_1(S)\xi, \xi^* \rangle_{L^2}, \\
h_{0102} &= \langle F''(S)\xi \cdot \zeta_{0002} + g'_2(S)\xi, \xi^* \rangle_{L^2}.
\end{aligned}$$

We perform a smooth invertible transformation:

$$\begin{aligned}
v &= q + V_{1100}qr + V_{1010}q\bar{r} + V_{1200}qr^2 + V_{1020}q\bar{r}^2 \\
w &= r + W_{0001}\eta_1 + W_{0002}\eta_2 + W_{2000}q^2 + W_{0200}r^2 + W_{0020}\bar{r}^2 + W_{0110}|r|^2 \\
&\quad + W_{0011}\eta_1\bar{r} + W_{0012}\eta_2\bar{r} + W_{2010}q^2\bar{r} + W_{0120}\bar{r}|r|^2 + W_{0300}r^3 + W_{0030}\bar{r}^3
\end{aligned} \tag{5}$$

where

$$\begin{aligned}
V_{1100} &= -\frac{g_{1100}}{i\omega}, \quad V_{1010} = \frac{g_{1010}}{i\omega}, \\
V_{1200} &= -\frac{g_{1200} + g_{1100}V_{1100} + h_{0200}V_{1100} + h_{0200}V_{1010}}{2i\omega}, \\
V_{1020} &= \frac{g_{1020} + g_{1010}V_{1010} + h_{0020}V_{1100} + h_{0020}V_{1010}}{2i\omega}, \\
W_{0001} &= \frac{h_{0001}}{i\omega}, \quad W_{0002} = \frac{h_{0002}}{i\omega}, \\
W_{0011} &= \frac{h_{0011} + 2W_{0020}\bar{h}_{0001} + W_{0110}h_{0001}}{2i\omega}, \\
W_{0012} &= \frac{h_{0012} + 2W_{0020}\bar{h}_{0002} + W_{0110}h_{0002}}{2i\omega}, \\
W_{2000} &= \frac{h_{2000}}{i\omega}, \quad W_{0200} = -\frac{h_{0200}}{i\omega}, \quad W_{0020} = \frac{h_{0020}}{3i\omega}, \quad W_{0110} = \frac{h_{0110}}{i\omega}, \\
W_{2010} &= \frac{h_{2010} + 2W_{2000}g_{1010} + 2W_{0020}h_{2000} + W_{0110}h_{2000}}{2i\omega}, \\
W_{0120} &= \frac{h_{0120} + 2W_{0200}h_{0020} + 2W_{0020}\bar{h}_{0110} + W_{0110}h_{0110} + W_{0110}h_{0020}}{2i\omega}, \\
W_{0300} &= -\frac{h_{0300} + 2W_{0200}h_{0200} + W_{0110}h_{0200}}{2i\omega}, \\
W_{0030} &= \frac{h_{0030} + 2W_{0020}h_{0020} + W_{0110}h_{0020}}{4i\omega}
\end{aligned}$$

Substituting (5) into (4), then we have

$$\begin{aligned}
\dot{v} &= G_{1001}\eta_1v + G_{1002}\eta_2v + G_{3000}v^3 + G_{1110}v|w|^2 \\
\dot{w} &= i\omega w + H_{0101}\eta_1w + H_{0102}\eta_2w + H_{2100}v^2w + H_{0210}w|w|^2
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
 G_{1001} &= g_{1001} + V_{1100}h_{0001} + V_{1010}\bar{h}_{0001}, \\
 G_{1002} &= g_{1002} + V_{1100}h_{0002} + V_{1010}\bar{h}_{0002}, \\
 G_{3000} &= g_{3000} + V_{1100}h_{2000} + V_{1010}\bar{h}_{2000}, \\
 G_{1110} &= g_{1110} + V_{1100}g_{1010} + V_{1100}h_{0110} + V_{1010}g_{1100} + V_{1010}\bar{h}_{0110}, \\
 H_{0101} &= h_{0101} + 2W_{0200}h_{0001} + W_{0110}\bar{h}_{0001}, \\
 H_{0102} &= h_{0102} + 2W_{0200}h_{0002} + W_{0110}\bar{h}_{0002}, \\
 H_{2100} &= h_{2100} + 2W_{2000}g_{1100} + 2W_{0200}h_{2000} + W_{0110}\bar{h}_{2000}, \\
 H_{0210} &= h_{0210} + 2W_{0200}h_{0110} + 2W_{0020}h_{0200} + W_{0110}h_{0200} + W_{0110}\bar{h}_{0110}.
 \end{aligned}$$

Let $w = Ae^{i\varphi}(A, \varphi \in \mathbf{R})$. (6) can be written as

$$\begin{aligned}
 \dot{\varphi} &= \omega_0 + \mu'_2 + h.o.t. \\
 \dot{v} &= (-\mu_1 + p_{11}v^2 + p_{12}A^2)v + h.o.t. \\
 \dot{A} &= (-\mu_2 + p_{21}v^2 + p_{22}A^2)A + h.o.t.
 \end{aligned} \tag{7}$$

where $p_{11} = \text{Re}G_{3000}$, $p_{12} = \text{Re}G_{1110}$, $p_{21} = \text{Re}H_{2100}$, $p_{22} = \text{Re}H_{0210}$. $\mu_1 = -\text{Re}G_{1001}\eta_1 - \text{Re}G_{1002}\eta_2$, $\mu_2 = -\text{Re}H_{0101}\eta_1 - \text{Re}H_{0102}\eta_2$, $\mu'_2 = -\text{Im}H_{0101}\eta_1 - \text{Im}H_{0102}\eta_2$

We consider bifurcation diagram of truncated equations of (7) near PH-point. Since the second and third equations of (7) are independent of φ , the bifurcation diagram is determined by

$$\begin{aligned}
 \dot{v} &= (-\mu_1 + p_{11}v^2 + p_{12}A^2)v, \\
 \dot{A} &= (-\mu_2 + p_{21}v^2 + p_{22}A^2)A.
 \end{aligned} \tag{8}$$

We assume p_{ij} satisfy the following:

$$p_{12}/p_{22} > 0, p_{21}/p_{11} > 0, p_{12}p_{21}/p_{11}p_{22} < 1, p_{11} < 0. \tag{9}$$

Remark 3.1 (9) can be checked numerically for (1).

$(v, A) = (0, 0)$ is a trivial equilibrium point for all μ_1 and μ_2 . Equilibria

$$EP_1^{\pm} : (v, A) = (0, \pm\sqrt{\frac{\mu_2}{p_{22}}}).$$

bifurcate at the bifurcation line $\mu_1 = 0$ and

$$EP_2^{\pm} : (v, A) = (\pm\sqrt{\frac{\mu_1}{p_{11}}}, 0)$$

bifurcate at the bifurcation line $\mu_2 = 0$. There also exist equilibria

$$EP_3^{\pm, \pm} : (v, A) = \left(\pm\sqrt{\frac{-p_{12}\mu_2 + p_{22}\mu_1}{p_{11}p_{22} - p_{12}p_{21}}}, \pm\sqrt{\frac{-p_{21}\mu_1 + p_{11}\mu_2}{p_{11}p_{22} - p_{12}p_{21}}} \right).$$

$EP_3^{\pm, \pm}$ bifurcate at the line

$$T_1 = \{(\mu_1, \mu_2) | \mu_1 = \frac{p_{12}}{p_{22}}\mu_2, \mu_2 > 0\}$$

and

$$T_2 = \{(\mu_1, \mu_2) \mid \mu_2 = \frac{p_{21}}{p_{11}}\mu_1, \mu_1 < 0\}.$$

Since q is velocity of single pulse, solution with $q \neq 0$ ($v \neq 0$) is corresponding to traveling pulse. By changing (μ_1, μ_2) like $6 \rightarrow 1 \rightarrow 2$ ($6 \rightarrow 5 \rightarrow 4$) in Fig.4 we have bifurcation diagram as shown in Fig.5.

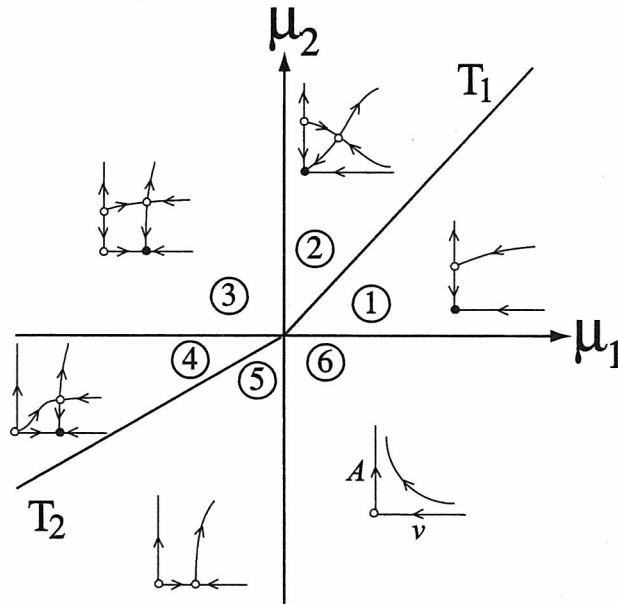


Figure 4: Phase portrait of (8) with (9) ([7]). There exist stable standing pulse solutions (stable traveling pulse solutions) for parameters in region 1 and region 2 (region 3 and region 4). The basin size of stable traveling pulse solution shrinks by taking parameters near T_2 . Loosely speaking the transition from region III to region IV in Fig.1 is caused by the shrinkage of the basin size.

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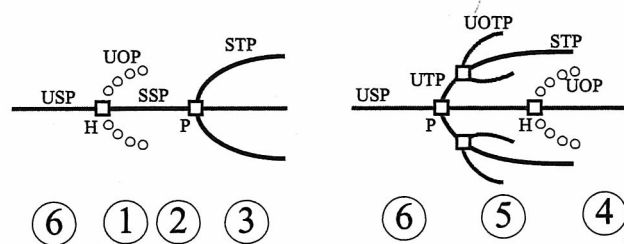


Figure 5: Schematic bifurcation diagram for (8). SSP: Stable standing pulse. USP: Unstable standing pulse. STP: Stable traveling pulse. UOP: Unstable oscillatory pulse. UOTP: Unstable oscillatory traveling pulse.

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