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A Study on Model Reference Adaptive Control in Economic Development (IV)

—Discrete Polynomic Non-Linear System—

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Abstract

We have investigated the adapsive process of the economic development.

Firstly, we have investigated the stability domain of propensity to consume and acceleration coefficient of economic development model.

Secondly, the recussive identification of dynamic parameters is conceived to be a useful method in specifying the regional income forecasting model.

Thirdly, the Model reference adaptive technique with discrete polynomic non-linear system is conceived to be a useful method in specifying the regional development model such as the Volterra-Lotka type.

Key Words: Model reference adaptive control. Stability domain, Regional income forecasting model. Discrete polynomic non-linear system, Volterra-Lotka type.

1. Introduction

In the previous papers,[1~4] we considered the main results of the economic development model by using Model reference adaptive techinique, and also with cenrtainty and uncertainty concerning the values of the adjustable model's parameters, the parametric identification and adaptive state observation, and Model reference adaptive I-O analysis.

In this paper, we shall consider some application models by using Model reference adaptive technique such as economic development model with parallel model reference adaptive system, regional income forecasting model and economic development model with discrete polynomic non-linear system.

2. Economic Development Model with Parallel Model Reference Adaptive System

Consider the following economic development model with a parallel model reference adaptive system.

(a) The developed regional economic model is given by

$$y_M(t) = (1+\beta)\, y_M(t-1) - \alpha\beta y_M(t+2) + g(t) \tag{1}$$

where

$y_M(t)$: regional output per regional resident at time t.
$g(t)$: government expenditure on goods and services per regional resident at time t.
α: propensity to consume of developed region.
β: acceleration coefficient of developed region.

(b) The developing regional economic model is given by

$$y_s(t)=(1+\hat{\beta})\,y_s(t-1)-\hat{\alpha}\hat{\beta}y_s(t-2)+g(t) \tag{2}$$

$y(t)$: regional output per regional resident at time t.
$g(t)$: government expenditure on goods and services per regional resident at time t.
$\hat{\alpha}$: propensity to consume of developing region.
$\hat{\beta}$: acceleration coefficient of developing region.

In order to track the developing regional economy to the developed regional economy, it is necessary to define the stability domain of α and β.

The generalized output difference.

$$\varepsilon^0(t)=y_M(t)-y_s^0(t) \tag{3}$$

$$\varepsilon(t)=y_M(t)-y_s(t) \tag{4}$$

The adaptation mechanism will contain a linear compensator generating a signal $\nu(t)$

$$\text{a prior} \qquad \nu^0(t)=\varepsilon^0(t)+\sum_{i=1}^{r}c_i\varepsilon(t-i) \tag{5}$$

$$\text{a posterior} \quad \nu(t)=\varepsilon(t)+\sum_{i=1}^{r}c_i\varepsilon(t-i) \tag{6}$$

The degree r and the coefficients c_i will be determined as part of the design.

The signal $\nu^0(t)$ will be used to implement the adaptation algorithms, which for this example will be chosen in the form.

$$\begin{aligned}\hat{\alpha}(t)\left(1+\hat{\beta}(t)\right)&=\hat{\alpha}(t-1)\left(1+\hat{\beta}(t-1)+\Gamma_{a_1}\left(\nu^0(t)\right)\right)\\&=\sum_{l=0}^{t}\Gamma_{a_1}\left(\nu^0(l)+\hat{\alpha}(-1)\left(1+\hat{\beta}(-1)\right)\right)\end{aligned} \tag{7}$$

$$\begin{aligned}-\hat{\alpha}(t)\,\hat{\beta}(t)&=-\hat{\alpha}(t-1)\,\hat{\beta}(t-1)+\Gamma_{a_2}\left(\nu^0(t)\right)\\&=\sum_{l=0}^{t}\Gamma_{a_2}\left(\nu^0(l)\right)-\hat{\alpha}(-1)\,\hat{\beta}(-1)\end{aligned} \tag{8}$$

In developing the design we shall use modified adaptation algorithms of the from.

$$\hat{\alpha}(t)\left(1+\hat{\beta}(t)\right)=\hat{\alpha}(t-1)\left(1+\hat{\beta}(t-1)\right)+\Gamma_{a_1}\left(\nu(t)\right) \tag{9}$$

$$-\hat{\alpha}(t)\,\hat{\beta}(t)=-\hat{\alpha}(t-1)\,\hat{\beta}(t-1)+\Gamma_{a_2}\left(\nu(t)\right) \tag{10}$$

First step

Subtracting Eq. (2) from (1) and also using Eq. (4), we obtain

$$\varepsilon(t)=\alpha(1+\beta)\,\varepsilon(t-1)-\alpha\beta\varepsilon(t-2)+\Big[\alpha(1+\beta)-\hat{\alpha}(t)\,\big(1+\hat{\beta}(t)\Big]\,y_s(t-1)$$
$$+\Big[-\alpha\beta+\hat{\alpha}(t)\,\hat{\beta}(t)\Big]\,y_s(t-2) \tag{11}$$

Also using Eqs. (6), (7) and (8), one abtains the following equivalent feedback system :

$$\varepsilon(t)=\alpha(1+\beta)\,\varepsilon(t-1)-\alpha\beta\varepsilon(t-2)+w_1(t) \tag{12}$$

$$\nu(t)=\varepsilon(t)+\sum_{i=1}^{r}c_i\,\varepsilon(t-i) \tag{13}$$

$$m(t)=-w_1(t)=\Big[\sum_{l=0}^{t}\Gamma'_{a_1}\big(\nu(l)\big)+\hat{\alpha}(-1)\,\big(1+\hat{\beta}(-1)\big)-\alpha(1+\beta)\Big]\,y_s(t-1)$$
$$+\Big[\sum_{l=0}^{t}\Gamma'_{a_2}\big(\nu(l)\big)-\hat{\alpha}(-1)\,\hat{\beta}(-1)+\alpha\beta\Big]\,y_s(t-2)$$
$$+\Big[\sum_{l=0}^{t}\Gamma'_{b_1}\big(\nu(l)\big)+P_1(-1)-1\Big]\,g(t) \tag{14}$$

Where Eqs. (12) and (13) defined a linear time invariant feedforward block and Eq (14) defines a nonlinear time varying feedback block.

Second step

To be able to apply the hyperstability theorem to the equivalent feedback system of Eqs. (12) to (14), one should first determine $\Gamma'_{a_i}(\nu(l))$ and $\Gamma'_{b_1}(\nu(l))$ such that the equivalent feedback block defined by Eq. (14)

$$\eta(0,t_1)=\sum_{t=0}^{t_1}\nu(t)\,w(t)\geq\gamma_0^2 \tag{14}$$

By using Eq. (14) the inequlity of Eq. (15) becomes.

$$\eta(0,t_1)=\sum_{t=0}^{t_1}\nu(t)\,y_s(t-1)\Big[\sum_{l=0}^{t}\Gamma_{a_1}\big(\nu(l)\big)+\hat{\alpha}(-1)\,\big(1+\hat{\beta}(-1)\big)-\alpha(1+\beta)\Big]$$
$$+\sum_{t=0}^{t_1}\nu(t)\,y_s(t-2)\Big[\sum_{l=0}^{t}\Gamma_{a_2}\big(\nu(l)\big)-\hat{\alpha}(-1)\,\hat{\beta}(-1)+\alpha\beta\Big]$$
$$+\sum_{t=0}^{t_1}\nu(t)\,g(t)\Big[\sum_{l=0}^{t}\Gamma_{b_1}\big(\nu(l)\big)\Big]\geq-r \tag{16}$$

One abtains the following particular sulution and :

$$\Gamma'_{a_i}\big(\nu(t)\big)=\alpha_i\,\nu(t)\,y_s(t-i)\qquad \alpha_i>0\quad i=1,2 \tag{17}$$

$$\Gamma'_{b_1}\big(\nu(t)\big)=\beta_1\,\nu(t)\,u(t)\qquad \beta_1>0 \tag{18}$$

Third step

The discrete transfer function of the equivalent feed forward block defined by Eqs. (12) and (13), which is,

$$h(z)=\frac{1+\sum_{i=1}^{r} c_i z^{-1}}{1-\alpha(1+\beta)\, z^{-1}+\alpha\beta z^{-2}} \tag{19}$$

should be strictly positive real.

So that the transfer function of Eq. (19) will be strictly positive real.

The stability domain is defined by the inequlities.

$$\begin{aligned} &1+\alpha+2\alpha\beta>0 \\ &1-\alpha>0 \\ &1-\alpha\beta>0 \end{aligned} \tag{20}$$

Applying the transformation $z(1+s)/(1-s)$ to Eq. (19) with $r=0$ (i. e., $c_i=0$)

$$h'(s)=\frac{1+2s+s^2}{(1-\alpha)+s(2-2\alpha\beta)+s(1+\alpha+2\alpha\beta)} \tag{21}$$

and the real part of $h'(s)$ for $s=jw$ is given by,

$$\mathrm{Re}\left\{h(jw)\right\}=\frac{(1-\alpha)+2(1-3\alpha\beta)\, w^2+(1+\alpha+2\alpha\beta)\, w^4}{[1-\alpha-w^2(1+\alpha+2\alpha\beta)]+4w^2(1-\alpha\beta)} \tag{22}$$

The real part of $h(jw)$ will be atrictly positive real for any real w, and therefore $h(s)$ will be atrictly positive real if, in addition to the condition of Eq. (19), it satisties one of the following two conditions.

$$\begin{aligned} &1-3\alpha\beta>0 \\ &(1-3\alpha\beta)^2-(1+\alpha+2\alpha\beta)(1-\alpha)<0 \end{aligned} \tag{23}$$

3. Regional Income Forecasting Model

We apply an Model Reference Adaptive identifier to the regional income forecasting model.

There exists two important keys to the forecasting success of the MRSA identifier; the suitable construction of regional income model and the proper use of an MRAS identifier.

Next, we shall consider the following regional income model with-input and single output parallel model reference adaptive system.

The regional income model with reference model.

$$\begin{aligned} y_P(t) &= \sum_{i=1}^{n} a_i y_P(t-i)+\sum_{i=0}^{m} b_i u(t-i) \\ &= P^T\phi(t-1) \end{aligned} \tag{24}$$

where

$$P^T=[a_1, a_2, \cdots, a_n, b_1, b_2, \cdots, b_m] \tag{25}$$

$$\phi^T(t-1)=\left[y_P(t-1), \cdots, y_P(t-n), u(t), \cdots, u(t-m)\right] \tag{26}$$

$y_P(t)$: the regional income at the instant t.

$u(t)$: the regional investment at the instant t.

P : the parameter vector.

We shall present the algorithm for the estimation of the regional income at the instant $t+1$ based on the regional income at the instant t and the regional investment at the instant $t+1$.

Now, we have the parameter vector P and the parameter vector C of adjustable compensator such as:

$$\hat{\theta}(t-1)=\left[\hat{p}(t-1)^T-\hat{c}(t-1)^T\right]^T \tag{27}$$

$$\hat{p}(t-1)=\left[\hat{a}_1(t-1), \cdots, \hat{a}_n(t-1), \hat{b}_0(t-1), \cdots, \hat{b}_m(t-1)\right] \tag{28}$$

$$\hat{c}(t-1)^T=\left[\hat{c}_1(t-1), \cdots, \hat{c}_n(t-1)\right] \tag{29}$$

$$F(t-1)>0$$

$$u(t-i):\ i=1, 2, \cdots\cdots m$$

$$y_p(t-i):\ i=1, 2, \cdots\cdots n$$

$$\varepsilon(t-i):\ i=1, 2, \cdots\cdots n$$

Firstly, we calculate the following vector with dimention $2n+m+1$ such as:

$$\phi(t)=\left[\psi(t)^T\cdot e(t-1)^T\right]^T \tag{30}$$

$$\psi(t)^T=\left[\hat{y}_p(t-1), \cdots, \hat{y}_p(t-n), u(t), \cdots, u(t-m)\right] \tag{31}$$

$$\varepsilon(t-1)^T=\left[\varepsilon(t-1), \cdots, \varepsilon(t-n)\right] \tag{32}$$

Secondly, we calculate the following output of adjustable model for generating a prior signal $\nu^0(t)$.

$$\hat{y}_p(t)=\hat{p}(t-1)^T\cdot\psi(t) \tag{33}$$

$$\varepsilon^0(t)=y_p(t)-\hat{y}_p^0(t) \tag{34}$$

From the equations (29), (32), (34), we abtain the following prior signal.

$$\nu^0(t)=\varepsilon^0(t)+\hat{c}(t-1)^T\cdot\varepsilon(t-1) \tag{35}$$

The estimation parameter $\hat{\theta}(t)$ and adaptive gain $F(t)$ are as follows:

$$\begin{aligned}\hat{\theta}(t)&=\left[\hat{p}(t)-\hat{c}(t)^T\right]^T\\&=\hat{\theta}(t-1)+\frac{F(t-1)\cdot\phi(t)\,\nu^0(t)}{1+\phi(t)^T\cdot F(t-1)\cdot\phi(t)}\end{aligned} \tag{36}$$

$$F(t)=\frac{1}{\lambda}\left\{F(t-1)-\frac{\mu F(t-1)\cdot\phi(t)\cdot\phi(t)^T\cdot F(t-1)}{\lambda+\mu\cdot\phi(t)\cdot F(t-1)\cdot\phi(t)}\right\} \tag{37}$$

Where, the conditions of λ and μ satisfied the stable of MRAS identifier are as follows:

$$0\leq\lambda\leq 1 \qquad 0\leq\mu\leq 2$$

Thirdly, we obtain the regional income of adjustable model.

$$y_p(t) = \hat{p}(t)^T \cdot \phi(t) \tag{38}$$

$$\varepsilon(t) = y_p(t) - \hat{y}_p(t) \tag{39}$$

At last, we abtain the regional income at instant $t+1$ based on the following vector $\hat{\phi}(t+1/t)$ at dimention $n+m+1$ based on the parameter $\hat{p}(t)$ and $\hat{u}(t+1/t)$.

$$\hat{\phi}(t+1/t) = \left[y_p(t), \cdots, y_p(t+1-n)\ \hat{u}(t+1/t), u(t), \cdots, u(t+1-m)\right]^T \tag{40}$$

$$\hat{y}_p(t+1/t) = \hat{p}(t)^T \cdot \hat{\phi}(t+1/t) \tag{41}$$

The flow chart of the algorithm represents in Figure 1.

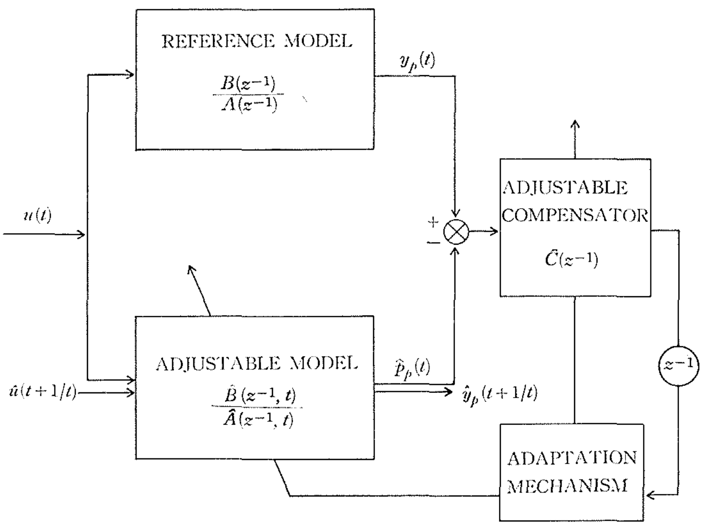

Figure 1. The flow chart of the algorithm.

4. Discrete Polynomic Non-linear System of Economic Development

In this chapter, we shall consider the model reference adaptive control scheme for multi-input multi-output discrete polynomic non-linear systems with input and output time delays. and present the regional development model such as the Votlerra-Lotka type.

Reference Model

$$\begin{aligned} x_M(t+1) = \sum_{j=1}^{n} \sum_{i=0}^{s} \Big\{ & A_{[i]j}\Big(\hat{a}(t-l_u)\Big)\, x_M^{[i]}(t+1-j) \\ & + \sum_{q=1}^{n} B_{[i]q,j}\Big(\hat{a}(t-l_u)\Big)\, x_M^{[i]}(t+1-j)\, \gamma_q(t+1-j)\Big\} + \hat{d}(t-l_u) \\ & + d(t-l_u) \end{aligned} \tag{42}$$

$$y_M(t) = L_y\, x_M(t)$$

where, y_M: output vector of reference model
x_M: state vector of reference model
$\hat{a}(t-l_u)$: estimated value of unknown parameter
$\hat{d}(t-l_u)$: assistant input

$$\gamma(t) = L_u u_M(t)$$

Adjustable Model

$$x_s(t+1) = \sum_{j=1}^{n} \sum_{i=0}^{s} \Big\{ A_{[i]j}(a)\, x_s^{[i]}(t+1-j)$$
$$+ \sum_{q=1}^{n} B_{[i]q,j}(a)\, x_s^{[i]}(t+1-j)\, \gamma_q(t+1-j) \Big\} \tag{43}$$
$$y_s(t) = L_y x_s(t)$$

where, x_s : state vector of adjustable model
y_s : output vector of adjustable model
a : vector of unknown parameter (r−dimentional)
$\gamma(t) = L_u u_s(t)$

$$L_u \triangleq \begin{bmatrix} z-l_{u_1} & & 0 \\ & \ddots & \\ 0 & & z-l_{u_m} \end{bmatrix} \qquad L_y \triangleq \begin{bmatrix} z-l_{y_1} & & 0 \\ & \ddots & \\ 0 & & z-l_{y_m} \end{bmatrix} \tag{44}$$
$$l_u \triangleq \max_i \{l_{u_i}\} \qquad l_y \triangleq \max_i \{l_{y_i}\}$$

A is satisfyed the following equations.

$$A_{[i]j}(a) = {}_0A_{[i]j} + \sum_{p=1}^{r} {}_pA_{[i]j}\, a_p \begin{pmatrix} i = 1, 2, \cdots, s \\ j = 1, 2, \cdots, n \end{pmatrix} \tag{45}$$

$$B_{[i]q,j}(a) = {}_0B_{[i]q,j} + \sum_{p=1}^{r} B_{[i]q,j}\, a_p \begin{pmatrix} i = 1, 2, \cdots, s \\ j = 1, 2, \cdots, n \\ q = 1, 2, \cdots, m \end{pmatrix} \tag{46}$$

$U(t)$ of the equation (42) is given the following equation by the existent assumption

$$u(t) = L_u^{-1} \Big[B^{-1}(t-l_u) \Big\{ x_M(t+1) - \sum_{i=0}^{s} \Big(\sum_{j=1}^{n} A_{[i]j}\big(\hat{a}(t-l_u)\big)\, x_M^{[i]}(t+1-j) \Big)$$
$$+ \sum_{j=2}^{n} \sum_{q=1}^{m} B_{[i]q,j}\big(\hat{a}(t-l_u)\big)\, x_M^{[i]}(t+1-j)\, \gamma_q\big((t+1-j)\big) - \hat{d}(t-l_u) \Big\} \Big] \tag{47}$$

where,

$$B(t) \triangleq B\big(\hat{a}(t),\, x_m(t+l_u)\big) = [b_1, \cdots, b_q, \cdots, b_m]$$
$$b_q(t) \triangleq \sum_{i=0}^{s} B_{[i]q,j}\big(\hat{a}(t)\big)\, x_m^{[i]}(t+l_u) \tag{48}$$

The equations (42), (43) are rewrited by the following signal vector Z_t.

$$L_y^{-1} y_M(t+1) = Z_{0,t}\big(\boldsymbol{y}_M(t+l_y),\, \boldsymbol{v}_M(t)\big)$$
$$+ Z_t\big(\boldsymbol{y}_M(t+l_y),\, \boldsymbol{v}_M(t)\big)\, \hat{a}(t-l_u)$$
$$+ \hat{d}(t-l_u) \tag{49}$$

$$L_y^{-1}y_s(t+1)=Z_{0t}\Big(\boldsymbol{y}_s(t+l_y),\,\boldsymbol{v}_s(t)\Big)$$
$$+Z_t\Big(\boldsymbol{y}_s(t+l_y),\,\boldsymbol{v}_s(t)\Big)\,a \tag{50}$$

where,

$$Z_t\triangleq Z_t\Big(\boldsymbol{y}_s(t+l_y),\,\boldsymbol{v}_s(t)\Big)=[Z_{1,t},\,Z_{2,t},\,\cdots,\,Z_{r,t}] \tag{51}$$

$$Z_{p,t}\Big(\boldsymbol{y}_s(t+l_y),\,\boldsymbol{v}_s(t)\Big)$$
$$=\sum_{j=1}^{n}\sum_{i=0}^{s}\Big\{{}_pA_{[i]j}\Big(L_y^{-1}y_s(t+1-j)^{[i]}$$
$$+\sum{}_pB_{[i]q,j}\Big(L_y^{-1}y_s(t+1-j)\Big)^{[i]}L_u u_q(t+1-j)\Big\} \tag{52}$$
$$(p=0,\,1,\,\cdots,\,r)$$

$$\boldsymbol{y}_M^T(t)=\Big[y_M^T(t),\,\cdots,\,y_M^T(t+1-n)\Big] \tag{53}$$

$$\boldsymbol{v}_S^T(t)=\Big[u_S^T(t-l_u),\,\cdots,\,u_S^T(t+1-n)\Big] \tag{54}$$

$$\boldsymbol{y}_S^T(t)=\Big[y_S^T(t),\,\cdots,\,y_S^T(t+1-n)\Big] \tag{55}$$

The output difference is represented as follows.

$$e(t)=y_M(t)-y_s(t) \tag{56}$$

The output generalized difference is defined as follows.

$$\varepsilon(t+1)=Z_{t-l_y}\Big(\boldsymbol{y}_s(t),\,\boldsymbol{v}_s(t-l_y)\Big)\,\phi(t) \tag{57}$$

where,

$$\phi(t)=\hat{a}(t)-a \tag{58}$$

The adaptation law is represented as follows.

$$\hat{a}(t+1)=\hat{a}(t)-\sigma_t P Z_{t-l_y}^T\,\varepsilon(t+1) \tag{59}$$

$$\sigma_t=\frac{\mu_t}{||H_t-\lambda_y||+r_t} \tag{60}$$

where,

$$H_t\triangleq Z_tPZ_t^T\,,\qquad P=P^T>0\,,$$
$$0<\mu<\mu_t<2-\mu\,,\qquad r_t>r>0 \tag{61}$$

The adaptive current controller is represented as follows.

$$\hat{e}^{(i)}(h+1)=\hat{\xi}(h-l_u)=\hat{d}(h-l_u)-\hat{d}(h-l_u)$$
$$=\hat{d}(h-l_u)+\sum_{j=1}^{n}\sum_{i=0}^{s}v_{[i]j}(h-l_u)\,\hat{e}^{[i]}(h+1-j) \tag{62}$$

$$\hat{e}^{(i)}(h+1)=G_{[i,1]}\Big(x_m(h+1),\,x_m(h+1)-\hat{e}^{(i)}(h+1)\Big)\,\hat{e}^{(i)}(h+1) \tag{63}$$
$$i=2,\,\cdots,\,s$$

We can get $\hat{e}^{(i)}(t+l_u)$ $(i=1, 2, \cdots, s)$ by using the adaptive current controller repeated l_u+l_y times, from $h=t-l_y$.

Then, we shall consider the following potential economic growth model with the concrete polynomic non-linear system analysis.

The developed country is depend on the petroleum produced in the developing country. The developing country is only produced the petroleum.

The developed country model:

$$y_M(t+1)=\left(1+\varepsilon_M-\lambda_M y_s(t)\right) y_M(t)+u_M(t-1) \tag{64}$$

y_M: the national output of the developed country.

ε_M: the potential economic growth rate such as

$$\left(y_M(t+1)-y_M(t)\right)/y_M(t) \tag{65}$$

λ_M: the rate of contribution to the economic growth of the developed country by the import of petroleum.

The developing country model

$$y_s(t+1)=\left(1-\varepsilon_s+\lambda_s y_s(t)\right) y_s(t)+u_s(t) \tag{66}$$

y_s: the national output of developing country.

ε_s: the potential economic growth rate such as

$$\left(y_s(t+1)-y_s(t)\right)/y_s(t)=-\varepsilon_s\,.$$

and the meaning of minus is represented by poor technology power as the developed controry does not buy the petroleum.

λ_s: the rate of contribution to the economic growth of the developing country by the export of petroleum.

the input $u_M(t)$, $u_s(t)$ is represented as follows.

$$\begin{aligned} u_M(t)=&\left(M_M(t+2)-M_M(t+1)-\hat{d}_M(t)\right)\\ &-\left(\varepsilon_M-\hat{\lambda}_M(t)\ M_s(t+1)\right) M_M(t+1) \end{aligned} \tag{67}$$

$$\begin{aligned} U_s(t)=&\left(M_s(t+1)-M_s(t)-\hat{d}_s(t-1)\right)\\ &+\left(\varepsilon_s-\hat{\lambda}_s(t-1)\ M_M(t)\right) M_s(t) \end{aligned} \tag{68}$$

where,

$$\begin{aligned} M_M(t+1)=&\left(1+\varepsilon_M-\hat{\lambda}_M(t-1)\ M_s(t)\right) M_M(t)\\ &+u_M(t-1)+\hat{d}_M(t-1) \end{aligned} \tag{69}$$

$$\begin{aligned} M_s(t+1)=&\left(1-\varepsilon_s+\hat{\lambda}_s(t-1)\ M_M(t)\right) M_s(t)\\ &+u_s(t)+\hat{d}_s(t-1) \end{aligned} \tag{70}$$

The state difference is represented as follows:

$$E_i^{(1)}(t) = M_i(t) - y_i(t) \tag{71}$$

The parameter difference is represented as follows:

$$\phi_i(t) = \hat{\lambda}_i(t) - \lambda_i \qquad (i = M, S) \tag{72}$$

The following equations $E_M^{(1)}(t-1)$,

$E_s^{(1)}(t-1)$ is obtained by the equation (64), (65) substitute to the equations (69), (70).

$$E_M^1(t-1) = -\phi_M(t)\, y_s(t-2)\, y_M(t-2) + \phi_M(t) + \xi_M(t-3) \tag{73}$$

$$E_s^1(t-1) = \phi_s(t)\, y_M(t-2)\, y_s(t-2) + \phi_s(t) + \xi_s(t-3) \tag{74}$$

where $\xi_i(t) = \hat{d}_i(t) - \hat{d}_i(t) \qquad (i = M, S)$

$$d_M(t-3)\left(E^{(1)}(t-2), E^{(1)}(t-2)\right)$$
$$= -(1+\varepsilon_M)\, E_M^{(1)}(t-2) + \hat{\lambda}_M(t-3)\, E_s^2(t-2) \tag{75}$$

$$d_s(t-3)\left(E^{(1)}(t-2), E^{(2)}(t-2)\right)$$
$$= -(1-\varepsilon_s)\, E_s^{(1)}(t-2) - \hat{\lambda}_s(t-3)\, E_s^{(2)}(t-2) \tag{76}$$

$$E_s^2(t-2) = \left\{G_{[2,1]}\left(M(t-2), M(t-2) - E(t-2)\right) E^{(1)}(t-2)\right\}^2 \tag{77}$$

$$\phi_M(t) = -\left(\hat{\lambda}_M(t-3) - \hat{\lambda}_M(t)\right) y_s(t-2)\, y_M(t-2) \tag{78}$$

$$\phi_s(t) = \left(\hat{\lambda}_s(t-3) - \hat{\lambda}_s(t)\right) y_M(t-2)\, y_s(t-2) \tag{79}$$

The adaptive current controller is represented as follows:

$$\hat{E}^{(1)}(h+1) = \hat{d}(h-1) - d(h-1)\left(\hat{E}^{(1)}(h), \hat{E}^{(2)}(h)\right) \tag{80}$$

$$\hat{E}^{(2)}(2h+1) = \left\{G_{[2,1]}\left(M(h+1), M(h+1) - \hat{E}(h+1)\right) \hat{E}^{(1)}(h+1)\right\}^2 \tag{81}$$

where

$$\hat{d}(t) = d(t)\left(\hat{E}^{(1)}(t+1), \hat{E}^2(t+1)\right) \tag{82}$$

The parameter estivation equations are represented as follows:

$$\hat{\lambda}_M(t+1) = \hat{\lambda}_M(t) + \alpha_t y_s(t-2)\, y_M(t-2)\, \varepsilon_M(t+1) \tag{83}$$

$$\hat{\lambda}_s(t+1) = \hat{\lambda}_s(t) - \alpha_t y_M(t-2)\, y_s(t-2)\, \varepsilon_s(t+1) \tag{84}$$

where $\varepsilon(t+1) = E^1(t-1) - \phi(t) - \xi(t-3)$ (85)

5. Conclusion

We have innestigated the adaptive process of some economic development model by using model reference adaptive technique with parallel system, parametric identification and discrete polynomic non-linear system.

The main results are as follows.
(1) The parallel model reference adaptive technique is conceived to be a useful method in specifying the stability domain of propensity to consume and acceleration coefficient of economic development model.
(2) The recussive identification of dynamic parameters is conceived to be a useful method in specifying the regional income forecasting model.
(3) The model reference adaptive technique with discrete polynomic non-linear system is conceived to be a useful method in specifying the regional development model such so the Volterra-Lotka type.

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