HOKKAIDO UNIVERSITY

Title	Analysis of rectangular building frames by the mechanical tabulation method
Author(s)	Takabeya, Fukuhei
Citation	Memoirs of the Faculty of Engineering, Hokkaido Imperial University, 1, 155-191
Issue Date	1928
Doc URL	https://hdl.handle.net/2115/37667
Type	departmental bulletin paper
File Information	1_155-191.pdf

Hokkaido University Collection of Scholarly and Academic Papers : HUSCAP

Analysis of Rectangular Building Frames by the Mechanical Tabulation Method

By

Prof. **Fukuhei Takabeya,** *Kogakuhakushi.*

(Received March 15, 1927)

The proper treatment of frames with stiff connections, composed of rectangular elements, is of constantly increasing importance in structural design. When loads are applied to frames of stiff connections at the joints, the members of the structure are subjected to deformations accompanied by secondary stresses. For the analysis of these stresses there are several methods which are however so involved and so tedious that most building ordinances are accustomed to make certain assumptions for the determination of the redundant stresses.

For such structures it is highly desirable that the most convenient methods of analysing the stresses and rigorous calculation should be developed.

We are hopeful that an easy and accurate method for obtaining joint moments of framed structures in stiff connection will eliminate current assumptions and lead to a rigorous design with speed and precision, contradicting the prevailing impression that the analysis is very tedious.

In some problems of elastic beams the method of area moment may often facilitate the calculation of the statically indeterminate stresses. As its application to the investigations of the stresses in rectangular building frames and continuous beams the slope deflection method[1] proposed by Prof. WILSON may be recommended.

The merit of this method consists in the large reduction in the number of unknowns. The solution of the equations is very much simplified therefore.

This paper offers fundamental tables, derived from the slope deflection method, tabulated mechanically by the rules upon which our mechanical solution is based. These standard tables lead us to the solution of statically indeterminate stresses with strict accuracy.

The advantages of the new method are simplicity and easy understanding of the principle as well as superiority in speed.

Assumptions upon which the Analysis is based, and the Conventional Notation and Signs used.

The analysis in this bulletin is based upon the following assumptions:

1). The connections between the columns and girders are perfectly rigid.

2). The length of a member is not changed by direct stress, and the deformation of a member due to the internal shearing stress is zero.

3). The settlement of the foundations and temperature change are not considered.

4). The vertical deflections of the ends of all girders are relatively equal to zero and the horizontal deflections of the tops of all columns of a story are equal.[2]

---

1) The principles of the slope-deflection method were given firstly by O. MOHR and the equations of the slope-deflection for a member acted upon by forces and couples at the ends were deduced by Manderla in 1878.

The application of these equations to high buildings has been made by several writers:

W. M. WILSON and G. A. MANEY, "Wind Stresses in the Steel Frames of Office Buildings", University of Illinois Bulletin, No. 80, 1915.

W. M. WILSON, F. E. RICHART and CAMILLO WEISS, "Analysis of Statically Indeterminate Structures by the Slope Deflection Method", University of Illinois Bulletin, No. 108, 1918.

2) Provided the change of span length caused by bending is not considered, the proposed method may be applicable without assumption (4). Some special problems, in

According to these assumptions all the columns and girders, in strained state, which intersect at one point are subjected to an equal change in slope and the moment at an end of a column is expressed as a function of the changes in the slopes and of the deflection of one end of the column relative to the other end. Likewise the moment at an end of a girder is expressed as a function of the changes in the slopes of the ends of the girder. Generally if a member AB restrained at the ends in flexure is subjected to any system of intermediate loads, the moments at the ends of the member is expressed by the well known equations, vis.,

$$(1)\ \begin{cases} M_{ab}=2E\xi_{ab}\left\{2\varphi_a+\varphi_b-3\mu_{ab}\right\}-C_{ab}, \\ M_{ba}=2E\xi_{ab}\left\{2\varphi_b+\varphi_a-3\mu_{ab}\right\}+C_{ba}, \end{cases}$$

where we denote by

M_{ab}, M_{ba} the resisting moments at A and B,

E the modulus of elasticity of the material,

φ_a, φ_b the changes in the slope of the tangent to the elastic curve at A and B,

$\xi_{ab}=\frac{I}{l}$, in which I=moment of inertia of the section of the member AB; l=length of the member AB,

$\mu_{ab}=\frac{\delta}{l}$, in which δ=deflection of the end A from its initial position,

$C_{ab}=\frac{2A}{l^2}\left\{3\lambda-l\right\}$; $C_{ba}=\frac{2A}{l^2}\left\{2l-3\lambda\right\}$, in which A=area of the moment diagram of a simple beam AB due to the intermediate loads; λ=distance of the centroid of the area A from the end B.

The conventional signs of the quantities used in the equations in this bulletin are as follows:

---

which the change of span length forms an essential part in working stress, have been treated by the same writer. To make this point of view more clear we have dared to write this (4). This paper is: Zur Berechnung des beiderseits eingemauerten Trägers unter besonderer Berücksichtigung der Längskraft, Berlin, 1924.

The resisting moment on a section is considered positive when the resisting couple acts in a clockwise direction upon the portion of the member considered and by this conventional rule each resisting moment of the portion of the member considered must always be indicated with the subscripts e.g. M_{ab}, M_{ba}.

The moment of an external force is positive when it tends to cause a clockwise rotation.

The change in slope is considered positive when the tangent to the elastic line of a member has been turned clockwise, measured from its initial position. The deflection is considered always positive to the rotation in the clockwise direction from the initial position of member. Here we give the definition of " deflection " for the movement of one end of a member relative to the other, measured perpendicularly to the initial position of member.

We denote the intersections of the neutral axes of the columns with the surface of foundation on the same level by I, II, III, IV, etc., and the values of ξ of the columns of the first story by ξ_I, ξ_{II}, ξ_{III}, ξ_{IV}, etc., beginning at the left and reading toward the right.

The intersections of the neutral axes of the girders with the neutral axes of the columns are denoted by 1, 2, 3, 4, etc., beginning at the left and reading toward the right and upward from the last intersection at the top of the first story, then along the girders of the top of the second story toward the left. From the last intersection at the top of the second story read upward again and toward the right and so on (Fig. 1).

The values of ξ of girders and some special columns in the way of reading are denoted by subscript of the letter of the intersection from which the member considered begins in the direction as mentioned above (Fig. 1). The values of ξ of columns are denoted by ξ' with subscript of the letter of the intersection upon which the neutral axis of the column considered stands. e.g., ξ'_1, ξ'_2, ξ'_3, etc. (Fig. 2).

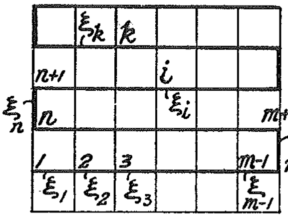


Fig. 1.

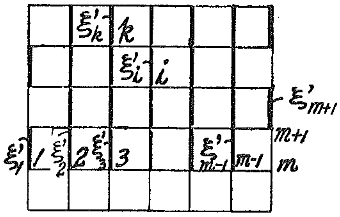


Fig. 2.

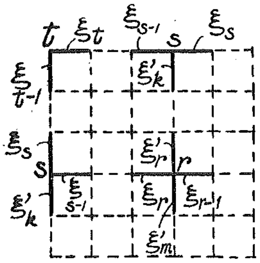


Fig. 3.

We denote by ρ_r two times the sum of ξ of all the members which intersect at r, e.g.: (Fig. 3).

$$(2)\quad \begin{cases} \rho_r = 2\left\{\xi_{r-1}+\xi_r+\xi'_r+\xi'_m\right\}, \\ \rho_s = 2\left\{\xi_{s-1}+\xi_s+\xi'_k\right\}, \\ \rho_t = 2\left\{\xi_{t-1}+\xi_t\right\}. \end{cases}$$

For all columns in a story we denote two times the sum of ξ by X e.g.:

$$(3)\quad \begin{cases} \text{For the first story:} \\ \quad X_1 = 2\left\{\xi_I+\xi_{II}+\xi_{III}+\dots\dots+\xi_M\right\} \\ \text{For the second story:} \\ \quad X_2 = 2\left\{\xi'_1+\xi'_2+\xi'_3+\dots\dots+\xi'_{m-1}+\xi_m\right\} \\ \text{For the third story:} \\ \quad X_3 = 2\left\{\xi'_{m+1}+\xi'_{m+2}+\dots\dots\xi'_{n-1}+\xi_n\right\} \\ \text{etc.} \end{cases}$$

We denote the load term C divided by $2E$ by p e.g.:

$$(4)\quad p_{r.\,r+1} = \frac{C_{r.\,r+1}}{2E};$$

the total horizontal shear due to wind loads in a bent at the first, second, third and nth stories by W_1, W_2, W_3, and W_n; and we put:

$$(5)\begin{cases} q_1=\dfrac{W_1h_1}{6E}, \\ q_2=\dfrac{W_2h_2}{6E}, \\ q_3=\dfrac{W_3h_3}{6E}, \\ \vdots \\ q_n=\dfrac{W_nh_n}{6E}. \end{cases}$$

where h_1, h_2, h_3, and h_n denote lengths of columns, in the first, second, third and nth stories, measured from neutral axis to neutral axis of the girders.

Condition of Equilibrium and Derivation of General Equations.

We treat in this bulletin a multiple storied bent with any number of spans. The stresses by wind loads and any system of vertical loads, in all of the members, are required to be calculated.

Since the moments at joints are expressed as a function of the slopes and the deflections of one end of the member relative to the other end, we firstly work out the slopes and the deflections by solving the equations obtained by such conditions of equilibrium as are explained in the following. With those values of the slopes and the deflections the moments can be then computed.

To obtain the equations from which the unknown slopes and deflections can be determined, we have two kinds of statical condition of equilibrium: one of which is of the equilibrium under the action of the moments at every point of intersection of the neutral axes of girders and columns, considering the point of intersection as a free body; the other kind of condition is of the equilibrium under the action of the moments at the two extremities of all the columns in a story, considering together all of the columns in a story as a free body.

From the former condition of equilibrium, each joint gives one equation;

from the latter each story gives one equation, i.e. for an n-storied bent with m-points of intersection of the neutral axes of girders and columns, the number of equations obtained, from which the slopes and the deflections can be determined, is $m+n$. The unknown quantities in the obtained equations are the changes in the slopes and the horizontal deflections of the columns. The changes in the slope at a joint, by the first assumption, are equal for all of the members which meet at the joint; and the horizontal deflections of the columns in a story are equal by the fourth assumption. We obtain therefore $m+n$ equations in which $m+n$ unknowns are contained. Solving these equations we can determine the slopes and the deflections, and then the moments required.

Single Storied Bent with any Number of Spans. Any System of Vertical Loads. Legs Fixed at the Bases.

Fig. 4 represents a single storied bent with any number of spans. All legs of the bent are fixed at I, II, III, ... *N-1*, and *N*. This bent carries any system of vertical loads. It is required to find the stresses in all of the members.

Representing the horizontal deflections of 1, 2, 3, ... *n-1* and *n* by δ_1 and $\mu_1=\dfrac{\delta_1}{h_1}$, we obtain from equation (1):

$$(6)\qquad M_{r.\,R}=2E\xi_R\left\{2\varphi_r-3\mu_1\right\},$$

$$(7)\qquad M_{r.\,r-1}=2E\xi_{r-1}\left\{2\varphi_r+\varphi_{r-1}\right\}+C_{r.\,r-1},$$

$$(8)\qquad M_{r.\,r+1}=2E\xi_r\left\{2\varphi_r+\varphi_{r+1}\right\}-C_{r.\,r+1},$$

$$(9)\qquad M_{R.\,r}=2E\xi_R\left\{\varphi_r-3\mu_1\right\}.$$

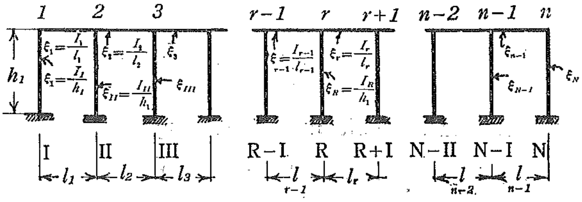


Fig. 4.

For the condition of equilibrium at joints:

$$(10)\quad M_{1I}+M_{12}=o,$$

$$(11)\quad \left\{\begin{array}{l} M_{21}+M_{2II}+M_{23}=o, \\ \cdot \\ \cdot \\ \cdot \\ M_{r,\,r-1}+M_{r,\,R}+M_{r,\,r+1}=o, \\ \cdot \\ \cdot \\ \cdot \\ M_{n-1,\,n-2}+M_{n-1,\,N-I}+M_{n-1,\,n}=o, \end{array}\right.$$

$$(12)\quad M_{n,\,n-1}+M_{n,\,N}=o.$$

For the condition of equilibrium for the bent as a whole:

$$M_{1,\,I}+M_{I,\,1}+M_{2,\,II}+M_{II,\,2}+\ldots M_{r,\,R}+M_{R,\,r}+\ldots+M_{n,\,N}+M_{N,\,n}=o,$$

or

$$(13)\quad \sum_{\binom{r=1}{R=I}}^{\binom{r=n}{R=N}}\left\{M_{r,\,R}+M_{R,\,r}\right\}=o.$$

The expressions of the moments M_{12}, M_{23}, M_{34},......$M_{n-2,\,n-1}$, and $M_{n-1,\,n}$ are obtained from equation (8); M_{21}, M_{32}, M_{43},......$M_{n-1,\,n-2}$, and $M_{n,\,n-1}$ from equation (7); M_{1I}, M_{2II}, M_{3III},......$M_{n-1,\,N-I}$ and $M_{n,\,N}$ from equation (6) and M_{I1}, M_{II2}, M_{III3},......$M_{N-I,\,n-1}$, and $M_{N,\,n}$ from equation (9).

Substituting the expressions of these moments in equations (10), (11), (12), and (13) gives the equations of Table I. Since we obtain $n+I$ equations from the Table we can determine therefrom $n+I$ unknowns, φ_1, φ_2, φ_3,......φ_n and μ_1.

Single Storied Bent with any Number of Spans, Carrying a Concentrated Horizontal Load at the Top. Legs Fixed at the Bases.

All legs of the same bent as shown in Fig. 4 are fixed at I, II, III,......$N-I$, and N. The bent carries a concentrated horizontal load ω_1 at the top i.e. at 1.

Since horizontal deflections of all the top joints are equal, we represent them by δ_1 and $\mu_1=\frac{\delta_1}{h_1}$.

For the expressions of $M_{r.\,R}$ and $M_{R.\,r}$, equations (6) and (9) are applicable. For $M_{r.\,r-1}$ and $M_{r.\,r+1}$, putting $C=o$ into equations (7) and (8) gives:

$$(14)\begin{cases} M_{r.\,r-1}=2E\xi_{r-1}\left\{2\varphi_r+\varphi_{r-1}\right\}, \\ M_{r.\,r+1}=2E\xi_r\left\{2\varphi_r+\varphi_{r+1}\right\}. \end{cases}$$

For equilibrium at 1, 2, 3, 4,……$n-1$, and n, equations (10), (11), and (12) are applicable.

For the bent as a whole to be in equilibrium

$$(15)\quad \sum_{\binom{r=1}{R=I}}^{\binom{r=n}{R=N}}\left\{M_{r.\,R}+M_{R.\,r}\right\}+\omega_1 h_1=o.$$

The expressions of moments M_{12}, M_{21}, M_{23}, M_{32},……$M_{n-1,\,n}$, and $M_{n,\,n-1}$ are obtained from equation (14); and M_{1I}, M_{I1}, M_{2II}, M_{II2},……$M_{n,\,N}$, and $M_{N,\,n}$ from equations (6) and (9).

Substituting the expressions of these moments in equations (10), (11), (12), and (15) gives the equations of Table II, from which we can determine φ and μ.

Single Storied Bent with any Number of Spans, Carrying any System of Vertical Loads and a Concentrated Horizontal Load at the Top. Legs Fixed at the Bases.

Under the law of superposition of stress and deformation, the previous two cases are applicable here. We represent the horizontal deflection of 1, 2, 3, 4,……$n-1$ and n by δ_1 and $\mu_1=\frac{\delta_1}{h_1}$.

For the expressions of $M_{r.\,R}$, $M_{R.\,r}$, $M_{r.\,r-1}$, and $M_{r.\,r+1}$, equations (6), (9), (7), and (8) are applicable.

For equilibrium at 1, 2, 3, 4,......$n-1$, and n, equations (10), (11), and (12) are applicable.

For the bent as a whole to be in equilibrium, equation (15) is applicable.

Table III gives $n+1$ equations from which $n+1$ unknowns φ_1, φ_2, φ_3,......φ_n and μ_1 can be determined.

Double Storied Rectangular Building Frame of any Number of Spans. Legs Fixed at the Bases.

Table IV gives the general equations to be used in determining the slopes and the deflections in a double storied rectangular building frame, whose legs are fixed at the bases, with any number of spans, carrying any system of vertical loads.

In Writing equation (1) of Table IV, which is

$$\rho_1\varphi_1+\xi_1\varphi_2+\xi'_1\varphi_n-3\xi_I\mu_1-3\xi'_1\mu_2=p_{12},$$

ρ_1, the coefficient of φ_1, is placed in the column under φ_1; ξ_1, the coefficient of φ_2, is placed under φ_2; ξ'_1, the coefficient of φ_n, is placed under φ_n; $-3\xi_I$, the coefficient of μ_1, is placed under μ_1; $-3\xi'_1$, the coefficient of μ_2, is placed under μ_2; and p_{12} is placed in the column headed "Right-hand member of equation".

In writing equation (2) of Table IV, which is

$$\xi_1\varphi_1+\rho_2\varphi_2+\xi_2\varphi_3+\xi'_2\varphi_{n-1}-3\xi_{II}\mu_1-3\xi'_2\mu_2=p_{23}-p_{21},$$

ξ_1, the coefficient of φ_1, is placed in the column under φ_1; ρ_2, the coefficient of φ_2, is placed under φ_2; ξ_2, the coefficient of φ_3, is placed under φ_3; ξ'_2, the coefficient of φ_{n-1}, is placed under φ_{n-1}, $-3\xi_{II}$, the coefficient of μ_1, is placed under μ_1; $-3\xi'_2$, the coefficient of μ_2, is placed under μ_2; and $p_{23}-p_{21}$ is placed in the column headed "Right-hand member of equation".

In a similar way we can write the equations from (3) to $(n+2)$ of Table IV, from which we can determine $n+2$ unknowns i.e. φ_1, φ_2, φ_3...... φ_n, μ_1 and μ_2.

Table V gives the general equations to be used in determining the slopes and the deflections in a double storied rectangular building frame fixed at the bases with any number of spans, carrying wind loads.[1]

In writing equation (1) of Table V, which is

$$\rho_1\varphi_1+\xi_1\varphi_2+\xi'_1\varphi_n-3\xi_I\mu_1-3\xi'_1\mu_2=o,$$

ρ_1, the coefficient of φ_1, is placed in the column under φ_1; ξ_1, the coefficient of φ_2, is placed under φ_2; ξ'_1, the coefficient of φ_n, is placed under φ_n; $-3\xi_I$, the coefficient of μ_1, is placed under μ_1; $-3\xi'_1$, the coefficient of μ_2, is placed under μ_2; and the right-hand member of this equation is zero, because the value in the column headed " Right-hand member of equation " is cipher.

In writing equation (2) of Table V, which is

$$\xi_1\varphi_1+\rho_2\varphi_2+\xi_2\varphi_3+\xi'_2\varphi_{n-1}-3\xi_{II}\mu_1-3\xi'_2\mu_2=o,$$

ξ_1, the coefficient of φ_1, is placed in the column under φ_1; ρ_2, the coefficient of φ_2, is placed under φ_2; ξ_2, the coefficient of φ_3, is placed under φ_3; ξ'_2, the coefficient of φ_{n-1}, is placed under φ_{n-1}; $-3\xi_{II}$, the coefficient of μ_1, is placed under μ_1; $-3\xi'_2$, the coefficient of μ_2, is placed under μ_2; and the right-hand member of this equation is zero, because the value in the column headed " Right-hand member of equation " is cipher.

In a similar way we can write the equations from (3) to $(n+2)$ of Table V, from which $n+2$ unknowns can be determined.

Table IV gives the general equations to be used in determining the slopes and deflections in a double storied rectangular building frame fixed at the bases with any number of spans, carrying any system of vertical loads and wind loads. In a similar way as explained above, we obtain, from Table VI, as many equations as there are unknowns.

Multiple Storied Rectangular Building Frame of any Number of Spans. Legs Fixed at the Bases.

Table VII gives the general equations to be used in determining the slopes and deflections in a triple storied rectangular building frame fixed at

---

[1] In this bulletin wind loads are assumed to be horizontal and concentrated at joints on one vertical side of the building.

the bases with any number of spans, carrying any system of vertical loads and wind loads.

For a rectangular building frame, any number of stories high and any number of spans long, we continue further to tabulate, in Table VII, the coefficients of unknown slopes and ratios of the deflection to story height, considering carefully the rules that exist, in the Table, on the symmetry of the places of coefficient of φ and μ, and the systematic arrangement of the suffix-number.

Bent with Legs Hinged at the Bases.

If the legs of the bent are hinged at I, II, III, IV,......$M-I$ and M, the equations of moments $M_{r.R}$ and $M_{R.r}$ are expressed as follows:

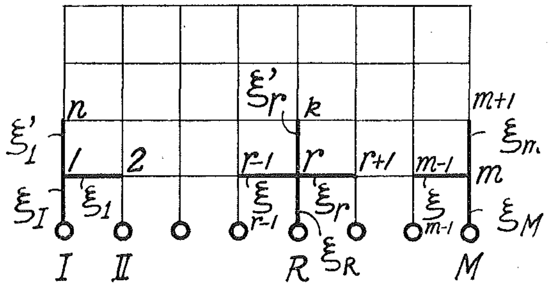


Fig. 5.

(16) $$M_{r.R}=2E\xi_R\left\{2\varphi_r+\varphi_R-3\mu_1\right\},$$

(17) $$M_{R.r}=2E\xi_R\left\{2\varphi_R+\varphi_r-3\mu_1\right\},$$

and the leg is hinged at the base; therefore: $M_{R.r}=0$. From this condition we get:

(18) $$\varphi_R=1{,}5\mu_1-0{,}5\varphi_r$$

Substituting this expression of φ_R in equation (16), we obtain:

(19) $$M_{r.R}=2E\xi_R\left\{1{,}5\varphi_r-1{,}5\mu_1\right\}.$$

For expressions of moments at the ends of all the girders and the columns, except the columns in the first story, the expressions, obtained in the case of a bent with fixed legs, due to vertical loads or wind loads, are applicable here.

For condition of equilibrium at 1:

$$M_{1I}+M_{12}+M_{1.\,n}=o$$

gives:

$$\tag{20}\begin{cases}\varphi_1\rho'_1+\varphi_2\xi_1+\varphi_n\xi'_1-1{,}5\mu_1\xi_I-3\mu_2\xi'_1=p_{12},\\ \text{where}\\ \rho'_1=2\left\{\xi_1+\xi'_1\right\}+1{,}5\xi_I.\end{cases}$$

For condition of equilibrium at r:

$$M_{r.\,r-1}+M_{r.\,R}+M_{r.\,k}+M_{r.\,r+1}=o$$

gives:

$$\tag{21}\begin{cases}\varphi_{r-1}\xi_{r-1}+\varphi_r\rho'_r+\varphi_{r+1}\xi_r+\varphi_k\xi'_r-1{,}5\mu_1\xi_R-3\mu_2\xi'_r=p_{r.\,r+1}-p_{r.\,r-1},\\ \text{where}\\ \rho'_r=2\left\{\xi_{r-1}+\xi_r+\xi'_r\right\}+1{,}5\xi_R.\end{cases}$$

For condition of equilibrium at m:

$$M_{m.\,m-1}+M_{m,\,M}+M_{m.\,m+1}=o$$

gives:

$$\tag{22}\begin{cases}\varphi_{m-1}\xi_{m-1}+\varphi_m\rho'_m+\varphi_{m+1}\xi_m-1{,}5\mu_1\xi_M-3\mu_2\xi_m=-p_{m.\,m-1},\\ \text{where}\\ \rho'_m=2\left\{\xi_{m-1}+\xi_m\right\}+1{,}5\xi_M.\end{cases}$$

For the sake of convenience we sum up the above obtained expressions for ρ' as follows:

$$\tag{23}\begin{cases}\rho'_1=2\left\{\xi_1+\xi'_1\right\}+1{,}5\xi_I,\\ \rho'_r=2\left\{\xi_{r-1}+\xi_r+\xi'_r\right\}+1{,}5\xi_R,\\ \rho'_m=2\left\{\xi_{m-1}+\xi_m\right\}+1{,}5\xi_M.\end{cases}$$

Single Storied Bent with any Number of Spans.
Legs Hinged at the Bases.

A single storied bent with any number of spans, hinged at the bases, carries any system of vertical loads.

Representing the horizontal deflections of the joints at the top by δ_1 and $\mu_1=\frac{\delta_1}{h_1}$, we obtain from equations (19), (7), and (8):

$$M_{r.\,r-1}=2E\xi_{r-1}\left\{2\varphi_r+\varphi_{r-1}\right\}+C_{r.\,r-1},$$

$$M_{r.\,R}=2E\xi_R\left\{1,5\varphi_r-1,5\mu_1\right\},$$

$$M_{r.\,r+1}=2E\xi_r\left\{2\varphi_r+\varphi_{r+1}\right\}-C_{r.\,r+1}.$$

For condition of equilibrium at joints, equations (10), (11), and (12) are applicable.

For condition of equilibrium for the bent as a whole:

$$\sum_{\binom{r=1}{R=I}}^{\binom{r=n}{R=N}}\left\{M_{r.\,R}\right\}=0. \tag{24}$$

These conditions of equilibrium give the equations from which we can determine unknown slopes and deflections. These equations are tabulated in Table VIII.

Table IX gives the general equations to be used in determining the slopes and the deflections in a single storied bent, whose legs are hinged at the bases, with any number of spans, carrying a wind load at the top.

In writing equation (1) of Table IX, which is

$$\rho_1'\varphi_1+\xi_1\varphi_2-1,5\xi_I\mu_1=0,$$

ρ_1', the coefficient of φ_1, is placed in the column under φ_1; ξ_1, the coefficient of φ_2, is placed under φ_2; $-1,5\xi_I$, the coefficient of μ_1, is placed under μ_1; and the right-hand member of this equation is zero, because the value in the column headed " Right-hand member of equation " is cipher.

In writing equation (2) of Table IX, which is

$$\xi_1\varphi_1+\rho_2'\varphi_2+\xi_2\varphi_3-1,5\xi_{II}\mu_1=0,$$

ξ_1, the coefficient of φ_1, is placed in the column under φ_1; ρ_2', the coefficient of φ_2, is placed under φ_2; ξ_2, the coefficient of φ_3, is placed under φ_3; $-1,5\xi_{II}$, the coefficient of μ_1, is placed under μ_1; and the right-hand member of this equation is zero, because the value in the column headed "Right-hand member of equation" is cipher.

In a similar way we can write the equation from (3) to $(n+1)$ of Table IX, from which $n+1$ unknowns can be determined.

Table X gives the general equations to be used in determining the slopes and deflections in a single storied bent hinged at the bases with any number of spans, carrying any system of vertical loads and a wind load. In a similar way as explained above, we obtain, from Table X, as many equations as there are unknowns.

Multiple Storied Rectangular Building Frame of any Number of Spans. Legs Hinged at the Bases.

Table XI and Table XII give respectively the general equations to be used in determining the slopes and deflections in a double storied and in a triple storied rectangular building frames hinged at the bases with any number of spans.

For a rectangular building frame, any number of stories high and any number of spans long, we further continue to tabulate, in Table XII, the coefficients of unknown slopes and ratios of the deflection to story height, considering carefully the rules that exist, in the Table, on the symmetry of the places of coefficient of φ and μ, and the systematical arrangement of the suffix-number.

Special Cases.

As a special cases of the above treated structure there are rectangular building frames with an axis of symmetry of the building, carrying vertical loads symmetrical about the vertical center line of the building. To this case no horizontal deflections come in calculation and therefore the equations are very much simplified.

The general equations to be used in determining the unknown slopes in triple storied bents, whose legs are fixed at the bases and the number of spans being 2m and 2m-1 are given from Table XIII and Table XIV respectively; to the hinged condition Table XV and Table XVI are applicable.

Further, to the symmetrical loads about the center of each span, Tables XVII, XVIII, XIX and XX are applicable.

For fixed condition and span of even number, the writer has already treated in the Memoir, Vol. 1, No. 2, 1926.

Continuous beams of any number of spans, carrying any system of vertical loads, may be considered as a special case of the rectangular building frame. There may be three types of continuous beam; the first of which is defined as a continuous beam fixed at both ends and supported simply at the intermediate points; the second of which is defined as a continuous beam hinged or supported simply at both ends and supported at the intermediate points; and the third is the combined type of the above two; i.e. at one end of the beam it is fixed and at the other end it is supported or hinged.

These three types of continuous beam are, in this article, treated firstly assuming that all of the supports keep, even after loading, their initial position on the same level. Fig. 6 represents the above mentioned types of continuous beam and the condition of equilibrium at supports gives the equations to be used in determining unknown slopes at the supports.

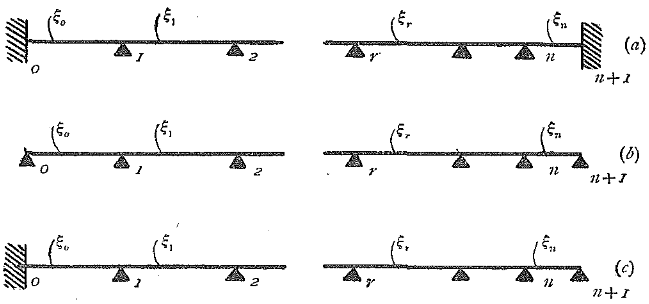


Fig. 6.

For condition of equilibrium at intermediate support r, $M_{r.\,r-1}+M_{r.\,r+1}=0$ gives:

$$(25)\qquad \left\{\begin{array}{l} \varphi_{r-1}\xi_{r-1}+\varphi_r\rho_r+\varphi_{r+1}\xi_r=p_{r.\,r+1}-p_{r.\,r-1}, \\ \text{where} \\ \rho_r=2\left\{\xi_{r-1}+\xi_r\right\}, \\ p_{r.\,r+1}=\dfrac{C_{r.\,r+1}}{2E} \\ p_{r,\,r-1}=\dfrac{C_{r.\,r-1}}{2E} \end{array}\right.$$

At support next to the fixed end, $M_{10}+M_{12}=0$ gives:

$$(26)\qquad \varphi_1\rho_1+\varphi_2\xi_1=p_{12}-p_{10}.$$

At support next to the hinged end, $M_{10}+M_{12}=0$ gives:

$$(27)\qquad \left\{\begin{array}{l} \varphi_1\rho_1'+\varphi_2\xi_1=p_{12}-p_{10}-0{,}5p_{01}, \\ \text{where} \\ \rho_1'=1{,}5\xi_0+2\xi_1. \end{array}\right.$$

At support before the fixed end, $M_{n.\,n-1}+M_{n.\,n+1}=0$ gives:

$$(28)\qquad \varphi_{n-1}\xi_{n-1}+\varphi_n\rho_n=p_{n.\,n+1}-p_{n.\,n-1}.$$

At support before the hinged end, $M_{n.\,n-1}+M_{n.\,n+1}=0$ gives:

$$(29)\qquad \left\{\begin{array}{l} \varphi_{n-1}\xi_{n-1}+\varphi_n\rho_n'=0{,}5p_{n+1.\,n}+p_{n.\,n+1}-p_{n.\,n-1}, \\ \text{where} \\ \rho_n'=2\xi_{n-1}+1{,}5\xi_n. \end{array}\right.$$

By proper combination of these equations from (25) to (29), corresponding to the type of continuous beam as shown in Fig. 6 (a), (b), and (c), we obtain Tables XXI, XXII and XXIII; or vice versa, from these Tables we get general equations to be used in determining the slopes at supports and with the values of slopes obtained we can determine moments at supports.

To speak more generally the moments in a continuous beam may be developed by the settlement of the supporting parts as well as the loads applied.

Assuming that the beam remains in contact with all supports even after the loading and settlement of the supports, we obtain, almost in the same way as explained previously, the general equations, which determine unknown slopes, for a continuous beam fixed at both ends and supported at the intermediate points on different levels, carrying any system of vertical loads. These equations are given in Table XXIV. Here it is noticed that the settlement of supports must be given or estimated for materials of the supports or properties of foundation used, and therefore the ratios of the vertical deflection to span length must be, in this case, known quantities for the calculation of redundant stress.

Some Examples on Application of the Proposed Method.

The following examples may be convenient so as to make it possible to apply the proposed method easily.

EXAMPLE 1. (Table XXVIII).

Triple Storied Bent of Six Spans. Legs Fixed at the Bases.
Cross-Sections of all Members Different
Each Story Height Different
Any System of Vertical Loads on Girders
Wind Loads Assumed at Joints on one Vertical Side

EXAMPLE 2. (Table XXIX).

Five Storied Bent of Triple Span. Legs Fixed at the Bases.
Cross-Sections of all Members Different
Each Story Height Different
Any System of Vertical Loads on Girders
Wind Loads Assumed at Joints on One Vertical Side

EXAMPLE 3. (Table XXX).

Symmetrical Eight-Span Bent Four Stories High.
Legs Fixed at the Bases
Any System of Symmetrical Vertical Loads on Girders

EXAMPLE 4. (Table XXXI).

Symmetrical Seven-Span Bent Five Stories High.
Legs Fixed at the Bases
Any System of Symmetrical Vertical Loads on Girders

EXAMPLE 5. (Table XXXII).

Triple Storied Bent of Six Spans.
Legs Hinged at the Bases
Cross-Sections of all Members Different
Each Story Height Different
Any System of Vertical Loads on Girders
Wind Loads Assumed at Joints on One Vertical Side

EXAMPLE 6. (Table XXXIII).

Five Storied Bent of Triple Span. Legs Hinged at the Bases.
Cross-Sections of all Members Different
Each Story Height Different
Any System of Vertical Loads on Girders
Wind Loads Assumed at Joints on one Vertical Side

EXAMPLE 7. (Table XXXIV)

Symmetrical Eight-Span Bent Four Stories Heigh. Legs Hinged at the Bases.
Any System of Symmetrical Vertical Loads on Girders

EXAMPLE 8. (Table XXXV).

Symmetrical Seven-Span Bent Five Stories High.
Legs Hinged at the Bases
Any System of Symmetrical Vertical Loads on Girders

Solution of Equations and General Formulas for Special Cases.

The general equations obtained from the Tables of this paper are solved by determinants or by process of elimination. To the calculation of a large number of unknowns, the solution by determinants is very long and the process of elimination is preferable, however it may be sometimes painstaking work.

As an example of the solution of a bent in symmetrical condition Mr. Y. YANO, assistant of the author's Institute, has designed and computed the stresses of a symmetrical six-span bent six stories high, carrying a system of symmetrical load.

This numerical example is added at the end of the note, the writer gratefully acknowledging indebtedness to him.

To the special cases as shown in Fig. 7, the general formulae for unknown quantities have been introduced as follows:

Dividing each equation by the coefficient of the first unknown of the equation, we obtain equations whose first terms in the left-hand member are plus unity. These equations are written in tabular form as shown in Table 1, in which a, b, and c are given in actual case by numerical values.

BENT WITH AXIS OF SYMMETRY OF THE STRUCTURE AND ANY SYSTEM OF SYMMETRICAL VERTICAL LOADS.

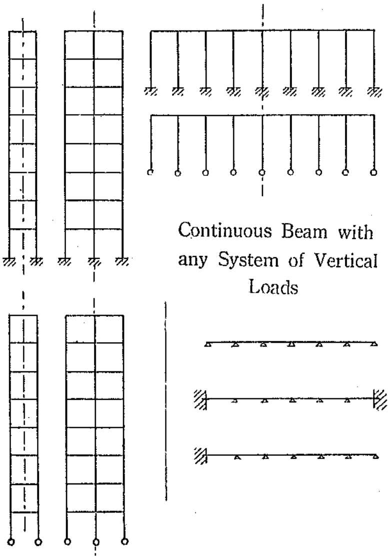


Fig. 7.

TABLE I.

No. of Equation	Left-Hand Member of Equation (Coefficients of Unknowns)												Right-Hand Member of Equation
	X_0	X_1	X_2	X_3	X_4	X_5	→	X_{n-4}	X_{n-3}	X_{n-2}	X_{n-1}	X_n	
1	1	a_1											C_1
2	1	b_1	a_2										C_2
3		1	b_2	a_3									C_3
4			1	b_3	a_4								C_4
5				1	b_4	a_5							C_5
↓						↘	↘						↓
						↘							
$n-2$								1	b_{n-3}	a_{n-2}			C_{n-2}
$n-1$									1	b_{n-2}	a_{n-1}		C_{n-1}
n										1	b_{n-1}	a_n	C_n
$n+1$											1	b_n	C_{n+1}

For convenience of calculation we use the notations as denoted in the following:

$$
(30)\left\{
\begin{aligned}
&\triangle_0 = 1, \\
&\triangle_1 = a_1 - b_1 = \frac{a_1}{\triangle_0} - b_1, \\
&\triangle_2 = -\frac{a_2}{a_1 - b_1} - b_2 = -\frac{a_2}{\triangle_1} - b_2, \\
&\triangle_3 = -\frac{a_3}{-\dfrac{a_2}{a_1 - b_1} - b_2} - b_3 = -\frac{a_3}{\triangle_2} - b_3, \\
&\triangle_4 = -\frac{a_4}{-\dfrac{a_3}{-\dfrac{a_2}{a_1 - b_1} - b_2} - b_3} - b_4 = -\frac{a_4}{\triangle_3} - b_4, \\
&\vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad\qquad \vdots
\end{aligned}
\right.
$$

$$\left|\begin{array}{l}\triangle_n=-\cfrac{a_n}{-\cfrac{a_{n-1}}{-\cfrac{a_{n-2}}{\cfrac{\vdots}{-\cfrac{a_3}{-\cfrac{a_2}{a_1-b_1}-b_2}-b_3}}-b_{n-2}}-b_{n-1}}-b_n=-\cfrac{a_n}{\triangle_{n-1}}-b_n.\end{array}\right.$$

Then we obtain general expressions for unknowns of Table I :

$$(31)\left\{\begin{array}{l}X_n=\cfrac{\cfrac{\cfrac{\cfrac{\cdots\cfrac{\cfrac{\cfrac{\cfrac{C_1}{\triangle_0}-C_2}{\triangle_1}-C_3}{\triangle_2}-C_4}{\triangle_3}-C_5\cdots}{\triangle_{n-3}}-C_{n-1}}{\triangle_{n-2}}-C_n}{\triangle_{n-1}}-C_{n+1}}{\triangle_n},\\ X_{n-1}=C_{n+1}-b_nX_n,\\ X_{n-2}=C_n-a_nX_n-b_{n-1}X_{n-1},\\ \vdots\\ X_2=C_4-a_4X_4-b_3X_3,\\ X_1=C_3-a_3X_3-b_2X_2,\\ X_0=C_2-a_2X_2-b_1X_1.\end{array}\right.$$

For continuous beams it may be more convenient to choose unknowns in moments than those chosen in slopes, i.e. the equation of three moments given by Clapeyron determines unknown moments directly. And the formulae introduced above are applicable to that.

Table XXV gives the general equations to be used in determining the moments at supports on the same level for a continuous beam of any number of spans, carrying any system of vertical loads. (Fig. 6b).

To the investigation of stresses of a continuous beam with a great number of spans, Table XXV is of the utmost importance for the computer who can therefrom easily work out the moments by the process of elimination for the numerical values of the coefficients of the unknown moments.

From Table XXV we can derive Table XXVI and Table XXVII; e.g. on substituting $m_0=o$ in Table XXV we obtain Table XXVI which gives the general equations to be used in determining the moments at supports on the same level for a continuous beam fixed at the left end and carrying any system of vertical loads.

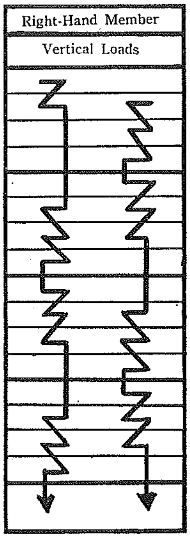


Fig. 8.

When we further snbstitute $l_n=o$ in Table XXVI i.e. dividing the last equation by m_{n-1} and then substituting in it $m_{n-1}=\infty$ we obtain Table XXVII which gives the general equations for the redundant moments of a continuous beam fixed at both ends.

Properties of Proposed Tables and Rules in the Tabulation.

Determination of Coefficients.

For coefficients of unknown slopes there are two kinds, one of which is expressed by ξ and can be determined by $I:l$ for girders and $I:h$ for columns; the other kind is denoted by ρ and can be determined by equations (2), (23), and the equations given in Tables.

More detailed instruction about ξ has been given above in "Conventional Notations and Signs Used".

Next, coefficients of the unknown ratios of the horizontal deflection to story height, i.e. μ always takes minus sign, and one kind of coefficient of μ is expressed by ξ with multiplier -3 for a bent with fixed legs, this multiplier being $-1,5$ for μ_1 (at the top of the first story) of a bent with hinged legs; the other kind is expressed by $-X$ except $-\frac{X_1}{2}$, the coefficient of μ_1 for the equation obtained by condition of equilibrium for the first story of a bent with hinged legs. (Tables VIII, IX, X, XI, XII, XXXII, and XXXIII).

The right-hand member of equation, denoted by p and q, is given by equations (4) and (5). This is the variable term due to the condition of applied loads: p due to any system of vertical loads; and q due to wind loads assumed horizontally at joints on one vertical side. q takes always minus sign and for a bent with hinged legs q_1 takes multiplier -2.

Diagonal Line of ρ and ξ, Other Properties of Table.

The coefficient ρ finds itself in each Table on a continuous line which may be expressed as a diagonal line of a square formed by columns of unknown slopes φ and rows of general equations, φ being taken horizontally and number of equation vertically; we name this square "Great Square". As shown e.g. in Table VII, ρ arrange themselves, along the diagonal line of the Great Square, in order of the suffix-number of ρ, from the top left corner of the Great Square to the bottom right corner of the same; in other words, ρ arranges itself, from ρ_1 to ρ_r, reading downward to the right along the diagonal line of the Great Square. In the same Table we find

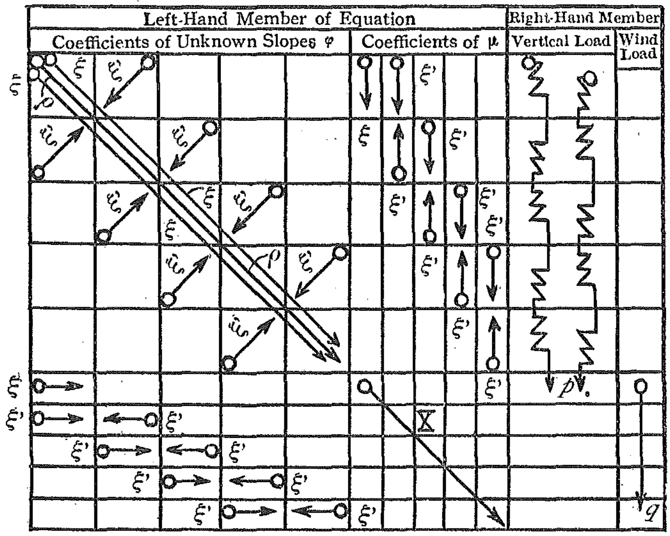


Fig. 9.

lines of ξ on both sides of the ρ-line, and the arrangement of the suffix-number is the same as in the case of ρ; while the line of ξ' with the mark (′) is placed intersecting at right angles with the above mentioned three lines of ρ and ξ, arranging themselves, in this case, towards the diagonal of Great Square, one part descending leftward and the other part ascending rightward; these two parts meet together in the diagonal of the Great Square and the line of ξ' with the mark (′) may be also considered as a diagonal line of a small square which is one part of Great Square. We call this the "Small Square".

On considering the diagonal of the Great Square as an axis of symmetry of the Table, we find the arrangements of ξ in stepped form in the position of symmetry; and on the prolonged line of the diagonal of Great Square X is placed in order of the suffix-number. For the arrangement ξ in stepped form, ξ with suffix of large letter firstly comes and next ξ' with the mark (′) comes in the same relation as shown in the diagonal of Small Square, in other words, as the horizontal projection of the ξ'—line of small Square ξ' finds itself in the columns under φ and as the vertical projection of the same ξ' finds itself in the column under μ (See e.g. Table VII).

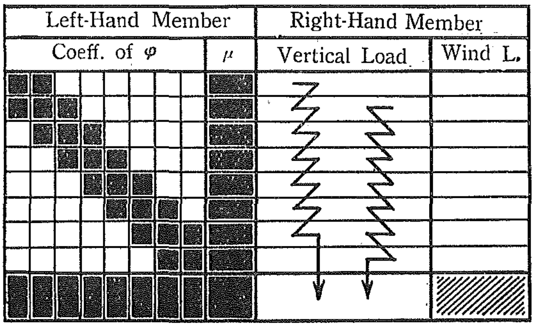


Fig. 10.

For rectangular building frames with any system of vertical loads, each equation, given by condition of equilibrium at joint, has p in right-hand member; the order of the suffix-number of p is indicated in Fig. 8 in the form of a zigzag line, of which there exist also systematic relations in strict order, the right-hand term of p taking minus sign.

For frames with wind loads, each equation, given by condition of equilibrium for a story as a whole has q in the right-hand member of the equation, q taking minus sign, regularly ordered and reading vertically downwards.

For special cases of rectangular building frames, the properties of the Table are almost the same as explained above, and will not be repeated here in detail.

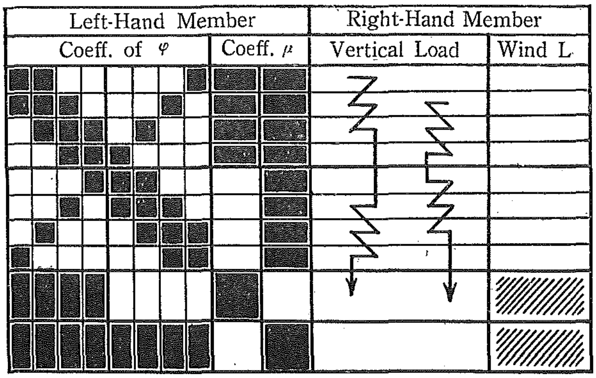


Fig. 11.

This method of mechanical solution of such a problem may be applicable to the investigation of trusses with stiff connection and the secondary stresses may be mechanically determined in a similar manner to that explained above.

To facilitate the understanding of the existing properties in the proposed Table, Fig. 9 is of importance, where the regularly ordered lines of each quantity mentioned above are shown graphically. From this figure we can deduce a relation between the number of stories and the number of diagonals of Small Squares.

The general type of figure of Tables due to loads and number of stories is given in Figs. 10, 11, 12, 13 and 14.

Fig. 10 shows the general type of Table for a single storied bent with any system of vertical loads and wind loads.

Fig. 11 shows the general type of Table for a double storied bent with any system of vertical loads and wind loads.

Figs. 12, 13 and 14 show respectively the general type of Table for triple, quadruple, and five storied bents with any system of vertical loads and wind loads.

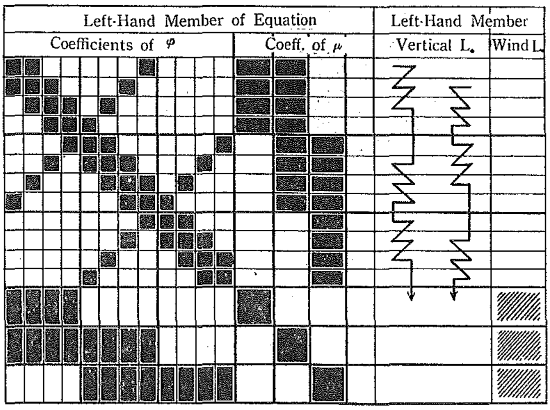


Fig. 12.

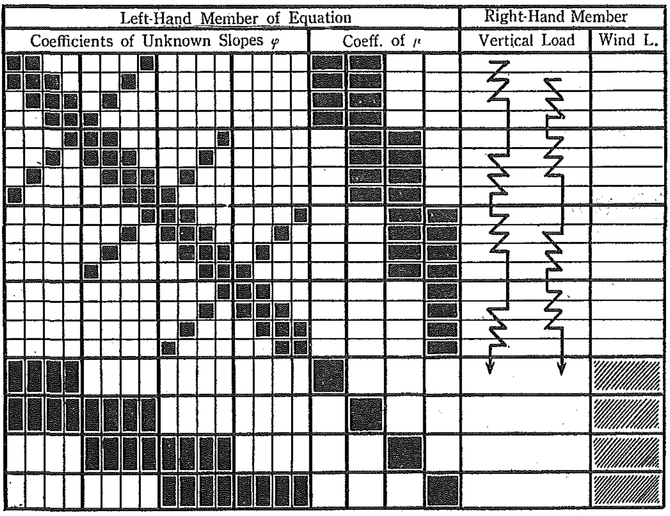

Left-Hand Member of Equation
Right-Hand Member
Coefficients of Unknown Slopes φ
Coeff. of μ
Vertical Load
Wind L.

Fig. 13.

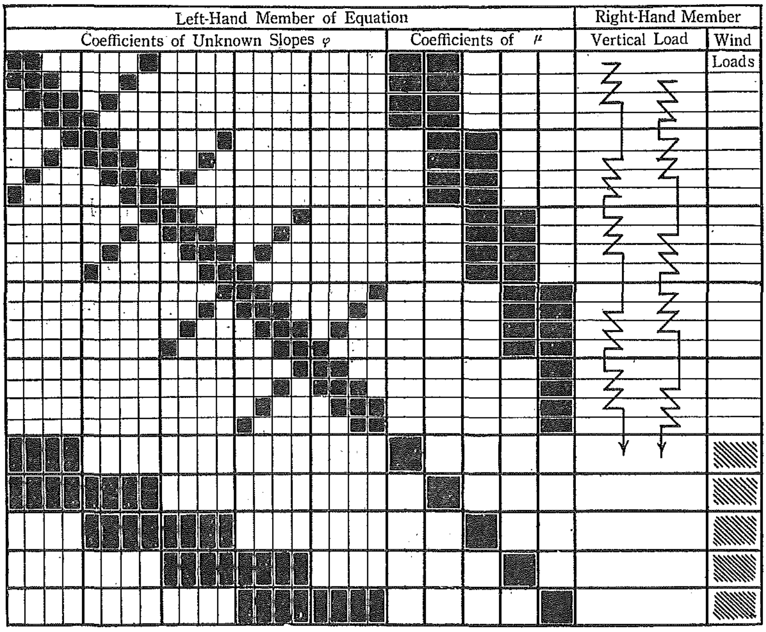

Left-Hand Member of Equation
Right-Hand Member
Coefficients of Unknown Slopes φ
Coefficients of μ
Vertical Load
Wind Loads

Fig. 14.

Analytical Index of Tables.

The Following index may be convenient so as to make it possible to locate the desired table in this bulletin quickly.

Fundamental Equations.

- General Equations for the Moments at the Ends of any Member (Table A)
- Values of Constants C and H to be used in the Equations of Table A.
  - For any System of Vertical Loads (Table B)
  - For Loads Symmetrical about Center of Member (Table C)

Tables of General Equations.

- Bent of Single Story with any Number of Spans.
  - Legs Fixed at the Bases (Table I, II, and III)
  - Legs Hinged at the Bases (Table VIII, IX, and X)
- Bent of Double Story with any Number of Spans.
  - Legs Fixed at the Bases (Table IV, V, and VI)
  - Legs Hinged at the Bases (Table XI)
- Bent of Triple Story with any Number of Spans
  - Legs Fixed at the Bases (Table VII)
  - Legs Hinged at the Bases (Table XII)
- Bent of Triple Story with an Axis of Symmetry of the Building
  - Legs Fixed at the Bases
    - Number of Spans 2m (Table XIII)
    - Number of Spans 2m-1 (Table XIV)
  - Legs Hinged at the Bases
    - Number of Spans 2m (Table XV)
    - Number of Spans 2m-1 (Table XVI)
- Bent of Triple Story with an Axis of Symmetry of the Building, Carrying any System of Symmetrical Vertical Loads about the Center of each Span.
  - Legs Fixed at the Bases
    - Number of Spans 2m (Table XVII)

Number of Spans 2m-1 (Table XVIII)
Legs Hinged at the Bases
Number of Spans 2m (Table XIX)
Number of Spans 2m-1 (Table XX)
Continuous Beam with any Number of Spans, Supports all on Same Level.
Beam Fixed at both Ends (Table XXI and XXVII)
Beam Hinged at both Ends (Table XXII and XXV)
Beam Fixed at one End and Hinged at the Other (Table XXIII and XXVI)
Continuous Beam with any Number of Spans, Supports on Different Levels.
Beam Fixed at both Ends (Table XXIV)

Tables of Examples.

Triple Storied Bent of Six Spans in Asymmetrical Condition (Table XXVIII and XXXII)
Four Storied Bent of Eight Spans in Asymmetrical Condition (Table XXX and XXXIV)
Five Storied Bent of Three Spans in Asymmetrical Condition (Table XXIX and XXXIII)
Five Storted Bent of Seven Spans in Symmetrical Condition (Table XXXI and XXXV)

Summary and Conclusions.

The General conclusions to be drawn from the results of the investigation described in this bulletin may be summarized as follows:

1). The general form of the fundamental Table is memorized with little effort and can be easily tabulated by arranging ξ, ρ, p, q, and X with suffix, in order of the suffix-number, in the columns marked.

2). No mistakes can be performed in writing the fundamental Table, on account of the systematic arrangement of the suffix-number and the symmetrical property of the places where the coefficients of unknown quantities are to be written.

For complicated calculations it is sometimes very difficult to avoid some mistakes when there is no way to check the obtained results; our systematic arrangement of the suffix-number and the symmetrical property of the Table are of great help in this difficulty.

3). The accuracy of the calculation is the same as that of the slope deflection method, because our solution has been derived from it, and therefore the method is very accurate, it having no approximations except those in the assumptions.

4). The knowledge of elementary algebra is sufficient for the calculation of the stresses of higher structures except in finding values of load terms p; for the load condition usually applied, there are Tables in current use, which give the values of p.

5). As in the computation by the slope deflection method, the calculation is long to be used in the actual design of a high building; it has however its greatest value as a standard calculation for checking the accuracy of another approximate rapid method.

6). The properties of our Tables are of great help as a process to check general equations of equilibrium obtained by slope-deflection method.

APPENDIX.

Numerical Calculation of Statically Indeterminate Stresses.

As the numerical example of proposed method, Mr. Y. YANO, assistant of my institute, has designed, after the regulations of the Japanese Government, a department store building of reinforced concrete. This skeleton building is symmetrical about the vertical center line of the frame, as shown in Fig. 15, six-stories high and six-spans long, carrying loads concentrated symmetrically and distributed uniformly. In calculation the average value of the moment of inertia of the central section and the end-sections has been taken as that of each girder-section. With such values of moment of inertia there are indicated in the figure the values of ξ for each member of the frame.

Table 2 gives the general equations to be used in determining the unknown slopes due to the concentrated loads and uniformly distributed loads; and the numerical values of the constants in these equations are given there. The figures given in the right-hand column are coefficients of .0001 as indicated at the head of the column. These simultaneous equations have been solved by the process of elimination and the results are as follows:

$\varphi_1 = 1.902\ 235\ 320\ 0$
$\varphi_2 = -.110\ 091\ 174\ 4$
$\varphi_3 = .006\ 634\ 501\ 9$
$\varphi_4 = .002\ 725\ 673\ 4$
$\varphi_5 = -.081\ 713\ 749\ 3$
$\varphi_6 = 1.720\ 532\ 363\ 0$
$\varphi_7 = 2.014\ 816\ 120\ 0$
$\varphi_8 = -.171\ 212\ 248\ 2$
$\varphi_9 = .014\ 246\ 846\ 0$
$\varphi_{10} = .029\ 958\ 992\ 0$
$\varphi_{11} = -.236\ 248\ 470\ 5$
$\varphi_{12} = 1.982\ 781\ 047\ 0$
$\varphi_{13} = 1.914\ 580\ 320\ 0$
$\varphi_{14} = -.175\ 120\ 211\ 0$
$\varphi_{15} = .005\ 133\ 375\ 5$
$\varphi_{16} = .188\ 981\ 556\ 3$
$\varphi_{17} = -.928\ 937\ 543\ 7$
$\varphi_{18} = 4.444\ 895\ 510\ 0$

The moments at the ends of the columns and girders given by the fundamental equation (1) are indicated in Table 3, where the "Total Moment" means the total sum of the moments due to concentrated loads and uniformly distributed loads.

The accuracy of the computations has been checked by the statical condition of equilibrium of the joint-moment and its percentage of error for the least moment is given in the right-hand column of the same table.

TABLE 2.

TABLE OF THE EQUATION USED TO DETERMINE THE UNKNOWN SLOPES OF THE SYMMETRICAL SIX-SPAN SIX-STORY BENT SHOWN IN Fig. 15 (Calculated By Y. YANO).

No. of Equation	Left-Hand Member of Equation																		Right-Hand Member of Equation Coefficient of 0.0001
	Coefficients of Unknown Slopes φ																		
	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_7	φ_8	φ_9	φ_{10}	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	φ_{16}	φ_{17}	φ_{18}	
1	483	55				88.5													1065
2	55	847.6	55		156														0
3		55	847.6	156															0
4			156	736	55				102										0
5		156		55	736	55		102											0
6	88.5				55	432	72.5												1065
7						72.5	400	55				72.5							1065
8					102		55	530	55		53								0
9				102				55	530	53									0
10									53	402	55				38				0
11								53		55	402	55		38					0
12							72.5				55	400	72.5						1065
13												72.5	362	55				53.5	1065
14											38		55	366	55		35		0
15										38				55	366	35			0
16															35	380	77.5		0
17														35		77.5	380	77.5	0
18													53.5				77.5	262	1195

TABLE 3.

TABLE OF JOINT-MOMENT

(Moments are Expressed in Ft.-lbs.)

	Total moment	Mc due to concentrated load	Mu due to uniform load	Percentage of error for the least moment
M_{18-17}	−28900	−24790	−4110	
M_{18-13}	28900	24790	4110	
M_{17-18}	69780	59850	9930	
M_{17-16}	−66220	−56800	−9420	
M_{17-14}	−3560	−3050	−510	
M_{16-17}	57620	49420	8200	
M_{16-c}	−58290	−50000	−8290	
M_{16-15}	670	580	90	
M_{15-14}	52800	46190	6610	
M_{15-c}	−53220	−46550	−6670	
M_{15-16}	350	300	50	
M_{15-10}	70	60	10	
M_{14-13}	57550	50340	7210	
M_{14-15}	−54200	−47410	−6790	
M_{14-17}	−2240	−1960	−280	
M_{14-11}	−1110	−970	−140	
M_{13-14}	−43200	−37790	−5410	

TABLE 3 (Continued)

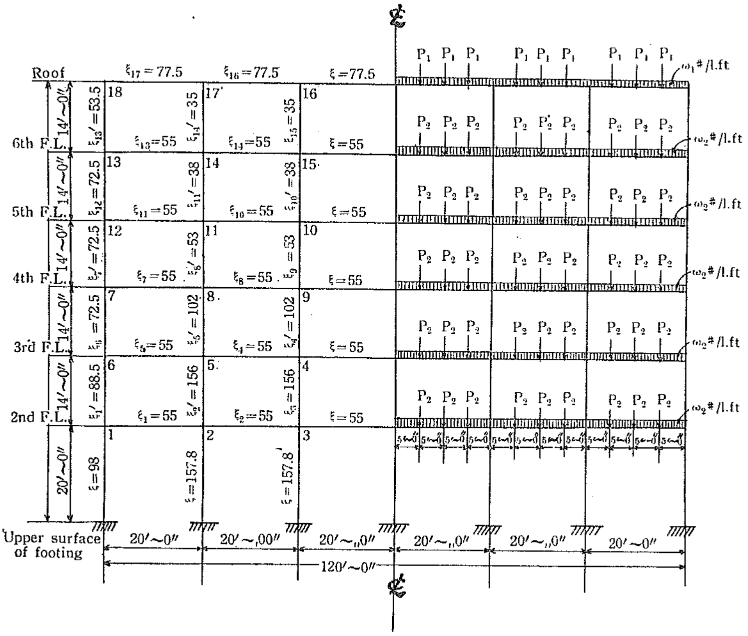
	Total moment	Mc due to concentrated load	Mu due to uniform load	Percentage of error for the least moment
M_{13-18}	22130	19360	2770	
M_{13-12}	21070	18430	2640	
M_{12-11}	−43000	−37610	−5390	
M_{12-13}	21320	18650	2670	
M_{12-7}	21680	18960	2720	
M_{11-12}	57400	50210	7190	
M_{11-10}	−54470	−47650	−6820	
M_{11-14}	−1230	−1070	−160	
M_{11-8}	−1700	−1490	−210	
M_{10-11}	52770	46160	6610	
M_{10-c}	−53090	−46440	−6650	
M_{10-15}	120	110	10	
M_{10-9}	200	170	30	
M_{9-8}	52860	46240	6620	
M_{9-c}	−53170	−46510	−6660	
M_{9-10}	150	130	20	
M_{9-4}	160	140	20	
M_{8-9}	−54150	−47370	−6780	

TABLE 3 (Continued)

	Total moment	Mc due to concentrated load	Mu due to uniform load	Percentage of error for the least moment
M_{8-7}	57850	50610	7240	
M_{8-11}	−1540	−1350	−190	
M_{8-5}	−2160	−1890	−270	
M_{7-8}	−42640	−37300	−5340	
M_{7-12}	21800	19070	2730	
M_{7-6}	20840	18230	2610	
M_{6-5}	−44010	−38500	−5510	M_{Total} (−570) 2.85% M_c (−510) 2.95% M_u (−90) 3.6%
M_{6-7}	19780	17300	2480	
M_{6-1}	23650	20690	2960	
M_{5-6}	57530	50320	7210	
M_{5-4}	−53690	−46970	−6720	
M_{5-8}	−1710	−1490	−220	
M_{5-2}	−2130	−1860	−270	
M_{4-5}	53040	46390	6650	
M_{4-c}	−53230	−46560	−6670	
M_{4-9}	100	90	10	
M_{4-3}	90	80	10	
M_{3-2}	52980	46340	6640	

TABLE 3 (Continued)

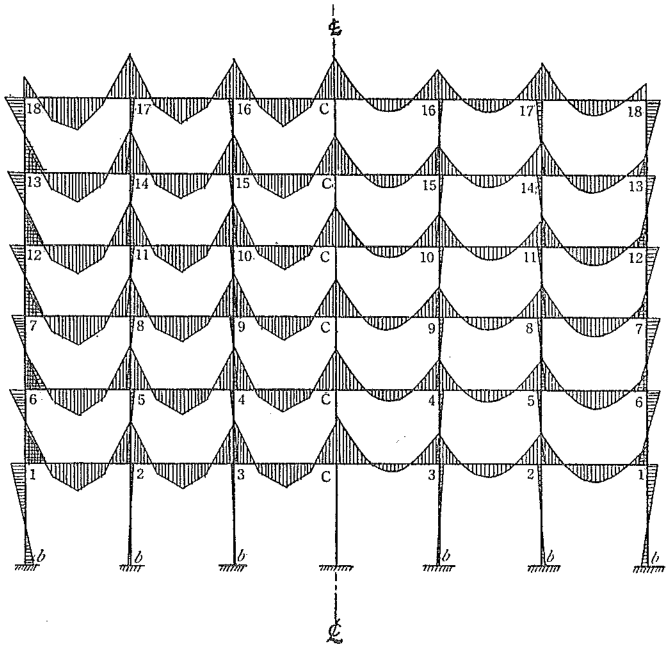
	Total moment	Mc due to concentrated load	Mu due to uniform load	Percentage of error for the least moment
M_{3-c}	−53210	−46550	−6660	
M_{3-4}	130	120	10	
M_{3-6}	100	90	10	
M_{2-3}	−53840	−47090	−6750	M_{Total} (50) 2.87% M_{c} (40) 2.63% M_{u} (10) 4.64%
M_{2-1}	57880	50630	7250	
M_{2-b}	−1740	−1520	−220	
M_{2-5}	−2350	−2060	−290	
M_{1-b}	18640	16300	2340	
M_{1-6}	24450	21390	3060	
M_{1-2}	−43090	−37690	−5400	
M_{b-1}	9320	8150	1170	
M_{b-2}	−870	−760	−110	
M_{b-3}	50	45	5	
M_{c-3}	53270	46600	6670	
M_{c-4}	53260	46590	6670	
M_{c-9}	53290	46620	6670	
M_{c-10}	53350	46670	6680	
M_{c-15}	53280	46610	6670	
M_{c-16}	60480	51880	8600	



Elevation of Frame.

Fig. 15.

$P_1=8200^{\#}$, $P_2=7450^{\#}$,
$\omega_1=255^{\#}/l.ft$,
$\omega_2=200^{\#}/l.ft$.



Bending Mt. Diagram
due to the Concentrated Load.
Scale of Mt. *10,0000 ft.#*

Fig. 16.

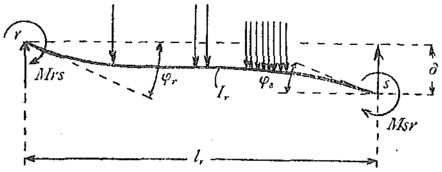
Bending Mt. Diagram
due to the Uniform Load.
Scale of Mt. *10,000 ft.#*

FUNDAMENTAL EQUATIONS

TABLE A

General Equations for the Moments at the Ends of any Member

Any System of Vertical Loads



$$M_{rs}=2E\xi_r\left\{2\varphi_r+\varphi_s-3\mu\right\}-C_{rs}$$

$$M_{sr}=2E\xi_r\left\{2\varphi_s+\varphi_r-3\mu\right\}+C_{sr}$$

Where

$$\xi_r=\frac{I_r}{l_r},\qquad \mu=\frac{\delta}{l_r}.$$

If end s is hinged,

$$M_{rs}=3E\xi_r\left\{\varphi_r-\mu\right\}-H_{rs}.$$

If end r is hinged,

$$M_{sr}=3E\xi_r\left\{\varphi_s-\mu\right\}+H_{sr}$$

Relation between C and H.

$$H_{rs}=C_{rs}+\frac{C_{sr}}{2},$$

$$H_{sr}=C_{sr}+\frac{C_{rs}}{2}.$$

Values of C and H for Various Loads are given in Tables B and C.

TABLE B

Values of Constants C and H to be Used in the Equations of Table A.

Any System of Vertical Loads

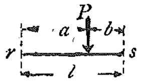
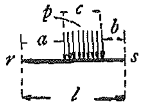
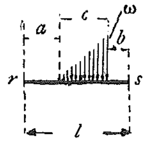
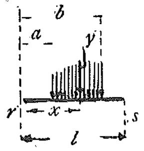
Condition of Loading	C_{rs}	C_{sr}	H_{rs}	H_{sr}
	$\frac{Pab^2}{l^2}$	$\frac{Pa^2b}{l^2}$	$\frac{Pab}{2l^2}\left\{a+2b\right\}$	$\frac{Pab}{2l^2}(2a+b)$
	$\frac{pc}{12l^2}\left\{12ab(b+c)+6b^2c+4c^2(a+b)+c^3\right\}$ Special Cases	$\frac{pc}{12l^2}\left\{12ab(a+c)+6a^2c+4c^2(a+b)+c^3\right\}$	$\frac{pc}{8l^2}\left\{4ab(a+2b+3c)+2c(a^2+2b^2)+4c^2(a+b)+c^3\right\}$	$\frac{pc}{8l^2}\left\{12abc+8a^2b+4ab^2+4a^2c+2b^2c+4c^2(a+b)+c^3\right\}$
	$a=0.$ $\frac{pc^2}{12l^2}\left\{6b^2+4bc+c^2\right\}$	$\frac{pc^3}{12l^2}\left\{4b+c\right\}$	$\frac{pc^2}{8l^2}\left\{4b^2+4bc+c^2\right\}$	$\frac{pc^2}{8l^2}\left\{2b^2+4bc+c^2\right\}$
	$b=0.$ $\frac{pc^3}{12l^2}\left\{4a+c\right\}$	$\frac{pc^2}{12l^2}\left\{6a^2+4ac+c^2\right\}$	$\frac{pc^2}{8l^2}\left\{2a^2+4ac+c^2\right\}$	$\frac{pc^2}{8l^2}\left\{4a^2+4ac+c^2\right\}$
	$a=b=0,$ $c=l.$ $\frac{pl^2}{12}$	$\frac{pl^2}{12}$	$\frac{pl^2}{8}$	$\frac{pl^2}{8}$
	$\frac{wc}{60l^2}\left\{20bc(a+b)+5c^2(a+2b)+30ab^2+2c^3\right\}$ Special Cases	$\frac{wc}{60l^2}\left\{10ac(a+c)+15b(2a^2+c^2)+40abc+3c^3\right\}$	$\frac{wc}{120l^2}\left\{80abc+40b^2c+20ac^2+35bc^2+60ab^2+10a^2c+30a^2b+7c^3\right\}$	$\frac{wc}{120l^2}\left\{100abc+20a^2c+20b^2c+25ac^2+30ab^2+60a^2b+40bc^2+8c^3\right\}$
	$a=0.$ $\frac{wc^2}{30l^2}\left\{10b^2+5bc+c^2\right\}$	$\frac{wc^3}{20l^2}\left\{5b+c\right\}$	$\frac{wc^2}{120l^2}\left\{40b^2+35bc+7c^2\right\}$	$\frac{wc^2}{30l^2}\left\{5b^2+10bc+2c^2\right\}$
	$b=0.$ $\frac{wc^3}{60l^2}\left\{5a+2c\right\}$	$\frac{wc^2}{60l^2}\left\{10a(a+c)+3c^2\right\}$	$\frac{wc^2}{120l^2}\left\{10a^2+20ac+7c^2\right\}$	$\frac{wc^2}{120l^2}\left\{20a^2+25ac+8c^2\right\}$
	$a=b=0,$ $c=l.$ $\frac{wl^2}{30}$	$\frac{wl^2}{20}$	$\frac{7}{120}wl^2$	$\frac{1}{15}wl^2$
	$\frac{1}{l^2}\int_a^b yx(l-x)^2dx$ Special Case	$\frac{1}{l^2}\int_a^b yx^2(l-x)dx$	$\frac{1}{2l^2}\int_a^b yx(l-x)(2l-x)dx$	$\frac{1}{2l^2}\int_a^b yx(l^2-x^2)dx$
	$a=0,\ b=l.$ $\frac{1}{l^2}\int_0^l yx(l-x)^2dx$	$\frac{1}{l^2}\int_0^l yx^2(l-x)dx$	$\frac{1}{2l^2}\int_0^l yx(l-x)(2l-x)dx$	$\frac{1}{2l^2}\int_0^l yx(l^2-x^2)dx$

TABLE C

Values of Constants C and H to be Used in the Equations of Table A.

Loads Symmetrical about Center of Member

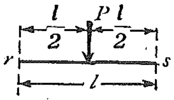
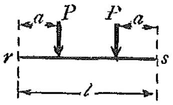
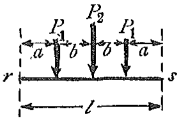
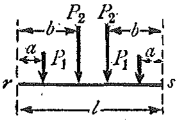
Condition of Loading	$C_{rs}=C_{sr}$	$H_{rs}=H_{sr}$
	$\frac{Pl}{8}$	$\frac{3}{16}Pl$
	$\frac{Pa(l-a)}{l}$	$\frac{3}{2l}Pa(l-a)$
	Special Case $a=\frac{l}{3}.\qquad P\frac{2l}{9}$	$\frac{Pl}{3}$
	$\frac{(a+b)}{l^2}\left\{2P_1a(a+2b)+P_2(a+b)^2\right\}$	$\frac{3(a+b)}{2l^2}\left\{2P_1a(a+2b)+P_2(a+b)^2\right\}$
	Special Case $\left\{\begin{array}{l}a=b=\frac{l}{4},\\P_1=P_2\end{array}\right.\qquad P\frac{5l}{16}$	$\frac{15}{32}Pl$
	$\frac{1}{l}\left\{P_1a(l-a)+P_2b(l-b)\right\}$	$\frac{3}{2l}\left\{P_1a(l-a)+P_2b(l-b)\right\}$
	Special Cases $1)\left\{\begin{array}{l}2\cdot a=b\\P_1=P_2\end{array}\right.\quad P\frac{a}{l}\left\{3l-5a\right\}$	$\frac{3Pa}{2l}\left\{3l-5a\right\}$
	$2)\left\{\begin{array}{l}a=\frac{b}{2}=\frac{l}{5}\\P_1=P_2.\end{array}\right.\quad P\frac{2l}{5}$	$\frac{3}{5}Pl$

TABLE C (Continued)

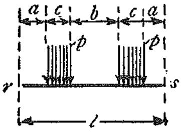
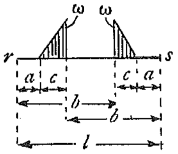
Condition of Loading	$C_{rs}=C_{sr}$	$H_{rs}=H_{sr}$
	$\frac{pc}{6l}\left\{6a(l-a)+3bc+4c^2\right\}$	$\frac{pc}{4l}\left\{6a(l-a)+3bc+4c^2\right\}$
	Special Cases	
	1) $a=0.\quad \frac{pc^2}{6l}(3b+4c)$	$\frac{pc^2}{4l}\left\{3b+4c\right\}$
	2) $b=0.\quad \frac{pc}{3l}\left\{3a(l-a)+2c^2\right\}$	$\frac{pc}{2l}\left\{3a(l-a)+2c^2\right\}$
	3) $a=b=c=\frac{l}{5}.\quad \frac{P31l^2}{750}$	$\frac{31}{500}pl^2$
	4) $a=b=0,\quad 2c=l.\frac{pl^2}{12}$	$\frac{pl^2}{8}$
	$\frac{wc}{12l}\left\{6ab+4bc+2ac+c^2\right\}$	$\frac{wc}{8l}\left\{6ab+4bc+2ac+c^2\right\}$
	Special Cases	
	1) $a=0.\quad \frac{wc^2}{12l}(4b+c)$	$\frac{wc^2}{8l}\left\{4b+c\right\}$
	2) $a+c=b.\quad \frac{wc}{12l}\left\{6a^2+12ac+5c^2\right\}$	$\frac{wc}{8l}\left\{6a^2+12ac+5c^2\right\}$
	3) $a=0,\ b=c=\frac{l}{2}.\quad w\frac{5l^2}{96}$	$\frac{5}{64}wl^2$

TABLE C (Continued)

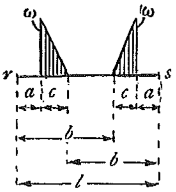
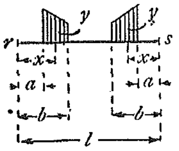
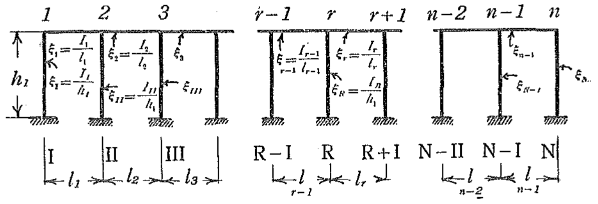
Condition of Loading	$C_{rs}=C_{sr}$	$H_{rs}=H_{sr}$
	$\frac{wc}{12l}\left\{6ab+4ac+2bc+c^2\right\}$	$\frac{wc}{8l}\left\{6ab+4ac+2bc+c^2\right\}$
	Special Cases 1) $a=o.$ $\frac{wc^2}{12l}(2b+c)$	$\frac{wc^2}{8l}\left\{2b+c\right\}$
	2) $a+c=b.$ $\frac{wc}{4l}\left\{2a^2+4ac+c^2\right\}$	$\frac{3wc}{8l}\left\{2a^2+4ac+c^2\right\}$
	3) $a=o,\ b=c=\frac{l}{2}.$ $w\frac{l^2}{32}$	$\frac{3}{64}wl^2$
	$\frac{1}{l}\int_a^b yx(l-x)dx$	$\frac{3}{2l}\int_a^b yx(l-x)dx$
	Special Cases $a=o,\ b=\frac{l}{2}.$ $\frac{1}{l}\int_o^{\frac{l}{2}} yx(l-x)dx$	$\frac{3}{2l}\int_o^{\frac{l}{2}} yx(l-x)dx$

TABLE I

General Equations for a Single Storied Bent with any Number of Spans.
Legs Fixed at the Bases.
Cross-Sections of all Members Different.
Any System of Vertical Loads on Girders.



Equation	Left-Hand Member of Equation													Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ												μ	Vertical Load
	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	→	φ_{n-4}	φ_{n-3}	φ_{n-2}	φ_{n-1}	φ_n	μ_1	
1	ρ_1	ξ_1											$-3\xi_I$	$p_{1.2}$
2	ξ_1	ρ_2	ξ_2										$-3\xi_{II}$	$p_{2.3}$ — $p_{2.1}$
3		ξ_2	ρ_3	ξ_3									$-3\xi_{III}$	$p_{3.4}$ — $p_{3.2}$
4			ξ_3	ρ_4	ξ_4								$-3\xi_{IV}$	$p_{4.5}$ — $p_{4.3}$
5				ξ_4	ρ_5	ξ_5							$-3\xi_V$	$p_{5.6}$ — $p_{5.4}$
↓						↘	↘	↘					↓	↓
$n-3$								ξ_{n-4}	ρ_{n-3}	ξ_{n-3}			$-3\xi_{N-III}$	$p_{n-3,\,n-2}$ — $p_{n-3,\,n-4}$
$n-2$									ξ_{n-3}	ρ_{n-2}	ξ_{n-2}		$-3\xi_{N-II}$	$p_{n-2,\,n-1}$ — $p_{n-2,\,n-3}$
$n-1$										ξ_{n-2}	ρ_{n-1}	ξ_{n-1}	$-3\xi_{N-I}$	$p_{n-1,\,n.}$ — $p_{n-1,\,n-2}$
n											ξ_{n-1}	ρ_n	$-3\xi_N$	— $p_{n,\,n-1}$
$n+1$	ξ_I	ξ_{II}	ξ_{III}	ξ_{IV}	ξ_V	ξ_{VI}	→	ξ_{N-IV}	ξ_{N-III}	ξ_{N-II}	ξ_{N-I}	ξ_N	$-X_1$	0

TABLE II (Figure of Table I)

General Equations for a Single Storied Bent with any Number of Spans.
Legs Fixed at the Bases.
Cross-Sections of all Members Different.
Wind Loads Assumed at the Top of one Vertical Side.

Equation	Left-Hand Member of Equation													Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ												μ	Wind Load
	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	→	φ_{n-4}	φ_{n-3}	φ_{n-2}	φ_{n-1}	φ_n	μ_1	
1	ρ_1	ξ_1											$-3\xi_I$	0
2	ξ_1	ρ_2	ξ_2										$-3\xi_{II}$	0
3		ξ_2	ρ_3	ξ_3									$-3\xi_{III}$	0
4			ξ_3	ρ_4	ξ_4								$-3\xi_{IV}$	0
5				ξ_4	ρ_5	ξ_5							$-3\xi_V$	0
↓						↘	↘	↘					↓	↓
$n-3$								ξ_{n-4}	ρ_{n-3}	ξ_{n-3}			$-3\xi_{N-III}$	0
$n-2$									ξ_{n-3}	ρ_{n-2}	ξ_{n-2}		$-3\xi_{N-II}$	0
$n-1$										ξ_{n-2}	ρ_{n-1}	ξ_{n-1}	$-3\xi_{N-I}$	0
n											ξ_{n-1}	ρ_n	$-3\xi_N$	0
$n+1$	ξ_I	ξ_{II}	ξ_{III}	ξ_{IV}	ξ_V	ξ_{VI}	→	ξ_{N-IV}	ξ_{N-III}	ξ_{N-II}	ξ_{N-I}	ξ_N	$-X_1$	$-q_1$

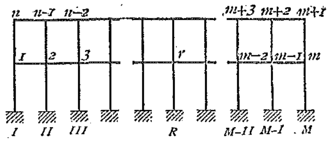
TABLE III (Figure of Table I)

General Equations for a Single Storied Bent with any Number of Spans.
Legs Fixed at the Bases.
Wind Loads Assumed at the Top of one Vertical Side
Any System of Vertical Loads on Girders

Equation	Left-Hand Member of Equation													Right-Hand Member of Equation	
	Coefficients of Unknown Slopes φ												μ	Vertical Load	Wind Load
	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	→	φ_{n-4}	φ_{n-3}	φ_{n-2}	φ_{n-1}	φ_n	μ_1		
1	ρ_1	ξ_1											$-3\xi_{I}$	$p_{1.2}$	0
2	ξ_1	ρ_2	ξ_2										$-3\xi_{II}$	$p_{2.3}$ — $p_{2.1}$	0
3		ξ_2	ρ_3	ξ_3									$-3\xi_{III}$	$p_{3.4}$ — $p_{3.2}$	0
4			ξ_3	ρ_4	ξ_4								$-3\xi_{IV}$	$p_{4.5}$ — $p_{4.3}$	0
5				ξ_4	ρ_5	ξ_5							$-3\xi_{V}$	$p_{5.6}$ — $p_{5.4}$	0
↓					↘	↘	↘						↓	↓	↓
$n-3$								ξ_{n-4}	ρ_{n-3}	ξ_{n-3}			$-3\xi_{N-III}$	$p_{n-3,\, n-2}$ — $p_{n-3,\, n-4}$	0
$n-2$									ξ_{n-3}	ρ_{n-2}	ξ_{n-2}		$-3\xi_{N-II}$	$p_{n-2,\, n-1}$ — $p_{n-2,\, n-3}$	0
$n-1$										ξ_{n-2}	ρ_{n-1}	ξ_{n-1}	$-3\xi_{N-I}$	$p_{n-1,\, n.}$ — $p_{n-1,\, n-2}$	0
n											ξ_{n-1}	ρ_n	$-3\xi_{N}$	— $p_{n,\, n-1}$	0
$n+1$	ξ_{I}	ξ_{II}	ξ_{III}	ξ_{IV}	ξ_{V}	ξ_{VI}	→	ξ_{N-IV}	ξ_{N-III}	ξ_{N-II}	ξ_{N-I}	ξ_{N}	$-X_1$	0	$-q_1$

TABLE IV

General Equations for a Double Storied Bent with any Number of Spans.
Legs Fixed at the Bases.
Cross-Sections of all Members and each Story Height Different
Any System of Vertical Loads on Girders



Equation	Left-Hand Member of Equation																						Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ																				Coeff. of μ		Vertical Load
	φ_1	φ_2	φ_3	φ_4	φ_5	→	φ_{m-3}	φ_{m-2}	φ_{m-1}	φ_m	φ_{m+1}	φ_{m+2}	φ_{m+3}	φ_{m+4}	→	φ_{n-4}	φ_{n-3}	φ_{n-2}	φ_{n-1}	φ_n	μ_1	μ_2	
1	ρ_1	ξ_1																		ξ'_1	$-3\xi_I$	$-3\xi'_1$	$p_{1.2}$
2	ξ_1	ρ_2	ξ_2																ξ'_2		$-3\xi_{II}$	$-3\xi'_2$	$p_{2.3}$ — $p_{2.1}$
3		ξ_2	ρ_3	ξ_3														ξ'_3			$-3\xi_{III}$	$-3\xi'_3$	$p_{3.4}$ — $p_{3.2}$
4			ξ_3	ρ_4	ξ_4												ξ'_4				$-3\xi_{IV}$	$-3\xi'_4$	$p_{4.5}$ — $p_{4.3}$
↓					↘	↘	↘								↙						↓	↓	↓
$m-2$							ξ_{m-3}	ρ_{m-2}	ξ_{m-2}				ξ'_{m-2}								$-3\xi_{M-II}$	$-3\xi'_{m-2}$	$p_{m-2.m-1}$ — $p_{m-2,m-3}$
$m-1$								ξ_{m-2}	ρ_{m-1}	ξ_{m-1}		ξ'_{m-1}									$-3\xi_{M-I}$	$-3\xi'_{m-1}$	$p_{m-1,m}$ — $p_{m-1,m-2}$
m									ξ_{m-1}	ρ_m	ξ_m										$-3\xi_M$	$-3\xi_m$	— $p_{m,m-1}$
$m+1$										ξ_m	ρ_{m+1}	ξ_{m+1}										$-3\xi_m$	— $p_{m+1,m+2}$
$m+2$									ξ'_{m-1}		ξ_{m+1}	ρ_{m+2}	ξ_{m+2}									$-3\xi'_{m-1}$	$p_{m+2,m+1}$ — $p_{m+2,m+3}$
$m+3$								ξ'_{m-2}				ξ_{m+2}	ρ_{m+3}	ξ_{m+2}								$-3\xi'_{m-1}$	$p_{m+3,m+2}$ — $p_{m+3,m+4}$
↓						↗								↘	↘	↘						↑	↓
$n-3$				ξ'_4												ξ_{n-4}	ρ_{n-3}	ξ_{n-3}				$-3\xi'_4$	$p_{n-3,n-4}$ — $p_{n-3,n-2}$
$n-2$			ξ'_3														ξ_{n-3}	ρ_{n-2}	ξ_{n-2}			$-3\xi'_3$	$p_{n-2,n-3}$ — $p_{n-2,n-1}$
$n-1$		ξ'_2																ξ_{n-2}	ρ_{n-1}	ξ_{n-1}		$-3\xi'_2$	$p_{n-1,n-2}$ — $p_{n-1,n}$
n	ξ'_1																		ξ_{n-1}	ρ_n		$-3\xi'_1$	$p_{n,n-1}$
$n+1$	ξ_I	ξ_{II}	ξ_{III}	ξ_{IV}	ξ_V	→	ξ_{M-III}	ξ_{M-II}	ξ_{M-I}	ξ_M											$-X_1$		0
$n+2$	ξ'_1	ξ'_2	ξ'_3	ξ'_4	ξ'_5	→	ξ'_{m-3}	ξ'_{m-2}	ξ'_{m-1}	ξ_m	ξ_m	ξ'_{m-1}	ξ'_{m-2}	ξ'_{m-3}	←	ξ'_5	ξ'_4	ξ'_3	ξ'_2	ξ'_1		$-X_2$	0

Table V (Figure of Table IV)

General Equations for a Double Storied Bent with any Number of Spans.
Legs Fixed at the Bases.
Cross-Sections of all Members and each Story Height Different
Wind Loads Assumed at Joints on one Vertical Side

Equation	Left-Hand Member of Equation																					Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ																			Coeff. of μ		Wind Load
	φ_1	φ_2	φ_3	φ_4	φ_5	→	φ_{m-3}	φ_{m-2}	φ_{m-1}	φ_m	φ_{m+1}	φ_{m+2}	φ_{m+3}	φ_{m+4}	→	φ_{n-4}	φ_{n-3}	φ_{n-2}	φ_{n-1}	φ_n	μ_1	μ_2	
1	ρ_1	ξ_1																		ξ'_1	$-3\xi_{I}$	$-3\xi'_1$	0
2	ξ_1	ρ_2	ξ_2																ξ'_2		$-3\xi_{II}$	$-3\xi'_2$	0
3		ξ_2	ρ_3	ξ_3														ξ'_3			$-3\xi_{III}$	$-3\xi'_3$	0
4			ξ_3	ρ_4	ξ_4												ξ'_4				$-3\xi_{IV}$	$-3\xi'_4$	0
↓					↘	↘	↘								↙						↓	↓	↓
$m-2$							ξ_{m-3}	ρ_{m-2}	ξ_{m-2}				ξ'_{m-2}								$-3\xi_{M-II}$	$-3\xi'_{m-2}$	0
$m-1$								ξ_{m-2}	ρ_{m-1}	ξ_{m-1}		ξ'_{m-1}									$-3\xi_{M-I}$	$-3\xi'_{m-1}$	0
m									ξ_{m-1}	ρ_m	ξ_m										$-3\xi_{M}$	$-3\xi_m$	0
$m+1$										ξ_m	ρ_{m+1}	ξ_{m+1}										$-3\xi_m$	0
$m+2$									ξ'_{m-1}		ξ_{m+1}	ρ_{m+2}	ξ_{m+2}									$-3\xi'_{m-1}$	0
$m+3$								ξ'_{m-2}				ξ_{m+2}	ρ_{m+3}	ξ_{m+3}								$-3\xi'_{m-2}$	0
↓					↗	↗								↘	↘	↘						↑	↓
$n-3$				ξ'_4												ξ_{n-4}	ρ_{n-3}	ξ_{n-3}				$-3\xi'_4$	0
$n-2$			ξ'_3														ξ_{n-3}	ρ_{n-2}	ξ_{n-2}			$-3\xi'_3$	0
$n-1$		ξ'_2																ξ_{n-2}	ρ_{n-1}	ξ_{n-1}		$-3\xi'_2$	0
n	ξ'_1																		ξ_{n-1}	ρ_n		$-3\xi'_1$	0
$n+1$	ξ_{I}	ξ_{II}	ξ_{III}	ξ_{IV}	ξ_{V}	→	ξ_{M-III}	ξ_{M-II}	ξ_{M-I}	ξ_{M}											$-X_1$		$-q_1$
$n+2$	ξ'_1	ξ'_2	ξ'_3	ξ'_4	ξ'_5	→	ξ'_{m-3}	ξ'_{m-2}	ξ'_{m-1}	ξ_m	ξ_m	ξ'_{m-1}	ξ'_{m-2}	ξ'_{m-3}	←	ξ'_5	ξ'_4	ξ'_3	ξ'_2	ξ'_1		$-X_2$	$-q_2$

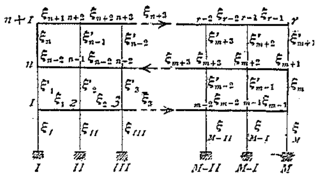
TABLE VI (Figure of Table IV)

General Equations for a Double Storied Bent with any Number of Spans.
Legs Fixed at the Bases.
Cross-Sections of all Members and each Story Height Different
Wind Loads Assumed at Joints on one Vertical Side
Any System of Vertical Loads on Girders

Equation	Left-Hand Member of Equation																						Right-Hand Member of Equation	
	Coefficients of Unknown Slopes φ																				Coeff. of μ		Vertical Load	Wind Load
	φ_1	φ_2	φ_3	φ_4	φ_5	→	φ_{m-3}	φ_{m-2}	φ_{m-1}	φ_m	φ_{m+1}	φ_{m+2}	φ_{m+3}	φ_{m+4}	→	φ_{n-4}	φ_{n-3}	φ_{n-2}	φ_{n-1}	φ_n	μ_1	μ_2		
1	ρ_1	ξ_1																		ξ'_1	$-3\xi_I$	$-3\xi'_1$	$p_{1.2}$	0
2	ξ_1	ρ_2	ξ_2																ξ'_2		$-3\xi_{II}$	$-3\xi'_2$	$p_{2.3}$ — $p_{2.1}$	0
3		ξ_2	ρ_3	ξ_3														ξ'_3			$-3\xi_{III}$	$-3\xi'_3$	$p_{3.4}$ — $p_{3.2}$	0
4			ξ_3	ρ_4	ξ_4												ξ'_4				$-3\xi_{IV}$	$-3\xi'_4$	$p_{4.5}$ — $p_{4.3}$	0
↓					↘	↘	↘							↙							↓	↓	↓	↓
$m-2$							ξ_{m-3}	ρ_{m-2}	ξ_{m-2}				ξ'_{m-2}								$-3\xi_{M-II}$	$-3\xi'_{m-2}$	$p_{m-2,\ m-1}$ — $p_{m-2,\ m-3}$	0
$m-1$								ξ_{m-2}	ρ_{m-1}	ξ_{m-1}		ξ'_{m-1}									$-3\xi_{M-I}$	$-3\xi'_{m-1}$	$p_{m-1,\ m}$ — $p_{m-1,\ m-2}$	0
m									ξ_{m-1}	ρ_m	ξ_m										$-3\xi_M$	$-3\xi_m$	— $p_{m,\ m-1}$	0
$m+1$										ξ_m	ρ_{m+1}	ξ_{m+1}										$-3\xi_m$	— $p_{m+1,\ m+2}$	0
$m+2$									ξ'_{m-1}		ξ_{m+1}	ρ_{m+2}	ξ_{m+2}									$-3\xi'_{m-1}$	$p_{m+2,\ m+1}$ — $p_{m+2,\ m+3}$	0
$m+3$								ξ'_{m-2}				ξ_{m+2}	ρ_{m+3}	ξ_{m+3}								$-3\xi'_{m-2}$	$p_{m+3,\ m+2}$ — $p_{m+3,\ m+4}$	0
↓						↗								↘	↘	↘						↑	↓	↓
$n-3$				ξ'_4												ξ_{n-4}	ρ_{n-3}	ξ_{n-3}				$-3\xi'_4$	$p_{n-3,\ n-4}$ — $p_{n-3,\ n-2}$	0
$n-2$			ξ'_3														ξ_{n-3}	ρ_{n-2}	ξ_{n-2}			$-3\xi'_3$	$p_{n-2,\ n-3}$ — $p_{n-2,\ n-1}$	0
$n-1$		ξ'_2																ξ_{n-2}	ρ_{n-1}	ξ_{n-1}		$-3\xi'_2$	$p_{n-1,\ n-2}$ — $p_{n-1,\ n.}$	0
n	ξ'_1																		ξ_{n-1}	ρ_n		$-3\xi'_1$	$p_{n,\ n-1}$	0
$n+1$	ξ_I	ξ_{II}	ξ_{III}	ξ_{IV}	ξ_V	→	ξ_{M-III}	ξ_{M-II}	ξ_{M-I}	ξ_M											$-X_1$		0	$-q_1$
$n+2$	ξ'_1	ξ'_2	ξ'_3	ξ'_4	ξ'_5	→	ξ'_{m-3}	ξ'_{m-2}	ξ'_{m-1}	ξ_m	ξ_m	ξ'_{m-1}	ξ'_{m-2}	ξ'_{m-3}	←	ξ'_5	ξ'_4	ξ'_3	ξ'_2	ξ'_1		$-X_2$	0	$-q_2$

TABLE VII

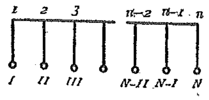
General Equations for a Triple Storied Bent with any Number of Spans.
Legs Fixed at the Bases.
Cross-Sections of all Members Different
Any System of Vectical Loads on Girders
Wind Loads Asssumed at Joints on one Vertical Side



Equation	Left-Hand Member of Equation																								Right-Hand Member of Equation	
	Coefficients of Unknown Slopes φ																					Coeff. of μ			Vertical Load	Wind Load
	φ_1	φ_2	φ_3	→	φ_{m-2}	φ_{m-1}	φ_m	φ_{m+1}	φ_{m+2}	φ_{m+3}	→	φ_{n-2}	φ_{n-1}	φ_n	φ_{n+1}	φ_{n+2}	φ_{n+3}	→	φ_{r-2}	φ_{r-1}	φ_r	μ_1	μ_2	μ_3		
1	ρ_1	ξ_1												ξ'_1								$-3\xi_I$	$-3\xi'_1$		p_{12}	0
2	ξ_1	ρ_2	ξ_2										ξ'_2									$-3\xi_{II}$	$-3\xi'_2$		p_{23} — p_{21}	0
3		ξ_2	ρ_3	ξ_2								ξ'_3										$-3\xi_{III}$	$-3\xi'_3$		p_{34} — p_{32}	0
↓				↘	↘						↙											↓	↓		↓ ↓	↓
				↘																						
$m-I$					ξ_{m-2}	ρ_{m-1}	ξ_{m-1}		ξ'_{m-1}													$-3\xi_{M-I}$	$-3\xi'_{m-1}$		$p_{m-1,m}$ — $p_{m-1,m-2}$	0
m						ξ_{m-1}	ρ_m	ξ_m														$-3\xi_M$	$-3\xi_m$		— $p_{m,m-1}$	0
$m+I$							ξ_m	ρ_{m+1}	ξ_{m+1}												ξ'_{m+1}		$-3\xi_m$	$-3\xi'_{m+1}$	— $p_{m+1,m+2}$	0
$m+2$						ξ'_{m-1}		ξ_{m+1}	ρ_{m+2}	ξ_{m+2}										ξ'_{m+2}			$-3\xi'_{m-1}$	$-3\xi'_{m+2}$	$p_{m+2,m+1}$ — $p_{m+2,m+3}$	0
$m+3$					ξ'_{m-2}				ξ_{m+2}	ρ_{m+3}	ξ_{m+3}								ξ'_{m+3}				$-3\xi'_{m-2}$	$-3\xi'_{m+3}$	$p_{m+3,m+2}$ — $p_{m+3,m+4}$	0
↓				↗							↘	↘						↙						↓	↓ ↓	↓
											↘												↑			
$n-I$		ξ'_2										ξ_{n-2}	ρ_{n-1}	ξ_{n-1}		ξ'_{n-1}							$-3\xi'_2$	$-3\xi'_{n-1}$	$p_{n-1,n-2}$ — $p_{n-1,n}$	0
n	ξ'_1												ξ_{n-1}	ρ_n	ξ_n								$-3\xi'_1$	$-3\xi_n$	$p_{n,n-1}$	0
$n+I$														ξ_n	ρ_{n+1}	ξ_{n+1}								$-3\xi_n$	$p_{n+1,n+2}$	0
$n+2$													ξ'_{n-1}		ξ_{n+1}	ρ_{n+2}	ξ_{n+2}							$-3\xi'_{n-1}$	$p_{n+2,n+3}$ — $p_{n+2,n+1}$	0
$n+3$												ξ'_{n-2}				ξ_{n+2}	ρ_{n+3}	ξ_{n+3}						$-3\xi'_{n-2}$	$p_{n+3,n+4}$ — $p_{n+3,n+2}$	0
↓											↗							↘	↘						↓ ↓	↓
																		↘						↑		
$r-I$									ξ'_{m+2}										ξ_{r-2}	ρ_{r-1}	ξ_{r-1}			$-3\xi'_{m+2}$	$p_{r-1,r}$ — $p_{r-1,r-2}$	0
r								ξ'_{m+1}												ξ_{r-1}	ρ_r			$-3\xi'_{m+1}$	— $p_{r,r-1}$	0
$r+I$	ξ_I	ξ_{II}	ξ_{III}	→	ξ_{M-II}	ξ_{M-I}	ξ_M															$-X_1$			0	$-q_1$
$r+2$	ξ'_1	ξ'_2	ξ'_3	→	ξ'_{m-2}	ξ'_{m-1}	ξ_m	ξ_n	ξ'_{m-1}	ξ'_{m-2}	←	ξ'_3	ξ'_2	ξ'_1									$-X_2$		0	$-q_2$
$r+3$								ξ'_{m+1}	ξ'_{m+2}	ξ'_{m+3}	→	ξ'_{n-2}	ξ'_{n-1}	ξ_n	ξ_n	ξ'_{n-1}	ξ'_{n-2}	←	ξ'_{m+3}	ξ'_{m+2}	ξ'_{m+1}			$-X_3$	0	$-q_3$

TABLE VIII

General Equations for a Single Storied Bent with any Number of Spans.
Legs Hinged at the Bases.
Cross-Sections of all Members Different.
Any System of Vertical Loads on Girders



Equation	Left-Hand Member of Equation													Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ												μ	Vertical Load
	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	→	φ_{n-4}	φ_{n-3}	φ_{n-2}	φ_{n-1}	φ_n	μ_1	
1	ρ_1'	ξ_1											$-1,5\xi_I$	$p_{1.2}$
2	ξ_1	ρ_2'	ξ_2										$-1,5\xi_{II}$	$p_{2.3}$ — $p_{2.1}$
3		ξ_2	ρ_3'	ξ_3									$-1,5\xi_{III}$	$p_{3.4}$ — $p_{3.2}$
4			ξ_3	ρ_4'	ξ_4								$-1,5\xi_{IV}$	$p_{4.5}$ — $p_{4.3}$
5				ξ_4	ρ_5'	ξ_5							$-1,5\xi_V$	$p_{5.6}$ — $p_{5.4}$
↓													↓	↓
						↘	↘	↘						
$n-3$								ξ_{n-4}	ρ_{n-3}'	ξ_{n-3}			$-1,5\xi_{N-III}$	$p_{n-3,\,n-2}$ — $p_{n-3,\,n-4}$
$n-2$									ξ_{n-3}	ρ_{n-2}'	ξ_{n-2}		$-1,5\xi_{N-II}$	$p_{n-2,\,n-1}$ — $p_{n-2,\,n-3}$
$n-1$										ξ_{n-2}	ρ_{n-1}'	ξ_{n-1}	$-1,5\xi_{N-I}$	$p_{n-1,\,n.}$ — $p_{n-1,\,n-2}$
n											ξ_{n-1}	ρ_n'	$-1,5\xi_N$	— $p_{n,\,n-1}$
$n+1$	ξ_I	ξ_{II}	ξ_{III}	ξ_{IV}	ξ_V	ξ_{VI}	→	ξ_{N-IV}	ξ_{N-III}	ξ_{N-II}	ξ_{N-I}	ξ_N	$-\frac{X_1}{2}$	0

TABLE IX (Figure of Table VIII)

General Equations for a Single Storied Bent with any Number of Span.
Legs Hinged at the Bases
Cross-Sections of all Members Different
Wind Loads Assumed at the Top of one Vertical Side

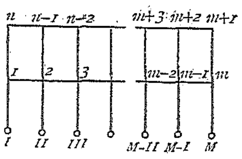
Equation	Left-Hand Member of Equation													Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ												μ	Wind Load
	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	→	φ_{n-4}	φ_{n-3}	φ_{n-2}	φ_{n-1}	φ_n	μ_1	
1	ρ_1'	ξ_1											$-1,5\xi_I$	0
2	ξ_1	ρ_2'	ξ_2										$-1,5\xi_{II}$	0
3		ξ_2	ρ_3'	ξ_3									$-1,5\xi_{III}$	0
4			ξ_3	ρ_4'	ξ_4								$-1,5\xi_{IV}$	0
5				ξ_4	ρ_5'	ξ_5							$-1,5\xi_V$	0
↓													↓	↓
						↘	↘	↘						
$n-3$								ξ_{n-4}	ρ_{n-3}'	ξ_{n-3}			$-1,5\xi_{N-III}$	0
$n-2$									ξ_{n-3}	ρ_{n-2}'	ξ_{n-2}		$-1,5\xi_{N-II}$	0
$n-1$										ξ_{n-2}	ρ_{n-1}'	ξ_{n-1}	$-1,5\xi_{N-I}$	0
n											ξ_{n-1}	ρ_n'	$-1,5\xi_N$	0
$n+1$	ξ_I	ξ_{II}	ξ_{III}	ξ_{IV}	ξ_V	ξ_{VI}	→	ξ_{N-IV}	ξ_{N-III}	ξ_{N-II}	ξ_{N-I}	ξ_N	$-\frac{X_1}{2}$	$-2q_1$

TABLE X (Figure of Table VIII)

General Equations for a Single Storied Bent with any Number of Spans.
Legs Hinged at the Bases
Cross-Sections of all Members Different
Wind Loads Assumed at the Top of one Vertical Side
Any System of Vertical Loads on Girders

Equation	Left-Hand Member of Equation													Right-Hand Member of Equation	
	Coefficients of Unknown Slopes φ												μ	Vertical Load	Wind Load
	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	→	φ_{n-4}	φ_{n-3}	φ_{n-2}	φ_{n-1}	φ_n	μ_1		
1	ρ'_1	ξ_1											$-1{,}5\xi_I$	$p_{1.2}$	0
2	ξ_1	ρ'_2	ξ_2										$-1{,}5\xi_{II}$	$p_{2.3}$ — $p_{2.1}$	0
3		ξ_2	ρ'_3	ξ_3									$-1{,}5\xi_{III}$	$p_{3.4}$ — $p_{3.2}$	0
4			ξ_3	ρ'_4	ξ_4								$-1{,}5\xi_{IV}$	$p_{4.5}$ — $p_{4.3}$	0
5				ξ_4	ρ'_5	ξ_5							$-1{,}5\xi_V$	$p_{5.6}$ — $p_{5.4}$	0
↓					↘	↘	↘						↓	↓	↓
$n-3$								ξ_{n-4}	ρ'_{n-3}	ξ_{n-3}			$-1{,}5\xi_{N-III}$	$p_{n-3,\, n-2}$ — $p_{n-3,\, n-4}$	0
$n-2$									ξ_{n-3}	ρ'_{n-2}	ξ_{n-2}		$-1{,}5\xi_{N-II}$	$p_{n-2,\, n-1}$ — $p_{n-2,\, n-3}$	0
$n-1$										ξ_{n-2}	ρ'_{n-1}	ξ_{n-1}	$-1{,}5\xi_{N-I}$	$p_{n-1,\, n,}$ — $p_{n-1,\, n-2}$	0
n											ξ_{n-1}	ρ'_n	$-1{,}5\xi_N$	— $p_{n,\, n-1}$	0
$n+1$	ξ_I	ξ_{II}	ξ_{III}	ξ_{IV}	ξ_V	ξ_{VI}	→	ξ_{N-IV}	ξ_{N-III}	ξ_{N-II}	ξ_{N-I}	ξ_N	$-\dfrac{X_1}{2}$	0	$-2q_1$

TABLE XI

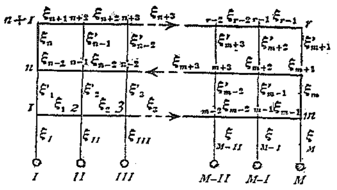


General Equations for a Double Storied Bent with any Number of Spans.
Legs Hinged at the Bases.
Cross-Sections of all Members and each Story Height Different
Wind Loads Assumed at Joints on one Vertical Side
Any System of Vertical Loads on Girders

Equation	Left-Hand Member of Equation																						Left-Hand Member of Equation	Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ																				Coeff. of μ			Wind Load
	φ_1	φ_2	φ_3	φ_4	φ_5	→	φ_{m-3}	φ_{m-2}	φ_{m-1}	φ_m	φ_{m+1}	φ_{m+2}	φ_{m+3}	φ_{m+4}	→	φ_{n-4}	φ_{n-3}	φ_{n-2}	φ_{n-1}	φ_n	μ_1	μ_2		
1	ρ'_1	ξ_1																		ξ'_1	$-1{,}5\xi_I$	$-3\xi'_1$	$p_{1.2}$	o
2	ξ_1	ρ'_2	ξ_2																ξ'_2		$-1{,}5\xi_{II}$	$-3\xi'_2$	$p_{2.3}$ — $p_{2.1}$	o
3		ξ_2	ρ'_3	ξ_3														ξ'_3			$-1{,}5\xi_{III}$	$-3\xi'_3$	$p_{3.4}$ — $p_{3.2}$	o
4			ξ_3	ρ'_4	ξ_4												ξ'_4				$-1{,}5\xi_{IV}$	$-3\xi'_4$	$p_{4.5}$ — $p_{4.3}$	o
↓																					↓	↓	↓	↓
						↘	↘	↘						↙										
m−2							ξ_{m-3}	ρ'_{m-2}	ξ_{m-2}				ξ'_{m-2}								$-3\xi_{M-II}$	$-3\xi'_{m-2}$	$p_{m-2,\,m-1}$ — $p_{m-2,\,m-3}$	o
m−1								ξ_{m-2}	ρ'_{m-1}	ξ_{m-1}		ξ'_{m-1}									$-3\xi_{M-I}$	$-3\xi'_{m-1}$	$p_{m-1,\,m}$ — $p_{m-1,\,m-2}$	o
m									ξ_{m-1}	ρ'_m	ξ_m										$-3\xi_M$	$-3\xi_m$	— $p_{m,\,m-1}$	o
m+1										ξ_m	ρ_{m+1}	ξ_{m+1}										$-3\xi_m$	— $p_{m+1,\,m+2}$	o
m+2									ξ'_{m-1}		ξ_{m+1}	ρ_{m+2}	ξ_{m+2}									$-3\xi'_{m-1}$	$p_{m+2,\,m+1}$ — $p_{m+2,\,m+3}$	o
m+3								ξ'_{m-2}				ξ_{m+2}	ρ_{m+3}	ξ_{m+3}								$-3\xi'_{m-2}$	$p_{m+3,\,m+2}$ — $p_{m+3,\,m+4}$	o
↓																						↑	↓	↓
						↗								↘	↘	↘								
n−3				ξ'_4												ξ_{n-4}	ρ_{n-3}	ξ_{n-3}				$-3\xi'_4$	$p_{n-3,\,n-4}$ — $p_{n-3,\,n-2}$	o
n−2			ξ'_3														ξ_{n-3}	ρ_{n-2}	ξ_{n-2}			$-3\xi'_3$	$p_{n-2,\,n-3}$ — $p_{n-2,\,n-1}$	o
n−1		ξ'_2																ξ_{n-2}	ρ_{n-1}	ξ_{n-1}		$-3\xi'_2$	$p_{n-1,\,n-2}$ — $p_{n-1,\,n.}$	o
n	ξ'_1																		ξ_{n-1}	ρ_n		$-3\xi'_1$	$p_{n,\,n-1}$	o
n+1	ξ_I	ξ_{II}	ξ_{III}	ξ_{IV}	ξ_V	→	ξ_{M-III}	ξ_{M-II}	ξ_{M-I}	ξ_M											$-\frac{X_1}{2}$		o	$-2q_1$
n+2	ξ'_1	ξ'_2	ξ'_3	ξ'_4	ξ'_5	→	ξ'_{m-3}	ξ'_{m-2}	ξ'_{m-1}	ξ_m	ξ_m	ξ'_{m-1}	ξ'_{m-2}	ξ'_{m-3}	←	ξ'_5	ξ'_4	ξ'_3	ξ'_2	ξ'_1		$-X_2$	o	$-q_2$

TABLE XII

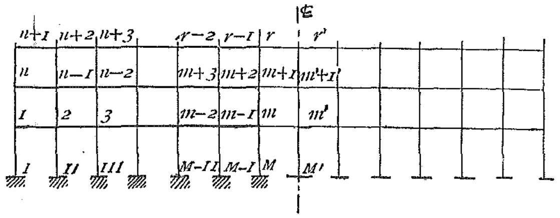
General Equations for a Triple Storied Bent with any Number of Spans.
Legs Hinged at the Bases
Cross-Sections of all Members Different
Any System of Vertical Loads on Girders
Wind Loads Assumed at Joints on one Vertical Side



Equation	Left-Hand Member of Equation																								Right-Hand Member of Equation	
	Coefficients of Unknown Slopes φ																					Coeff. of μ			Vertical Load	Wind Load
	φ_1	φ_2	φ_3	→	φ_{m-2}	φ_{m-1}	φ_m	φ_{m+1}	φ_{m+2}	φ_{m+3}	→	φ_{n-2}	φ_{n-1}	φ_n	φ_{n+1}	φ_{n+2}	φ_{n+3}	→	φ_{r-2}	φ_{r-1}	φ_r	μ_1	μ_2	μ_3		
1	ρ'_1	ξ_1												ξ'_1								$-1{,}5\xi_I$	$-3\xi'_1$		p_{12}	o
2	ξ_1	ρ'_2	ξ_2										ξ'_2									$-1{,}5\xi_{II}$	$-3\xi'_2$		p_{23} — p_{21}	o
3		ξ_2	ρ'_3	ξ_3								ξ'_3										$-1{,}5\xi_{III}$	$-3\xi'_3$		p_{34} — p_{32}	o
↓				↘	↘						↙											↓	↓		↓ ↓	↓
				↘																						
$m-1$					ξ_{m-2}	ρ'_{m-1}	ξ_{m-1}		ξ_{m-1}													$-1{,}5_{M-1}$	$-3\xi'_{m-1}$		$p_{m-1,\,m}$ — $p_{m-1,\,m-2}$	o
m						ξ_{m-1}	ρ'_m	ξ_m														$-1{,}5\xi_M$	$-3\xi_m$		— $p_{m,\,m-1}$	o
$m+1$							ξ_m	ρ_{m+1}	ξ_{m+1}												ξ'_{m+1}		$-3\xi_m$	$-3\xi'_{m+1}$	— $p_{m+1,\,m+2}$	o
$m+2$						ξ'_{m-1}		ξ_{m+1}	ρ_{m+2}	ξ_{m+2}										ξ'_{m+2}			$-3\xi'_{m-1}$	$-3\xi'_{m+2}$	$p_{m+2,\,m+1}$ — $p_{m+2,\,m+3}$	o
$m+3$					ξ'_{m-2}				ξ_{m+2}	ρ_{m+3}	ξ_{m+3}								ξ'_{m+3}				$-3\xi'_{m-2}$	$-3\xi'_{m+3}$	$p_{m+3,\,m+2}$ — $p_{m+3,\,m+4}$	o
↓				↗							↘	↘						↙						↓	↓ ↓	↓
											↘												↑			
$n-1$		ξ'_2										ξ_{n-2}	ρ_{n-1}	ξ_{n-1}		ξ'_{n-1}							$-3\xi'_2$	$-3\xi'_{n-1}$	$p_{n-1,\,n+2}$ — $p_{n-1,\,n}$	o
n	ξ'_1												ξ_{n-1}	ρ_n	ξ_n								$-3\xi'_1$	$-3\xi_n$	$p_{n,\,n-1}$	o
$n+1$														ξ_n	ρ_{n+1}	ξ_{n+1}								$-3\xi_n$	$p_{n+1,\,n+2}$	o
$n+2$													ξ'_{n-1}		ξ_{n+1}	ρ_{n+2}	ξ_{n+2}							$-3\xi'_{n-1}$	$p_{n+2,\,n+3}$ — $p_{n+2,\,n+1}$	o
$n+3$												ξ'_{n-2}				ξ_{n+2}	ρ_{n+3}	ξ_{n+3}						$-3\xi'_{n-2}$	$p_{n+3,\,n+4}$ — $p_{n+3,\,n+2}$	o
↓											↗							↘	↘						↓ ↓	↓
																		↘						↑		
$r-1$									ξ'_{m+2}										ξ_{r-2}	ρ_{r-1}	ξ_{r-1}			$-3\xi'_{m+2}$	$p_{r-1,}$ — $p_{r-1,\,r-2}$	o
r								ξ'_{m+1}												ξ_{r-1}	ρ_r			$-3\xi'_{m+1}$	— $p_{r,\,r-1}$	o
$r+1$	ξ_I	ξ_{II}	ξ_{III}	→	ξ_{M-II}	ξ_{M-I}	ξ_M															$-\frac{X_1}{2}$			o	$-2q_1$
$r+2$	ξ'_1	ξ'_2	ξ'_3	→	ξ'_{m-2}	ξ'_{m-1}	ξ_m	ξ_m	ξ'_{m-1}	ξ'_{m-2}	←	ξ'_3	ξ'_2	ξ'_1									$-X_2$		o	$-q_2$
$r+3$								ξ'_{m+1}	ξ'_{m+2}	ξ'_{m+3}	→	ξ'_{n-2}	ξ'_{n-1}	ξ_n	ξ_n	ξ'_{n-1}	ξ'_{n-2}	←	ξ'_{m+3}	ξ'_{m+2}	ξ'_{m+1}			$-X_3$	o	$-q_3$

TABLE XIII

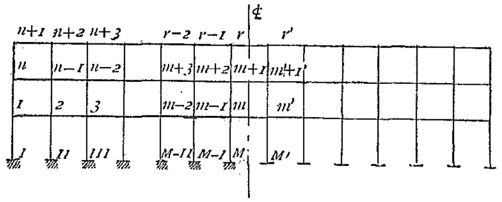
General Equations for a Triple Storied Bent with an Axis of Symmetry of the Building.
Legs Fixed at the Bases.
Any System of Symmetrical Vertical Loads about the Axis of Symmetry
Number of Spans 2m



Equation	Left-Hand Member of Equation																					Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ																					Vertical Load
	φ_1	φ_2	φ_3	→	φ_{m-2}	φ_{m-1}	φ_m	φ_{m+1}	φ_{m+2}	φ_{m+3}	→	φ_{n-2}	φ_{n-1}	φ_n	φ_{n+1}	φ_{n+2}	φ_{n+3}	→	φ_{r-2}	φ_{r-1}	φ_r	
1	ρ_1	ξ_1												ξ'_1								p_{12}
2	ξ_1	ρ_2	ξ_2										ξ'_2									p_{23} — p_{21}
3		ξ_2	ρ_3	ξ_3								ξ'_3										p_{34} — p_{32}
↓				↘	↘						↙											↓ ↓
				↘																		
$m-1$					ξ_{m-2}	ρ_{m-1}	ξ_{m-1}		ξ'_{m-1}													$p_{m-1,m}$ — $p_{m-1,m-2}$
m						ξ_{m-1}	ρ_m	ξ_m														$p_{m,m'}$ — $p_{m,m-1}$
$m+1$							ξ_m	ρ_{m+1}	ξ_{m+1}												ξ'_{m+1}	$p_{m+1,m'+1'}$ — $p_{m+1,m+2}$
$m+2$						ξ'_{m-1}		ξ_{m+1}	ρ_{m+2}	ξ_{m+2}										ξ'_{m+2}		$p_{m+2,m+1}$ — $p_{m+2,m+3}$
$m+3$					ξ'_{m-2}				ξ_{m+2}	ρ_{m+3}	ξ_{m+3}								ξ'_{m+3}			$p_{m+3,m+2}$ — $p_{m+3,m+4}$
↓				↗							↘	↘						↙				↓ ↓
											↘											
$n-1$		ξ'_2										ξ_{n-2}	ρ_{n-1}	ξ_{n-1}		ξ'_{n-1}						$p_{n-1,n-2}$ — $p_{n-1,n}$
n	ξ'_1												ξ_{n-1}	ρ_n	ξ_n							$p_{n,n-1}$
$n+1$														ξ_n	ρ_{n+1}	ξ_{n+1}						$p_{n+1,n+2}$
$n+2$													ξ'_{n-1}		ξ_{n+1}	ρ_{n+2}	ξ_{n+2}					$p_{n+2,n+3}$ — $p_{n+2,n+1}$
$n+3$												ξ'_{n-2}				ξ_{n+2}	ρ_{n+3}	ξ_{n+3}				$p_{n+3,n-4}$ — $p_{n+3,n+2}$
↓											↗							↘	↘			↓ ↓
																		↘				
$r-1$									ξ'_{m+2}										ξ_{r-2}	ρ_{r-1}	ξ_{r-1}	$p_{r-1,r}$ — $p_{r-1,r-2}$
r								ξ'_{m+1}												ξ_{r-1}	ρ_r	$p_{r,r'}$ — $p_{r,r-1}$

Table XIV

General Equations for a Triple Storied Bent with an Axis of Symmetry of the Building.
Legs Fixed at the Bases
Any System of Symmetrical Vertical Loads about the Axis of Symmetry
Number of Spans 2m–1



Equation	Left-Hand Member of Equation																					Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ																					Vertical Load
	φ_1	φ_2	φ_3	→	φ_{m-2}	φ_{m-1}	φ_m	φ_{m+1}	φ_{m+2}	φ_{m+3}	→	φ_{n-2}	φ_{n-1}	φ_n	φ_{n+1}	φ_{n+2}	φ_{n+3}	→	φ_{r-2}	φ_{r-1}	φ_r	
1	ρ_1	ξ_1												ξ'_1								p_{12}
2	ξ_1	ρ_2	ξ_2										ξ'_2									p_{23} — p_{21}
3		ξ_2	ρ_3	ξ_3								ξ'_3										p_{34} — p_{32}
↓				↘	↘						↙											↓ ↓
				↘																		
$m-1$					ξ_{m-2}	ρ_{m-1}	ξ_{m-1}		ξ'_{m-1}													$p_{m-1,\,m}$ — $p_{m-1,\,m-2}$
m						ξ_{m-1}	ρ''_m	ξ_m														$p_{m,\,m'}$ — $p_{m,\,m-1}$
$m+1$							ξ_m	ρ''_{m+1}	ξ_{m+1}												ξ'_{m+1}	$p_{m+1,\,m'+1'}$ — $p_{m+1,\,m+2}$
$m+2$						ξ'_{m-1}		ξ_{m+1}	ρ_{m+2}	ξ_{m+2}										ξ'_{m+2}		$p_{m+2,\,m+1}$ — $p_{m+2,\,m+3}$
$m+3$					ξ'_{m-2}				ξ_{m+2}	ρ_{m+3}	ξ_{m+3}								ξ'_{m+3}			$p_{m+3,\,m+2}$ — $p_{m+3,\,m+4}$
↓				↗							↘	↘						↙				↓ ↓
											↘											
$n-1$		ξ'_2										ξ_{n-2}	ρ_{n-1}	ξ_{n-1}		ξ'_{n-1}						$p_{n-1,\,n-2}$ — $p_{n-1,\,n}$
n	ξ'_1												ξ_{n-1}	ρ_n	ξ_n							$p_{n,\,n-1}$
$n+1$														ξ_n	ρ_{n+1}	ξ_{n+1}						$p_{n+1,\,n+2}$
$n+2$													ξ'_{n-1}		ξ_{n+1}	ρ_{n+2}	ξ_{n+2}					$p_{n+2,\,n+3}$ — $p_{n+2,\,n+1}$
$n+3$												ξ'_{n-2}				ξ_{n+2}	ρ_{n+3}	ξ_{n+3}				$p_{n+3,\,n+4}$ — $p_{n+3,\,n+2}$
↓											↗							↘	↘			↓ ↓
																		↘				
$r-1$									ξ'_{m+2}										ξ_{r-2}	ρ_{r-1}	ξ_{r-1}	$p_{r-1,\,r}$ — $p_{r-1,\,r-2}$
r								ξ'_{m+1}												ξ_{r-1}	ρ''_r	$p_{r,\,r'}$ — $p_{r,\,r-1}$

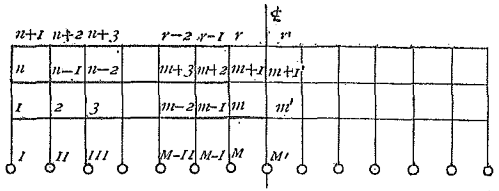
$$\rho''_m \;= 2(\xi_{m-1}+\xi_m+\xi_M)+\xi \text{ of } m-m',$$

$$\rho''_{m+1}=2(\xi_m+\xi_{m+1}+\xi'_{m+1})+\xi \text{ of } m+1,\ m'+1',$$

$$\rho''_r \;= 2(\xi'_{m+1}+\xi_{r-1})+\xi \text{ of } r-r'.$$

TABLE XV

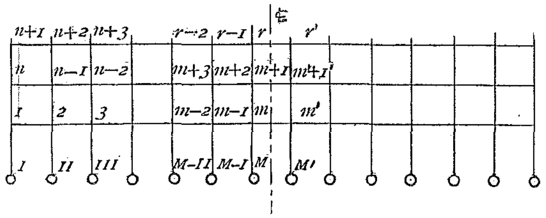
General Equations for a Triple Storied Bent with an Axis of Symmetry of the Building.
Legs Hinged at the Bases
Any System of Symmetrical Vertical Loads about the Axis of Symmetry
Number of Spans 2m



Equation	Left-Hand Member of Equation																					Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ																					Vertical Load
	φ_1	φ_2	φ_3	→	φ_{m-2}	φ_{m-1}	φ_m	φ_{m+1}	φ_{m+2}	φ_{m+3}	→	φ_{n-2}	φ_{n-1}	φ_n	φ_{n+1}	φ_{n+2}	φ_{n+3}	→	φ_{r-2}	φ_{r-1}	φ_r	
1	ρ_1'	ξ_1												ξ'_1								p_{12}
2	ξ_1	ρ_2'	ξ_2										ξ'_2									p_{23} — p_{21}
3		ξ_2	ρ_3'	ξ_3								ξ'_3										p_{34} — p_{32}
↓				↘	↘						↙											↓ ↓
				↘																		
m−1					ξ_{m-2}	ρ'_{m-1}	ξ_{m-1}		ξ'_{m-1}													$p_{m-1,m}$ — $p_{m-1,m-2}$
m						ξ_{m-1}	ρ'_m	ξ_m														$p_{m,m'}$ — $p_{m,m-1}$
m+1							ξ_m	ρ_{m+1}	ξ_{m+1}												ξ'_{m+1}	$p_{m+1,m'+1'}$ — $p_{m+1,m+2}$
m+2						ξ'_{m-1}		ξ_{m+1}	ρ_{m+2}	ξ_{m+2}										ξ'_{m+2}		$p_{m+2,m+1}$ — $p_{m+2,m+3}$
m+3					ξ'_{m-2}				ξ_{m+2}	ρ_{m+3}	ξ_{m+3}								ξ'_{m+3}			$p_{m+3,m+2}$ — $p_{m+3,m+4}$
↓				↗							↘	↘						↙				↓ ↓
											↘											
n−1		ξ'_2										ξ_{n-2}	ρ_{n-1}	ξ_{n-1}		ξ'_{n-1}						$p_{n-1,n-2}$ — $p_{n-1,n}$
n	ξ'_1												ξ_{n-1}	ρ_n	ξ_n							$p_{n,n-1}$
n+1														ξ_n	ρ_{n+1}	ξ_{n+1}						$p_{n+1,n+2}$
n+2													ξ'_{n-1}		ξ_{n+1}	ρ_{n+2}	ξ_{n+2}					$p_{n+2,n+3}$ — $p_{n+2,n+1}$
n+3												ξ'_{n-2}				ξ_{n+2}	ρ_{n+3}	ξ_{n+3}				$p_{n+3,n+4}$ — $p_{n+3,n+2}$
↓											↗							↘	↘			↓ ↓
																		↘				
r−1									ξ'_{m+2}										ξ_{r-2}	ρ_{r-1}	ξ_{r-1}	$p_{r-1,r}$ — $p_{r-1,r-2}$
r								ξ'_{m+1}												ξ_{r-1}	ρ_r	$p_{r,r'}$ — $p_{r,r-1}$

TABLE XVI

General Equations for a Triple Storied Bent with an Axis of Symmetry of the Building.
Legs Hinged at the Bases.
Any System of Symmetrical Vertical Loads about the Axis of Symmetry.
Number of Spans 2m—1.



Equation	Left-Hand Member of Equation																					Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ																					
	φ_1	φ_2	φ_3	→	φ_{m-2}	φ_{m-1}	φ_m	φ_{m+1}	φ_{m+2}	φ_{m+3}	→	φ_{n-2}	φ_{n-1}	φ_n	φ_{n+1}	φ_{n+2}	φ_{n+3}	→	φ_{r-2}	φ_{r-1}	φ_r	Vertical Load
1	ρ'_1	ξ_1												ξ'_1								p_{12}
2	ξ_1	ρ'_2	ξ_2										ξ'_2									p_{23} — p_{21}
3		ξ_2	ρ'_3	ξ_3								ξ'_3										p_{34} — p_{32}
↓				↘	↘						↙											↓ ↓
				↘																		
$m-1$					ξ_{m-2}	ρ'_{m-1}	ξ_{m-1}		ξ'_{m-1}													$p_{m-1,\,m}$ — $p_{m-1,\,m-2}$
m						ξ_{m-1}	ρ''_m	ξ_m														$p_{m,\,m'}$ — $p_{m,\,m-1}$
$m+1$							ξ_m	ρ''_{m+1}	ξ_{m+1}												ξ'_{m+1}	$p_{m+1,\,m'+1'}$ — $p_{m+1,\,m+2}$
$m+2$						ξ'_{m-1}		ξ_{m+1}	ρ_{m+2}	ξ_{m+2}										ξ'_{m+2}		$p_{m+2,\,m+1}$ — $p_{m+2,\,m+3}$
$m+3$					ξ'_{m-2}				ξ_{m+2}	ρ_{m+3}	ξ_{m+3}								ξ'_{m+3}			$p_{m+3,\,m+2}$ — $p_{m+3,\,m+4}$
↓				↗							↘	↘						↙				↓ ↓
											↘											
$n-1$		ξ'_2										ξ_{n-2}	ρ_{n-1}	ξ_{n-1}		ξ'_{n-1}						$p_{n-1,\,n-2}$ — $p_{n-1,\,n}$
n	ξ'_1												ξ_{n-1}	ρ_n	ξ_n							$p_{n,\,n-1}$
$n+1$														ξ_n	ρ_{n+1}	ξ_{n+1}						$p_{n+1,\,n+2}$
$n+2$													ξ'_{n-1}		ξ_{n+1}	ρ_{n+2}	ξ_{n+2}					$p_{n+2,\,n+3}$ — $p_{n+2,\,n+1}$
$n+3$												ξ'_{n-2}				ξ_{n+2}	ρ_{n+3}	ξ_{n+3}				$p_{n+3,\,n+4}$ — $p_{n+3,\,n+2}$
↓											↗							↘	↘			↓ ↓
																		↘				
$r-1$									ξ'_{m+2}										ξ_{r-2}	ρ_{r-1}	ξ_{r-1}	$p_{r-1,\,r}$ — $p_{r-1,\,r-2}$
r								ξ'_{m+1}												ξ_{r-1}	ρ''_r	$p_{r,\,r'}$ — $p_{r,\,r-1}$

$$\rho''_m \;=2(\xi_{m-1}+\xi_m)+1.5\ \xi_M+\xi \text{ of } m-m',$$

$$\rho''_{m+1}=2(\xi_m+\xi_{m+1}+\xi'_{m+1})+\xi \text{ of } m+1,\ m'+1',$$

$$\rho''_r \;=2(\xi'_{m+1}+\xi_{r-1})+\xi \text{ of } r-r'.$$

TABLE XVII (Figure of Table XIII)

General Equations for a Triple Storied Bent with an Axis of Symmetry of the Building.
Legs Fixed at the Bases
Any System of Symmetrical Vertical Loads about the Center of each Span
Number of Spans 2m

Equation	Left-Hand Member of Equation																						Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ																						Vertical Load
	φ_1	φ_2	φ_3	→	φ_{m-2}	φ_{m-1}	φ_m	φ_{m+1}	φ_{m+2}	φ_{m+3}	→	φ_{n-2}	φ_{n-1}	φ_n	φ_{n+1}	φ_{n+2}	φ_{n+3}	→	φ_{r-2}	φ_{r-1}	φ_r		
1	ρ_1	ξ_1												ξ'_1									p_1
2	ξ_1	ρ_2	ξ_2										ξ'_2										p_2 — p_1
3		ξ_2	ρ_3	ξ_3								ξ'_3											p_3 — p_2
↓				↘	↘						↙												↓
				↘																			
$m-1$					ξ_{m-2}	ρ_{m-1}	ξ_{m-1}		ξ'_{m-1}														p_{m-1} — p_{m-2}
m						ξ_{m-1}	ρ_m	ξ_m															p_m — p_{m-1}
$m+1$							ξ_m	ρ_{m+1}	ξ_{m+1}												ξ'_{m+1}		p_{m+1} — p_{m+2}
$m+2$						ξ'_{m-1}		ξ_{m+1}	ρ_{m+2}	ξ_{m+2}										ξ'_{m+2}			p_{m+2} — p_{m+3}
$m+3$					ξ'_{m-2}				ξ_{m+2}	ρ_{m+3}	ξ_{m+3}								ξ'_{m+3}				p_{m+3} — p_{m+4}
↓				↗							↘	↘						↙					↓
											↘												
$n-1$		ξ'_2										ξ_{n-2}	ρ_{n-1}	ξ_{n-1}		ξ'_{n-1}							p_{n-1} — p_n
n	ξ'_1												ξ_{n-1}	ρ_n	ξ_n								p_n
$n+1$														ξ_n	ρ_{n+1}	ξ_{n+1}							p_{n+1}
$n+2$													ξ'_{n-1}		ξ_{n+1}	ρ_{n+2}	ξ_{n+2}						p_{n+2} — p_{n+1}
$n+3$												ξ'_{n-2}				ξ_{n+2}	ρ_{n+3}	ξ_{n+3}					p_{n+3} — p_{n+2}
↓											↗							↘	↘				↓
																		↘					
$r-1$									ξ'_{m+2}										ξ_{r-2}	ρ_{r-1}	ξ_{r-1}		p_{r-1} — p_{r-2}
r								ξ'_{m+1}												ξ_{r-1}	ρ_r		p_r — p_{r-1}

(The similar problem has been tried by the author. This memoir, Vol. I, No. 2, 1926).

TABLE XVIII (Figure of Table XIV)

General Equations for a Triple Storied Bent with an Axis of Symmetry of the Building.
Legs Fixed at the Bases
Any System of Symmetrical Vertical Loads about the Center of each Span
Number of Spans 2m−1

Equation	Left-Hand Member of Equation																					Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ																					Vertical Load
	φ_1	φ_2	φ_3	→	φ_{m-2}	φ_{m-1}	φ_m	φ_{m+1}	φ_{m+2}	φ_{m+3}	→	φ_{n-2}	φ_{n-1}	φ_n	φ_{n+1}	φ_{n+2}	φ_{n+3}	→	φ_{r-2}	φ_{r-1}	φ_r	
1	ρ_1	ξ_1												ξ'_1								p_1
2	ξ_1	ρ_2	ξ_2										ξ'_2									p_2 — p_1
3		ξ_2	ρ_3	ξ_3								ξ'_3										p_3 — p_2
↓				↘	↘						↙											↓
				↘																		
$m-1$					ξ_{m-2}	ρ_{m-1}	ξ_{m-1}		ξ'_{m-1}													p_{m-1} — p_{m-2}
m						ξ_{m-1}	ρ''_m	ξ_m														p_m — p_{m-1}
$m+1$							ξ_m	ρ''_{m+1}	ξ_{m+1}												ξ'_{m+1}	p_{m+1} — p_{m+2}
$m+2$						ξ'_{m-1}		ξ_{m+1}	ρ_{m+2}	ξ_{m+2}										ξ'_{m+2}		p_{m+2} — p_{m+3}
$m+3$					ξ'_{m-2}				ξ_{m+2}	ρ_{m+3}	ξ_{m+3}								ξ'_{m+3}			p_{m+3} — p_{m+4}
↓				↗							↘	↘						↙				↓
											↘											
$n-1$		ξ'_2										ξ_{n-2}	ρ_{n-1}	ξ_{n-1}		ξ'_{n-1}						p_{n-1} — p_n
n	ξ'_1												ξ_{n-1}	ρ_n	ξ_n							p_n
$n+1$														ξ_n	ρ_{n+1}	ξ_{n+1}						p_{n+1}
$n+2$													ξ'_{n-1}		ξ_{n+1}	ρ_{n+2}	ξ_{n+2}					p_{n+2} — p_{n+1}
$n+3$												ξ'_{n-2}				ξ_{n+2}	ρ_{n+3}	ξ_{n+3}				p_{n+3} — p_{n+2}
↓											↗							↘	↘			↓
																		↘				
$r-1$									ξ'_{m+2}										ξ_{r-2}	ρ_{r-1}	ξ_{r-1}	p_{r-1} — p_{r-2}
r								ξ'_{m+1}												ξ_{r-1}	ρ''_r	p_r — p_{r-1}

$$\rho''_m \;=2(\xi_{m-1}+\xi_m+\xi_M)+\xi \text{ of } m-m',$$

$$\rho''_{m+1}=2(\xi_m+\xi_{m+1}+\xi'_{m+1})+\xi \text{ of } m+1,\ m'+1',$$

$$\rho''_r \;=2(\xi'_{m+1}+\xi_{r-1})+\xi \text{ of } r-r'.$$

TABLE XIX (Figure of Table XV)

General Equations for a Triple Storied Bent with an Axis of Symmetry of the Building.
Legs Hinged at the Bases
Any System of Symmetrical Vertical Loads about the Center of each Span
Number of Spans 2m

Equation	Left-Hand Member of Equation																					Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ																					Vertical Load
	φ_1	φ_2	φ_3	→	φ_{m-2}	φ_{m-1}	φ_m	φ_{m+1}	φ_{m+2}	φ_{m+3}	→	φ_{n-2}	φ_{n-1}	φ_n	φ_{n+1}	φ_{n+2}	φ_{n+3}	→	φ_{r-2}	φ_{r-1}	φ_r	
1	ρ'_1	ξ_1												ξ'_1								p_1
2	ξ_1	ρ'_2	ξ_2										ξ'_2									p_2 — p_1
3		ξ_2	ρ'_3	ξ_3								ξ'_3										p_3 — p_2
↓				↘	↘						↙											↓
				↘																		
m-1					ξ_{m-2}	ρ'_{m-1}	ξ_{m-1}		ξ'_{m-1}													p_{m-1} — p_{m-2}
m						ξ_{m-1}	ρ'_m	ξ_m														p_m — p_{m-1}
m+1							ξ_m	ρ_{m+1}	ξ_{m+1}												ξ'_{m+1}	p_{m+1} — p_{m+2}
m+2						ξ'_{m-1}		ξ_{m+1}	ρ_{m+2}	ξ_{m+2}										ξ'_{m+2}		p_{m+2} — p_{m+3}
m+3					ξ'_{m-2}				ξ_{m+2}	ρ_{m+3}	ξ_{m+3}								ξ'_{m+3}			p_{m+3} — p_{m+4}
↓				↗							↘	↘						↙				↓
											↘											
n-1		ξ'_2										ξ_{n-2}	ρ_{n-1}	ξ_{n-1}		ξ'_{n-1}						p_{n-1} — p_n
n	ξ'_1												ξ_{n-1}	ρ_n	ξ_n							p_n
n+1														ξ_n	ρ_{n+1}	ξ_{n+1}						p_{n+1}
n+2													ξ'_{n-1}		ξ_{n+1}	ρ_{n+2}	ξ_{n+2}					p_{n+2} — p_{n+1}
n+3												ξ'_{n-2}				ξ_{n+2}	ρ_{n+3}	ξ_{n+3}				p_{n+3} — p_{n+2}
↓											↗							↘	↘			↓
																		↘				
r-1									ξ'_{m+2}										ξ_{r-2}	ρ_{r-1}	ξ_{r-1}	p_{r-1} — p_{r-2}
r								ξ'_{m+1}												ξ_{r-1}	ρ_r	p_r — p_{r-1}

TABLE XX (Figure of Table XVI)

General Equations for a Triple Storied Bent with an Axis of Symmetry of the Building.
Legs Hinged at the Bases
Any System of Symmetrical Vertical Loads about the Center of each Span
Number of Spans 2m–1

Equation	Left-Hand Member of Equation																					Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ																					Vertical Load
	φ_1	φ_2	φ_3	→	φ_{m-2}	φ_{m-1}	φ_m	φ_{m+1}	φ_{m+2}	φ_{m+3}	→	φ_{n-2}	φ_{n-1}	φ_n	φ_{n+1}	φ_{n+2}	φ_{n+3}	→	φ_{r-2}	φ_{r-1}	φ_r	
1	ρ'_1	ξ_1												ξ'_1								p_1
2	ξ_1	ρ'_2	ξ_2										ξ'_2									p_2 — p_1
3		ξ_2	ρ'_3	ξ_3								ξ'_3										p_3 — p_2
↓				↘	↘						↙											↓
				↘																		
$m-1$					ξ_{m-2}	ρ'_{m-1}	ξ_{m-1}		ξ'_{m-1}													p_{m-1} — p_{m-2}
m						ξ_{m-1}	ρ''_m	ξ_m														p_m — p_{m-1}
$m+1$							ξ_m	ρ''_{m+1}	ξ_{m+1}												ξ'_{m+1}	p_{m+1} — p_{m+2}
$m+2$						ξ'_{m-1}		ξ_{m+1}	ρ_{m+2}	ξ_{m+2}										ξ'_{m+2}		p_{m+2} — p_{m+3}
$m+3$					ξ'_{m-2}				ξ_{m+2}	ρ_{m+3}	ξ_{m+3}								ξ'_{m+3}			p_{m+3} — p_{m+4}
↓				↗							↘	↘						↙				↓
											↘											
$n-1$		ξ'_2										ξ_{n-2}	ρ_{n-1}	ξ_{n-1}		ξ'_{n-1}						p_{n-1} — p_n
n	ξ'_1												ξ_{n-1}	ρ_n	ξ_n							p_n
$n+1$														ξ_n	ρ_{n+1}	ξ_{n+1}						p_{n+1}
$n+2$													ξ'_{n-1}		ξ_{n+1}	ρ_{n+2}	ξ_{n+2}					p_{n+2} — p_{n+1}
$n+3$												ξ'_{n-2}				ξ_{n+2}	ρ_{n+3}	ξ_{n+3}				p_{n+3} — p_{n+2}
↓											↗							↘	↘			↓
																		↘				
$r-1$									ξ'_{m+2}										ξ_{r-2}	ρ_{r-1}	ξ_{r-1}	p_{r-1} — p_{r-2}
r								ξ'_{m+1}												ξ_{r-1}	ρ''_r	p_r — p_{r-1}

$$\rho''_m = 2(\xi_{m-1}+\xi_m)+1{,}5\xi_M+\xi \text{ of } m-m',$$

$$\rho''_{m+1} = 2(\xi_m+\xi_{m+1}+\xi'_{m+1})+\xi \text{ of } m+1,\ m'+1',$$

$$\rho''_r = 2(\xi'_{m+1}+\xi_{r-1})+\xi \text{ of } r-r'.$$

TABLE XXI (Fig. 6a)

General Equations for a Continuous Beam Fixed at both Ends.
Supports all on same Level
Any Number of Spans
Cross-Sections and Span Lengths all Different
Any System of Vertical Loads on Beams

Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ												Vertical Load
	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	→	φ_{n-4}	φ_{n-3}	φ_{n-2}	φ_{n-1}	φ_n	
1	ρ_1	ξ_1											$p_{1.2}$ — $p_{1.0}$
2	ξ_1	ρ_2	ξ_2										$p_{2.3}$ — $p_{2.1}$
3		ξ_2	ρ_3	ξ_3									$p_{3.4}$ — $p_{3.2}$
4			ξ_3	ρ_4	ξ_4								$p_{4.5}$ — $p_{4.3}$
5				ξ_4	ρ_5	ξ_5							$p_{5.6}$ — $p_{5.4}$
↓													↓
							↘	↘	↘				
$n-3$								ξ_{n-4}	ρ_{n-3}	ξ_{n-3}			$p_{n-3,\,n-2}$ — $p_{n-3,\,n-4}$
$n-2$									ξ_{n-3}	ρ_{n-2}	ξ_{n-2}		$p_{n-2,\,n-1}$ — $p_{n-2,\,n-3}$
$n-1$										ξ_{n-2}	ρ_{n-1}	ξ_{n-1}	$p_{n-1,\,n,}$ — $p_{n-1,\,n-2}$
n											ξ_{n-1}	ρ_n	$p_{n,\,n+1}$ — $p_{n,\,n-1}$

TABLE XXII (Fig. 6b)

General Equations for a Continuous Beam Hinged at both Ends.
Supports all on same Level
Any Number of Spans
Cross-Sections and Span Lengths all Different
Any System of Vertical Loads on Beams

Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ												Vertical Load
	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	→	φ_{n-4}	φ_{n-3}	φ_{n-2}	φ_{n-1}	φ_n	
1	ρ_1'	ξ_1											$p_{1,2}$ — $p_{1.0}$ $-0{,}5 \cdot p_{0,1}$
2	ξ_1	ρ_2	ξ_2										$p_{2.3}$ — $p_{2.1}$
3		ξ_2	ρ_3	ξ_3									$p_{3.4}$ — $p_{3.2}$
4			ξ_3	ρ_4	ξ_4								$p_{4.5}$ — $p_{4.3}$
5				ξ_4	ρ_5	ξ_5							$p_{5.6}$ — $p_{5.4}$
↓													↓
							↘	↘	↘				
$n-3$								ξ_{n-4}	ρ_{n-3}	ξ_{n-3}			$p_{n-3,\,n-2}$ — $p_{n-3,\,n-4}$
$n-2$									ξ_{n-3}	ρ_{n-2}	ξ_{n-2}		$p_{n-2,\,n-1}$ — $p_{n-2,\,n-3}$
$n-1$										ξ_{n-2}	ρ_{n-1}	ξ_{n-1}	$p_{n-1,\,n,}$ — $p_{n-1,\,n-2}$
n											ξ_{n-1}	ρ_n'	$0{,}5\; p_{n+1,\,n} + p_{n,\,n+1}$ — $p_{n,\,n-1}$

TABLE XXIII (Fig. 6c)

General Equations for a Continuous Beam Fixed at one End and Hinged at the other.
Supports all on same Level.
Any Number of Spans.
Cross-Sections and Span Lengths all Different.
Any System of Vertical Loads on Beams.

Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ												Vertical Load
	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	→	φ_{n-4}	φ_{n-3}	φ_{n-2}	φ_{n-1}	φ_n	
1	ρ_1	ξ_1											$p_{1.2} - p_{1.0}$
2	ξ_1	ρ_2	ξ_2										$p_{2.3} - p_{2.1}$
3		ξ_2	ρ_3	ξ_3									$p_{3.4} - p_{3.2}$
4			ξ_3	ρ_4	ξ_4								$p_{4.5} - p_{4.3}$
5				ξ_4	ρ_5	ξ_5							$p_{5.6} - p_{5.4}$
↓													↓
						↘	↘	↘					
$n-3$								ξ_{n-4}	ρ_{n-3}	ξ_{n-3}			$p_{n-3,\,n-2} - p_{n-3,\,n-4}$
$n-2$									ξ_{n-3}	ρ_{n-2}	ξ_{n-2}		$p_{n-2,\,n-1} - p_{n-2,\,n-3}$
$n-1$										ξ_{n-2}	ρ_{n-1}	ξ_{n-1}	$p_{n-1,\,n} - p_{n-1,\,n-2}$
n											ξ_{n-1}	ρ'_n	$0.5\,p_{n+1,\,n} + p_{n,\,n+1} - p_{n,\,n-1}$

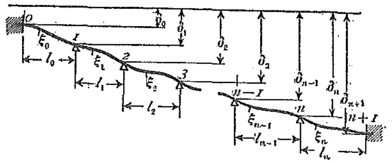


TABLE XXIV

General Equations for a Continuous Beam Fixed at both Ends.
Supports on Different Levels. Any Number of Spans.
Cross-Sections and Span Lengths all Different.
Any System of Vertical Loads on Beams.

Equation	Left-Hand Member of Equation												Right-Hand Member of Equation	
	Coefficients of Unknown Slopes φ												Vertical Load	Settlement of Foundation
	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	→	φ_{n-4}	φ_{n-3}	φ_{n-2}	φ_{n-1}	φ_n		
1	ρ_1	ξ_1											$p_{1.2} - p_{1.0}$	$3\,\xi_0(\partial_1-\partial_0):l_0 + 3\,\xi_1(\partial_2-\partial_1):l_1$
2	ξ_1	ρ_2	ξ_2										$p_{2.3} - p_{2.1}$	$3\,\xi_1(\partial_2-\partial_1):l_1 + 3\,\xi_2(\partial_3-\partial_2):l_2$
3		ξ_2	ρ_3	ξ_3									$p_{3.4} - p_{3.2}$	$3\,\xi_2(\partial_3-\partial_2):l_2 + 3\,\xi_3(\partial_4-\partial_3):l_3$
4			ξ_3	ρ_4	ξ_4								$p_{4.5} - p_{4.3}$	$3\,\xi_3(\partial_4-\partial_3):l_3 + 3\,\xi_4(\partial_5-\partial_4):l_4$
5				ξ_4	ρ_5	ξ_5							$p_{5.6} - p_{5.4}$	$3\,\xi_4(\partial_5-\partial_4):l_4 + 3\,\xi_5(\partial_6-\partial_5):l_5$
↓													↓	↓
						↘	↘	↘						
$n-3$								ξ_{n-4}	ρ_{n-3}	ξ_{n-3}			$p_{n-3,\,n-2} - p_{n-3,\,n-4}$	$3\,\xi_{n-4}(\partial_{n-3}-\partial_{n-4}):l_{n-4} + 3\,\xi_{n-3}(\partial_{n-2}-\partial_{n-3}):l_{n-3}$
$n-2$									ξ_{n-3}	ρ_{n-2}	ξ_{n-2}		$p_{n-2,\,n-1} - p_{n-2,\,n-3}$	$3\,\xi_{n-3}(\partial_{n-2}-\partial_{n-3}):l_{n-3} + 3\,\xi_{n-2}(\partial_{n-1}-\partial_{n-2}):l_{n-2}$
$n-1$										ξ_{n-2}	ρ_{n-1}	ξ_{n-1}	$p_{n-1,\,n} - p_{n-1,\,n-2}$	$3\,\xi_{n-2}(\partial_{n-1}-\partial_{n-2}):l_{n-2} + 3\,\xi_{n-1}(\partial_n-\partial_{n-1}):l_{n-1}$
n											ξ_{n-1}	ρ_n	$p_{n,\,n+1} - p_{n,\,n-1}$	$3\,\xi_{n-1}(\partial_n-\partial_{n-1}):l_{n-1} + 3\,\xi_n(\partial_{n+1}-\partial_n):l_n$

TABLE XXV

General Equations for a Continuous Beam
Supports all on same Level.
Any Number of Spans
Any System of Vertical Loads

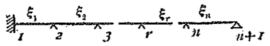
Equation of Three Moments

Equation	Left-Hand Member of Equation (Coefficients of Unknown Moments)													Right-Hand Member of Equation
	M_{12}	M_{23}	M_{34}	M_{45}	M_{56}	M_{67}	M_{78}	→	$M_{n-4,n-3}$	$M_{n-3,n-2}$	$M_{n-2,n-1}$	$M_{n-1,n}$	$M_{n,n+1}$	
1	$\overset{2}{(m_0+1)}$	1												$-2m_0H_{10}-2H_{12}$
2	m_1	$\overset{2}{(m_1+1)}$	1											$-2m_1H_{21}-2H_{23}$
3		m_2	$\overset{2}{(m_2+1)}$	1										$-2m_2H_{32}-2H_{34}$
4			m_3	$\overset{2}{(m_3+1)}$	1									$-2m_3H_{43}-2H_{45}$
5				m_4	$\overset{2}{(m_4+1)}$	1								$-2m_4H_{54}-2H_{56}$
6					m_5	$\overset{2}{(m_5+1)}$	1							$-2m_5H_{65}-2H_{67}$
↓							↘	↘	↘					↓
n-3									m_{n-4}	$\overset{2}{(m+1)}_{n-4}$	1			$-2m_{n-4}H_{n-3,n-4}-2H_{n-3,n-2}$
n-2										m_{n-3}	$\overset{2}{(m+1)}_{n-3}$	1		$-2m_{n-3}H_{n-2,n-3}-2H_{n-2,n-1}$
n-1											m_{n-2}	$\overset{2}{(m+1)}_{n-2}$	1	$-2m_{n-2}H_{n-1,n-2}-2H_{n-1,n}$
n												m_{n-1}	$\overset{2}{(m+1)}_{n-1}$	$-2m_{n-1}H_{n,n-1}-2H_{n,n+1}$

TABLE XXVI

General Equations for a Continuous Beam Fixed at one End.
Supports all on same Level.
Any Number of Spans.
Any System of Vertical Loads.

Equation of Three Moments.



Equation	Left-Hand Member of Equation (Coefficients of Unknown Moments)													Right-Hand Member of Equation
	M_{12}	M_{23}	M_{34}	M_{45}	M_{56}	M_{67}	M_{78}	→	$M_{n-4,n-3}$	$M_{n-3,n-2}$	$M_{n-2,n-1}$	$M_{n-1,n}$	$M_{n,n+1}$	
1	2	1												$-2H_{12}$
2	m_1	$\overset{2}{(m_1+1)}$	1											$-2m_1H_{21}-2H_{23}$
3		m_2	$\overset{2}{(m_2+1)}$	1										$-2m_2H_{32}-2H_{34}$
4			m_3	$\overset{2}{(m_3+1)}$	1									$-2m_3H_{43}-2H_{45}$
5				m_4	$\overset{2}{(m_4+1)}$	1								$-2m_4H_{54}-2H_{56}$
6					m_5	$\overset{2}{(m_5+1)}$	1							$-2m_5H_{65}-2H_{67}$
↓							↘	↘	↘					↓
n-3									m_{n-4}	$\overset{2}{(m+1)}_{n-4}$	1			$-2m_{n-4}H_{n-3,n-4}-2H_{n-3,n-2}$
n-2										m_{n-3}	$\overset{2}{(m+1)}_{n-3}$	1		$-2m_{n-3}H_{n-2,n-3}-2H_{n-2,n-1}$
n-1											m_{n-2}	$\overset{2}{(m+1)}_{n-2}$	1	$-2m_{n-2}H_{n-1,n-2}-2H_{n-1,n}$
n												m_{n-1}	$\overset{2}{(m+1)}_{n-1}$	$-2m_{n-1}H_{n,n-1}-2H_{n,n+1}$

In Tables XXV, XXVI, and XXVII

For ‘ $\overset{2}{(m+1)}$ ’ read ‘ $2(m+1)$ ’.
and

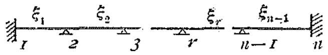
$$H_{r,s}=C_{r,s}+\frac{C_{s,r}}{2},$$

$$m_0=\frac{\xi_1}{\xi_0},\qquad m_1=\frac{\xi_2}{\xi_1},\qquad m_2=\frac{\xi_3}{\xi_2}\cdots\qquad m_r=\frac{\xi_{r+1}}{\xi_r}.$$

TABLE XXVII

General Equations for a Continuous Beam Fixed at both Ends.
Supports all on same Level
Any Number of Spans
Any System of Vertical Loads

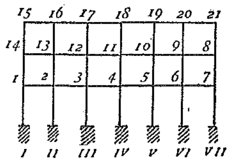
Equation of Three Moments



Equation	Left-Hand Member of Equation (Coefficients of Unknown Moments)													Right-Hand Member of Equation
	M_{12}	M_{23}	M_{34}	M_{45}	M_{56}	M_{67}	M_{78}	→	$M_{n-4,n-3}$	$M_{n-3,n-2}$	$M_{n-2,n-1}$	$M_{n-1,n}$	$M_{n,n+1}$	
1	2	1												$-2H_{12}$
2	m_1	$2(m_1+1)$	1											$-2m_1H_{21}-2H_{23}$
3		m_2	$2(m_2+1)$	1										$-2m_2H_{32}-2H_{34}$
4			m_3	$2(m_3+1)$	1									$-2m_3H_{43}-2H_{45}$
5				m_4	$2(m_5+1)$	1								$-2m_4H_{54}-2H_{56}$
6					m_5	$2(m_5+1)$	1							$-2m_5H_{65}-2H_{67}$
↓							↘	↘	↘					↓
$n-3$									m_{n-4}	$2(m+1)_{n-4}$	1			$-2m_{n-4}H_{n-3,n-4}-2H_{n-3,n-2}$
$n-2$										m_{n-3}	$2(m+1)_{n-3}$	1		$-2m_{n-3}H_{n-2,n-3}-2H_{n-2,n-1}$
$n-1$											m_{n-2}	$2(m+1)_{n-2}$	1	$-2m_{n-2}H_{n-1,n-2}-2H_{n-1,n}$
n												1	2	$-2H_{n,n-1}$

TABLE XXVIII

General Equations for a Triple Storied Bent of Six Spans. Legs Fixed at the Bases.
Cross-Sections of all Members Different.
Each Story Height Different.
Wind Loads Assumed at Joints on one Vertical Side.
Any System of Vertical Loads on Girders.



Equation	Left-Hand Member of Equation																								Right-Hand Member of Equation	
	Coefficients of Unknown Slopes φ																					Coeff. of μ				
	φ_{1}	φ_{2}	φ_{3}	φ_{4}	φ_{5}	φ_{6}	φ_{7}	φ_{8}	φ_{9}	φ_{10}	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	φ_{16}	φ_{17}	φ_{18}	φ_{19}	φ_{20}	φ_{21}	μ_{1}	μ_{2}	μ_{3}	Vertical Load	Wind Load
1	ρ_{1}	ξ_{1}												ξ'_{1}								$-3\xi_{I}$	$-3\xi'_{1}$		p_{12}	
2	ξ_{1}	ρ_{2}	ξ_{2}										ξ'_{2}									$-3\xi_{II}$	$-3\xi'_{2}$		$p_{23}-p_{21}$	
3		ξ_{2}	ρ_{3}	ξ_{3}								ξ'_{3}										$-3\xi_{III}$	$-3\xi'_{3}$		$p_{34}-p_{32}$	
4			ξ_{3}	ρ_{4}	ξ_{4}						ξ'_{4}											$-3\xi_{IV}$	$-3\xi'_{4}$		$p_{45}-p_{43}$	
5				ξ_{4}	ρ_{5}	ξ_{5}				ξ'_{5}												$-3\xi_{V}$	$-3\xi'_{5}$		$p_{56}-p_{54}$	
6					ξ_{5}	ρ_{6}	ξ_{6}		ξ'_{6}													$-3\xi_{VI}$	$-3\xi'_{6}$		$p_{67}-p_{65}$	
7						ξ_{6}	ρ_{7}	ξ_{7}														$-3\xi_{VII}$	$-3\xi_{7}$		$-p_{76}$	
8							ξ_{7}	ρ_{8}	ξ_{8}												ξ'_{8}		$-3\xi_{7}$	$-3\xi'_{8}$	$-p_{89}$	
9						ξ'_{6}		ξ_{8}	ρ_{9}	ξ_{9}										ξ'_{9}			$-3\xi'_{6}$	$-3\xi'_{9}$	$p_{98}-p_{9,10}$	
10					ξ'_{5}				ξ_{9}	ρ_{10}	ξ_{10}								ξ'_{10}				$-3\xi'_{5}$	$-3\xi'_{10}$	$p_{10,9}-p_{10,11}$	
11				ξ'_{4}						ξ_{10}	ρ_{11}	ξ_{11}						ξ'_{11}					$-3\xi'_{4}$	$-3\xi'_{11}$	$p_{11,10}-p_{11,12}$	
12			ξ'_{3}								ξ_{11}	ρ_{12}	ξ_{12}				ξ'_{12}						$-3\xi'_{3}$	$-3\xi'_{12}$	$p_{12,11}-p_{12,13}$	
13		ξ'_{2}										ξ_{12}	ρ_{13}	ξ_{13}		ξ'_{13}							$-3\xi'_{2}$	$-3\xi'_{13}$	$p_{13,12}-p_{13,14}$	
14	ξ'_{1}												ξ_{13}	ρ_{14}	ξ_{14}								$-3\xi'_{1}$	$-3\xi_{14}$	$p_{14,13}$	
15														ξ_{14}	ρ_{15}	ξ_{15}								$-3\xi_{14}$	$p_{15,16}$	
16													ξ'_{13}		ξ_{15}	ρ_{16}	ξ_{16}							$-3\xi'_{13}$	$p_{16,17}-p_{16,15}$	
17												ξ'_{12}				ξ_{16}	ρ_{17}	ξ_{17}						$-3\xi'_{12}$	$p_{17,18}-p_{17,16}$	
18											ξ'_{11}						ξ_{17}	ρ_{18}	ξ_{18}					$-3\xi'_{11}$	$p_{18,19}-p_{18,17}$	
19										ξ'_{10}								ξ_{18}	ρ_{19}	ξ_{19}				$-3\xi'_{10}$	$p_{19,20}-p_{19,18}$	
20									ξ'_{9}										ξ_{19}	ρ_{20}	ξ_{20}			$-3\xi'_{9}$	$p_{20,21}-p_{20,19}$	
21								ξ'_{8}												ξ_{20}	ρ_{21}				$-p_{21,20}$	
22	ξ_{I}	ξ_{II}	ξ_{III}	ξ_{IV}	ξ_{V}	ξ_{VI}	ξ_{VII}															$-X_{1}$				$-q_{1}$
23	ξ'_{1}	ξ'_{2}	ξ'_{3}	ξ'_{4}	ξ'_{5}	ξ'_{6}	ξ_{7}	ξ_{7}	ξ'_{6}	ξ'_{5}	ξ'_{4}	ξ'_{3}	ξ'_{2}	ξ'_{1}									$-X_{2}$			$-q_{2}$
24								ξ'_{8}	ξ'_{9}	ξ'_{10}	ξ'_{11}	ξ'_{12}	ξ'_{13}	ξ_{14}	ξ_{14}	ξ'_{13}	ξ'_{12}	ξ'_{11}	ξ'_{10}	ξ'_{9}	ξ'_{8}			$-X_{3}$		$-q_{3}$

TABLE XXIX

General Equations for a Five Storied Bent of Triple Span. Legs Fixed at the Bases.
Cross-Sections of all Members Different
Each Story Height Different
Wind Loads Assumed at Joints on one Vertical Side
Any System of Vertical Loads on Girders

Equation	Left-Hand Member of Equation																									Right-Hand Member of Equation	
	Coefficients of Unknown Slopes φ																				Coeff. of μ						
	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_7	φ_8	φ_9	φ_{10}	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	φ_{16}	φ_{17}	φ_{18}	φ_{19}	φ_{20}	μ_1	μ_2	μ_3	μ_4	μ_5	Vertical Load	Wind Load
1	ρ_1	ξ_1						ξ'_1													$-3\xi_I$	$-3\xi'_1$				p_{12}	
2	ξ_1	ρ_2	ξ_2				ξ'_2														$-3\xi_{II}$	$-3\xi'_2$				$p_{23}-p_{21}$	
3		ξ_2	ρ_3	ξ_3		ξ'_3															$-3\xi_{III}$	$-3\xi'_3$				$p_{34}-p_{32}$	
4			ξ_3	ρ_4	ξ_4																$-3\xi_{IV}$	$-3\xi_4$				$-p_{43}$	
5				ξ_4	ρ_5	ξ_5						ξ'_5										$-3\xi_4$	$-3\xi'_5$			$-p_{56}$	
6			ξ'_3		ξ_5	ρ_6	ξ_6				ξ'_6											$-3\xi'_3$	$-3\xi'_6$			$p_{65}-p_{67}$	
7		ξ'_2				ξ_6	ρ_7	ξ_7		ξ'_7												$-3\xi'_2$	$-3\xi'_7$			$p_{76}-p_{78}$	
8	ξ'_1						ξ_7	ρ_8	ξ_8													$-3\xi'_1$	$-3\xi_8$			p_{87}	
9								ξ_8	ρ_9	ξ_9						ξ'_9							$-3\xi_8$	$-3\xi'_9$		$p_{9,10}$	
10							ξ'_7		ξ_9	ρ_{10}	ξ_{10}				ξ'_{10}								$-3\xi'_7$	$-3\xi'_{10}$		$p_{10,11}-p_{10,9}$	
11						ξ'_6				ξ_{10}	ρ_{11}	ξ_{11}		ξ'_{11}									$-3\xi'_6$	$-3\xi'_{11}$		$p_{11,12}-p_{11,10}$	
12					ξ'_5						ξ_{11}	ρ_{12}	ξ_{12}										$-3\xi'_5$	$-3\xi_{12}$		$-p_{12,11}$	
13												ξ_{12}	ρ_{13}	ξ_{13}						ξ'_{13}				$-3\xi_{12}$	$-3\xi'_{13}$	$-p_{13,14}$	
14											ξ'_{11}		ξ_{13}	ρ_{14}	ξ_{14}				ξ'_{14}					$-3\xi'_{11}$	$-3\xi'_{14}$	$p_{14,13}-p_{14,15}$	
15										ξ'_{10}				ξ_{14}	ρ_{15}	ξ_{15}		ξ'_{15}						$-3\xi'_{10}$	$-3\xi'_{15}$	$p_{15,14}-p_{15,16}$	
16									ξ'_9						ξ_{15}	ρ_{16}	ξ_{16}							$-3\xi'_9$	$-3\xi_{16}$	$p_{16,15}$	
17																ξ_{16}	ρ_{17}	ξ_{17}							$-3\xi_{16}$	$p_{17,18}$	
18															ξ'_{15}		ξ_{17}	ρ_{18}	ξ_{18}						$-3\xi'_{15}$	$p_{18,19}-p_{18,17}$	
19														ξ'_{14}				ξ_{18}	ρ_{19}	ξ_{19}					$-3\xi'_{14}$	$p_{19,20}-p_{19,18}$	
20													ξ_{13}						ξ_{19}	ρ_{20}					$-3\xi'_{13}$	$-p_{20,19}$	
21	ξ_I	ξ_{II}	ξ_{III}	ξ_{IV}																	$-X_1$						$-q_1$
22	ξ'_1	ξ'_2	ξ'_3	ξ_4	ξ_4	ξ'_3	ξ'_2	ξ'_1														$-X_2$					$-q_2$
23					ξ'_5	ξ'_6	ξ'_7	ξ_8	ξ_8	ξ'_7	ξ'_6	ξ'_5											$-X_3$				$-q_3$
24									ξ'_9	ξ'_{10}	ξ'_{11}	ξ_{12}	ξ_{12}	ξ'_{11}	ξ'_{10}	ξ'_9								$-X_4$			$-q_4$
25													ξ'_{13}	ξ'_{14}	ξ'_{15}	ξ_{16}	ξ_{16}	ξ'_{15}	ξ'_{14}	ξ'_{13}					$-X_5$		$-q_5$

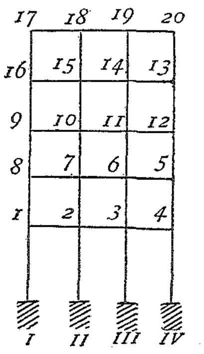


TABLE XXX

General Equations for a Symmetrical Eight Span Bent Four Stories High.
Legs Fixed at the Bases.
Any System of Symmetrical Vertical Loads on Girders.

Equation	Left-Hand Member of Equation																Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ																
	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_7	φ_8	φ_9	φ_{10}	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	φ_{16}	Vertical Load
1	ρ_1	ξ_1						ξ'_1									p_{12}
2	ξ_1	ρ_2	ξ_2				ξ'_2										$p_{23}-p_{21}$
3		ξ_2	ρ_3	ξ_3		ξ'_3											$p_{34}--p_{32}$
4			ξ_3	ρ_4	ξ_4												$p_{44'}-p_{43}$
5				ξ_4	ρ_5	ξ_5						ξ'_5					$p_{55'}-p_{56}$
6			ξ'_3		ξ_5	ρ_6	ξ_6				ξ'_6						$p_{65}-p_{67}$
7		ξ'_2				ξ_6	ρ_7	ξ_7		ξ'_7							$p_{76}-p_{78}$
8	ξ'_1						ξ_7	ρ_8	ξ_8								p_{87}
9								ξ_8	ρ_9	ξ_9						ξ'_9	$p_{9,10}$
10							ξ'_7		ξ_9	ρ_{10}	ξ_{10}				ξ'_{10}		$p_{10,11}-p_{10,9}$
11						ξ'_6				ξ_{10}	ρ_{11}	ξ_{11}		ξ'_{11}			$p_{11,12}-p_{11,10}$
12					ξ'_5						ξ_{11}	ρ_{12}	ξ_{12}				$p_{12,12'}-p_{12,11}$
13												ξ_{12}	ρ_{13}	ξ_{13}			$p_{13,13'}-p_{13,14}$
14											ξ'_{11}		ξ_{13}	ρ_{14}	ξ_{14}		$p_{14,13}-p_{14,15}$
15										ξ'_{10}				ξ_{14}	ρ_{15}	ξ_{15}	$p_{15,14}-p_{15,16}$
16									ξ'_9						ξ_{15}	ρ_{16}	$p_{16,15}$

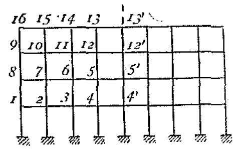
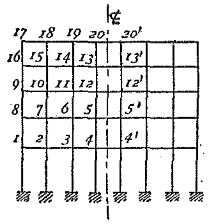


TABLE XXXI

General Equations for a Symmetrical Seven-Span Bent Five Stories High.
Legs Fixed at the Bases.
Any System of Symmetrical Vertical Loads on Girders.

Equation	Left-Hand Member of Equation																				Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ																				
	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_7	φ_8	φ_9	φ_{10}	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	φ_{16}	φ_{17}	φ_{18}	φ_{19}	φ_{20}	Vertical Load
1	ρ_1	ξ_1						ξ'_1													p_{12}
2	ξ_1	ρ_2	ξ_2				ξ'_2														$p_{23}-p_{21}$
3		ξ_2	ρ_3	ξ_3		ξ'_3															$p_{34}-p_{32}$
4			ξ_3	ρ'_4	ξ_4																$p_{44'}-p_{43}$
5				ξ_4	ρ''_5	ξ_5						ξ'_5									$p_{55'}-p_{56}$
6			ξ'_3		ξ_5	ρ_6	ξ_6				ξ'_6										$p_{65}-p_{67}$
7		ξ'_2				ξ_6	ρ_7	ξ_7		ξ'_7											$p_{76}-p_{78}$
8	ξ'_1						ξ_7	ρ_8	ξ_8												p_{87}
9								ξ_8	ρ_9	ξ_9						ξ'_9					$p_{9,10}$
10							ξ'_7		ξ_9	ρ_{10}	ξ_{10}				ξ'_{10}						$p_{10,11}-p_{10,9}$
11						ξ'_6				ξ_{10}	ρ_{11}	ξ_{11}		ξ'_{11}							$p_{11,12}-p_{11,10}$
12					ξ'_5						ξ_{11}	ρ''_{12}	ξ_{12}								$p_{12,12'}-p_{12,11}$
13												ξ_{12}	ρ''_{13}	ξ_{13}						ξ'_{13}	$p_{13,13'}-p_{13,14}$
14											ξ'_{11}		ξ_{13}	ρ_{14}	ξ_{14}				ξ'_{14}		$p_{14,13}-p_{14,15}$
15										ξ'_{10}				ξ_{14}	ρ_{15}	ξ_{15}		ξ'_{15}			$p_{15,14}-p_{15,16}$
16									ξ'_9						ξ_{15}	ρ_{16}	ξ_{16}				$p_{16,16}$
17																ξ_{16}	ρ_{17}	ξ_{17}			$p_{17,18}$
18															ξ'_{15}		ξ_{17}	ρ_{18}	ξ_{18}		$p_{18,19}-p_{18,17}$
19														ξ'_{14}				ξ_{18}	ρ_{19}	ξ_{19}	$p_{19,20}-p_{19,18}$
20													ξ'_{13}						ξ_{19}	ρ''_{20}	$p_{20,20'}-p_{20,19}$



$\rho'_4=2(\xi_3+\xi_4+\xi_{IV})+\xi$ of 4–4'.

$\rho''_5=2(\xi_4+\xi_5+\xi'_5)+\xi$ of 5–5',

$\rho''_{12}=2(\xi'_5+\xi_{11}+\xi_{12})+\xi$ of 12–12',

$\rho''_{13}=2(\xi_{12}+\xi_{13}+\xi'_{13})+\xi$ of 13–13',

$\rho''_{20}=2(\xi'_{13}+\xi_{19})+\xi$ of 20–20'.

TABLE XXXII

General Equations for a Triple Storied Bent of Six Spans. Legs Hinged at the Bases.
Cross-Sections of all Members Different.
Each Story Height Different
Wind Loads Assumed at Joints on one Vertical Side
Any System of Vertical Loads on Girders

Equation	Left-Hand Member of Equation																								Right-Hand Member of Equation	
	Coefficients of Unknown Slopes φ																					Coeff. of μ				
	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_7	φ_8	φ_9	φ_{10}	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	φ_{16}	φ_{17}	φ_{18}	φ_{19}	φ_{20}	φ_{21}	μ_1	μ_2	μ_3	Vertical Load	Wind Load
1	ρ_1	ξ_1												ξ'_1								$-1,5\xi_I$	$-3\xi'_1$		p_{12}	
2	ξ_1	ρ_2	ξ_2										ξ'_2									$-1,5\xi_{II}$	$-3\xi'_2$		$p_{23}-p_{21}$	
3		ξ_2	ρ_3	ξ_3								ξ'_3										$-1,5\xi_{III}$	$-3\xi'_3$		$p_{34}-p_{32}$	
4			ξ_3	ρ_4	ξ_4						ξ'_4											$-1,5\xi_{IV}$	$-3\xi'_4$		$p_{45}-p_{43}$	
5				ξ_4	ρ_5	ξ_5				ξ'_5												$-1,5\xi_V$	$-3\xi'_5$		$p_{56}-p_{54}$	
6					ξ_5	ρ_6	ξ_6		ξ'_6													$-1,5\xi_{VI}$	$-3\xi'_6$		$p_{67}-p_{65}$	
7						ξ_6	ρ_7	ξ_7														$-1,5\xi_{VII}$	$-3\xi_7$		$-p_{76}$	
8							ξ_7	ρ_8	ξ_8												ξ'_8		$-3\xi_7$	$-3\xi'_8$	$-p_{89}$	
9						ξ'_6		ξ_8	ρ_9	ξ_9										ξ'_9			$-3\xi'_6$	$-3\xi'_9$	$p_{98}-p_{9,10}$	
10					ξ'_5				ξ_9	ρ_{10}	ξ_{10}								ξ'_{10}				$-3\xi'_5$	$-3\xi'_{10}$	$p_{10,9}-p_{10,11}$	
11				ξ'_4						ξ_{10}	ρ_{11}	ξ_{11}						ξ'_{11}					$-3\xi'_4$	$-3\xi'_{11}$	$p_{11,10}-p_{11,12}$	
12			ξ'_3								ξ_{11}	ρ_{12}	ξ_{12}				ξ'_{12}						$-3\xi'_3$	$-3\xi'_{12}$	$p_{12,11}-p_{12,13}$	
13		ξ'_2										ξ_{12}	ρ_{13}	ξ_{13}		ξ'_{13}							$-3\xi'_2$	$-3\xi'_{13}$	$p_{13,12}-p_{13,14}$	
14	ξ'_1												ξ_{13}	ρ_{14}	ξ_{14}								$-3\xi'_1$	$-3\xi_{14}$	$p_{14,13}$	
15														ξ_{14}	ρ_{15}	ξ_{15}								$-3\xi_{14}$	$p_{15,16}$	
16													ξ'_{13}		ξ_{15}	ρ_{16}	ξ_{16}							$-3\xi'_{13}$	$p_{16,17}-p_{16,15}$	
17												ξ'_{12}				ξ_{16}	ρ_{17}	ξ_{17}						$-3\xi'_{12}$	$p_{17,18}-p_{17,16}$	
18											ξ'_{11}						ξ_{17}	ρ_{18}	ξ_{18}					$-3\xi'_{11}$	$p_{18,19}-p_{18,17}$	
19										ξ'_{10}								ξ_{18}	ρ_{19}	ξ_{19}				$-3\xi'_{10}$	$p_{19,20}-p_{19,18}$	
20									ξ'_9										ξ_{19}	ρ_{20}	ξ_{20}			$-3\xi'_9$	$p_{20,21}-p_{20,19}$	
21								ξ'_8												ξ_{20}	ρ_{21}				$-p_{21,20}$	
22	ξ_I	ξ_{II}	ξ_{III}	ξ_{IV}	ξ_V	ξ_{VI}	ξ_{VII}															$-\frac{X_1}{2}$				$-2q_1$
23	ξ'_1	ξ'_2	ξ'_3	ξ'_4	ξ'_5	ξ'_6	ξ_7	ξ_7	ξ'_6	ξ'_5	ξ'_4	ξ'_3	ξ'_2	ξ'_1									$-X_2$			$-q_2$
24								ξ'_8	ξ'_9	ξ'_{10}	ξ'_{11}	ξ'_{12}	ξ'_{13}	ξ_{14}	ξ_{14}	ξ'_{13}	ξ'_{12}	ξ'_{11}	ξ'_{10}	ξ'_9	ξ'_8			$-X_3$		$-q_3$

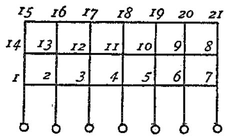


TABLE XXXIII

General Equations for a Five Storied Bent of Triple Span. Legs Hinged at the Bases
Cross-Sections of all Members Different
Each Story Height Different
Wind Loads Assumed at Joints on one Vertical Side
Any System of Vertical Loads on Girders

Equation	Left-Hand Member of Equation																									Right-Hand Member of Equation	
	Coefficients of Unknown Slopes φ																				Coeff. of μ						
	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_7	φ_8	φ_9	φ_{10}	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	φ_{16}	φ_{17}	φ_{18}	φ_{19}	φ_{20}	μ_1	μ_2	μ_3	μ_4	μ_5	Vertical Load	Wind Load
1	ρ'_1	ξ_1						ξ'_1													$-1{,}5\xi_I$	$-3\xi'_1$				p_{12}	
2	ξ_1	ρ'_2	ξ_2				ξ'_2														$-1{,}5\xi_{II}$	$-3\xi'_2$				$p_{23}-p_{21}$	
3		ξ_2	ρ'_3	ξ_3		ξ'_3															$-1{,}5\xi_{III}$	$-3\xi'_3$				$p_{34}-p_{32}$	
4			ξ_3	ρ'_4	ξ_4																$-1{,}5\xi_{IV}$	$-3\xi_4$				$-p_{43}$	
5				ξ_4	ρ_5	ξ_5						ξ'_5										$-3\xi_4$	$-3\xi'_5$			$-p_{56}$	
6			ξ'_3		ξ_5	ρ_6	ξ_6				ξ'_6											$-3\xi'_3$	$-3\xi'_6$			$p_{65}-p_{67}$	
7		ξ'_2				ξ_6	ρ_7	ξ_7		ξ'_7												$-3\xi'_2$	$-3\xi'_7$			$p_{76}-p_{78}$	
8	ξ'_1						ξ_7	ρ_8	ξ_8													$-3\xi'_1$	$-3\xi_8$			p_{87}	
9								ξ_8	ρ_9	ξ_9						ξ'_9							$-3\xi_8$	$-3\xi'_9$		$p_{9.10}$	
10							ξ'_7		ξ_9	ρ_{10}	ξ_{10}				ξ'_{10}								$-3\xi'_7$	$-3\xi'_{10}$		$p_{10.11}-p_{10,9}$	
11						ξ'_6				ξ_{10}	ρ_{11}	ξ_{11}		ξ'_{11}									$-3\xi'_6$	$-3\xi'_{11}$		$p_{11,12}-p_{11,10}$	
12					ξ'_5						ξ_{11}	ρ_{12}	ξ_{12}										$-3\xi'_5$	$-3\xi_{12}$		$-p_{12,11}$	
13												ξ_{12}	ρ_{13}	ξ_{13}						ξ'_{13}				$-3\xi_{12}$	$-3\xi'_{13}$	$-p_{13.14}$	
14											ξ'_{11}		ξ_{13}	ρ_{14}	ξ_{14}				ξ'_{14}					$-3\xi'_{11}$	$-3\xi'_{14}$	$p_{14,13}-p_{14,15}$	
15										ξ'_{10}				ξ_{14}	ρ_{15}	ξ_{15}		ξ'_{15}						$-3\xi'_{10}$	$-3\xi'_{15}$	$p_{15.14}-p_{15,16}$	
16									ξ'_9						ξ_{15}	ρ_{16}	ξ_{16}							$-3\xi'_9$	$-3\xi_{16}$	$p_{16.15}$	
17																ξ_{16}	ρ_{17}	ξ_{17}							$-3\xi_{16}$	$p_{17.18}$	
18															ξ'_{15}		ξ_{17}	ρ_{18}	ξ_{18}						$-3\xi'_{15}$	$p_{18,19}-p_{18,17}$	
19														ξ'_{14}				ξ_{18}	ρ_{19}	ξ_{19}					$-3\xi'_{14}$	$p_{19,20}-p_{19,18}$	
20													ξ'_{13}						ξ_{19}	ρ_{20}					$-3\xi'_{13}$	$-p_{20,19}$	
21	ξ_I	ξ_{II}	ξ_{III}	ξ_{IV}																	$-\frac{X_1}{2}$						$-2q_1$
22	ξ'_1	ξ'_2	ξ'_3	ξ_4	ξ_4	ξ'_3	ξ'_2	ξ'_1														$-X_2$					$-q_2$
23					ξ'_5	ξ'_6	ξ'_7	ξ_8	ξ_8	ξ'_7	ξ'_6	ξ'_5											$-X_3$				$-q_3$
24									ξ'_9	ξ'_{10}	ξ'_{11}	ξ_{12}	ξ_{12}	ξ'_{11}	ξ'_{10}	ξ'_9								$-X_4$			$-q_4$
25													ξ'_{13}	ξ'_{14}	ξ'_{15}	ξ_{16}	ξ_{16}	ξ'_{15}	ξ'_{14}	ξ'_{13}					$-X_5$		$-q_5$

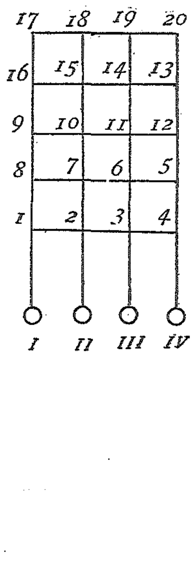


TABLE XXXIV

General Equations for a Symmetrical Eight-Span Bent
Four Stories High. Legs Hinged at the Bases
Any System of Symmetrical Vertical Loads on Girders

Equation	Left-Hand Member of Equation																Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ																
	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_7	φ_8	φ_9	φ_{10}	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	φ_{16}	Vertical Load
1	ρ'_1	ξ_1						ξ'_1									p_{12}
2	ξ_1	ρ'_2	ξ_2				ξ'_2										$p_{23}-p_{21}$
3		ξ_2	ρ'_3	ξ_3		ξ'_3											$p_{34}-p_{32}$
4			ξ_3	ρ'_4	ξ_4												$p_{44'}-p_{43}$
5				ξ_4	ρ_5	ξ_5						ξ'_5					$p_{55'}-p_{56}$
6			ξ'_3		ξ_5	ρ_6	ξ_6				ξ'_6						$p_{65}-p_{67}$
7		ξ'_2				ξ_6	ρ_7	ξ_7		ξ'_7							$p_{76}-p_{78}$
8	ξ'_1						ξ_7	ρ_8	ξ_8								p_{87}
9								ξ_8	ρ_9	ξ_9						ξ'_9	$p_{9,10}$
10							ξ'_7		ξ_9	ρ_{10}	ξ_{10}				ξ'_{10}		$p_{10,11}-p_{10,9}$
11						ξ'_6				ξ_{10}	ρ_{11}	ξ_{11}		ξ'_{11}			$p_{11.12}-p_{11,10}$
12					ξ'_5						ξ_{11}	ρ_{12}	ξ_{12}				$p_{12,12'}-p_{12,11}$
13												ξ_{12}	ρ_{13}	ξ_{13}			$p_{13,13'}-p_{13,14}$
14											ξ'_{11}		ξ_{13}	ρ_{14}	ξ_{14}		$p_{14,13}-p_{14,15}$
15										ξ'_{10}				ξ_{14}	ρ_{15}	ξ_{15}	$p_{15,14}-p_{15,16}$
16									ξ'_9						ξ_{15}	ρ_{16}	$p_{16,15}$

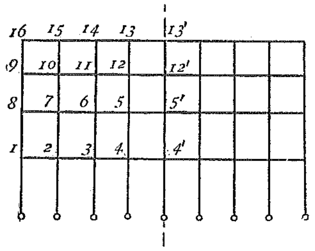
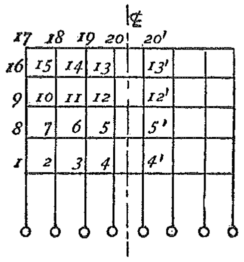


TABLE XXXV

General Equations for a Symmetrical Seven-Span Bent Five Stories High. Legs
Hinged at the Bases
Any System of Symmetrical Vertical Loads on Girders

Equation	Left-Hand Member of Equation																				Right-Hand Member of Equation
	Coefficients of Unknown Slopes φ																				
	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_7	φ_8	φ_9	φ_{10}	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	φ_{16}	φ_{17}	φ_{18}	φ_{19}	φ_{20}	Vertical Load
1	ρ'_1	ξ_1						ξ'_1													p_{12}
2	ξ_1	ρ'_2	ξ_2				ξ'_2														$p_{23}-p_{21}$
3		ξ_2	ρ'_3	ξ_3		ξ'_3															$p_{34}-p_{32}$
4			ξ_3	ρ''_4	ξ_4																$p_{44'}-p_{43}$
5				ξ_4	ρ''_5	ξ_5						ξ'_5									$p_{55'}-p_{56}$
6			ξ'_3		ξ_5	ρ_6	ξ_6				ξ'_6										$p_{65}-p_{67}$
7		ξ'_2				ξ_6	ρ_7	ξ_7		ξ'_7											$p_{76}-p_{78}$
8	ξ'_1						ξ_7	ρ_8	ξ_8												p_{87}
9								ξ_8	ρ_9	ξ_9						ξ'_9					$p_{9,10}$
10							ξ'_7		ξ_9	ρ_{10}	ξ_{10}				ξ'_{10}						$p_{10,11}-p_{10,9}$
11						ξ'_6				ξ_{10}	ρ_{11}	ξ_{11}		ξ'_{11}							$p_{11,12}-p_{11,10}$
12					ξ'_5						ξ_{11}	ρ''_{12}	ξ_{12}								$p_{12,12'}-p_{12,11}$
13												ξ_{12}	ρ''_{13}	ξ_{13}						ξ'_{13}	$p_{13,13'}-p_{13,14}$
14											ξ'_{11}		ξ_{13}	ρ_{14}	ξ_{14}				ξ'_{14}		$p_{14,13}-p_{14,15}$
15										ξ'_{10}				ξ_{14}	ρ_{15}	ξ_{15}		ξ'_{15}			$p_{15,14}-p_{15,16}$
16									ξ'_9						ξ_{15}	ρ_{16}	ξ_{16}				$p_{16,15}$
17																ξ_{16}	ρ_{17}	ξ_{17}			$p_{17,18}$
18															ξ'_{15}		ξ_{17}	ρ_{18}	ξ_{18}		$p_{18,19}-p_{18,17}$
19														ξ'_{14}				ξ_{18}	ρ_{19}	ξ_{19}	$p_{19,20}-p_{19,18}$
20													ξ'_{13}						ξ_{19}	ρ''_{20}	$p_{20'21}-p_{20,19}$



$\rho''_4=2(\xi_3+\xi_4)+1{,}5\xi_{IV}+\xi$ of $4-4'$, $\rho''_5=2(\xi_4+\xi_5+\xi'_5)+\xi$ of $5-5'$,

$\rho''_{12}=2(\xi'_5+\xi_{11}+\xi_{12})+\xi$ of $12-12'$, $\rho''_{13}=2(\xi_{12}+\xi_{13}+\xi'_{13})+\xi$ of $13-13'$,

$\rho''_{20}=2(\xi'_{13}+\xi_{19})+\xi$ of $20-20'$.