# On the crack of a thin glass plate clamped at the edge and bent by uniform pressure

By

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## Introduction.

When we were investigating some vacuum phenomena, the glass floor of the vacuum vessel was broken by chance. By improving the vacuum apparatus, we have found a convenient method to study the cracking of a thin elastic plate bent by uniform pressure.

It may be considered that the crack begins from the point of the greatest stress, but the crack figure depends not ouly upon the boundary conditions, but also upon the rate of increasing pressure. Among many experiments which we have carried out, we are going to describe the crack of a thin glass plate clamped at the edge and bent by uniform pressure when the rate of increasing pressure is infinitely slow, since these experiments can be done very easily without any incertitude of the boundary conditions.

The object of this essay is to show that

1. by using this apparatus, we can easily study the crack figure of the plate clamped at the edge in an arbitrary form,

2. we can obtain the point of the greatest stress without any calculation of the elastic theory.

Indeed we can calculate the deflection and stress distribution in some simple cases, but in general they can not be ascertained exactly. In order to show

how far this method can indicate the point of the greatest stress, we have studied mostly those cases having boundaries in the shapes of a circle, an ellipse, a square and a rectangle.

## Apparatus.

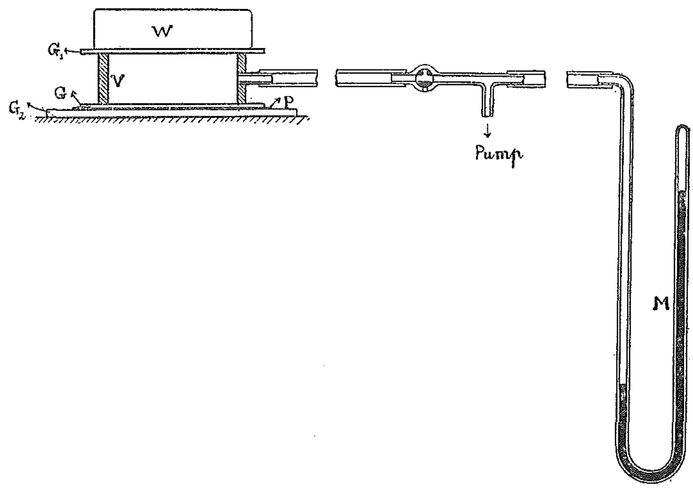


Fig. 1.

W: heavy weight,
$G_1$: thick plate glass,
V : vacuum vessel,
G : thin glass plate,
P : thin hard porous paper,
$G_2$: thick plate glass,
M : Manometer.

We use those vacuum vessels which have

1, circular edge with diameter $2r=21$ cm.,
2, elliptic edge with radii $a=12.4$, $b=9.7$ cm.,
3, square edge with sides $2a=18.6$ cm.,
4, rectangular edge with sides $2a=13.2$, $2b=24.2$ cm.,
5, rectangular edge with sides $2a=30$, $2b=50$ cm.,
6, combined edge of the above two rectangles.

## Experiment.

On a thick plate glass was placed a thin hard porous paper, which allows the air to go in very slowly. To fulfil the condition of the clamping a heavy weight was set on the vacuum vessel. By means of a manometer, the pressure inside of the vacuum vessel was measured. The difference of the atmospheric pressure is the cause of the bending of the thin glass plate. The experiment is so simple that we need not describe the details. But it is only necessary to note that the pressure of the vacuum vessel should diminish very slowly. The crack occurs some minutes (frequently 10 or 15 minutes) after the pump is stopped. As soon as the sound of cracking was heard, the air cock was opened and the pressure difference reduced in order to diminish the secondary effects of the first crack.

## Result of the experiments.

As it was desired to study the relation between the point of the greatest stress and the crack figure, commercial window glass was used as specimen. Indeed the breaking strength is very variable, but the crack figure is nearly the same, so long as the edge is the same, as is shown in the following photographs.

| Plate No. | Form of edge | Pressure | Thickness | The greatest str. at edge / The max stress at center | Crack begun from edge. | Crack begun from center. |
|---|---|---|---|---|---|---|
| 1 | Circle | 10 cm in Hg | 1.79 | 1.6 | ,, | |
| 2 | ,, | 15 | 1.74 | ,, | ,, | |
| 3 | ,, | 10 | 1.50 | ,, | ,, | |
| 4 | ,, | 16 | 1.63 | ,, | ,, | |
| | | | | | | |
| 5 | ellipse | 14 | 1.35 | 1.7 | ,, | |
| 6 | ,, | 18 | 1.55 | ,, | ,, | |
| 7 | ,, | 14 | 1.55 | ,, | ,, | |
| 8 | ,, | 10 | 1.63 | ,, | ,, | |
| 9 | ,, | 18 | 1.65 | ,, | ,, | |
| 10 | ,, | 10 | 1.76 | ,, | ,, | |
| 11 | ,, | 16 | 1.50 | ,, | ,, | |
| | | | | | | |
| 12 | square | 16 | 1.47 | 2.4 | ,, | |
| 13 | ,, | 12 | 1.79 | ,, | ,, | |
| 14 | ,, | 14 | 1.75 | ,, | ,, | |
| 15 | ,, | 14 | 1.58 | ,, | ,, | |
| 16 | ,, | 12 | 1.66 | ,, | | ,, |
| 17 | ,, | 22 | 1.80 | ,, | | ,, |
| 18 | ,, | 10 | 1.50 | ,, | ,, | |
| 19 | ,, | 14 | 1.80 | ,, | ,, | |
| 20 | ,, | 18 | 1.40 | ,, | ,, | |
| | | | | | | |
| 21 | rectangle | 24 | 1.55 | 2.2 | ,, | |
| 22 | ,, | 22 | 1.50 | ,, | ,, | |
| 23 | ,, | 16 | 1.60 | ,, | ,, | |
| 24 | ,, | 12 | 1.63 | ,, | ,, | |
| 25 | ,, | 12 | 1.55 | ,, | ,, | |
| 26 | ,, | 17 | 1.50 | ,, | ,, | |
| 27 | ,, | 23 | 1.60 | ,, | | ,, |
| 28 | ,, | 11 | 1.58 | ,, | | ,, |
| 29 | ,, | 16 | 1.64 | ,, | ,, | ,, |
| | | | | | | |
| 30 | great rectangle | 4 | 1.55 | 2.1 | ,, | |
| 31 | ,, | 4 | 1.60 | ,, | ,, | |

The fifth column is calculated from the elastic theory.

The Poisson's ratio is assumed to be 0.25.

## Discussion of the results.

Cracks can be considered to be due to the stress, but they depend also upon defects in the material. Though the elastic theory can not hold strictly in the breaking phenomena, the stress may be estimated approximately from the elastic theory. Any material defect, however, is difficult to detect before the experiment.

Suppose that there is a point of greatest bending stress in one direction and there is another point in which the stress is smaller than that of the first point, but the amount of stress is equal in all directions around that point. From which point does the crack begin under these conditions?

We must consider the ratio of the stress amounts and the probability with which the weak point in the material happens to exist in the neighbourhood of the points in question. These circumstances occur in the case of square edge, since the stress is the same in all directions at the middle point of the plane, while the greatest stress is at the middle point of the longer side and crack must occur only parallel to that side.

I. Circular edge.

The greatest stress occurs at the edge.[1] Indeed the crack begins from some point of the edge.

If the material is exactly uniform and homogeneous, the crack begins at all points of the edge at the same time, and after the cracking the stress distribution will be the same as that of the case where the edge is supported, for some duration of time in which the pressure keeps constant.

Then the stress at the center increases very rapidly. This can not occur exactly in nature, but imperfectly. We may conclude from the photographs that the crack begins at some point at the edge and in consequence of this crack, the stress at the center increases so rapidly that some radial cracks occur just after the first crack.

II. Elliptic edge.

Similar to the circle. The calculation is well known.[2]

1) Love, The mathematical theory of elasticity, p. 484. Morley, Strength of materials, p. 430.

2) Love, l.c. p. 484. Morley, l.c. p. 440.

III. Square edge.

In the fourth edition of Love, *Mathematical Theory of Elasticity*[1] there is a section upon this problem. We calculate the stress after that method. The greatest stress occurs at the middle point of the longer side. The ratio between that and the stress at the middle point of the plate is about 2.4, but the crack occurs from the center. Sometimes, though very rarely, it occurs as in Fig. 10. If the crack begins from only one side then the boundary condition will become different and a secondary crack will occur as is shown in Fig. 8.

IV. Rectangular edge.

Similar to the square.

It is remarkable that in Fig. 15 the crack at the edge begins nearly at the same time as the crack from the middle point of the rectangles. Cracks from the middle point of the rectangle are transversed by the crack from the edge, but one crack from the edge on the opposite side is transversed by a crack from the middle point of the rectangle.

V. Clamped at the edge and at the middle point of the plate.

The crack begins from the middle point of the plate, In the case of square edge the crack is parallel to one of the sides, and in the case of rectangle it is parallel to the shorter side. The pressure difference is nearly the same as the case where it is not clamped at the middle point of the plate.

VI. Combined edge of the rectangles.

As is shown in Fig. 16, the crack occurs from the convex corner of inside rectangle. Therefore we made the corner round in order to remove the convex corner effect. Comparing Fig. 16 with Fig. 17 we can conclude that the former cracks are the cracks of the latter case plus the effect of the corner.

VII. Other conditions.

If we use a light weight instead of a heavy one, the boundary will be clamped imperfectly. If we diminish the pressure of the vessel very quickly,

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1) Love, l.c. p. 494.

the rate of increasing pressure must be considerable. Under these conditions we can have many different complicated crack figures, one of which is shown in Fig. 18.

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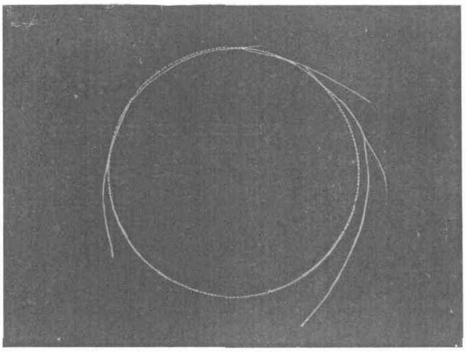
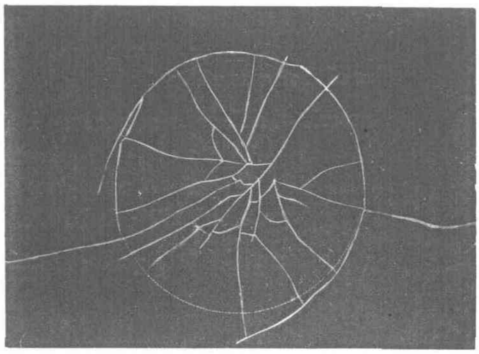
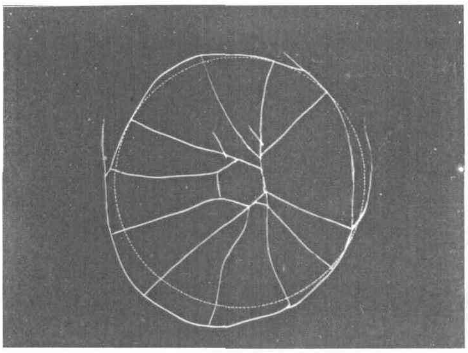
The Crack figure of thin glass plate.
The dotted line denotes the clamped edge.



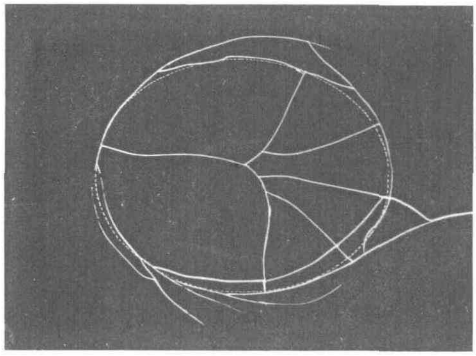
No. 1. Fig. 2. $\times 6.25^{-1}$



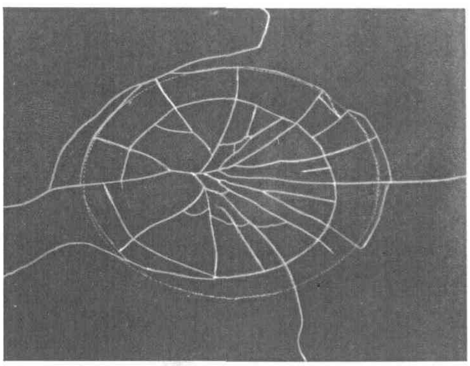
No. 2. Fig. 3. $\times 6.25^{-1}$



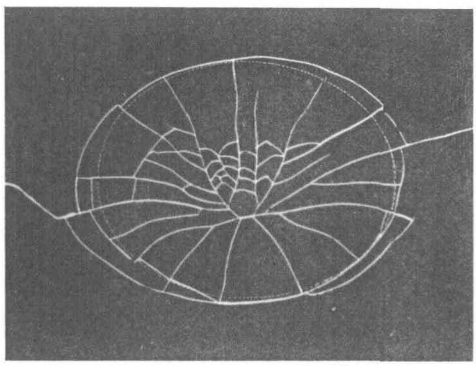
No. 3. Fig. 4. $\times 6.25^{-1}$



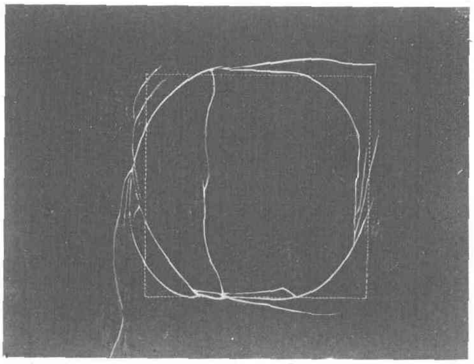
No. 8. Fig. 5. $\times 6.25^{-1}$



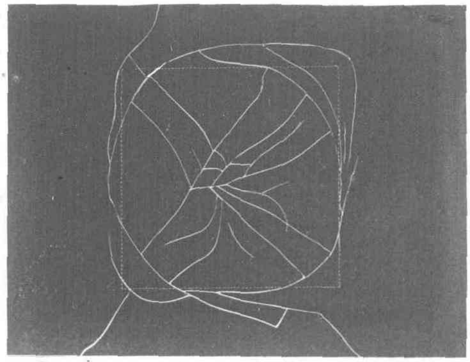
No. 10. Fig. 6. $\times 6.25^{-1}$



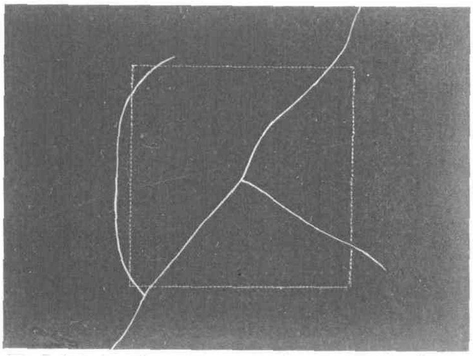
No. 11. Fig. 7. $\times 6.25^{-1}$



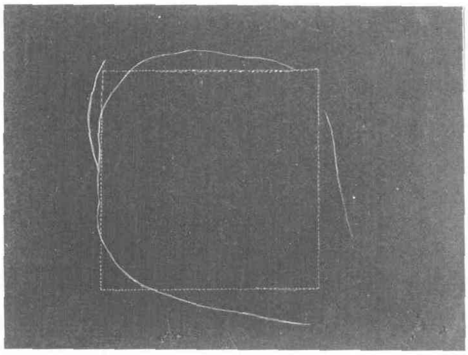
No. 12. Fig. 8. $\times 6.25^{-1}$



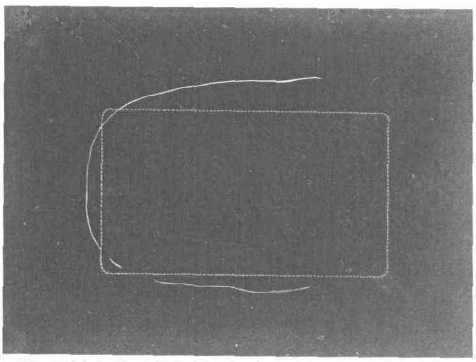
No. 14. Fig. 9. $\times 6.25^{-1}$



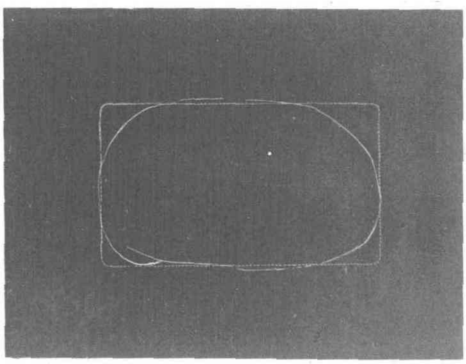
No. 16. Fig. 10. $\times 6.25^{-1}$



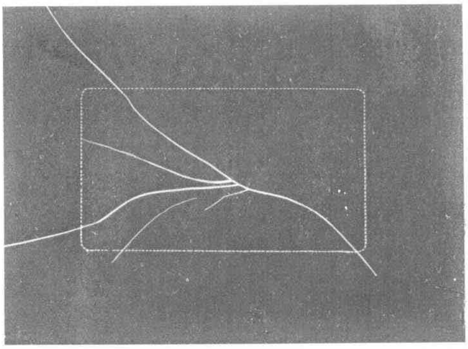
No. 19. Fig. 11. $\times 6.25^{-1}$



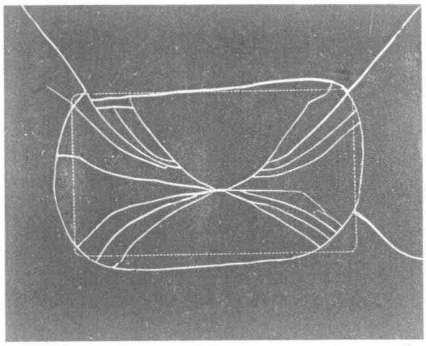
No. 24. Fig. 12. $\times 6.25^{-1}$



No. 25. Fig. 13. $\times 6.25^{-1}$



No. 28. Fig. 14. $\times 6.25^{-1}$



No. 29. Fig. 15. $\times 6.25^{-1}$

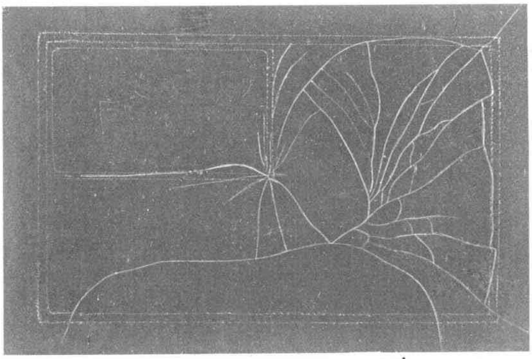


Fig. 16. $\times 8.8^{-1}$

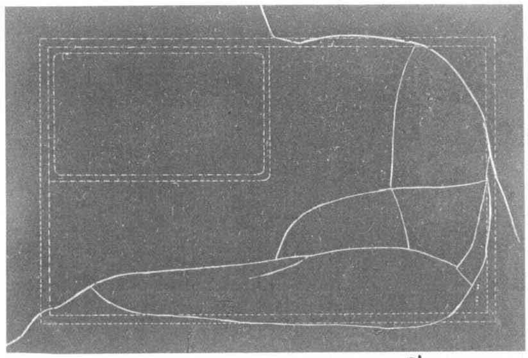


Fig. 17. $\times 8.8^{-1}$

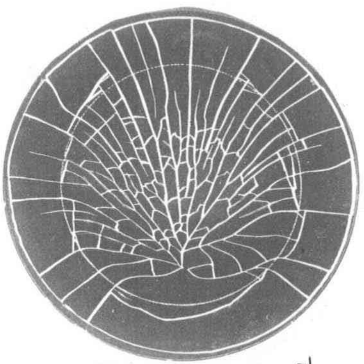


Fig. 18. $\times 6.25^{-1}$