Vertical and Horizontal Load Stresses in the Building Frames with Constant Ratio of Stiffness.

By

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Introductory

The coefficients derived from the computation results of the joint-rotation angles, especially relating to the multiple storied building frames with a large number of bays, are the chief object of study in the present paper.

The investigation of the effect of vertical loads and also wind loads on the elastic deformations is of the utmost importance for the analysis of the statically-indeterminate structures, while the notice of investigators seems especially to have escaped up to this time, the exact solution of multiple storied frames with a large number of bays.

The Mechanical Tabulation Method facilitates the process of finding the equilibrium equations from which the unknowns are to be determined. For example, the complete calculation of a twenty storied building frame with ten bays, carrying loads distributed vertically and symmetrically on each story in the same manner, was accomplished by a single person within about seven hours, in a single day.

The calculations performed during the last few weeks in the direction mentioned have been collected in the present note.

First we solved the above problem of a twenty storied frame with ten bays and secondly the similar problem of a six storied frame with twenty bays, both under the condition of constant K, i.e. constant value of moments of inertia divided by the length of the members. Next, as a combination of the above two, the similar problem of a twenty storied frame with twenty bays was tried. It was found that for such problems it might have been more practical to make the calculation proceed as the extension of the first problem above mentioned.

The relation between the individual coefficients in the last problem, may lead to the solution of the similar problem of an infinite number of stories and bays under the rigorous estimation of the unknown slopes, with striking facility as well as speed.

Finally we treated a twenty storied bent with three bays, carrying a horizontal load concentrated at each joint on the left-hand side of the structure.

As appendix we add sixty tables of the utmost importance for the calculation of the statically-indeterminate stresses in question. The diagrams in the tables show the relation between the value of distorsions and number of bays as well as stories for any rectangular building frames with constant moments of inertia divided by the length of the members. From the diagrams we may determine the values of slope and deflection, with which directly the value of joint-moments may be estimated and at the same time, some characteristics of the variation of elastic distorsions may be cleared up.

The present calculations have been made by the effort of my assistants, Messrs. H. Matsuzaka and S. Ono, to whom my thanks are obvious.

1. Twenty Storied Building Frame with Ten Bays. Loads Vertical and Symmetrical.

A proceeding paragraph by the present author in the *Memoirs,* contains the derivation of the joint-equilibrium equations to be used to solve the general case shown in the accompanying sketch (Fig. 1). This frame carries any system of the same vertical loads symmetrical on each bay of equal length.

This exact solution, with the same accuracy as by the Slope Deflection Method, seems to be quite a complicated and tedious problem.

But the Mechanical Tabulation Method determines with striking facility, without any calculations, as many simultaneous equations as we have unknown slope terms to solve for.

Moreover, the Mechanical Tabulation Method accompanied by the process of Repeated Trials solves the same problem with the same accuracy as by the process of elimination, or perhaps with more accurate results, because it may be said that it is very interesting to find that a solution of a large set of equations, e.g. sixty, seventy and more may be obtained

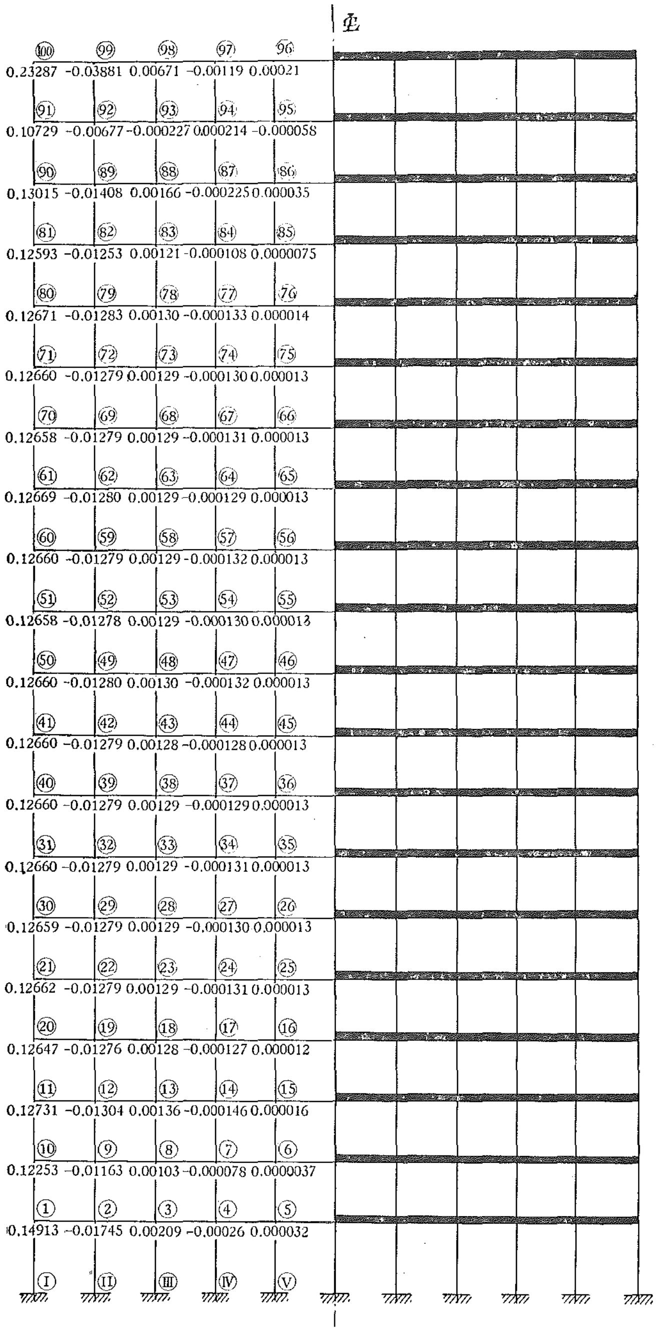

℄
100 99 98 97 96
0.23287 -0.03881 0.00671 -0.00119 0.00021
91 92 93 94 95
0.10729 -0.00677 -0.000227 0.000214 -0.000058
90 89 88 87 86
0.13015 -0.01408 0.00166 -0.000225 0.000035
81 82 83 84 85
0.12593 -0.01253 0.00121 -0.000108 0.0000075
80 79 78 77 76
0.12671 -0.01283 0.00130 -0.000133 0.000014
71 72 73 74 75
0.12660 -0.01279 0.00129 -0.000130 0.000013
70 69 68 67 66
0.12658 -0.01279 0.00129 -0.000131 0.000013
61 62 63 64 65
0.12669 -0.01280 0.00129 -0.000129 0.000013
60 59 58 57 56
0.12660 -0.01279 0.00129 -0.000132 0.000013
51 52 53 54 55
0.12658 -0.01278 0.00129 -0.000130 0.000013
50 49 48 47 46
0.12660 -0.01280 0.00130 -0.000132 0.000013
41 42 43 44 45
0.12660 -0.01279 0.00128 -0.000128 0.000013
40 39 38 37 36
0.12660 -0.01279 0.00129 -0.000129 0.000013
31 32 33 34 35
0.12660 -0.01279 0.00129 -0.000131 0.000013
30 29 28 27 26
0.12659 -0.01279 0.00129 -0.000130 0.000013
21 22 23 24 25
0.12662 -0.01279 0.00129 -0.000131 0.000013
20 19 18 17 16
0.12647 -0.01276 0.00128 -0.000127 0.000012
11 12 13 14 15
0.12731 -0.01304 0.00136 -0.000146 0.000016
10 9 8 7 6
0.12253 -0.01163 0.00103 -0.000078 0.0000037
1 2 3 4 5
0.14913 -0.01745 0.00209 -0.00026 0.000032
I II III IV V

Fig. 1.

approximately even by the process of elimination, on account of the errors accumulated by the neglecting of the higher decimal part during the successive numerical calculations and the capacity of the calculating machine being limited; while for the process of Repeated Trial, slide-rules are employable for the true values up to three figures: e.g. 0.149 instead of 0.1491347.

Thus continue successive substitutions until all the values of unknowns are obtained expecting to repeat the trials many times until approximately equal values are obtained at least two successive times after the first. Then the results obtained are nothing but the true values which must coincide with the results computed by the process of elimination, however, the results are carried only to the third decimal place.

For practical purposes, the above mentioned calculation results are accurate enough to apply, if in the case where the purposes require the further decimal part of the data, it is a trifling matter to continue the same process of calculation, considering the obtained results as the approximate value of the unknowns.

For the frame shown in Fig. 1, we may write out, by the Mechanical Tabulation Method, with no calculations, the hundred simultaneous equations for the hundred joints in a systematic arrangement and in a symmetrical relation as shown in Table I.

In the Mechanical Tabulation Method, it is a small matter to fill in the numerical values of the coefficients for any particular equation involving the unknowns, and as soon as the mere writing operation is done, the process of Repeated Trials is available. For the computation by the process of Repeated Trials, the form of equation (3) given by the present author in the preceding paragraph above mentioned is practical, e.g. for joint 1 in Fig. 1 we get the equation:

$$\overset{\overset{\varphi_{10}}{+}}{\varphi_1}(6)+\varphi_2=\tau_1$$

For joint 2, we get the equation:

$$\varphi_1+\overset{\overset{\varphi_9}{+}}{\varphi_2}(8)+\varphi_3=\tau_2=0 \qquad \text{for the frame of equal bays.}$$

etc.

These equilibrium equations do nothing but show the arrangement of the unknowns φ and two times the number of members which come in

Table I.

Equation	Right-Hand Member of Equation*	Left-Hand Member of Equation
1	1	φ_{1}: 6, φ_{2}: 1, φ_{10}: 1
2	0	φ_{1}: 1, φ_{2}: 8, φ_{3}: 1, φ_{9}: 1
3	0	φ_{2}: 1, φ_{3}: 8, φ_{4}: 1, φ_{8}: 1
4	0	φ_{3}: 1, φ_{4}: 8, φ_{5}: 1, φ_{7}: 1
5	0	φ_{4}: 1, φ_{5}: 8, φ_{6}: 1
6	0	φ_{5}: 1, φ_{6}: 8, φ_{7}: 1, φ_{15}: 1
7	0	φ_{4}: 1, φ_{6}: 1, φ_{7}: 8, φ_{8}: 1, φ_{14}: 1
8	0	φ_{3}: 1, φ_{7}: 1, φ_{8}: 8, φ_{9}: 1, φ_{13}: 1
9	0	φ_{2}: 1, φ_{8}: 1, φ_{9}: 8, φ_{10}: 1, φ_{12}: 1
10	1	φ_{1}: 1, φ_{9}: 1, φ_{10}: 6, φ_{11}: 1
11	1	φ_{10}: 1, φ_{11}: 6, φ_{12}: 1, φ_{20}: 1
12	0	φ_{9}: 1, φ_{11}: 1, φ_{12}: 8, φ_{13}: 1, φ_{19}: 1
13	0	φ_{8}: 1, φ_{12}: 1, φ_{13}: 8, φ_{14}: 1, φ_{18}: 1
14	0	φ_{7}: 1, φ_{13}: 1, φ_{14}: 8, φ_{15}: 1, φ_{17}: 1
15	0	φ_{6}: 1, φ_{14}: 1, φ_{15}: 8, φ_{16}: 1
16	0	φ_{15}: 1, φ_{16}: 8, φ_{17}: 1, φ_{25}: 1
17	0	φ_{14}: 1, φ_{16}: 1, φ_{17}: 8, φ_{18}: 1, φ_{24}: 1
18	0	φ_{13}: 1, φ_{17}: 1, φ_{18}: 8, φ_{19}: 1, φ_{23}: 1
19	0	φ_{12}: 1, φ_{18}: 1, φ_{19}: 8, φ_{20}: 1, φ_{22}: 1
20	1	φ_{11}: 1, φ_{19}: 1, φ_{20}: 6, φ_{21}: 1
21	1	φ_{20}: 1, φ_{21}: 6, φ_{22}: 1, φ_{30}: 1
22	0	φ_{19}: 1, φ_{21}: 1, φ_{22}: 8, φ_{23}: 1, φ_{29}: 1
23	0	φ_{18}: 1, φ_{22}: 1, φ_{23}: 8, φ_{24}: 1, φ_{28}: 1
24	0	φ_{17}: 1, φ_{23}: 1, φ_{24}: 8, φ_{25}: 1, φ_{27}: 1
25	0	φ_{16}: 1, φ_{24}: 1, φ_{25}: 8, φ_{26}: 1
26	0	φ_{25}: 1, φ_{26}: 8, φ_{27}: 1, φ_{35}: 1
27	0	φ_{24}: 1, φ_{26}: 1, φ_{27}: 8, φ_{28}: 1, φ_{34}: 1
28	0	φ_{23}: 1, φ_{27}: 1, φ_{28}: 8, φ_{29}: 1, φ_{33}: 1
29	0	φ_{22}: 1, φ_{28}: 1, φ_{29}: 8, φ_{30}: 1, φ_{32}: 1
30	1	φ_{21}: 1, φ_{29}: 1, φ_{30}: 6, φ_{31}: 1
31	1	φ_{30}: 1, φ_{31}: 6, φ_{32}: 1, φ_{40}: 1
32	0	φ_{29}: 1, φ_{31}: 1, φ_{32}: 8, φ_{33}: 1, φ_{39}: 1
33	0	φ_{28}: 1, φ_{32}: 1, φ_{33}: 8, φ_{34}: 1, φ_{38}: 1
34	0	φ_{27}: 1, φ_{33}: 1, φ_{34}: 8, φ_{35}: 1, φ_{37}: 1
35	0	φ_{26}: 1, φ_{34}: 1, φ_{35}: 8, φ_{36}: 1
36	0	φ_{35}: 1, φ_{36}: 8, φ_{37}: 1, φ_{45}: 1
37	0	φ_{34}: 1, φ_{36}: 1, φ_{37}: 8, φ_{38}: 1, φ_{44}: 1
38	0	φ_{33}: 1, φ_{37}: 1, φ_{38}: 8, φ_{39}: 1, φ_{43}: 1
39	0	φ_{32}: 1, φ_{38}: 1, φ_{39}: 8, φ_{40}: 1, φ_{42}: 1
40	1	φ_{31}: 1, φ_{39}: 1, φ_{40}: 6, φ_{41}: 1
41	1	φ_{40}: 1, φ_{41}: 6, φ_{42}: 1, φ_{50}: 1
42	0	φ_{39}: 1, φ_{41}: 1, φ_{42}: 8, φ_{43}: 1, φ_{49}: 1
43	0	φ_{38}: 1, φ_{42}: 1, φ_{43}: 8, φ_{44}: 1, φ_{48}: 1
44	0	φ_{37}: 1, φ_{43}: 1, φ_{44}: 8, φ_{45}: 1, φ_{47}: 1
45	0	φ_{36}: 1, φ_{44}: 1, φ_{45}: 8, φ_{46}: 1
46	0	φ_{45}: 1, φ_{46}: 8, φ_{47}: 1, φ_{55}: 1
47	0	φ_{44}: 1, φ_{46}: 1, φ_{47}: 8, φ_{48}: 1, φ_{54}: 1
48	0	φ_{43}: 1, φ_{47}: 1, φ_{48}: 8, φ_{49}: 1, φ_{53}: 1
49	0	φ_{42}: 1, φ_{48}: 1, φ_{49}: 8, φ_{50}: 1, φ_{52}: 1
50	1	φ_{41}: 1, φ_{49}: 1, φ_{50}: 6, φ_{51}: 1
51	1	φ_{50}: 1, φ_{51}: 6, φ_{52}: 1, φ_{60}: 1
52	0	φ_{49}: 1, φ_{51}: 1, φ_{52}: 8, φ_{53}: 1, φ_{59}: 1
53	0	φ_{48}: 1, φ_{52}: 1, φ_{53}: 8, φ_{54}: 1, φ_{58}: 1
54	0	φ_{47}: 1, φ_{53}: 1, φ_{54}: 8, φ_{55}: 1, φ_{57}: 1
55	0	φ_{46}: 1, φ_{54}: 1, φ_{55}: 8, φ_{56}: 1
56	0	φ_{55}: 1, φ_{56}: 8, φ_{57}: 1, φ_{65}: 1
57	0	φ_{54}: 1, φ_{56}: 1, φ_{57}: 8, φ_{58}: 1, φ_{64}: 1
58	0	φ_{53}: 1, φ_{57}: 1, φ_{58}: 8, φ_{59}: 1, φ_{63}: 1
59	0	φ_{52}: 1, φ_{58}: 1, φ_{59}: 8, φ_{60}: 1, φ_{62}: 1
60	1	φ_{51}: 1, φ_{59}: 1, φ_{60}: 6, φ_{61}: 1
61	1	φ_{60}: 1, φ_{61}: 6, φ_{62}: 1, φ_{70}: 1
62	0	φ_{59}: 1, φ_{61}: 1, φ_{62}: 8, φ_{63}: 1, φ_{69}: 1
63	0	φ_{58}: 1, φ_{62}: 1, φ_{63}: 8, φ_{64}: 1, φ_{68}: 1
64	0	φ_{57}: 1, φ_{63}: 1, φ_{64}: 8, φ_{65}: 1, φ_{67}: 1
65	0	φ_{56}: 1, φ_{64}: 1, φ_{65}: 8, φ_{66}: 1
66	0	φ_{65}: 1, φ_{66}: 8, φ_{67}: 1, φ_{75}: 1
67	0	φ_{64}: 1, φ_{66}: 1, φ_{67}: 8, φ_{68}: 1, φ_{74}: 1
68	0	φ_{63}: 1, φ_{67}: 1, φ_{68}: 8, φ_{69}: 1, φ_{73}: 1
69	0	φ_{62}: 1, φ_{68}: 1, φ_{69}: 8, φ_{70}: 1, φ_{72}: 1
70	1	φ_{61}: 1, φ_{69}: 1, φ_{70}: 6, φ_{71}: 1
71	1	φ_{70}: 1, φ_{71}: 6, φ_{72}: 1, φ_{80}: 1
72	0	φ_{69}: 1, φ_{71}: 1, φ_{72}: 8, φ_{73}: 1, φ_{79}: 1
73	0	φ_{68}: 1, φ_{72}: 1, φ_{73}: 8, φ_{74}: 1, φ_{78}: 1
74	0	φ_{67}: 1, φ_{73}: 1, φ_{74}: 8, φ_{75}: 1, φ_{77}: 1
75	0	φ_{66}: 1, φ_{74}: 1, φ_{75}: 8, φ_{76}: 1
76	0	φ_{75}: 1, φ_{76}: 8, φ_{77}: 1, φ_{85}: 1
77	0	φ_{74}: 1, φ_{76}: 1, φ_{77}: 8, φ_{78}: 1, φ_{84}: 1
78	0	φ_{73}: 1, φ_{77}: 1, φ_{78}: 8, φ_{79}: 1, φ_{83}: 1
79	0	φ_{72}: 1, φ_{78}: 1, φ_{79}: 8, φ_{80}: 1, φ_{82}: 1
80	1	φ_{71}: 1, φ_{79}: 1, φ_{80}: 6, φ_{81}: 1
81	1	φ_{80}: 1, φ_{81}: 6, φ_{82}: 1, φ_{90}: 1
82	0	φ_{79}: 1, φ_{81}: 1, φ_{82}: 8, φ_{83}: 1, φ_{89}: 1
83	0	φ_{78}: 1, φ_{82}: 1, φ_{83}: 8, φ_{84}: 1, φ_{88}: 1
84	0	φ_{77}: 1, φ_{83}: 1, φ_{84}: 8, φ_{85}: 1, φ_{87}: 1
85	0	φ_{76}: 1, φ_{84}: 1, φ_{85}: 8, φ_{86}: 1
86	0	φ_{85}: 1, φ_{86}: 8, φ_{87}: 1, φ_{95}: 1
87	0	φ_{84}: 1, φ_{86}: 1, φ_{87}: 8, φ_{88}: 1, φ_{94}: 1
88	0	φ_{83}: 1, φ_{87}: 1, φ_{88}: 8, φ_{89}: 1, φ_{93}: 1
89	0	φ_{82}: 1, φ_{88}: 1, φ_{89}: 8, φ_{90}: 1, φ_{92}: 1
90	1	φ_{81}: 1, φ_{89}: 1, φ_{90}: 6, φ_{91}: 1
91	1	φ_{90}: 1, φ_{91}: 6, φ_{92}: 1, φ_{100}: 1
92	0	φ_{89}: 1, φ_{91}: 1, φ_{92}: 8, φ_{93}: 1, φ_{99}: 1
93	0	φ_{88}: 1, φ_{92}: 1, φ_{93}: 8, φ_{94}: 1, φ_{98}: 1
94	0	φ_{87}: 1, φ_{93}: 1, φ_{94}: 8, φ_{95}: 1, φ_{97}: 1
95	0	φ_{86}: 1, φ_{94}: 1, φ_{95}: 8, φ_{96}: 1
96	0	φ_{95}: 1, φ_{96}: 6, φ_{97}: 1
97	0	φ_{94}: 1, φ_{96}: 1, φ_{97}: 6, φ_{98}: 1
98	0	φ_{93}: 1, φ_{97}: 1, φ_{98}: 6, φ_{99}: 1
99	0	φ_{92}: 1, φ_{98}: 1, φ_{99}: 6, φ_{100}: 1
100	1	φ_{91}: 1, φ_{99}: 1, φ_{100}: 4

* For the convenience of the representation, the right-hand member of the equation has been written in the left-hand side of the table.

the joint under consideration. It may therefore be very easy to get all the joint-equilibrium equations by mere looking at the joints and the boundary loading conditions shown in Fig. 1.

To the computor who is quite familiar with the method, it may be a matter of no great difficulty to make the calculation proceed, considering Fig. 1 itself as if it were a representation of one set of the simultaneous equations, without writing any equilibrium equations. The author thinks therefore that it may be very practical and of much importance that as soon as we have the sketch of the building frame upon which the joint numbers have been indicated, the sketch itself will show directly the representation of the joint-equilibrium equations to be used to determine the unknowns.

The calculation results with multiplicator: C, by the process of Repeated Trials are shown in the left half of the sketch (Fig. 1).

The joint moment M_{1-2} is given as follows:

$$M_{1-2}=2\varphi_1+\varphi_2-C$$
$$=\{2(0.14913)-0.01745-1\}C=-0.719\,C$$
$$M_{1-10}=2\varphi_1+\varphi_{10}=\{2(0.14913)+0.12253\}C$$
$$=0.421\,C$$
$$M_{1-I}=2\varphi_1=0.298\,C$$

etc.

For the case shown in Fig. 1, there is a fairly regular variation in the coefficients from joint to joint. The elastic distorsions at all the joints on the first column line from the left side of the structure, are of positive rotations and those of the second column joints are negative. Except at the joints of the next to the top floor, the elastic distorsions occur in the same direction on the same column line, positive and negative alternately, because, very interestingly the elastic distorsions are irregular at some of the joints on the next to the top floor.

In the author's experience, the coefficient 0.149 is the definite number at joint 1 for an arbitrary multiple storied bent with multiple number of bays. The coefficients at joints 1 to 15, i.e. at the joints on the lower three stories, are different but in definite relation to each other. The coefficients at joints 86 to 100, i.e. at the joints on the upper three stories, are also different but definitely related to each other, while all the coefficients at the remaining joints have a fairly regular variation and are considered as constant numbers for multiple storied frames in question. For example, for a frame of ten stories and four equal bays, with constant K, as shown in Fig. 2, the elastic distorsions have the values written in the figure at its corresponding joint. But these constants are practically derived at once from the results in Fig. 1.

It may be said that it is a matter of great importance as well as very interesting that the number "13" comes frequently in the calculation results. Readers may have already noticed from Fig. 1 that the joints on the comparatively central stories have approximately the following values:

$$0.13 \quad -0.013 \quad 0.0013 \quad -0.00013 \quad 0.000013$$

All the joints on the lower floors deviate from the above said constant values, on account of the fixed condition at the foundation and for the similar reason, the joints on the upper floors are effected by the free condition of the roof.

By this example it may be said that the coefficient-variation in the direction of height has become obvious, while in the following problems, the coefficient variation may be cleared in the horizontal direction.

2. Six Storied Building Frame with Twenty Bays. Loads Vertical and Symmetrical.

Fig. 2 shows a symmetrical building frame of twenty equal bays, six stories high, carrying loads symmetrical and vertical on each girder of the structure.

Calculation similar to that in the above problem determines the unknown slope terms as shown in the left half of Fig. 2.

Readers may find in Figs. 2 and 3 the following abridged notations:

0.0^4314 instead of 0.0000314

0.0^5397 instead of 0.00000397 etc.

By this example it may be said that the coefficients of the outside joints remain in the almost constant values which we have seen in the previous problem, while the variation of the coefficients in the inner joints are different, on account of the fixed condition at the lower ends, modified by the effect of the free condition of the roof ends.

The results obtained in the problem may be applied to the similar problem of comparatively low built structures.

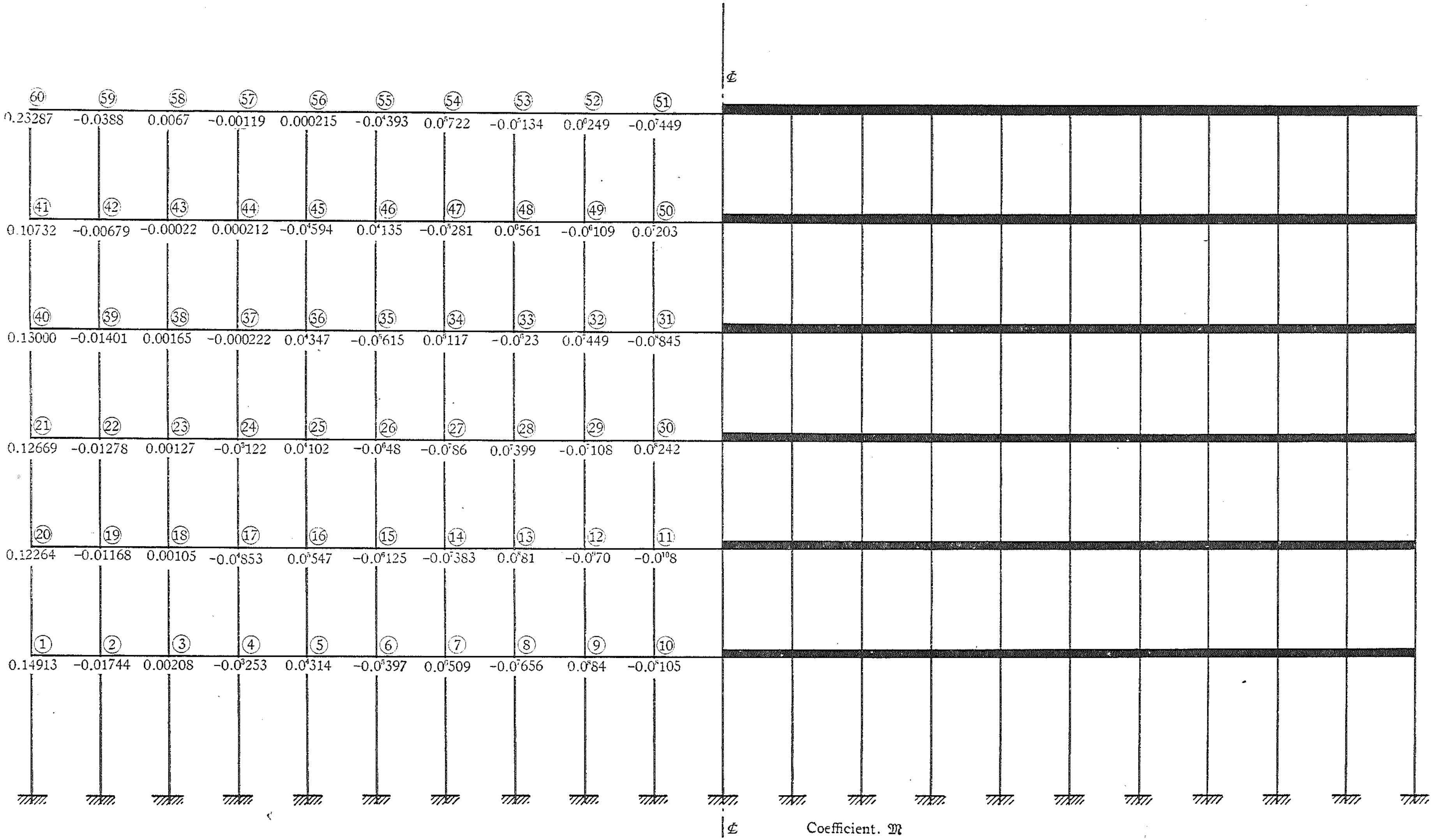


Fig. 2.

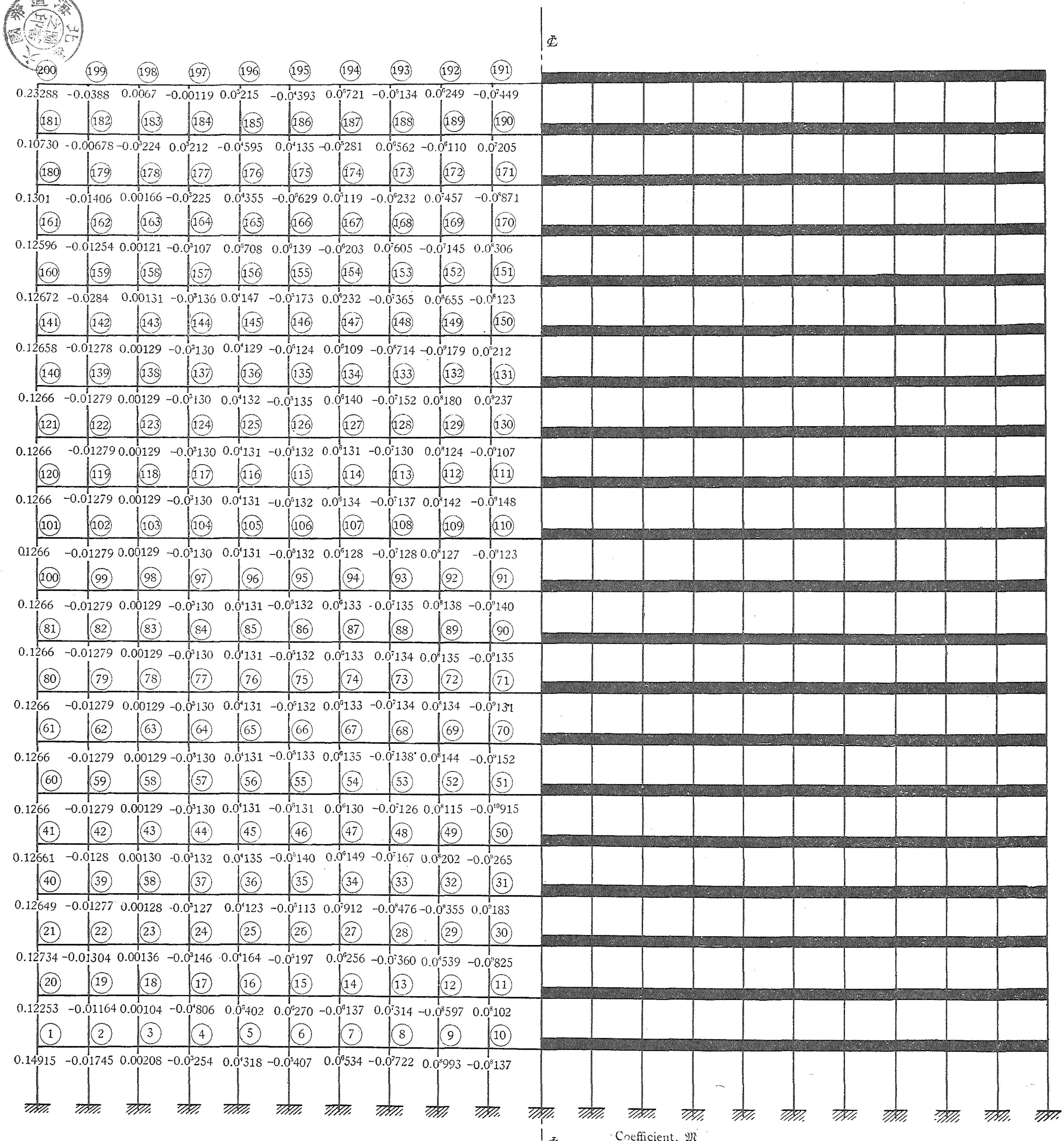
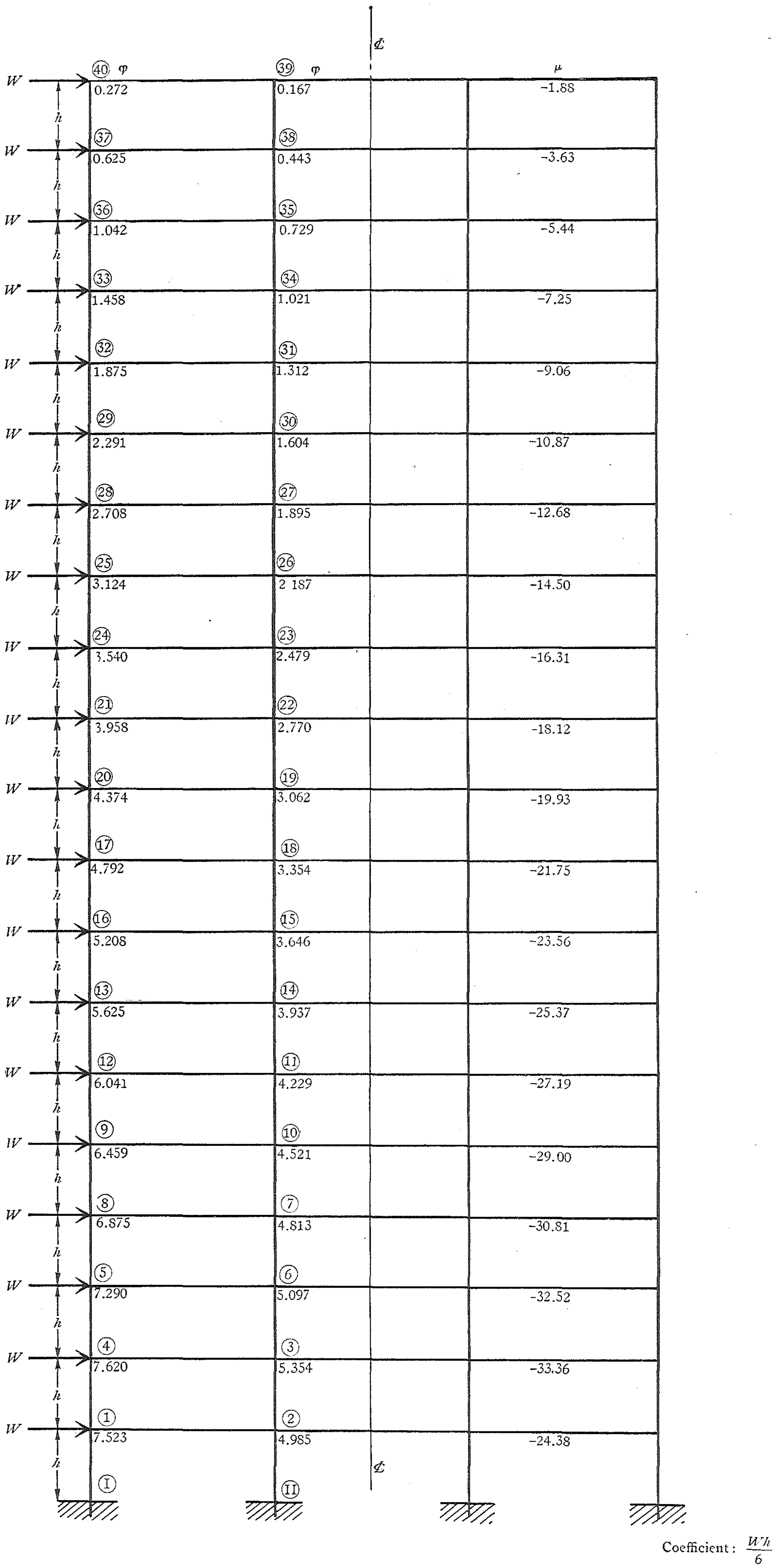


Fig. 3.

(See Memoirs Vol. I, No. 3, Table C)



℄
40 φ 0.272
39 φ 0.167
μ -1.88
37 0.625
38 0.443
-3.63
36 1.042
35 0.729
-5.44
33 1.458
34 1.021
-7.25
32 1.875
31 1.312
-9.06
29 2.291
30 1.604
-10.87
28 2.708
27 1.895
-12.68
25 3.124
26 2 187
-14.50
24 3.540
23 2.479
-16.31
21 3.958
22 2.770
-18.12
20 4.374
19 3.062
-19.93
17 4.792
18 3.354
-21.75
16 5.208
15 3.646
-23.56
13 5.625
14 3.937
-25.37
12 6.041
11 4.229
-27.19
9 6.459
10 4.521
-29.00
8 6.875
7 4.813
-30.81
5 7.290
6 5.097
-32.52
4 7.620
3 5.354
-33.36
1 7.523
2 4.985
-24.38
I
II
W
h
Coefficient: $\frac{Wh}{6}$

Fig. 4.

3. Twenty Storied Building Frame with Twenty Bays. Loads Vertical and Symmetrical.

Fig. 3 shows a symmetrical building frame of twenty equal bays, twenty stories high, carrying loads symmetrical and vertical on each girder of the structure. It may be noticed that, as in the above two cases, the system of vertical symmetrical loads is arbitrary and the results obtained may have a quite wide range of application.

The computation of the problem was commenced under the expectation of combining the above two problems, having deduced the approximate values from the known data of the two cases. After the calculation of successive substitution, we found that the coefficients computed are approximately the same as the values which may be assumed and derived from the extension of the first problem of the twenty storied frame with ten bays.

The calculation was, however, quite long in spite of the simplicity and regularity of the mothed, on account of the large number of equations treated. In other words, the problem was to find two hundred unknowns from two hundred equations. It was completed, by my assistant, within about fifteen hours.

It may be here said that almost the same resumé as in the first problem is worked out from the calculation. The coefficients variation in the horizontal and also in the vertical direction may be estimated from that resumé.

4. Twenty Storied Building Frame with Three Bays. Loads Horizontal and Concentrated at Each Joint on the Left Side of the Structure.

Fig. 4 shows a symmetrical building frame of three bays, twenty stories high, carrying loads horizontal and concentrated on the left side of the structure. The method of solution I, mentioned in another note of the present author in the *Memoirs,* solves the problem very easily.

The computation results of the unknown slope and deflection terms are shown in Fig. 4, where we find some important properties of regular arrangement of the results; the variation of the coefficients in the vertical

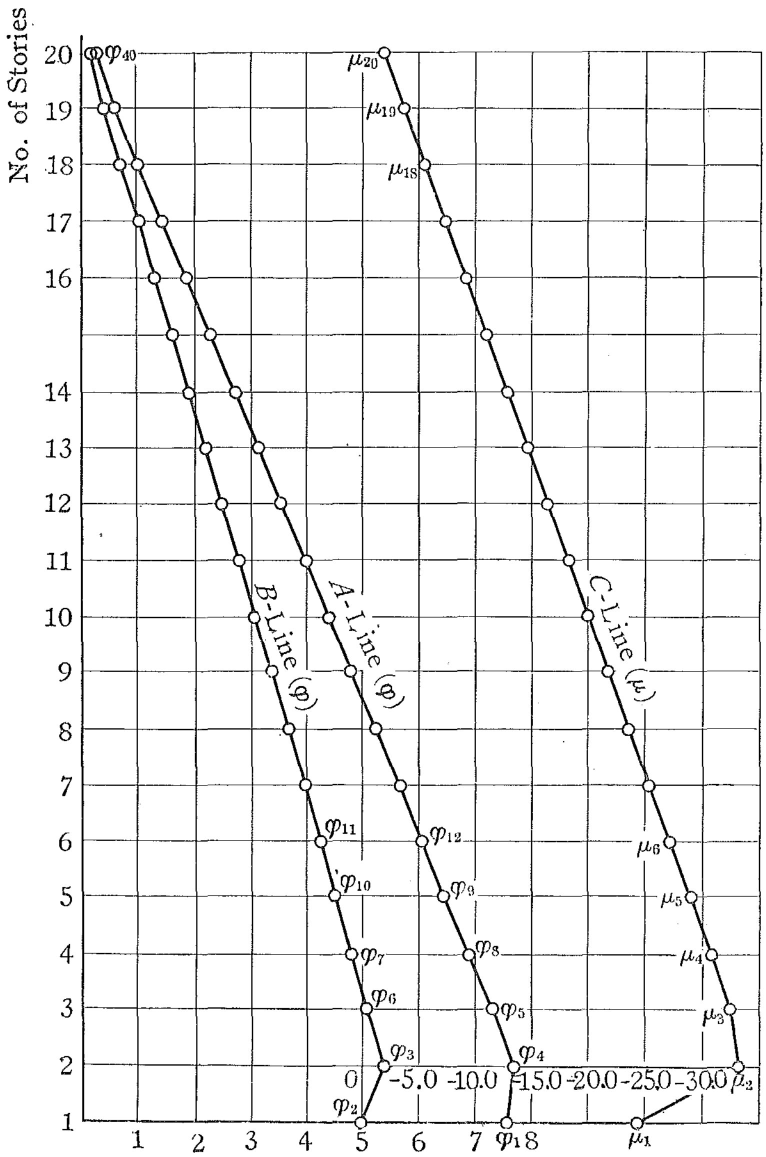


Fig. 5.

direction is quite linear, except for the coefficients of the bottom and top two.

The linear variation of the coefficients is shown graphically in Fig. 5. In the figure, Line-A and Line-B show the slope variation and we call therefore such a line as represents the tendency of the variation for slopes, the "Slope Variation Line." While Line-C shows the tendency of the variation for deflections; we therefore call it "Deflection Variation Line." Both variation lines may be very important to investigate the similar problem of multiple storied bent with multiple bays.

In this example, the common difference may be calculated as follows:

For Line-A, the common difference is approximately 0.42; for Line-B 0.29 and for Line-C 1.8.

This complete calculation may be made by a single person in about six hours, using a slide-rule.

To find the joint moments, the calculation is as follows:

At joint 1

$$M_{1-2}=2\varphi_1+\varphi_2=\left\{2(7.523)+4.985\right\}\frac{Wh}{6}=3.34Wh$$

$$M_{1-4}=2\varphi_1+\varphi_4+\mu_2=\left\{2(7.523)+7.620-33.36\right\}\frac{Wh}{6}=-1.78Wh$$

$$M_{1-I}=2\varphi_1+\mu_1=\left\{2(7.523)-24\ 38\right\}\frac{Wh}{6}=-1.56\,Wh$$

etc.

It may be said that it is a matter of great importance firstly to find the unknown slope and deflection terms but that it is a trifling matter to determine joint moments with the values of elastic redundancies already known.

The relation between the individual coefficients in the problem, may lead us to the solution of the similar problem of an infinite number of stories with three bays with little effort of calculation.

Summary and Conclusions.

The principal conclusions to be drawn from the calculation results of the investigations described in this note may be summarized as follows:

1. Caused by the fixed condition of the foundation and also by the free condition of the roof, the regularity of the variation for the elastic distorsions seems to be disturbed and the smooth line of variation seems to have abrupt changes near the foundation and roof.
2. In the wide central part of the structure considered in a plane, the elastic distorsions seems to have fairly regular variations.
3. For a system of vertical symmetrical loading, the coefficients 0.13,

0.013, 0.0013, 0.00013 etc. seem to play some important rôle to the slope-term variation of the central zone of the structure.

4. For a system of vertical symmetrical loading, the Slope Variation Line may probably be drawn as in the horizontal loadings. In this case, however, the slope terms have almost a constant value in the vertical directions.
5. For a system of horizontal loading, concentrated at joints on the left side of the structure, the Slope and also Deflection Variation Line may be considered straight and the inclination angle of the Variation Line is influenced by the number of bays; in other words, it may be said that each building frame with a definite number of bays has its own original inclination for its Variation Line.

In the concluding paragraph we may be able to quote an interesting problem as follows, on considering the twenty storied frame with twenty bays, carrying a system of vertical loads, as a special block composed of elastic members with stiff connecting joints as a whole.

The deformation phenomena of such a special block under flexure caused by vertical symmetrical loads may be investigated from the calculation results above treated while the ordinary concrete specimens under compression test have some resemblance to it.

It is a well known fact that the concrete block shows the breaking phenomena, a principal part of the test specimen being left in the form of an hour-glass. In the case under consideration, the frame block, as we call it, is subjected to flexure under vertical loadings on each story, i.e. the loadings are uniformly distributed in the inner portion of the block, as if it were under an imaginary compression test whose internal loading effects are transmitted uniformly, and even the portion under the largest deformation remains, in the case, without breaking down. Our computation results explain eloquently the deformation phenomena of such frame block; the largest deformation occurs at the top corners of the block (at joint 200) and the next largest deformation at the bottom corners (at joint 1). The amount of the deformations at the joints on the third floor is greater than that of the joints on the second. Each story as a whole ranges smaller and larger values of deformation alternately.

The portion (A) in Fig. 6 has a regular variation of the slope; while the portion (B) in the same figure is out of regularity and its distribution aspect of the distorsion is very complicated.

It may be said that the former portion is subjected to the regular distribution of the bending effect and in its portion all the boundary condition is almost similar to each other; while on the later portion, the stress distribution is subjected to the fixing or free boundary effects and deviates from the regularity of the former.

This deformation aspect may be more interesting, on comparison of Fig. 6 with Figs. 7 and 8, to deep investigation on a variety of similar problems.

The fact that each frame has its own original Variation Line of Slope and Deflection, shall lead us to the rapid and exact solution of similar problems never previously treated.

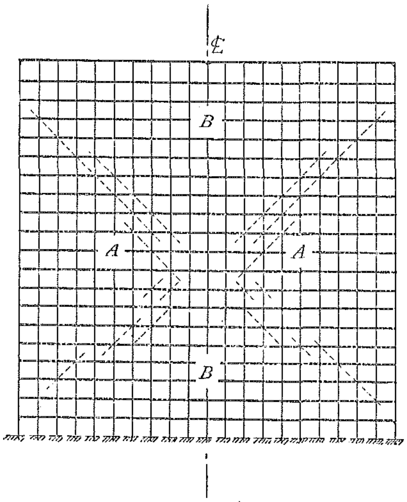


Fig. 6.

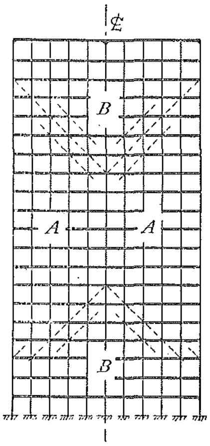


Fig. 7.

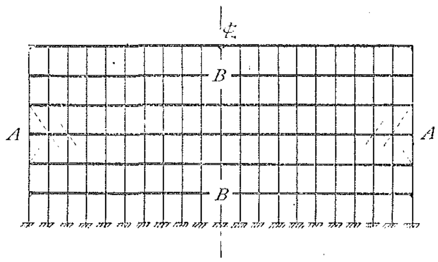


Fig. 8.

APPENDIX.

The diagrams shown in Tables I to LX are very important data to work out the statically-indeterminate joint-moments of building frames with constant K, with respect to the vertical symmetrical loads and horizontal loads of equal magnitude, concentrated at all the joints on the left side of the structure.

Some of the diagrams may perhaps give the approximate value of φ and μ, nevertheless the values obtained from them may be very convenient, and practical enough to warrant the employment of the process of Repeated Trials, finally to get the true value of joint-moments.

The following index is given so as to make it possible to locate any desired diagram in this bulletin quickly. The conventional notations in the tables are as shown in Fig. 9.

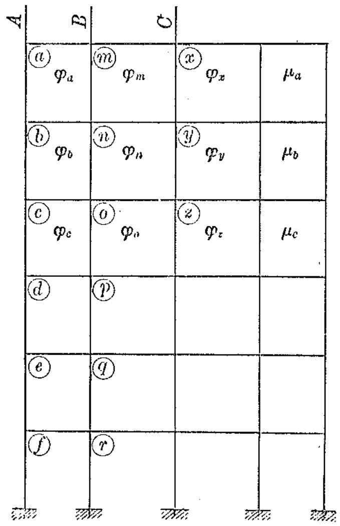
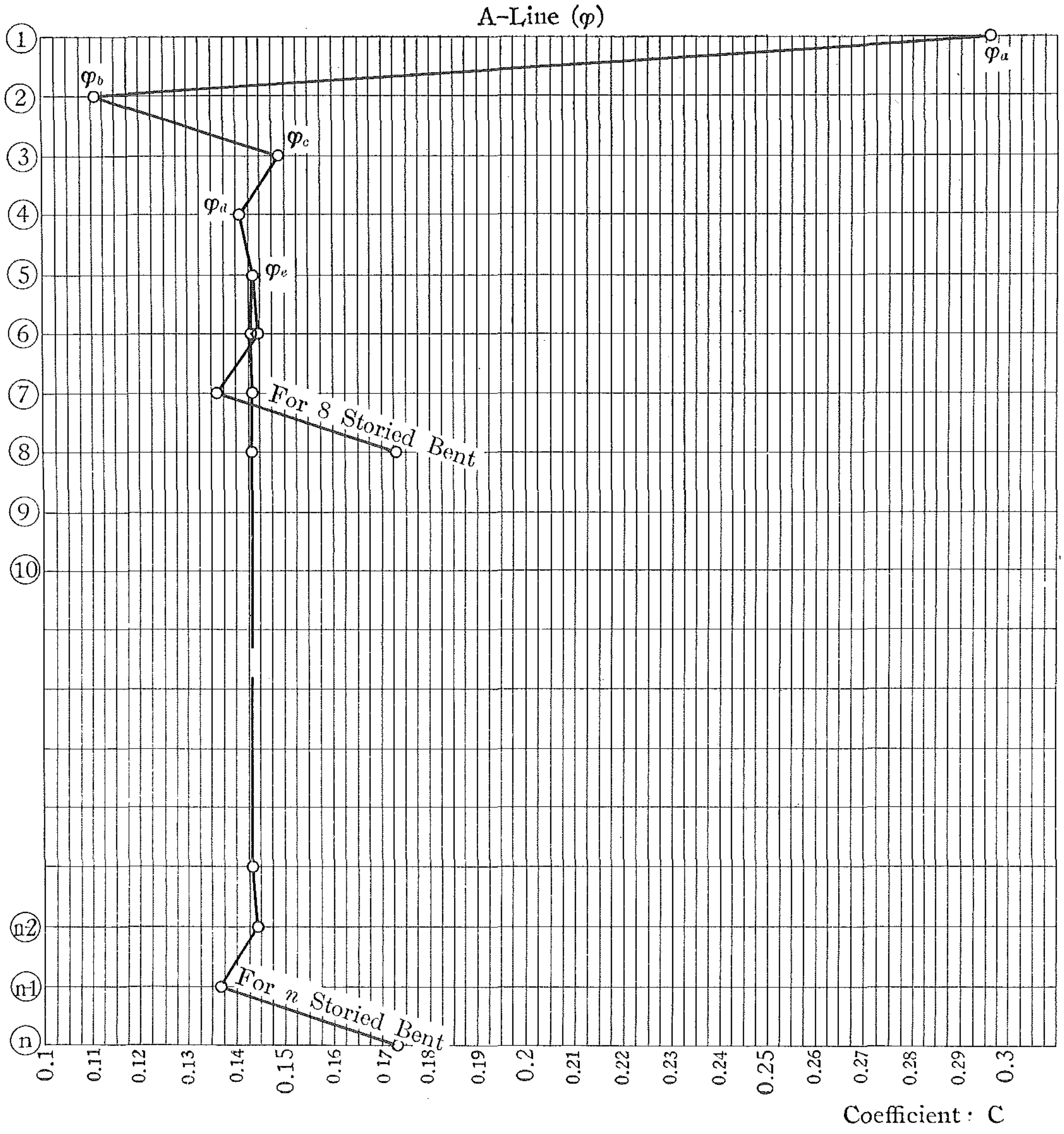


Fig. 9.

Multiple Storied Bent with Single Bay. Loads Vertical and Symmetrical.
Legs Fixed at Bases.

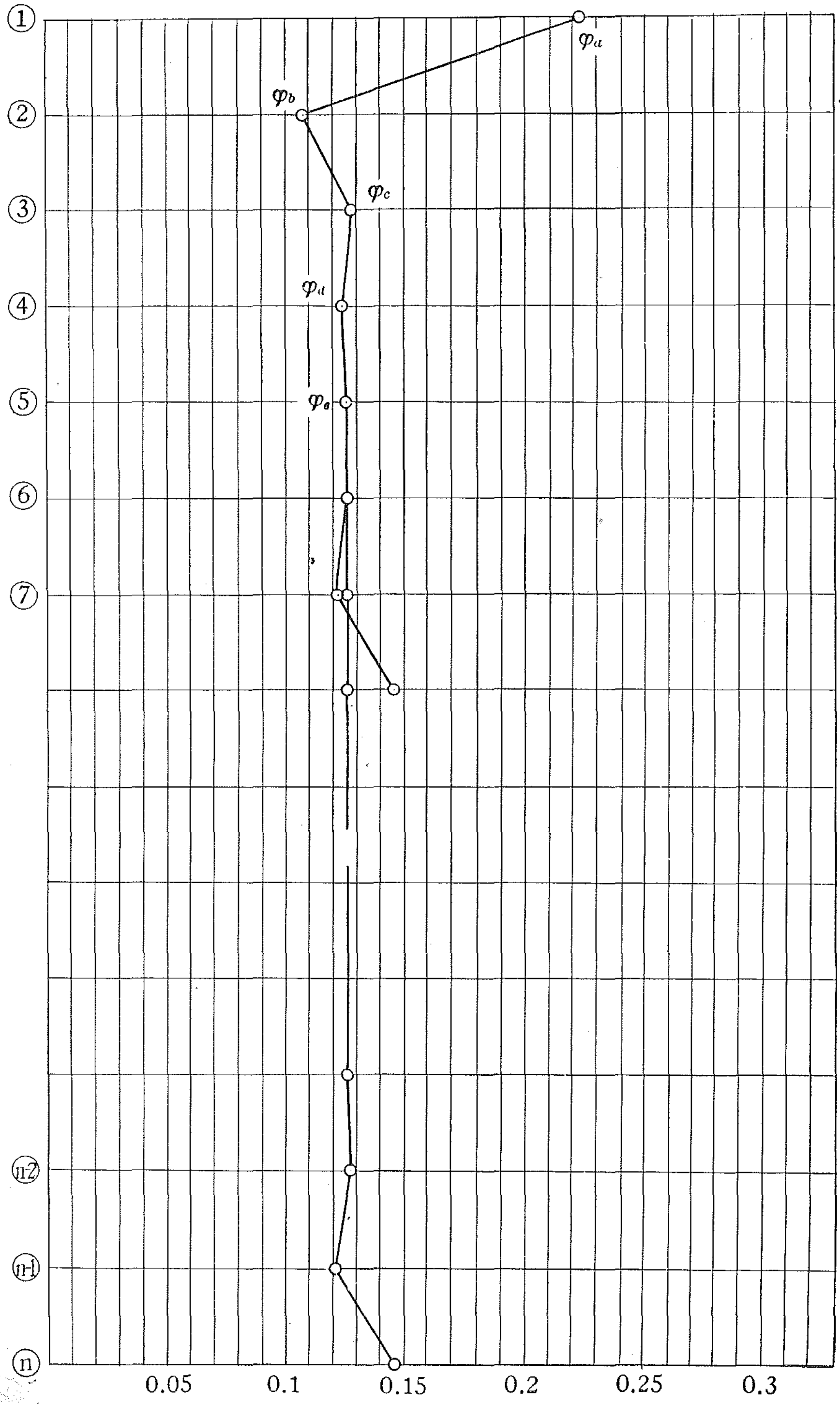
Table I.



Multiple Storied Bent with Two Bays. Legs Fixed at Bases.
Loads Vertical and Symmetrical.

Table II.

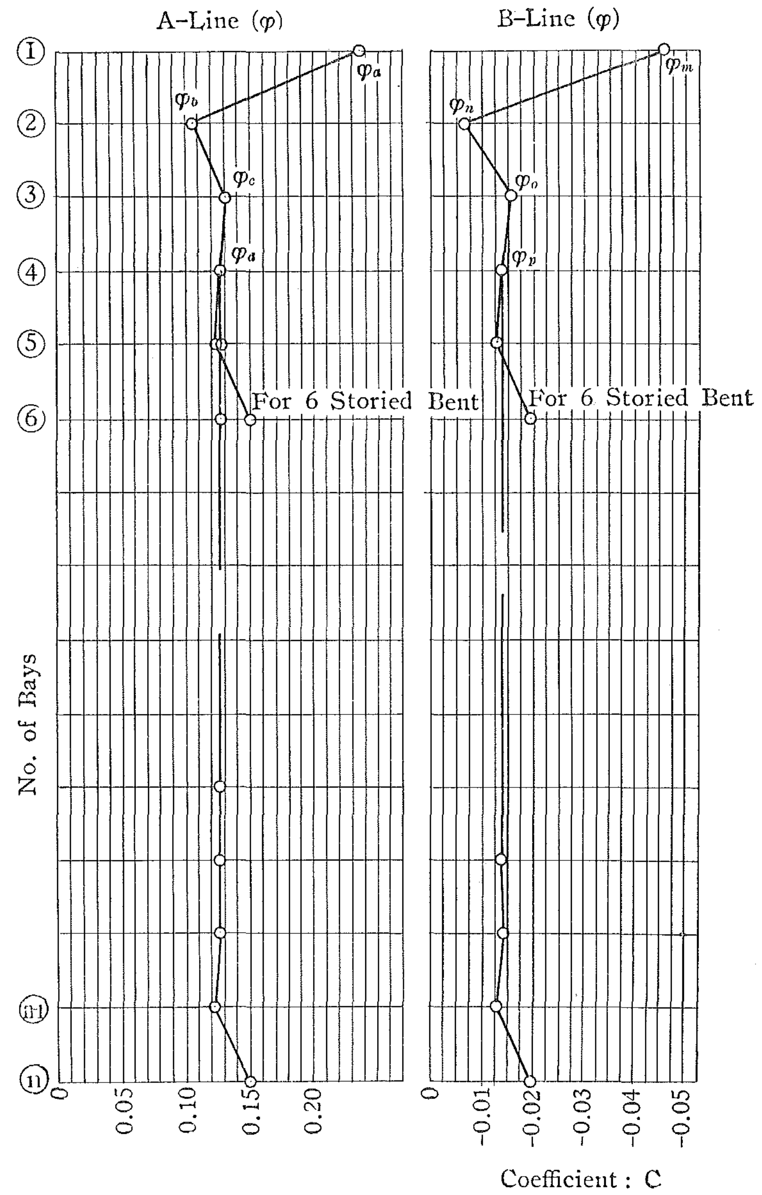
A-Line (φ)



Coefficient : C

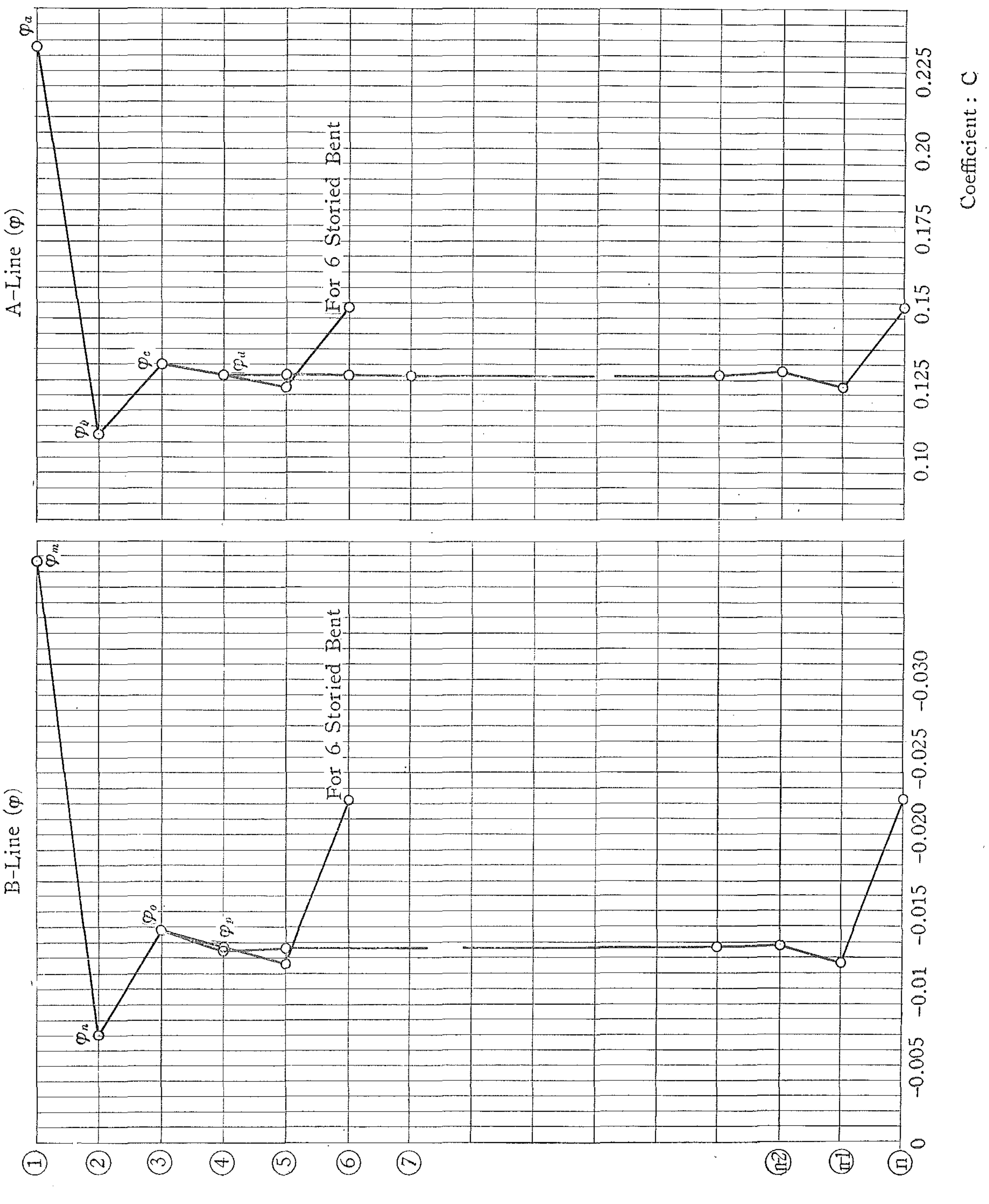
Multiple Storied Bent with Three Bays. Legs Fixed at Bases.
Loads Vertical and Symmetrical.

Table III.

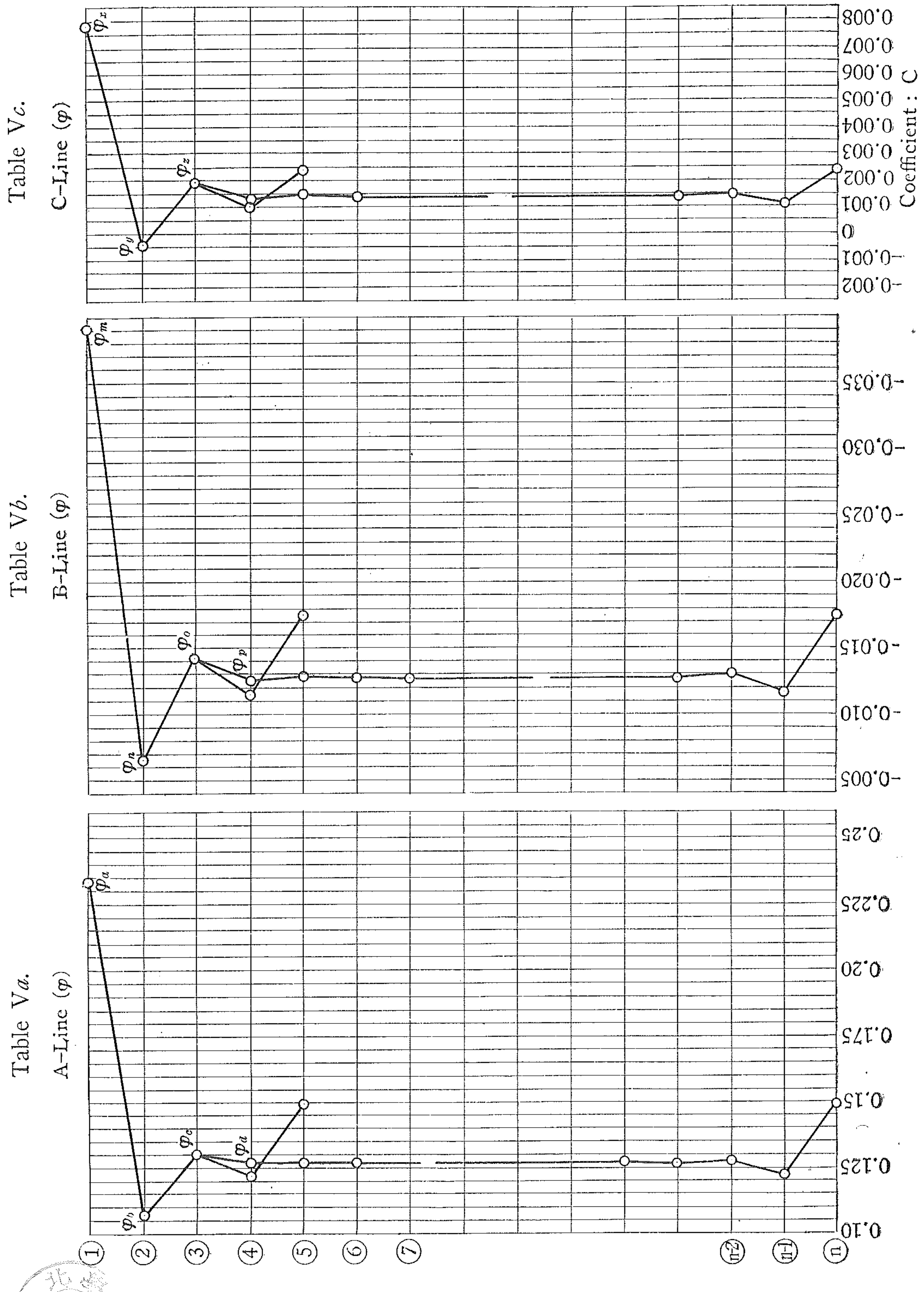


Multiple Storied Bent with Four Bays. Legs Fixed at Bases.
Loads Vertical and Symmetrical.

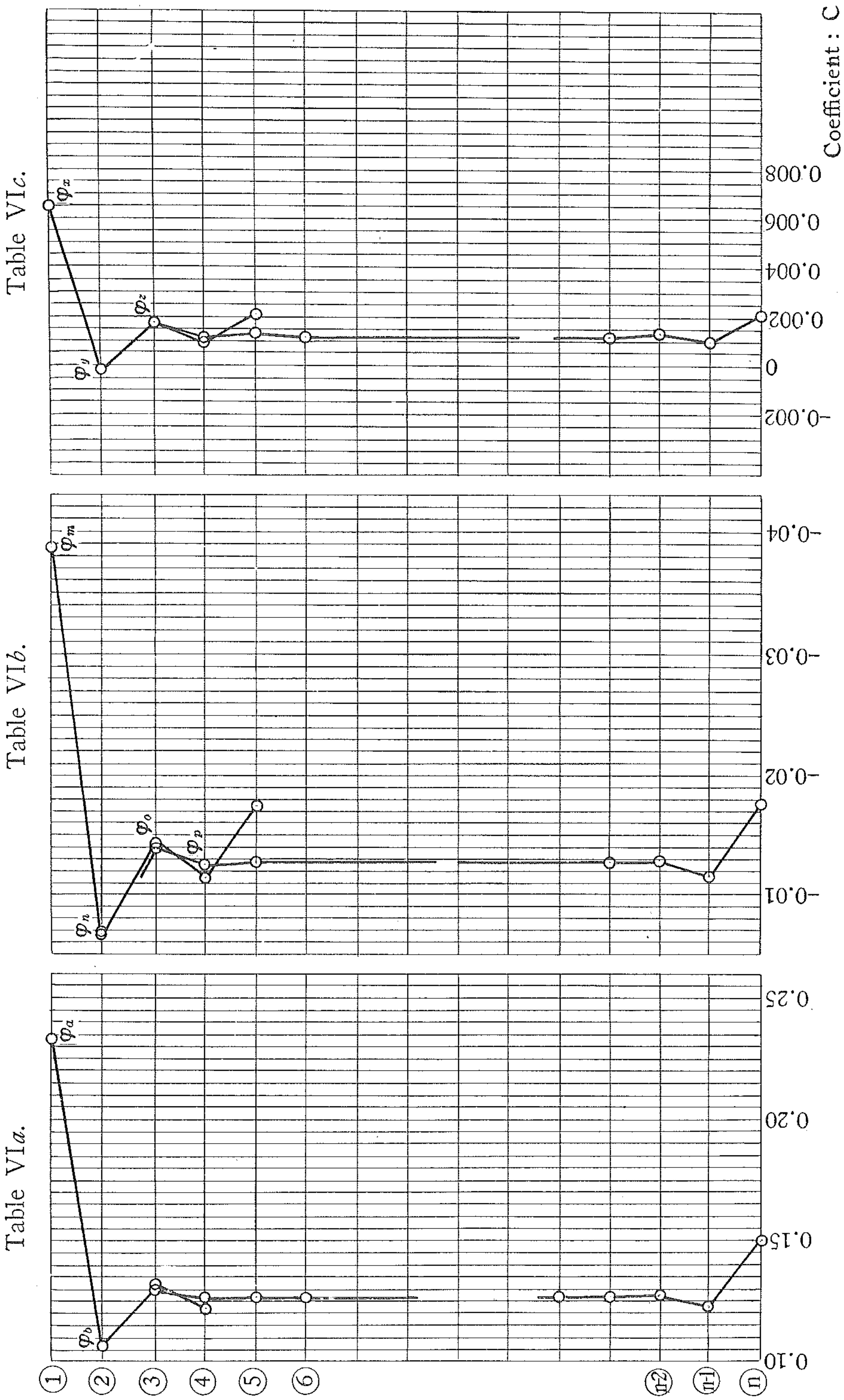
Table IV.



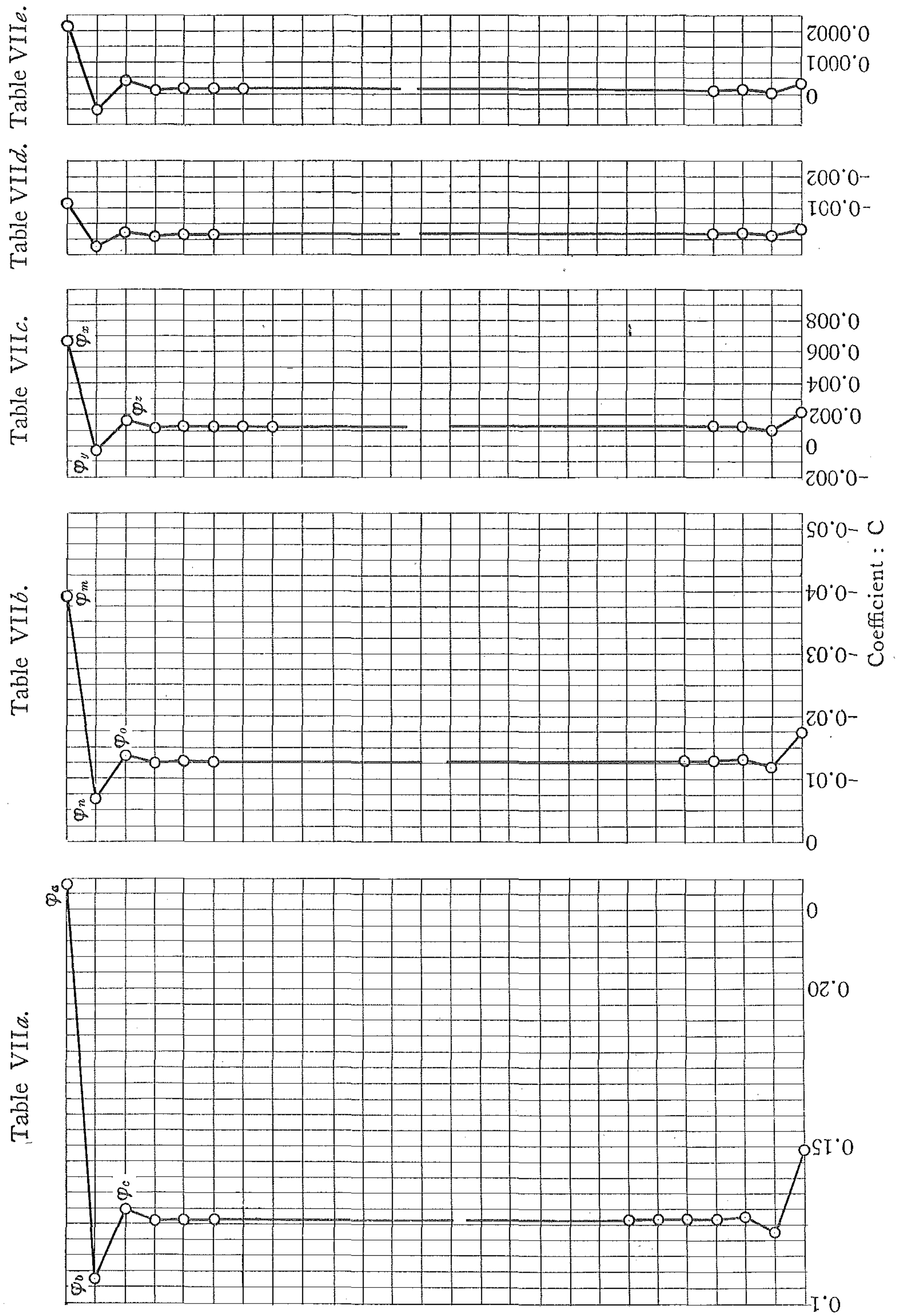
Multiple Storied Bent with Five Bays. Legs Fixed at Bases.
Loads Vertical and Symmetrical.



Multiple Storied Bent with Six Bays. Legs Fixed at Bases.
Loads Vertical and Symmetrical.



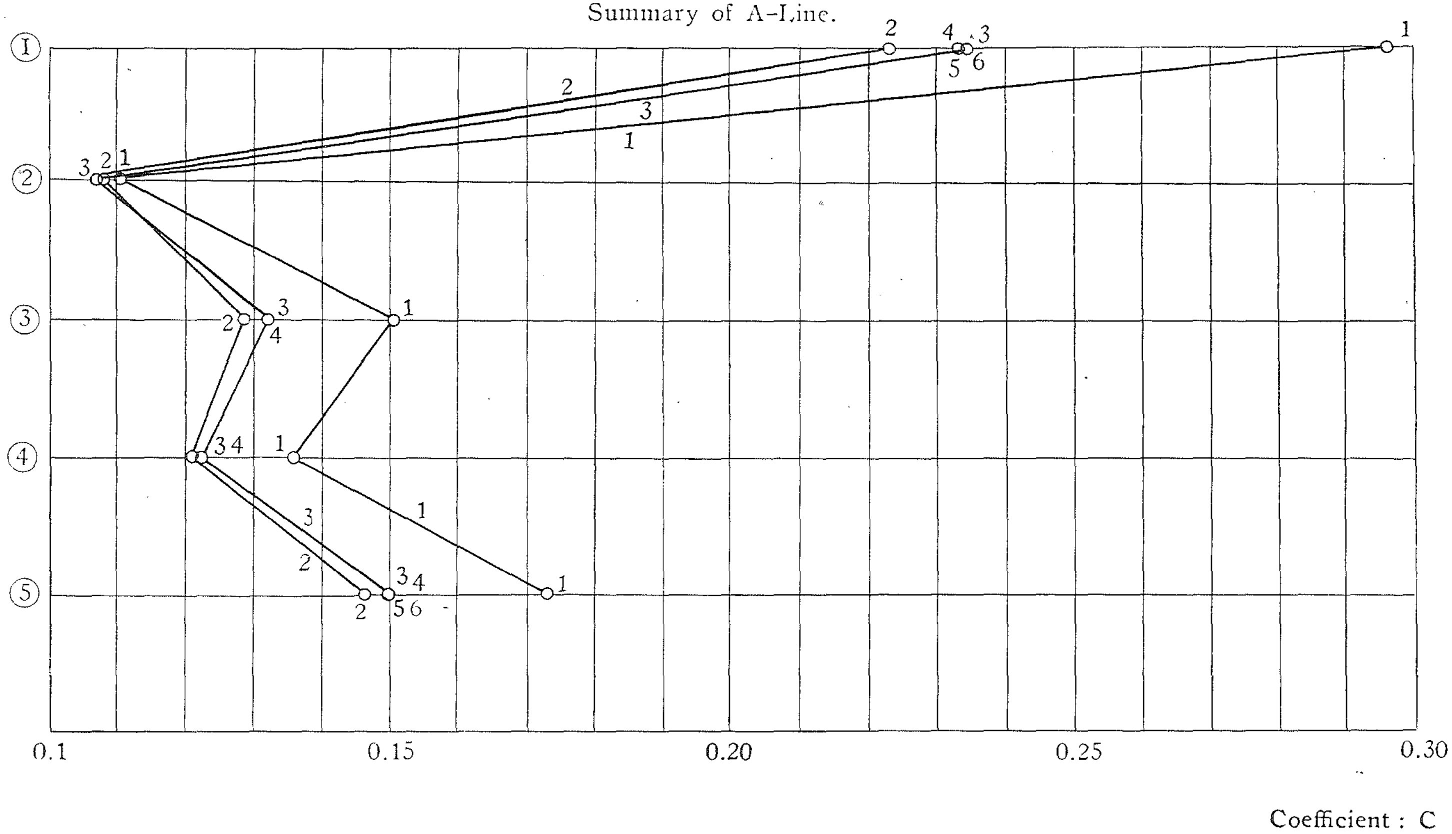
Multiple Storied Bent with Ten Bays. Legs Fixed at Bases.
Loads Vertical and Symmetrical.



Five Storied Bent with Multiple Bays, Carrying Loads Vertical and Symmetrical. Legs Fixed at Bases.

Table VIII.

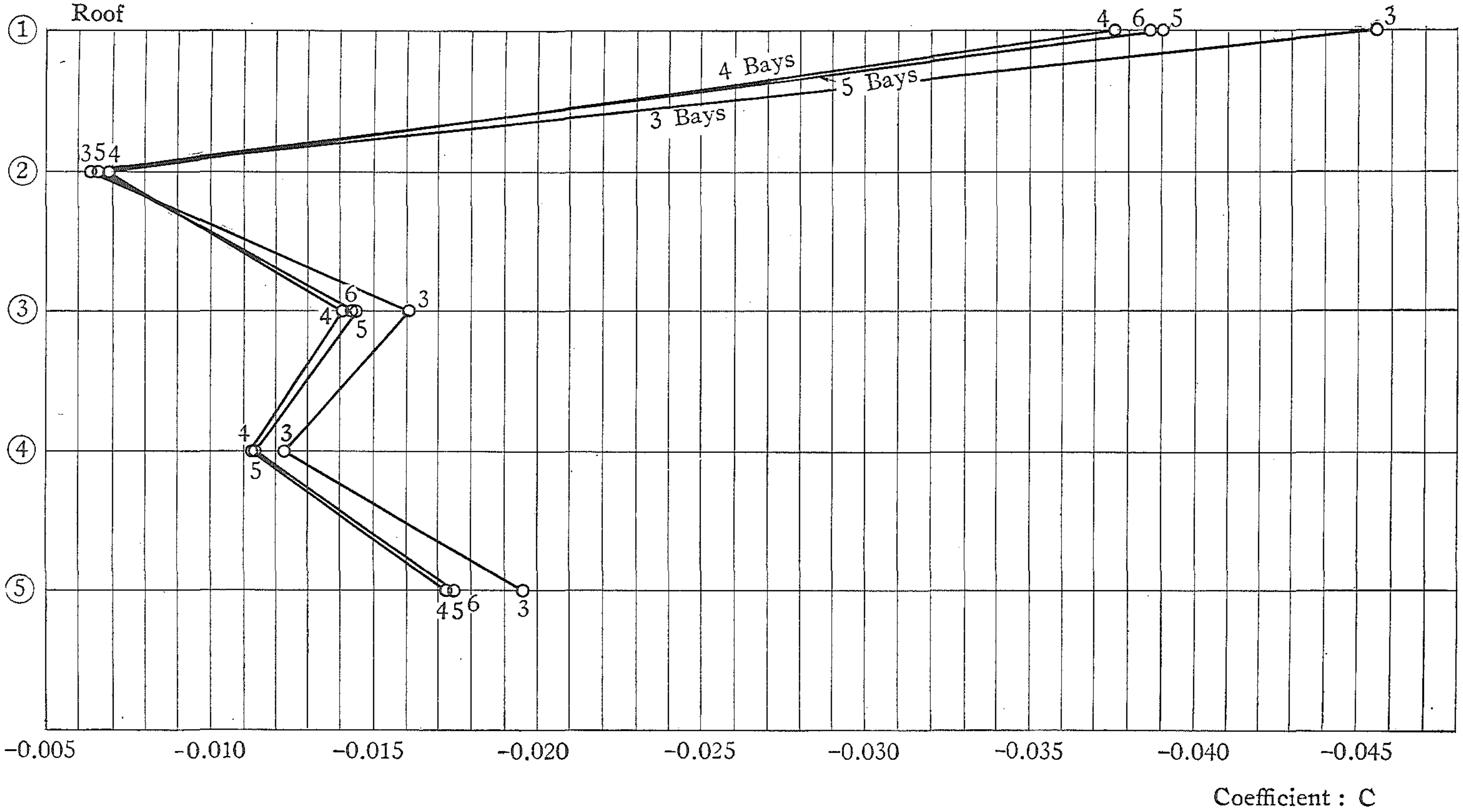
Summary of A-Line.



Five Storied Bent with Vertical and Symmetrical Loads.
Legs Fixed at Bases.

Table IX.

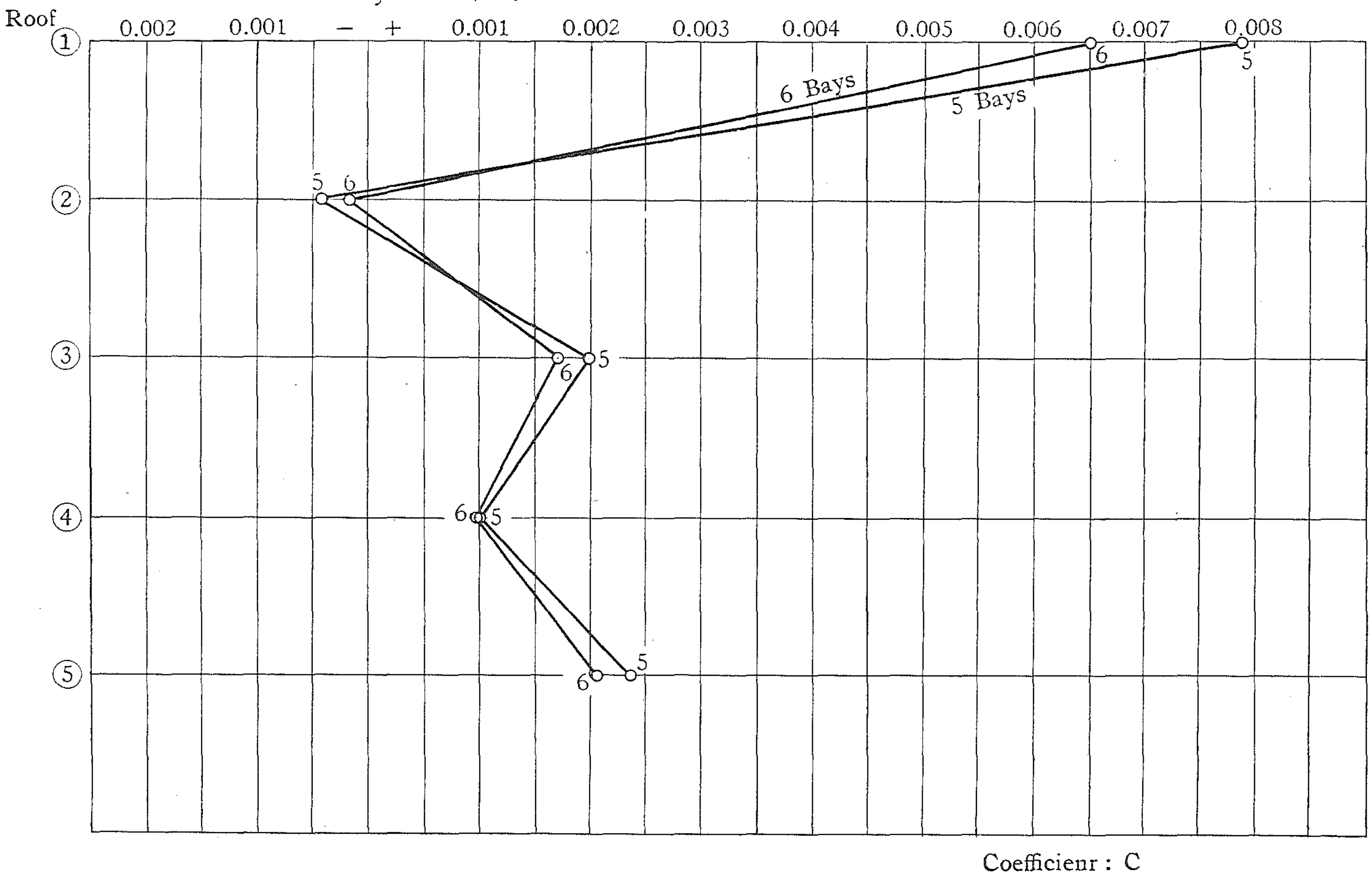
Summary of B-Line.



Five Storied Bent with Multiple Bays, Carrying Loads Vertical and Symmetrical. Legs Fixed at Bases.

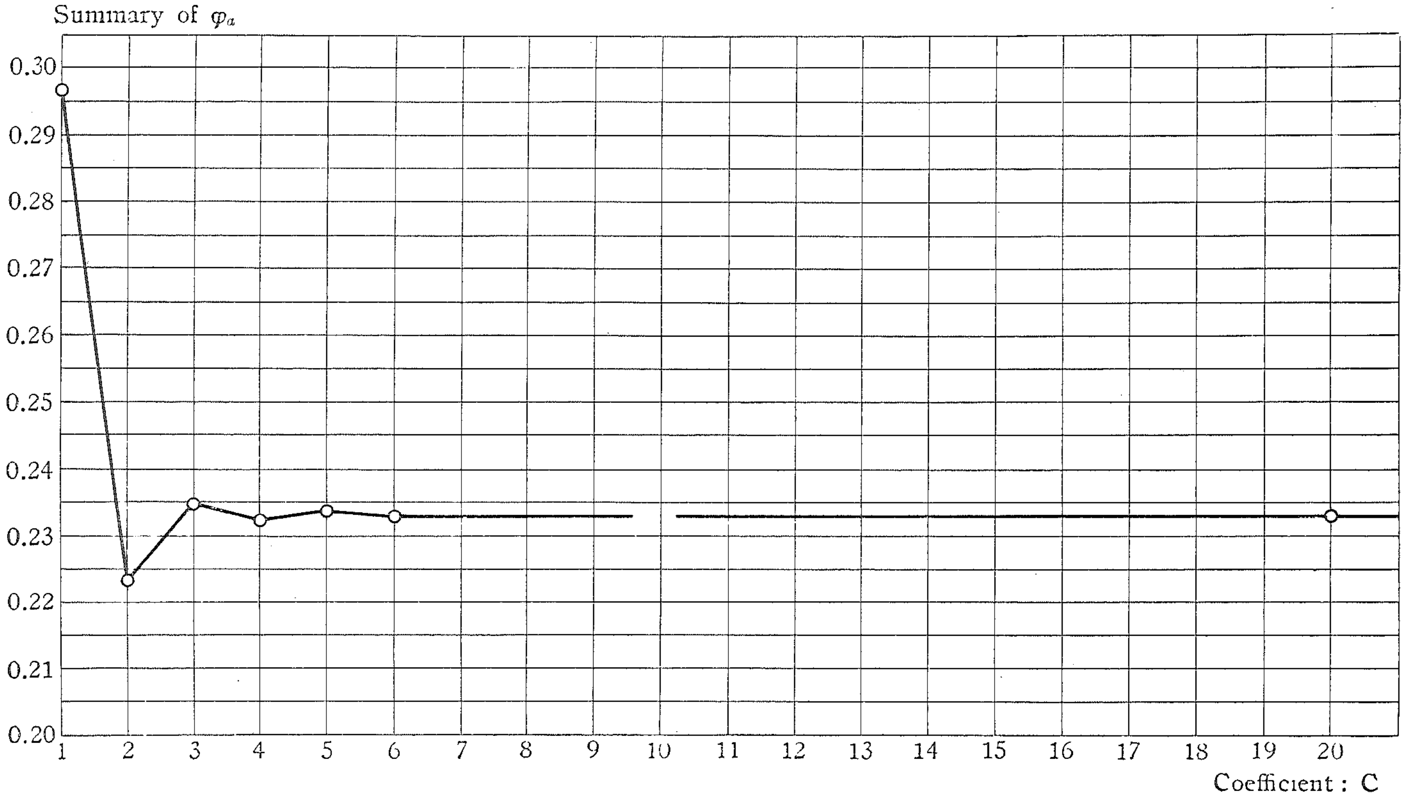
Table X.

Summary ot C-Line.



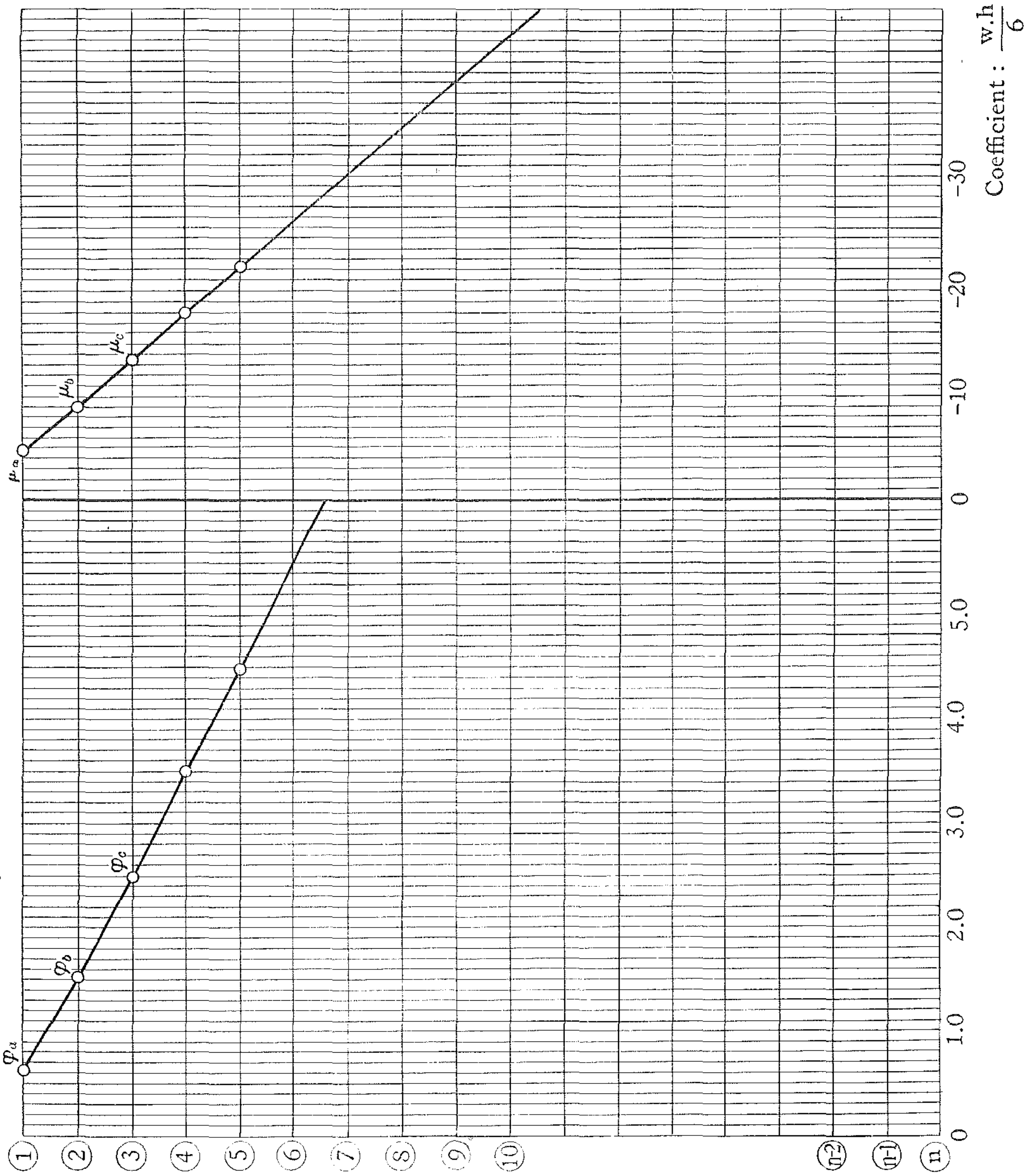
Multiple Storied Bent with Vertical and Symmetrical Loads.
Legs Fixed at Bases.

Table XI.



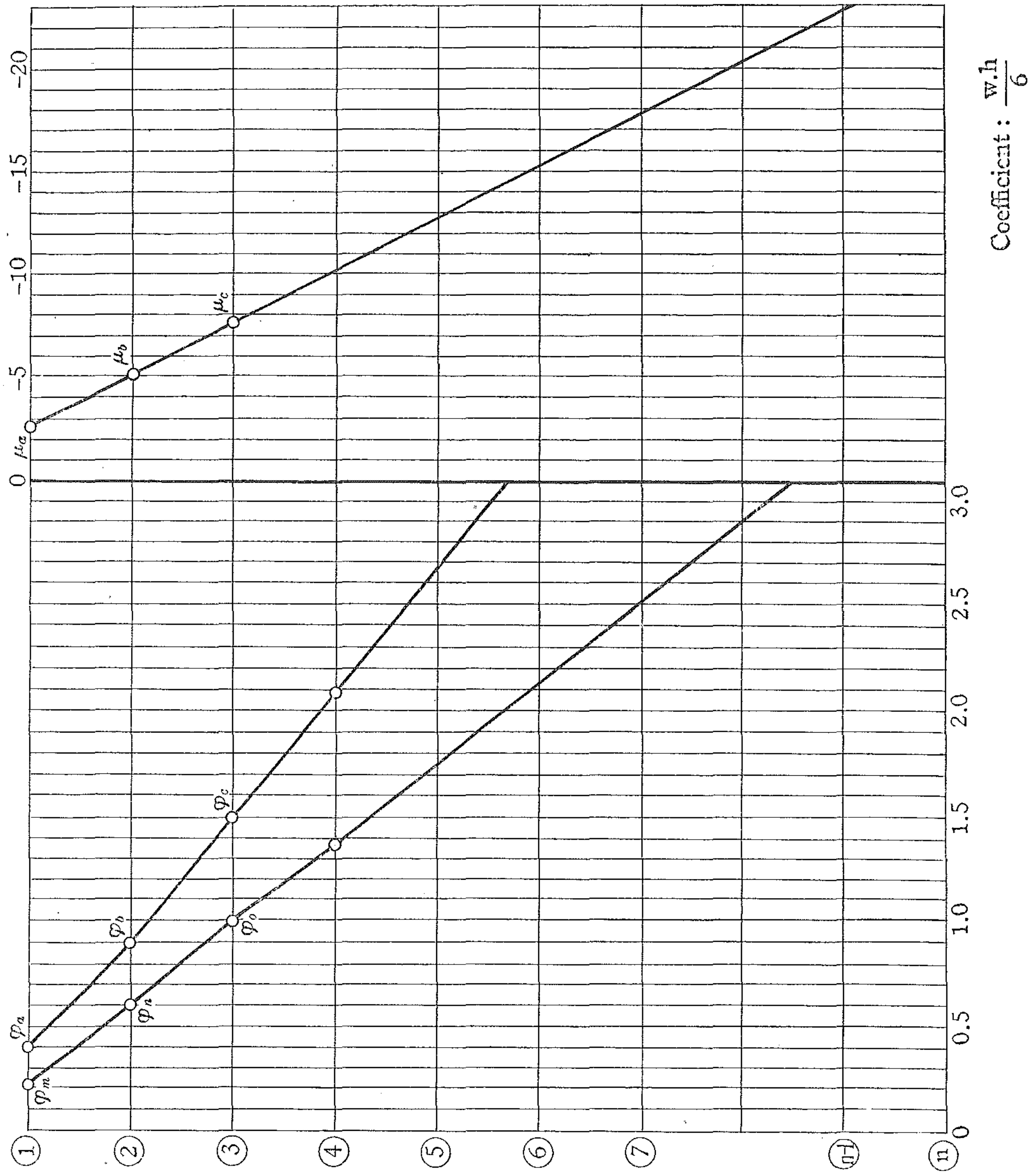
Multiple Storied Bent with Single Bay. Legs Fixed at Bases.
Loads Horizontal and Concentrated at Each Joint
on Left Side of Structure.

Table XII.



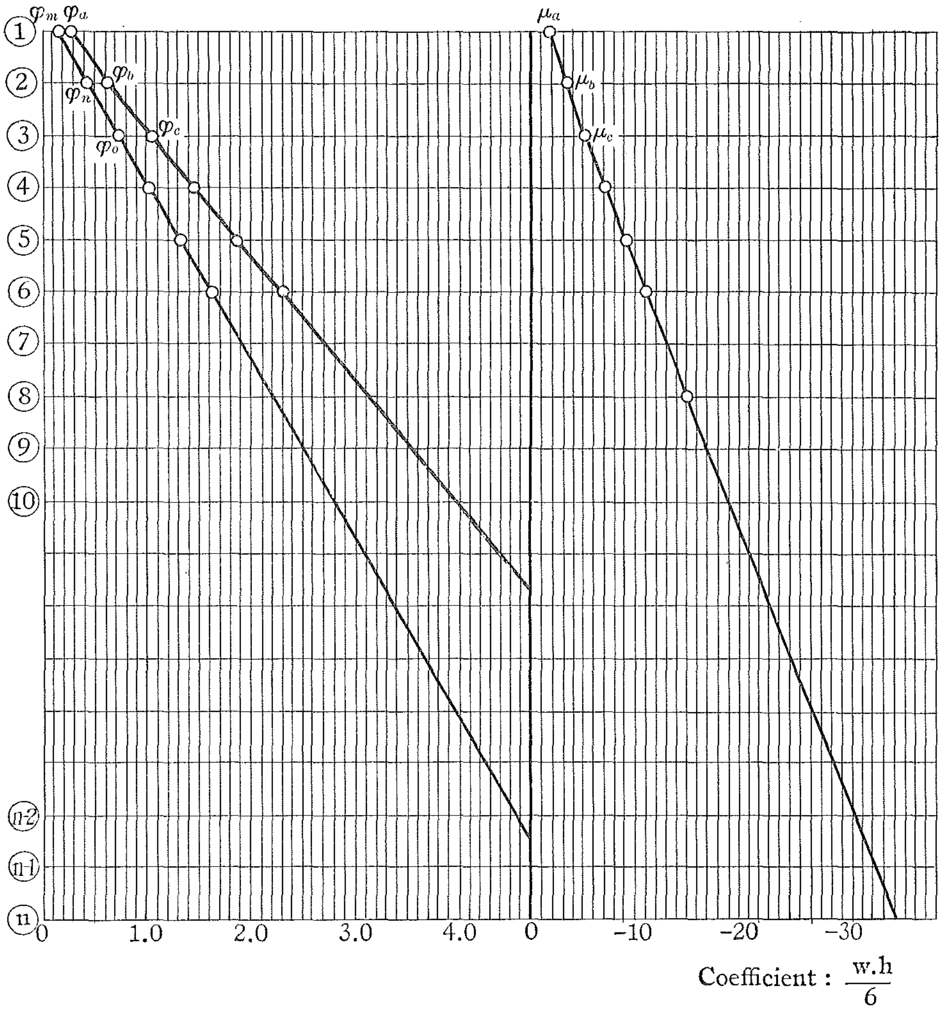
Multiple Storied Bent with Two Bays. Legs Fixed at Bases.
Loads Horizontal and Concentrated at Each Joint
on Left Side of Structure.

Table XIII.



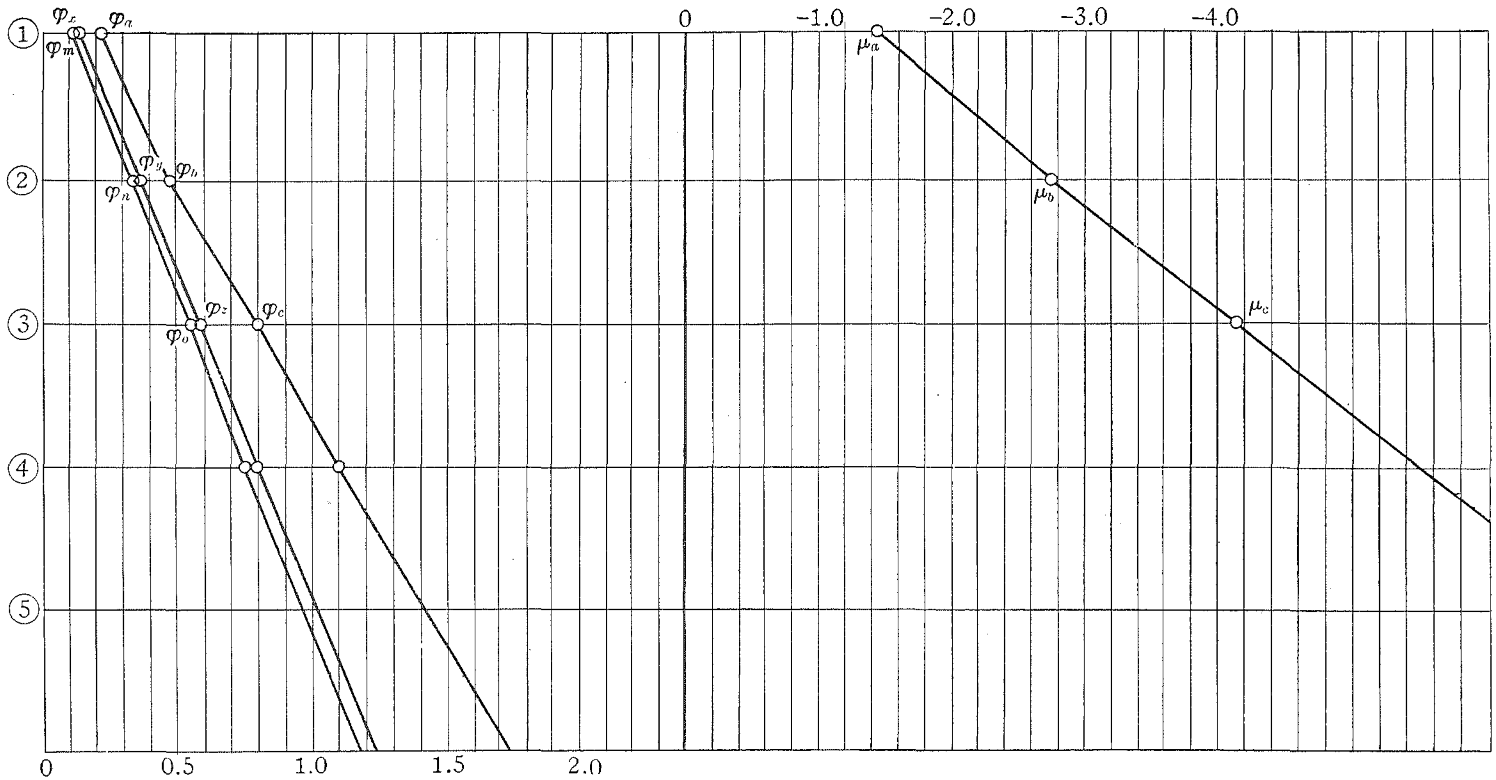
Multiple Storied Bent with Three Bays. Legs Fixed at Bases.
Loads Horizontal and Concentrated at Each Joint
on Left Side of Structure.

Table XIV.



Multiple Storied Bent with Four Bays. Legs Fixed at Bases.
Loads Horizontal and Concentrated at Each Joint
on Left Side of Structure.

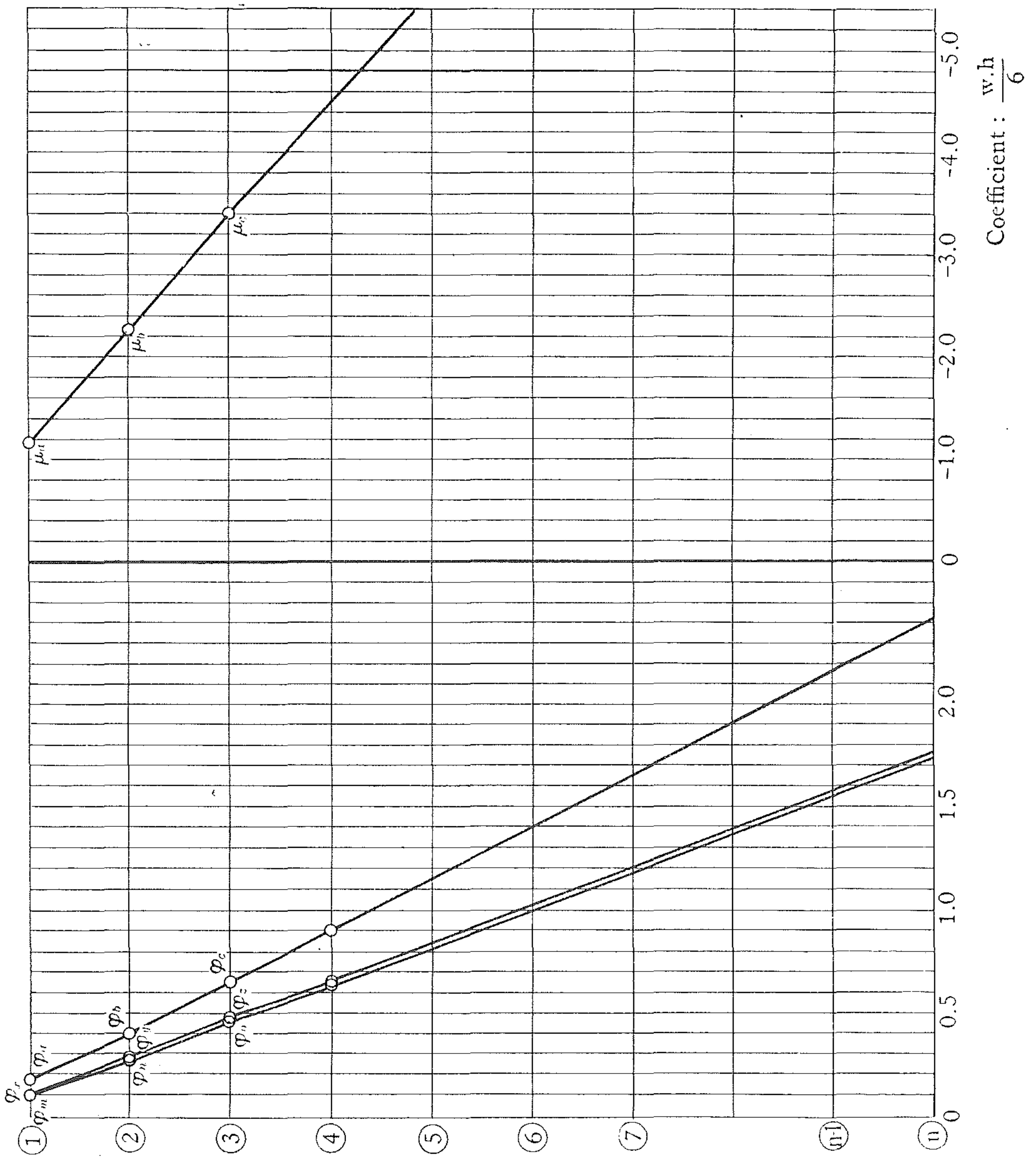
Table XV.



Coefficient : $\frac{w.h}{6}$

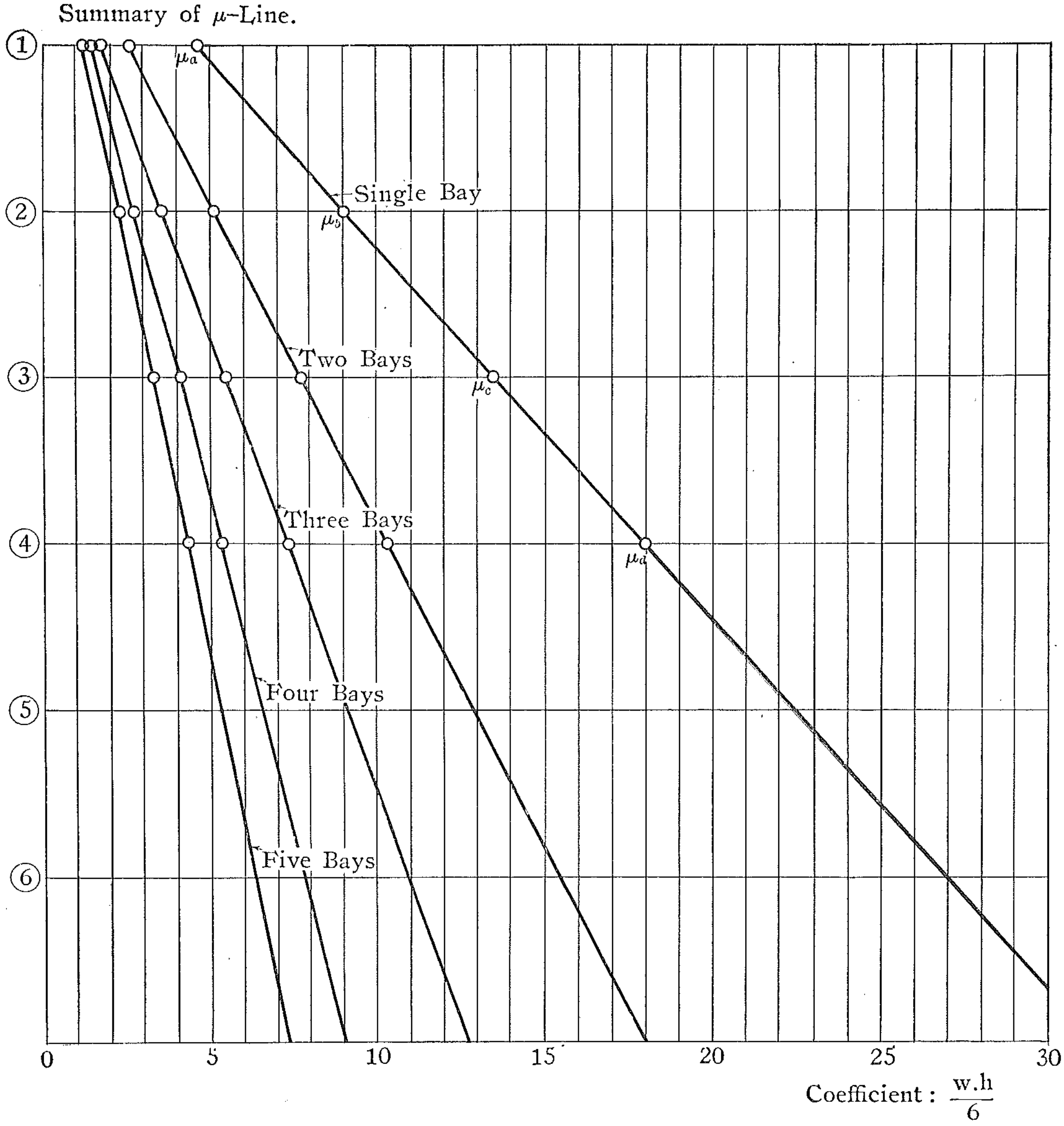
Multiple Storied Bent with Five Bays. Legs Fixed at Bases.
Loads Horizontal and Concentrated at Each Joint
on Left Side of Structure.

Table XVI.



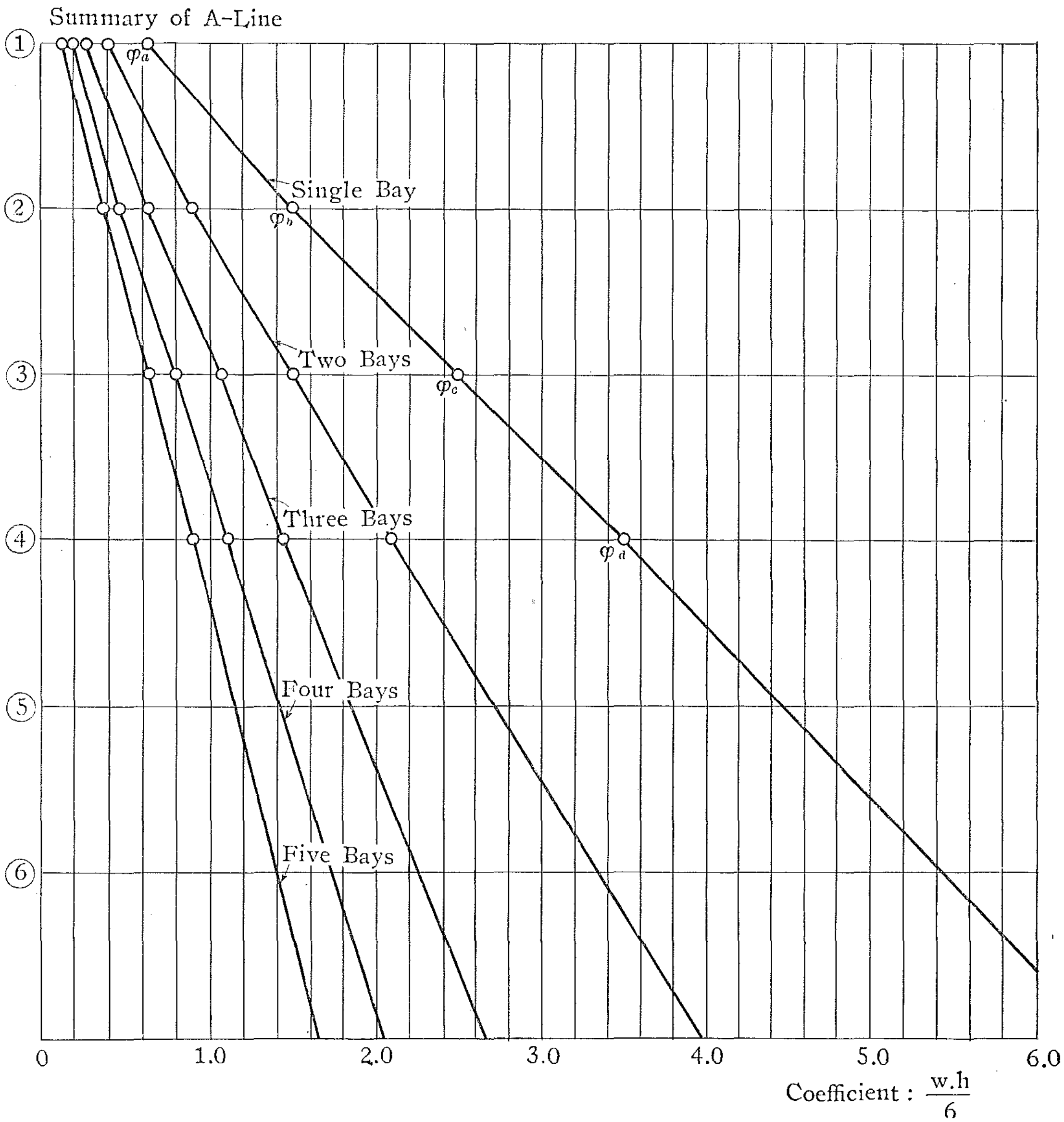
Multiple Storied Bent with Multiple Bays. Legs Fixed at Bases.
Loads Horizontal and Concentrated at Each Joint
on Left Side of Structure.

Table XVII.



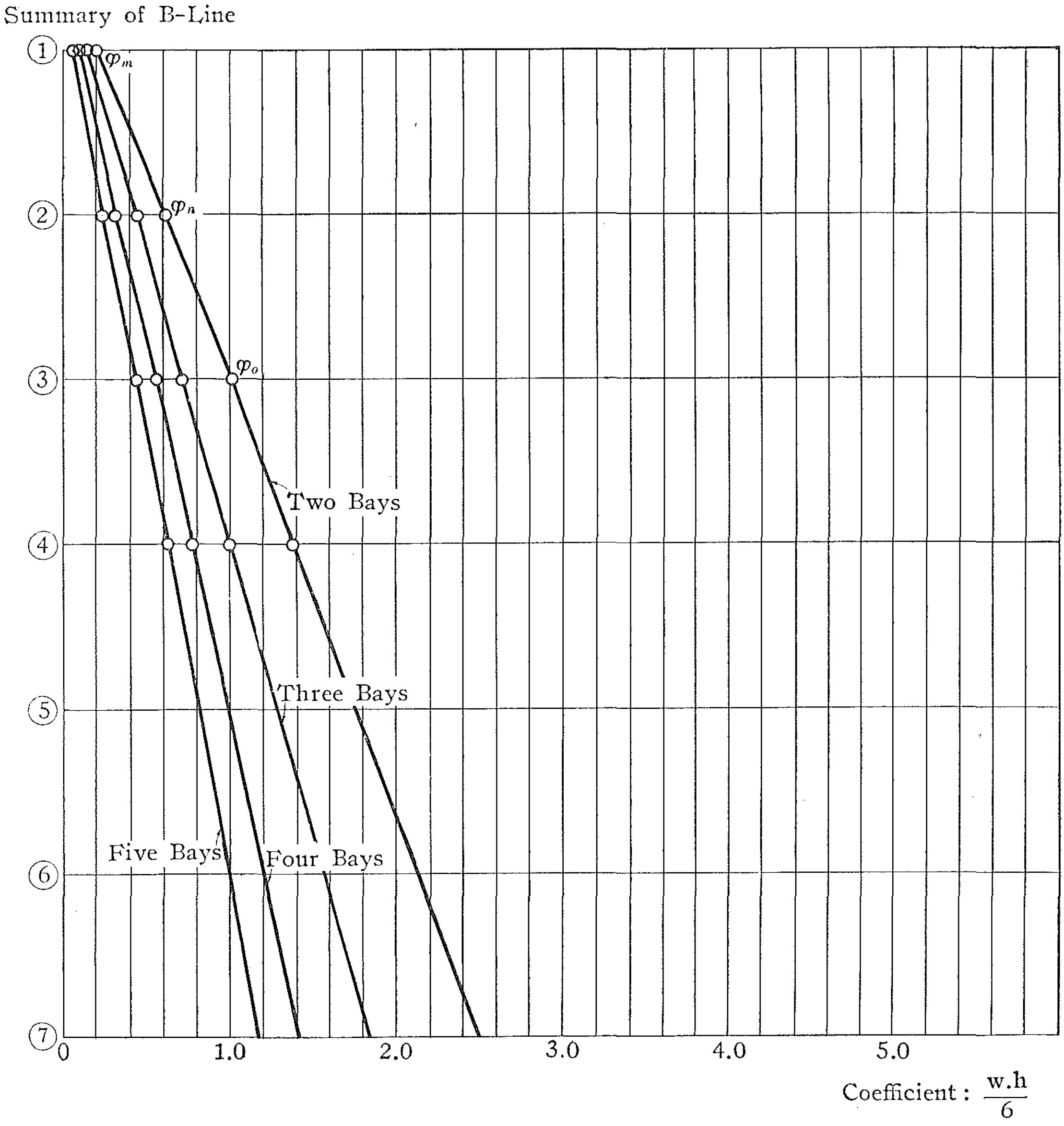
Multiple Storied Bent with Multiple Bays. Legs Fixed at Bases.
Loads Horizontal and Concentrated at Each Joint
on Left Side of Structure.

Table XVIII.



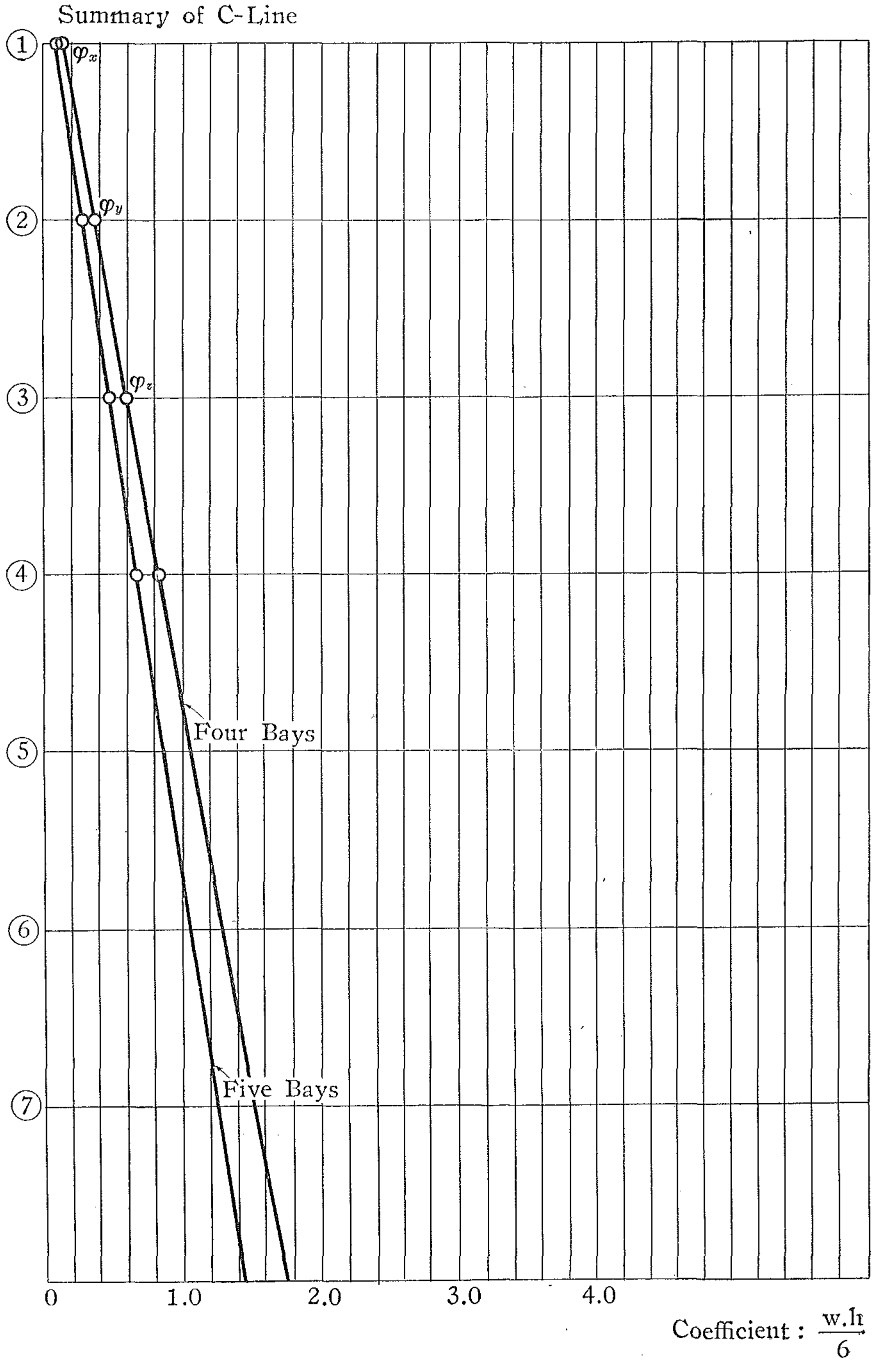
Multiple Storied Bent with Multiple Bays. Legs Fixed at Bases.
Loads Horizontal and Concentrated at Each Joint
on Left Side of Structure.

Table XIX.



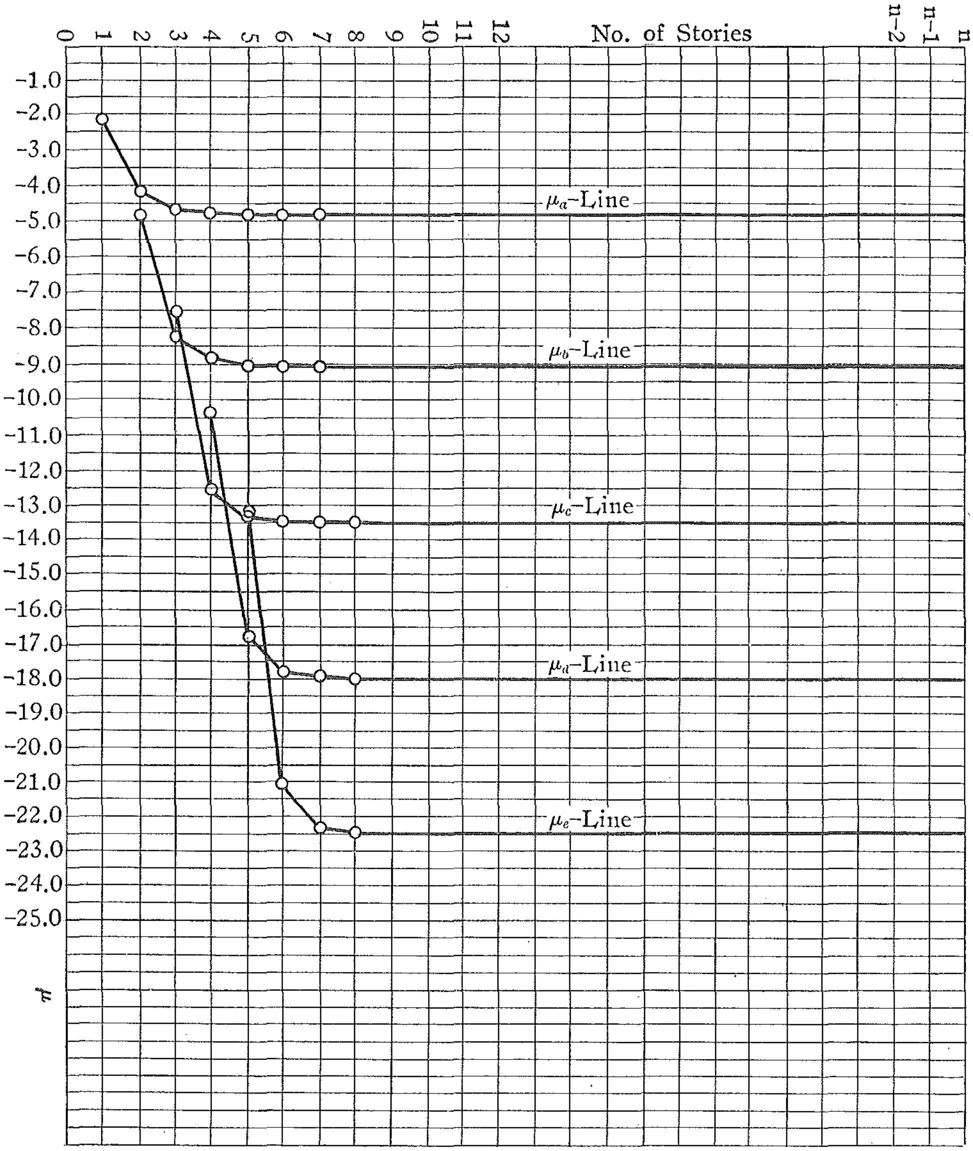
Multiple Storied Bent with Multiple Bays. Legs Fixed at Bases.
Loads Horizontal and Concentrated at Each Joint
on Left Side of Structure.

Table XX.



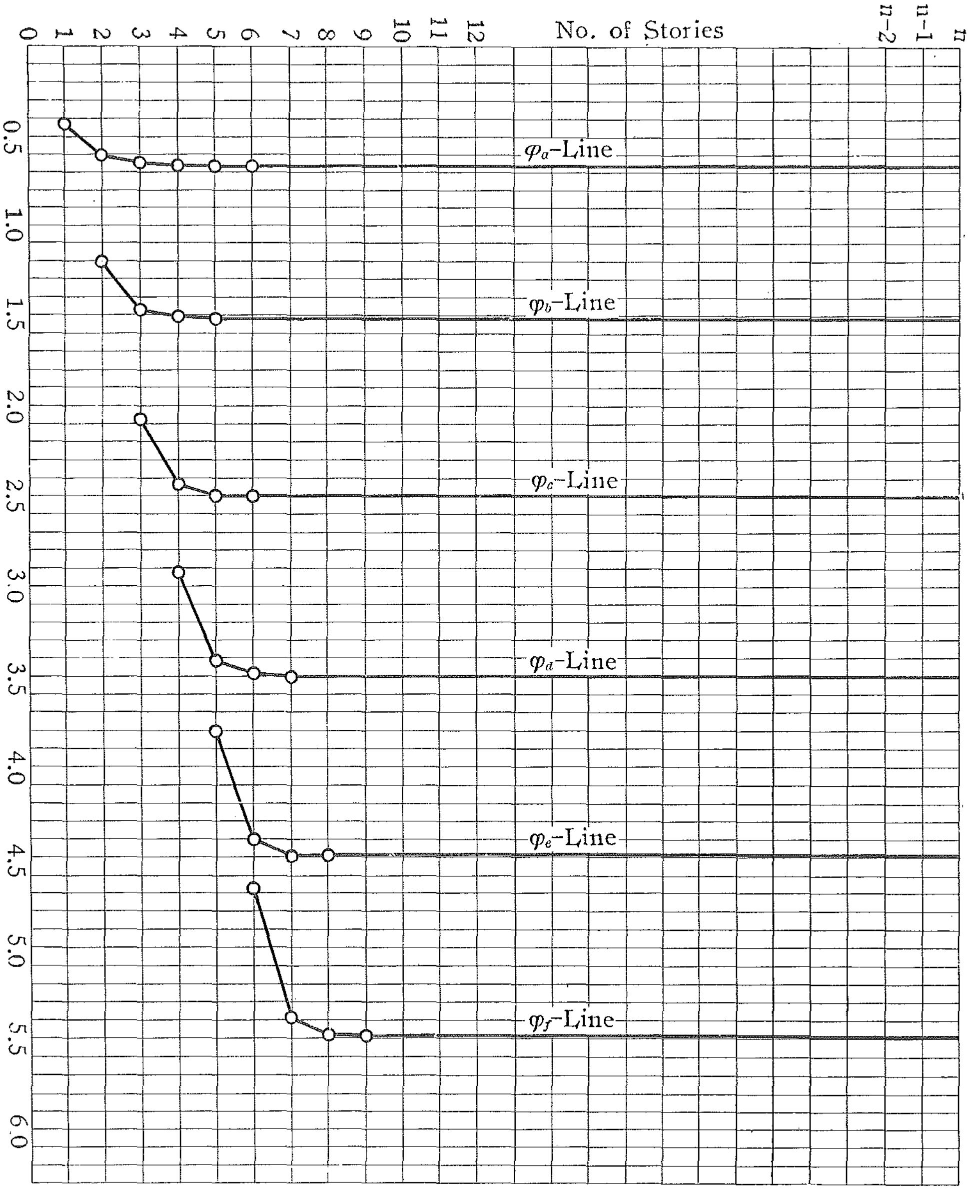
Multiple Storied Bent with Single Bay. Legs Fixed at Bases.
Loads Horizontal and Concentrated at Each Joint
on Left Side of Structure.

Table XXI.



Multiple Storied Bent with Single Bays. Legs Fixed at Bases
Loads Horizontal and Concentrated at Each Joint
on Left Side of Structure.

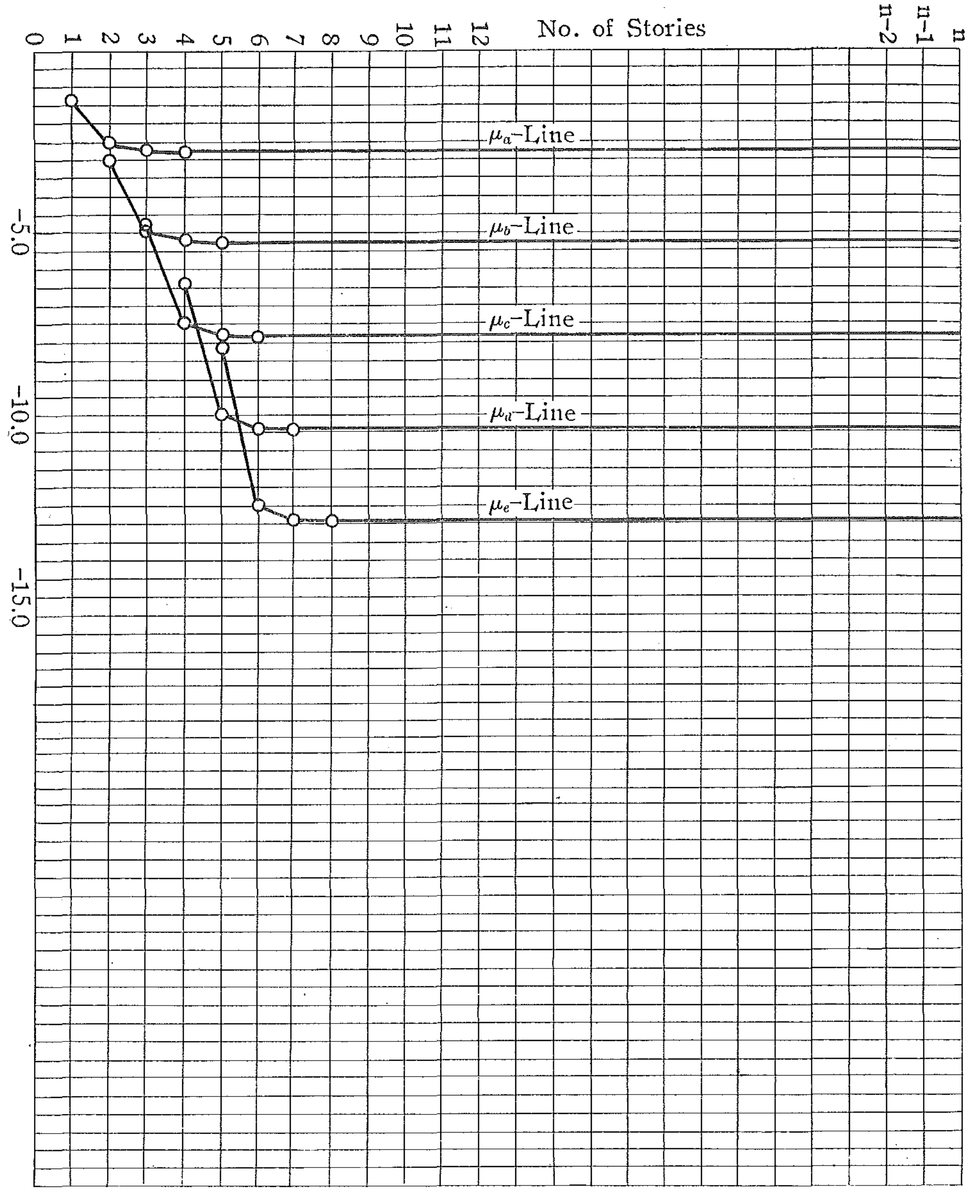
Table XXII.



Coefficient: $\frac{w.h}{6}$

Multiple Storied Bent with Two Bays. Legs Fixed at Bases.
Loads Horizontal and Concentrated at Each Joint
on Left Side of Structure.

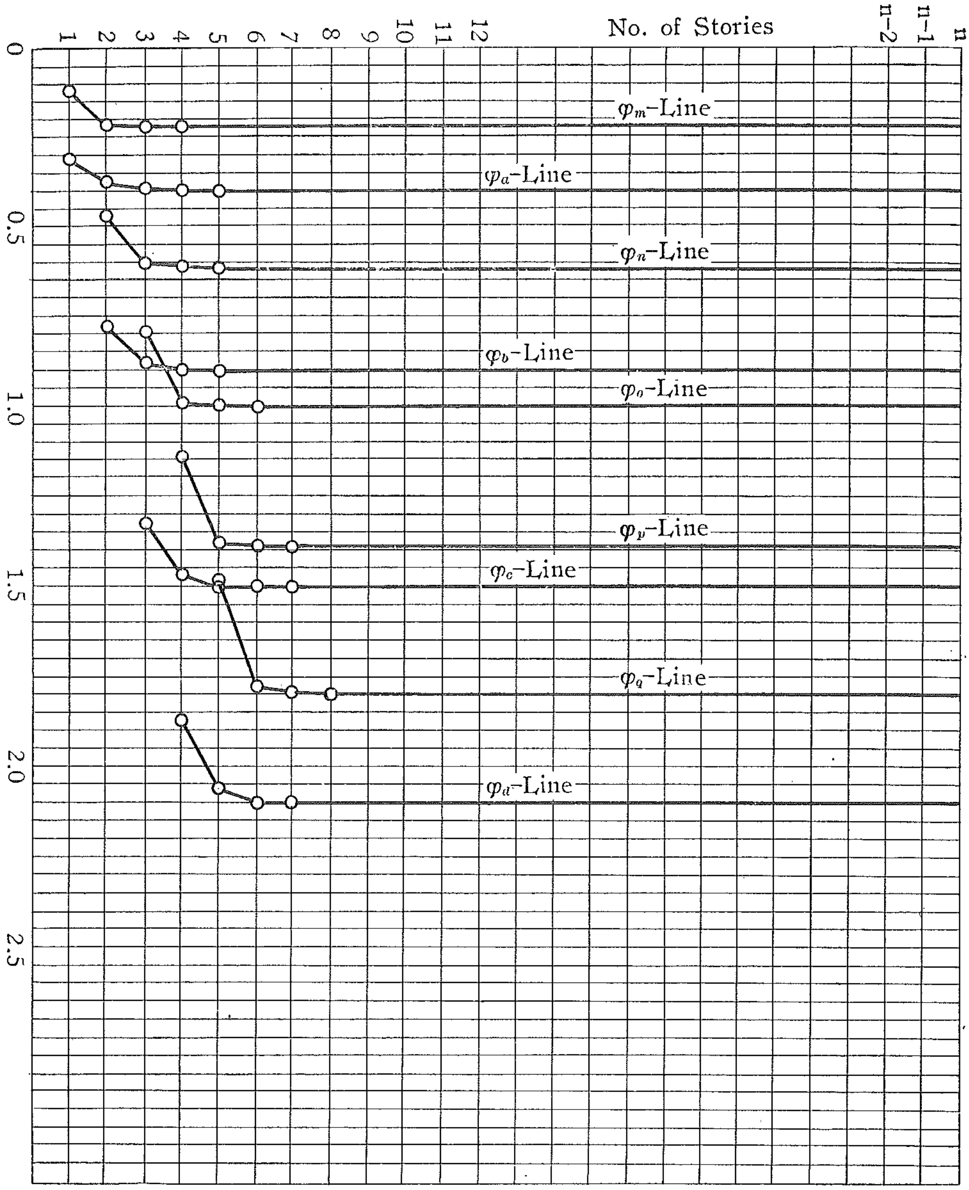
Table XXIII.



Coefficient: $\frac{w.h}{6}$

Multiple Storied Bent with Two Bays. Legs Fixed at Bases.
Loads Horizontal and Concentrated at Each Joint
on Left Side of Structure.

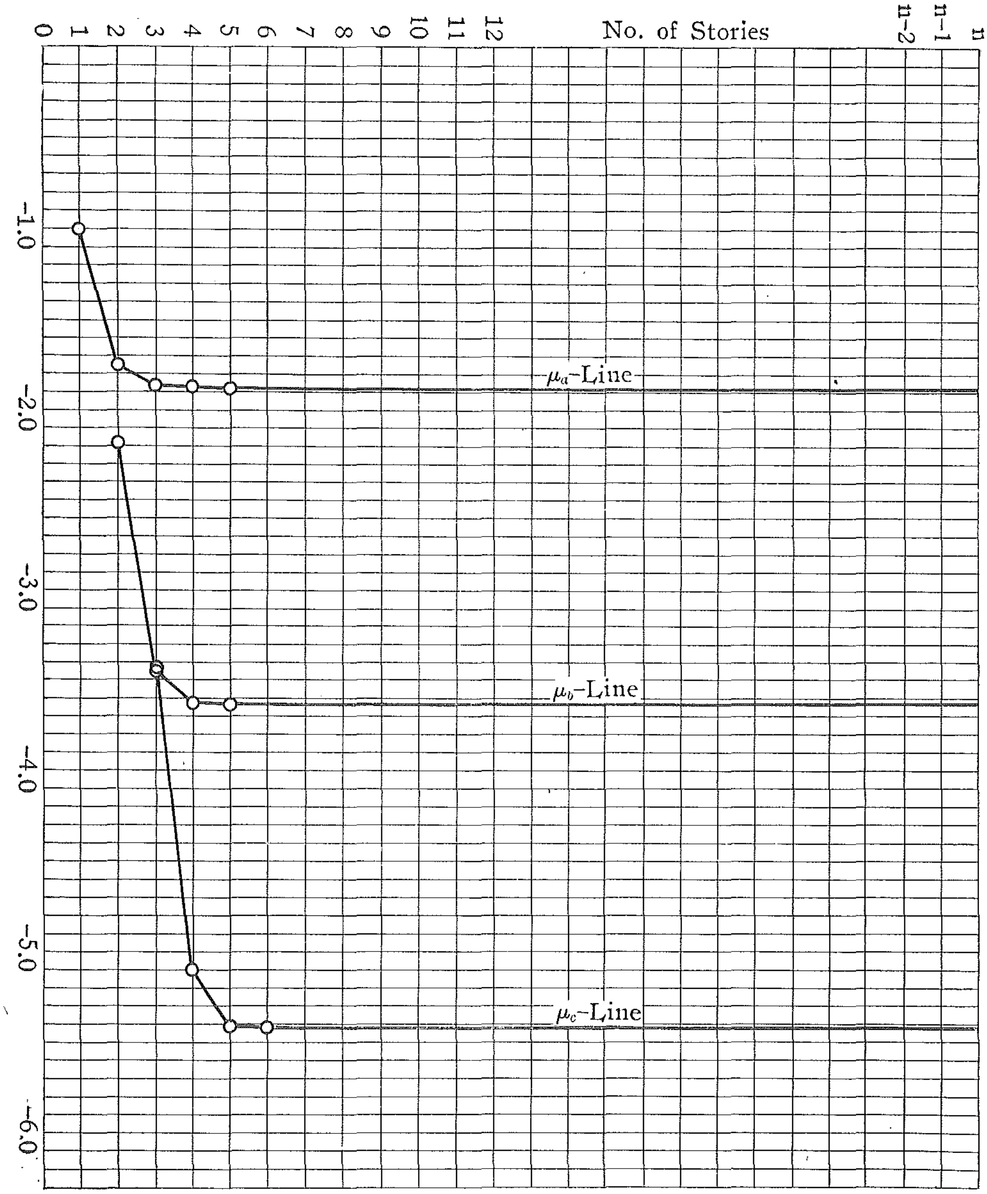
Table XXIV.



Coefficient: $\frac{wh}{6}$

Multiple Storied Bent with Three Bays. Legs Fixed at Bases.
Loads Horizontal and Concentrated at Each Joint
on Left Side of Structure.

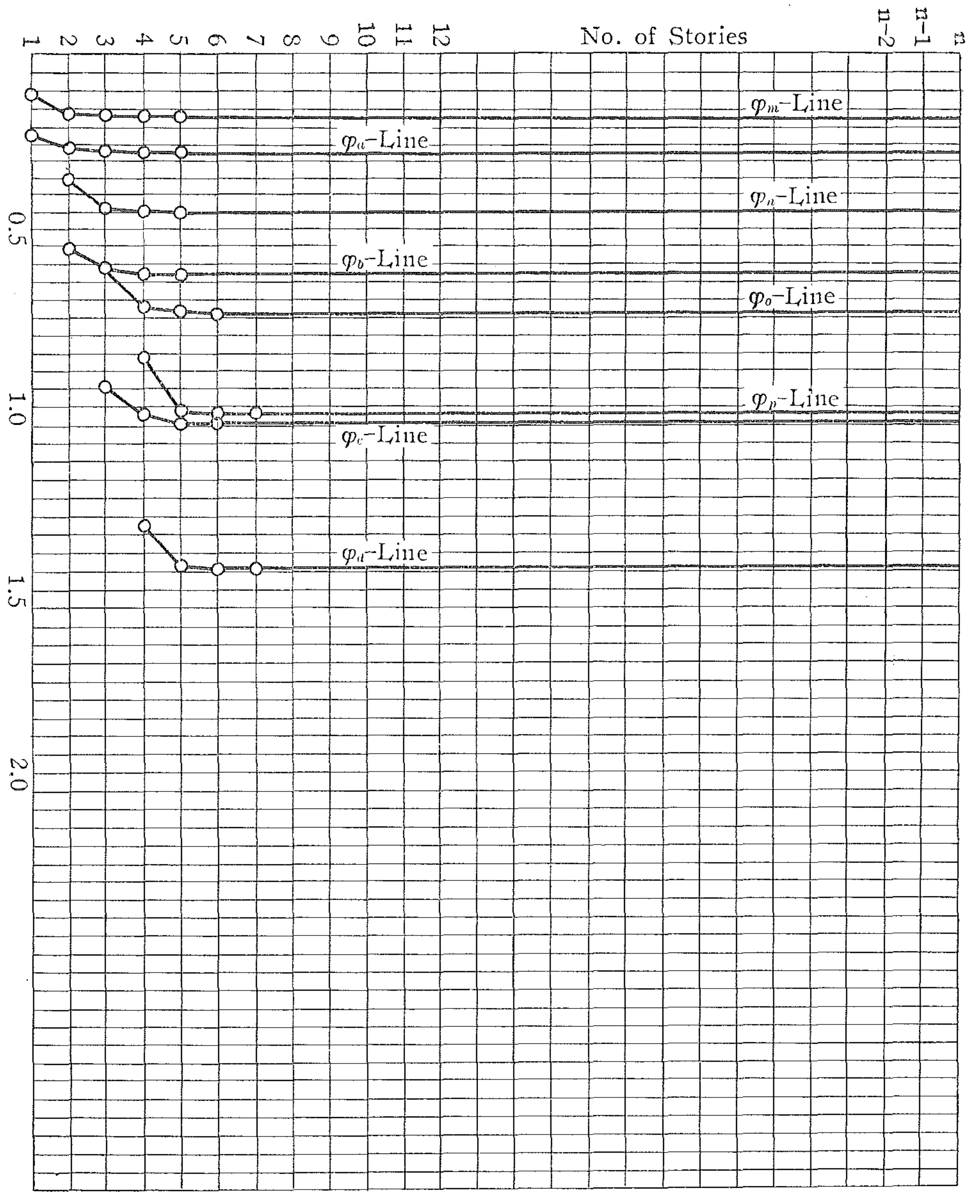
Table XXV.



Coefficient: $\frac{w.h}{6}$

Multiple Storied Bent with Three Bays. Legs Fixed at Bases.
Loads Horizontal and Concentrated at Each Joint
on Left Side of Structure.

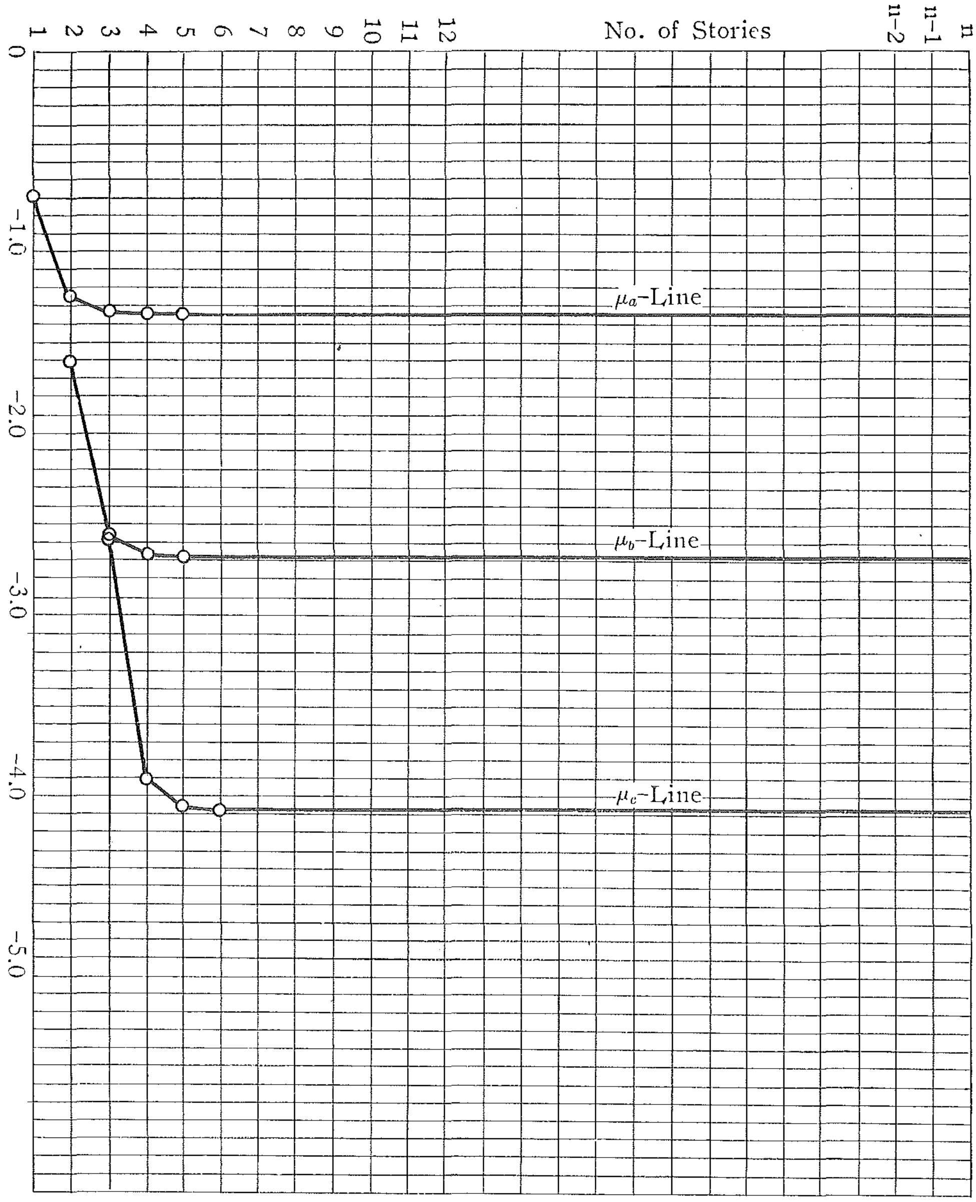
Table XXVI.



Coefficient: $\frac{w.h}{6}$

Multiple Storied Bent with Four Bays. Legs Fixed at Bases.
Loads Horizontal and Concentrated at Each Joint
on Left Side of Structure.

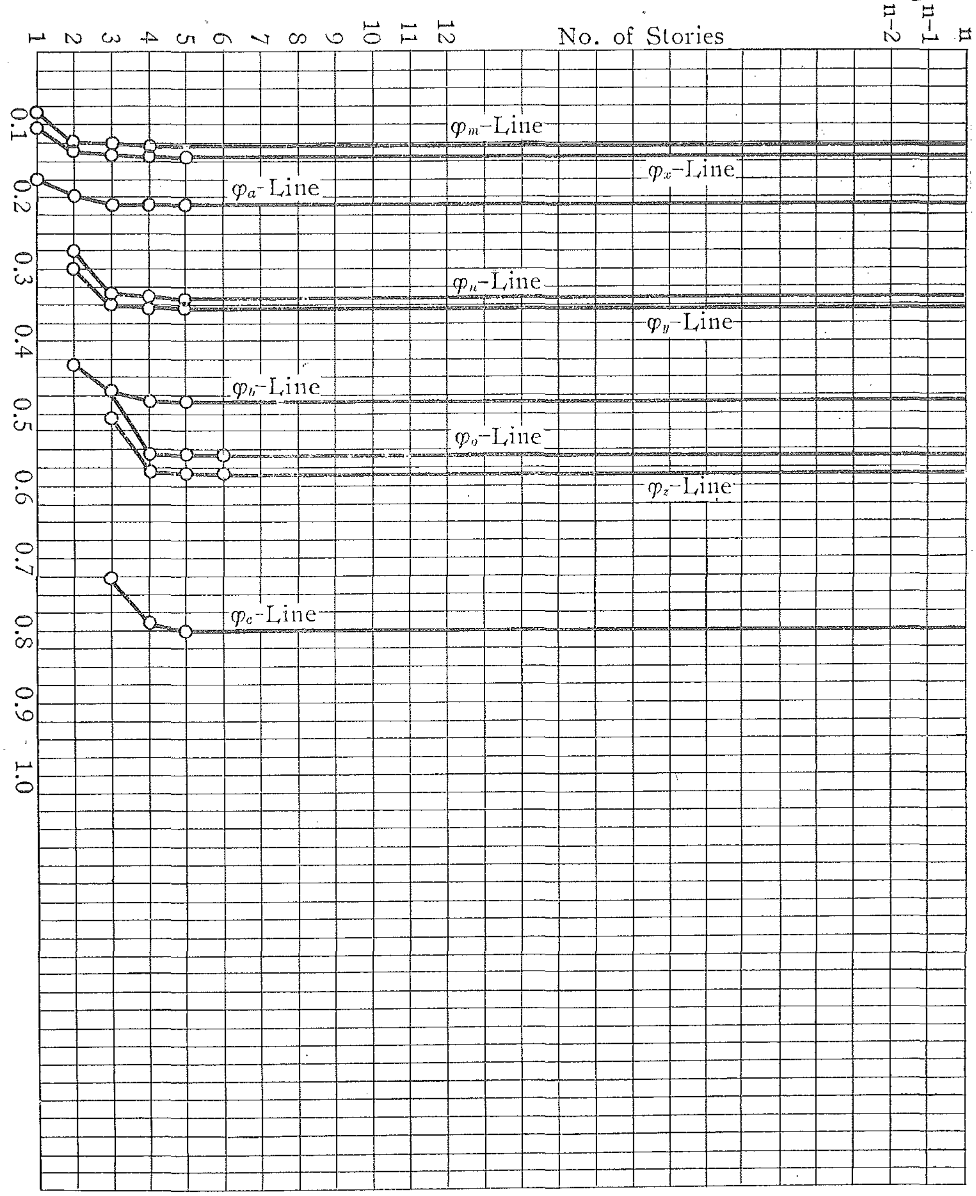
Table XXVII.



Coefficient: $\frac{w.h}{6}$

Multiple Storied Bent with Four Bays. Legs Fixed at Bases.
Loads Horizontal and Concentrated at Each Joint
on Left Side of Structure.

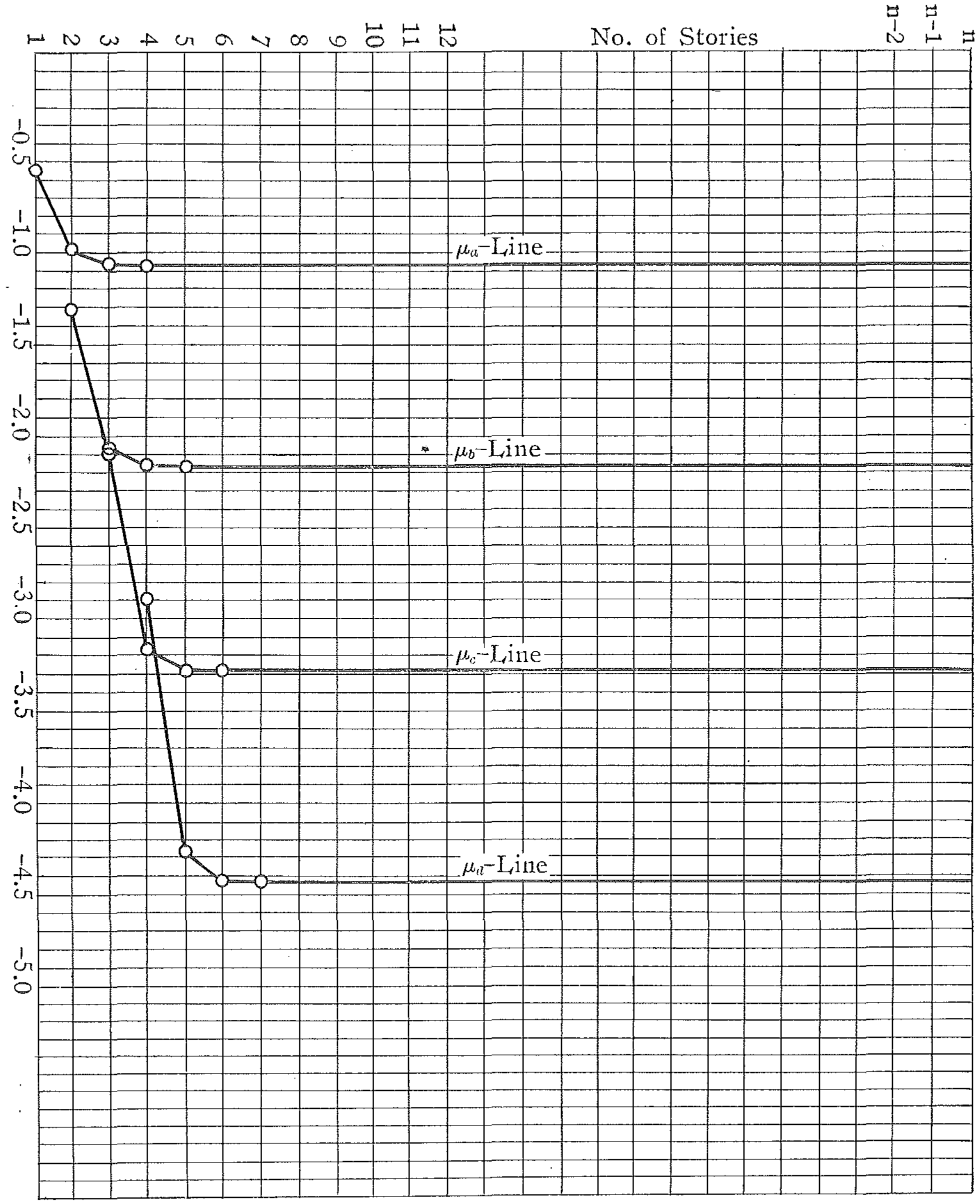
Table XXVIII.



Coefficient: $\frac{w.h}{6}$

Multiple Storied Bent with Five Bays. Legs Fixed at Bases.
Loads Horizontal and Concentrated at Each Joint
on Left Side of Structure.

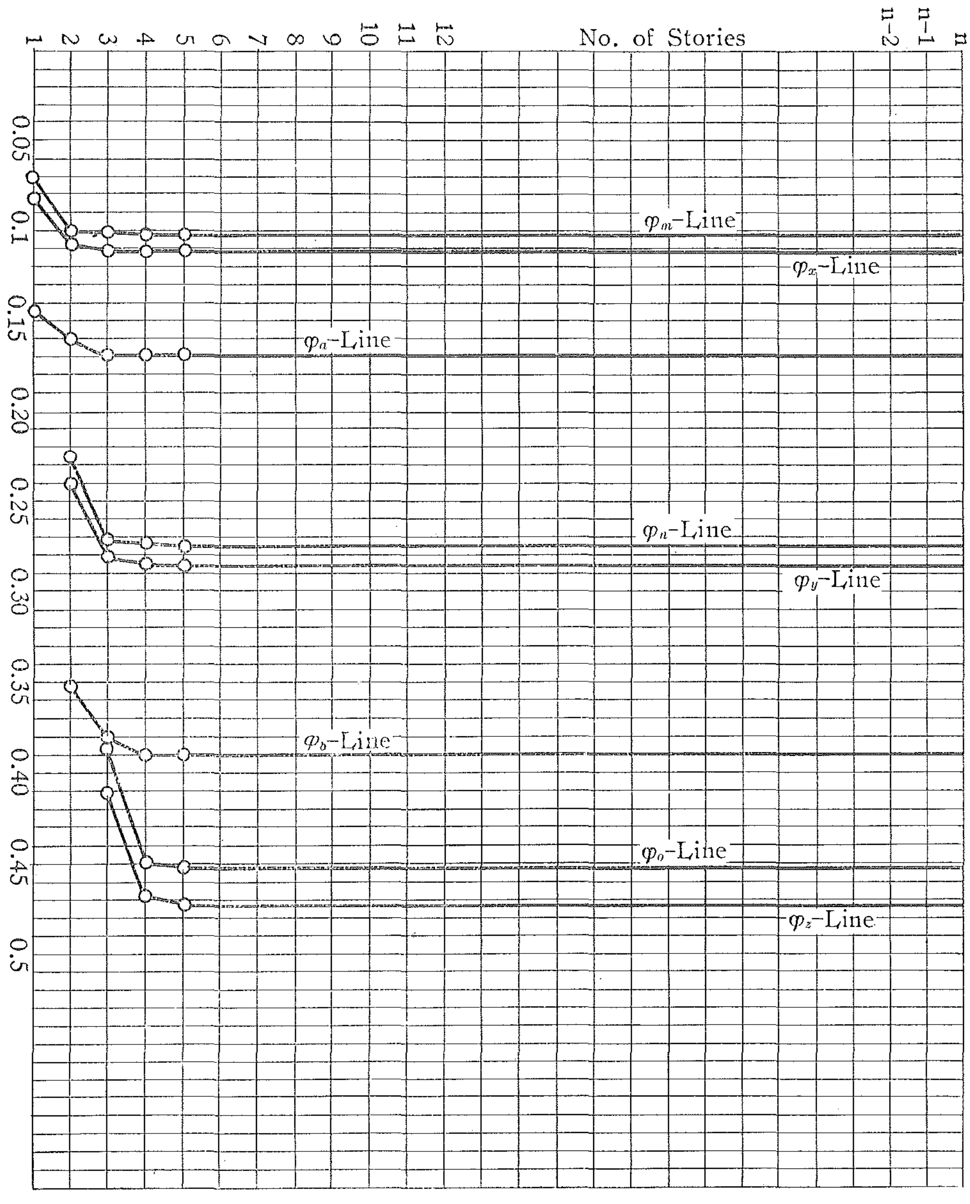
Table XXIX.



Coefficient: $\frac{w.h}{6}$

Multiple Storied Bent with Five Bays. Legs Fixed at Bases.
Loads Horizontal and Concentrated at Each Joint
on Left Side of Structure.

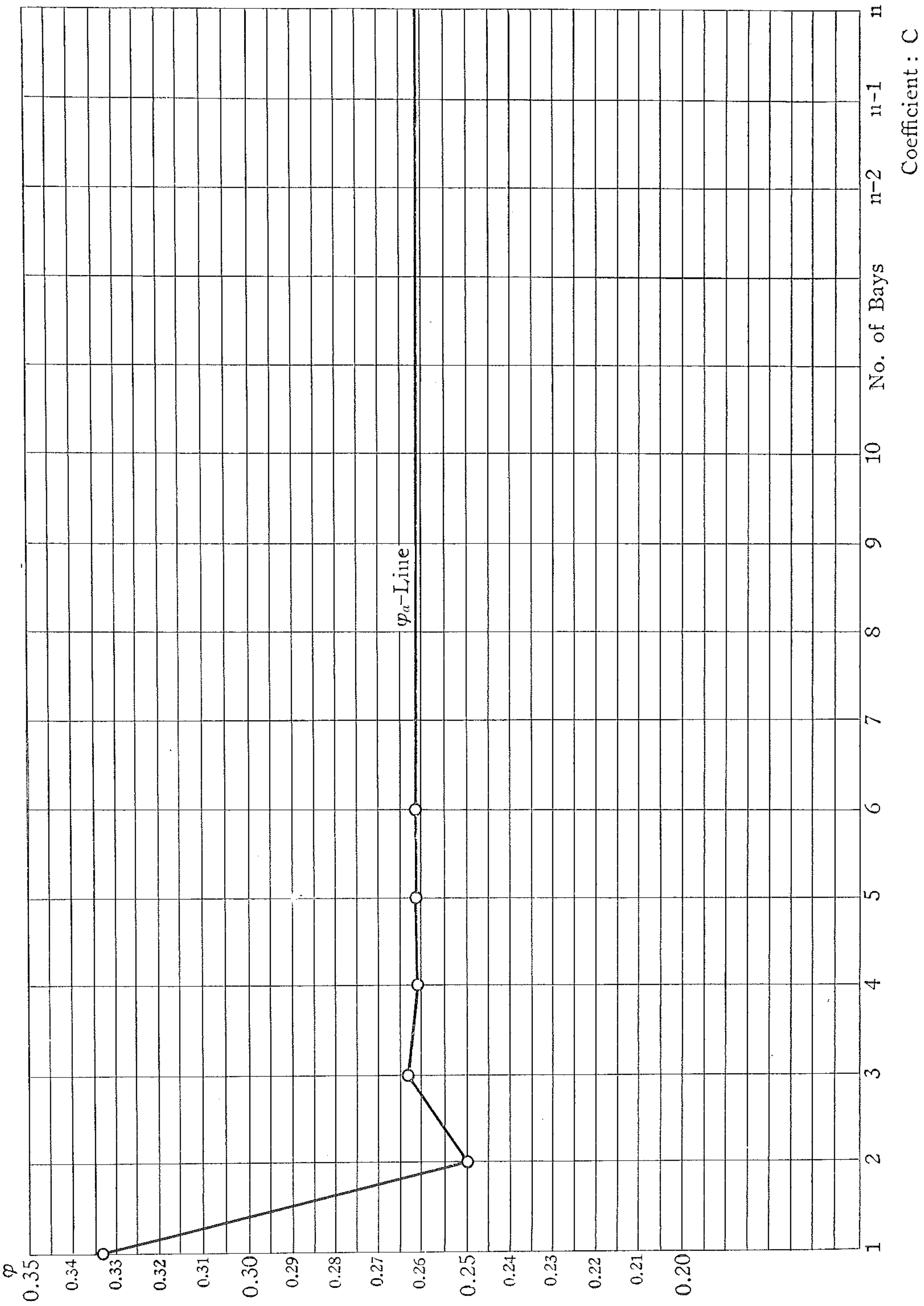
Table XXX.



Coefficient: $\frac{w.h}{6}$

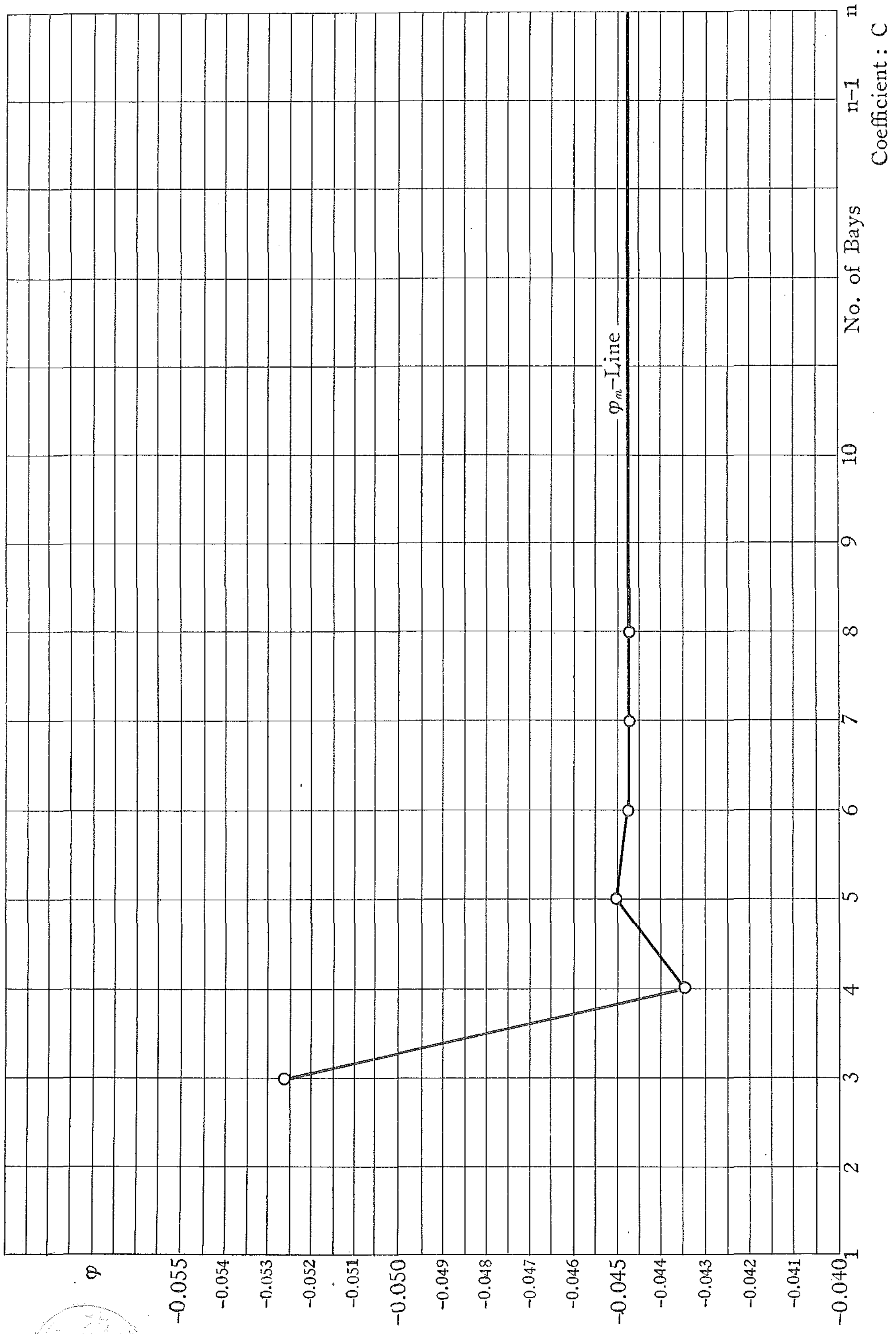
Single Storied Bent with Multiple Bays, Carrying Loads Vertical and Symmetrical. Legs Fixed at Bases.

Table XXXI.



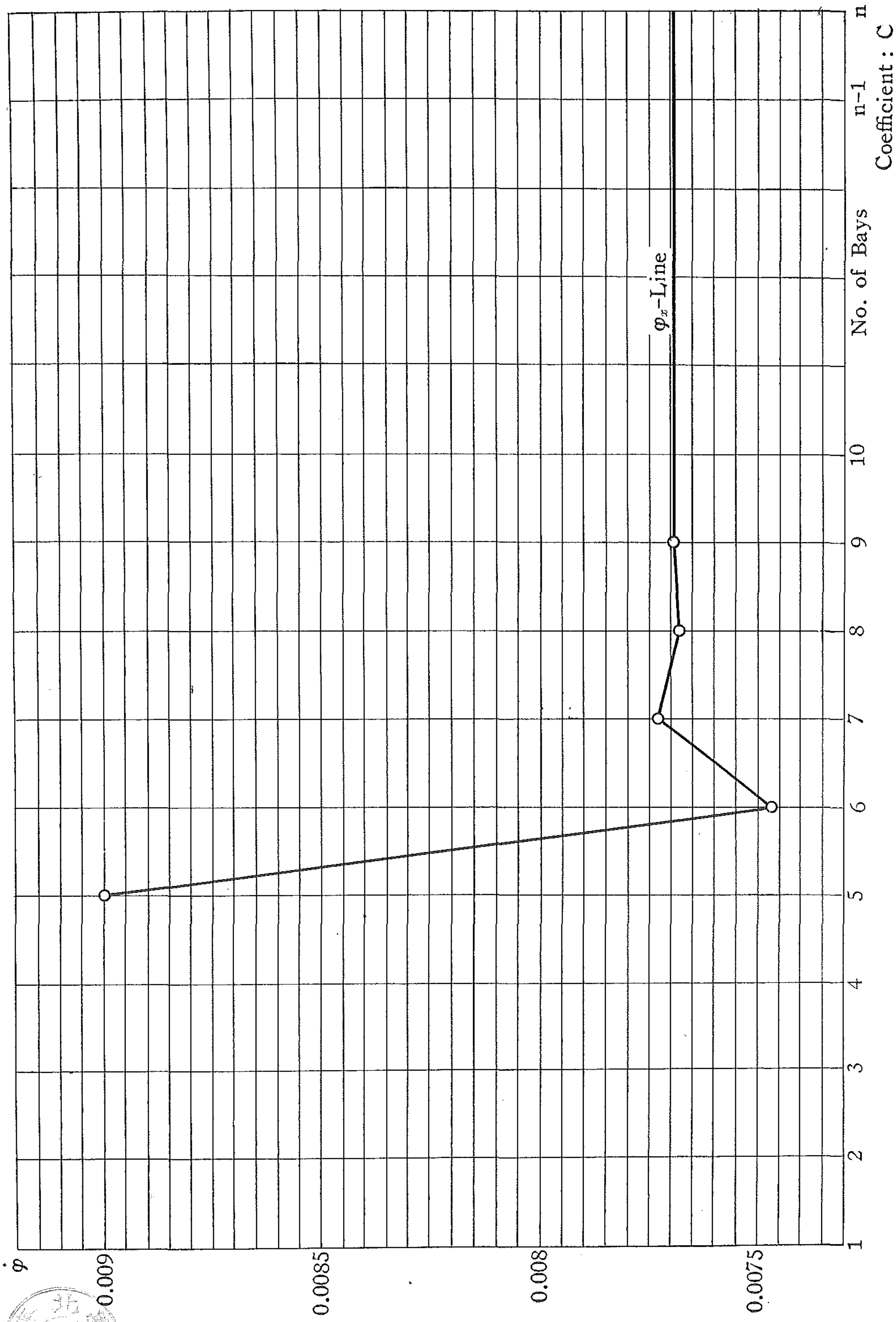
Single Storied Bent with Multiple Bays, Carrying Loads Vertical and Symmetrical. Legs Fixed at Bases.

Table XXXII.



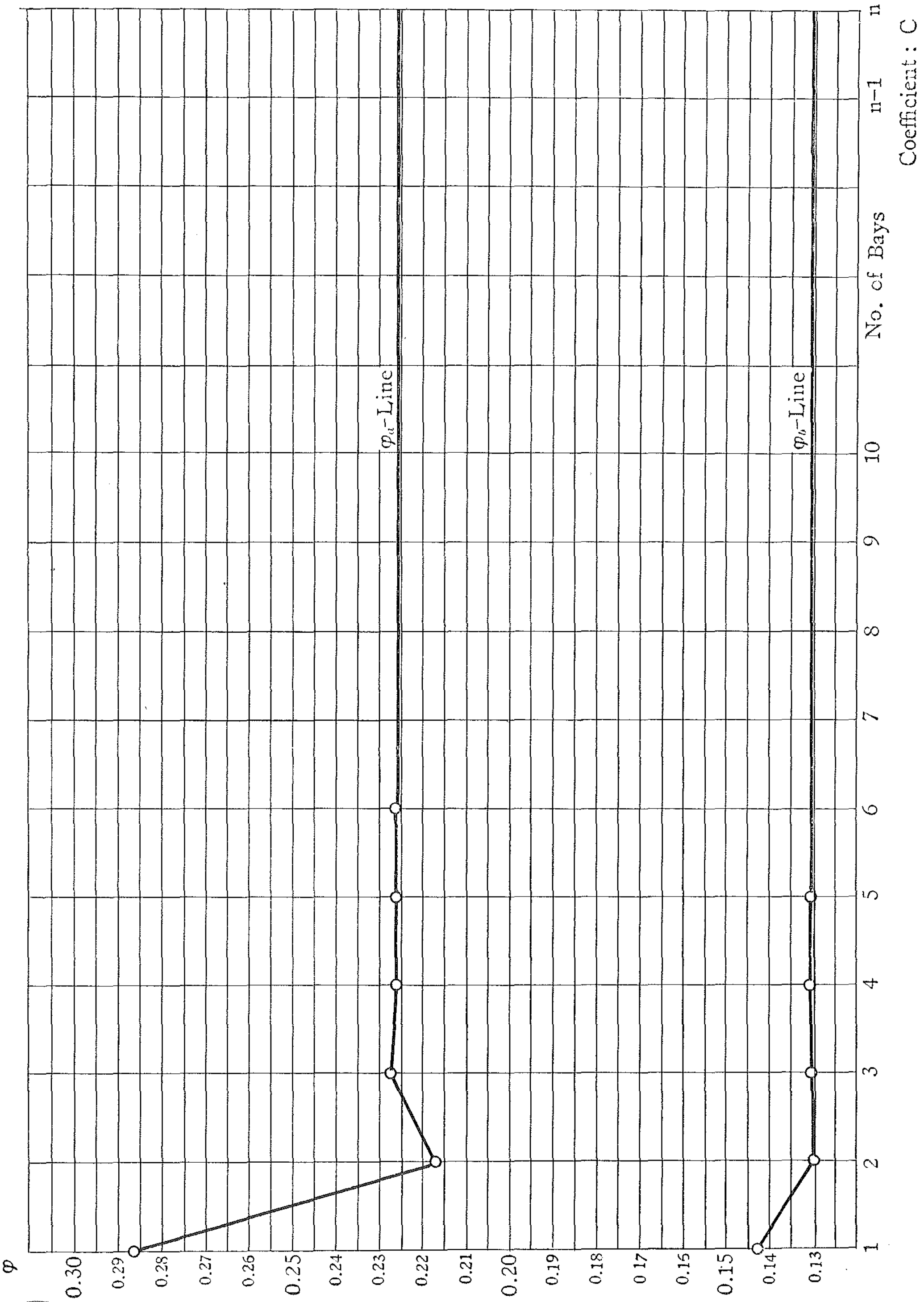
Single Storied Bent with Multiple Bays, Carrying Loads Vertical
and Symmetrical. Legs Fixed at Bases.

Table XXXIII.



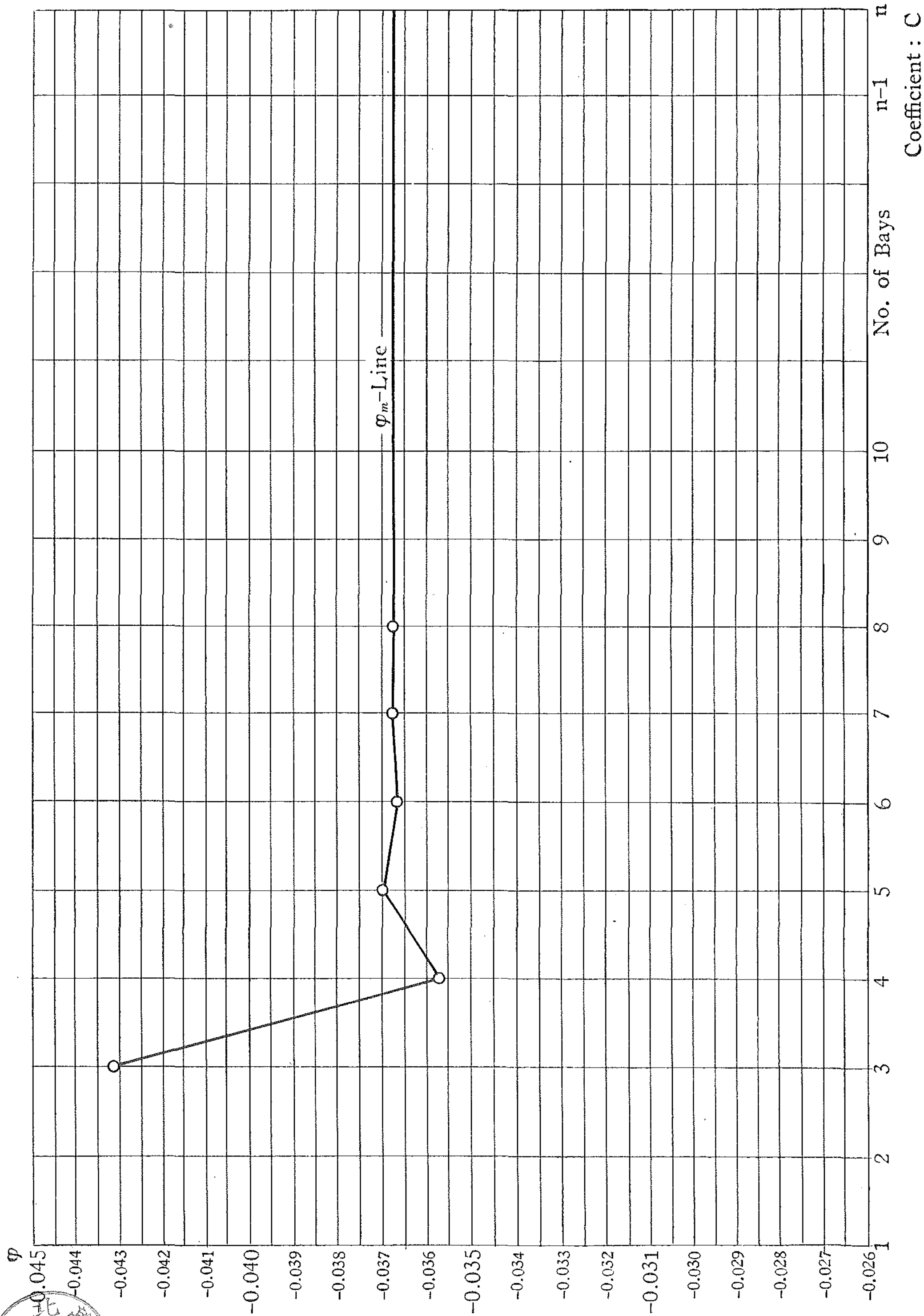
Two Storied Bent with Multiple Bays, Carrying Loads Vertical and Symmetrical. Legs Fixed at Bases.

Table XXXIV.



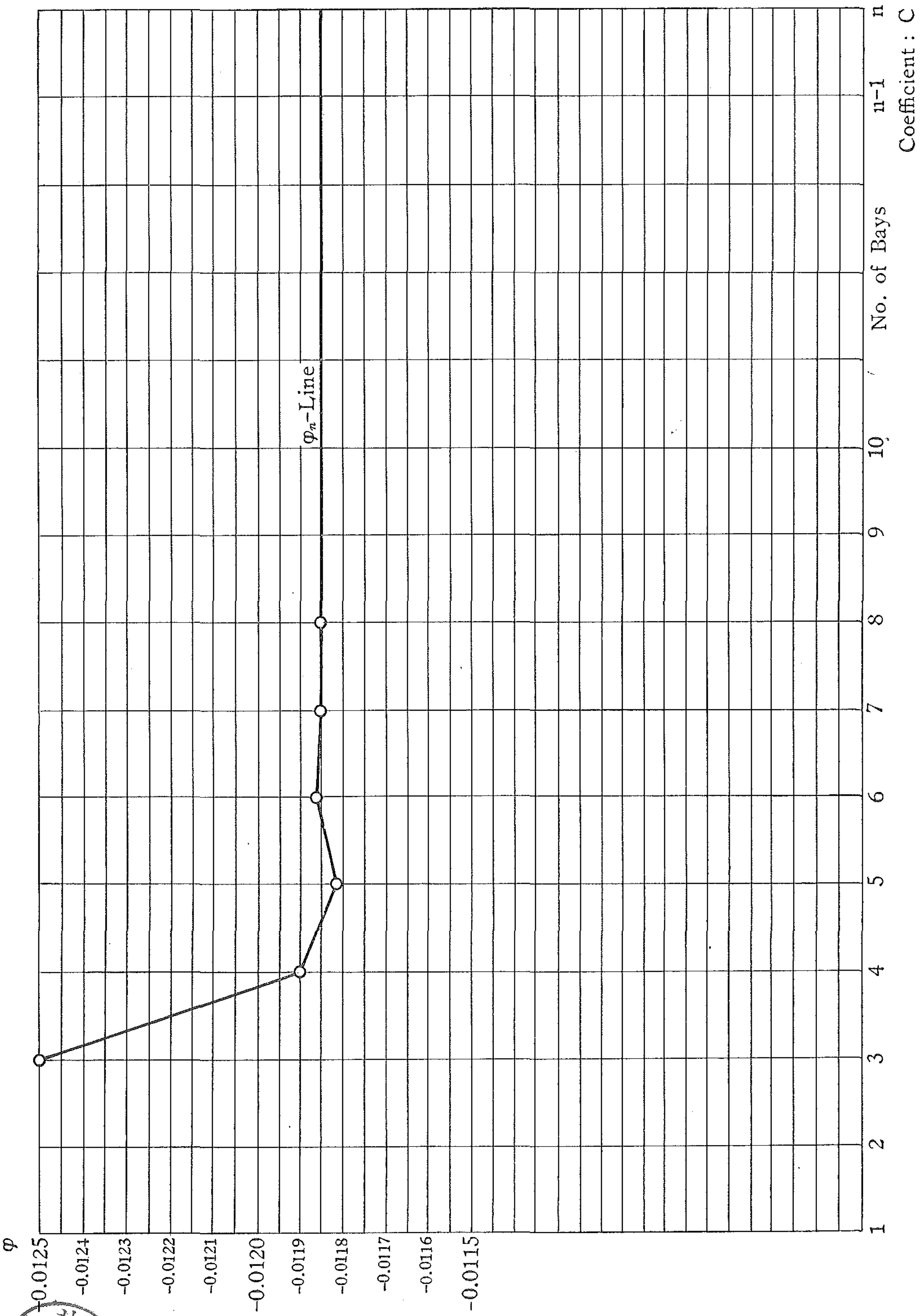
Two Storied Bent with Multiple Bays, Carrying Loads Vertical
and Symmetrical. Legs Fixed at Bases.

Table XXXV.



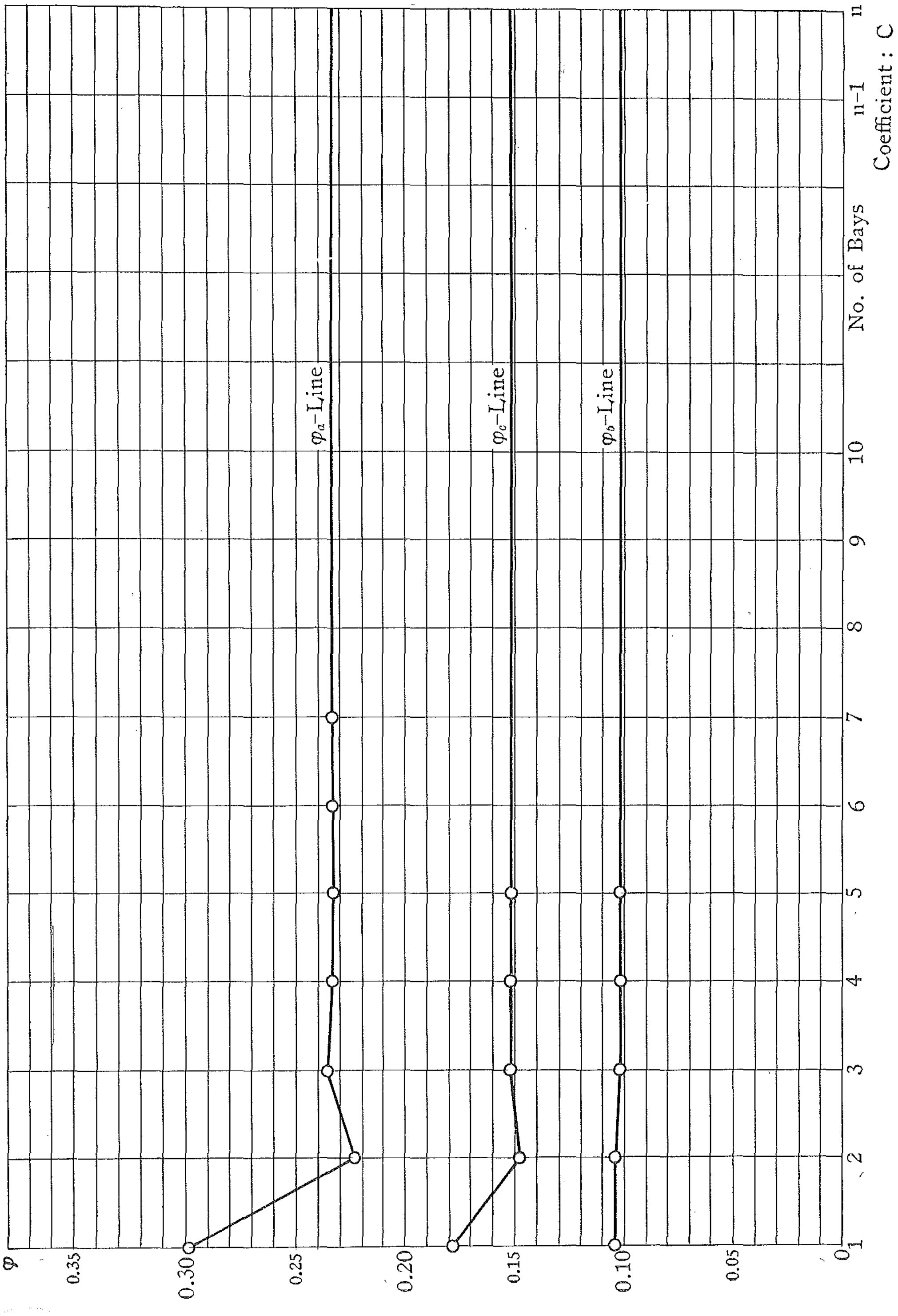
Two Storied Bent with Multiple Bays, Carrying Loads Vertical and Symmetrical. Legs Fixed at Bases.

Table XXXVI.



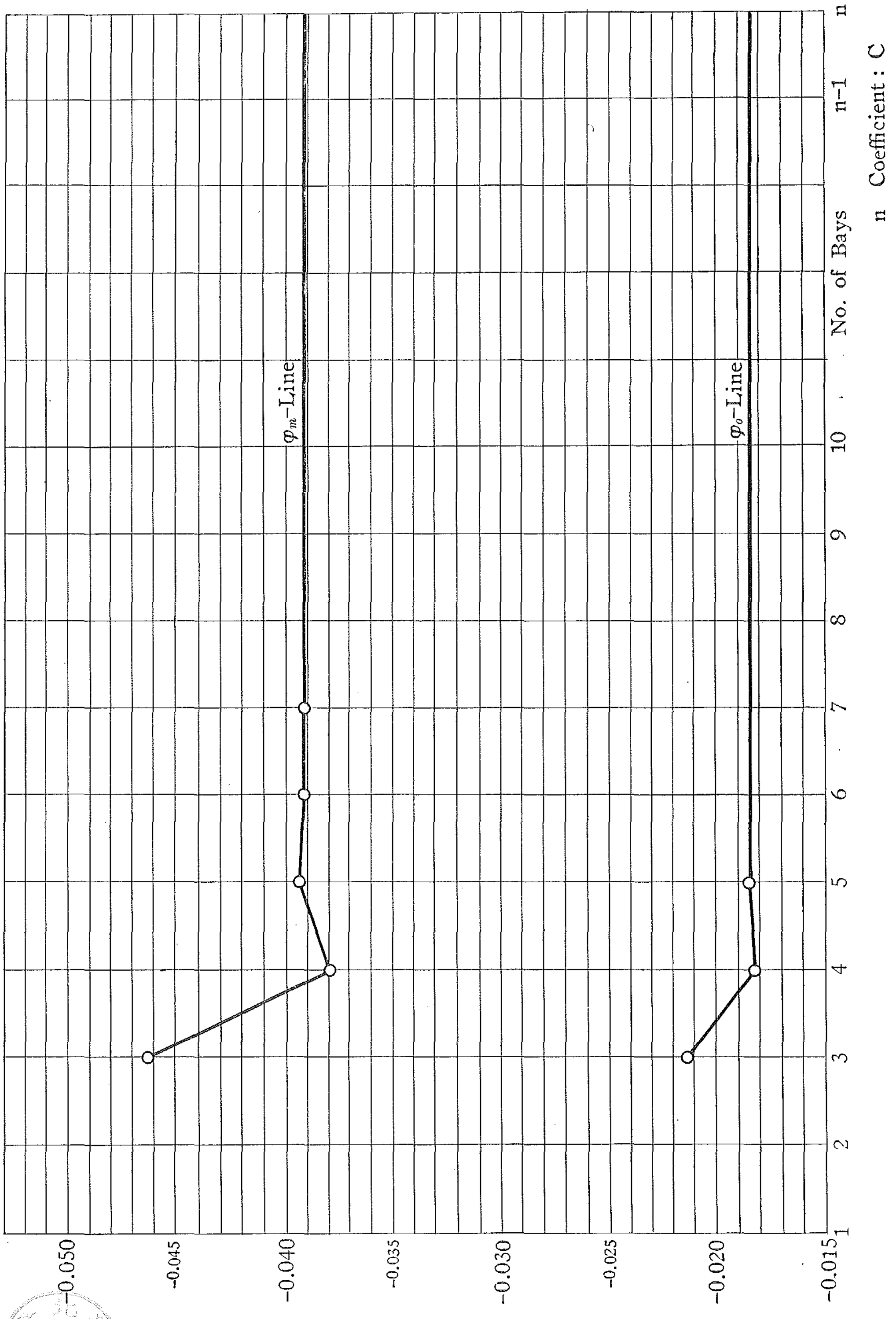
Three Storied Bent with Multiple Bays, Carrying Loads Vertical and Symmetrical. Legs Fixed at Bases.

Table XXXVII.



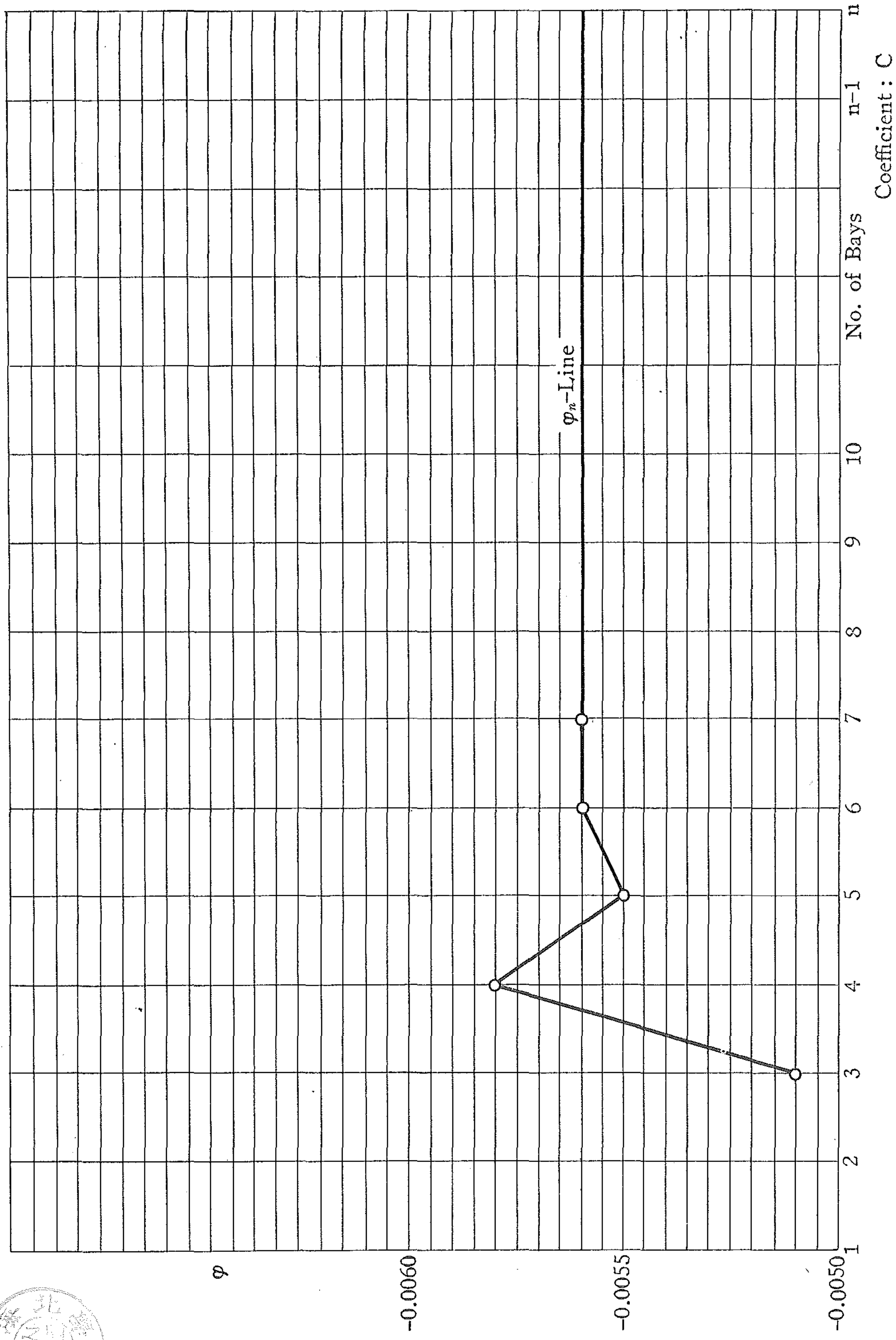
Three Storied Bent with Multiple Bays, Carrying Loads Vertical and Symmetrical. Legs Fixed at Bases.

Table XXXVIII.



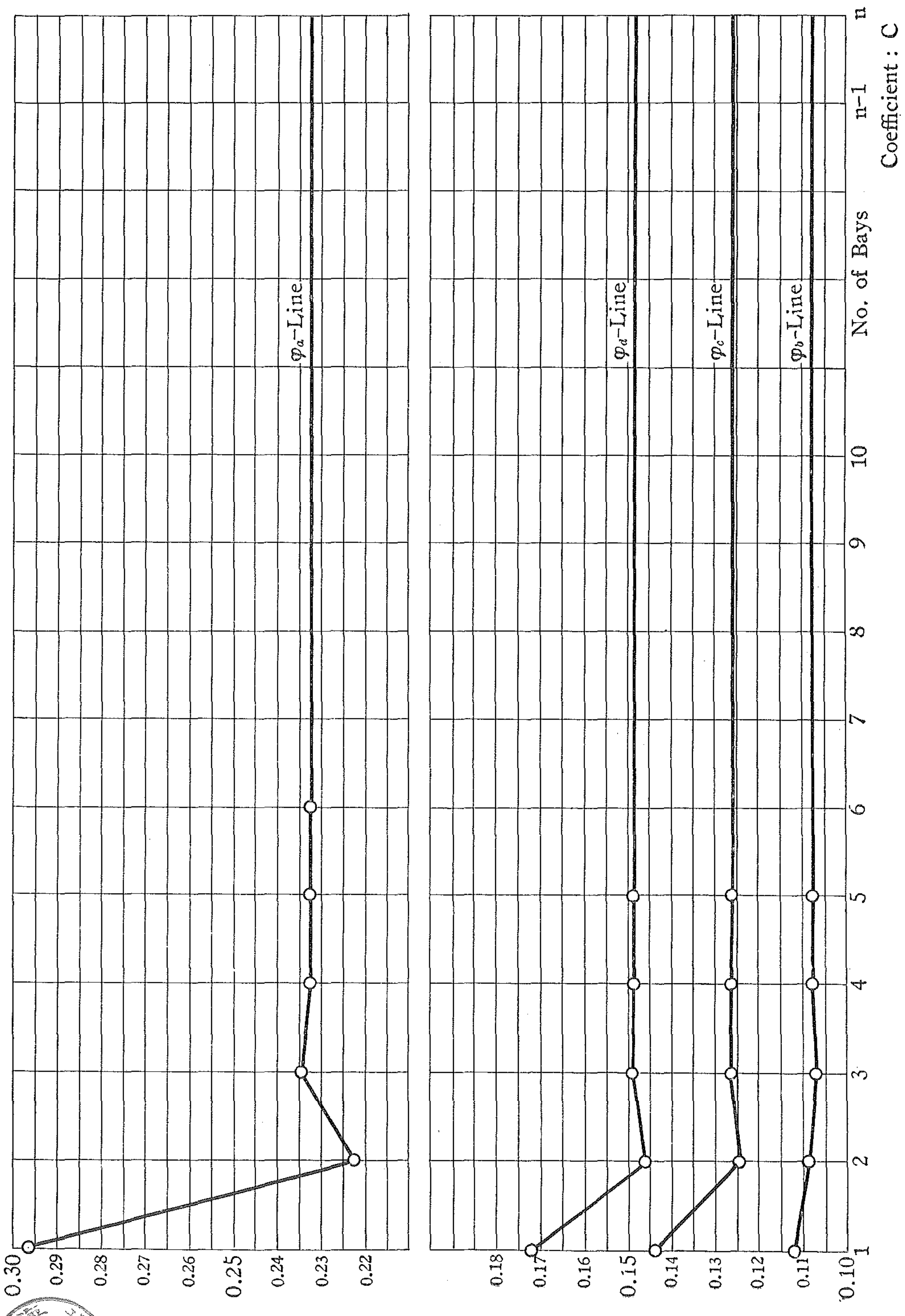
Three Storied Bent with Multiple Bays, Carrying Loads Vertical and Symmetrical. Legs Fixed at Bases.

Table XXXIX.



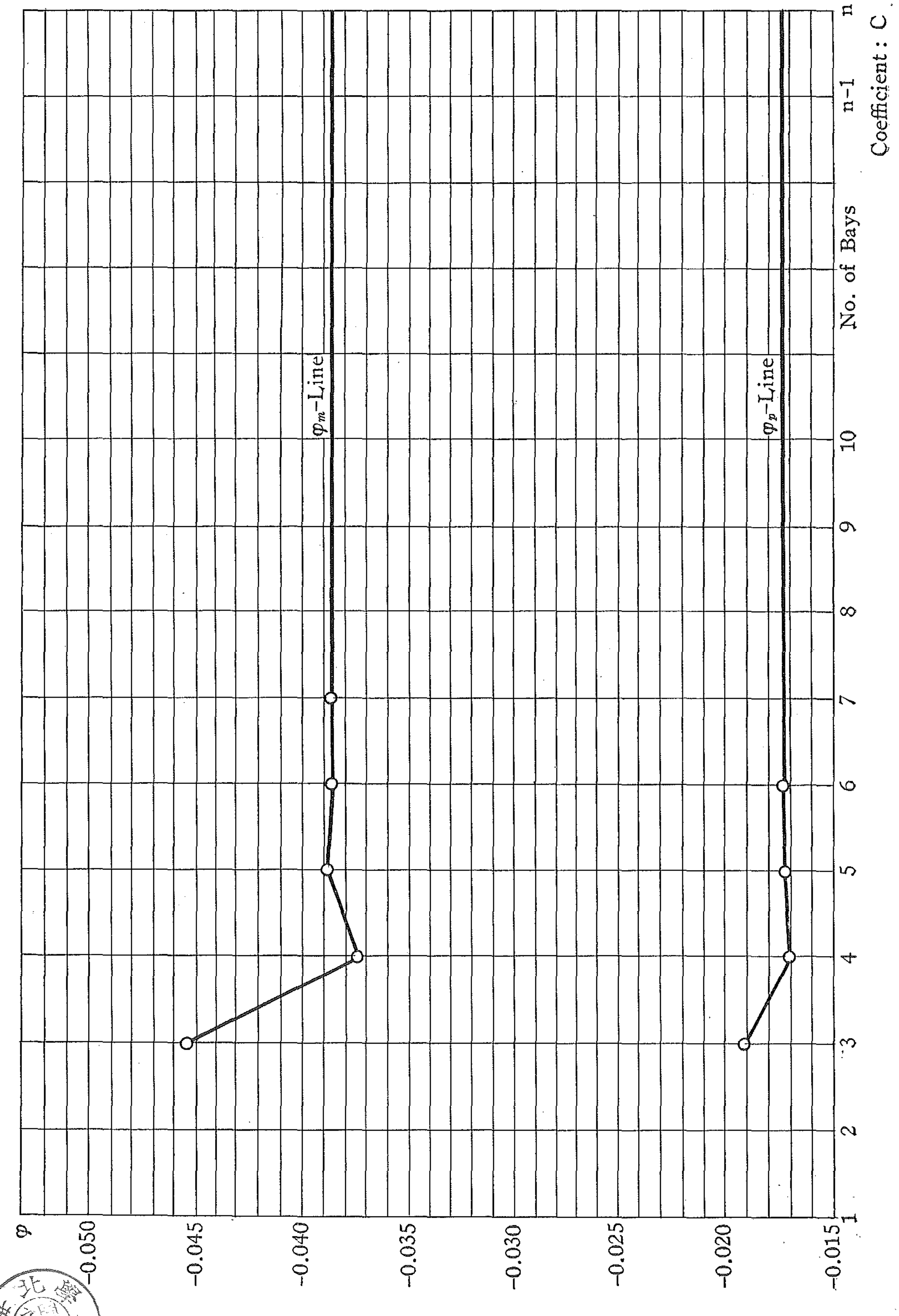
Four Storied Bent with Multiple Bays, Carrying Loads Vertical and Symmetrical. Legs Fixed at Bases.

Table XL.



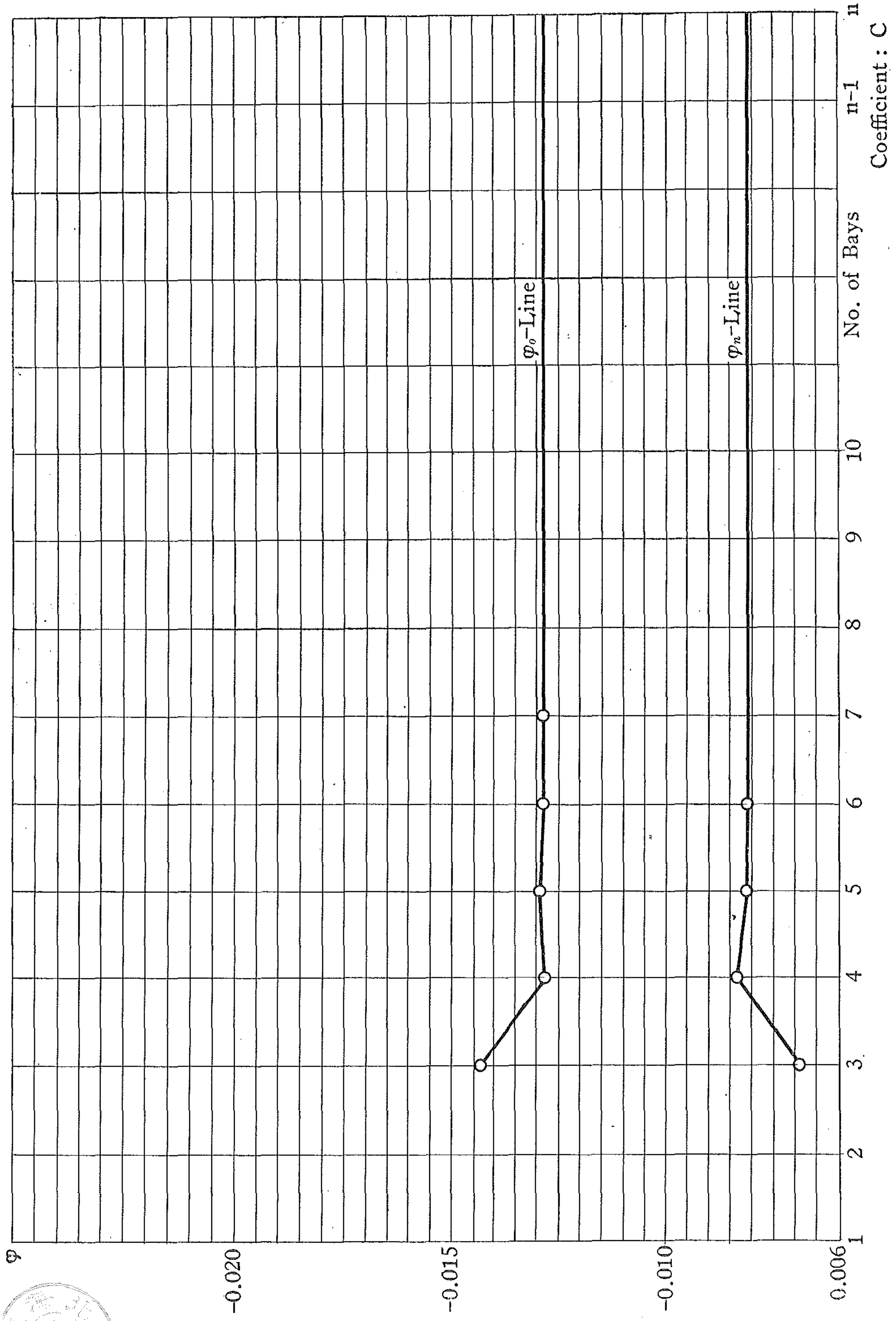
Four Storied Bent with Multiple Bays, Carrying Loads Vertical and Symmetrical. Legs Fixed at Bases.

Table XLI.



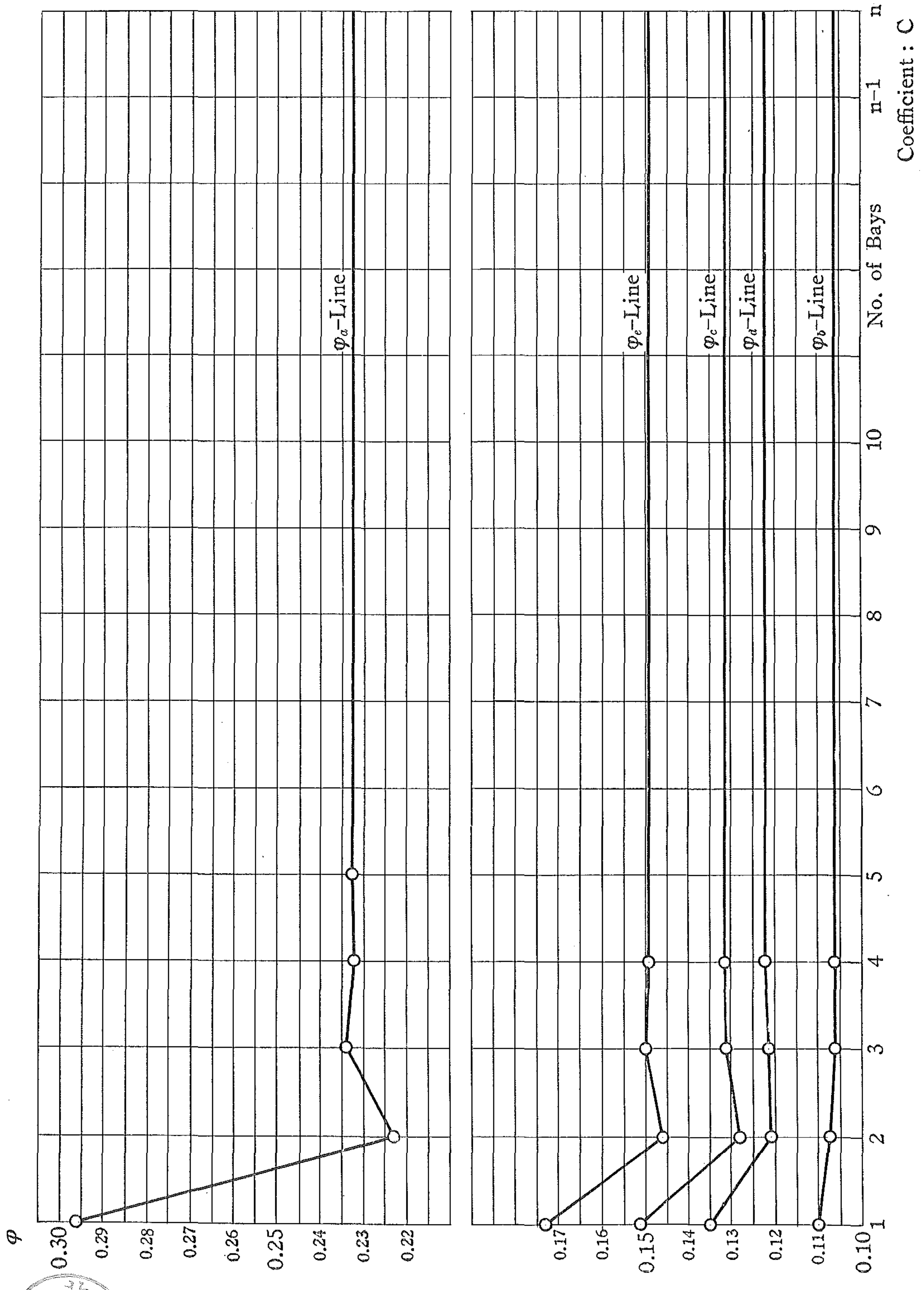
Four Storied Bent with Multiple Bays, Carrying Loads Vertical and Symmetrical. Legs Fixed at Bases.

Table XLII.



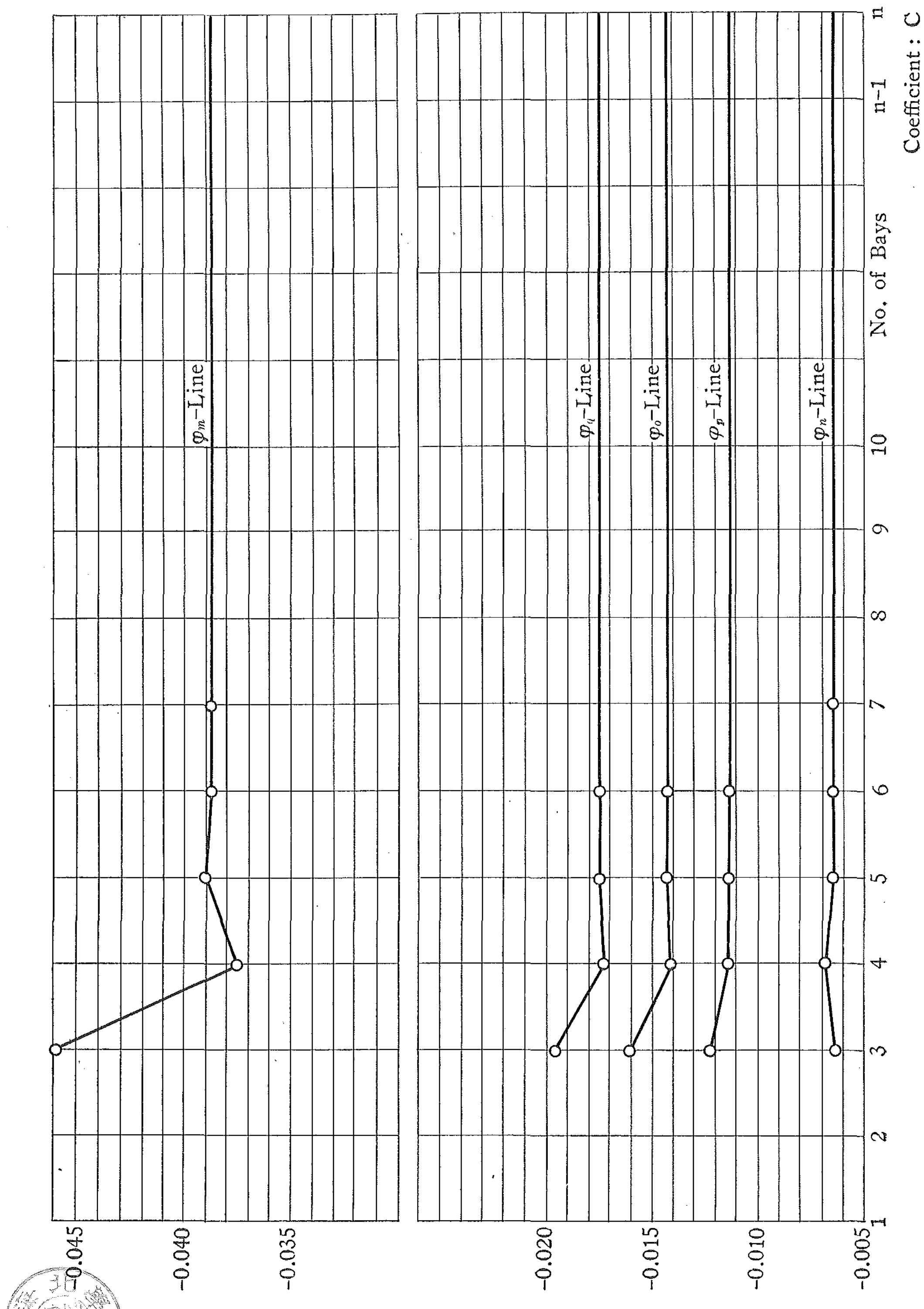
Five Storied Bent with Multiple Bays, Carrying Loads Vertical and Symmetrical. Legs Fixed at Bases.

Table XLIII.



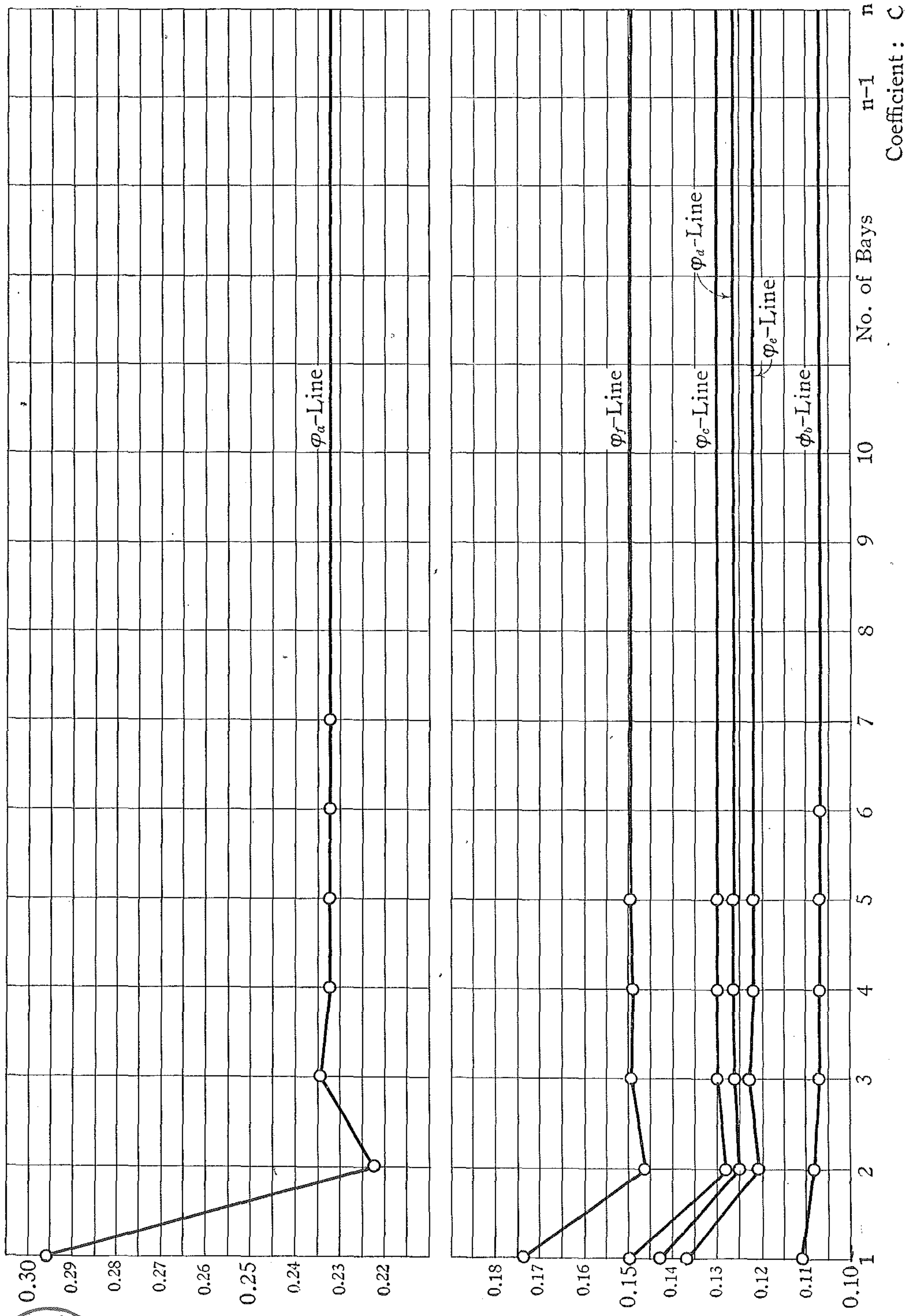
Five Storied Bent with Multiple Bays, Carrying Loads Vertical and Symmetrical. Legs Fixed at Bases.

Table XLIV



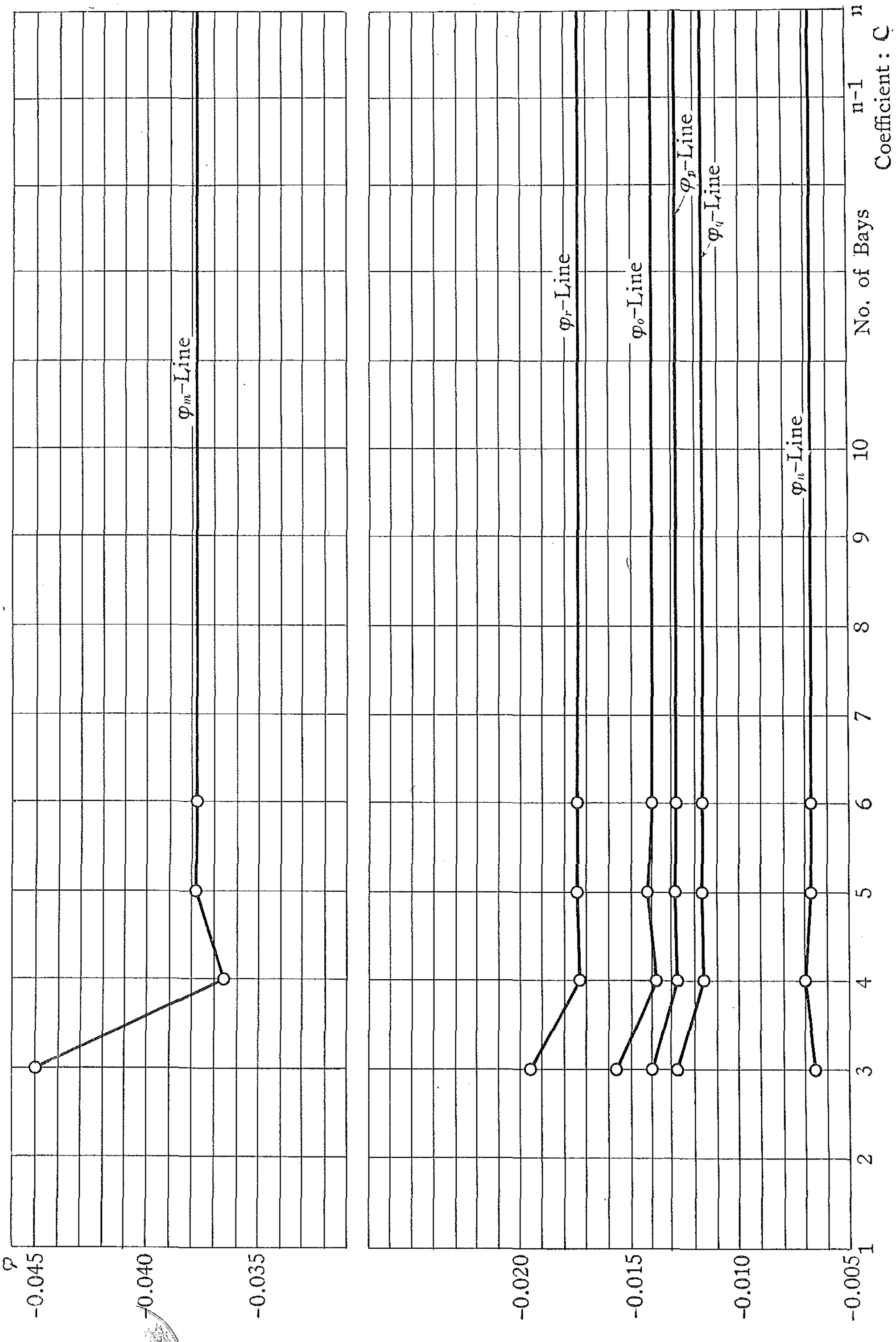
Six Storied Bent with Multiple Bays, Carrying Loads Vertical and Symmetrical. Legs Fixed at Bases.

Table XLV.



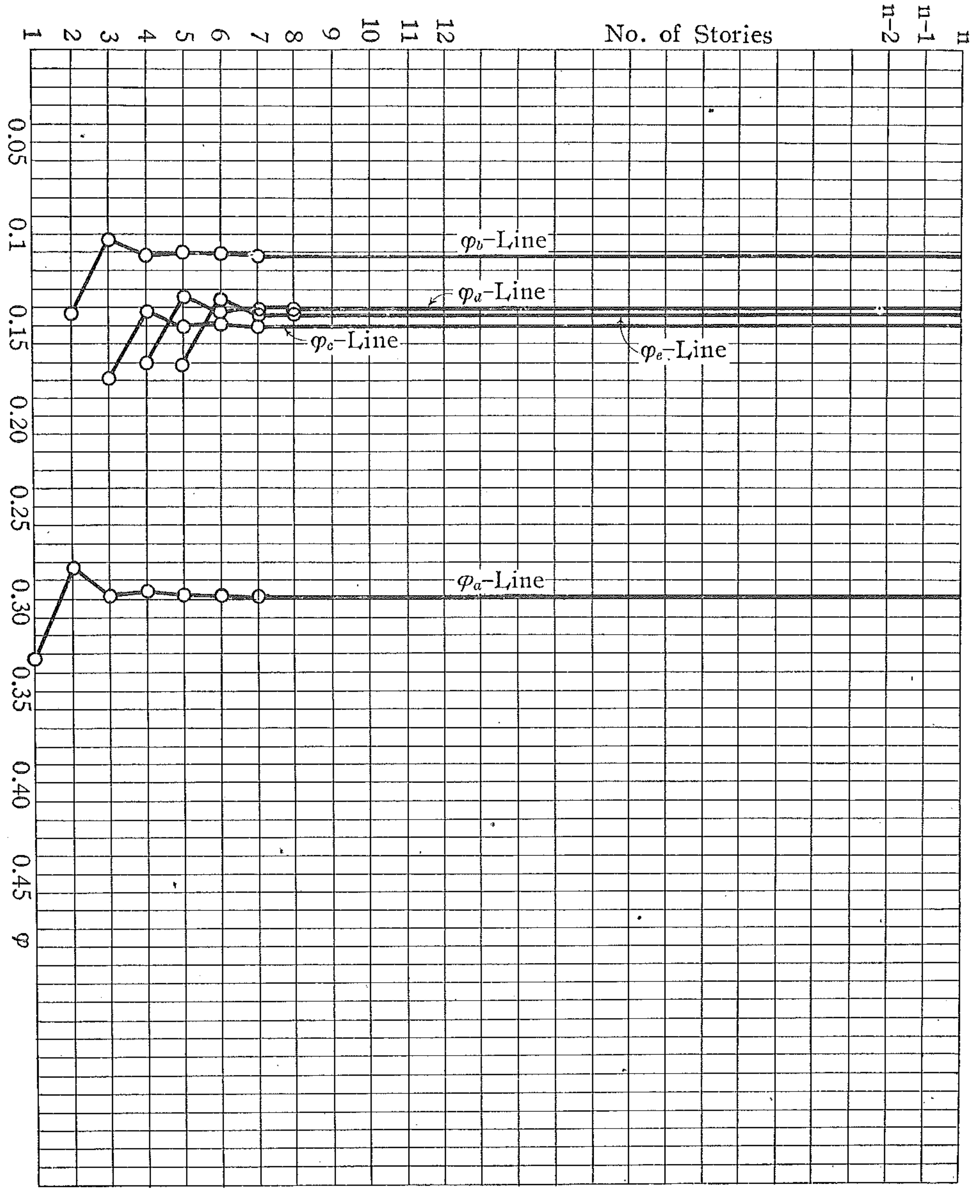
Six Storied Bent with Multiple Bays, Carrying Loads Vertical and Symmetrical. Legs Fixed at Bases.

Table XLVI.



Multiple Storied Bent with Single Bay. Legs Fixed at Bases.
Loads Vertical and Symmetrical.

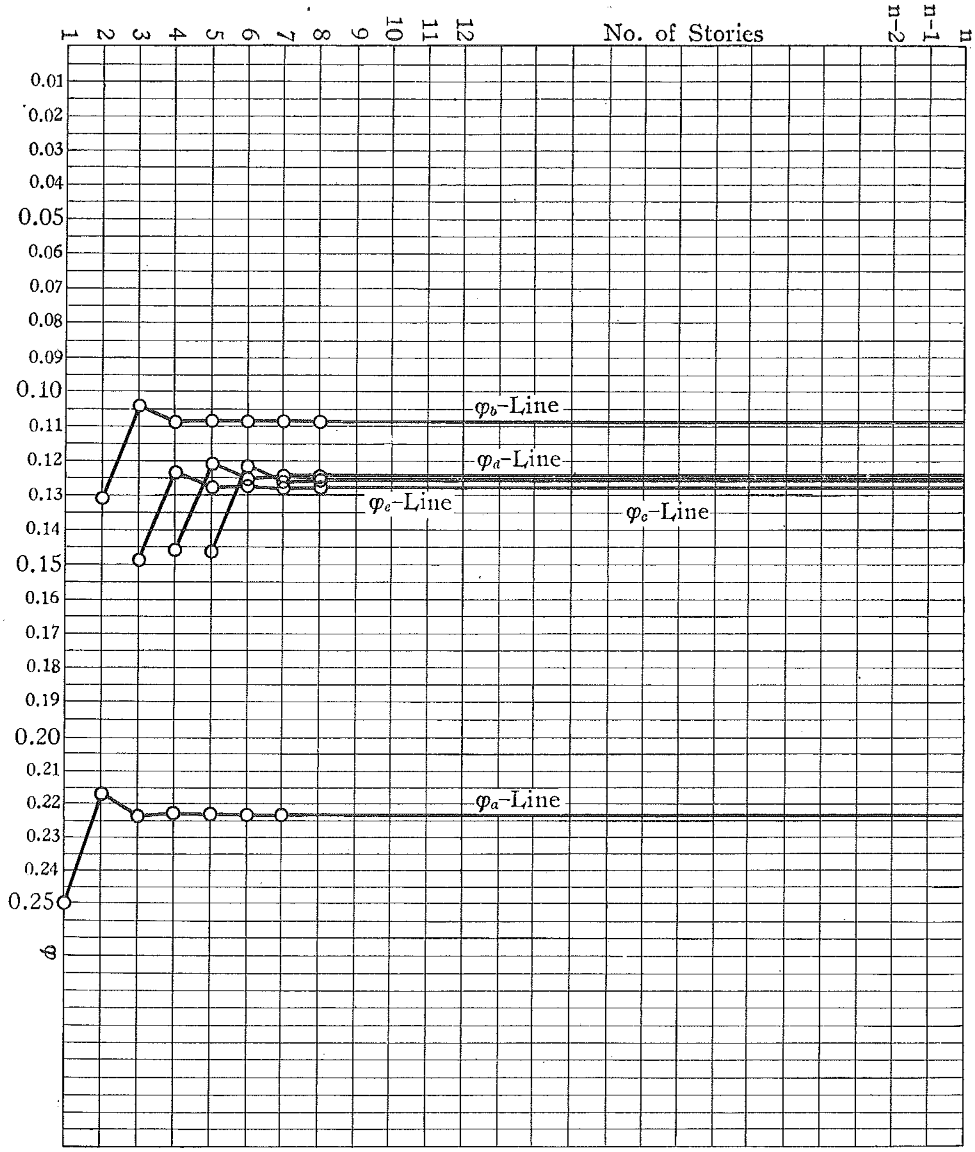
Table XLVII.



Coefficient: C

Multiple Storied Bent with Two Bays. Legs Fixed at Bases.
Loads Vertical and Symmetrical.

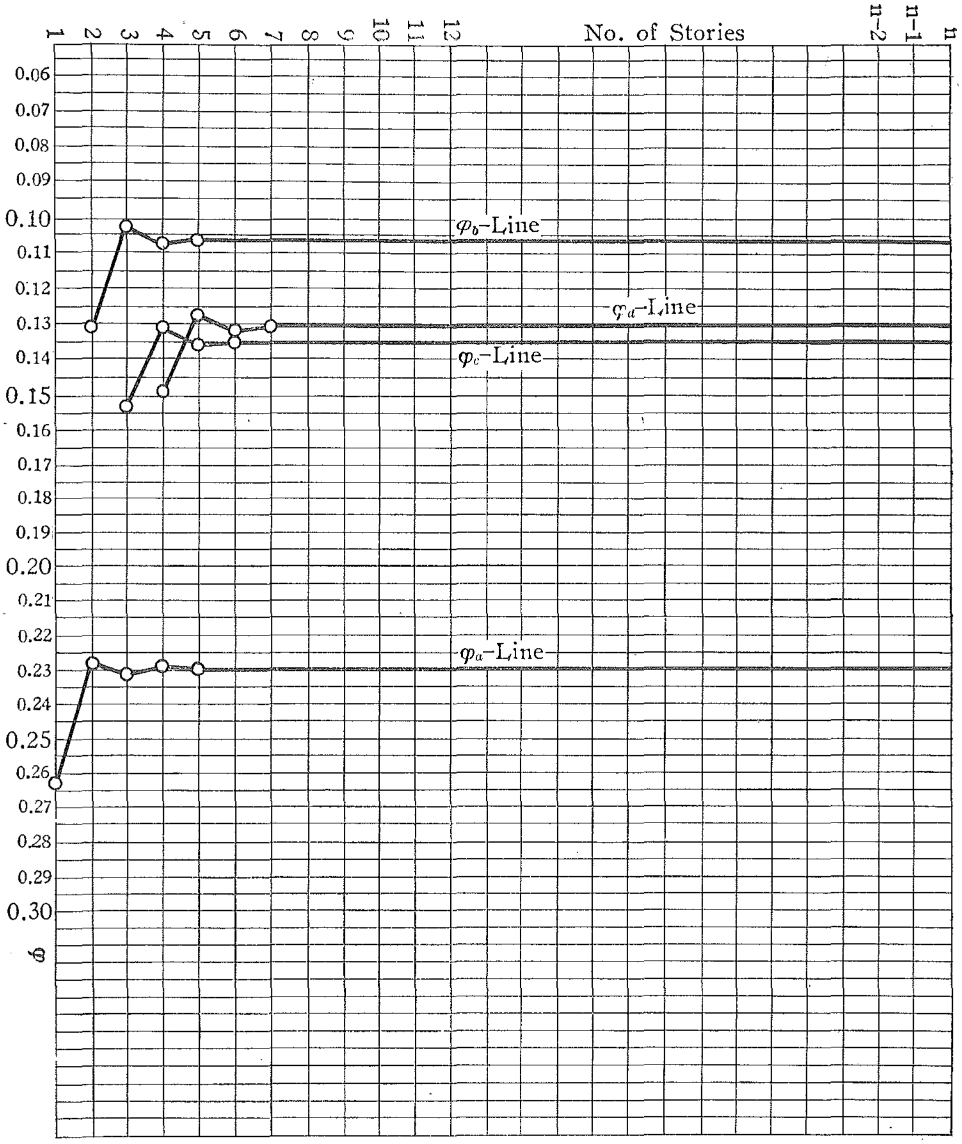
Table XLVIII.



Coefficient: C

Multiple Storied Bent with Three Bays. Legs Fixed at Bases.
Loads Vertical and Symmetrical.

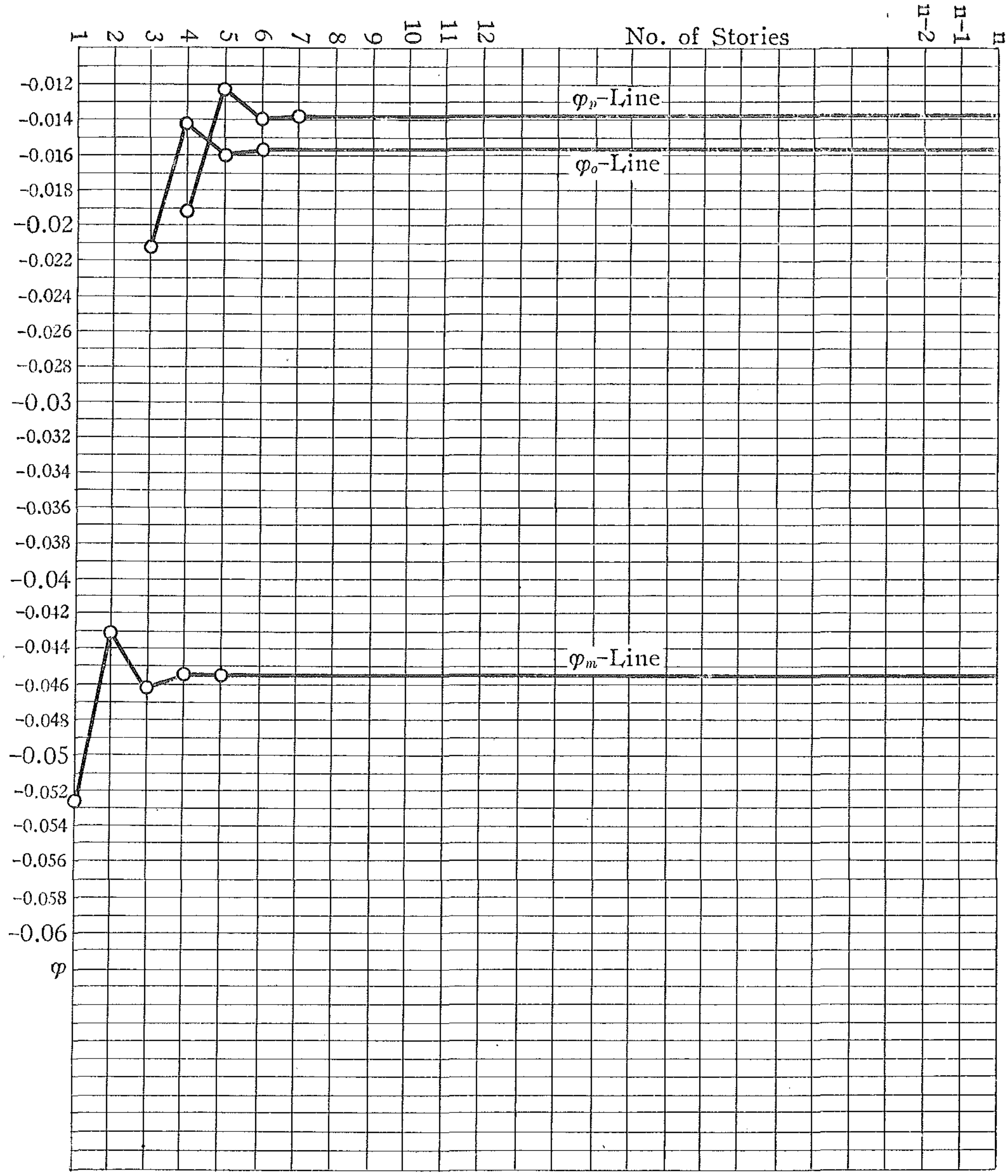
Table XLIX.



Coefficient: C

Multiple Storied Bent with Three Bays. Legs Fixed at Bases.
Loads Vertical and Symmetrical.

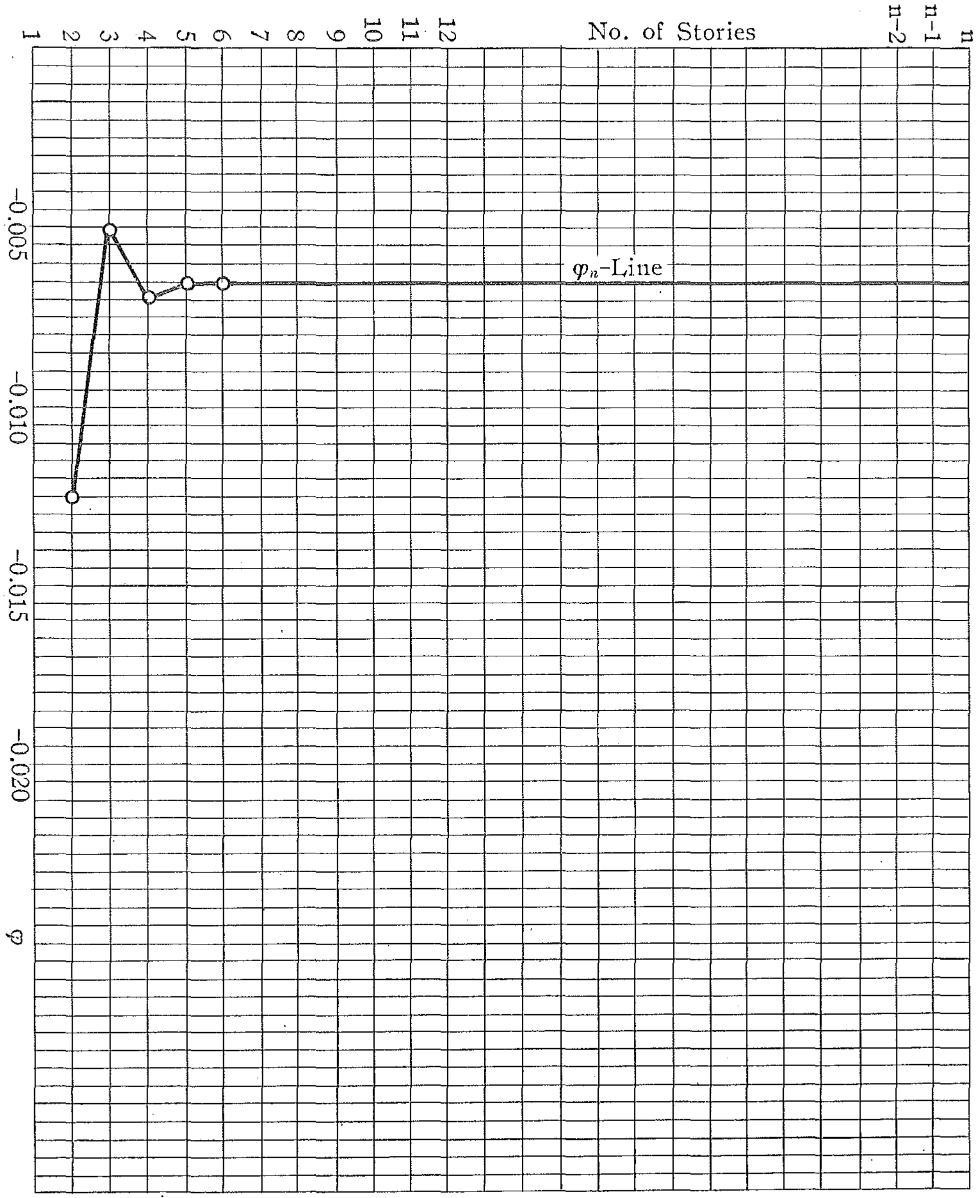
Table L.



Coefficient: C

Multiple Storied Bent with Three Bays. Legs Fixed at Bases.
Loads Vertical and Symmetrical.

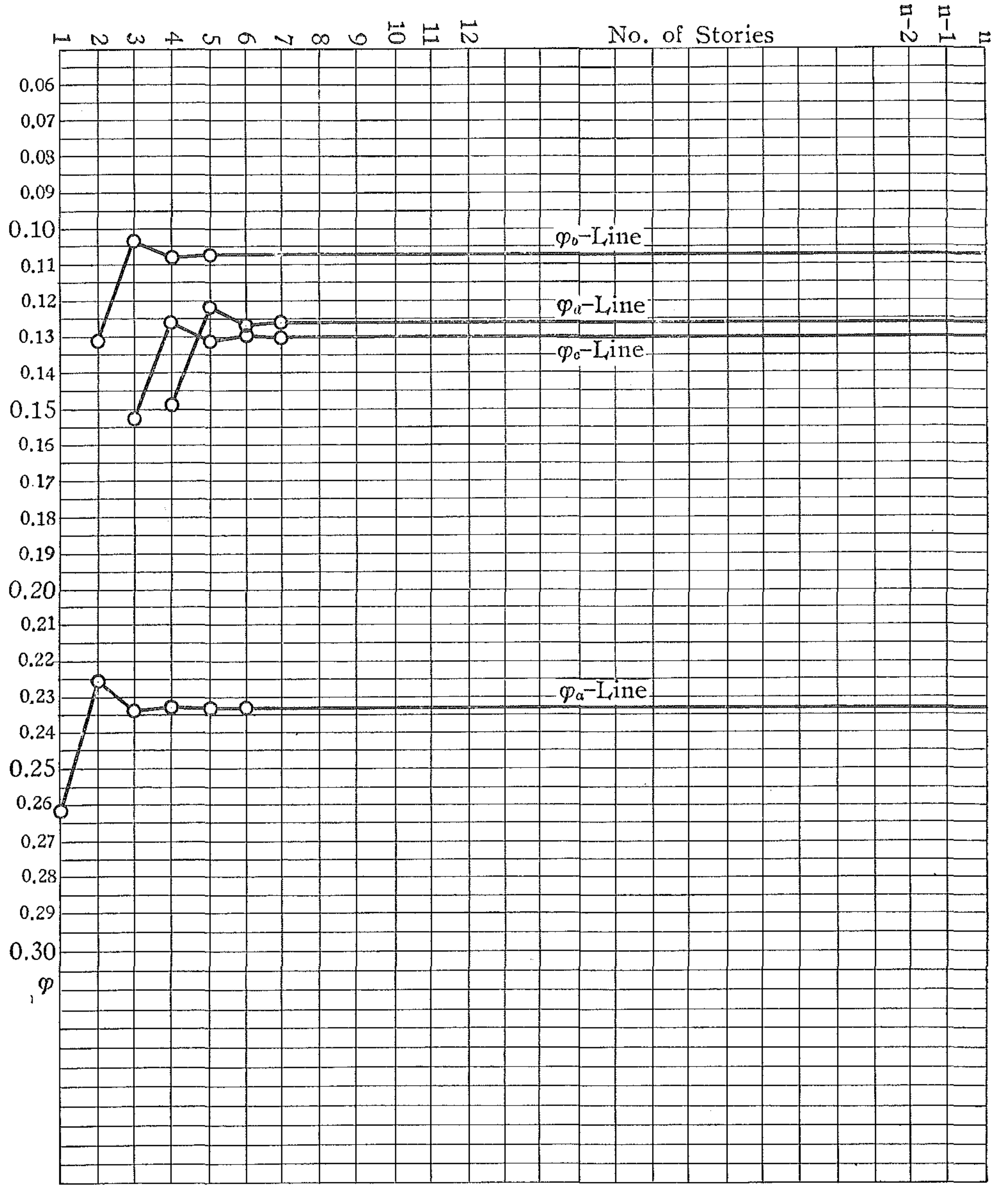
Table LI.



Coefficient: C

Multiple Storied Bent with Four Bays. Legs Fixed at Bases.
Loads Vertical and Symmetrical.

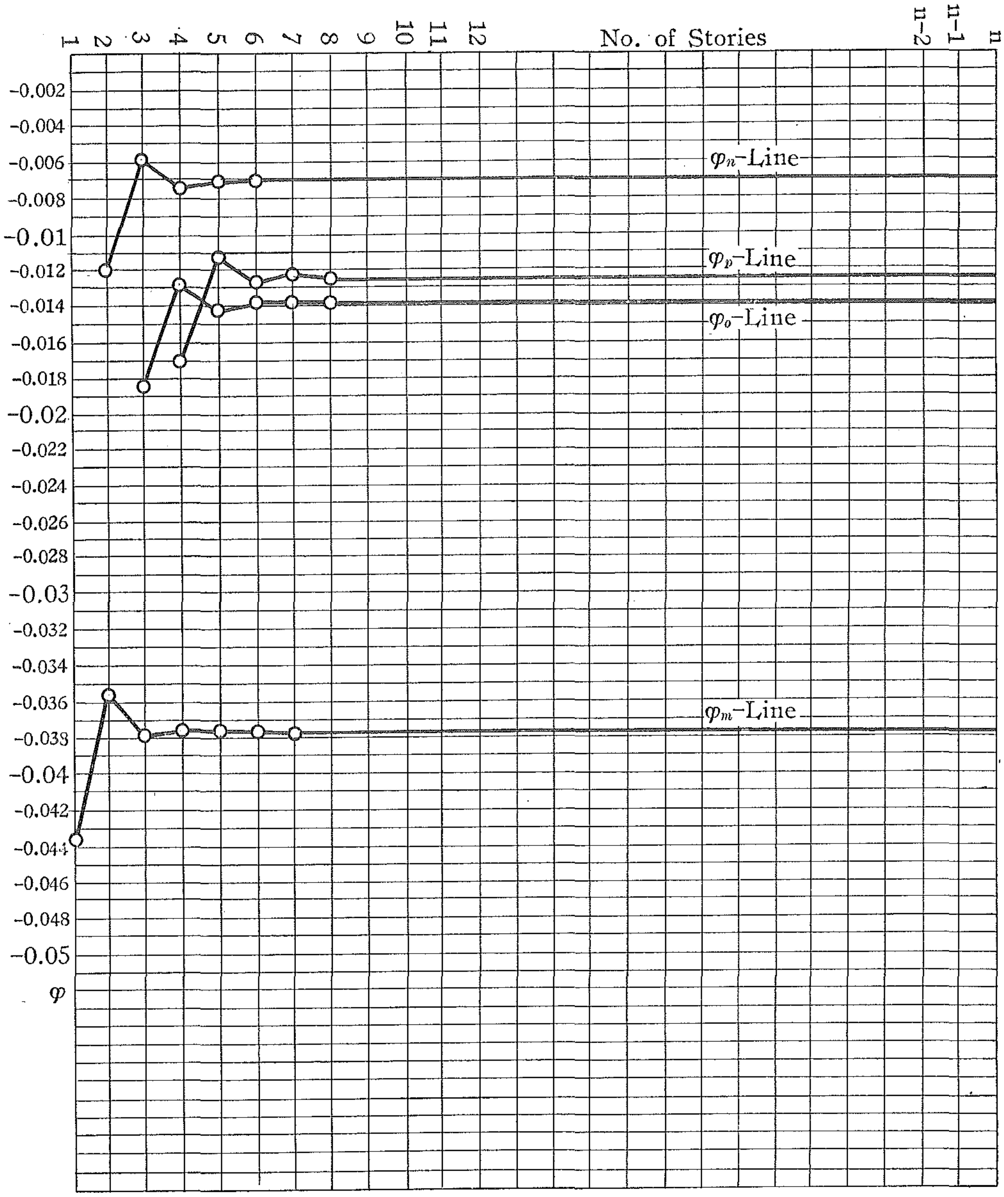
Table LII.



Coefficient: C

Multiple Storied Bent with Four Bays. Legs Fixed at Bases.
Loads Vertical and Symmetrical.

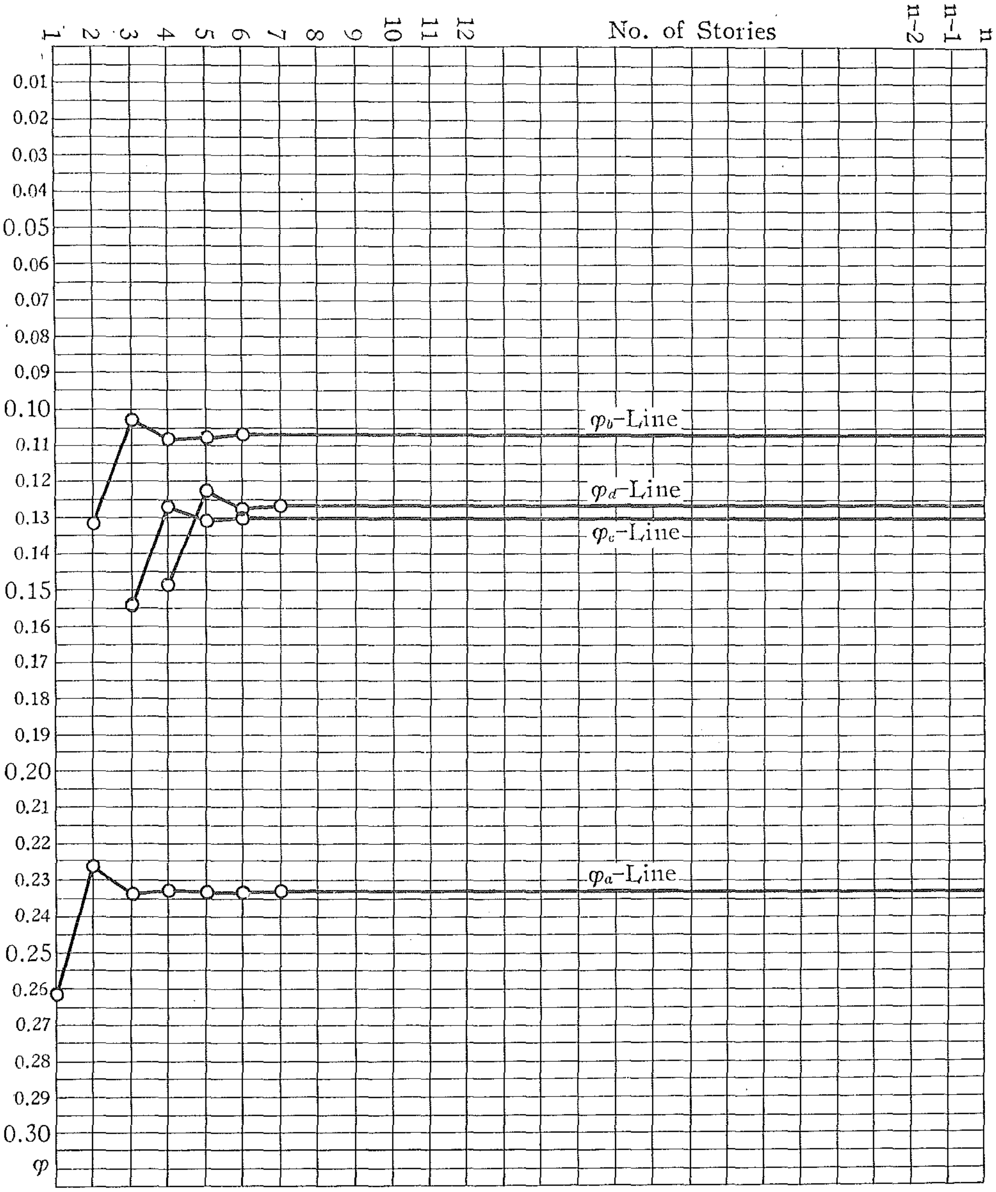
Table LIII.



Coefficient: C

Multiple Storied Bent with Five Bays. Legs Fixed at Bases.
Loads Vertical and Symmetrical.

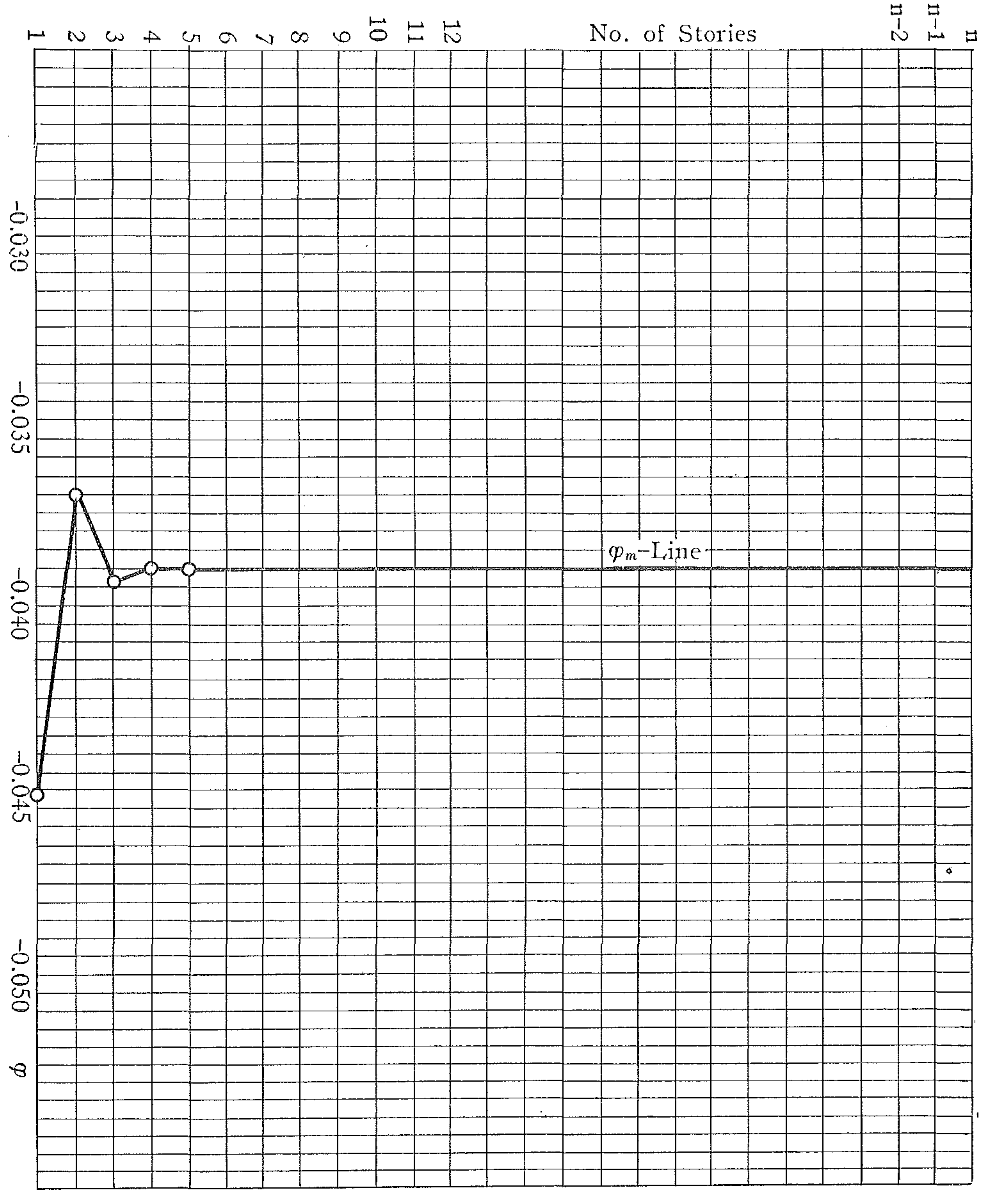
Table LIV.



Coefficient: C

Multiple Storied Bent with Five Bays. Legs Fixed at Bases.
Loads Vertical and Symmetrical.

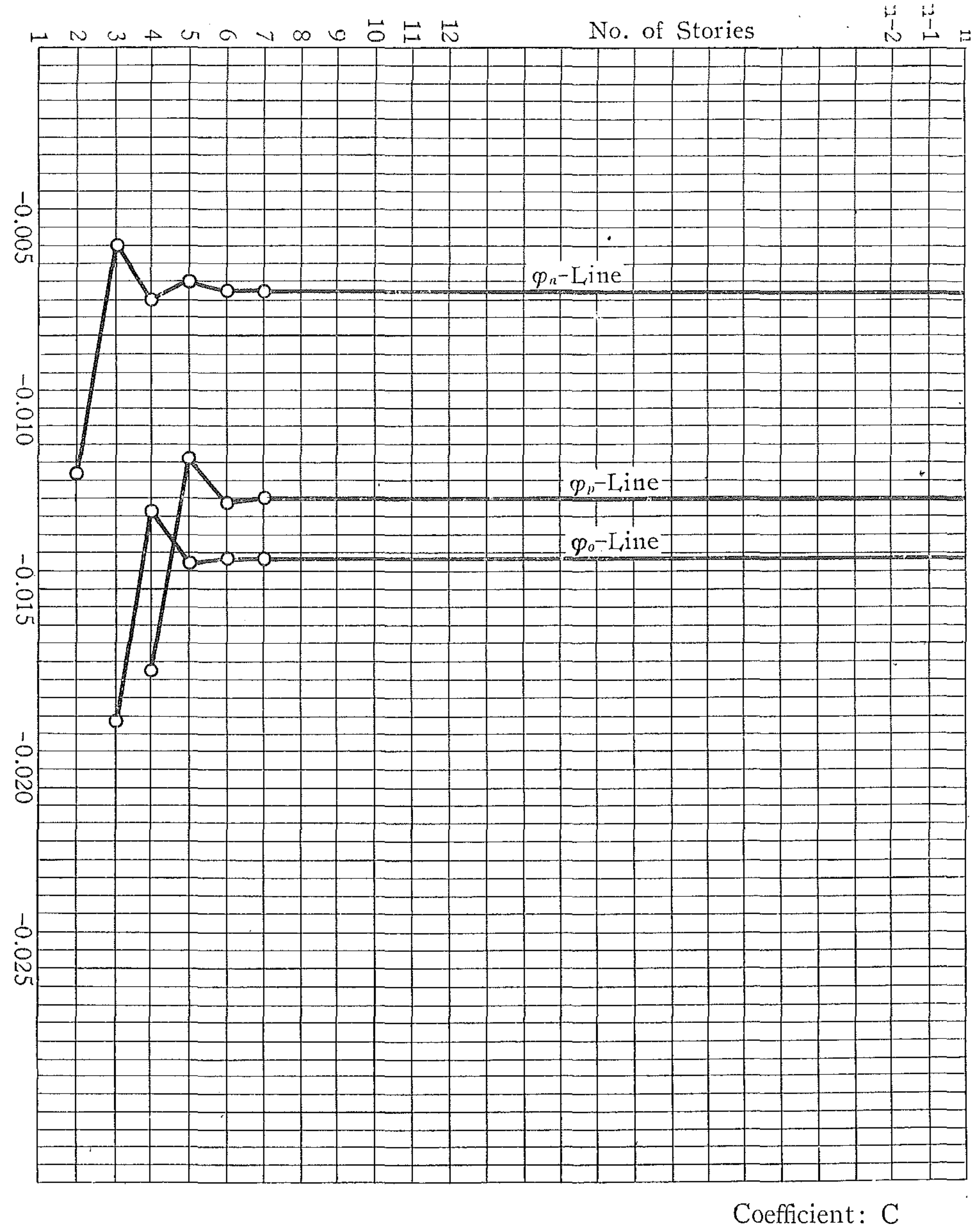
Table LV.



Coefficient: C

Multiple Storied Bent with Five Bays. Legs Fixed at Bases.
Loads Vertical and Symmetrical.

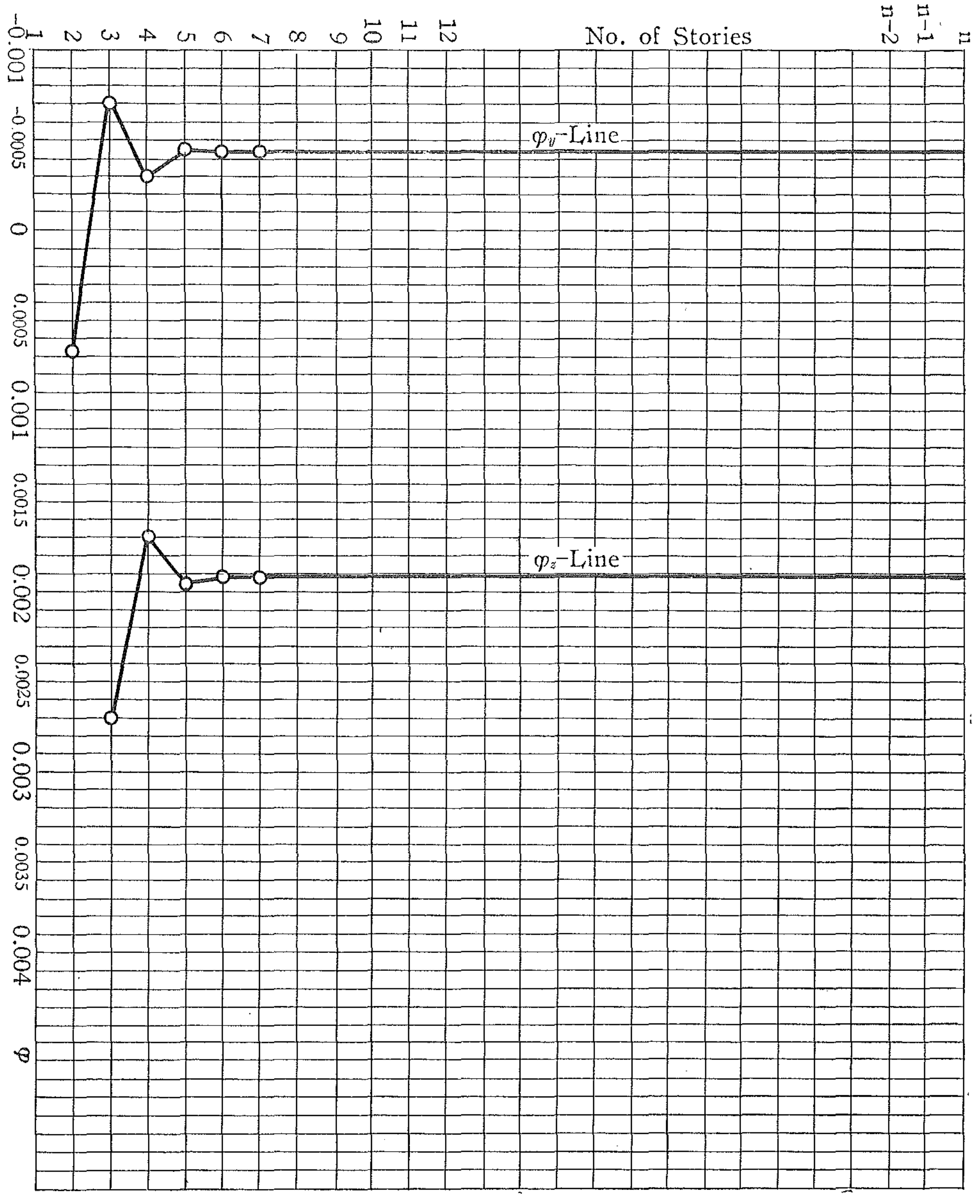
Table LVI.



Coefficient: C

Multiple Storied Bent with Five Bays. Legs Fixed at Bases.
Loads Vertical and Symmetrical.

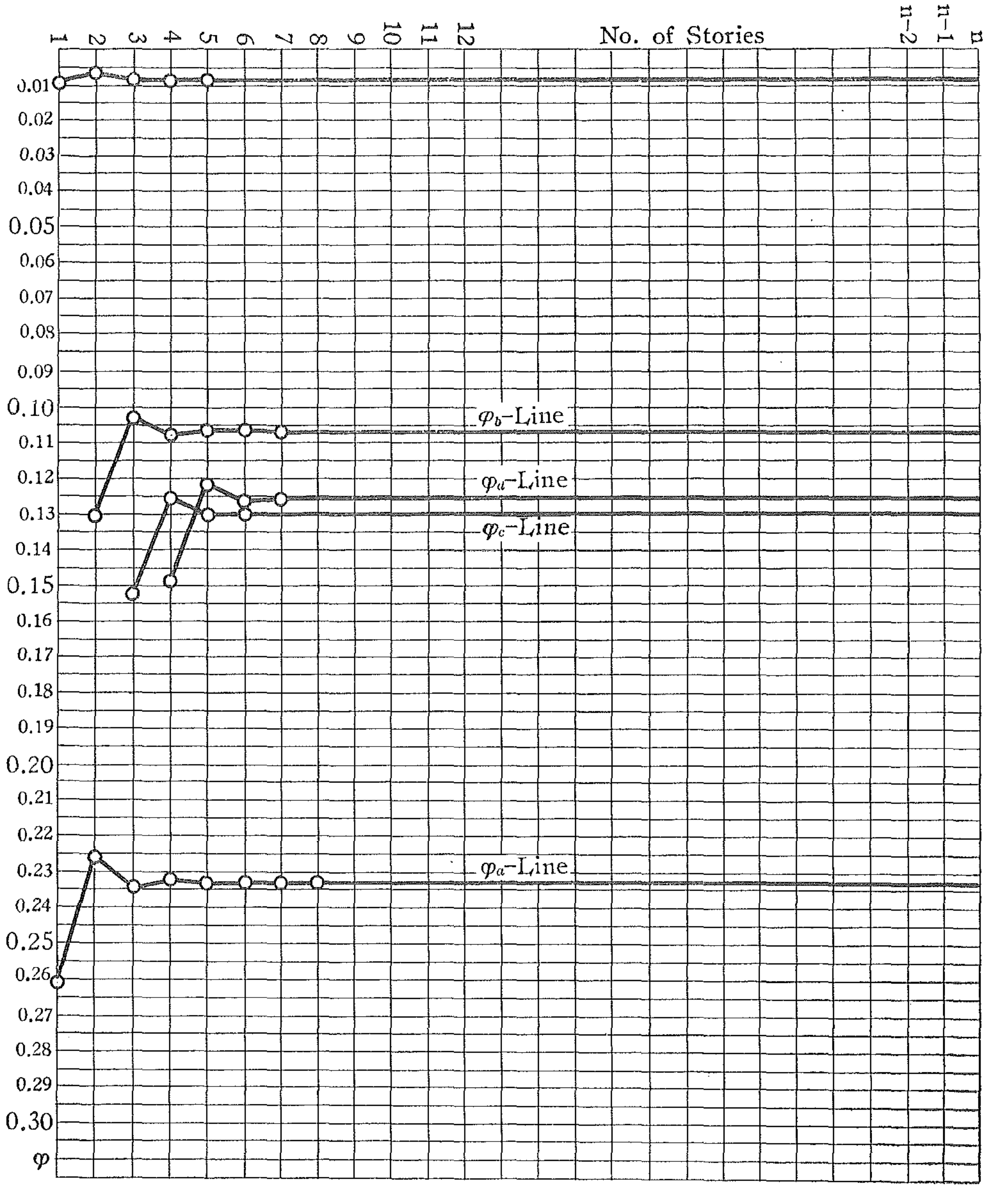
Table LVII.



Coefficient: C

Multiple Storied Bent with Six Bays. Legs Fixed at Bases.
Loads Vertical and Symmetrical.

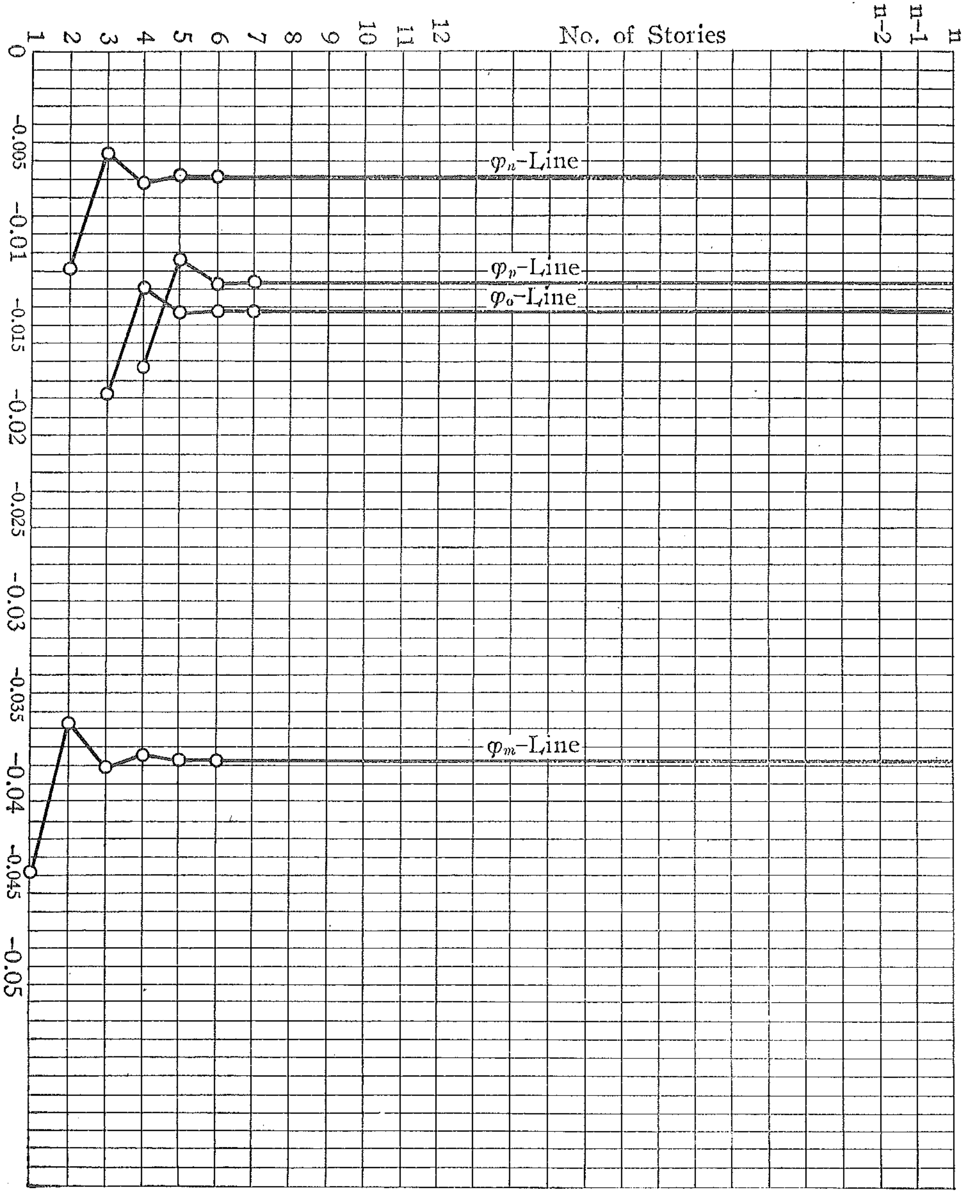
Table LVIII.



Coefficient: C

Multiple Storied Bent with Six Bays. Legs Fixed at Bases.
Loads Vertical and Symmetrical.

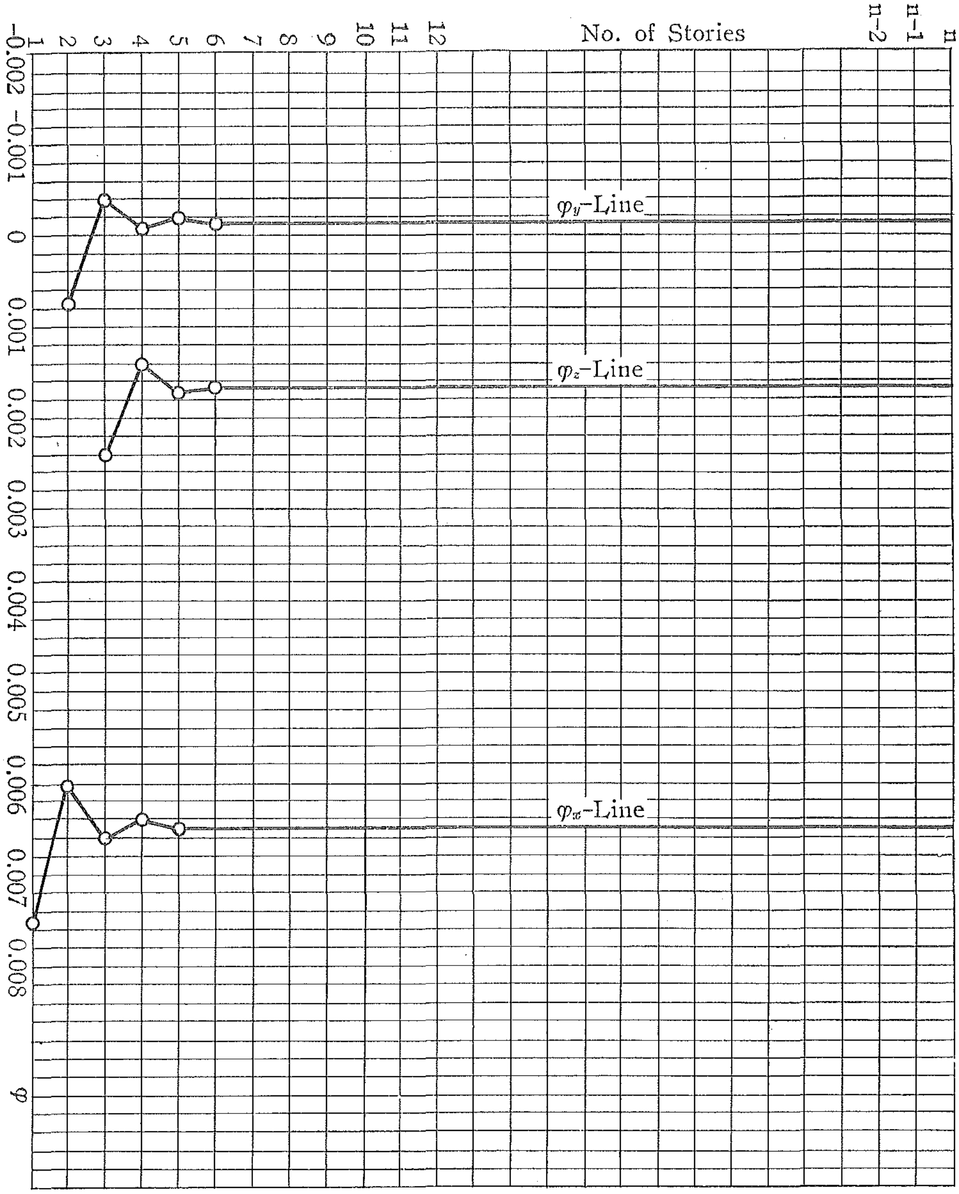
Table LIX.



Coefficient: C

Multiple Storied Bent with Six Bays. Legs Fixed at Bases.
Loads Vertical and Symmetrical.

Table LX.



Coefficient: C