A Speedy and Accurate Method of Wind Stress Calculation for High Storied Bents.

By

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Introduction.

With the increase in the number of stories of modern tall building frames with stiff connecting joints composed of rectangular elements, the proper treatment of wind stress analysis has been given considerable attention in recent years. Many authorities have presented various methods of solution, from the standpoint both of accuracy and of rapidity.

The analysis of these stresses, when viewed from a theoretical standpoint, is so complicated that for many years there have been difficulties in the way of solving unknown factors in the problems of tall built structures. In order to overcome these difficulties it seems necessary to develop some practical and workable method simple in theoretical analysis, which at the same time will provide a means of calculation, that both lessens the mathematical difficulties and shortens the tedious labour of detailed computation.

The new equations obtained from some hundred problems for a series of similar bents which have been solved by aid of the Mechanical Tabulation Method* (whose accuracy is quite the same as the Slope Deflection Method) give quickly the bending moments at any joint for a tall building frame subjected to horizontal joint loads.

Frames are arbitrary for number of stories and bays. For example, for a frame 150 stories high and 5 bays long, all the moments at the upper ends of the columns, which are on the outward side of the structure are determined from the following equation :

$$M = (0.055\,n - 0.016)\,W.\,h,$$

* See Proceedings, American Society of Civil Engineers, Oct., 1934. Vol. 60, No. 8, Part, 2, P. 650, Footnote and Takabeya " Rahmentafeln ", Julius Springer, Berlin, 1930.

where n indicates the number of the story which the required moment belongs to and n is to be numbered from the roof side. W shows intensity of the horizontal joint load and h height of column:

If $n = 100$, then we have from the above equation:

$$M = (0.055 \times 100 - 0.016)\, W.h = 5.48\, W.h$$

In general, all the joint moments for columns and girders are calculated from the following equation:

$$M = (a.n - b)\, W.h$$

Description of the coefficients a and b are given below in the present paper.

The bending moments calculated from the Moment Linear Equations in a similar expression to that given above are accurate enough to furnish the data for designing a tall building frame in theoretical proportion.

The Moment Linear Equations are therefore to contribute to the preliminary design of a tall building frame with an enormous majority of statically indeterminate quantities.

To solve such problems of many redundancies, it is in general theoretically necessary to know the sectional dimensions of all the members for the structures which have not yet been designed. Accordingly the sectional dimensioning may be forced to be assumed for the preliminary design.

In that very sense, the proposed equations have the merit of being free from the knowledge of sectional dimensions and the method makes the problem, to a certain extent, to be statically determinate.

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* F. Takabeya: A New Method of Wind Stress Calculation in High Building Frames, The Memoirs of the Faculty of Engineering, The Hokkaido Imperial University, Vol. 3, No. 2., 1932.

F. Takabeya: Nouvelle Methode de calcul des ossatures a grand nombre d'étages sous l'action du vent et des sollicitations sismiques, "L'OSSATURE METALLIQUE", No. 5, 1935, Bruxelles.

A. Habel: Winddruck und Stockwerkrahmenbau, HDI-Mitteilungen des Hauptvereines deutscher Ingenieure in der Tschechoslowakischen Republik, Jahrgang 1934 (Heft 13/14).

A. Habel: Berechnung der waagerechten Grundschwingungen von Stockwerkrahmenbauten nach der Energiemethode, "Der Bauingenieur" 1935. Heft 49/50, Springer, Berlin.

What degree of accuracy may be acquired from the speedy calculation, when compared with the results obtained from other exact method of solution is of very great importance.

As a numerical example, the Habel Bent is important, which is 25 stories 100 m high and 5 bays 30 m long, solved by Professor Habel, applying the old equations which were reported by the present author in 1932.* This example is described below in the present paper.

For the final design, or for an exact calculation of a tall building frame, the author proposes here the Mechanical Tabulation Method and the Slope Distribution Method, both of which give the same accuracy as the Slope Deflection Method does. As the numerical example, the Wilson-Maney Bent was very important for comparison.

For the verification of the exactness of the proposed method, the following works have been used. Winddruck und Stockwerkrahmenbau von Prof. Dr. A. Habel, HDI Mitteilung des Hauptvereines deutscher Ingenieure in der Tschechoslowakischen Republik, Jahrgang 1934 (Heft 13/14) and Wind Stresses in the Steel Frames of Office Buildings, University of Illinois Bulletin No. 80, 1915.

The author of the present paper gratefully acknowledges indebtedness to the authors of those Bulletins.

I. Moment Formulae for Preliminary Design.

About ten years ago, the author tried to investigate the stress characteristics of tall building frames subjected to wind pressure.

For that purpose. frames of constant stiffness ($I : s =$ constant) and with columns of the same height were considered.

The author assumed wind load as concentrated horizontally at every joint on the left side of the structure, carrying equal intensity of wind pressure.

After having solved an enormous number of problems for a series of similar frames, he unexpectedly found some convenient and practically important characters of the moment variation in the vertical direction of the structure as a whole. Namely, when the problems have been solved by exact calculation all the joint moments have been found to be distributed linearly in the direction of the story, in other words, the increase of the amount of joint moments, from the roof to the basement, shows a straight-line variation.

This peculiarity simplifies all the solutions of problems of the same kind and it plays a very important rôle. In a similar manner, tall wind frames designed in theoretical proportion, instead of constant

K, as shown in Fig. 1, show the similar characteristics of straight line variation. The bending moment at any joint of such frames may be calculated thereby easily and directly from the equation:

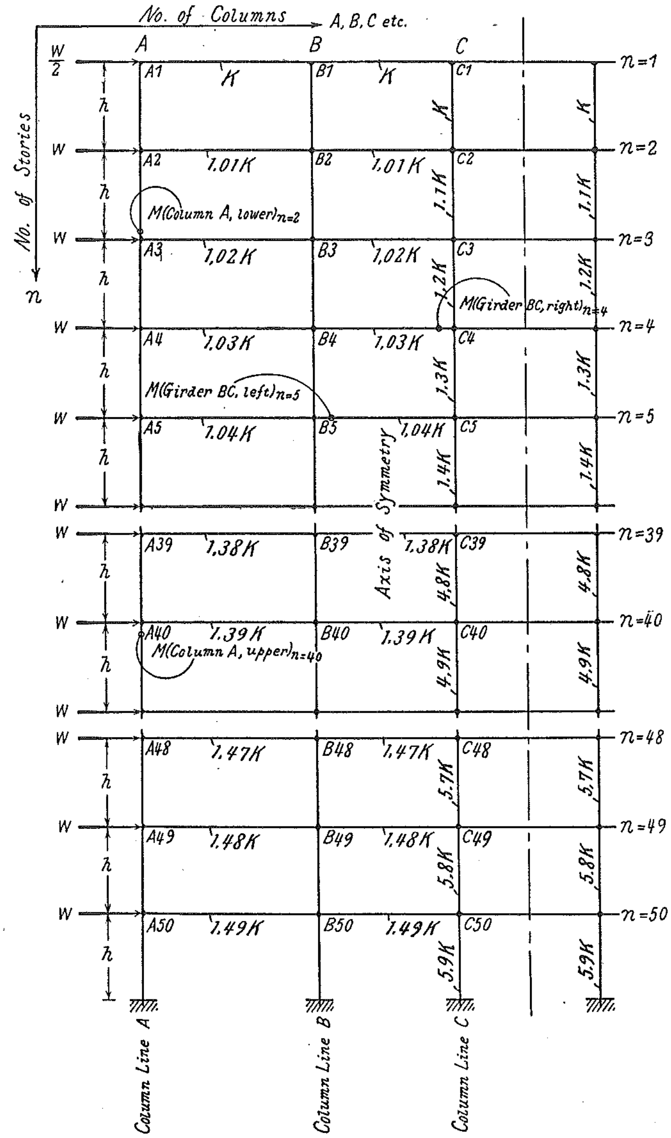

Fig. 1.

$M = (an - b)\, W.h$, in which M expresses the required joint moment, "a" and "b", as mentioned before, two constants which are to be given by the present investigation. Each constant has a proper numerical value depending on the columns and girders, especially on the number of bays, but independent of the number of stories; h and W are respectively the height of columns and the joint load assumed horizontally and concentrated at each joint on the left side of the frame.

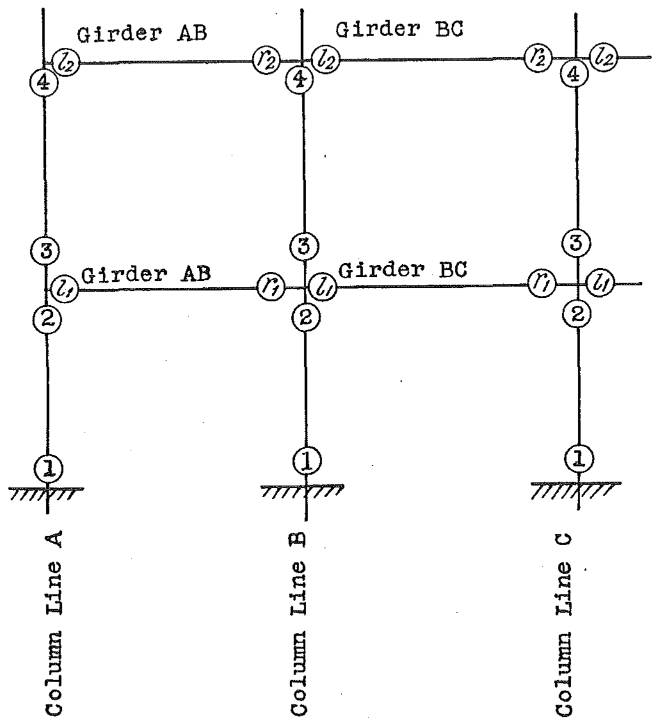

Fig. 2.

For example, with respect to the symmetrical frame of five bays, fifty stories high, each joint moment in the upper end of the left, outside column, for each story, is expressed by the following equation:

$$M = (0.055\,\mathrm{n} - 0.016)\ W.\,h$$

If $n = 20$, then

$$M = (0.055 \times 20 - 0.016)\ W.\,h = 1.08\ W.\,h$$

(From Slope-Deflection Calculation 1.09 $W.\,h$)

If $n=30$, then

$$M=(0.055\times 30-0.016)\,W.\,h=1.63\,W.\,h$$

(From Slope-Deflection Calculation $1.62\,W.\,h$)

Similar characteristics of linear variation of moment exist for all the other columns and girders.

In order to differentiate one joint moment from another one, all the joint moments which occur at the upper end of the column in Column Line A (See Fig. 1) are denoted by M(Column A, upper) and likewise M(Column A, lower), M(Column B, upper), etc. (see Fig. 1).

Similarly the joint moments at the left and right ends of the girders in Girder Line AB are denoted by M(Girder AB, left) and M(Girder AB, right), respectively.

Thus, for columns:

$$M(\text{column A, upper})=(an-b)\,W.\,h,$$

$$M(\text{column A, lower})=(an-b)\,W.\,h,$$

$$M(\text{column B, upper})=(an-b)\,W.\,h,$$

$$M(\text{column B, lower})=(an-b)\,W.\,h,$$

etc.

For girders:

$$M(\text{girder AB, left})=(an-b)\,W.\,h,$$

$$M(\text{girder AB, right})=(an-b)\,W.\,h,$$

$$M(\text{girder BC, left})=(an-b)\,W.\,h,$$

$$M(\text{girder BC, right})=(an-b)\,W.\,h$$

etc.

The values of coefficient "a" and "b" in the above equations, concerned with a symmetrical five bay frame, an arbitrary number of stories high, are tabulated in Table 1.

Table 1.

(Frame with 5 bays, and any number of stories)

Coefficient		a	b	Coefficient		a	b
Column	A upper	0.055	0.016	Girder	AB left	0.106	0.0685
	A lower	0.054	0.0435		AB right	0.10	0.0669
	B upper	0.10	0.0378		BC left	0.096	0.0676
	B lower	0.10	0.0596		BC right	0.0965	0.0673
	C upper	0.096	0.0365		CD left	0.0966	0.0666
	C lower	0.096	0.0685				

For the local effect of base fixation, the above formulae are not applicable near the basement and the moments in the lowest two stories near the basement are directly located from Tables A to L, and these moments are denoted by M(column A, (1)), M(column A, (2)) etc, and by M(girder AB (l_1)), M(girder AB (r_1)) etc. with suffix number (1), (2), (l_1) and (r_1), respectively, as shown in Fig. 2.

The similar local effect near the roof, depending on the abrupt change of the boundary condition, gives another form of these equations. But practically the values of the joint moments near roof have very little importance and the correction of those has been disregarded in this paper.

Table A. All column moments in negative directions
All girder moments in positive directions

Frames of Single Bay — Coeff. : W.h

Column and Girder Moments	No. of Stories 10	20	30	40	50
Column A (1)	3.406	7.775	12.654	17.896	23.411
Column A (2)	1.343	1.976	2.098	1.852	1.341
Column A (3)	2.212	5.167	8.382	11.800	15.379
Column A (4)	2.039	4.082	5.870	7.451	8.869
Girder AB (l_1)	3.552	7.140	10.475	13.654	16.721
Girder AB (l_2)	3.830	8.451	12.932	17.307	21.592

Table B. All column moments in negative directions
All girder moments in positive directions

Frames of 2 Bays Coeff: W.h

Column and Girder Moments	No. of Stories				
	10	20	30	40	50
Column A (1)	2.038	4.600	7.447	10.498	13.699
Column A (2)	0.800	1.222	1.378	1.377	1.164
Column A (3)	1.143	2.663	4.315	6.081	7.927
Colomn A (4)	1.094	2.192	3.171	4.050	4.868
Column B (1)	2.367	5.278	8.457	11.828	15.336
Column B (2)	1.457	2.578	3.399	3.998	4.439
Column B (3)	2.051	4.630	7.313	10.086	12.909
Column B (4)	1.977	4.163	6.215	8.152	10.002
Girder AB (l_1)	1.943	3.880	5.688	7.418	9.083
Girder AB (r_1)	1.754	3.602	5.354	7.041	8.670
Girder AB (l_2)	2.028	4.442	6.784	9.089	11.325
Girder AB (r_2)	1.856	4.165	6.437	8.686	10.876

Table C. All column moments in negative directions
All girder moments in positive directions

Frames of 3 Bays Coeff: W.h

Column and Girder Moments	No. of Stories				
	10	20	30	40	50
Column A (1)	1.463	3.287	5.305	7.468	9.735
Column A (2)	0.593	0.919	1.064	1.0751	0.994
Column A (3)	0.784	1.812	2.924	4.112	5.357
Column A (4)	0.764	1.524	2.210	2.830	3.408
Column B (1)	1.676	3.733	5.976	8.356	10.832
Column B (2)	1.017	1.812	2.406	2.851	3.189
Column B (3)	1.371	3.102	4.905	6.770	8.676
Column B (4)	1.333	2.811	4.211	5.538	6.810
Girder AB (l_1)	1.377	2.731	3.987	5.189	6.345
Girder AB (r_1)	1.255	2.548	3.765	4.937	6.068
Girder AB (l_2)	1.411	3.074	4.670	6.243	7.774
Girder AB (r_2)	1.301	2.857	4.443	5.978	7.477
Girder BC (l_1)	1.133	2.365	3.543	4.685	5.791
Girder BC (l_2)	1.191	2.713	4.216	5.714	7.181

Table D. All column moments in negative directions
All girder moments in positive directions

Column and Girder Moments	Frames of 4 Bays — No. of Stories				Coeff.: W.h
	10	20	30	40	50
Column A (1)	1.142	2.558	4.124	5.797	7.560
Column A (2)	0.469	0.734	0.864	0.891	0.834
Column A (3)	0.595	1.368	2.207	3.100	4.037
Column A (4)	0.586	1.167	1.693	2.173	2.622
Column B (1)	1.307	2.904	4.642	6.482	8.410
Column B (2)	0.800	1.426	1.900	2.260	2.534
Column B (3)	1.049	2.364	3.730	5.140	6.595
Column B (4)	1.023	2.156	3.224	4.240	5.232
Column C (1)	1.292	2.880	4.612	6.447	8.371
Column C (2)	0.770	1.378	1.840	2.191	2.456
Column C (3)	1.009	2.299	3.648	5.045	6.455
Column C (4)	0.987	2.096	3.147	4.151	5.066
Girder AB (l_1)	1.063	2.104	3.064	3.981	4.881
Girder AB (r_1)	0.968	1.962	2.892	3.787	4.667
Girder AB (l_2)	1.079	2.343	3.557	4.744	5.919
Girder AB (r_2)	0.995	2.205	3.384	4.544	5.692
Girder BC (l_1)	0.881	1.830	2.731	3.603	4.462
Girder BC (r_1)	0.890	1.839	2.741	3.612	4.472
Girder BC (l_2)	0.917	2.075	3.219	4.351	5.482
Girder BC (r_2)	0.924	2.083	3.227	4.359	5.498

Table E. All column moments in negative directions
All girder moments in positive directions

Column and Girder Moments	Frames of 5 Bays — No. of Stories				Coeff.: W.h
	10	20	30	40	50
Column A (1)	0.936	2.094	3.372	4.744	6.180
Column A (2)	0.389	0.612	0.725	0.752	0.722
Column A (3)	0.478	1.098	1.770	2.485	3.236
Column A (4)	0.476	0.946	1.374	1.759	2.121
Column B (1)	1.071	2.375	3.794	5.302	6.8688
Column B (2)	0.659	1.174	1.568	1.867	2.100
Column B (3)	0.847	1.905	3.004	4.142	5.304
Column B (4)	0.829	1.744	2.610	3.434	4.224
Column C (1)	1.059	2.357	3.771	5.274	6.8397
Column C (2)	0.636	1.138	1.522	1.813	2.040
Column C (3)	0.818	1.856	2.941	4.067	5.220
Column C (4)	0.803	1.699	2.551	3.364	4.145
Girder AB (l_1)	0.864	1.709	2.488	3.239	3.961
Girder AB (r_1)	0.787	1.593	2.348	3.081	3.787
Girder AB (l_2)	0.870	1.890	2.867	3.837	4.779
Girder AB (r_2)	0.802	1.779	2.728	3.675	4.597
Girder BC (l_1)	0.716	1.485	2.216	2.931	3.620
Girder BC (r_1)	0.722	1.493	2.224	2.938	3.628
Girder BC (l_2)	0.739	1.674	2.596	3.519	4.422
Girder BC (r_2)	0.744	1.680	2.602	3.525	4.428
Girder CD (l_1)	0.729	1.500	2.231	2.946	3.636
Girder CD (l_2)	0.748	1.686	2.608	3.531	4.434

Table F. All column moments in negative directions
All girder moments in positive directions

Column and Girder Moments	Frames of 6 Rays — No. of Stories (Coeff.: W. h)				
	10	20	30	40	50
Column A (1)	0.793	1.772	2.854	4.015	5.226
Column A (2)	0.331	0.524	0.621	0.653	0.633
Column A (3)	0.399	0.918	1.476	2.072	2.699
Column A (4)	0.398	0.795	1.153	1.479	1.786
Column B (1)	0.907	2.009	3.210	4.485	5.807
Column B (2)	0.559	0.939	1.333	1.593	1.794
Column B (3)	0.711	1.597	2.516	3.465	4.437
Column B (4)	0.697	1.465	2.192	2.884	3.551
Column C (1)	0.898	1.995	3.191	4.463	5.783
Column C (2)	0.541	0.970	1.297	1.549	1.746
Column C (3)	0.688	1.557	2.465	3.405	4.370
Column C (4)	0.676	1.429	2.144	2.829	3.486
Girder AB (l_1)	0.730	1.439	2.097	2.728	3.334
Girder AB (r_1)	0.665	1.342	1.980	2.595	3.187
Girder AB (l_2)	0.733	1.586	2.408	3.216	4.003
Girder AB (r_2)	0.676	1.503	2.292	3.080	3.851
Girder BC (l_1)	0.604	1.250	1.868	2.468	3.047
Girder BC (r_1)	0.614	1.262	1.880	2.480	3.052
Girder BC (l_2)	0.622	1.405	2.180	2.949	3.704
Girder BC (r_2)	0.630	1.414	2.190	2.959	3.709

Table G. All column moments in negative directions
All girder moments in positive directions

Column and Girder Moments	Frames of 7 Bays — No. of Stories (Coeff.: W. h)				
	10	20	30	40	50
Column A (1)	0.689	1.537	2.475	3.478	4.528
Column A (2)	0.289	0.458	0.547	0.575	0.564
Column A (3)	0.344	0.788	1.267	1.778	2.314
Column A (4)	0.344	0.686	0.995	1.277	1.543
Column B (1)	0.787	1.743	2.782	3.884	5.030
Column B (2)	0.486	0.869	1.162	1.387	1.568
Column B (3)	0.613	1.374	2.164	2.980	3.814
Column B (4)	0.602	1.264	1.891	2.487	3.062
Column C (1)	0.779	1.729	2.765	3.864	5.007
Column C (2)	0.470	0.842	1.128	1.348	1.523
Column C (3)	0.591	1.338	2.118	2.927	3.752
Column C (4)	0.583	1.231	1.848	2.437	3.004
Girder AB (l_1)	0.631	1.244	1.811	2.355	2.877
Girder AB (r_1)	0.575	1.160	1.709	2.240	2.750
Girder AB (l_2)	0.631	1.366	2.072	2.768	3.443
Girder AB (r_2)	0.582	1.286	1.972	2.652	3.312
Girder BC (l_1)	0.523	1.081	1.613	2.130	2.629
Girder BC (r_1)	0.527	1.087	1.619	2.136	2.634
Girder BC (l_2)	0.536	1.210	1.876	2.539	3.186
Girder BC (r_2)	0.539	1.214	1.880	2.544	3.190
Girder CD (l_1)	0.532	1.092	1.624	2.141	2.640
Girder CD (l_2)	0.543	1.218	1.884	2.548	3.195

Table H. All column moments in negative directions
All girder moments in positive directions

Column and Girder Moments	Frames of 8 Bays				Coeff.: W.h
	No. of Stories				
	10	20	30	40	50
Column A (1)	0.608	1.353	2.183	3.067	3.997
Column A (2)	0.256	0.396	0.487	0.517	0.505
Column A (3)	0.300	0.680	1.110	1.557	2.026
Column A (4)	0.312	0.600	0.873	1.126	1.358
Column B (1)	0.695	1.540	2.454	3.424	4.439
Column B (2)	0.429	0.771	1.028	1.231	1.390
Column B (3)	0.537	1.207	1.879	2.612	3.345
Column B (4)	0.528	1.112	1.660	2.186	2.692
Column C (1)	0.688	1.529	2.440	3.408	4.420
Column C (2)	0.416	0.747	1.000	1.198	1.351
Column C (3)	0.520	1.176	1.860	2.566	3.293
Column C (4)	0.512	1.084	1.624	2.143	2.644
Girder AB (l_1)	0.556	1.101	1.594	2.069	2.534
Girder AB (r_1)	0.506	1.024	1.504	1.968	2.422
Girder AB (l_2)	0.555	1.199	1.821	2.424	3.024
Girder AB (r_2)	0.512	1.130	1.733	2.322	2.910
Girder BC (l_1)	0.460	0.952	1.419	1.871	2.316
Girder BC (r_1)	0.468	0.962	1.428	1.876	2.321
Girder BC (l_2)	0.471	1.063	1.649	2.224	2.799
Girder BÇ (r_2)	0.477	1.070	1.656	2.232	2.803

Table I. All column moments in negative directions
All girder moments in positive directions

Column and Girder Moments	Frames of 9 Bays				Coeff.: W.h
	No. of Stories				
	10	20	30	40	50
Column A (1)	0.545	1.215	1.954	2.745	3.575
Column A (2)	0.230	0.365	0.439	0.467	0.460
Column A (3)	0.268	0.613	0.986	1.385	1.802
Column A (4)	0.270	0.536	0.780	1.005	1.212
Column B (1)	0.623	1.377	2.197	3.064	3.969
Column B (2)	0.385	0.688	0.923	1.105	1.248
Column B (3)	0.480	1.075	1.691	2.327	2.977
Column B (4)	0.473	0.991	1.481	1.951	2.401
Column C (1)	0.617	1.367	2.183	3.048	3.952
Column C (2)	0.372	0.668	0.897	1.074	1.214
Column C (3)	0.463	1.047	1.655	2.284	2.930
Column C (4)	0.457	0.965	1.449	1.911	2.356
Girder AB (l_1)	0.499	0.981	1.424	1.848	2.260
Girder AB (r_1)	0.454	0.914	1.344	1.757	2.161
Girder AB (l_2)	0.497	1.073	1.622	2.161	2.693
Girder AB (r_2)	0.458	1.010	1.544	2.070	2.591
Girder BC (l_1)	0.413	0.852	1.268	1.671	2.065
Girder BC (r_1)	0.417	0.856	1.272	1.676	2.070
Girder BC (l_2)	0.422	0.951	1.469	1.982	2.492
Girder BC (r_2)	0.425	0.954	1.472	1.986	2.496
Girder CD (l_1)	0.421	0.861	1.277	1.680	2.074
Girder CD (l_2)	0.428	0.957	1.475	1.989	2.499

Table J. All column moments in negative directions
All girder moments in positive directions

Frames of 10 Bays Coeff.: W.h

Column and Girder Moments	No. of Stories				
	10	20	30	40	50
Column A (1)	0.493	1.099	1.769	2.482	3.234
Column A (2)	0.208	0.332	0.397	0.426	0.420
Column A (3)	0.241	0.552	0.887	1.246	1.622
Column A (4)	0.243	0.484	0.703	0.908	1.093
Column B (1)	0.563	1.246	1.987	2.771	3.591
Column B (2)	0.349	0.624	0.835	1.003	1.133
Column B (3)	0.432	0.969	1.523	2.095	2.684
Column B (4)	0.425	0.893	1.337	1.760	2.166
Column C (1)	0.558	1.237	1.976	2.757	3.575
Column C (2)	0.338	0.606	0.812	0.976	1.102
Column C (3)	0.418	0.944	1.492	2.058	2.641
Column C (4)	0.413	0.871	1.308	1.726	2.126
Girder AB (l_1)	0.450	0.885	1.288	1.668	2.042
Girder AB (r_1)	0.409	0.825	1.216	1.586	1.952
Girder AB (l_2)	0.448	0.967	1.465	1.946	2.430
Girder AB (r_2)	0.413	0.910	1.394	1.865	2.338
Girder BC (l_1)	0.372	0.768	1.147	1.508	1.866
Girder BC (r_1)	0.378	0.776	1.155	1.516	1.870
Girder BC (l_2)	0.380	0.856	1.326	1.786	2.249
Girder BC (r_2)	0.385	0.862	1.333	1.792	2.252

Table K. All column moments in negative directions
All girder moments in positive directions

Frames of 11 Bays Coeff.: W.h

Column and Girder Moments	No. of Stories				
	10	20	30	40	50
Column A (1)	0.451	1.004	1.616	2.268	2.953
Column A (2)	0.191	0.305	0.365	0 390	0.387
Column A (3)	0.220	0.503	0.807	1.131	1.474
Column A (4)	0.222	0.442	0.640	0.824	0.995
Column B (1)	0 515	1.137	1.815	2.532	3.279
Column B (2)	0.319	0.571	0.764	0.917	1.038
Column B (3)	0.395	0.882	1.388	1.909	2.443
Column B (4)	0.389	0.815	1.218	1.604	1.973
Column C (1)	0.510	1.129	1.805	2.519	3.265
Column C (2)	0.308	0.554	0.743	0.891	1.010
Column C (3)	0.381	0.859	1.358	1.874	2.404
Column C (4)	0.376	0.794	1.191	1.572	1.937
Girder AB (l_1)	0.411	0.806	1.175	1.524	1.862
Girder AB (r_1)	0.374	0.752	1.109	1.449	1.780
Girder AB (l_2)	0.409	0.879	1.335	1.777	2.214
Girder AB (r_2)	0.377	0.828	1.271	1.702	2.130
Girder BC (l_1)	0.341	0.701	1.047	1.378	1.701
Girder BC (r_1)	0.344	0.704	1.050	1.382	1.705
Girder BC (l_2)	0.348	0.779	1.209	1.630	2.049
Girder BC (r_2)	0.350	0.782	1.212	1.633	2.052
Girder CD (l_1)	0.347	0.708	1.054	1.385	1.708
Girder CD (l_2)	0.352	0.785	1.215	1.636	2.054

Table L. All column moments in negative directions
All girder moments in positive directions

Frames of 12 Bays					Coeff.: W.h
Column and Girder Moments	No. of Stories				
	10	20	30	40	50
Column A (1)	0.415	0.924	1.485	2.087	2.716
Column A (2)	0.176	0.280	0.339	0.360	0.359
Column A (3)	0.201	0.461	0.740	1.038	1.353
Column A (4)	0.203	0.405	0.590	0.756	0.914
Column B (1)	0.474	1.047	1.668	2.329	3.015
Column B (2)	0.294	0.526	0.705	0.843	0.956
Column B (3)	0.361	0.810	1.272	1.751	2.242
Column B (4)	0.356	0.748	1.119	1.472	1.811
Column C (1)	0.469	1.039	1.659	2.318	3.003
Column C (2)	0.285	0.510	0.686	0.822	0.932
Column C (3)	0.349	0.789	1.246	1.720	2.207
Column C (4)	0.346	0.729	1.094	1.443	1.778
Girder AB (l_1)	0.377	0.742	1.077	1.401	1.710
Girder AB (r_1)	0.343	0.692	1.016	1.333	1.635
Girder AB (l_2)	0.373	0.809	1.221	1.633	2.033
Girder AB (r_2)	0.344	0.762	1.162	1.564	1.956
Girder BC (l_1)	0.312	0.645	0.959	1.267	1.563
Girder BC (r_1)	0.317	0.651	0.965	1.274	1.566
Girder BC (l_2)	0.317	0.717	1.105	1.498	1.881
Girder BC (r_2)	0.321	0.722	1.111	1.503	1.884

(1) *Moment Linear Equations and Effect of Base Fixation.*

For practical purposes, the following equations may be convenient so as to make it possible to locate the desired joint moments.

Frames of Single Bay. Coeff.: W. h

Column Moments: (All negative directions)

$$M(\text{column A, upper}) = 0.2548\,n - 0.0733$$

$$M(\text{column A, lower}) = 0.2451\,n - 0.1777$$

Girder Moments: (All positive directions)

$$M(\text{girder AB, left}) = 0.498\,n - 0.316$$

Frames of Two Bays. Coeff. : W. h

Column Moments : (All negative directions)

M(column A, upper) $= 0.130\,n - 0.037$
M(column A, lower) $= 0.128\,n - 0.101$
M(column B, upper) $= 0.248\,n - 0.093$
M(column B, lower) $= 0.242\,n - 0.137$

Girder moments : (All positive directions)

M(girder AB, left) $= 0.255\,n - 0.162$
M(girder AB, right) $= 0.244\,n - 0.166$

Frames of Three Bays. Coeff. : W. h

Column Moments : (All negative directions)

M(column A, upper) $= 0.091\,n - 0.027$
M(column A, lower) $= 0.0876\,n - 0.0703$
M(column B, upper) $= 0.1652\,n - 0.0632$
M(column B, lower) $= 0.1652\,n - 0.0991$

Girder Moments : (All positive directions)

M(girder AB, left) $= 0.171\,n - 0.1067$
M(girder AB, right) $= 0.1655\,n - 0.1101$
M(girder BC, left) $= 0.162\,n - 0.1155$

Frames of Four Bays. Coeff. : W. h

Column Moments : (All negative directions)

M(column A, upper) $= 0.069\,n - 0.0205$
M(column A, lower) $= 0.066\,n - 0.053$
M(column B, upper) $= 0.127\,n - 0.050$
M(column B, lower) $= 0.125\,n - 0.074$
M(column C, upper) $= 0.122\,n - 0.049$
M(column C, lower) $= 0.123\,n - 0.076$

Girder Moments : (All positive directions)

M(girder AB, left) $= 0.131\,n - 0.083$
M(girder AB, right) $= 0.127\,n - 0.086$
M(girder BC, left) $= 0.122\,n - 0.086$
M(girder BC, right) $= 0.122\,n - 0.085$

Frames of Five Bays. Coeff. : W. h

Column moments : (All negative directions)

M(column A, upper) = 0.055 n—0.016

M(column A, lower) = 0.053 n—0.043

M(column B, upper) = 0.103 n—0.0388

M(column B, lower) = 0.101 n—0.0593

M(column C, upper) = 0.102 n—0.041

M(column C, lower) = 0.101 n—0.06

Girder Moments : (All positive directions)

M(girder AB, left) = 0.105 n—0.0664

M(girder AB, right) = 0.10 n—0.068

M(girder BC, left) = 0.097 n—0.0686

M(girder BC, right) = 0.0966 n—0.0674

M(girder CD, left) = 0.0968 n—0.0668

Frames of Six Bays. Coeff. : W. h

Column Moments : (All negative directions)

M(column A, upper) = 0.046 n—0.013

M(column A, lower) = 0.044 n—0.036

M(column B, upper) = 0.084 n—0.032

M(column B, lower) = 0.084 n—0.05

M(column C, upper) = 0.083 n—0.033

M(column C; lower) = 0.082 n—0.050

M(column D, upper) = M(column C, upper)

M(column D, lower) = M(column C, lower)

Girder Moments : (All positive directions)

M(girder AB, left) = 0.088 n—0.055

M(girder AB, right) = 0.086 n—0.057

M(girder BC, left) = 0.081 n—0.057

M(girder BC, right) = 0.082 n—0.057

M(girder CD, left) = M(girder BC, left)

Frames of Seven Bays. Coeff. : W. h

Column Moments: (All negative directions)

M(column A, upper) $= 0.039\,n - 0.011$
M(column A, lower) $= 0.038\,n - 0.0309$
M(column B, upper) $= 0.072\,n - 0.0272$
M(column B, lower) $= 0.071\,n - 0.042$
M(column C, upper) $= 0.0707\,n - 0.0277$
M(column C, lower) $= 0.0701\,n - 0.0433$
M(column D, upper) $= M$(column C, upper)
M(column D, lower) $= M$(column D, lower)

Girder Moments: (All positive directions)

M(girder AB, left) $= 0.076\,n - 0.047$
M(girder AB, right) $= 0.073\,n - 0.049$
M(girder BC, left) $= 0.07\,n - 0.049$
M(girder BC, right) $= 0.071\,n - 0.048$
M(girder CD, left) $= M$(girder BC, left)
M(girder CD, right) $= M$(girder BC, right)

Frames of Eight Bays. Coeff. : W. h

Column Moments: (All negative directions)

M(column A, upper) $= 0.034\,n - 0.009$
M(column A, lower) $= 0.033\,n - 0.027$
M(column B, upper) $= 0.063\,n - 0.024$
M(column B, lower) $= 0.063\,n - 0.037$
M(column C, upper) $= 0.0625\,n - 0.0245$
M(column C, lower) $= 0.062\,n - 0.038$
M(column D, upper) $= M$(column C, upper)
M(column D, lower) $= M$(column C, lower)

Girder Moments: (All positive directions)

M(girder AB, left) $= 0.067\,n - 0.04$
M(girder AB, right) $= 0.064\,n - 0.043$
M(girder BC, left) $= 0.061\,n - 0.043$
M(girder BC, right) $= 0.061\,n - 0.042$
M(girder CD, left) $= M$(girder BC, left)
M(girder CD, right) $= M$(girder BC, right)

Frames of Nine Bays. Coeff. : W. h

Column Moments: (All negative directions)

M(column A, upper) = 0.030 n−0.008
M(column A, lower) = 0.029 n−0.0234
M(column B, upper) = 0.056 n−0.021
M(column B, lower) = 0.056 n−0.033
M(column C, upper) = 0.0556 n−0.022
M(column C, lower) = 0.055 n−0.034
M(column D, upper) = M(column C, upper)
M(column D, lower) = M(column C, lower)

Girder Moments: (All positive directions)

M(girder AB, left) = 0.059 n−0.036
M(girder AB, right) = 0.056 n−0.037
M(girder BC, left) = 0.055 n−0.0385
M(girder BC, right) = 0.055 n−0.0384
M(girder CD, left) = M(girder BC, left)
M(girder CD, right) = M(girder BC, right)

Frames of Ten Bays. Coeff. : W. h

Column Moments: (All negative directions)

M(column A, upper) = 0.0275 n−0.0075
M(column A, lower) = 0.0265 n−0.0215
M(column B, upper) = 0.051 n−0.019
M(column B, lower) = 0.051 n−0.030
M(column C, upper) = 0.0498 n−0.0193
M(column C, lower) = 0.0498 n−0.0307
M(column D, upper) = M(column C, upper)
M(column D, lower) = M(column C, lower)

Girder Moments: (All positive directions)

M(girder AB, left) = 0.0538 n−0.0328
M(girder AB, right) = 0.0515 n−0.034
M(girder BC, left) = 0.049 n−0.034
M(girder BC, right) = 0.0495 n−0.0345
M(girder CD, left) = M(girder BC, left)
M(girgdr CD, right) = M(girder BC, right)

Frames of Eleven Bays. Coeff. : W. h.

Column Moments : (All negative directions)

M(column A, upper) = 0.025 n—0.007
M(column A, lower) = 0.024 n—0.019
M(column B, upper) = 0.046 n—0.017
M(column B, lower) = 0.046 n—0.027
M(column C, upper) = 0.0455 n—0.0175
M(column C, lower) = 0.0455 n—0.0285
M(column D, upper) = M(column C, upper)
M(column D, lower) = M(column C, lower)

Girder Moments : (All positive directions)

M(girder AB, left) = 0.049 n—0.030
M(girder AB, right) = 0.047 n—0.032
M(girder BC, left) = 0.045 n—0.0315
M(girder BC, right) = 0.045 n—0.031
M(girder CD, left) = M(girder BC, left)
M(girder CD, right) = M(girder BC, right)

Frames of Twelve Bays. Coeff. : W. h

Column Moments : (All negative directions)

M(column A, upper) = 0.023 n—0.006
M(column A, lower) = 0.0225 n—0.0175
M(column B, upper) = 0.0425 n—0.0165
M(column B, lower) = 0.042 n—0.025
M(column C, upper) = 0.0417 n—0.0163
M(column C, lower) = 0.0414 n—0.0257
M(column D, upper) = M(column C, upper)
M(column D, lower) = M(column C, lower)

Girder Moments : (All positive directions)

M(girder AB, left) = 0.0448 n—0.028
M(girder AB, right) = 0.043 n—0.029
M(girder BC, left) = 0.0408 n—0.0288
M(girder BC, right) = 0.0409 n—0.0283
M(girder CD, left) = M(girder BC, left)
M(girder CD, right) = M(girder BC, right)

For the effect of base-fixation, Table *A* to Table *L* are applicable to the same practical purposes.

(2) *Exact Calculation Results Compared with the Results by Moment Linear Equations.*

The present author proposed[1] another formulae of Moment-Linear Equations obtained for skyscraper frames of constant ratio of stiffness from roof to basement, having equal height of columns in each story, subjected to wind pressures. Those formulae were compared with the calculation results for an actual building frame of reinforced concrete, 25 storied 100 meter high and 5 bays 30 meter long, (see Fig. 3) designed and reported in 1934 by A. Habel, professor of the Deutsche Technische Hochschüle, Brünn. To the unexpected delight of the present author, notwithstanding that he treated the frames as of constant ratio of stiffness from the roof to the basement, the Moment Linear Equations gave, in most values, a good coincidence, except near the roof and basement, to the exact calculation for the reinforced concrete frame of variable stiffness ratio as shown in Fig. 3. The base-fixation gave a large discrepancy near the basement, because at that time the author did not report the effect of the base-fixation.

Thenceforth the author treated the frame of variable stiffness ratio as shown in the frame of Fig. 1 and studied further the effect of base-fixation for the lowest two stories. Table I and Table II show the comparison between the bending moment values computed by a rapid calculation of Moment-Linear Equations and those by exact calculation (by Habel) of the Mechanical Tabulation Method whose fundamental equation is the same as the Slope Deflection Equation.

In the author's opinion, when the frame has the same height of column in every story, joint moments seem to be given approximately by the Moment Linear Equations with a quite good coincidence, but when each story has a different height of column the effect of the column height as well as the load intensity is not trifling, so its correction must be necessary. For these problems under different conditions, the calculation naturally can not be made in a fraction of time and some exact methods of calculation are necessary; the following two methods are applicable to that purposes:

(1) The Method of Slope Distribution and (2) The Mechanical Tabulation Method.*

(1) Cf. Mem. Fac. Engin., H. I. U., Vol. 3, No. 2, 1932.
* See Mem. Fac. Engin., H. I. U., Vol. 1, No. 3, 1927.

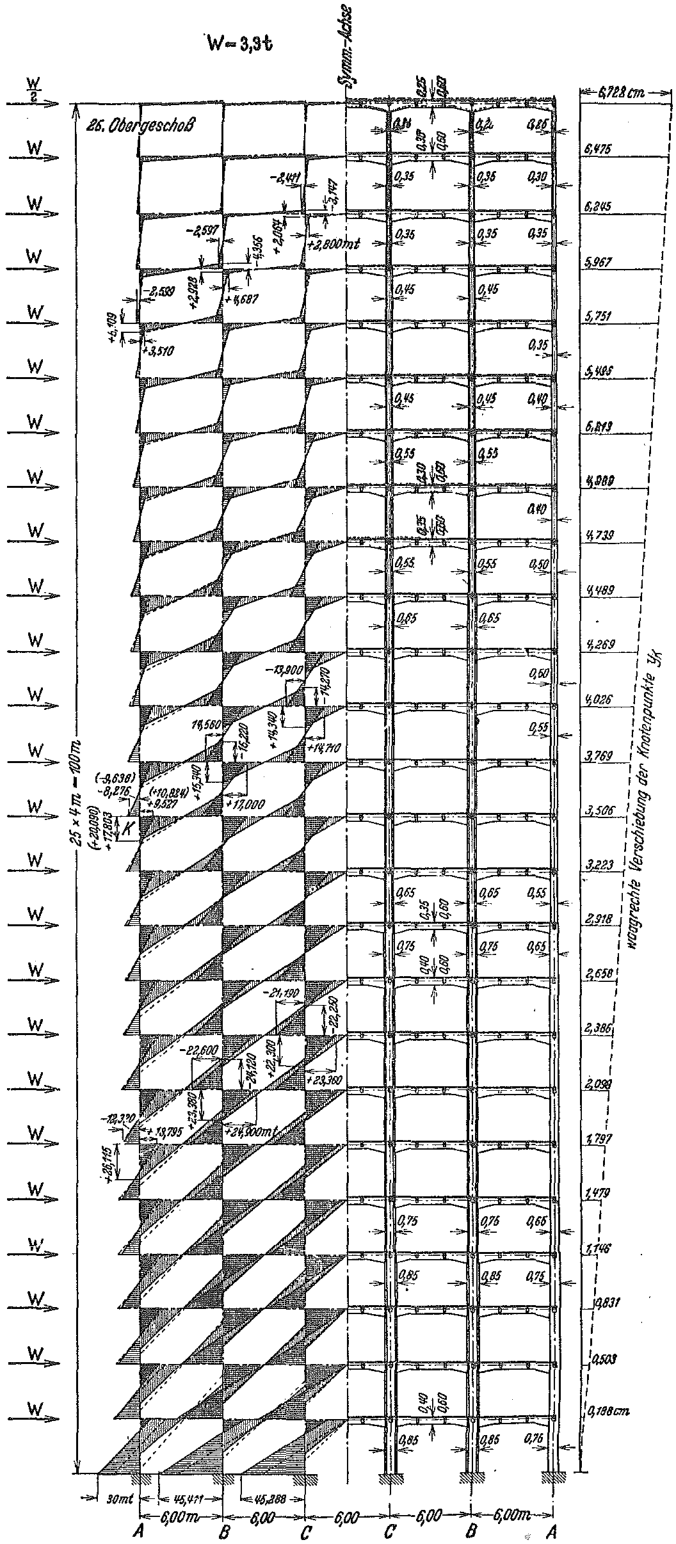

Fig. 3. (nach Habel)

Table I,

No. of Stories numbered from Roof	$M\left(\text{column A } \begin{matrix}\text{upper}\\ \text{lower}\end{matrix}\right)$ (m. t)		$M\left(\text{column B } \begin{matrix}\text{upper}\\ \text{lower}\end{matrix}\right)$ (m. t)		$M\left(\text{column C } \begin{matrix}\text{upper}\\ \text{lower}\end{matrix}\right)$ (m. t)	
	Moment Linear Equation	Exact Value	Moment Linear Equation	Exact Value	Moment Linear Equation	Exact Value
$n=1$	0.515 0.132	1.002 0.965	0.847 0.546	0.337 0.341	0.805 0.541	0.328 0.327
2	1.241 0.832	1.345 1.245	2.207 1.880	1.677 1.651	2.152 1.874	1.571 2.411
3	1.967 1.531	3.003 2.980	3.567 3.213	2.653 2.597	3.498 3.208	2.800 2.467
4	2.693 2.231	2.533 2.599	4.926 4.546	4.687 4.542	4.844 4.541	4.457 4.282
5	3.419 2.930	3.510 3.280	6.286 5.879	5.966 5.766	6.191 5.874	5.684 5.494
6	4.145 3.630	5.099 4.042	7.645 7.212	7.050 6.794	7.537 7.207	6.747 6.568
7	4.871 4.330	4.592 4 040	9.005 8.546	8.853 8.415	8.884 8.540	8.648 8.352
8	5.597 5.029	5.210 4.756	10.365 9.879	9.934 9.965	10.230 9.874	9.810 9.825
9	6.323 5.729	7.227 7.083	11.724 11.212	10.810 10.410	11.576 11.207	10.430 10.140
10	7.049 6.428	5.664 5.916	13.084 12.545	13.240 12.400	12.923 12.540	13.060 12.420
11	7.775 7.128	6.802 6.118	14.443 13.878	14.180 13.990	14.269 13.873	14.310 13.900
12	8.501 7.828	9.094 8.306	15.803 15.212	14.980 14.560	15.616 15.206	14.710 14.250
13	9.227 8.527	8.277 8.276	17.163 16.545	17.000 16.268	16.962 16.540	16.641 16.038
14	9.953 9.227	9.527 9.103	18.522 17.878	18.045 17.470	18.308 17.873	17.775 17.180
15	10.679 9.926	10.144 9.798	19.882 19.211	19.336 18.865	19.655 19.206	19.006 18.551
16	11.405 10.626	10.671 10.535	21.241 20.544	20.565 20.360	21.001 20.539	20.027 20.142
17	12.131 11.326	11.817 11.273	22.601 21.878	22.160 21.380	22.348 21.872	21.080 21.190

Table I—(*Continued*).

No. of Stories numbered from Roof	$M\left(\text{column A}\ {\text{upper} \atop \text{lower}}\right)$ (m. t)		$M\left(\text{column B}\ {\text{upper} \atop \text{lower}}\right)$ (m. t)		$M\left(\text{column C}\ {\text{upper} \atop \text{lower}}\right)$ (m. t)	
	Moment Linear Equation	Exact Value	Moment Linear Equation	Exact Value	Moment Linear Equation	Exact Value
18	12.857 12.025	12.288 12.020	23.961 23.211	23.361 22.600	23.694 23.206	23.360 21.871
19	13.583 12.725	13.270 12.320	25.320 24.544	24.900 23.965	25.040 24.539	24.045 23.600
20	14.309 13.424	13.795 13.080	26.680 25.877	26.085 25.210	26.387 25.872	25.690 24.840
21	15.035 14.124	14.733 14.059	28.039 27.210	27.260 26.458	27.733 27.205	26.980 25.810
22	15.761 14.824	15.170 14.280	29.399 28.544	28.670 27.850	29.080 28.538	28.740 27.190
23	16.487 15.523	15.860 15.540	30.759 29.877	29.678 29.248	30.426 29.872	29.317 28.857
24	15.312 18.929	14.920 18.100	28.736 32.399	29.140 32.365	28.050 31.660	28.700 31.875
25	8.824 36.076	8.068 30.000	18.097 40.715	16.585 45.411	17.556 40.445	16.348 45.288

Table II.

No. of Stories numbered from Roof	$M\left(\text{girder AB}\ {\text{left} \atop \text{right}}\right)$ (m. t)		$M\left(\text{girder BC}\ {\text{left} \atop \text{right}}\right)$ (m. t)		M (girder CD left) (m. t)	
	Moment Linear Equation	Exact Value	Moment Linear Equation	Exact Value	Moment Linear Equation	Exact Value
$n=1$	0.509 0.422	1.002 0.449	0.375 0.385	0.112 0.048	0.396	0.280
2	1.896 1.742	2.310 1.611	1.655 1.661	0.407 0.726	1.674	1.172
3	3.282 3.062	4.248 3.084	2.936 2.936	1.220 3.147	2.952	2.064
4	4.668 4.382	5.513 4.356	4.216 4.211	2.928 3.232	4.229	3.692
5	6.054 5.702	6.109 5.604	5.496 5.486	4.904 4.919	5.507	5.047

Table II—(*Continued*).

No. of Stories numbered from Roof	$M\left(\text{girder AB } \begin{smallmatrix}\text{left}\\ \text{right}\end{smallmatrix}\right)$ (m. t)		$M\left(\text{girder BC } \begin{smallmatrix}\text{left}\\ \text{right}\end{smallmatrix}\right)$ (m. t)		M (girder CD left) (m. t)	
	Moment Linear Equation	Exact Value	Moment Linear Equation	Exact Value	Moment Linear Equation	Exact Value
6	7.440 7.022	8.379 7.253	6.777 6.761	5.563 5.885	6.785	6.356
7	8.826 8.342	8.634 8.267	8.057 8.036	7.370 7.507	8.063	7.709
8	10.212 9.662	9.250 9.170	9.338 9.311	9.179 9.100	9.340	9.062
9	11.598 10.982	11.783 10.917	10.618 10.586	9.858 10.038	10.618	10.217
10	12.984 12.302	12.747 12.220	11.898 11.862	11.430 11.520	11.896	11.680
11	14.370 13.622	12.718 13.110	13.179 13.137	13.470 13.400	13.174	13.330
12	15.756 14.942	15.212 14.780	14.459 14.412	14.190 14.270	14.451	14.340
13	17.142 16.262	16.583 16.220	15.740 15.687	15.340 15.396	15.729	15.495
14	18.528 17.582	17.803 17.392	17.020 16.962	16.921 16.892	17.007	16.921
15	19.914 18.902	19.247 18.730	18.300 18.237	18.093 18.053	18.285	18.133
16	21.300 20.222	20.469 19.990	19.581 19.512	19.430 19.209	19.562	19.369
17	22.686 21.542	22.352 21.640	20.861 20.787	20.880 20.591	20.840	20.631
18	24.072 22.862	23.561 22.620	22.142 22.062	22.121 22.250	22.118	22.300
19	25.458 24.182	25.290 24.120	23.422 23.338	23.380 22.928	23.396	22.988
20	26.844 25.502	26.115 25.445	24.702 24.613	24.605 24.595	24.673	24.695
21	28.230 26.822	27.813 26.640	25.983 25.888	25.830 25.760	25.951	26.060
22	29.616 28.142	29.229 27.854	27.263 27.163	27.274 27.250	27.229	27.300
23	31.002 29.462	30.140 29.179	28.544 28.438	28.349 28.248	28.507	28.259
24	31.396 29.746	30.460 29.639	28.182 28.261	28.749 28.689	28.340	28.868
25	27.700 26.011	26.168 24.875	24.427 24.532	24.075 24.111	24.625	24.112

II. The Method of Slope Distribution.

In this paper the problem of building frames with no yielding at the foundation is treated. Therefore all the girders remain always horizontal, while the columns deflect due to the system of horizontal joint loads. The moment at the end A for the column Aa in Fig. 4 is expressed by the well known equation:

$$M_{Aa} = 2EK_{Aa}\left\{2\theta_A+\theta_a-3\frac{d_r}{h_r}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

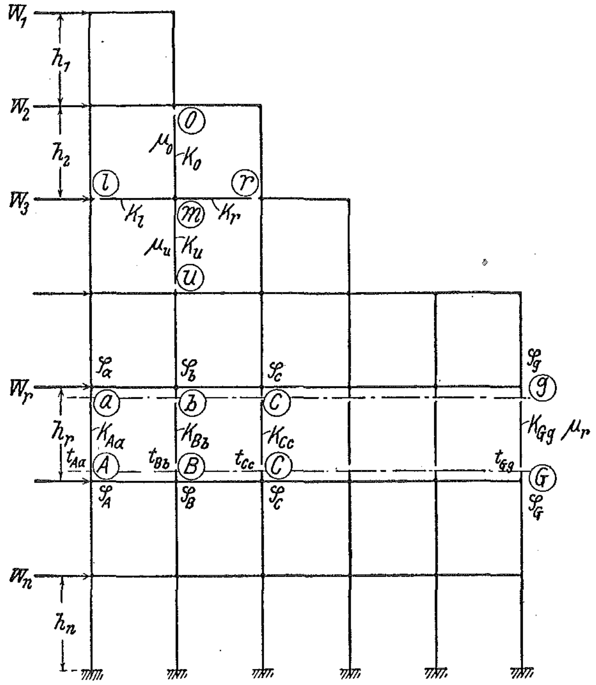

Fig. 4.

In this equation, M_{Aa} denotes the moment at the end A of the column Aa; θ_A and θ_a the joint-rotation angle (or slope) of the ends A and a respectively of the column; d_r the deflection (horizontal displacement) of one end of the column Aa relative to the other; h_r the length of the column Aa; E the modulus of elasticity of the material; K_{Aa} the moment of inertia of the section divided by the length of the column Aa.

The resisting moment on a section is considered positive when the couple acts in a clockwise direction upon the portion of the member considered; the joint-rotation angle θ is considered positive when the angle has been turned clockwise, measured from its initial position; the member-revolution angle (or deflection angle) d/h is also positive to the revolution in the clockwise direction from the initial position of the member.

For the analysis of high built frames the following equation is introduced instead of equation (1):

$$M_{Aa} = K_{Aa}\{2\varphi_A+\varphi_a-\mu_r\} \quad\dots\dots\dots\dots\dots\dots \quad (2)$$

In this equation

$$\left.\begin{aligned} \varphi_A &= 2E\theta_A \\ \varphi_a &= 2E\theta_a \\ \mu_r &= 6E\frac{d_r}{h_r} \end{aligned}\right\} \quad\dots\dots\dots\dots\dots\dots\dots\dots \quad (3)$$

For the purpose of obtaining M_{Aa}, it is more practical to consider φ and μ rather than θ and d/h as unknowns; in this paper φ and μ are called "slope" and "deflection" respectively, even if they are not reasonably defined in the strict meaning.

To obtain the equations from which the unknown slopes and deflections can be determined, we have two kinds of statical conditions of equilibrium: one of these is of equilibrium under the action of the moments at every point of intersection of the neutral axes of girders and columns, considering the point of intersection as a free body; the other condition is of equilibrium under the action of the shears and moments at the two extremities of all the columns in a story, considering together all of the columns in a story as a free body.

From the former condition of equilibrium, each joint gives one equation; from the latter each story gives one equation, i. e., for an n-storied bent with m-points of intersection of the neutarl axes of girders and columns, the number of equations obtained, from which the slopes and the deflections can be determined, is $m+n$.

The unknown quantities in the equations obtained are the changes in the slopes and the horizontal deflections of the columns. The changes in the slope at a joint, by the rigid connection, are equal for all of the members which meet at the joint; and the horizontal deflections of the columns in a story are equal in the case in question.

One obtains therefore $m+n$ equations in which $m+n$ unknowns are contained. Solving these equations simultaneously, in case of ordinary method, the unknowns are determined and then the moments required, while in the proposed method these equilibrium equations are not solved simultaneously but each equation determines one unknown, distributing unbalanced slopes and deflections by a slide-rule calculation on the figure of the bent as shown in Fig. 9.

(1) *Derivations of Equations of φ and μ.*

For the joint m shown in Fig. 4, the equilibrium condition of the total moment gives:

$$\left.\begin{array}{l} \sum M_m = 0\,, \\ \text{or} \\ M_{m0}+M_{mr}+M_{mu}+M_{ml} = 0 \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Applying equation (2) one obtains from equation (4):

$$\varphi_m \rho_m + (\varphi_0-\mu_0)K_0 + \varphi_r K_r + (\varphi_u-\mu_u)K_u + \varphi_l K_l = 0\,, \quad \ldots \quad (4)_0$$

where

$$\rho_m = 2(K_0+K_r+K_u+K_l)\,.$$

The above equation is called the "joint-equilibrium equation", which gives

$$\varphi_m = -\{(\varphi_0-\mu_0)\gamma_{0(m)} + \varphi_r\gamma_{r(m)} + (\varphi_u-\mu_u)\gamma_{u(m)} + \varphi_l\gamma_{l(m)}\} \quad \ldots. \quad (5)$$

where

$$\left.\begin{array}{l} \gamma_{0(m)} = \dfrac{K_0}{\rho_m}\,, \quad \gamma_{r(m)} = \dfrac{K_r}{\rho_m}\,, \\ \gamma_{u(m)} = \dfrac{K_u}{\rho_m}\,, \quad \gamma_{l(m)} = \dfrac{K_l}{\rho_m}\,, \quad \rho_m = 2(K_0+K_r+K_u+K_l) \end{array}\right\} \quad (6)$$

Equation (5) may be memorized easily by the following graphical representation:

$$\varphi_m = -\left\{\begin{array}{ccc} & (\varphi_0-\mu_0)\gamma_{0(m)} & \\ \varphi_l\gamma_{l(m)} & + & \varphi_r\gamma_{r(m)} \\ & (\varphi_u-\mu_u)\gamma_{u(m)} & \end{array}\right\} \quad \ldots\ldots\ldots\ldots \quad (7)$$

In order to obtain the equation of μ, one may consider two horizontal sections at both ends of the columns at the r-th story numbered from the roof side of the frame shown in Fig. 4.

The equilibrium condition of the r-th story as a whole gives:

$$\left.\begin{matrix} M_{aA}+M_{bB}+M_{cC}+\cdots+M_{gG} \\ M_{Aa}+M_{Bb}+M_{Cc}+\cdots+M_{Gg} \end{matrix}\right\}+h_r\sum_{s=1}^{r}W_s=0 \quad \ldots\ldots\ldots (8)$$

Applying equation (2) one obtains from equation (8):

$$3\{(\varphi_A+\varphi_a)K_{Aa}+(\varphi_B+\varphi_b)K_{Bb}+(\varphi_C+\varphi_c)K_{Cc}+\cdots+(\varphi_G+\varphi_g)K_{Gg}\}$$
$$-2\mu_r(K_{Aa}+K_{Bb}+K_{Cc}+\cdots+K_{Gg})+h_r\sum_{s=1}^{r}W_s=0 \quad \ldots\ldots (8)_0$$

The above equation is called the "story-equilibrium equation", which gives:

$$\mu_r=\frac{H_r}{T_r}+\{(\varphi_A+\varphi_a)t_{Aa}+(\varphi_B+\varphi_b)t_{Bb}+(\varphi_C+\varphi_c)t_{Cc}$$
$$+\cdots+(\varphi_G+\varphi_g)t_{Gg}\} \quad \ldots\ldots\ldots\ldots (9)$$

where

$$\left.\begin{aligned} H_r &= h_r(W_1+W_2+W_3+\cdots+W_r)\,. \\ T_r &= 2(K_{Aa}+K_{Bb}+\cdots+K_{Gg})\,, \\ t_{Aa} &= \frac{3K_{Aa}}{T_r}\,,\quad t_{Bb}=\frac{3K_{Bb}}{T_r}\,,\quad \cdots\, t_{Gg}=\frac{3K_{Gg}}{T_r}\,. \end{aligned}\right\} \ldots (10)$$

Equation (9) may be memorized easily by the following representation:

$$\mu_r=\frac{H_r}{T_r}+\left\{\begin{pmatrix}\varphi_a\\+\\\varphi_A\end{pmatrix}t_{Aa}+\begin{pmatrix}\varphi_b\\+\\\varphi_B\end{pmatrix}t_{Bb}+\cdots+\begin{pmatrix}\varphi_g\\+\\\varphi_G\end{pmatrix}t_{Gg}\right\} \ldots (11)$$

In equations (7) and (11), the values of γ, t and H/T are to be given as known values; therefore, in equation (7), if the true values of φ and μ are known, one can determine the true value of φ_m.

Even if one could not find the true values of φ and μ, if he could find their approximate values: $\varphi_0^{(0)}$, $\varphi_r^{(0)}$, $\varphi_u^{(0)}$, $\varphi_l^{(0)}$, $\mu_0^{(0)}$ and $\mu_u^{(0)}$, he can determine the approximate value $\varphi_m^{(1)}$, substituting $\varphi^{(0)}$ and $\mu^{(0)}$ for φ and μ in equation (7):

$$\varphi_m^{(1)} = -\left\{\begin{matrix} & (\varphi_0^{(0)}-\mu_0^{(0)})\gamma_{0(m)} & \\ \varphi_l^{(0)}\gamma_{l(m)} & + & \varphi_r^{(0)}\gamma_{r(m)} \\ & (\varphi_u^{(0)}-\mu_u^{(0)})\gamma_{u(m)} & \end{matrix}\right\}\cdots\cdots\cdots\cdots (12)$$

In the same way, in equation (11), if one could find the true values of φ_a, φ_A, φ_b, φ_B etc., he can determine the true value of μ_r.

Even if one could not find the true value of φ, if he could find the approximate value of φ, say $\varphi_a^{(1)}$, $\varphi_A^{(1)}$, $\varphi_b^{(1)}$, $\varphi_B^{(1)}$ etc., he can determine the approximate value $\mu_r^{(1)}$, substituting $\varphi^{(1)}$ for φ in equation (11):

$$\mu_r^{(1)} = \frac{H_r}{T_r}+\left\{\begin{pmatrix}\varphi_a^{(1)}\\+\\\varphi_A^{(1)}\end{pmatrix}t_{Aa}+\begin{pmatrix}\varphi_b^{(1)}\\+\\\varphi_B^{(1)}\end{pmatrix}t_{Bb}+\cdots\cdots+\begin{pmatrix}\varphi_g^{(1)}\\+\\\varphi_G^{(1)}\end{pmatrix}t_{Gg}\right\}\cdots (13)$$

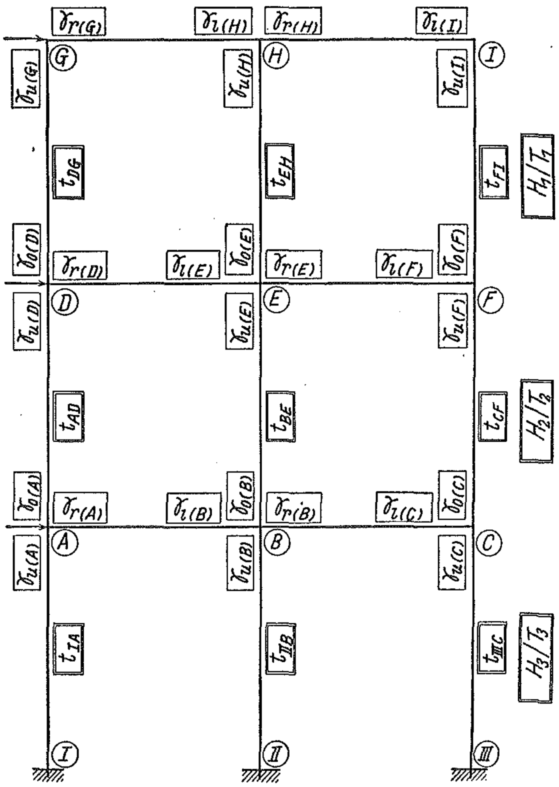

Fig. 5.

The calculation for equations (12) and (13) is worked out, in the present method, on the frame-schema, for example as shown in Figs. 5 and 9.

Fig. 5 shows the position of the values: γ, t, and H/T to be written and Fig. 9 shows the numerical example of a four storied symmetrical bent with three bays, carrying a load horizontal and concentrated at each joint on the left side of the structure. This example will be treated in the following section.

(2) *Derivation of the Approximate Values of μ and φ.*

For the cantilever beam shown in Fig. 6, carrying a load $\sum^{r} W$, the deflection $\varDelta$ under the load is given by the formula:

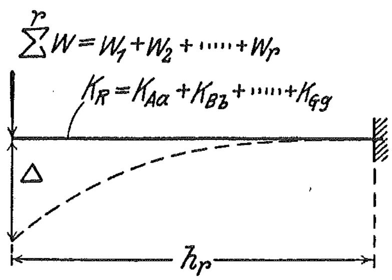

Fig. 6.

$$\varDelta = \frac{\sum^{r} W \cdot h_r^3}{3EJ_R} = \frac{\sum^{r} W \cdot h_r^2}{3EK_R}$$

From the equation one gets:

$$\mu_r = 6E\frac{\varDelta}{h_r} = \frac{2h_r \sum^{r} W}{K_R}$$

$$= \frac{4H_r}{T_r} \cdots\cdots (14)$$

While, for the bent shown in Fig. 4 the expression of μ_r is approximately given by the formula:

$$\mu_r^{(0)} = \alpha_r \frac{H_r}{T_r} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (15)$$

in which α_r is given by equation (18) below.

Assuming the positions of inflexion points at the middle of each girder and column, one gets for equation of φ at the joint m in the bent shown in Fig. 4:

$$\varphi_m^{(0)} = \frac{2}{3}\{\mu_0 \gamma_{0(m)} + \mu_u \gamma_{u(m)}\} \cdots\cdots\cdots\cdots (16)$$

(see Fig. 4)

Assuming farther μ_0 and μ_u to be equal and writing them as μ_r gives:

$$\varphi_m^{(0)} = \frac{2}{3}\mu_r \{\gamma_{0(m)} + \gamma_{u(m)}\} \cdots\cdots\cdots\cdots\cdots (17)$$

Substituting the expressions of φ from equation (17) in equation (9) gives:

$$\mu_r^{(0)} = a_r \frac{H_r}{T_r}$$

where

$$a_r = \frac{1}{1-Q}$$

and

$$Q = \frac{2}{3}\Big\{ \left[(\gamma_{0(A)}+\gamma_{u(A)}) + (\gamma_{0(a)}+\gamma_{u(a)}) \right] t_{Aa} + (\gamma_{0(B)}+\gamma_{u(B)}+\gamma_{0(b)}+\gamma_{u(b)})\, t_{Bb} \cdots\cdots + (\gamma_{0(R)}+\gamma_{u(R)}+\gamma_{0(r)}+\gamma_{u(r)})\, t_{Rr} + \cdots\cdots + (\gamma_{0(G)}+\gamma_{u(G)}+\gamma_{0(g)}+\gamma_{u(g)})\, t_{Gg} \Big\} \quad \ldots\ldots\ldots\ldots \quad (18)$$

The values of γ and t are to be given as numerical values as shown in Fig. 9 and a_r is therefore worked out from equation (18) and the approximate value of μ can be determined from equation (15). When all the values of μ are determined on each story, all the approximate values of φ at every joint are calculated from equation (16).

(3) *Bents with an Axis of Symmetry of the Building.*

For the bents with an axis of symmetry, the calculation is simplified enough to be done for the half of the structure, i. e., on the left side of the central axis.

For the symmetrical bents with the number of bays $2m+1$, the symmetrical axis of the building passes through the middle of the central bay, and the joints on both sides of the axis are to have the slope of equal amount and the same direction.

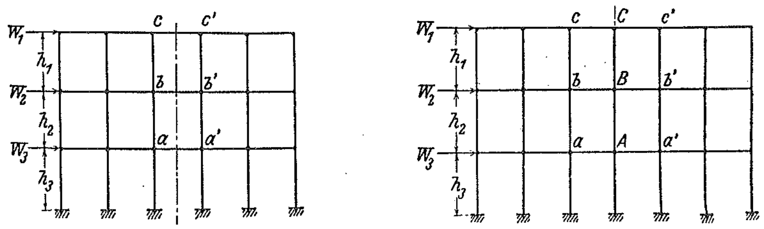

Fig. 7.

For example, in Fig. 7 (a), $\varphi_a = \varphi_{a'}$, $\varphi_b = \varphi_{b'}$ etc., and therefore on such joints as a, b etc., substituting $\varphi_r = \varphi_m$ in equation (5) gives:

$$\left.\begin{aligned} \varphi_m &= -\left\{(\varphi_0-\mu_0)[\gamma_{0(m)}]+(\varphi_u-\mu_u)[\gamma_{u(m)}]+\varphi_l[\gamma_{l(m)}]\right\}, \\ &\text{where} \\ [\gamma_{0(m)}] &= \frac{K_0}{\rho_m+K_r}, \quad [\gamma_{u(m)}]=\frac{K_u}{\rho_m+K_r} \\ [\gamma_{l(m)}] &= \frac{K_l}{\rho_m+K_r}, \quad \rho_m = 2(K_0+K_r+K_u+K_l) \end{aligned}\right\} \quad (19)$$

In the case of symmetrical bents with $2m$ bays, the symmetrical axis of the building coincides with the gravity axis of the central column and for example, in Fig. 7 (b) $\varphi_a=\varphi_{a'}$, $\varphi_b=\varphi_{b'}$ etc., and on such central joints as A, B etc., putting $\varphi_r=\varphi_l$ and $\gamma_r=\gamma_l$ in equation (5) gives:

$$\varphi_m = -\left\{(\varphi_0-\mu_0)\gamma_{0(m)}+(\varphi_u-\mu_u)\gamma_{u(m)}+2\varphi_l\gamma_{l(m)}\right\} \ldots\ldots \quad (20)$$

If the approximate values of φ and μ are known and then applied in equations (19) and (20), equations similar to equation (12) may be obtained.

III. Application of the Slope Distribution Method.

As the numerical example of the proposed method, a symmetrical four storied bent with three bays, carrying horizontal loads concentrated at every joint on the left side of the structure is shown in Fig. 8.

The numerical values written in the side of each member show the numerical value of K (or J/s) for the corresponding column and girder.

It is required to find all the resisting moments at each joint of the bent.

(1) *Preparatory Calculation.*

From equation (6):

$\rho_{01}=2(0.8+0.7+1.0)=5.0$

$\rho_{02}=2(0.8+0.7+0.8)\doteq 4.6$

$\rho_{03}=2(0.7+0.6+0.8)=4.2$

$\rho_{04}=2(0.6+0.7)=2.6$

$\rho_{11}=2(0.8+1.5+1.0+0.7)=8.0$

$\rho_{12}=2(0.8+1.5+0.8+0.7)=7.6$

$\rho_{13}=2(0.7+1.2+0.8+0.6)=6.6$

$\rho_{14}=2(1.2+0.7+0.6)=5.0$

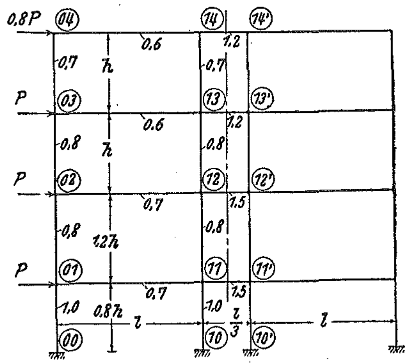

Fig. 8.

The same equation gives:

at joint 01:

$$\gamma_0 = \frac{0.8}{5.0} = 0.16\,, \qquad \gamma_r = \frac{0.7}{5.0} = 0.14$$

$$\gamma_u = \frac{1.0}{5.0} = 0.2$$

at joint 02:

$$\gamma_0 = \frac{0.8}{4.6} = 0.174\,, \qquad \gamma_r = \frac{0.7}{4.6} = 0.152$$

$$\gamma_u = \frac{0.8}{4.6} = 0.174$$

at joint 03:

$$\gamma_0 = \frac{0.7}{4.2} = 0.167\,, \qquad \gamma_r = \frac{0.6}{4.2} = 0.143$$

$$\gamma_u = \frac{0.8}{4.2} = 0.19$$

at joint 04:

$$\gamma_r = \frac{0.6}{2.6} = 0.23 \qquad \gamma_u = \frac{0.7}{2.6} = 0.27$$

From equation (19):

at joint 11:

$$[\gamma_0] = \frac{0.8}{8.0+1.5} = 0.084\,, \quad [\gamma_u] = \frac{1.0}{9.5} = 0.105$$

$$[\gamma_l] = \frac{0.7}{9.5} = 0.074$$

at joint 12:

$$[\gamma_0] = \frac{0.8}{7.6+1.5} = 0.088 \qquad [\gamma_u] = \frac{0.8}{9.1} = 0.088$$

$$[\gamma_l] = \frac{0.7}{9.1} = 0.077$$

at joint 13:

$$[\gamma_0] = \frac{0.7}{6.6+1.2} = 0.09 \qquad [\gamma_u] = \frac{0.8}{7.8} = 0.1025$$

$$[\gamma_l] = \frac{0.6}{7.8} = 0.077$$

at joint 14:

$$[\gamma_u] = \frac{0.7}{5.0+1.2} = 0.113$$

$$[\gamma_l] = \frac{0.6}{6.2} = 0.097$$

Equation (10) gives:

$$H_1 = h \cdot 0.8P = 0.8Ph$$

$$H_2 = h(0.8P+P) = 1.8Ph$$

$$H_3 = 1.2h(0.8P+P+P) = 3.36Ph$$

$$H_4 = 0.8h(0.8P+P+P+P) = 3.04Ph$$

$$T_1 = 2(0.7+0.7+0.7+0.7) = 5.6$$

$$T_2 = T_3 = 2(0.8+0.8+0.8+0.8) = 6.4$$

$$T_4 = 2(1.0+1.0+1.0+1.0) = 8.0$$

$$\frac{H_1}{T_1} = \frac{0.8Ph}{5.6} = 0.143Ph$$

$$\frac{H_2}{T_2} = \frac{1.8}{6.4}Ph = 0.282Ph$$

$$\frac{H_3}{T_3} = \frac{3.36}{6.4}Ph = 0.525Ph$$

$$\frac{H_4}{T_4} = \frac{3.04}{8.0}Ph = 0.38Ph$$

$$t_{1(Aa)} = t_{1(Bb)} = \cdots\cdot = \frac{3(0.7)}{5.6} = 0.375$$

$$t_{2(Aa)} = t_{2(Bb)} = \cdots\cdot = \frac{3(0.8)}{6.4} = 0.375$$

$$t_{3(Aa)} = t_{3(Bb)} = \cdots = \frac{3(0.8)}{6.4} = 0.375$$

$$t_{4(Aa)} = t_{4(Bb)} = \cdots = \frac{3(1.0)}{8.0} = 0.375$$

In the present case, all the values of t are equal and take 0.375, because the expression of t is given:

$$t = \frac{3K}{2(4K)} = \frac{3}{8} = 0.375.$$

From equation (18)

$$\alpha_1 = \frac{1}{1-\frac{2}{3}(0.375)(2)\{(0.167+0.19+0.27)+(0.09+0.1025+0.113)\}}$$

$$= 1.9$$

$$\alpha_2 = \frac{1}{1-\frac{2}{3}(0.375)(2)\{(0.174+0.174+0.167+0.19)+(0.088+0.88+0.09+0.1025)\}}$$

$$= 2.2$$

$$\alpha_3 = \frac{1}{1-\frac{2}{3}(0.375)(2)\{(0.16+0.2+0.174+0.174)+(0.084+0.105+0.088+0.088)\}}$$

$$= 2.2$$

Similarly:

$$\alpha_4 = 1.4$$

From equation (15):

$$\mu_1^{(0)} = \alpha_1 \frac{H_1}{T_1} = 1.9 \frac{0.8}{5.6} Ph = 0.272Ph = 0.3Ph$$

$$\mu_2^{(0)} = \alpha_2 \frac{H_2}{T_2} = 2.2 \frac{1.8}{6.4} Ph = 0.6Ph$$

$$\mu_3^{(0)} = \alpha_3 \frac{H_3}{T_3} = 2.2 \frac{3.36}{6.4} Ph = 1.2Ph$$

$$\mu_4^{(0)} = \alpha_4 \frac{H_4}{T_4} = 1.4 \frac{3.04}{8.0} Ph = 0.5Ph$$

From equation (16):

$$\varphi_{01}^{(0)} = \frac{2}{3}\{(1.2)(0.16)+(0.5)(0.2)\}\,Ph = 0.20Ph$$

$$\varphi_{02}^{(0)} = \frac{2}{3}\{(0.6)(0.174)+(1.2)(0.174)\}\,Ph = 0.21Ph$$

$$\varphi_{03}^{(0)} = \frac{2}{3}\{(0.3)(0.167)+(0.6)(0.19)\}\,Ph = 0.11Ph$$

$$\varphi_{04}^{(0)} = \frac{2}{3}\{(0.3)(0.27)\} = 0.054Ph$$

Similarly:

$$\varphi_{11}^{(0)} = \frac{2}{3}\{(1.2)(0.084)+(0.5)(0.105)\}\,Ph$$

$$= 0.10Ph$$

$$\varphi_{12}^{(0)} = 0.11Ph\,, \qquad \varphi_{13}^{(0)} = 0.059Ph\,,$$

$$\varphi_{14}^{(0)} = 0.023Ph\,.$$

The values of γ, $\varphi^{(0)}$, $\mu^{(0)}$ and H/T, which have been worked out in the preparatory calculation above, are written down in the corresponding joint and story, as shown in Fig. 9.

In the figure, one finds, by the side of the columns, the values of $2t$ instead of t, because the symmetrical condition of the construction of the given bent makes it convenient in calculating equation (13). The reason why will be explained later on.

(2) *Calculation on Frame-Schema (Fig. 9).*

It is required in this section to find the hyperstatical quantities φ and μ. This calculation is not to be worked by solving equations simultaneously, but is, in the present method, to be worked out mechanically on the frame-schema.

The following calculation is only to trace the calculation procedure and not to show the proposed method of calculation itself. The present method is to be worked out on Fig. 9.

From equation (12), looking at Fig. 9:

at joint 01 (Coeff.: P. h disregarded)

$$\varphi_{01}^{(1)} = -\left\{\begin{matrix} (\varphi_{02}^{(0)}-\mu_3^{(0)})\gamma_{0(01)} & \\ + & \varphi_{11}^{(0)}\gamma_{r(01)} \\ (\varphi_{00}^{(0)}-\mu_4^{(0)})\gamma_{u(01)} & \end{matrix}\right\}$$

$$= -\left\{\begin{matrix} (0.21-1.2)(0.16) & \\ + & (0.10)(0.14) \\ (-0.5)(0.2) & \end{matrix}\right\}$$

$$= +0.159-0.014+0.10 = +0.245 \text{ (Coeff.: } Ph \text{ disregarded)}$$

The multiplication of the calculation is to bé done by slide rule and there are written down the products in the side of joint 01 as shown in Fig. 9.

In the same way, at joint 02:

$$\varphi_{02}^{(1)} = -\left\{\begin{matrix} (0.11-0.6)(0.174) & \\ + & (0.11)(0.152) \\ (0.245^{*}-1.2)(0.174) & \end{matrix}\right\}$$

$$= +0.085-0.017+0.166 = +0.234$$

*Here it is noted that $\varphi_{01}^{(1)}$ was used instead of $\varphi_{01}^{(0)}$.

The order of calculation in considering the adjacent joints is to be made in clockwise direction, i. e., the upper, right, lower and the left joints.

In the same way, all the values of $\varphi^{(0)}$ are obtained:

$$\varphi_{03}^{(1)} = +0.102,$$

$$\varphi_{04}^{(1)} = +0.049, \quad \varphi_{11}^{(1)} = +0.127, \quad \varphi_{12}^{(1)} = +0.125$$

$$\varphi_{13}^{(1)} = +0.066, \quad \varphi_{14}^{(1)} = +0.02$$

Next from equation (13):

$$\mu_1^{(1)} = \frac{H_1}{T_1} + 2\left\{\begin{pmatrix} \varphi_{04}^{(1)} \\ + \\ \varphi_{03}^{(1)} \end{pmatrix} t_{03-04} + \begin{pmatrix} \varphi_{14}^{(1)} \\ + \\ \varphi_{13}^{(1)} \end{pmatrix} t_{13-14}\right\}$$

$$= \frac{H_1}{T_1} + (2t)\left\{\begin{pmatrix} \varphi_{04}^{(1)} \\ + \\ \varphi_{03}^{(1)} \end{pmatrix} + \begin{pmatrix} \varphi_{14}^{(1)} \\ + \\ \varphi_{13}^{(1)} \end{pmatrix}\right\}$$

$$= 0.143 + (0.75)\left\{\begin{pmatrix} 0.049 \\ + \\ 0.102 \end{pmatrix} + \begin{pmatrix} 0.02 \\ + \\ 0.06 \end{pmatrix}\right\} = 0.143+0.178 = 0.321$$

This calculation is to be done reading the numerical values written in Fig. 9 and it may have become clear that $(2t)$ was written by the side of each column, instead of writing (t).

In the same way:

$$\mu_2^{(1)} = 0.282+(0.75)\left\{\begin{pmatrix}0.102\\+\\0.234\end{pmatrix}+\begin{pmatrix}0.066\\+\\0.125\end{pmatrix}\right\}$$

$$= 0.282+0.395 = 0.677$$

$$\mu_3^{(1)} = 0.525+(0.75)\left\{\begin{pmatrix}0.234\\+\\0.245\end{pmatrix}+\begin{pmatrix}0.125\\+\\0.127\end{pmatrix}\right\}$$

$$= 0.525+0.549 = 1.074$$

$$\mu_4^{(1)} = 0.38+(0.75)\{(0.245+0.127)\}$$

$$= 0.38+0.279 = 0.659$$

Again from equation (12):

$$\varphi_{01}^{(2)} = -\left\{\begin{matrix}(0.234-1.074)(0.16)\\+\\(-0.659)(0.2)\end{matrix}\quad (0.127)(0.14)\right\}$$

$$= +0.135-0.018+0.132 = +0.249$$

$$\varphi_{02}^{(2)} = +0.10-0.019+0.144 = +0.225$$

etc.

Such calculations for φ and μ, are continued on Fig. 9 until the consecutive values are obtained with small difference between them.

The result of the final calculation is as follows: (Coeff. P. h)

$\varphi_{01} = 0.25$, $\quad\varphi_{02} = 0.223$

$\varphi_{03} = 0.127$, $\quad\varphi_{04} = 0.056$

$\varphi_{11} = 0.131$, $\quad\varphi_{12} = 0.12$

$\varphi_{13} = 0.077$, $\quad\varphi_{14} = 0.026$

$\mu_1 = 0.357$, $\quad\mu_2 = 0.692$

$\mu_3 = 1.068$, $\quad\mu_4 = 0.665$ (Coeff.: Ph)

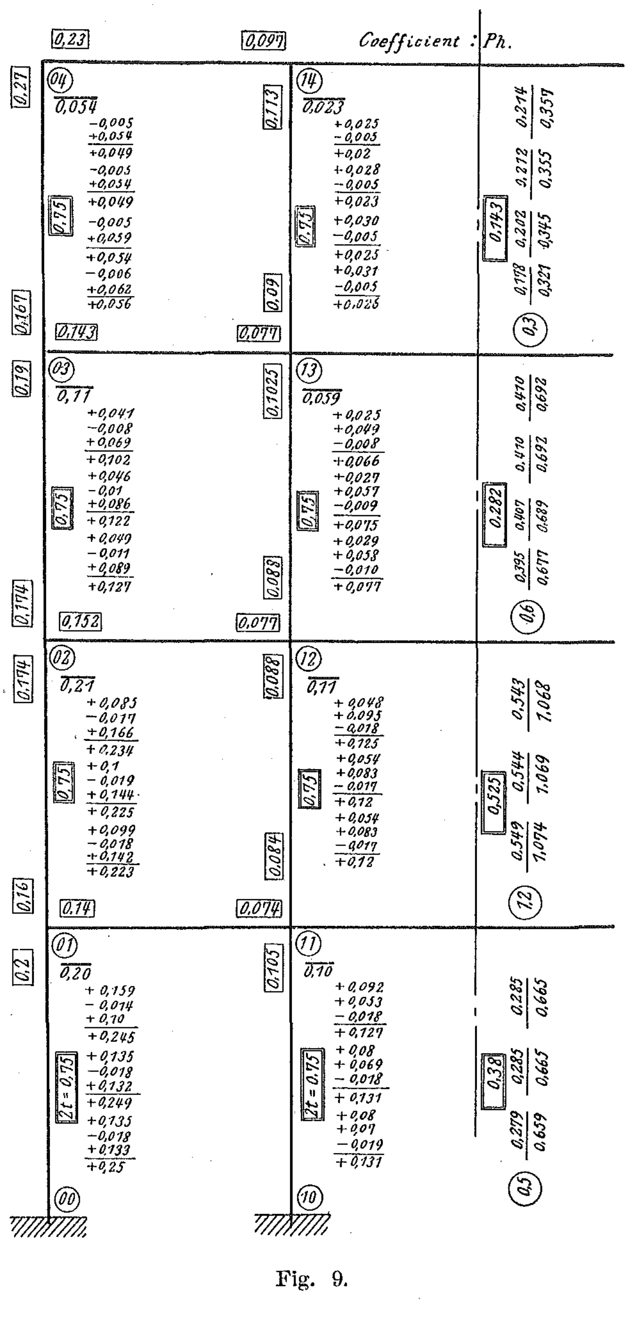

Fig. 9.

(3) *Calculation of Joint Moments.*

With the values of φ and μ, which were obtained in the foregoing calculation in Fig. 9, the joint moments can be worked out as follows:

From equation (2)

$$M_{01-02} = K_{01-02}\{2\varphi_{01}+\varphi_{02}-\mu_3\}$$
$$= 0.8\{2(0.25)+0.223-1.068\}\,Ph$$
$$= -0.28Ph$$

$$M_{01-11} = K_{01-11}\{2\varphi_{01}+\varphi_{11}\}$$
$$= 0.7\{2(0.25)+0.131\}\,Ph = 0.44Ph$$

$$M_{01-00} = K_{01-00}\{2\varphi_{01}-\mu_4\} = 1.0\{2(0.25)-0.665\}\,Ph$$
$$= -0.16Ph$$

etc.

IV. Sectional Summary and Conculusions.

The general conclusions to be drawn from the proposed method of solution may be summarized as follows:

(a) The accuracy of the Slope Distribution Method is the same as that of the Slope Deflection Method.

(b) A knowledge of arithmetical culculation is sufficient for the calculation of the stresses of statically indeterminate structures.

(c) A frame-schema itself, so to say, embodies a system of elastic equations to be solved and all the complete calculations to find redundancies are worked out on the figure of the frame, applying only multiplication, addition and subtraction.

(d) This method can be applied quite mechanically. For determination of redundancies one is not restricted to the solution of simultaneous equations, but the method is nothing but to solve one unknown from one equation.

(e) The proposed method has the characteristic that it is easy to become aware of miscalculation during the course of intermediate calculation and sometimes even if the miscalculations were continued without coming to notice and without correcting small mistakes, little effect is exerted upon the result of the calculation only causing further repetitions, while in the case of process of elimination of redundancies even a small miscalculation yields a fatal effect, without any chance of its coming to notice.