A Speedy Method of Calculation for Secondary Stresses of Simple Bridge Trusses by Numerical Moment Formulae.

By

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(Received April 18, 1938.)

Contents.

Introduction.

The use of rigid joint frames, either machine riveted or electric welded, has been greatly increased in recent years and the practical importance of the proper treatment of secondary stresses which are induced by joint rigidity has likewise remarkably grown.

The well-known calculation method of secondary stresses by Manderla, and those of Müller-Breslau, Ritter and Mohr are among the current methods of tedious calculation for secondary stresses; on the other hand the Slope Deflection Equation principally applied by Wilson and by Maney as well as by Gehler, simplified the redundancies. The Mechanical Tabulation Method* proposed about ten years ago by F. Takabeya made the calculation process quite mechanical, that is, the simultaneous equations, from which the unknown quantities are to be determined, have been obtained without considering any equilibrium equations, and those have been solved by the process of iteration.

Recently there have sometimes been applied the method of Distributing End-Moments first proposed by H. Cross and the Slop Distribution Method** proposed by F. Takabeya. These Methods of

* See Proceedings, American Society of Civil Engineers, Oct., 1934. Vol. 60, No. 8, Part 2, P. 650, Footnote and Takabeya "Rahmentafeln", Julius Springer, Berlin, 1930.

** F. Takabeya: On the Secondary Stress Calculation of Trusses by the Slope Distribution Method, Journal of the Civil Engineering Society, Japan, Vol. XXI. No. 1, January, 1935.

F. Takabeya and T. Sakai: A New Method of Rapid Computation of Secondary Stresses of Simple Bridge Trusses, The Memoirs of the Faculty of Engineering. The Hokkaido Imperial University, Vol. 4, No. 2, P. 121-139. December, 1937.

secondary stress calculation eliminated completely the first process of writing the simultaneous equation and made the second process of iteration a simple and mechanical method of calculation on the frame-schema itself.

The Cross method seems to the present author to have its merit for the treatment of problems with a relative minority of members at each joint such as a continuous beam while the Slope Distribution Method seems to have comparatively greater usefulness for the problems with a relative majority of members at each joint such as trusses and building frames.

All the above mentioned analyses of secondary stresses are very complicated and require considerable tedious labour as compared with that of primary direct stresses. In order to overcome these difficulties it seems necessary to develop some practical and workable method simple in theoretical analysis, which at the same time will provide a means of calculation, that both lessens the mathematical difficulties and shortens the tedious labour of detailed computations.

The new nearly eight hundred formulae proposed in this paper quickly give the bending moments at any joint by which the secondary stresses are caused, for four kinds of trusses which are most frequently adopted in bridge trusses.

For example, for a through Pratt truss with parallel chords of any number of panels, the bending moment at the left end of the lower chord member 2-4 due to the full lower joint loads is determined from the following numerical moment formula:

$$M_{24} = \left\{(-0.42\,n+4.50)+2.42/a_t\right\} P \frac{a_{\text{max.}}}{A_{\text{max.}}} \frac{K_{24}}{k_{24}}$$

In the above formula,

$M_{24} =$ bending moment at end 2 of member 2–4,

$n =$ number of panels in the span,

$P =$ intensity of lower joint loads,

$a_t = \dfrac{A_t \cdot a_{\text{max.}}}{A_{\text{max.}}}$

$A_t =$ sectional area of tensile sub-verticals of the truss in question,

$a_{\text{max.}} =$ the maximum sectional area in upper chord members of the standard truss which is proposed in this paper,

$A_{\max.}$ = the maximum sectional area in upper chord members of the truss in question,

K_{24} = stiffness (I/l) of member 2–4 in the truss in question,

k_{24} = stiffness of member 2–4 in the standard truss.

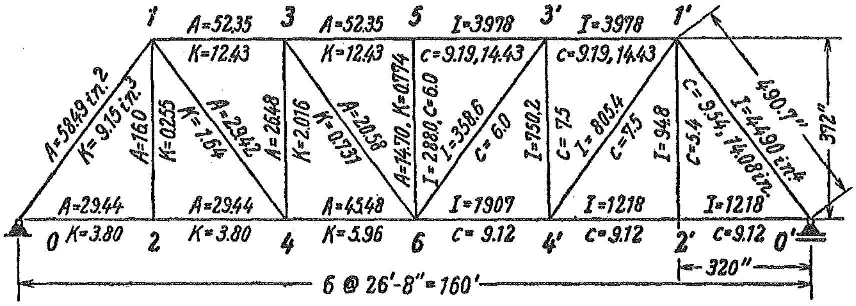

Fig. 1.

Fig. 1 shows a Pratt truss with 6 panels, carring live joint loads of 1000 lbs. . The sectional area of member, denoted by (A), the moment of inertia of the section (I), the stiffness value of member (K) and the distance of fibre from neutral axis of the section of member (c) are as written in Fig. 1.

In this example,

$$n = 6$$
$$P = 1000 \text{ lbs.}$$
$$K_{24} = 3.80 \text{ in.}^3$$
$$A_{\max.} = A_{35} = 52.35 \text{ in.}^2$$
$$A_t = A_{12} = 16.00 \text{ in.}^2$$

and in the standard truss $a_{\max.}$ and k_{24} are respectively 0.85 and 0.40 as given later.

Therefore, one gets

$$a_t = \frac{(16.00)(0.85)}{52.35} = 0.26\,, \qquad \frac{a_{\max.}}{A_{\max.}} = \frac{0.85}{52.35} = 0.0162$$

$$\frac{K_{24}}{k_{24}} = \frac{3.80}{0.40} = 9.5$$

Putting these values into the above numerical moment formula, the bending moment at the left end of member 2–4 due to the full lower joint loads is obtained as follows:

$$\begin{aligned} M_{24} &= \{(-0.24)(6)+4.50+(2.42)/(0.26)\}(1000)(0.0162)(9.5) \\ &= (11.28)(1000)(0.0162)(9.5) \\ &= 1740 \text{ in.-lbs.} \end{aligned}$$

In a similar way, the maximum moment at the upper end of diagonal 1–4 of a through Pratt truss due to live load is determined from the following numerical moment formula:

$$M_{14} = \{(0.69\,n-2.30)+0.00/a_t\}P \cdot \frac{a_{\text{max.}}}{A_{\text{max.}}} \frac{K_{14}}{k_{14}}$$

In the truss of the above example $K_{14} = 1.64$ in.^{3} and its corresponding value in the standard truss $k_{14} = 0.07$ which is also given latter.

Therefore, the maximum bending moment at the upper end of diagonal 1–4 due to live load is obtained as follows:

$$\begin{aligned} M_{14} &= \{(0.69)(6)-2.30\}(1000)(0.0162)(1.64/0.07) \\ &= (1.84)(1000)(0.0162)(23.4) \\ &= 700 \text{ in.-lbs.} \end{aligned}$$

The secondary stresses σ due to the above obtained moments become respectively 13.03(13.86) lbs./sq. in. and 6.52(6.34) lbs./sq. in. from the equation $\sigma = Mc/I$. The values in brackets are those calculated by Johnson for the same example.

In general, moments due to full loads, maximum moments and moments due to single load which is placed at any relatively effective joint are all obtained from numerical moment formulae having the following forms:

$$M_{mn} = \left\{(a \cdot n+b)+c \cdot \frac{1}{a_t}\right\}P \cdot \frac{a_{\text{max.}}}{A_{\text{max.}}} \frac{K_{mn}}{k_{mn}}$$

or

$$M_{mn} = \left\{(a \cdot n+b)+c \cdot \frac{1}{a_c}\right\}P \cdot \frac{a_{\text{max.}}}{A_{\text{max.}}} \frac{K_{mn}}{k_{mn}}$$

In the above formulae,

$$a_c = \frac{A_c \cdot a_{\text{max.}}}{A_{\text{max.}}}$$

where A_c denotes the sectional area of compressive sub-verticals of the truss in question.

The former formula is for the upper joint loads and the latter for lower joint loads.

Descriptions of the derivation of the numerical moment formnlae and the coefficients a, b and c are given below in the present paper.

The bending moments calculated from the numerical moment formulae in a similar expression to that given above are accurate enough to use for all practical purposes and yet the calculation time estimated is given in minutes.

I. Derivation of Numerical Moment Formulae.

In the preceding Memoirs* the author investigated the rapid calculation of secondary stresses by the joint moment diagrams of the standard trusses and the characteristics of the same stresses. As a result of more progressed investigation, the author proposes here the numerical moment formulae from which the bending moments at the ends of any member of the bridge trusses can be quickly computed and describes the derivation of these numerical moment formulae dividing his report into several items as follows:

1. *Determination of Standard Trusses.* After investigation of a great many examples of bridge trusses in actual use, it become obvious that the members of the trusses have a constant ratio, on the whole, both for stiffness and sectional area, between one member and another.

Considering these stiffness values and sectional area-distributions as well as the effect of the direct stresses, there were chosen the standard trusses for the Warren and Pratt trusses with parallel chords in the preceding Memoirs.** The standard trusses likewise can be chosen for trusses with polygonal chords.

Fig. 2 shows the standard through Warren truss with parallel chords, Fig. 3 the standard deck Warren truss with parallel chords, Fig. 4 the standard through Pratt truss with parallel chords and Fig. 5 the standard through Warren truss with polygonal chords.

These 4 types of bridges are most usually adopted among simple bridge trusses.

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* ** F. Takabeya and T. Sakai: A New Method of Rapid Computation of Secondary Stresses of Simple Bridge Trusses, The Memoirs of the Faculty of Engineering, The Hokkaido Imperial University, Vol. 4, No. 2, December, 1937.

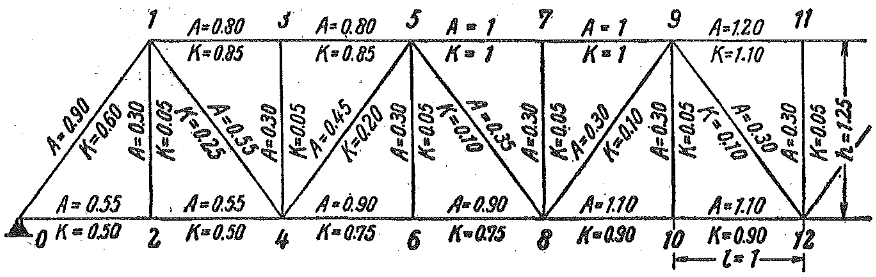

Fig. 2.

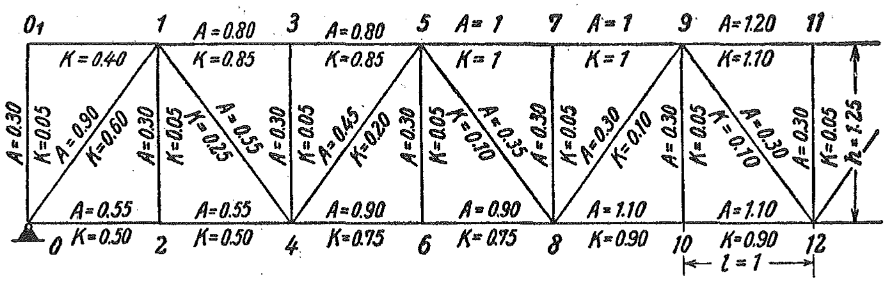

Fig. 3.

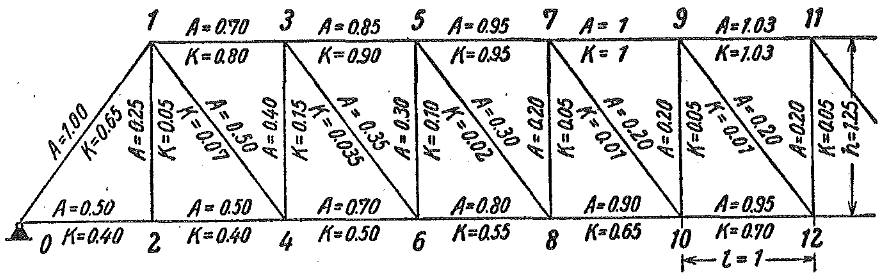

Fig. 4.

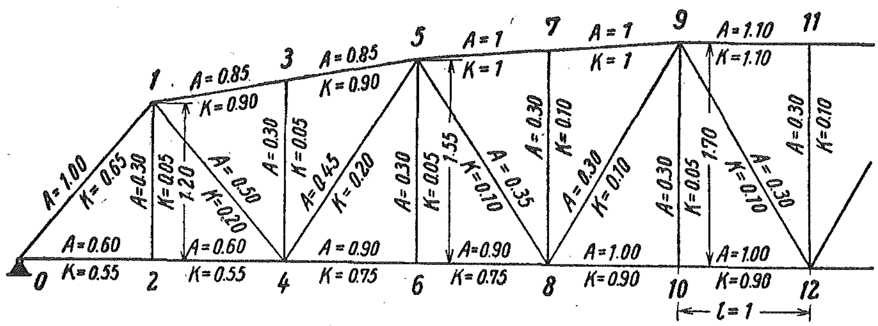

Fig. 5.

Some of the actual examples were different to some degree from the standard sectional ratio chosen here by the author, especially in respect to stiffness in web members and also to sectional area in sub-verticals. However the effect of these divergences from the standard in these members can be easily corrected and accordingly these differences are not an important matter.

Height and panel length of the standard trusses were chosen as shown in the above Figs. 2 to 5 which correspond to many actual examples.

The standard trusses with any number of panels are obtained from the above Figs. 2 to 5. Fer example, the standard trusses with six panels are obtained as shown Figs. 6 to 9 from Figs. 2 to 5 respectively.

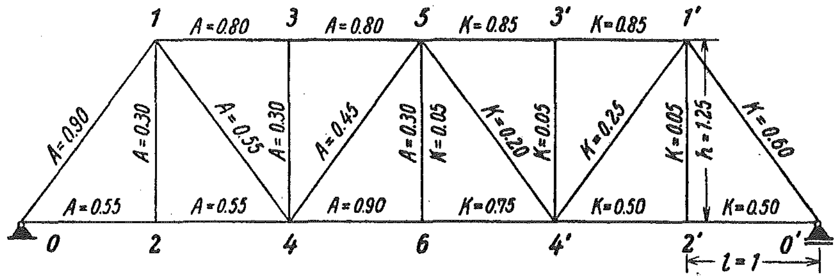

Fig. 6.

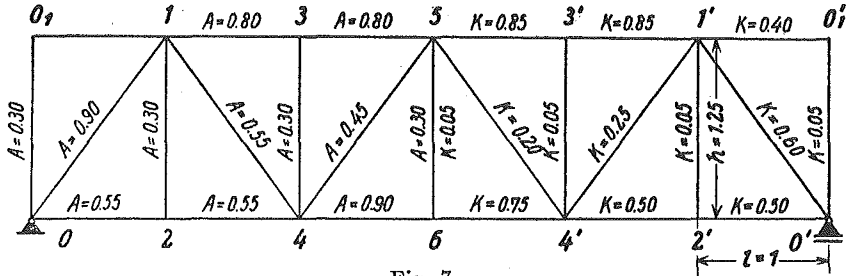

Fig. 7.

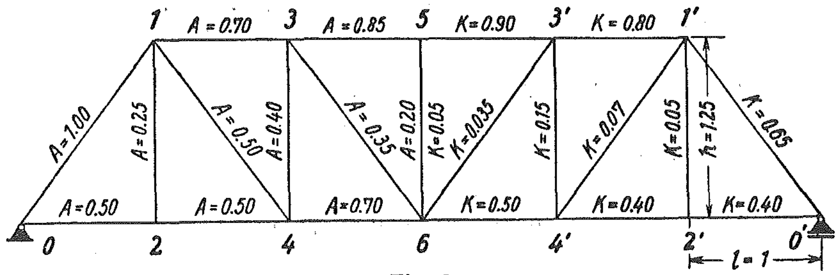

Fig. 8.

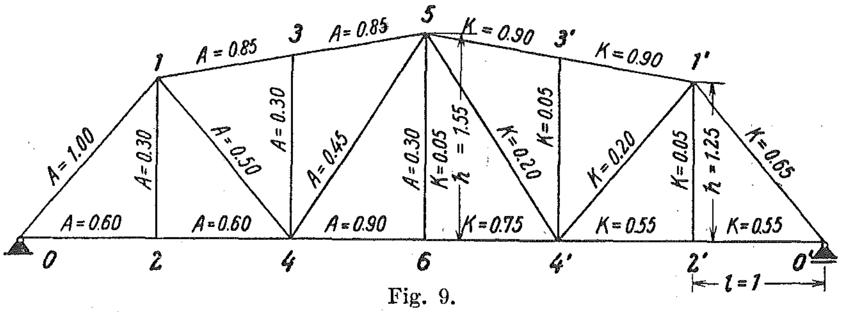

Fig. 9.

2. *Numerical Moment Formulae for Standard Trusses.* In this paper the author defines "numerical moment formulae" as an equation which represents numerically the relation between the bending moment at the end of one member and the number of panels in a span.

The relations between the bending moments and the number of panels in a span will next be found for the following three cases:

A. Bending Moments due to full loads.

B. Maximum bending moments.

C. Bending moments due to a single load.

A. *Bending Moments due to Full Loads.* The variation of the bending moments due to the increase of the number of panels in a span is considered to be linear for any kind of truss from the facts that the direct stress in any member varies linearly by the variation of the number of panels in a span as already mentioned in the preceding Memoirs* and the bending moments are induced due to the direct stresses.

In the standard through Warren truss with parallel chords, for example, the variation of the moments is as indicated in Tables 1 and 2. Again, in the standard through Warren truss with polygonal chords, the variation of the moment becomes as shown in Tables 3 and 4.

All the values of moments in these tables are for full joint loads having unit intensity and they were obtained accurately by the Slope Distibution Method.

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* F. Takabeya and T. Sakai: A New Method of Rapid Computation of Secondary Stresses of Simple Bridge Trusses, The Memoirs of the Faculty of Engineering, The Hokkaido Imperial University, Vol. 4, No. 2, P. 173-175. December, 1937.

Table 1.

Moment	Number of Panels in Span					
	16	12	10	8	6	4
M_{01}	21.9 (21.60)	18.35 (17.80)	15.42 (15.90)	13.66 (14.00)	12.03 (12.10)	10.06 (10.20)
M_{10}	8.3 (9.43)	7.08 (6.91)	5.87 (5.61)	4.32 (4.39)	2.95 (3.13)	1.84 (1.87)
M_{02}	−21.8 (−21.60)	−18 35 (−17.80)	−15.42 (−15.90)	−13.66 (−14.00)	−12.03 (−12.10)	−10.06 (−10.20)
M_{20}	−19.2 (−18.80)	−17.95 (−17.40)	−16.12 (−16.70)	−15.93 (−16.00)	−15.85 (−15.30)	−14.99 (−14.60)
M_{13}	−30.0 (−30.20)	−20.93 (−21.40)	−17.20 (−17.00)	−12.62 (−12.60)	− 8.09 (− 8.20)	− 3.73 (− 3.80)
M_{31}	−31.6 (−31.78)	−23.24 (−24.86)	−21.76 (−21.40)	−17.82 (−17.94)	−14.79 (−14.48)	−12.25 (−10.92)
M_{24}	11.5 (11.19)	12.62 (12.03)	11.80 (12.45)	12.82 (12.87)	13.87 (13.29)	14.10 (13.71)
M_{42}	−14.4 (−14.51)	− 5.59 (− 6.92)	− 3.36 (− 3.10)	0.82 (0.72)	4.81 (4.54)	7.18 (8.36)
M_{35}	22.0 (22.20)	16.66 (18.00)	16.80 (16.50)	14.46 (14.60)	13.07 (12.70)	
M_{53}	−35.0 (−35.56)	−27.01 (−25.32)	−19.45 (−20.20)	−15.44 (−15.08)	− 8.46 (− 9.96)	
M_{46}	−21.0 (−21.32)	−18.91 (−17.24)	−15.20 (−15.20)	−13.35 (−13.20)	−11.25 (−11.10)	

Table 2.

Moment	Number of Panels in Span					
	16	12	10	8	6	4
M_{12}	7.4 (6.49)	4.37 (4.53)	3.56 (3.55)	2.54 (2.57)	1.59 (1.59)	0.69 (0.61)
M_{21}	7.7 (7.61)	5.33 (5.35)	4.31 (4.25)	3.11 (3.13)	1.98 (2.01)	0.89 (0.89)
M_{14}	14.3 (14.28)	9.47 (9.96)	7.77 (7.80)	5.76 (5.64)	3.55 (3.48)	1.19 (1.32)
M_{41}	8.5 (8.40)	5.21 (5.40)	3.84 (3.90)	2.55 (2.40)	0.90 (0.90)	−1.32 (−0.60)
M_{34}	9.6 (9.68)	6.58 (6.46)	4.97 (4.90)	3.39 (3.34)	1.73 (1.78)	0 (0.22)
M_{43}	8.5 (8.48)	5.86 (5.76)	4.49 (4.40)	3.06 (3.04)	1.61 (1.68)	0 (0.32)
M_{45}	18.4 (18.90)	13.43 (13.00)	10.25 (10.00)	6.93 (7.00)	3.94 (4.00)	
M_{54}	9.1 (9.50)	5.99. (5.50)	3.78 (3.50)	1.26 (1.50)	−0.60 (−0.50)	
M_{56}	8.7 (9.00)	5.31 (5.40)	3.58 (3.60)	1.65 (1.80)	0 (0)	
M_{65}	9.5 (9.70)	5.75 (5.82)	3.92 (3.88)	1.83 (1.94)	0 (0)	

Table 3.

Moment	Number of Panels in Span				
	12	10	8	6	4
M_{01}	16.85 (16.85)	14.95 (15.27)	13.89 (13.70)	12.12 (12.11)	10.35 (10.53)
M_{10}	1.66 (1.72)	2.05 (1.45)	1.15 (1.18)	0.88 (0.91)	2.37 (0.64)
M_{02}	−16.85 (−16.85)	−14.95 (−15.27)	−13.89 (−13.70)	−12.12 (−12.11)	−10.35 (−10.35)
M_{20}	−20.58 (−20.58)	−18.84 (−19.50)	−19.16 (−18.42)	−18.09 (−17.34)	−15.27 (−16.26)
M_{13}	−11.99 (−12.00)	−10.04 (− 9.50)	− 6.88 (− 7.00)	− 4.43 (− 4.50)	− 4.08 (− 2.00)
M_{31}	−20.22 (−20.61)	−19.48 (−18.95)	−17.32 (−17.29)	−16.05 (−15.63)	−13.57 (−13.97)
M_{24}	16.29 (16.04)	15.52 (15.92)	16.72 (15.80)	16.55 (15.68)	14.35 (15.56)
M_{42}	− 0.43 (− 0.32)	1.47 (2.10)	4.64 (4.52)	6.80 (6.94)	6.73 (9.36)
M_{35}	16.04 (16.46)	16.48 (16.00)	15.60 (15.54)	15.54 (15.08)	
M_{53}	−12.83 (−13.10)	− 7.90 (− 7.40)	− 1.59 (− 1.50)	4.73 (4.00)	
M_{46}	−18.32 (−18.24)	−15.16 (−15.80)	−13.31 (−13.36)	−10.67 (−10.92)	

Table 4.

Moment	Number of Panels in Span				
	12	10	8	6	4
M_{12}	3.42 (3.44)	2.68 (2.70)	1.96 (1.96)	1.22 (1.22)	0.74 (0.48)
M_{21}	4.28 (4.30)	3.32 (3.38)	2.45 (2.46)	1.54 (1.54)	0.92 (0.62)
M_{14}	6.91 (6.90)	5.31 (5.40)	3.77 (3.90)	2.33 (2.40)	0.97 (0.09)
M_{41}	4.13 (4.05)	2.73 (2.75)	1.38 (1.45)	0.07 (0.15)	−1.14 (−1.15)
M_{34}	4.18 (4.15)	3.00 (2.95)	1.72 (1.75)	0.51 (0.55)	0 (−0.65)
M_{43}	3.94 (3.95)	2.89 (2.85)	1.70 (1.75)	0.60 (0.65)	0 (−0.45)
M_{45}	10.68 (10.56)	8.08 (8.10)	5.59 (5.64)	3.20 (3.18)	
M_{54}	5.19 (4.90)	3.13 (3.40)	1.86 (1.90)	0.42 (0.40)	
M_{56}	3.24 (3.10)	1.92 (2.02)	1.02 (0.96)	0 (−0.10)	
M_{65}	3.59 (3.50)	2.17 (2.30)	1.12 (1.10)	0 (−0.10)	

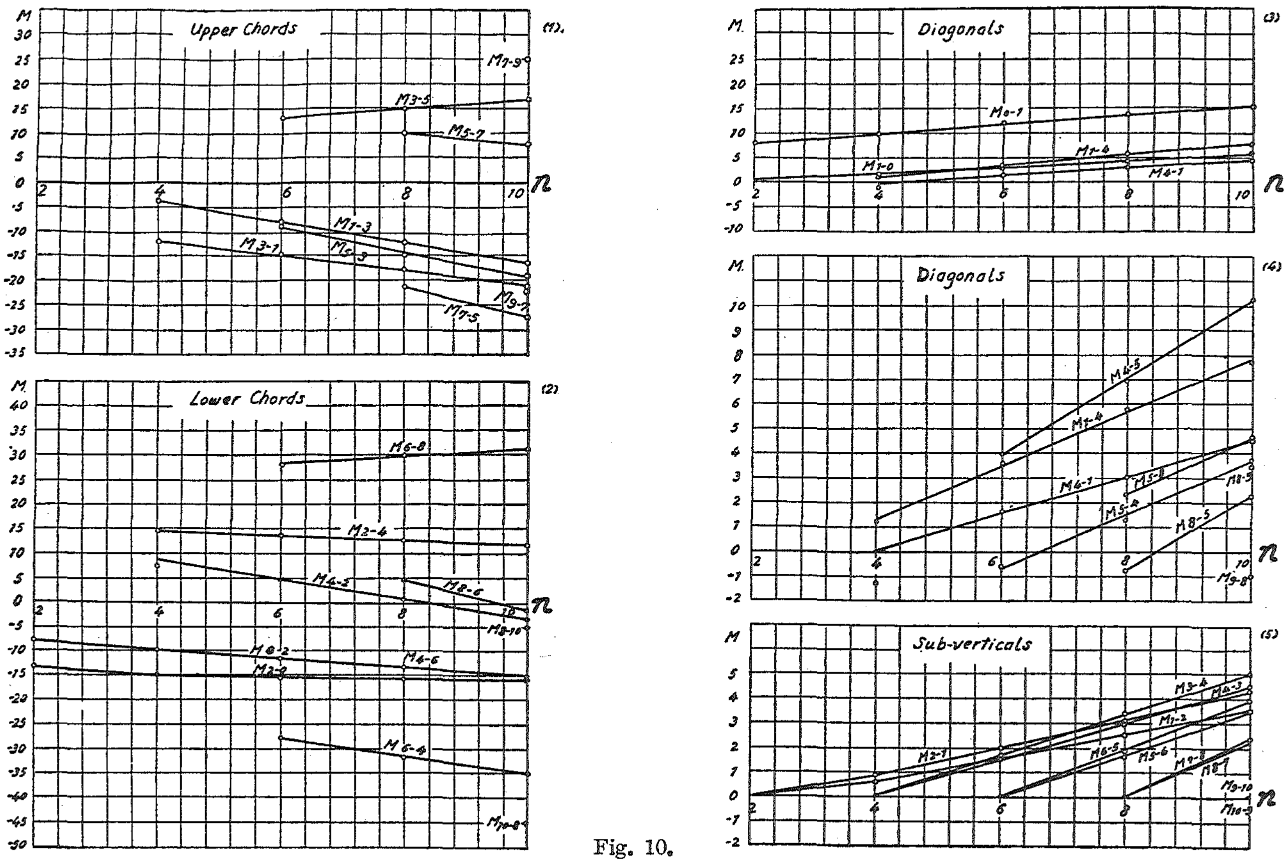
Upper Chords
(1).
M
n
Lower Chords
(2)
Diagonals
(3)
Diagonals
(4)
Sub-verticals
(5)

Fig. 10.

In Fig. 10 there is shown graphically this linear variation of the moments for the standard through Warren truss with parallel chords.

Therefore by the use of this characteristic of the linear variation of moments, the numerical moment formulae are obtained in the form

$$M_{mn} = (a \cdot n + b) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(A)}$$

where M_{mn} denotes the bending moment at the end m of member $m-n$, n number of panels and a and b are constants which are concerned with the type of truss and loading condition and independent of the number of panels in a span.

For examples, the numerical moment formulae for the standard through Warren truss with parallel chords and the same with polygonal chords are obtained by the use of the values in Tables 1 to 4 as follows:

Standard Through Warren Truss with Parallel Chords.

Moments of Chord Members:

M_{01} (portal 0–1, lower) $= (0.95\,n + 6.40)$
M_{10} (portal 0–1, upper) $= (0.63\,n - 0.65)$
M_{02} (lower chord 0–2, left) $= (-0.95\,n - 6.40)$
M_{20} (lower chord 0–2, right) $= (-0.35\,n - 13.20)$
M_{13} (upper chord 1–3, left) $= (-2.20\,n + 5.00)$
M_{31} (upper chord 1–3, right) $= (-1.73\,n - 4.10)$
M_{24} (lower chord 2–4, left) $= (-0.21\,n + 14.55)$
M_{42} (lower chord 2–4, right) $= (-1.91\,n + 16.00)$
M_{35} (upper chord 3–5, left) $= (0.95\,n + 7.00)$
M_{53} (upper chord 3–5, right) $= (-2.56\,n + 5.40)$
M_{46} (lower chord 4–6, left) $= (-1.02\,n - 5.00)$
etc.

Moments of Web Members:

M_{12} (sub-vertical 1–2, upper) $= (0.49\,n - 1.35)$
M_{21} (sub-vertical 1–2, lower) $= (0.56\,n - 1.35)$
M_{14} (diagonal 1–4, upper) $= (1.08\,n - 3.00)$
M_{41} (diagonal 1–4, lower) $= (0.75\,n - 3.60)$
M_{34} (sub-vertical 3–4, upper) $= (0.78\,n - 2.90)$
M_{43} (sub-vertical 3–4, lower) $= (0.68\,n - 2.40)$
M_{45} (diagonal 4–5, lower) $= (1.50\,n - 5.00)$
M_{54} (diagonal 4–5, upper) $= (1.00\,n - 6.50)$
M_{56} (sub-vertical 5–6, upper) $= (0.90\,n - 5.40)$
M_{65} (sub-vertical 5–6, lower) $= (0.97\,n - 5.82)$
etc.

Standard Through Warren Truss with Polygonal Chords.

Moments of Chord Members:

M_{01} (portal 0–1, lower) $= (0.79\,n+7.37)$
M_{10} (portal 0–1, upper) $= (0.135\,n+0.10)$
M_{02} (lower chord 0–2, left) $= (-0.79\,n-7.37)$
M_{20} (lower chord 0–2, right) $= (-0.54\,n-14.10)$
M_{13} (upper chord 1–3, left) $= (-1.25\,n+3.00)$
M_{31} (upper chord 1–3, right) $= (-0.83\,n-10.65)$
M_{24} (lower chord 2–4, left) $= (0.06\,n+15.32)$
M_{42} (lower chord 2–4, right) $= (-1.21\,n+14.20)$
M_{35} (upper chord 3–5, left) $= (0.23\,n+13.70)$
M_{53} (upper chord 3–5, right) $= (-2.85\,n+21.10)$
M_{46} (lower chord 4–6, left) $= (-1.22\,n-3.60)$
etc.

Moments of Web Members:

M_{12} (sub-vertical 1–2, upper) $= (0.37\,n-1.00)$
M_{21} (sub-vertical 1–2, lower) $= (0.46\,n-1.22)$
M_{14} (diagonal 1–4, upper) $= (0.75\,n-2.10)$
M_{41} (diagonal 1–4, lower) $= (0.65\,n-3.75)$
M_{34} (sub-vertical 3–4, upper) $= (0.60\,n-3.05)$
M_{43} (sub-vertical 3–4, lower) $= (0.55\,n-2.65)$
M_{45} (diagonal 4–5, lower) $= (1.23\,n-4.20)$
M_{54} (diagonal 4–5, upper) $= (0.75\,n-4.10)$
M_{56} (sub-vertical 5–6, upper) $= (0.53\,n-3.28)$
M_{65} (sub-vertical 5–6, lower) $= (0.60\,n-3.70)$
etc.

In Tables 1 to 4, the values in brackets show the values calculated by the above numerical moment formulae.

B. *Maximum Bending Moments.* In the chord members the relation between the maximum bending moment at the end of a member and the variation of the number of panels in a span is generally like that for full joint loads, as the bending moment is nearly the maximum for full joint loads in general.

In the web members, the maximum bending moment is produced by the loads at the joints on the right side of the member in question, that is, the maximum bending moment occurs for the same partial loading as the maximum direct stress. The variation of the maximum bending moment due to the increase of the number of panels in a span is also condidered to be linear from the fact that the maximum direct stress by which the maximum bending moment is produced, can be also considered to vary linearly by the variation of the number of panels in a span as already mentioned in the preceding Memoirs*.

In the standard through Warren truss with parallel chords, for example, the variation of the maximum bending moments, from which the maximum secondary stresses are produced, is as shown in Table 5. Again, in the standard through Warren truss with polygonal chords the variation of the bending moments is as indicated in Table 6.

All the moment values are for joint loads having unit intensity.

Table 5.

Moment	Number of Panels in Span				
	12	10	8	6	4
M_{12}	4.40 (4.63)	3.59 (3.65)	2.60 (2.67)	1.69 (1.69)	0.86 (0.71)
M_{21}	5.33 (5.37)	4.31 (4.25)	3.11 (3.13)	1.98 (2.01)	0.93 (0.89)
M_{14}	10.41 (11.06)	8.70 (8.90)	6.76 (6.74)	4.62 (4.58)	2.52 (2.42)
M_{41}	8.67 (8.67)	7.45 (7.43)	6.17 (6.19)	4.94 (4.95)	2.61 (3.71)
M_{34}	6.91 (6.91)	5.34 (5.35)	3.80 (3.79)	2.23 (2.23)	0.63 (0.67)
M_{43}	6.40 (6.41)	5.06 (5.05)	3.66 (3.69)	2.29 (2.33)	0.81 (0.97)
M_{45}	15.07 (14.80)	11.95 (11.80)	8.72 (8.80)	5.82 (5.80)	
M_{54}	8.17 (7.80)	6.06 (5.80)	3.79 (3.80)	1.83 (1.80)	
M_{56}	5.90 (5.95)	4.25 (4.25)	2.51 (2.55)	0.93 (0.85)	
M_{65}	6.29 (6.37)	4.55 (4.55)	2.65 (2.73)	0.94 (0.91)	
M_{58}	7.23 (7.20)	5.18 (5.20)	3.21 (3.20)		

* F. Takabeya and T. Sakai: A New Method of Rapid Computation of Secondary Stresses of Simple Bridge Trusses, The Memoirs of the Faculty of Engineering, The Hokkaido Imperial University, Vol, 4, No. 2, P. 176. December, 1937.

Table 6.

Moment	Number of Panels in Span				
	12	10	8	6	4
M_{12}	3.57 (3.55)	2.83 (2.85)	2.13 (2.15)	1.41 (1.45)	0.95 (0.75)
M_{21}	4.28 (4.30)	3.33 (3.38)	2.47 (2.46)	1.59 (1.54)	1.00 (0.62)
M_{14}	7.93 (7.90)	6.35 (6.40)	4.85 (4.90)	3.47 (3.40)	2.17 (1.90)
M_{41}	7.62 (7.40)	6.31 (6.40)	5.13 (5.30)	4.07 (4.20)	2.33 (3.10)
M_{34}	4.83 (4.83)	3.68 (3.75)	2.54 (2.67)	1.55 (1.59)	0.71 (0.51)
M_{43}	4.80 (4.80)	3.77 (3.80)	2.72 (2.80)	1.81 (1.80)	0.91 (0.80)
M_{45}	12.80 (12.75)	10.26 (10.35)	7.84 (7.95)	5.51 (5.55)	
M_{54}	8.05 (8.04)	6.13 (6.40)	4.92 (4.76)	3.25 (3.12)	
M_{56}	4.21 (4.25)	2.97 (3.15)	2.15 (2.05)	1.05 (0.95)	
M_{65}	4.55 (4.57)	3.18 (3.35)	2.21 (2.13)	1.07 (0.91)	

In Fig. 11 there is shown graphically the above linear variation of the moments for the standard through Warren truss with parallel chords.

Therefore the numerical moment formulae for the maximum bending moments are also obtained in the same form as for the full loads.

For examples, the moment equations for the standard through Warren trusses with parallel and polygonal chords are obtained by the use of the values in Tables 5 and 6 as follows:

Standard Through Warren Truss with Parallel Chords.

Maximum Moments of Web Members:

M_{12} (sub-vertical 1–2, upper) $= (0.49\,n - 1.25)$
M_{21} (sub-vertical 1–2, lower) $= (0.56\,n - 1.35)$
M_{14} (diagonal 1–4, upper) $= (1.08\,n - 1.90)$
M_{41} (diagonal 1–4, lower) $= (0.62\,n + 1.23)$
M_{34} (sub-vertical 3–4, upper) $= (0.78\,n - 2.45)$
M_{43} (sub-vertical 3–4, lower) $= (0.68\,n - 1.75)$
M_{45} (diagonal 4–5, lower) $= (1.50\,n - 3.20)$
M_{54} (diagonal 4–5, upper) $= (1.00\,n - 4.20)$

M_{56} (sub-vertical 5–6, upper) = $(0.85\,n-4.25)$
M_{65} (sub-vertical 5–6, lower) = $(0.91\,n-4.55)$
M_{58} (diagonal 5–8, upper) = $(1.00\,n-4.80)$
etc.

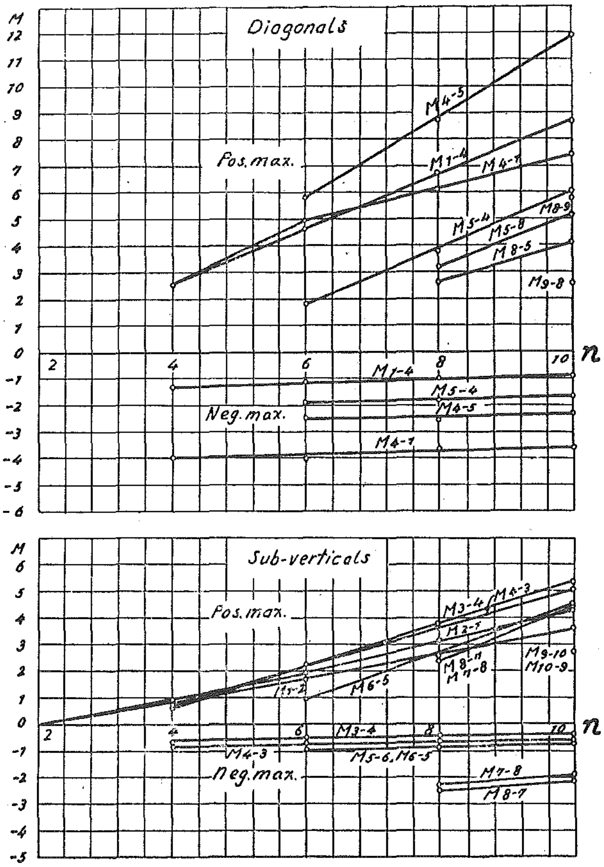

Fig. 11.

Standard Through Warren Truss with Polygonal Chords.

Maximum Moments of Web Members:

M_{12} (sub-vertical 1–2, upper) = $(0.35\,n-0.65)$
M_{21} (sub-vertical 1–2, lower) = $(0.46\,n-1.22)$
M_{14} (diagonal 1–4, upper) = $(0.75\,n-1.10)$
M_{41} (diagonal 1–4, lower) = $(0.55\,n+0.90)$
M_{34} (sub-vertical 3–4, upper) = $(0.54\,n-1.65)$
M_{43} (sub-vertical 3–4, lower) = $(0.50\,n-1.20)$

M_{45} (diagonal 4–5, lower) $= (1.20\,n-1.65)$
M_{54} (diagonal 4–5, upper) $= (0.82\,n-1.80)$
M_{56} (sub-vertical 5–6, upper) $= (0.55\,n-2.35)$
M_{65} (sub-vertical 5–6, lower) $= (0.61\,n-2.75)$
etc.

In Tables 5 and 6, the values in brackets show the values calculated by the above formulae.

C. *Bending Moments due to a Single Load.* In the chord members, the amount of the maximum bending moment at the end of a member due to a single load is little affected by the variation of the number of panels in a span.

Thus, in the standard through Warren truss, M_{01}, M_{02}, M_{20}, M_{24} and M_{42} attain the maximum values with the single load at joint 2. These values are considered to be nearly constant for any number of panels, as shown in Table 7.

Moments M_{13}, M_{31} and M_{35} attain the maximum values with the single load at joint 4. These amounts for any number of panels are written in Table 8. They also are nearly constant.

Tables 9 and 10 are for the standard through Warren truss with polygonal chords. Moments in Table 9 are for a single load at joint 2 and those in Table 10 for a single load at joint 4.

Table 7.

Moment	Number of Panels in Span				
	12	10	8	6	4
M_{01}	8.64	8.50	8.49	8.49	8.23
M_{02}	− 8.64	− 8.50	− 8.49	− 8.49	− 8.23
M_{20}	−14.78	−14.55	−14.64	−14.69	−14.45
M_{24}	14.65	14.42	14.55	14.64	14.49
M_{42}	9.45	9.25	9.47	9.57	9.61

Table 8.

Moment	Number of Panels in Span				
	12	10	8	6	4
M_{13}	− 5.39	− 5.48	− 5.28	− 4.91	− 4.20
M_{31}	−11.10	−11.58	−11.36	−11.37	−10.31
M_{35}	10.62	11.16	11.01	11.20	

Table 9.

Moment	Number of Panels in Span				
	12	10	8	6	4
M_{01}	8.69	8.68	8.68	8.61	8.47
M_{02}	− 8 69	− 8.68	− 8.68	− 8.61	− 8.47
M_{20}	−15.68	−15.58	−15.69	−15.63	−15.27
M_{24}	15.67	15.59	15.72	15.68	15.35
M_{42}	9.62	9.58	9.68	9.67	9.70

Table 10.

Moment	Number of Panels in Span				
	12	10	8	6	4
M_{13}	− 4.37	− 4.42	− 4.14	− 3.86	− 4.81
M_{31}	−12.21	−12.41	−12.24	−12.33	
M_{35}	12.25	12.47	12.39	12.63	

In the web members the maximum bending moment at the end of a member due to a single load is slightly affected by variation in the number of panels in a span. But it is considered to be approximately constant as for the chord members.

Therefore, for the maximum bending moment at the end of any member due to a single load constant a in numerical moment formulae is considered to be zero.

3. *Numerical Moment Formulae for Trusses with the Same Ratio, both of Stiffness and Sectional Area as Standard Trusses.*

In ordinary construction of bridge trusses, panel points carry all the loads and the loaded chords carry no intermediate loads; in consequence of the loading condition the moment at the end A of the member AB restrained at the ends is expressed by the well-known equation,

$$M_{ab} = K_{ab}(2\varphi_a+\varphi_b+\Psi_{ab}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(a)}$$

in this equation, we denote by K_{ab} the moment of inertia of the section divided by the length of the member AB, i.e, K_{ab} the stiffness value of the member AB; φ_a and φ_b the joint-rotation angle of the ends A and B, respectively, multiplied by $2E$; E the modulus of elasticity of the material; and by Ψ_{ab} the member revolution angle multiplied by $-6E$.

The bending moment on a section is considered positive when the couple acts in a clockwise direction upon the portion of the member considered; the joint-rotation angle is considered positive when the angle has turned clockwise, measured from its initial position; the member revolution angle is also positive in case of revolution in clockwise direction from the initial position of the member.

As the value Ψ_{ab} for an absolutely stiff connecting joint is but slightly different from that for a pin connecting joint, it is usually found by Williot-Mohr's Displacement Diagram, considering it as of the same value and it is substituted as a known quantity into the Slope Deflection Equation (a).

For a joint m shown in Fig. 12 (a) the equilibrium condition of the total moments gives:

$$\left.\begin{array}{l} \Sigma M_m = 0 \\ M_{m1}+M_{m2}+\cdots\cdots+M_{mn}=0 \end{array}\right\} \quad \cdots\cdots\cdots\cdots \text{(b)}$$

in which

$$\left.\begin{array}{l} M_{m1} = K_{m1}(2\varphi_m+\varphi_1+\Psi_{m1}) \\ M_{m2} = K_{m2}(2\varphi_m+\varphi_2+\Psi_{m2}) \\ \quad\vdots \\ M_{mn} = K_{mn}(2\varphi_m+\varphi_n+\Psi_{mn}) \end{array}\right\} \quad \cdots\cdots\cdots\cdots \text{(c)}$$

Substituting equation (c) in (b) gives:

$$\begin{aligned} 2\varphi_m(K_{m1}+K_{m2}+\cdots\cdots+K_{mn})+\varphi_1K_{m1}+\varphi_2K_{m2}+\cdots\cdots+\varphi_nK_{mn} \\ = -(\Psi_{m1}K_{m1}+\Psi_{m2}K_{m2}+\cdots\cdots+\Psi_{mn}K_{mn}) \quad \cdots\cdots\cdots\cdots \text{(d)} \end{aligned}$$

From this condition of equilibrium, each joint gives one equation and m joints give therefore m equations, in which m unknown φ are contained.

Next, let another frame of similar form be considered, having the same stiffness ratio as well as the same sectional area-ratio, as shown in Fig. 12 (b), while the stiffness values are α times the former frame in Fig. 12 (a) and the sectional area κ times the same.

For the joint m in Fig. 12 (b), the equilibrium condition gives in a similar way:

$$2\varphi'_m\kappa(K_{m1}+K_{m2}+\cdots\cdots+K_{mn})+\varphi'_1\kappa K_{m1}+\varphi'_2\kappa K_{m2}+\cdots\cdots+\varphi'_n\kappa K_{mn}$$
$$=-\kappa(\Psi'_{m1}K_{m1}+\Psi'_{m2}K_{m2}+\cdots\cdots+\Psi'_{mn}K_{mn}) \quad \dots\dots\dots\dots \text{(e)}$$

in which φ' and Ψ' are respectively the joint-rotation angle and member revolution angle for the frame shown in Fig. 12 (b).

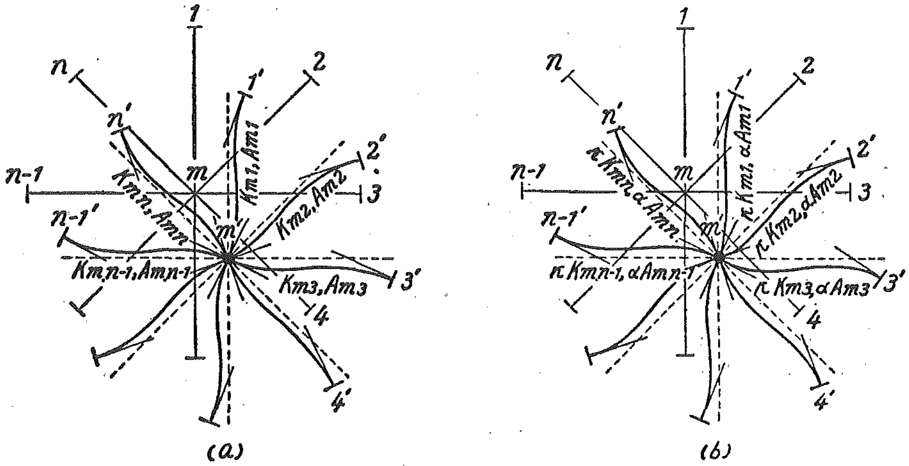

Fig. 12.

Mohr's Displacement Diagram shows generally:

$$\Psi' = \frac{\Psi}{\alpha} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(f)}$$

Equation (e) therefore becomes:

$$2\varphi'_m(K_{m1}+K_{m2}+\cdots\cdots+K_{mn})+\varphi'_1K_{m1}+\varphi'_2K_{m2}+\cdots\cdots+\varphi'_nK_{mn}$$
$$=-\frac{1}{\alpha}(\Psi_{m1}K_{m1}+\Psi_{m2}K_{m2}+\cdots\cdots+\Psi_{mn}K_{mn}) \quad \dots\dots\dots \text{(g)}$$

From equations (d) and (g) it is obvious that:

$$\varphi'_m=\frac{\varphi_m}{\alpha}\,,\quad \varphi'_1=\frac{\varphi_1}{\alpha}\,,\quad \varphi'_2=\frac{\varphi_2}{\alpha}\cdots\cdots\varphi'_n=\frac{\varphi_n}{\alpha} \quad \dots\dots\dots \text{(h)}$$

and

$$M'_{m1}=\kappa K_{m1}\left(2\frac{\varphi_m}{\alpha}+\frac{\varphi_1}{\alpha}+\frac{\Psi_{m1}}{\alpha}\right)$$
$$M'_{m2}=\kappa K_{m2}\left(2\frac{\varphi_m}{\alpha}+\frac{\varphi_2}{\alpha}+\frac{\Psi_{m2}}{\alpha}\right)$$
$$\vdots$$
$$M'_{mn}=\kappa K_{mn}\left(2\frac{\varphi_m}{\alpha}+\frac{\varphi_n}{\alpha}+\frac{\Psi_{mn}}{\alpha}\right)$$

or

$$\left.\begin{aligned} M'_{m1} &= \frac{\kappa}{\alpha} M_{m1} \\ M'_{m2} &= \frac{\kappa}{\alpha} M_{m2} \\ &\vdots \\ M'_{mn} &= \frac{\kappa}{\alpha} M_{mn} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad \text{(i)}$$

In the above equations M' is the moment at the end of a member of Fig. 12 (b) due to the same loading as the frame of Fig. 12 (a).

Equations (i) are found to be very important relations and very practical of application to the calculation of secondary stresses.

Considering any truss of similar form, having the same stiffness ratio as well as the same sectional area-ratio as the standard one, while the stiffness values are κ times the standard one and the sectional areas α times the same, the numerical moment formulae for this truss becomes as follows by the use of the relation of equations (i) and the numerical moment formula of the standard truss which is represented by equation (A):

$$M_{mn} = (a \cdot n + b)\frac{\kappa}{\alpha} \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad \text{(B)}$$

in which constants a and b are the same as those for the standard truss, and n the number of panels in a span.

4. *Numerical Moment Formulae for Trusses with Sub-Verticals of Different Sectional Area-Ratio from Standard Trusses.*

The variation in the area of sub-verticals has great effect upon the secondary stresses in the chord members and some of the actual examples are different from the area-ratio in the standard truss as the area of these sub-verticals is independent of the number of panels in a span. Hence it is necessary to find this effect.

In the standard truss, let

a_{ot} = sectional area of tensile sub-verticals,

M_l = bending moment at the end of any member due to lower joint loads,

M_o = bending moment at the end of the same member in the case when sub-verticals are assumed not to extend due to the same loading.

Then the bending moment at the end of any member in the standard truss due to lower joint loads is expressed as follows:

$$M_l = M_o + (M_l - M_o) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(j)}$$

In the above equation, M_o is constant for any value of the sectional area of the sub-verticals when their stiffness is considered to be constant, and the difference between M_l and M_o, namely, $M_l - M_o$ gives the moment arising from the elastic elongation of the sub-verticals.

Therefore, this amount of $M_l - M_o$ should be directly proportional to the elastic elongation of the sub-verticals and accordingly inversely proportional to the sectional area of those members.

Therefore, when the sectional area of the tensile sub-verticals in the standard truss takes any amount a_t instead a_{ot}, the moment due to lower joint loads becomes

$$M = M_o + (M_l - M_o)\frac{a_{ot}}{a_t} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(k)}$$

In a similar way, when the sectional area of the compressive sub-verticals takes any amount of a_c instead of a_{oc}, the moment due to upper joint loads becomes

$$M = M_o + (M_u - M_o)\frac{a_{oc}}{a_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(l)}$$

where M_u is the moment of any member in the standard truss due to the upper joint loads.

Now, for example, a Warren truss of 10 panels with parallel chords is taken as shown in Fig. 13. The sectional areas of sub-verticals in this truss are all equally 0.15 which value is one-half of that in the standard truss. The stiffness values are assumed to be equal to the standard one.

In the standard truss, moments due to full lower joint loads having unit intensity are as follows:

$$M_{02} = -15.42, \quad M_{20} = -16.12, \quad M_{24} = 11.80, \quad M_{42} = -3.36 \quad \text{etc.}$$

When the sub-verticals are assumed not to extend, these moments become as follows:

$$M_{02} = -8.40, \quad M_{20} = -4.81, \quad M_{24} = 0.22, \quad M_{42} = -17.17 \quad \text{etc.}$$

Hence by equation (k) the moments of the truss of this example are computed as follows:

$$M_{02} = -8.40+(-15.42+8.40)\frac{0.30}{0.15} = -22.44(-22.44)$$

$$M_{20} = -4.81+(-16.12+4.81)\frac{0.30}{0.15} = -27.43(-27.45)$$

$$M_{24} = 0.22+(11.80-0.22)\frac{0.30}{0.15} = 23.38(23.40)$$

$$M_{42} = -17.17+(-3.36+17.17)\frac{0.30}{0.15} = 10.45(10.47)$$

Etc.

The above values in brackets are those calculated directly by the Slope Distribution Method.

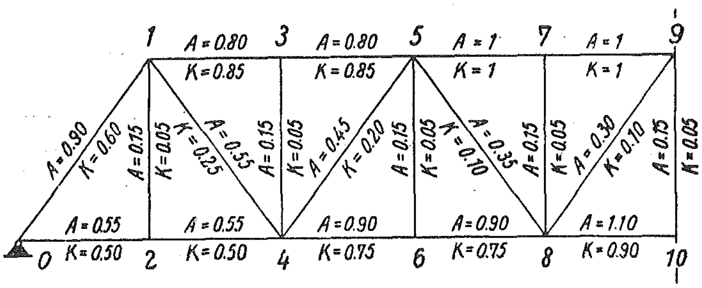

Fig. 13.

Even when the sub-verticals are assumed not to extend under any loads, the numerical moment formulae are also given for the standard trusses in the same form as equation (A), and yet, as the effect of the variation of the sectional area of the sub-verticals is little influenced by the variation of the number of panels in a span, the coefficient of n in the numerical moment formulae is considered to take the same value in both cases.

For example, moments and numerical moment formulae for the standard through Warren trusses with either parallel or polygonal chords having the full lower joint loads under the above mentioned assumption that the sub-verticals do not extend under any loads, are as follows:

Tables 11 and 12 show the moments for the truss with parallel chords and Tables 13 and 14 the moments for the truss with polygonal chords.

The intensity of the lower joint loads is taken as unity.

Table 11.

Moment	Number of Panels in Span					
	16	12	10	8	6	4
M_{01}	15.1 (14.50)	11.34 (10.70)	8.40 (8.80)	6.64 (6.90)	5.02 (5.00)	3.13 (3.10)
M_{10}	6.2 (7.08)	4.81 (4.56)	3.56 (3.30)	2.00 (2.04)	0.62 (0.78)	− 0.66 (− 0.48)
M_{02}	−15.1 (−14.50)	−11.34 (−10.70)	− 8.40 (− 8.80)	− 6.64 (− 6.90)	− 5.02 (− 5.00)	− 3.13 (− 3.10)
M_{20}	− 8.4 (− 8.10)	− 6.64 (− 6.70)	− 4.81 (− 6.00)	− 4.62 (− 5.30)	− 4.57 (− 4.60)	− 4.16 (− 3.90)
M_{13}	−27.3 (−28.00)	−18.99 (−19.40)	−15.20 (−15.00)	−10.64 (−10.60)	− 7.08 (− 6.20)	− 1.93 (− 1.80)
M_{31}	−30.9 (−13.23)	−22.68 (−24.30)	−21.20 (−20.85)	−17.25 (−17.39)	−14.17 (−13.93)	−11.34 (−10.47)
M_{24}	0.4 (0.24)	1.03 (1.08)	0.22 (1.50)	1.24 (1.92)	2.32 (2.34)	3.08 (2.76)
M_{42}	−27.5 (−28.36)	−20.28 (−20.72)	−17.17 (−16.90)	−12.94 (−13.08)	− 8.84 (− 9.26)	− 4.58 (− 5.44)
M_{35}	21.5 (21.80)	16.25 (18.00)	16.39 (16.10)	14.02 (14.20)	12.58 (12.30)	
M_{53}	−34.8 (−35.56)	−26.98 (−25.32)	−19.39 (−20.20)	−15.43 (−15.08)	− 8.70 (− 9.96)	
M_{46}	− 5.70 (− 5.72)	− 3.09 (− 1.64)	0.63 (0.40)	2.37 (2.44)	4.26 (4.48)	

Table 12.

Moment	Number of Panels in Span					
	16	12	10	8	6	4
M_{12}	6.6 (6.74)	4.61 (4.78)	3.80 (3.80)	2.78 (2.82)	1.83 (1.84)	0.87 (0.86)
M_{21}	8.0 (7.86)	5.61 (5.62)	4.58 (4.50)	3.38 (3.38)	2.25 (2.26)	1.09 (1.14)
M_{14}	14.5 (14.38)	9.57 (10.00)	7.84 (7.90)	5.86 (5.74)	3.66 (3.58)	1.72 (1.42)
M_{41}	7.6 (7.65)	4.38 (4.65)	2.99 (3.15)	1.72 (1.65)	0.14 (0.15)	−1.04 (−1.35)
M_{34}	9.4 (9.6)	6.43 (6.31)	4.82 (4.75)	3.24 (3.19)	1.59 (1.63)	0 (0.07)
M_{43}	8.2 (8.33)	5.60 (5.61)	4.25 (4.25)	3.06 (2.89)	1.38 (1.58)	0 (0.20)
M_{45}	17.2 (18.10)	12.52 (12.10)	9.32 (9.10)	6.03 (6.10)	3.07 (3.10)	
M_{54}	8.6 (9.00)	7.66 (5.00)	3.36 (3.00)	0.86 (1.00)	−1.04 (−1.00)	
M_{56}	8.8 (9.00)	5.35 (5.40)	3.65 (3.60)	1.70 (1.80)	0 (0)	
M_{65}	9.5 (9.70)	5.85 (5.82)	4.00 (3.88)	1.88 (1.94)	0 (0)	

Table 13.

Moment	Number of Panels in Span				
	12	10	8	6	4
M_{01}	9.41 (9.43)	7.50 (7.86)	6.46 (6.27)	4.70 (4.69)	3.09 (3.11)
M_{10}	− 0.65 (− 0.66)	− 0.29 (− 0.93)	− 1.20 (− 1.20)	− 1.49 (− 1.47)	− 0.20 (− 1.74)
M_{02}	− 9.41 (− 9.43)	− 7.50 (− 7.86)	− 6.46 (− 6.27)	− 4.70 (− 4.69)	− 3.09 (− 3.11)
M_{20}	− 8.30 (− 8.30)	− 6.56 (− 7.22)	− 6.93 (− 6.14)	− 5.88 (− 5.06)	− 3.85 (− 3.98)
M_{13}	− 9.77 (− 9.68)	− 7.82 (− 7.18)	− 4.66 (− 4.68)	− 2.19 (− 2.18)	− 2.13 (0.32)
M_{31}	−19.60 (−20.00)	−18.87 (−18.34)	−16.68 (−16.68)	−15.34 (−15.02)	−12.60 (−13.36)
M_{24}	3.71 (3.46)	2.93 (3.34)	4.16 (3.22)	4.05 (3.10)	2.74 (2.98)
M_{42}	−16.34 (−16.22)	−14.42 (−13.80)	−11.14 (−11.38)	− 8.80 (− 8.96)	− 5.67 (− 6.54)
M_{35}	15.65 (16.07)	16.07 (15.61)	15.15 (15.15)	15.01 (14.69)	
M_{53}	−12.64 (−12.90)	− 7.74 (− 7.20)	− 1.52 (− 1.30)	4.46 (4.20)	
M_{46}	0.54 (0.60)	3.67 (3.05)	5.31 (5.49)	7.63 (7.93)	

Table 14.

Moment	Number of Panels in Span				
	12	10	8	6	4
M_{12}	3.68 (3.70)	2.94 (2.96)	2.22 (2.22)	1.46 (1.48)	0.92 (0.74)
M_{21}	4.58 (4.60)	3.63 (3.68)	2.77 (2.76)	1.83 (1.84)	1.11 (0.92)
M_{14}	6.75 (6.70)	5.16 (5.20)	3.64 (3.70)	2.22 (2.20)	1.41 (0.70)
M_{41}	2.94 (2.90)	1.57 (1.60)	0.25 (0.30)	−0.97 (−1.00)	−0.93 (−2.30)
M_{34}	3.95 (3.93)	2.80 (2.73)	1.51 (1.53)	0.33 (0.33)	0 (−0.86)
M_{43}	3.56 (3.55)	2.50 (2.45)	1.34 (1.35)	0.25 (0.25)	0 (−0.85)
M_{45}	9.27 (9.16)	6.67 (6.70)	4.24 (4.24)	1.89 (1.78)	
M_{54}	4.55 (4.25)	2.52 (2.75)	1.24 (1.25)	−0.24 (−0.25)	
M_{56}	3.34 (3.20)	2.03 (2.12)	1.07 (1.06)	0 (0)	
M_{65}	3.77 (3.60)	2.31 (2.40)	1.21 (1.20)	0 (0)	

Numerical moment formulae are obtained by the use of the values in Tables 11 to 14 as follows:

Standard Through Warren Truss with Parallel Chords.

Moments of Chord Members:

M_{01} (portal 0–1, lower) = $(0.95\,n-0.70)$
M_{10} (portal 0–1, upper) = $(0.63\,n-3.00)$
M_{02} (lower chord 0–2, left) = $(-0.95\,n+0.70)$
M_{20} (lower chord 0–2, right) = $(-0.35\,n-2.50)$
M_{13} (upper chord 1–3, left) = $(-2.20\,n+7.00)$
M_{31} (upper chord 1–3, right) = $(-1.73\,n-3.55)$
M_{24} (lower chord 2–4, left) = $(-0.21\,n+3.60)$
M_{42} (lower chord 2–4, right) = $(-1.91\,n+2.20)$
M_{35} (upper chord 3–5, left) = $(0.95\,n+6.60)$
M_{53} (upper chord 3–5, right) = $(-2.56\,n+5.40)$
M_{46} (lower chord 4–6, left) = $(-1.02\,n+10.60)$
etc.

Moments of Web Members:

M_{12} (sub-vertical 1–2, upper) = $(0.49\,n-1.10)$
M_{21} (sub-vertical 1–2, lower) = $(0.56\,n-1.10)$
M_{14} (diagonal 1–4, upper) = $(1.08\,n-2.90)$
M_{41} (diagonal 1–4, lower) = $(0.75\,n-4.35)$
M_{34} (sub-vertical 3–4, upper) = $(0.78\,n-3.05)$
M_{43} (sub-vertical 3–4, lower) = $(0.68\,n-2.55)$
M_{45} (diagonal 4–5, lower) = $(1.50\,n-5.90)$
M_{54} (diagonal 4–5, upper) = $(1.00\,n-7.00)$
M_{56} (sub-vertical 5–6, upper) = $(0.90\,n-5.40)$
M_{65} (sub-vertical 5–6, lower) = $(0.97\,n-5.82)$
etc.

Standard Through Warren Truss with Polygonal Chords.

Moments of Chord Members:

M_{01} (portal 0–1, lower) = $(0.79\,n-0.05)$
M_{10} (portal 0–1, upper) = $(0.135\,n-2.28)$
M_{02} (lower chord 0–2, left) = $(-0.79\,n+0.05)$
M_{20} (lower chord 0–2, right) = $(-0.54\,n-1.82)$
M_{13} (upper chord 1–3, left) = $(-1.25\,n+5.32)$
M_{31} (upper chord 1–3, right) = $(-0.83\,n-10.04)$
M_{24} (lower chord 2–4, left) = $(0.06\,n+2.74)$

M_{42} (lower chord 2–4, right) $= (-1.21\,n-1.70)$
M_{35} (upper chord 3–5, left) $= (0.23\,n+13.31)$
M_{53} (upper chord 3–5, right) $= (-2.85\,n+21.30)$
M_{46} (lower chord 4–6, left) $= (-1.22\,n+15.25)$
etc.

Moments of Web Members:

M_{12} (sub-vertical 1–2, upper) $= (0.37\,n-0.74)$
M_{21} (sub-vertical 1–2, lower) $= (0.46\,n-0.92)$
M_{14} (diagonal 1–4, upper) $= (0.75\,n-2.30)$
M_{41} (diagonal 1–4, lower) $= (0.65\,n-4.90)$
M_{34} (sub-vertical 3–4, upper) $= (0.60\,n-3.27)$
M_{43} (sub-vertical 3–4, lower) $= (0.55\,n-3.05)$
M_{45} (diagonal 4–5, lower) $= (1.23\,n-5.60)$
M_{54} (diagonal 4–5, upper) $= (0.75\,n-4.75)$
M_{56} (sub-vertical 5–6, upper) $= (0.53\,n-3.18)$
M_{65} (sub-vertical 5–6, lower) $= (0.60\,n-3.60)$
etc.

Now, let the numerical moment formulae of M_o and M_l or M_u be respectively as follows:

$$\left.\begin{aligned} M_o &= a_o n + b_o \\ M_l \ \text{or} \ M_u &= an+b \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(m)}$$

Then the numerical moment formulae of the standard truss with sub-verticals of any area a_t or a_c become from equation (k) or (l) as follows:

$$\left.\begin{aligned} M &= (a_o n+b_o)+(an+b-a_o n-b_o)\frac{a_{ot}}{a_t} \\ \text{or} \quad M &= (a_o n+b_o)+(an+b-a_o n-b_o)\frac{a_{oc}}{a_c} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(n)}$$

The former is for the lower joint loads and the latter for the upper joint loads.

In the above formula the coefficients a_o and a are the same because of the above mentioned characteristic of them.

Therefore the above equation becomes

$$\left.\begin{aligned} &M=(a_o n+b_o)+(b-b_o)\frac{a_{ot}}{a_t} \\ \text{or} \qquad &M=(a_o n+b_o)+(b-b_o)\frac{a_{oc}}{a_c} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \text{(o)}$$

Since a_{ot} and a_{oc} are constants, equation (o) becomes

$$\left.\begin{aligned} &M=(a_o n+b_o)+c_t\frac{1}{a_t} \\ \text{or} \qquad &M=(a_o n+b_o)+c_c\frac{1}{a_c} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \text{(p)}$$

Omitting the suffixes of a, b and c, the numerical moment formula of any member $m-n$ in the standard truss with sub-verticals of any area a_t and a_c is generally represented in the form

$$\left.\begin{aligned} &M_{mn}=(a\cdot n+b)+c\cdot\frac{1}{a_t} \\ \text{or} \qquad &M_{mn}=(a\cdot n+b)+c\cdot\frac{1}{a_c} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \text{(C)}$$

in which, the term in bracket shows the moment of the standard truss in the case when sub-verticals are assumed not to extend and the last term shows the effect of the extention or shortening of sub-verticals.

For any truss of the same ratio both of stiffness and sectional area as the standard truss with sub-verticals of any area a_t and a_c, the numerical moment formulae become

$$\left.\begin{aligned} &M_{mn}=\left\{(a\cdot n+b)+c\cdot\frac{1}{a_t}\right\}\frac{\kappa}{\alpha} \\ \text{or} \qquad &M_{mn}=\left\{(a\cdot n+b)+c\cdot\frac{1}{a_c}\right\}\frac{\kappa}{\alpha} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \text{(D)}$$

When one lets

$a_{\max.}$ = the maximum sectional area in upper chord members of the standard truss,

$A_{\max.}$ = the maximum sectional area in upper chord members of the truss in question,

and

A_t , A_c = sectional areas of the tensile sub-venticals and compressive sub-verticals respectively,

in the above equation α, a_t and a_c are determined from the following relations:

$$\left.\begin{aligned} \alpha &= \frac{A_{\text{max.}}}{a_{\text{max.}}} \\ a_t &= \frac{A_t \cdot a_{\text{max.}}}{A_{\text{max.}}} \\ a_c &= \frac{A_c \cdot a_{\text{max.}}}{A_{\text{max.}}} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots \quad \text{(E)}$$

5. *Numerical Moment Formulae for Trusses with Members of Different Stiffness Ratio from Standard Trusses.* Even in the same sectional area the web members take different stiffness values according to the form of the sections. Some of them are stiffly built up to resist either tention or compression and their stiffness values are large. On the contrary, others are slenderly built to resist only tension and their stiffness values are small.

Hence, some of the actual examples are different from the standard trusses in their stiffess ratio.

The bending moment at the end A of member AB is expressed, as above, by equation (a), that is,

$$M_{ab} = K_{ab}(2\varphi_a + \varphi_b + \Psi_{ab})$$

The stiffness of web members is very small as compared with that of chord members and the effect of the variation of the stiffness of the web members is negligible upon the joint-rotation angles, φ_a and φ_b. Also the member rotation angle Ψ_{ab} is considered to be independent of the stiffness of the members of trusses and it has relation only to the sectional areas of the same.

Hence it may be said that the end moments of the chord members are not influenced by the stiffness values of the web members and these end moments of the web members are proportional to their stiffness values under the assumption that the sectional areas of the web members are constant for any stiffness value of them.

Because of this important characteristic, the numerical moment formulae of any truss which in its web members has different stiffness values from the standard truss is obtained from the formulae of the standard truss. These numerical moment formulae become

$$\left.\begin{aligned} M_{mn} &= \left\{(a\cdot n+b)+c\cdot\frac{1}{a_t}\right\}\frac{a_{\text{max.}}}{A_{\text{max.}}}\cdot\frac{K_{mn}}{k_{mn}} \\ \text{or}\qquad M_{mn} &= \left\{(a\cdot n+b)+c\cdot\frac{1}{a_c}\right\}\frac{a_{\text{max.}}}{A_{\text{max.}}}\cdot\frac{K_{mn}}{k_{mn}} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(F)}$$

in which

K_{mn} = stiffness of the member $m-n$ in the truss in question

and

k_{mn} = stiffness of the member $m-n$ in the standard truss.

Since the equilibrium condition at a joint, namely, $\Sigma M=0$, is disturbed by the above correction, this discrepancy of the equilibrium condition should be distributed to each member which meets at this joint, proportionary to their stiffness values.

Now, for example, let a through Warren truss be considered with 10 panels and stiff sub-verticals, having the stiffness value of 0.15, while the other members are the same as those of the standard truss, as shown in Fig. 14. This truss is subjected to full lower joint loads having unit intensity

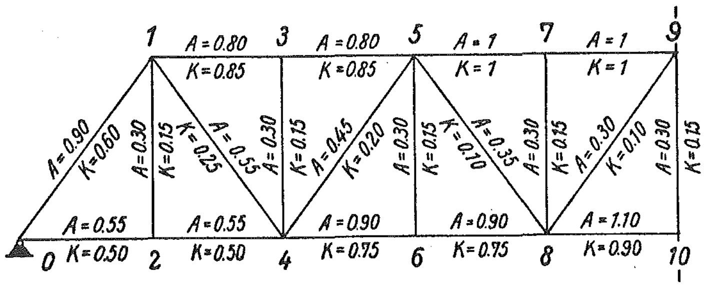

Fig 14.

The bending moments at the end of any members in such a truss are easily and rapidly determined by the use of the results obtained for the standard truss as follows:

The moments of the standard truss are such follows;

At joint 0,

$M_{01}=15.42,\qquad M_{02}=-15.42$

At joint 1,

$M_{10}=5.87,\qquad M_{13}=-17.20,\qquad M_{14}=7.77,\qquad M_{12}=3.56$

At joint 2,

$$M_{20} = -16.12, \qquad M_{24} = 11.80, \qquad M_{21} = 4.31$$

At joint 3,

$$M_{31} = -21.76, \qquad M_{35} = 16.80, \qquad M_{34} = 4.97$$

Etc.

From the above values, the moments of the truss in this example are determined as follows:

At joint 0, all the members which meet at this joint are the same as in the standard truss. Hence, the moments at this joint are also considered to be the same as those of the standard one with values as follows:

$$M_{01} = 15.42(16.04), \qquad M_{02} = -15.42(-16.04)$$

At joint 1, as the stiffness values of members, 1–0, 1–3, and 1–4 are the same as of the standard truss, the moments of these members are also the same as of the standard truss. However, the stiffness value of member, 1–2, is 3 times that of the standard truss. Hence, the moment of this member is also considered to be 3 times that of the standard truss. These values are as follows:

$$M_{10} = 5.87, \quad M_{13} = -17.20, \quad M_{14} = 7.77, \quad M_{12} = (3.56)\frac{0.15}{0.05} = 10.68$$

The discrepancy of the equilibrium condition at joint 1 due to the stiffness correction becomes as follows:

$$\Sigma M = 5.87 - 17.20 + 7.77 + 10.68 = 7.12$$

Distributing this discrepancy to each member in proportion to their stiffness values, the final moments at joint 1 can be determined as follows:

$$M_{10} = 5.87 - (7.12)\frac{0.60}{0.60+0.85+0.25+0.15} = 5.87 - (7.12)\frac{0.60}{1.87}$$

$$= 5.87 - 2.31 = 3.56(4.93)$$

$$M_{13} = -17.20 - (7.12)\frac{0.85}{1.85} = -17.20 - 3.27 = -20.47(21.41)$$

$$M_{14}=7.77-(7.12)\frac{0.25}{1.85}=7.77-0.96=6.81(6.73)$$

$$M_{12}=10.68-(7.12)\frac{0.15}{1.85}=10.68-0.58=10.10(9.75)$$

In the same way, at joint 2 one gets,

$$M_{20}=-19.87(19.11),\qquad M_{24}=8.05(7.50),\qquad M_{21}=11.81(11.61)$$

At joint 3,

$$M_{35}=12.30(12.43),\qquad M_{31}=-26.26(26.60),\qquad M_{34}=14.00(14.17)$$

Etc.

The values in brackets are the ones directly calculated by the Slope Distribution Method.

When the intensity of the joint loads is P instead of unity, the numerical moment formula (F) becomes

$$\left.\begin{aligned} M_{mn}&=\left\{(a\cdot n+b)+c\cdot\frac{1}{a_t}\right\}P\cdot\frac{a_{\max.}}{A_{\max.}}\cdot\frac{K_{mn}}{k_{mn}}\\ \text{or}\qquad M_{mn}&=\left\{(a\cdot n+b)+c\cdot\frac{1}{a_c}\right\}P\cdot\frac{a_{\max.}}{A_{\max.}}\cdot\frac{K_{mn}}{k_{mn}}\end{aligned}\right\}\quad\cdots\cdots\cdots\cdots\;(G)$$

This is the general expression of the numerical moment formulae.

When a, b and c are determined for the standard trusses, the moments of any truss can be rapidly calculated from equation (G).

In the following sections, there will be shown the values of a, b and c of any members of the following four most usually adopted types of truss for the full loads, maximum moments and single load which is placed at a relatively effective joint:

I. Through Warren truss with parallel chords.
II. Deck Warren truss with parallel chords.
III. Through Pratt truss with parallel chords.
IV. Through Warren truss with polygonal chords.

In chord members it will be seen that for some members, the secondary stresses are not at their maximum when the primary stresses are maximum, but inasmuch as the primary stress is generally much larger than the secondary stress, the total is generally a maximum when the primary stress is a maximum, that is, when a truss is fully loaded. Accordingly, the values a, b and c for maximum moments of the chord members are omitted here.

II. Numerical Moment Formulae for Through Warren Truss with Parallel Chords.

1. *Moments due to Full Lower Joint Loads.* Coeff. : $P\dfrac{a_{\max.}}{A_{\max.}}\dfrac{K_{mn}}{k_{mn}}$

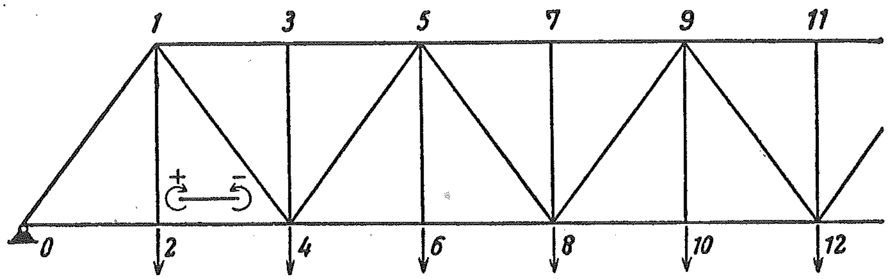

Fig. 15.

Moments of Chord Members :

M (portal 0–1, lower) = $(0.95\,n-0.70)+2.13/a_t$ .... (1)
M (portal 0–1, upper) = $(0.63\,n-3.00)+0.70/a_t$ .... (2)
M (lower chord 0–2, left) = $(-0.95\,n+0.70)-2.13/a_t$ .... (3)
M (lower chord 0–2, right) = $(-0.35\,n-2.50)-3.21/a_t$ .... (4)
M (lower chord 2–4, left) = $(-0.21\,n+3.60)+3.28/a_t$ .... (5)
M (lower chord 2–4, right) = $(-1.91\,n+2.20)+4.14/a_t$ .... (6)
M (lower chord 4–6, left) = $(-1.02\,n+10.60)-4.68/a_t$ .... (7)
M (lower chord 4–6, right) = $(-1.50\,n-2.25)-5.32/a_t$ .... (8)
M (lower chord 6–8, left) = $(0.53\,n+8.07)+5.32/a_t$ .... (9)
M (lower chord 6–8, right) = $(-3.20\,n+12.10)+5.49/a_t$ ...... (10)
M (lower chord 8–10, left) = $(0.02\,n+13.50)-5.58/a_t$ .... (11)
M (lower chord 8–10, right) = $(-2.33\,n-0.25)-6.43/a_t$ ...... (12)
M (lower chord 10–12, left) = $(1.00\,n+13.55)+6.43/a_t$ .... (13)
M (lower chord 10–12, right) = $(-6.40\,n+52.60)+6.66/a_t$ .... (14)
M (upper chord 1–3, left) = $(-2.20\,n+7.00)-0.60/a_t$ .... (15)
M (upper chord 1–3, right) = $(-1.73\,n-3.55)-0.16/a_t$ .... (16)
M (upper chord 3–5, left) = $(0.95\,n+6.60)+0.12/a_t$ .... (17)
M (upper chord 3–5, right) = $(-2.56\,n+5.40)+0.00/a_t$ .... (18)
M (upper chord 5–7, left) = $(-0.50\,n+14.00)-0.15/a_t$ .... (19)
M (upper chord 5–7, right) = $(-2.65\,n+0.50)-0.00/a_t$ ...... (20)
M (upper chord 7–9, left) = $(1.50\,n+8.70)+0.00/a_t$ .... (21)
M (upper chord 7–9, right) = $(-4.10\,n+18.60)+0.00/a_t$ ...... (22)
M (upper chord 9–11, left) = $(0.56\,n+16.22)-0.00/a_t$ .... (23)
M (upper chord 9–11, right) = $(-5.45\,n+31.90)-0.00/a_t$ ..... (24)

Moments of Web Members:

M (diagonal 1–4, upper) $= (1.08\,n-2.90)-0.03/a_t$ .... (25)
M (diagonal 1–4, lower) $= (0.75\,n-4.30)+0.22/a_t$ ... . (26)
M (diagonal 4–5, lower) $= (1.50\,n-5.90)+0.27/a_t$ .... (27)
M (diagonal 4–5, upper) $= (1.00\,n-7.00)+0.15/a_t$ .... (28)
M (diagonal 5–8, upper) $= (1.16\,n-7.00)+0.00/a_t$ .... (29)
M (diagonal 5–8, lower) $= (0.78\,n-7.10)+0.03/a_t$ .... (30)
M (diagonal 8–9, lower) $= (1.30\,n-9.70)+0.06/a_t$ .... (31)
M (diagonal 8–9, upper) $= (0.90\,n-10.00)+0.00/a_t$ .... (32)
M (diagonal 9–12, upper) $= (1.39\,n-12.32)+0.00/a_t$ .... (33)
M (diagonal 9–12, lower) $= (0.79\,n-10.65)+0.00/a_t$ .... (34)
M (sub-vertical 1–2, upper) $= (0.49\,n-1.10)-0.075/a_t$ .... (35)
M (sub-vertical 1–2, lower) $= (0.56\,n-1.10)-0.075/a_t$ .... (36)
M (sub-vertical 3–4, upper) $= (0.78\,n-3.05)+0.045/a_t$ .... (37)
M (sub-vertical 3–4, lower) $= (0.68\,n-2.55)+0.045/a_t$ .... (38)
M (sub-vertical 5–6, upper) $= (0.90\,n-5.40)-0.00/a_t$ .... (39)
M (sub-vertical 5–6, lower) $= (0.97\,n-5.82)-0.00/a_t$ .... (40)
M (sub-vertical 7–8, upper) $= (1.15\,n-9.20)+0.00/a_t$ .... (41)
M (sub-vertical 7–8, lower) $= (1.10\,n-8.80)+0.00/a_t$ .... (42)
M (sub-vertical 9–10, upper) $= (1.25\,n-12.50)-0.00/a_t$ .... (43)
M (sub-vertical 9–10, lower) $= (1.33\,n-13.30)-0.00/a_t$ .... (44)
M (sub-vertical 11–12, upper) $= (1.37\,n-16.44)+0.00/a_t$ ...... (45)
M (sub-vertical 11–12, lower) $= (1.28\,n-15.36)+0.00/a_t$ ...... (46)

2. *Moments due to Full Upper Joint Loads.* Coeff.: $P\dfrac{a_{\max.}}{A_{\max.}}\dfrac{K_{mn}}{k_{mn}}$

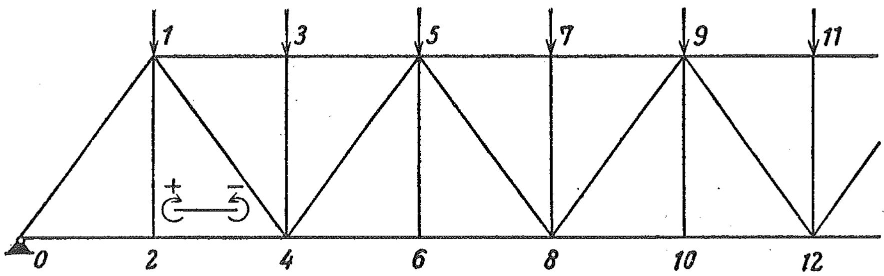

Fig. 16.

Moments of Chord Members:

M (portal 0–1, lower) $= (0.95\,n-0.70)+0.51/a_c$ .... (47)
M (portal 0–1, upper) $= (0.63\,n-3.00)+2.16/a_c$ .... (48)
M (lower chord 0–2, left) $= (-0.95\,n+0.70)-0.51/a_c$ .... (49)

M (lower chord 0–2, right) $= (-0.35\,n-3.00)+0.15/a_c$ .... (50)
M (lower chord 2–4, left) $= (-0.21\,n+4.10)-0.24/a_c$ .... (51)
M (lower chord 2–4, right) $= (-1.91\,n+2.20)-0.12/a_c$ .... (52)
M (lower chord 4–6, left) $= (-1.02\,n+10.60)-0.22/a_c$ .... (53)
M (lower chord 4–6, right) $= (-1.50\,n-2.25)-0.075/a_c$ ...... (54)
M (lower chord 6–8, left) $= (0.53\,n+8.07)+0.075/a_c$ .... (55)
M (lower chord 6–8, right) $= (-3.20\,n+12.10)+0.00/a_c$ ...... (56)
M (lower chord 8–10, left) $= (0.02\,n+13.50)+0.00/a_c$ .... (57)
M (lower chord 8–10, right) $= (-2.33\,n-0.25)+0.00/a_c$ ...... (58)
M (lower chord 10–12, left) $= (1.00\,n+13.55)+0.00/a_c$ .... (59)
M (lower chord 10–12, right) $= (-6.40\,n+52.60)+0.00/a_c$ .... (60)
M (upper chord 1–3, left) $= (-2.20\,n+7.00)-3.34/a_c$ .... (61)
M (upper chord 1–3, right) $= (-1.73\,n-3.55)-5.47/a_c$ .... (62)
M (upper chord 3–5, left) $= (0.95\,n+6.60)+5.52/a_c$ .... (63)
M (upper chord 3–5, right) $= (-2.56\,n+5.40)+6.66/a_c$ .... (64)
M (upper chord 5–7, left) $= (-0.50\,n+14.00)-6.84/a_c$ .... (65)
M (upper chord 5–7, right) $= (-2.65\,n+0.50)-7.35/a_c$ .... (66)
M (upper chord 7–9, left) $= (1.50\,n+8.70)+7.35/a_c$ .... (67)
M (upper chord 7–9, right) $= (-4.10\,n+18.60)+7.50/a_c$ ...... (68)
M (upper chord 9–11, left) $= (0.56\,n+16.22)-7.50/a_c$ .... (69)
M (upper chord 9–11, right) $= (-5.45\,n+31.90)-7.92/a_c$ .... (70)

Moments of Web Members:

M (diagonal 1–4, upper) $= (1.08\,n-2.90)+0.99/a_c$ .... (71)
M (diagonal 1–4, lower) $= (0.75\,n-4.35)+0.43/a_c$ .... (72)
M (diagonal 4–5, lower) $= (1.50\,n-5.90)-0.03/a_c$ .... (73)
M (diagonal 4–5, upper) $= (1.00\,n-7.00)+0.12/a_c$ .... (74)
M (diagonal 5–8, upper) $= (1.16\,n-7.00)+0.06/a_c$ .... (75)
M (diagonal 5–8, lower) $= (0.78\,n-7.10)+0.00/a_c$ .... (76)
M (diagonal 8–9, lower) $= (1.30\,n-9.70)+0.00/a_c$ .... (77)
M (diagonal 8–9, upper) $= (0.90\,n-10.00)+0.00/a_c$ .... (78)
M (diagonal 9–12, upper) $= (1.39\,n-12.32)+0.00/a_c$ .... (79)
M (diagonal 9–12, lower) $= (0.79\,n-10.65)+0.00/a_c$ .... (80)
M (sub-vertical 1–2, upper) $= (0.49\,n-1.10)+0.19/a_c$ .... (81)
M (sub-vertical 1–2, lower) $= (0.56\,n-1.10)+0.09/a_c$ .... (82)
M (sub-vertical 3–4, upper) $= (0.78\,n-3.05)-0.045/a_c$ .... (83)
M (sub-vertical 3–4, lower) $= (0.68\,n-2.55)-0.06/a_c$ .... (84)
M (sub-vertical 5–6, upper) $= (0.90\,n-5.40)+0.00/a_c$ .... (85)
M (sub-vertical 5–6, lower) $= (0.97\,n-5.82)+0.00/a_c$ .... (86)
M (sub-vertical 7–8, upper) $= (1.15\,n-9.20)-0.00/a_c$ .... (87)
M (sub-vertical 7–8, lower) $= (1.10\,n-8.80)-0.00/a_c$ .... (88)

M (sub-vertical 9–10, upper) $= (1.25\,n-12.50)+0.00/a_c$ .... (89)
M (sub-vertical 9–10, lower) $= (1.33\,n-13.30)+0.00/a_c$ .... (90)
M (sub-vertical 11–12, upper) $= (1.37\,n-16.44)-0.00/a_c$ ...... (91)
M (sub-vertical 11–12, lower) $= (1.28\,n-15.36)-0.00/a_c$ ...... (92)

3. *Maximum Moments due to Live Loads.* Coeff.: $P\dfrac{a_{\max.}}{A_{\max.}}\dfrac{K_{mn}}{k_{mn}}$

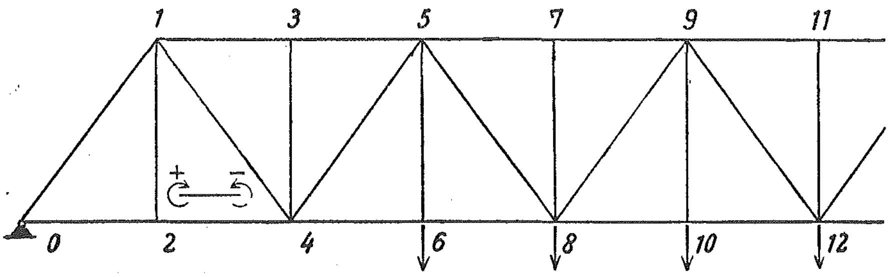

Fig. 17.

Moments of Web Members:

M (diagonal 1–4, upper) $= (1.08\,n-2.80)+0.27/a_t$ .... (93)
M (diagonal 1–4, lower) $= (0.62\,n-1.62)+0.855/a_t$ .... (94)
M (diagonal 4–5, lower) $= (1.50\,n-5.53)+0.70/a_t$ .... (95)
M (diagonal 4–5, upper) $= (1.00\,n-4.55)+0.105/a_t$ .... (96)
M (diagonal 5–8, upper) $= (1.00\,n-4.90)+0.03/a_t$ .... (97)
M (diagonal 5–8, lower) $= (0.70\,n-4.10)+0.36/a_t$ .... (98)
M (diagonal 8–9, lower) $= (1.28\,n-8.30)+0.39/a_t$ .... (99)
M (diagonal 8–9, upper) $= (0.74\,n-5.10)+0.075/a_t$ .... (100)
M (diagonal 9–12, upper) $= 5.97+0.00/a_t(n=12)$ .... (101)
M (diagonal 9–12, lower) $= 2.70+0.365/a_t(n=12)$ .... (102)
M (sub-vertical 1–2, upper) $= (0.49\,n-1.12)-0.04/a_t$ .... (103)
M (sub-vertical 1–2, lower) $= (0.56\,n-1.10)-0.06/a_t$ .... (104)
M (sub-vertical 3–4, upper) $= (0.78\,n-2.75)+0.09/a_t$ .... (105)
M (sub-vertical 3–4, lower) $= (0.68\,n-2.35)+0.18/a_t$ .... (106)
M (sub-vertical 5–6, upper) $= (0.85\,n-4.15)-0.03/a_t$ .... (107)
M (sub-vertical 5–6, lower) $= (0.91\,n-4.40)-0.045/a_t$ .... (108)
M (sub-vertical 7–8, upper) $= (1.07\,n-6.61)+0.09/a_t$ .... (109)
M (sub-vertical 7–8, lower) $= (1.01\,n-6.30)+0.20/a_t$ .... (110)
M (sub-vertical 9–10, upper) $= (0.98\,n-7.00)-0.03/a_t$ .... (111)
M (sub-vertical 9–10, lower) $= (1.04\,n-7.50)-0.05/a_t$ .... (112)
M (sub-vertical 11–12, upper) $= 3.41+0.09/a_t(n=12)$ .... (113)
M (sub-vertical 11–12, lower) $= 3.27+0.18/a_t(n=12)$ .... (114)

4. *Moments due to a Single Load.* Coeff.: $P\frac{a_{\max.}}{A_{\max.}}\frac{K_{mn}}{k_{mn}}$

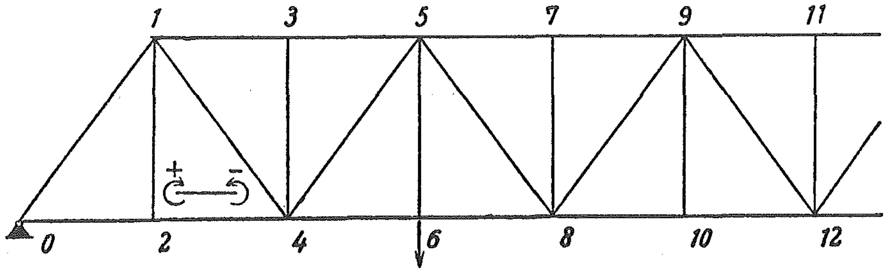

Fig. 18.

Moments of Chord Members:

M (portal 0–1, lower)(load at joint 2) $= 1.70+2.02/a_t$ .... (115)
M (portal 0–1, upper)(load at joint 2) $= -3.30+0.78/a_t$ .... (116)
(load at joint 4) $= 2.90$ .... (117)
M (lower chord 0–2, left)(load at joint 2) $= -1.70-2.02/a_t$ ... (118)
M (lower chord 0–2, right)(load at joint 2) $= -4.55-3.04/a_t$. (119)
M (lower chord 2–4, left)(load at joint 2) $= 4.40+3.04/a_t$ ..... (120)
M (lower chord 2–4, right)(load at joint 2) $= 0.85+2.58/a_t$ ... (121)
(load at joint 4) $= -7.30$ .... (122)
M (lower chord 4–6, left)(load at joint 4) $= 7.00$ .... (123)
(load at joint 6) $= -3.80-3.11/a_t$ ... (124)
M (lower chord 4–6, right)(load at joint 6) $= -11.70-4.38/a_t$. (125)
M (lower chord 6–8, left)(load at joint 6) $= 11.90+4.40/a_t$ .... (126)
M (lower chord 6–8, right)(load at joint 6) $= 2.15+3.10/a_t$ .... (127)
(load at joint 8) $= -13.80$ .... (128)
(load at joint 10) $= -0.15+2.49/a_t$. (129)
M (lower chord 8–10, left)(load at joint 6) $= 0.70-2.47/a_t$ .... (130)
(load at joint 8) $= 14.20$ .... (131)
(load at joint 10) $= -4.10-3.39/a_t$. (132)
M (lower chord 8–10, right)(load at joint 10) $= -17.80-5.08/a_t$ (133)
M (lower chord 10–12, left)(load at joint 10) $= 17.80+5.13/a_t$ .. (134)
M (lower chord 10–12, right)(load at joint 10) $= 3.10+3.60/a_t$. (135)
(load at joint 12) $= -16.55$ .... (136)
(load at joint 14) $= -0.35+2.71/a_t$ ...... (137)
M (upper chord 1–3, left)(load at joint 2) $= 3.00-0.36/a_t$ ..... (138)
(load at joint 4) $= -5.05$ .... (139)
M (upper chord 1–3, right)(load at joint 4) $= -11.15$ .... (140)
M (upper chord 3–5, left)(load at joint 4) $= 11.00$ .... (141)

M (upper chord 3–5, right)(load at joint 6) $= -13.25-0.075/a_t$ (142)
M (upper chord 5–7, left)(load at joint 6) $= 13.60-0.075/a_t$ ... (143)
M (upper chord 5–7, right)(load at joint 8) $= -18.35$ .... (144)
M (upper chord 7–9, left)(load at joint 8) $= 18.40$ .... (145)
M (upper chord 7–9, right)(load at joint 10) $= -19.25-0.015/a_t$ (146)
M (upper chord 9–11, left)(load at joint 10) $= 19.45+0.015/a_t$. (147)
M (upper chord 9–11, right)(load at joint 12) $= -21.00$ ...... (148)

Moments of Web Members:

M (diagonal 1–4, upper)(load at joint 2) $= 0.30-0.39/a_t$ ...... (149)
(load at joint 6) $= 1.50+0.36/a_t$ .... (150)
M (diagonal 1–4, lower)(load at joint 2) $= -0.75-0.61/a_t$ .... (151)
(load at joint 6) $= 1.80+0.81/a_t$ .... (152)
M (diagonal 4–5, lower)(load at joint 2) $= -0.30-0.43/a_t$ .... (153)
(load at joint 6) $= 1.57+0.67/a_t$ .... (154)
M (diagonal 4–5, upper)(load at joint 2) $= -0.68-0.20/a_t$ .... (155)
(load at joint 8) $= 2.20$ .... (156)
M (diagonal 5–8, upper)(load at joint 4) $= -0.45$ .... (157)
(load at joint 10) $= 1.25+0.18/a_t$ .... (158)
M (diagonal 5–8, lower)(load at joint 6) $= -0.75-0.31/a_t$ .... (159)
(load at joint 10) $= 1.25+0.37/a_t$ .... (160)
M (diagonal 8–9, lower)(load at joint 6) $= -0.72-0.31/a_t$ .... (161)
(load at joint 10) $= 1.25+0.37/a_t$ .... (162)
M (diagonal 8–9, upper)(load at joint 6) $= -0.95-0.15/a_t$ .... (163)
(load at joint 12) $= 1.25$ .... (164)
M (diagonal 9–12, upper)(load at joint 8) $= -0.90$ .... (165)
(load at joint 14) $= 1.45+0.16/a_t$ .... (166)
M (diagonal 9–12, lower)(load at joint 10) $= -0.95-0.34/a_t$ .. (167)
(load at joint 14) $= 1.20+0.34/a_t$ .... (168)
M (sub-vertical 1–2, upper)(load at joint 4) $= 1.00$ .... (169)
M (sub-vertical 1–2, lower)(load at joint 4) $= 1.10$ .... (170)
M (sub-vertical 3–4, upper)(load at joint 2) $= -0.25-0.045/a_t$. (171)
(load at joint 6) $= 1.35+0.09/a_t$ .... (172)
M (sub-vertical 3–4, lower)(load at joint 2) $= -0.23-0.11/a_t$.. (173)
(load at joint 6) $= 1.25+0.16/a_t$ .... (174)
M (sub-vertical 5–6, upper)(load at joint 4) $= -0.60$ .... (175)
(load at joint 8) $= 1.35$ .... (176)
M (sub-vertical 5–6, lower)(load at joint 4) $= -0.60$ .... (177)
(load at joint 8) $= 1.40$ .... (178)
M (sub-vertical 7–8, upper)(load at joint 6) $= -0.95-0.075/a_t$. (179)
(load at joint 10) $= 1.50+0.10/a_t$ .... (180)

M (sub-vertical 7–8, lower)(load at joint 6) $= -0.95 - 0.15/a_t$ .. (181)
(load at joint 10) $= 1.50 + 0.18/a_t$ .... (182)
M (sub-vertical 9–10, upper)(load at joint 8) $= -1.20$ .... (183)
(load at joint 12) $= 1.40$ .... (184)
M (sub-vertical 9–10, lower)(load at joint 8) $= -1.10$ .... (185)
(load at joint 12) $= 1.60$ .... (186)
M (sub-vertical 11–12, upper)(load at joint 10) $= -1.35 - 0.09/a_t$ (187)
(load at joint 14) $= 1.35 + 0.09/a_t$ .... (188)
M (sub-vertical 11–12, lower)(load at joint 10) $= -1.30 - 0.18/a_t$ (189)
(load at joint 14) $= 1.30 + 0.18/a_t$ .... (190)

III. Numerical Moment Formulae for Deck Warren Truss with Parallel Chords.

1. *Moments due to Full Upper Joint Loads (except O_1 and O_1')*

Coeff. : $P\dfrac{a_{\max.}}{A_{\max.}}\dfrac{K_{mn}}{k_{mn}}$

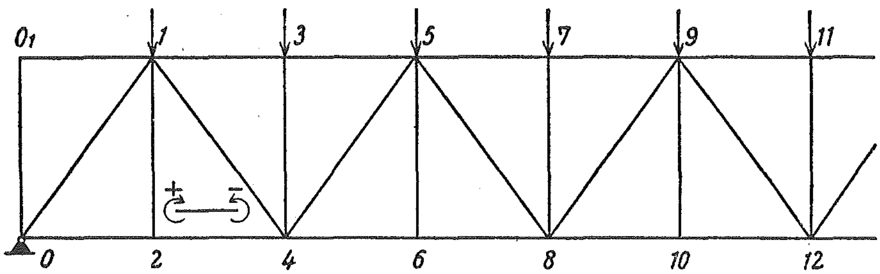

Fig. 19.

Moments of Chord Members:

M (lower chord 0–2, left) $= (-1.15\,n + 0.58) - 0.40/a_c$ .... (191)
M (lower chord 0–2, right) $= (-0.42\,n - 2.43) - 0.13/a_c$ .... (192)
M (lower chord 2–4, left) $= (-0.18\,n + 3.63) + 0.04/a_c$ .... (193)
M (lower chord 2–4, right) $= (-1.80\,n + 2.10) - 0.06/a_c$ .... (194)
M (lower chord 4–6, left) $= (-1.21\,n + 10.80) - 0.21/a_c$ .... (195)
M (lower chord 4–6, right) $= (-1.50\,n - 2.25) - 0.075/a_c$ ...... (196)
M (lower chord 6–8, left) $= (0.53\,n + 8.07) + 0.075/a_c$ .... (197)
M (lower chord 6–8, right) $= (-3.20\,n + 12.10) + 0.00/a_c$ ...... (198)
M (lower chord 8–10, left) $= (0.02\,n + 13.50) + 0.00/a_c$ .... (199)
M (lower chord 8–10, right) $= (-2.33\,n - 0.25) + 0.00/a_c$ ...... (200)
M (lower chord 10–12, left) $= (1.00\,n + 13.35) + 0.00/a_c$ .... (201)

M (lower chord 10–12, right) = $(-6.40\,n+52.60)+0.00/a_c$ .... (202)
M (upper chord 0_1–1, left) = $(-0.39\,n+0.23)+0.10/a_c$ .... (203)
M (upper chord 0_1–1, right) = $(-0.91\,n-0.90)+1.03/a_c$ ...... (204)
M (upper chord 1–3, left) = $(-1.66\,n+7.40)-3.82/a_c$ .... (205)
M (npper chord 1–3, right) = $(-1.58\,n-2.85)-5.65/a_c$ ...... (206)
M (upper chord 3–5, left) = $(0.83\,n+5.70)+5.70/a_c$ .... (207)
M (upper chord 3–5, right) = $(-2.56\,n+5.40)+6.66/a_c$ ...... (208)
M (upper chord 5–7, left) = $(-0.50\,n+14.00)-6.84/a_c$ .... (209)
M (upper chord 5–7, right) = $(-2.65\,n+0.50)-7.35/a_c$ .... (210)
M (upper chord 7–9, left) = $(1.50\,n+8.70)+7.35/a_c$ .... (211)
M (upper chord 7–9, right) = $(-4.10\,n+18.60)+7.50/a_c$ ..... (212)
M (upper chord 9–11, left) = $(0.56\,n+16.22)-7.50/a_c$ .... (213)
M (upper chord 9–11, right) = $(-5.45\,n+31.9)-7.92/a_c$ ..... (214)

Moments of Web Members:

M (diagonal 0–1, lower) = $(0.83\,n-0.43)+0.48/a_c$ .... (215)
M (diagonal 0–1, upper) = $(0.85\,n-2.80)+1.78/a_c$ .... (216)
M (diagonal 1–4, upper) = $(1.20\,n-2.60)+0.84/a_c$ .... (217)
M (diagonal 1–4, lower) = $(0{\cdot}83\,n-4.45)+0.36/a_c$ .... (218)
M (diagonal 4–5, lower) = $(1.50\,n-5.90)-0.03/a_c$ .... (219)
M (diagonal 4–5, upper) = $(1.00\,n-7.00)+0.12/a_c$ .... (220)
M (diagonal 5–8, upper) = $(1.16\,n-7.00)+0.06/a_c$ .... (221)
M (diagonal 5–8, lower) = $(0.78\,n-7.10)+0.00/a_c$ .... (222)
M (diagonal 8–9, lower) = $(1.30\,n-9.70)+0.00/a_c$ .... (223)
M (diagonal 8–9, upper) = $(0.90\,n-10.00)+0.00/a_c$ .... (224)
M (diagonal 9-12, upper) = $(1.39\,n-12.32)+0.00/a_c$ .... (225)
M (diagonal 9–12, lower) = $(0.79\,n-10.65)+0.00/a_c$ .... (226)
M (sub-vertical 0_1–0, upper) = $(0.39\,n-0.23)-0.10/a_c$ .... (227)
M (sub-vertical 0_1–0, lower) = $(0.32\,n-0.15)-0.08/a_c$ .... (228)
M (sub-vertical 1–2, upper) = $(0.52\,n-1.10)+0.17/a_c$ .... (229)
M (sub-vertical 1–2, lower) = $(0.60\,n-1.20)+0.09/a_c$ .... (230)
M (sub-vertical 3–4, upper) = $(0.75\,n-2.85)-0.045/a_c$ .... (231)
M (sub-vertical 3–4, lower) = $(0.68\,n-2.55)-0.06/a_c$ .... (232)
M (sub-vertical 5–6, upper) = $(0.90\,n-5.40)+0.00/a_c$ .... (233)
M (sub-vertical 5–6, lower) = $(0.97\,n-5.82)+0.00/a_c$ .... (234)
M (sub-vertical 7–8, upper) = $(1.15\,n-9.20)-0.00/a_c$ .... (235)
M (sub-vertical 7–8, lower) = $(1.10\,n-8.80)-0.00/a_c$ .... (236)
M (sub-vertical 9–10, upper) = $(1.25\,n-12.50)+0.00/a_c$ .... (237)
M (sub-vertical 9–10, lower) = $(1.33\,n-13.30)+0.00/a_c$ .... (238)
M (sub-vertical 11–12, upper) = $(1.37\,n-16.44)-0.00/a_c$ ..... (239)
M (sub-vertical 11–12, lower) = $(1.28\,n-15.36)-0.00/a_c$ ...... (240)

2. *Moments due to Full Lower Joint Loads.* Coeff. : $P\frac{a_{\max.}}{A_{\max.}}\frac{K_{mn}}{k_{mn}}$

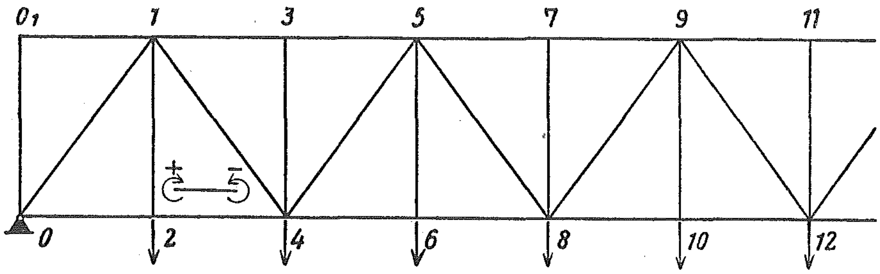

Fig. 20.

Moments of Chord Members:

M (lower chord 0–2, left) = $(-1.15\,n+0.58)-2.20/a_t$ .... (241)
M (lower chord 0–2, right) = $(-0.42\,n-2.43)-3.41/a_t$ .... (242)
M (lower chord 2–4, left) = $(-0.18\,n+3.63)+3.48/a_t$ .... (243)
M (lower chord 2–4, right) = $(-1.80\,n+2.10)+4.11/a_t$ .... (244)
M (lower chord 4–6, left) = $(-1.21\,n+10.80)-4.63/a_t$ .... (245)
M (lower chord 4–6, right) = $(-1.50\,n-2.25)-5.32/a_t$ .... (246)
M (lower chord 6–8, left) = $(0.53\,n+8.07)+5.32/a_t$ .... (247)
M (lower chord 6–8, right) = $(-3.20\,n+12.10)+5.49/a_t$ ...... (248)
M (lower chord 8–10, left) = $(0.02\,n+13.50)-5.58/a_t$ .... (249)
M (lower chord 8–10, right) = $(-2.33\,n-0.25)-6.43/a_t$ ...... (250)
M (lower chord 10–12, left) = $(1.00\,n+13.55)+6.43/a_t$ .... (251)
M (lower chord 10–12, right) = $(-6.40\,n+52.60)+6.66/a_t$ .... (252)
M (upper chord 0_1–1, left) = $(-0.39\,n+0.23)-0.10/a_t$ .... (253)
M (upper chord 0_1–1, right) = $(-0.91\,n-0.90)-0.21/a_t$ ...... (254)
M (upper chord 1–3, left) = $(-1.66\,n+7.40)-0.49/a_t$ .... (255)
M (upper chord 1–3, right) = $(-1.58\,n-2.85)-0.15/a_t$ .... (256)
M (upper chord 3–5, left) = $(0.83\,n+5.70)+0.12/a_t$ .... (257)
M (upper chord 3–5, right) = $(-2.56\,n+5.40)+0.00/a_t$ ...... (258)
M (upper chord 5–7, left) = $(-0.50\,n+14.00)-0.15/a_t$ .... (259)
M (upper chord 5–7, right) = $(-2.65\,n+0.50)-0.00/a_t$ .... (260)
M (upper chord 7–9, left) = $(1.50\,n+8.70)+0.00/a_t$ .... (261)
M (upper chord 7–9, right) = $(-4.10\,n+18.60)+0.00/a_t$ ...... (262)
M (upper chord 9–11, left) = $(0.56\,n+16.22)-0.00/a_t$ .... (263)
M (upper chord 9–11, right) = $(-5.45\,n+31.9)-0.00/a_t$ ...... (264)

Moments of Web Members:

M (diagonal 0–1, lower) = $(0.83\,n-0.43)+2.02/a_t$ .... (265)
M (diagonal 0–1, upper) = $(0.85\,n-2.80)+0.76/a_t$ .... (266)

M (diagonal 1–4, upper) $= (1.20\,n-2.60)+0.00/a_t$ .... (267)
M (diagonal 1–4, lower) $= (0.83\,n-4.45)+0.24/a_t$ .... (268)
M (diagonal 4–5, lower) $= (1.50\,n-5.90)+0.24/a_t$ .... (269)
M (diagonal 4–5, upper) $= (1.00\,n-7.00)+0.15/a_t$ .... (270)
M (diagonal 5–8, upper) $= (1.16\,n-7.00)+0.00/a_t$ .... (271)
M (diagonal 5–8, lower) $= (0.78\,n-7.10)+0.03/a_t$ .... (272)
M (diagonal 8–9, lower) $= (1.30\,n-9.70)+0.06/a_t$ .... (273)
M (diagonal 8–9, upper) $= (0.90\,n-10.00)+0.00/a_t$ .... (274)
M (diagonal 9–12, upper) $= (1.39\,n-12.32)+0.00/a_t$ .... (275)
M (diagonal 9–12, lower) $= (0.79\,n-10.65)+0.00/a_t$ .... (276)
M (sub-vertical 0_1–0, upper) $= (0.39\,n-0.23)+0.10/a_t$ .... (277)
M (sub-vertical 0_1–0, lower) $= (0.32\,n-0.15)+0.18/a_t$ .... (278)
M (sub-vertical 1–2, upper) $= (0.52\,n-1.10)-0.06/a_t$ .... (279)
M (sub-vertical 1–2, lower) $= (0.60\,n-1.20)-0.075/a_t$ .... (280)
M (sub-vertical 3–4, upper) $= (0.75\,n-2.85)+0.03/a_t$ .... (281)
M (sub-vertical 3–4, lower) $= (0.68\,n-2.55)+0.045/a_t$ .... (282)
M (sub-vertical 5–6, upper) $= (0.90\,n-5.40)-0.00/a_t$ .... (283)
M (sub-vertical 5–6, lower) $= (0.97\,n-5.82)-0.00/a_t$ .... (284)
M (sub-vertical 7–8, upper) $= (1.15\,n-9.20)+0.00/a_t$ .... (285)
M (sub-vertical 7–8, lower) $= (1.10\,n-8.80)+0.00/a_t$ .... (286)
M (sub-vertical 9–10, upper) $= (1.25\,n-12.50)-0.00/a_t$ .... (287)
M (sub-vertical 9–10, lower) $= (1.33\,n-13.30)-0.00/a_t$ .... (288)
M (sub-vertical 11–12, upper) $= (1.37\,n-16.44)+0.00/a_t$ ...... (289)
M (sub-vertical 11–12, lower) $= (1.28\,n-15.36)+0.00/a_t$ ...... (290)

3. *Maximum Moments due to Live Loads.* Coeff.: $P\dfrac{a_{\max.}}{A_{\max.}}\dfrac{K_{mn}}{k_{mn}}$

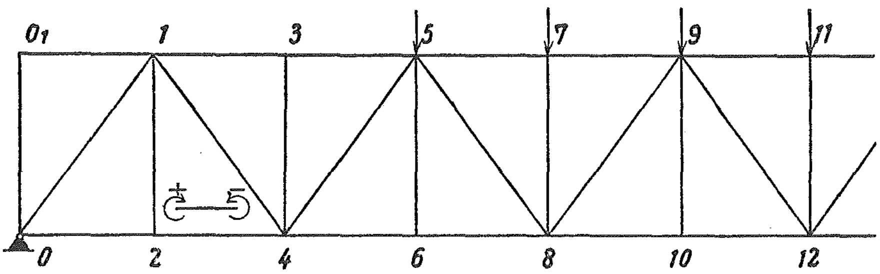

Fig. 21.

Moments of Web Members:

M (diagonal 0–1, lower) $= (0.83\,n-0.43)+0.48/a_c$ .... (291)
M (diagonal 0–1, upper) $= (0.85\,n-0.60)+1.75/a_c$ .... (292)

M (diagonal 1–4, upper) $= (1.20\,n-2.60)+0.84/a_c$ .... (293)
M (diagonal 1–4, lower) $= (0.66\,n-1.70)+0.075/a_c$ .... (294)
M (diagonal 4–5, lower) $= (1.50\,n-5.50)+0.06/a_c$ .... (295)
M (diagonal 4–5, upper) $= (1.13\,n-5.60)+0.70/a_c$ .... (296)
M (diagonal 5–8, upper) $= (1.00\,n-4.90)+0.37/a_c$ .... (297)
M (diagonal 5–8, lower) $= (0.70\,n-4.10)+0.03/a_c$ .... (298)
M (diagonal 8–9, lower) $= (1.28\,n-8.30)+0.06/a_c$ .... (299)
M (diagonal 8–9, upper) $= (0.74\,n-5.10)+0.37/a_c$ .... (300)
M (diagonal 9–12, upper) $= 5.97+0.39/a_c (n = 12)$ .... (301)
M (diagonal 9–12, lower) $= 2.70+0.09/a_c (n = 12)$ .... (302)
M (sub-vertical 0_1–0, upper) $= (0.39\,n-0.23)-0.10/a_c$ .... (303)
M (sub-vertical 0_1–0, lower) $= (0.32\,n-0.15)-0.08/a_c$ .... (304)
M (sub-vertical 1–2, upper) $= (0.52\,n-1.10)+0.17/a_c$ .... (305)
M (sub-vertical 1–2, lower) $= (0.60\,n-1.20)+0.09/a_c$ .... (306)
M (sub-vertical 3–4, upper) $= (0.73\,n-2.35)-0.045/a_c$ .... (307)
M (sub-vertical 3–4, lower) $= (0.65\,n-2.15)-0.045/a_c$ .... (308)
M (sub-vertical 5–6, upper) $= (0.85\,n-4.15)+0.18/a_c$ .... (309)
M (sub-vertical 5–6, lower) $= (0.91\,n-4.40)+0.09/a_c$ .... (310)
M (sub-vertical 7–8, upper) $= (1.07\,n-6.61)-0.06/a_c$ .... (311)
M (sub-vertical 7–8, lower) $= (1.01\,n-6.30)-0.015/a_c$ .... (312)
M (sub-vertical 9–10, upper) $= (0.98\,n-7.00)+0.18/a_c$ .... (313)
M (sub-vertical 9–10, lower) $= (1.04\,n-7.50)+0.09/a_c$ .... (314)
M (sub-vertical 11–12, upper) $= 3.41-0.05/a_c (n = 12)$ .... (315)
M (sub-vertical 11–12, lower) $= 3.27-0.03/a_c (n = 12)$ .... (316)

4. *Moments due to a Single Load.* Coeff.: $P\dfrac{a_{\max.}}{A_{\max.}}\dfrac{K_{mn}}{k_{mn}}$

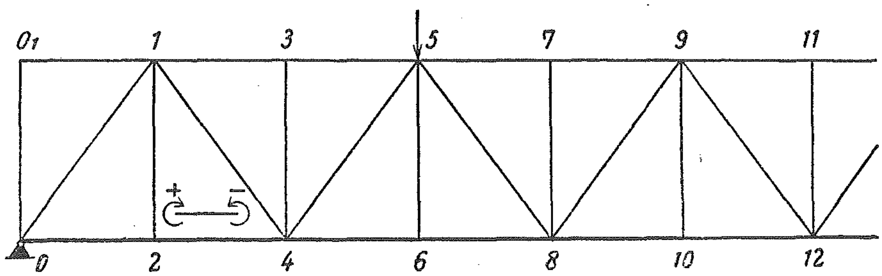

Fig. 22.

Moments of Chord Members:

M (lower chord 0–2, left)(load at joint 1) $= -2.48$ .... (317)
(load at joint 3) $= -1.65-0.39/a_c$ ...... (318)
M (lower chord 0–2, right)(load at joint 1) $= -4.80$ .... (319)

M (lower chord 2–4, left)(load at joint 1) = 4.50 .... (320)
M (lower chord 2–4, right)(load at joint 3) = $-7.30-0.00/a_c$. (321)
M (lower chord 4–6, left)(load at joint 3) = $7.05-0.09/a_c$ ..... (322)
M (lower chord 4–6, right)(load at joint 5) = -12.00 .... (323)
M (lower chord 6–8, left)(load at joint 5) = 11.75 .... (324)
M (lower chord 6–8, right)(load at joint 7) = $-14.40-0.00/a_c$. (325)
M (lower chord 8–10, left)(load at joint 7) = $14.20+0.00/a_c$ ... (326)
M (lower chord 8–10, right)(load at joint 9) = -17.80 .... (327)
M (lower chord 10–12, left)(load at joint 9) = 17.80 .... (328)
M (lower chord 10–12, right)(load at joint 11) = $-16.55-0.015/a_c$ (329)
M (upper chord 0_1–1, right)(load at joint 1) = -4.20 .... (330)
(load at joint 3) = $-0.15+1.00/a_c$ ...... (331)
M (upper chord 1–3, left)(load at joint 1) = 5.05 .... (332)
(load at joint 3) = $-5.00-3.66/a_c$ ...... (333)
M (upper chord 1–3, right)(load at joint 3) = $-11.40-5.04/a_c$. (334)
M (upper chord 3–5, left)(load at joint 3) = $11.10+5.04/a_c$ .... (335)
(load at joint 7) = $2.00+0.75/a_c$ .... (336)
M (upper chord 3–5, right)(load at joint 3) = $1.50+3.75/a_c$ ... (337)
(load at joint 5) = -13.00 .... (338)
(load at joint 7) = $0.50+2.67/a_c$ .... (339)
M (upper chord 5–7, left)(load at joint 3) = $1.10-2.64/a_c$ ...... (340)
(load at joint 5) = 13.50 .... (341)
(load at joint 7) = $-5.00-3.90/a_c$ ...... (342)
M (upper chord 5–7, right)(load at joint 7) = $-18.40-5.73/a_c$. (343)
M (upper chord 7–9, left)(load at joint 7) = $18.40+5.73/a_c$ .... (344)
M (upper chord 7–9, right)(load at joint 7) = $3.00+4.02/a_c$ ... (345)
(load at joint 9) = -19.35 .... (346)
(load at joint 11) = $-0.40+3.15/a_c$ ..... (347)
M (upper chord 9–11, left)(load at joint 7) = $0.33-3.25/a_c$ .... (348)
(load at joint 9) = 19.50 .... (349)
(load at joint 11) = $-3.35-4.17/a_c$ ..... (350)
M (upper chord 9–11, right)(load at joint 11) = $-21.20-6.21/a_c$ (351)

Moments of Web Members:

M (diagonal 0–1, lower)(load at joint 1) = 1.65 .... (352)
(load at joint 3) = $1.35+0.46/a_c$ .... (353)
M (diagonal 0–1, upper)(load at joint 1) = -2.05 .... (354)
(load at joint 3) = $2.95+1.68/a_c$ .... (355)
M (diagonal 1–4, upper)(load at joint 3) = $1.30+0.79/a_c$ ...... (356)
M (diagonal 1–4, lower)(load at joint 1) = -0.52 .... (357)
(load at joint 5) = 1.80 .... (358)

M (diagonal 4–5, lower)(load at joint 1) = -0.32 .... (359)
(load at joint 7) = $2.15+0.31/a_c$ .... (360)
M (diagonal 4–5, upper)(load at joint 3) = $-1.00-0.63/a_c$ .... (361)
(load at joint 7) = $2.20+0.70/a_c$ .... (362)
M (diagonal 5–8, upper)(load at joint 3) = $-0.45-0.30/a_c$ .... (363)
(load at joint 7) = $1.00+0.36/a_c$ .... (364)
M (diagonal 5–8, lower)(load at joint 3) = $-0.70-0.14/a_c$ .... (365)
(load at joint 9) = 1.25 .... (366)
M (diagonal 8–9, lower)(load at joint 5) = -0.70 .... (367)
(load at joint 11) = $1.40+0.18/a_c$ .... (368)
M (diagonal 8–9, upper)(load at joint 7) = $-0.95-0.34/a_c$ .... (369)
(load at joint 11) = $1.25+0.36/a_c$ .... (370)
M (diagonal 9–12, upper)(load at joint 7) = $-0.90-0.33/a_c$ ... (371)
(load at joint 11) = $1.30+0.36/a_c$ .... (372)
M (diagonal 9–12, lower)(load at joint 7) = $-1.10-0.16/a_c$ ... (373)
(load at joint 13) = 1.20 .... (374)
M (sub-vertical 0_1–0, upper)(load at joint 1) = 0.95 .... (375)
M (sub-vertical 0_1–0, lower)(load at joint 1) = 0.85 .... (376)
M (sub-vertical 1–2, upper)(load at joint 3) = $1.00+0.16/a_c$ ... (377)
M (sub-vertical 1–2, lower)(load at joint 3) = $1.10+0.09/a_c$ ... (378)
M (sub-vertical 3–4, upper)(load at joint 1) = -0.32 .... (379)
(load at joint 5) = 1.20 .... (380)
M (sub-vertical 3–4, lower)(load at joint 1) = -0.25 .... (381)
(load at joint 5) = 1.20 .... (382)
M (sub-vertical 5–6, upper)(load at joint 3) = $-0.65-0.13/a_c$. (383)
(load at joint 7) = $1.40+0.18/a_c$ .... (384)
M (sub-vertical 5–6, lower)(load at joint 3) = $-0.60-0.07/a_c$. (385)
(load at joint 7) = $1.50+0.09/a_c$ .... (386)
M (sub-vertical 7–8, upper)(load at joint 5) = -0.95 .... (387)
(load at joint 9) = 1.50 .... (388)
M (sub-vertical 7–8, lower)(load at joint 5) = -0.90 .... (389)
(load at joint 9) = 1.50 .... (390)
M (sub-vertical 9–10, upper)(load at joint 7) = $-1.20-0.16/a_c$. (391)
(load at joint 11) = $1.45+0.18/a_c$ .... (392)
M (sub-vertical 9–10, lower)(load at joint 7) = $-1.20-0.09/a_c$. (393)
(load at joint 11) = $1.50+0.07/a_c$ .... (394)
M (sub-vertical 11–12, upper)(load at joint 9) = -1.35 .... (395)
(load at joint 13) = 1.35 .... (396)
M (sub-vertical 11–12, lower)(load at joint 9) = -1.30 .... (397)
(load at joint 13) = 1.30 .... (398)

5. *Moments due to a Single Load at Joint O_1.* Coeff.: $P\frac{a_{\max.}}{A_{\max.}}\frac{K_{mn}}{k_{mn}}$

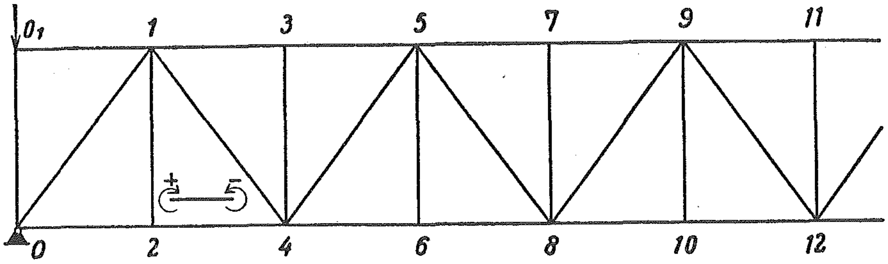

Fig. 23.

Moments of Chord Members:

M (lower chord 0–2, left) $= 0.17/a_c$ .... (399)
M (lower chord 0–2, right) $= 0.05/a_c$ .... (400)
M (lower chord 2–4, left) $= -0.03/a_c$ .... (401)
M (lower chord 2–4, right) $= 0.02/a_c$ .... (402)
M (lower chord 4–6, left) $= 0.05/a_c$ .... (403)
M (lower chord 4–6, right) $= 0.015/a_c$ .... (404)
M (upper chord 0_1–1, left) $= 0.30/a_c$ .... (405)
M (upper chord 0_1–1, right) $= 1.38/a_c$ .... (406)
M (upper chord 1–3, left) $= -0.68/a_c$ .... (407)
M (upper chord 1–3, right) $= -0.19/a_c$ .... (408)
M (upper chord 3–5, left) $= 0.18/a_c$ .... (409)
M (upper chord 3–5, right) $= 0.06/a_c$ .... (410)
M (upper chord 5–7, left) $= 0.04/a_c$ .... (411)

Moments of Web Members:

M (diagonal 0–1, lower) $= -0.035/a_c$ .... (412)
M (diagonal 0–1, upper) $= -0.43/a_c$ .... (413)
M (diagonal 1–4, upper) $= -0.22/a_c$ .... (414)
M (diagonal 1–4, lower) $= -0.09/a_c$ .... (415)
M (sub-vertical 0_1–0, upper) $= -0.30/a_c$ .... (416)
M (sub-vertical 0_1–0, lower) $= -0.14/a_c$ .... (417)
M (sub-vertical 1–2, upper) $= -0.05/a_c$ .... (418)
M (sub-vertical 1–2, lower) $= -0.03/a_c$ .... (419)

IV. Numerical Moment Formulae for Through Pratt Truss with Parallel Chords.

1. *Moments due to Full Lower Joint Loads.* Coeff.: $P\frac{a_{\max.}}{A_{\max.}}\frac{K_{mn}}{k_{mn}}$

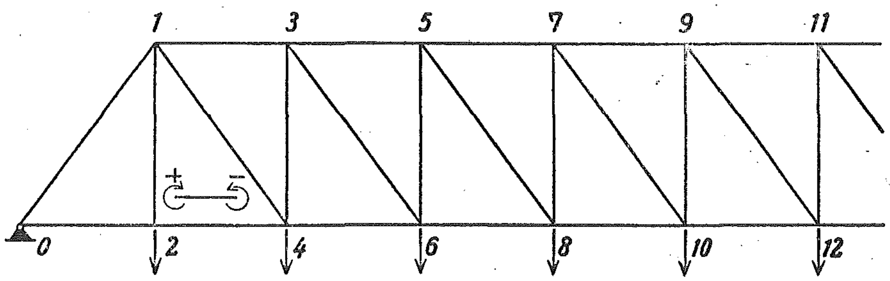

Fig. 24.

Moments of Chord Members:

M (portal 0–1, lower) $= (1.20\,n-2.15)+1.80/a_t$ .... (420)
M (portal 0–1, upper) $= (2.70\,n-11.30)+0.57/a_t$ .... (421)
M (lower chord 0–2, left) $= (-1.20\,n+2.15)-1.80/a_t$ .... (422)
M (lower chord 0–2, right) $= (-0.20\,n-3.40)-2.40/a_t$ .... (423)
M (lower chord 2–4, left) $= (-0.42\,n+4.50)+2.42/a_t$ .... (424)
M (lower chord 2–4, right) $= (-0.70\,n-1.90)+1.85/a_t$ .... (425)
M (lower chord 4–6, left) $= (-3.15\,n+18.60)-1.25/a_t$ .... (426)
M (lower chord 4–6, right) $= (-3.10\,n+6.65)-0.38/a_t$ .... (427)
M (lower chord 6–8, left) $= (-0.78\,n+16.85)+0.30/a_t$ .... (428)
M (lower chord 6–8, right) $= (0.54\,n-23.10)+0.08/a_t$ .... (429)
M (lower chord 8–10, left) $= (-3.00\,n+42.80)-0.07/a_t$ ...... (430)
M (lower chord 8–10, right) $= (-4.60\,n+18.70)-0.025/a_t$ .... (431)
M (upper chord 1–3, left) $= (-4.04\,n+15.05)-0.42/a_t$ .... (432)
M (upper chord 1–3, right) $= (-2.51\,n-5.40)-0.05/a_t$ .... (433)
M (upper chord 3–5, left) $= (-1.45\,n+21.20)+0.20/a_t$ .... (434)
M (upper chord 3–5, right) $= (-4.10\,n+11.80)+0.05/a_t$ ...... (435)
M (upper chord 5–7, left) $= (0.49\,n+10.25)-0.07/a_t$ .... (436)
M (upper chord 5–7, right) $= (-4.00\,n+11.93)-0.015/a_t$ ..... (437)
M (upper chord 7–9, left) $= (1.82\,n+5.67)+0.015/a_t$ .... (438)
M (upper chord 7–9, right) $= (-9.00\,n+59.27)+0.01/a_t$ ..... (439)

Moments of Web Members:

M (diagonal 1–4, upper) $= (0.69\,n-2.13)-0.14/a_t$ .... (440)
M (diagonal 1–4, lower) $= (0.64\,n-3.22)-0.22/a_t$ .... (441)
M (diagonal 3–6, upper) $= (0.69\,n-2.98)+0.015/a_t$ .... (442)

M (diagonal 3–6, lower) $= (0.68\,n - 4.05) + 0.025/a_t$ .... (443)
M (diagonal 5–8, upper) $= (0.56\,n - 3.70) - 0.00/a_t$ .... (444)
M (diagonal 5–8, lower) $= (0.52\,n - 4.25) - 0.00/a_t$ .... (445)
M (diagonal 7–10, upper) $= (0.38\,n - 3.35) + 0.00/a_t$ .... (446)
M (diagonal 7–10, lower) $= (0.35\,n - 3.60) + 0.00/a_t$ .... (447)
M (sub-vertical 1–2, upper) $= (0.65\,n - 1.57) - 0.03/a_t$ .... (448)
M (sub-vertical 1–2, lower) $= (0.62\,n - 1.13) - 0.015/a_t$ .... (449)
M (vertical 3–4, upper) $= (3.27\,n - 12.82) - 0.17/a_t$ .... (450)
M (vertical 3–4, lower) $= (3.21\,n - 13.40) - 0.40/a_t$ .... (451)
M (vertical 5–6, upper) $= (3.05\,n - 18.40) + 0.025/a_t$ .... (452)
M (vertical 5–6, lower) $= (3.20\,n - 19.43) + 0.06/a_t$ .... (453)
M (vertical 7–8, upper) $= (1.80\,n - 14.40) - 0.00/a_t$ .... (454)
M (vertical 7–8, lower) $= (1.94\,n - 15.50) - 0.00/a_t$ .... (455)
M (vertical 9–10, upper) $= (2.03\,n - 20.30) + 0.00/a_t$ .... (456)
M (vertical 9–10, lower) $= (2.31\,n - 23.10) + 0.00/a_t$ .... (457)

2. *Moments due to Full Upper Joint Loads.* Coeff.: $P\dfrac{a_{\max.}}{A_{\max.}}\dfrac{K_{mn}}{k_{mn}}$

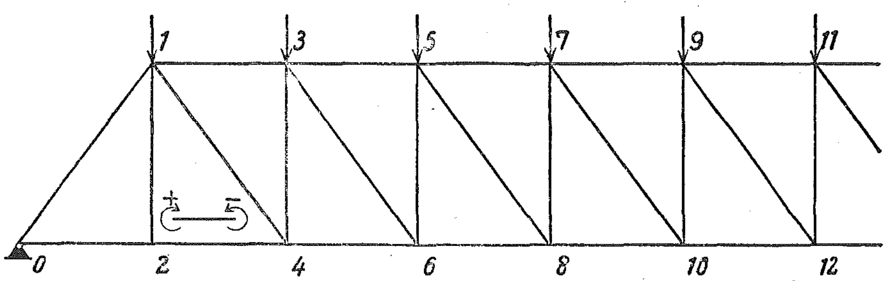

Fig. 25.

Moments of Chord Members:

M (portal 0–1, lower) $= 1.38\,n - 2.30$ .... (458)
M (portal 0–1, upper) $= 3.00\,n - 9.95$ .... (459)
M (lower chord 0–2, left) $= -1.38\,n + 2.30$ .... (460)
M (lower chord 0–2, right) $= -0.55\,n - 1.70$ .... (461)
M (lower chord 2–4, left) $= -0.15\,n + 3.40$ .... (462)
M (lower chord 2–4, right) $= -0.69\,n + 0.10$ .... (463)
M (lower chord 4–6, left) $= -3.35\,n + 16.09$ .... (464)
M (lower chord 4–6, right) $= -3.10\,n + 2.80$ .... (465)

$$M \text{ (lower chord 6–8, left)} = -1.24\,n + 22.73 \quad (466)$$
$$M \text{ (lower chord 6–8, right)} = 0.54\,n - 22.75 \quad (467)$$
$$M \text{ (lower chord 8–10, left)} = -3.89\,n + 49.45 \quad (468)$$
$$M \text{ (lower chord 8–10, right)} = -45.00 (n = 10) \quad (469)$$
$$M \text{ (upper chord 1–3, left)} = -4.44\,n + 13.70 \quad (470)$$
$$M \text{ (upper chord 1–3, right)} = -2.83\,n - 5.00 \quad (471)$$
$$M \text{ (upper chord 3–5, left)} = -1.82\,n + 23.83 \quad (472)$$
$$M \text{ (upper chord 3–5, right)} = -3.00\,n + 0.00 \quad (473)$$
$$M \text{ (upper chord 5–7, left)} = -1.33\,n + 26.38 \quad (474)$$
$$M \text{ (upper chord 5–7, right)} = -4.00\,n + 8.00 \quad (475)$$
$$M \text{ (upper chord 7–9, left)} = 0.95\,n + 16.90 \quad (476)$$
$$M \text{ (upper chord 7–9, right)} = -26.00 (n = 10) \quad (477)$$

Moments of Web Members:

$$M \text{ (diagonal 1–4, upper)} = 0.72\,n - 1.90 \quad (478)$$
$$M \text{ (diagonal 1–4, lower)} = 0.63\,n - 2.55 \quad (479)$$
$$M \text{ (diagonal 3–6, upper)} = 0.83\,n - 3.55 \quad (480)$$
$$M \text{ (diagonal 3–6, lower)} = 0.71\,n - 3.75 \quad (481)$$
$$M \text{ (diagonal 5–8, upper)} = 0.70\,n - 4.60 \quad (482)$$
$$M \text{ (diagonal 5–8, lower)} = 0.70\,n - 5.50 \quad (483)$$
$$M \text{ (diagonal 7–10, upper)} = 0.55\,n - 4.90 \quad (484)$$
$$M \text{ (diagonal 7–10, lower)} = 0.55\,n - 5.50 \quad (485)$$
$$M \text{ (sub-vertical 1–2, upper)} = 0.72\,n - 1.85 \quad (486)$$
$$M \text{ (sub-vertical 1–2, lower)} = 0.70\,n - 1.70 \quad (487)$$
$$M \text{ (vertical 3–4, upper)} = 3.82\,n - 15.28 \quad (488)$$
$$M \text{ (vertical 3–4, lower)} = 3.41\,n - 13.64 \quad (489)$$
$$M \text{ (vertical 5–6, upper)} = 3.63\,n - 21.78 \quad (490)$$
$$M \text{ (vertical 5–6, lower)} = 3.63\,n - 21.78 \quad (491)$$
$$M \text{ (vertical 7–8, upper)} = 2.50\,n - 20.00 \quad (492)$$
$$M \text{ (vertical 7–8, lower)} = 2.65\,n - 21.20 \quad (493)$$
$$M \text{ (vertical 9–10, upper)} = 2.80\,n - 28.00 \quad (494)$$
$$M \text{ (vertical 9–10, lower)} = 3.35\,n - 33.50 \quad (495)$$

Correction for Compressive Sub-vertical.

For the moments of upper chord members near the centre of a span, the following amounts shall be added to them as the effect of the compressive sub-vertical.

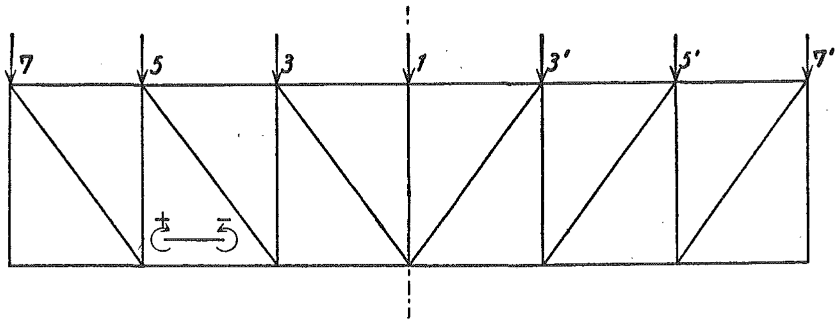

Fig. 26.

M (upper chord 1–3, right) : $-5.4/a_c$ .... (496)
M (upper chord 1–3, left) : $-3.6/a_c$ .... (497)
M (upper chord 3–5, right) : $2.8/a_c$ .... (498)
M (upper chord 3–5, left) : $0.9/a_c$ .... (499)
M (upper chord 5–7, right) : $-0.8/a_c$ .... (500)
M (upper chord 5–7, left) : $-0.2/a_c$ .... (501)

3. *Maximum Moments due to Live Loads.* Coeff. : $P\frac{a_{\text{max.}}}{A_{\text{max.}}}\frac{K_{mn}}{k_{mn}}$

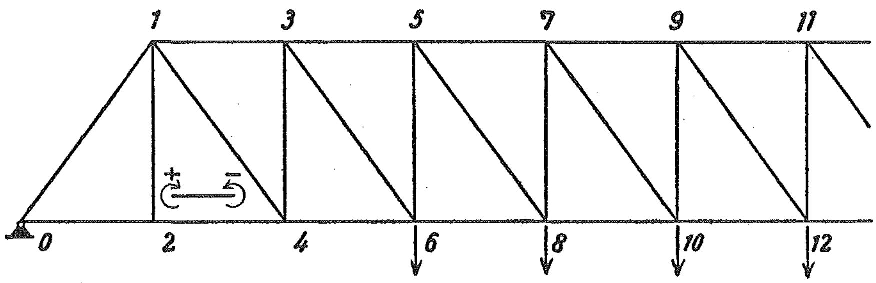

Fig. 27.

Moments of Web Members:

M (diagonal 1–4, upper) $= (0.69\,n-2.30)+0.00/a_t$ .... (502)
M (diagonal 1–4, lower) $= (0.54\,n-1.70)+0.00/a_t$ .... (503)
M (diagonal 3–6, upper) $= (0.73\,n-3.15)+0.00/a_t$ .... (504)
M (diagonal 3–6, lower) $= (0.65\,n-3.30)+0.00/a_t$ .... (505)
M (diagonal 5–8, upper) $= (0.56\,n-3.30)+0.00/a_t$ .... (506)
M (diagonal 5–8, lower) $= (0.65\,n-4.45)+0.00/a_t$ .... (507)
M (diagonal 7–10, upper) $= (0.25\,n-1.55)+0.00/a_t$ .... (508)
M (diagonal 7–10, lower) $= (0.30\,n-2.25)+0.00/a_t$ .... (509)
M (sub-vertical 1–2, upper) $= (0.63\,n-1.40)+0.00/a_t$ .... (510)

M (sub-vertical 1–2, lower) $= (0.62\,n-1.20)+0.00/a_t$ .... (511)
M (vertical 3–4, upper) $= (3.45\,n-13.00)+0.00/a_t$ .... (512)
M (vertical 3–4, lower) $= (3.05\,n-11.10)+0.00/a_t$ .... (513)
M (vertical 5–6, upper) $= (3.30\,n-17.80)+0.00/a_t$ .... (514)
M (vertical 5–6, lower) $= (3.40\,n-18.40)+0.00/a_t$ .... (515)
M (vertical 7–8, upper) $= (2.32\,n-15.80)+0.00/a_t$ .... (516)
M (vertical 7–8, lower) $= (2.25\,n-14.90)+0.00/a_t$ .... (517)
M (vertical 9–10, upper) $= 4.75(n=10)$ .... (518)
M (vertical 9–10, lower) $= 5.60(n=10)$ .... (519)

4. *Moments due to a Single Load.* Coeff.: $P\frac{a_{\max.}}{A_{\max.}}\frac{K_{mn}}{k_{mn}}$

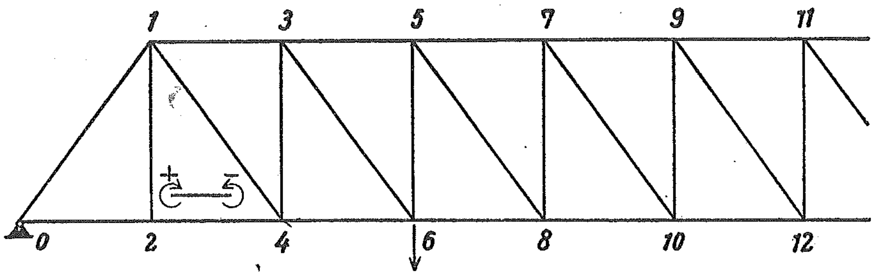

Fig. 28.

Moments of Chord Members:

M (portal 0–1, lower)(load at joint 2) $= 1.30+1.78/a_t$ .... (520)
M (portal 0–1, upper)(load at joint 2) $= -4.20+0.57/a_t$ ...... (521)
(load at joint 4) $= 3.00$ .... (522)
(load at joint 6) $= 4.40$ .... (523)
M (lower chord 0–2, left)(load at joint 2) $= -1.30-1.78/a_t$ ... (524)
M (lower chord 0–2, right)(load at joint 2) $= -3.52-2.41/a_t$. (525)
M (lower chord 2–4, left)(load at joint 2) $= 3.40+2.43/a_t$ ..... (526)
M (lower chord 2–4, right)(load at joint 2) $= 0.25+1.87/a_t$ ... (527)
(load at joint 4) $= -7.20$ .... (528)
M (lower chord 4–6, left)(load at joint 2) $= 1.20-1.25/a_t$ ..... (529)
(load at joint 4) $= 7.32$ .... (530)
(load at joint 6) $= -7.05$ .... (531)
M (lower chord 4–6, right)(load at joint 6) $= -13.65$ .... (532)
(load at joint 10) $= -4.35$ .... (533)
M (lower chord 6–8, left)(load at joint 6) $= 13.20$ .... (534)
(load at joint 8) $= -5.50$ .... (535)
M (lower chord 6–8, right)(load at joint 8) $= -19.00$ .... (536)
(load at joint 10) $= 5.00$ .... (537)

M (lower chord 8–10, left)(load at joint 8) = 19.60 .... (538)
(load at joint 10) = −8.70 .... (539)
M (lower chord 8–10, right)(load at joint 8) = 3.85 .... (540)
(load at joint 10) = −26.20 .... (541)
M (upper chord 1–3, left)(load at joint 2) = $4.10-0.40/a_t$ .... (542)
(load at joint 4) = 4.22 .... (543)
(load at joint 6) = −6.10 .... (544)
(load at joint 8) = −4.60 .... (545)
M (upper chord 1–3, right)(load at joint 4) = −12.15 .... (546)
M (upper chord 3–5, left)(load at joint 4) = 11.90 .... (547)
M (upper chord 3–5, right)(load at joint 6) = −14.00 .... (548)
M (upper chord 5–7, left)(load at joint 6) = 14.00 .... (549)
M (upper chord 5–7, right)(load at joint 8) = −17.30 .... (550)
M (upper chord 7–9, left)(load at joint 10) = 17.50 .... (551)
M (upper chord 7–9, right)(load at joint 10) = −25.25 .... (552)

Moments of Web Members:

M (diagonal 1–4, upper)(load at joint 2) = $0.02-0.14/a_t$ ...... (553)
(load at joint 6) = 1.05 .... (554)
M (diagonal 1–4, lower)(load at joint 2) = $-0.30-0.22/a_t$ .... (555)
(load at joint 6) = 1.10 .... (556)
M (diagonal 3–6, upper)(load at joint 2) = $-0.15+0.025/a_t$ ... (557)
(load at joint 8) = 1.00 .... (558)
M (diagonal 3–6, lower)(load at joint 4) = −0.30 .... (559)
(load at joint 8) = 1.00 .... (560)
M (diagonal 5–8, upper)(load at joint 4) = −0.25 .... (561)
(load at joint 10) = 0.70 .... (562)
M (diagonal 5–8, lower)(load at joint 6) = −0.40 .... (563)
(load at joint 10) = 0.75 .... (564)
M (diagonal 7–10, upper)(load at joint 6) = −0.20 .... (565)
(load at joint 12) = 0.31 .... (566)
M (diagonal 7–10, lower)(load at joint 8) = −0.32 .... (567)
(load at joint 12) = 0.34 .... (568)
M (sub-vertical 1–2, upper)(load at joint 2) = $-0.02-0.03/a_t$. (569)
(load at joint 4) = 1.00 .... (570)
M (sub-vertical 1–2, lower)(load at joint 4) = 1.15 .... (571)
M (vertical 3–4, upper)(load at joint 2) = $-1.17-0.17/a_t$ ..... (572)
(load at joint 6) = 4.80 .... (573)
M (vertical 3–4, lower)(load at joint 2) = $-1.15-0.40/a_t$ ..... (574)
(load at joint 6) = 4.70 .... (575)

M (vertical 5–6, upper)(load at joint 4) = —1.85 .... (576)
(load at joint 8) = 3.75 .... (577)
M (vertical 5–6, lower)(load at joint 4) = —1.70 .... (578)
(load at joint 8) = 4.00 .... (579)
M (vertical 7–8, upper)(load at joint 6) = —1.50 .... (580)
(load at joint 10) = 2.40 .... (581)
M (vertical 7–8, lower)(load at joint 6) = —1.70 .... (582)
(load at joint 10) = 2.80 .... (583)
M (vertical 9–10, upper)(load at joint 8) = —2.15 .... (584)
M (vertical 9–10, lower)(load at joint 8) = —2.50 .... (585)

V. Numerical Moment Formulae for Through Warren Truss with Polygonal Chords.

1. *Moments due to Full Lower Joint Loads.* Coeff.: $P\frac{a_{\text{max.}}}{A_{\text{max.}}}\frac{K_{mn}}{k_{mn}}$

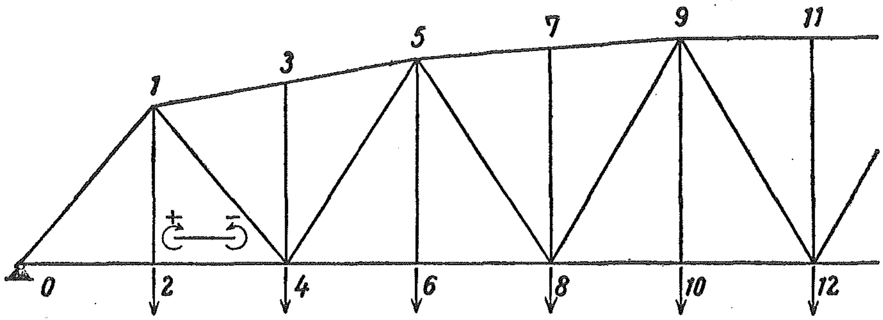

Fig. 29.

Moments of Chord Members:

M (portal 0–1, lower) = $(0.79\,n-0.05)+2.23/a_t$ .... (586)
M (portal 0–1, upper) = $(0.13\,n-2.28)+0.71/a_t$ .... (587)
M (lower chord 0–2, left) = $(-0.79\,n+0.05)-2.23/a_t$ .... (588)
M (lower chord 0–2, right) = $(-0.54\,n-1.82)-3.68/a_t$ .... (589)
M (lower chord 2–4, left) = $(0.06\,n+2.74)+3.77/a_t$ .... (590)
M (lower chord 2–4, right) = $(-1.21\,n-1.70)+4.77/a_t$ .... (591)
M (lower chord 4–6, left) = $(-1.22\,n+15.25)-5.65/a_t$ .... (592)
M (lower chord 4–6, right) = $(-1.80\,n+10.60)-6.50/a_t$ ...... (593)
M (lower chord 6–8, left) = $(1.20\,n-7.00)+6.53/a_t$ .... (594)
M (lower chord 6–8, right) = $(-0.60\,n-12.80)+7.71/a_t$ ...... (595)

M (lower chord 8–10, left) $= (-2.33\,n+39.70)-8.04/a_t$ ...... (596)
M (lower chord 8–10, right) $= (-5.45\,n+52.40)-8.67/a_t$ ..... (597)
M (lower chord 10–12, left) $= 11.25+8.68/a_t(n=12)$ .... (598)
M (lower chord 10–12, right) $= -20.90+8.50/a_t(n=12)$ ..... (599)
M (upper chord 1–3, left) $= (-1.25\,n+5.32)-0.70/a_t$ .... (600)
M (upper chord 1–3, right) $= (-0.83\,n-10.04)-0.18/a_t$ ..... (601)
M (upper chord 3–5, left) $= (0.23\,n+13.31)+0.12/a_t$ .... (602)
M (upper chord 3–5, right) $= (-2.85\,n+21.30)-0.06/a_t$ ..... (603)
M (upper chord 5–7, left) $= (0.82\,n-8.72)-0.16/a_t$ .... (604)
M (upper chord 5–7, right) $= (-0.50\,n-18.50)-0.03/a_t$ ..... (605)
M (upper chord 7–9, left) $= (-0.56\,n+28.50)-0.00/a_t$ .... (606)
M (upper chord 7–9, right) $= (-7.00\,n+72.00)-0.03/a_t$ ...... (607)
M (upper chord 9–11, left) $= 7.00-0.03/a_t(n=12)$ .... (608)
M (upper chord 9–11, right) $= -28.90-0.00/a_t(n=12)$ ...... (609)

Moments of Web Members:

M (diagonal 1–4, upper) $= (0.75\,n-2.30)+0.06/a_t$ .... (610)
M (diagonal 1–4, lower) $= (0.65\,n-4.90)+0.34/a_t$ .... (611)
M (diagonal 4–5, lower) $= (1.23\,n-5.60)+0.42/a_t$ .... (612)
M (diagonal 4–5, upper) $= (0.75\,n-4.75)+0.19/a_t$ .... (613)
M (diagonal 5–8, upper) $= (0.75\,n-4.65)+0.06/a_t$ .... (614)
M (diagonal 5–8, lower) $= (0.65\,n-6.20)+0.13/a_t$ .... (615)
M (diagonal 8–9, lower) $= (1.26\,n-11.00)+0.12/a_t$ .... (616)
M (diagonal 8–9, upper) $= (0.81\,n-8.60)+0.06/a_t$ .... (617)
M (diagonal 9–12, upper) $= 2.38+0.00/a_t(n=12)$ .... (618)
M (diagonal 9–12, lower) $= -0.90+0.00/a_t(n=12)$ .... (619)
M (sub-vertical 1–2, upper) $= (0.37\,n-0.74)-0.08/a_t$ .... (620)
M (sub-vertical 1–2, lower) $= (0.46\,n-0.92)-0.09/a_t$ .... (621)
M (sub-vertical 3–4, upper) $= (0.60\,n-3.27)+0.06/a_t$ .... (622)
M (sub-vertical 3–4, lower) $= (0.55\,n-3.05)+0.12/a_t$ .... (623)
M (sub-vertical 5–6, upper) $= (0.53\,n-3.18)-0.03/a_t$ .... (624)
M (sub-vertical 5–6, lower) $= (0.60\,n-3.60)-0.03/a_t$ .... (625)
M (sub-vertical 7–8, upper) $= (1.06\,n-10.00)+0.03/a_t$ .... (626)
M (sub-vertical 7–8, lower) $= (1.02\,n-9.70)+0.075/a_t$ .... (627)
M (sub-vertical 9–10, upper) $= (0.79\,n-7.90)-0.015/a_t$ .... (628)
M (sub-vertical 9–10, lower) $= (0.86\,n-8.60)-0.015/a_t$ .... (629)
M (sub-vertical 11–12, upper) $= 0(n=12)$ .... (630)
M (sub-vertical 11–12, lower) $= 0(n=12)$ .... (631)

2. *Moments due to Full Upper Joint Loads.* Coeff.: $P\frac{a_{\max.}}{A_{\max.}}\frac{K_{mn}}{k_{mn}}$

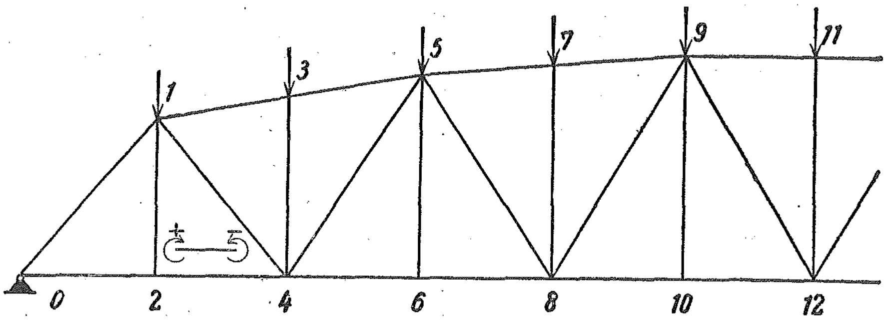

Fig. 30.

Moments of Chord Members:

M (portal 0–1, lower) $= (0.79\,n-0.05)+0.65/a_c$ .... (632)
M (portal 0–1, upper) $= (0.13\,n-2.28)+2.60/a_c$ .... (633)
M (lower chord 0–2, left) $= (-0.79\,n+0.05)-0.65/a_c$ .... (634)
M (lower chord 0–2, right) $= (-0.54\,n-1.82)-0.29/a_c$ .... (635)
M (lower chord 2–4, left) $= (0.06\,n+2.74)+0.17/a_c$ .... (636)
M (lower chord 2–4, right) $= (-1.21\,n-1.70)-0.10/a_c$ .... (637)
M (lower chord 4–6, left) $= (-1.22\,n+15.25)-0.30/a_c$ .... (638)
M (lower chord 4–6, right) $= (-1.80\,n+10.60)-0.09/a_c$ ...... (639)
M (lower chord 6–8, left) $= (1.20\,n-7.00)+0.045/a_c$ .... (640)
M (lower chord 6–8, right) $= (-0.60\,n-12.80)-0.03/a_c$ ...... (641)
M (lower chord 8–10, left) $= (-2.33\,n+39.70)-0.06/a_c$ ...... (642)
M (lower chord 8–10, right) $= (-5.45\,n+52.40)-0.03/a_c$ ..... (643)
M (lower chord 10–12, left) $= 11.25+0.00/a_c (n=12)$ .... (644)
M (lower chord 10–12, right) $= -20.90+0.00/a_c (n=12)$ ..... (645)
M (upper chord 1–3, left) $= (-1.25\,n+5.32)-3.74/a_c$ .... (646)
M (upper chord 1–3, right) $= (-0.83\,n-10.04)-6.54/a_c$ ..... (647)
M (upper chord 3–5, left) $= (0.23\,n+13.31)+6.61/a_c$ .... (648)
M (upper chord 3–5, right) $= (-2.85\,n+21.30)+7.86/a_c$ ..... (649)
M (upper chord 5–7, left) $= (0.82\,n-8.72)-8.36/a_c$ .... (650)
M (upper chord 5–7, right) $= (-0.50\,n-18.50)-9.54/a_c$ ..... (651)
M (upper chord 7–9, left) $= (-0.56\,n+28.50)+9.57/a_c$ .... (652)
M (upper chord 7–9, right) $= (-7.00\,n+72.00)+10.20/a_c$ .... (653)
M (upper chord 9–11, left) $= 7.00-10.38/a_c (n=12)$ .... (654)
M (upper chord 9–11, right) $= -28.90-10.80/a_c (n=12)$ ..... (655)

Moments of Web Members:

M (diagonal 1–4, upper) $= (0.75\,n-2.30)+0.90/a_c$ .... (656)
M (diagonal 1–4, lower) $= (0.65\,n-4.90)+0.40/a_c$ .... (657)
M (diagonal 4–5, lower) $= (1.23\,n-5.60)+0.045/a_c$ .... (658)
M (diagonal 4–5, upper) $= (0.75\,n-4.75)+0.27/a_c$ .... (659)
M (diagonal 5–8, upper) $= (0.75\,n-4.65)+0.15/a_c$ .... (660)
M (diagonal 5–8, lower) $= (0.65\,n-6.20)+0.075/a_c$ .... (661)
M (diagonal 8–9, lower) $= (1.26\,n-11.00)+0.03/a_c$ .... (662)
M (diagonal 8–9, upper) $= (0.81\,n-8.60)+0.03/a_c$ .... (663)
M (diagonal 9–12, upper) $= 2.38+0.08/a_c(n = 12)$ .... (664)
M (diagonal 9–12, lower) $= -0.90+0.04/a_c(n = 12)$ .... (665)
M (sub-vertical 1–2, upper) $= (0.37\,n-0.74)+0.24/a_c$ .... (666)
M (sub-vertical 1–2, lower) $= (0.46\,n-0.92)+0.12/a_c$ .... (667)
M (sub-vertical 3–4, upper) $= (0.60\,n-3.27)-0.07/a_c$ .... (668)
M (sub-vertical 3–4, lower) $= (0.55\,n-3.05)-0.045/a_c$ .... (669)
M (sub-vertical 5–6, upper) $= (0.53\,n-3.18)+0.08/a_c$ .... (670)
M (sub-vertical 5–6, lower) $= (0.60\,n-3.60)+0.045/a_c$ .... (671)
M (sub-vertical 7–8, upper) $= (1.06\,n-10.00)-0.03/a_o$ .... (672)
M (sub-vertical 7–8, lower) $= (1.02\,n-9.70)-0.015/a_c$ .... (673)
M (sub-vertical 9–10, upper) $= (0.79\,n-7.90)+0.03/a_c$ .... (674)
M (sub-vertical 9–10, lower) $= (0.86\,n-8.60)+0.03/a_c$ .... (675)
M (sub-vertical 11–12, upper) $= 0+0.00/a_c(n = 12)$ .... (676)
M (sub-vertical 11–12, lower) $= 0+0.00/a_c(n = 12)$ .... (677)

3. *Maximum Moments due to Live Loads.* Coeff.: $P\dfrac{a_{\max.}}{A_{\max.}}\dfrac{K_{mn}}{k_{mn}}$

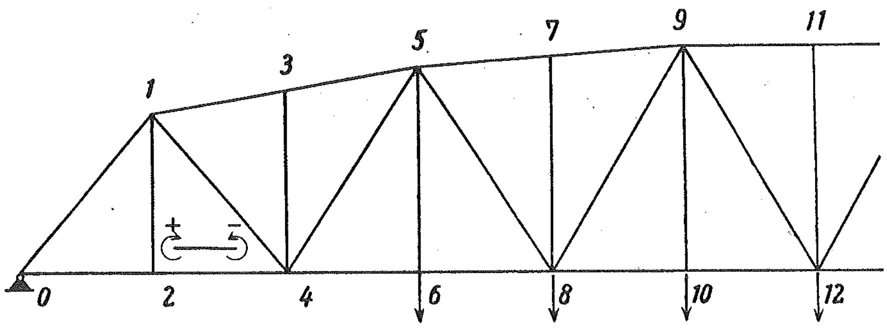

Fig. 31.

Moments of Web Members:

M (diagonal 1–4, upper) $= (0.75\,n-2.30)+0.36/a_t$ .... (678)
M (diagonal 1–4, lower) $= (0.55\,n-2.00)+0.87/a_t$ .... (679)

M (diagonal 4–5, lower) $= (1.20\,n-4.60)+0.88/a_t$ .... (680)
M (diagonal 4–5, upper) $= (0.82\,n-2.80)+0.30/a_t$ .... (681)
M (diagonal 5–8, upper) $= (0.75\,n-3.50)+0.22/a_t$ .... (682)
M (diagonal 5–8, lower) $= (0.70\,n-4.10)+0.51/a_t$ .... (683)
M (diagonal 8–9, lower) $= (1.05\,n-6.60)+0.54/a_t$ .... (684)
M (diagonal 8–9, upper) $= (0.70\,n-3.90)+0.27/a_t$ .... (685)
M (diagonal 9–12, upper) $= 5.50+0.24/a_t(n=12)$ .... (686)
M (diagonal 9–12, lower) $= 3.65+0.50/a_t(n=12)$ .... (687)
M (sub-vertical 1–2, upper) $= (0.35\,n-0.50)-0.03/a_t$ .... (688)
M (sub-vertical 1–2, lower) $= (0.46\,n-0.92)-0.09/a_t$ .... (689)
M (sub-vertical 3–4, upper) $= (0.54\,n-2.05)+0.12/a_t$ .... (690)
M (sub-vertical 3–4, lower) $= (0.50\,n-1.95)+0.22/a_t$ .... (691)
M (sub-vertical 5–6, upper) $= (0.55\,n-2.20)-0.045/a_t$ .... (692)
M (sub-vertical 5–6, lower) $= (0.61\,n-2.50)-0.06/a_t$ .... (693)
M (sub-vertical 7–8, upper) $= (0.67\,n-3.55)+0.13/a_t$ .... (694)
M (sub-vertical 7–8, lower) $= (0.65\,n-3.50)+0.27/a_t$ .... (695)
M (sub-vertical 9–10, upper) $= (0.70\,n-4.10)-0.03/a_t$ .... (696)
M (sub-vertical 9–10, lower) $= (0.77\,n-4.65)-0.075/a_t$ .... (697)
M (sub-vertical 11–12, upper) $= 3.46+0.12/a_t(n=12)$ .... (698)
M (sub-vertical 11–12, lower) $= 3.28+0.25/a_t(n=12)$ .... (699)

4. *Moments due to a Single Load.* Coeff.: $P\frac{a_{\max.}}{A_{\max.}}\frac{K_{mn}}{k_{mn}}$

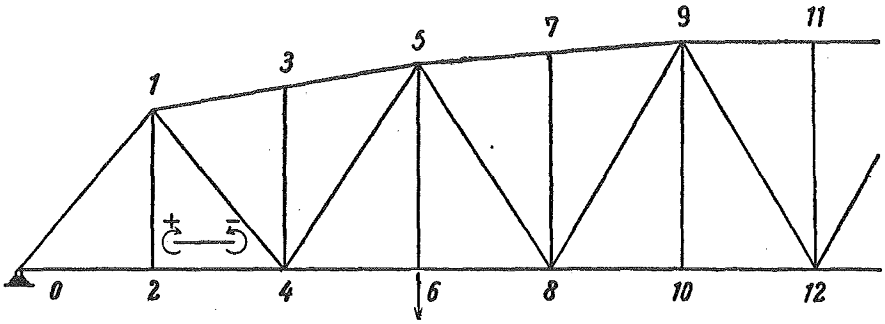

Fig. 32.

Moments of Chord Members:

M (portal 0–1, lower)(load at joint 2) $= 1.65+2.10/a_t$ .... (700)
M (portal 0–1, upper)(load at joint 2) $= -4.10+0.78/a_t$ ...... (701)
(load at joint 4) $= 2.50$ .... (702)
M (lower chord 0–2, left)(load at joint 2) $= -1.65-2.10/a_t$ ... (703)
M (lower chord 0–2, right)(load at joint 2) $= -5.15-3.15/a_t$. (704)

M (lower chord 2–4, left)(load at joint 2) $= 5.10+3.16/a_t$ ..... (705)
M (lower chord 2–4, right)(load at joint 2) $= 0.95+2.61/a_t$ ... (706)
(load at joint 4) $= -7.60$ .... (707)
(load at joint 6) $= -0.40+2.01/a_t$ ...... (708)
M (lower chord 4–6, left)(load at joint 2) $= 1.10-1.53/a_t$ ..... (709)
(load at joint 4) $= 8.50$ .... (710)
(load at joint 6) $= -3.40-3.84/a_t$ ...... (711)
M (lower chord 4–6, right)(load at joint 6) $= -9.90-5.43/a_t$ .. (712)
M (lower chord 6–8, left)(load at joint 6) $= 9.80+5.46/a_t$ ..... (713)
M (lower chord 6–8, right)(load at joint 6) $= 1.50+4.02/a_t$ .... (714)
(load at joint 8) $= -13.70$ .... (715)
(load at joint 10) $= 1.30+3.33/a_t$ .... (716)
M (lower chord 8–10, left)(load at joint 6) $= 1.80-3.06/a_t$ .... (717)
(load at joint 8) $= 14.40$ .... (718)
(load at joint 10) $= -5.00-4.65/a_t$ ..... (719)
M (lower chord 8–10, right)(load at joint 10) $= -15.20-6.90/a_t$ (720)
M (lower chord 10–12, left)(load at joint 10) $= 16.20+6.96/a_t$. (721)
M (lower chord 10–12, right)(load at joint 10) $= 4.30+4.89/a_t$. (722)
(load at joint 12) $= -16.20$ .... (723)
(load at joint 14) $= 0.15+3.70/a_t$ .... (724)
M (upper chord 1–3, left)(load at joint 2) $= 4.10-0.42/a_t$ ..... (725)
(load at joint 4) $= -4.40$ .... (726)
(load at joint 6) $= -1.40-0.24/a_t$ ...... (727)
M (upper chord 1–3, right)(load at joint 4) $= -12.30$ .... (728)
M (upper chord 3–5, left)(load at joint 4) $= 12.35$ .... (729)
M (upper chord 3–5, right)(load at joint 4) $= 4.50$ .... (730)
(load at joint 6) $= -11.00-0.09/a_t$ ..... (731)
M (upper chord 5–7, left)(load at joint 6) $= 11.40-0.12/a_t$ .... (732)
(load at joint 8) $= -6.50$ .... (733)
M (upper chord 5–7, right)(load at joint 8) $= -18.00$ .... (734)
M (upper chord 7–9, left)(load at joint 8) $= 18.25$ .... (735)
M (upper chord 7–9, right)(load at joint 8) $= 7.00$ .... (736)
(load at joint 10) $= -16.00-0.03/a_t$ .... (737)
M (upper chord 9–11, left)(load at joint 10) $= 17.30-0.00/a_t$ .. (738)
(load at joint 12) $= -6.50$ .... (739)
M (upper chord 9–11, right)(load at joint 12) $= -20.85$ ...... (740)

Moments of Web Members:

M (diagonal 1–4, upper)(load at joint 2) $= 0.10-0.34/a_t$ ...... (741)
(load at joint 6) $= 1.15+0.36/a_t$ .... (742)

M (diagonal 1–4, lower)(load at joint 2) $= -0.95-0.52/a_t$ .... (743)

(load at joint 6) $= 1.40+0.81/a_t$ .... (744)

M (diagonal 4–5, lower)(load at joint 2) $= -0.65-0.46/a_t$ .... (745)

(load at joint 6) $= 1.40+0.82/a_t$ .... (746)

M (diagonal 4–5, upper(load at joint 2) $= -0.85-0.22/a_t$ ..... (747)

(load at joint 8) $= 2.10$ .... (748)

M (diagonal 5–8, upper)(load at joint 4) $= -0.85$ .... (749)

(load at joint 10) $= 1.15+0.24/a_t$ .... (750)

M (diagonal 5–8, lower)(load at joint 6) $= -0.95-0.40/a_t$ .... (751)

(load at joint 10) $= 1.30+0.51/a_t$ .... (752)

M (diagonal 8–9, lower)(load at joint 6) $= -1.10-0.39/a_t$ .... (753)

(load at joint 10) $= 1.15+0.51/a_t$ .... (754)

M (diagonal 8–9, upper)(load at joint 6) $= -1.15-0.19/a_t$ .... (755)

(load at joint 12) $= 1.55$ .... (756)

M (diagonal 9–12, upper)(load at joint 8) $= -1.45$ .... (757)

(load at joint 14) $= 1.50+0.22/a_t$ .... (758)

M (diagonal 9–12, lower)(load at joint 10) $= -1.30-0.46/a_t$ .. (759)

(load at joint 14) $= 1.50+048/a_t$ .... (760)

M (sub-vertical 1–2, upper)(load at joint 4) $= 0.90$ .... (761)

M (sub-vertical 1–2, lower)(load at joint 4) $= 1.00$ .... (762)

M (sub-vertical 3–4, upper)(load at joint 2) $= -0.45-0.06/a_t$. (763)

(load at joint 6) $= 1.00+0.10/a_t$ .... (764)

M (sub-vertical 3–4, lower)(load at joint 2) $= -0.45-0.12/a_t$. (765)

(load at joint 6) $= 0.95+0.21/a_t$ .... (766)

M (sub-vertical 5–6, upper)(load at joint 4) $= -0.80$ .... (767)

(load at joint 8) $= 1.15$ .... (768)

M (sub-vertical 5–6, lower)(load at joint 4) $= -0.80$ .... (769)

(load at joint 8) $= 1.20$................. (770)

M (sub-vertical 7–8, upper)(load at joint 6) $= -1.10-0.09/a_t$. (771)

(load at joint 10) $= 1.25+0.13/a_t$ .... (772)

M (sub-vertical 7–8, lower)(load at joint 6) $= -1.05-0.19/a_t$. (773)

(load at joint 10) $= 1.20+0.25/a_t$ .... (774)

M (sub-vertical 9–10, upper)(load at joint 8) $= -1.30$......... (775)

(load at joint 12) $= 1.40$ .... (776)

M (sub-vertical 9–10, lower)(load at joint 8) $= -1.35$ .... (777)

(load at joint 12) $= 1.45$ .... (778)

M (sub-vertical 11–12, upper)(load at joint 10) $= -1.40-0.12/a_t$ (779)

(load at joint 14) $= 1.40+0.12/a_t$ .... (780)

M (sub-vertical 11–12, lower)(load at joint 10) $= -1.35-0.24/a_t$ (781)

(load at joint 14) $= 1.35+0.24/a_t$ .... (782)

VI. Application of Numerical Moment Formulae.

1. *Numerical Example I.* As the first numerical example of the proposed method, a through Pratt truss with six panels, carrying live joint loads of 1000 lbs. is shown in Fig. 33.

This example is quoted from "The Theory and Practice of Modern Framed Structures" by J. B. Johnson, C. W. Bryan and F. E. Turneaure*.

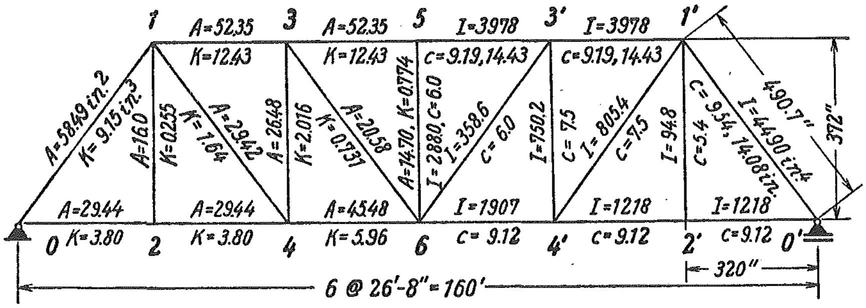

Fig. 33.

It will be seen that for some members the secondary stresses are not at the maximum when the primary stresses are maximum, but inasmuch as the primary stress is generally much larger than the secondary stress, the total is generally at the maximum when the primary stresses is maximum. On this basis, the secondary stresses which occur for the same loading as the maximum primary stresses will be computed.

The length of each member, denoted by (l), the sectional area of member (A), the moment of inertia of a section (I), the stiffness value of member (K) and the distance of the extreme fibre from neutral axis of the section of member (c) are written in Fig. 33.

In Fig. 34 there are shown the stiffness ratio and sectional area-ratio of this truss on the corresponding members while Fig. 35 shows the standard through Pratt truss with six panels which is obtained from Fig. 4.

* Part II, P. 441-455, John Wiley & Sons, New York, 1917.

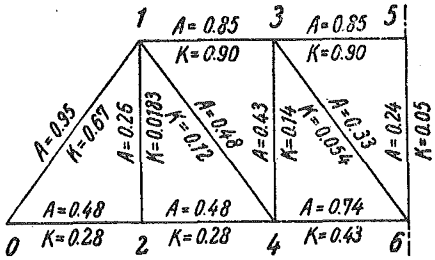

Fig. 34.

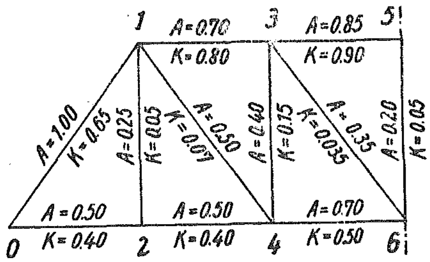

Fig. 35.

In this example, n, P, a_t, $\dfrac{a_{\max.}}{A_{\max.}}$ and $\dfrac{K_{mn}}{k_{mn}}$ become as follows:

n (number of panels) $= 6$

P (intensity of joint loads) $= 1000$ lbs.

$$a_t = \frac{A_t \cdot a_{\max.}}{A_{\max.}} = \frac{(16.00)(0.85)}{52.35} = 0.26$$

$$\frac{a_{\max.}}{A_{\max.}} = \frac{0.85}{52.35} = 0.0162$$

$$\frac{K_{01}}{k_{01}}, \quad \frac{K_{13}}{k_{13}}, \quad \frac{K_{46}}{k_{46}}, \quad \frac{K_{35}}{k_{35}} \doteqdot \frac{K_{\max.}}{k_{\max.}} = \frac{12.43}{0.90} = 13.8$$

$$\frac{K_{02}}{k_{02}}, \quad \frac{K_{24}}{k_{24}} = 9.5, \quad \frac{K_{12}}{k_{12}} = 5.1, \quad \frac{K_{14}}{k_{14}} = 23.4, \quad \frac{K_{34}}{k_{34}} = 13.5$$

$$\frac{K_{36}}{k_{36}} = 20.9, \quad \frac{K_{56}}{k_{56}} = 15.5$$

Substituting the above values into the numerical moment formulae for the through Pratt truss with parallel chords, the bending moments at the ends of any of the members can be obtained.

The primary stress in any of the members is at the maximum under full joint loads and the bending moments in chord members due to such a loading are obtained using the numerical moment formulae for full lower joint loads as follows:

From numerical moment formula (420),

$$M_{01} = M(\text{portal } 0\text{–}1, \text{ lower}) = \left\{(1.20\,n - 2.15) + 1.80/a_t\right\} P \frac{a_{\max.}}{A_{\max.}} \frac{K_{01}}{k_{01}}$$

$$= \left\{(1.20)(6) - 2.15 + (1.80)/(0.26)\right\}(1000)(0.0162)(13.8)$$

$$= 2683$$

From numerical moment formula (421),

$$M_{10} = M(\text{portal 0–1, upper}) = \left\{(2.70\,n - 11.30) + 0.57/a_t\right\} P \frac{a_{\max.}}{A_{\max.}} \frac{K_{01}}{k_{01}}$$

$$= \left\{(2.70)(6) - 11.30 + (0.57)/(0.26)\right\}(1000)(0.0162)(13.8)$$

$$= 1588$$

Etc.

All the bending moments in chord members are therefore as follows:

$$\begin{array}{lll} M_{01} = 2683, & M_{10} = 1588, & M_{02} = -1845 \\ M_{20} = -2130, & M_{24} = 1740, & M_{42} = 157 \\ M_{46} = -1145, & M_{64} = -3004, & M_{13} = -2421 \\ M_{31} = -4625, & M_{35} = 2970, & M_{53} = -2825 \end{array}.$$

The sum of the moments around joint 0 is 2683–1845 = 838. Hence by distribution of this discrepancy to members 0–1 and 0–2 proportionally to their stiffness values so as to satisfy the equilibrium condition, the corrected values of M_{01} and M_{02} are obtained as follows:

$$M_{01} = 2683 - (838)\frac{9.15}{9.15 + 3.80} = 2683 - 590 = 2093$$

$$M_{02} = -1845 - (838)\frac{3.80}{9.15 + 3.80} = -1845 - 250 = -2095$$

For all other joints such a correction can be neglected, as the amount of the discrepancy is negligible for all practical purposes.

In web members the loading for the maximum primary stress causes nearly the maximum secondary stress, and the maximum bending moments in web members are obtained from the numerical moment formulae of maximum moments as follows:

From numerical moment formula (502),

$$M_{14} = M(\text{diagonal 1–4, upper}) = \left\{(0.69\,n - 2.30) + 0.00/a_t\right\} P \frac{a_{\max.}}{A_{\max.}} \frac{K_{14}}{k_{14}}$$

$$= \left\{(0.69)(6) - 2.30\right\}(1000)(0.0162)(23.4)$$

$$= 700$$

From numerical moment formula (503),

$$M_{41} = M(\text{diagonal 1–4, lower}) = \left\{(0.54\,n-1.70)+0.00/a_t\right\} P\frac{a_{\max.}}{A_{\max.}}\,\frac{K_{14}}{k_{14}}$$

$$= \left\{(0.54)(6)-1.70\right\}(1000)(0.0162)(23.4)$$

$$= 585$$

Etc.

All the bending moments in web members are therefore as follows:

$$\begin{array}{lll} M_{14} = 700\,, & M_{41} = 585\,, & M_{36} = 417 \\ M_{63} = 203\,, & M_{12} = 197\,, & M_{21} = 209 \\ M_{34} = 1686\,, & M_{43} = 1577\,, & M_{56} = 502 \\ M_{65} = 502 & & \end{array}$$

All the above values of moments are given in in.-lbs.

After the values of bending moments have been obtained, the secondary stresses can be obtained from the general equation

$$\sigma_{ab} = \frac{M_{ab}\cdot c_{ab}}{I_{ab}} \qquad \text{(q)}$$

In this equation M_{ab} and σ_{ab} are respectively the bending moment and secondary stress at the end A of the member AB, C_{ab} the distance of the extreme fibre from the neutral axis of the section and I_{ab} the moment of inertia of the section.

The calculation is as follows:

$$\sigma_{01} = (2093)\frac{9.54}{4490} = 4.45 \quad \text{(compression)}$$

$$\sigma_{10} = (1588)\frac{14.08}{4490} = 5.00 \quad \text{(compression)}$$

Etc.

In Table 15 there are tabulated the secondary stresses which occur for the same loading as the maximum primary stresses. For unsymmetrical members, such as the portal and upper chord, the top and bottom fibre stresses have different values and properties, but in this table only one value which has the same property as of the maximum primary stress is recorded.

These are given in pounds per sq. in and also percentages of the maximum primary stresses. In the table the values in brackets are those calculated by Johnson.

In regard to signs, the sign of the primary stresses indicates the properties of the stress, so a plus sign will signify a tensile stress and a minus sign a compressive stress, while the sign of secondary stresses indicates the directions of moment, by which the secondary stress is caused.

Table 15.

Member		Secondary Stress	Maximum Primary Stress	Percent Secondary of Maximum Primary
Portal	0 1	+ 4.45	− 56.40	7.9 (7.5)
	1 0	+ 5.00		8.8 (5.1)
Lower Chords	0 2	−15.70	+ 73.10	21.4 (20.3)
	2 0	−15.95		21.8 (21.2)
	2 4	+13.03	+ 73.10	17.8 (19.0)
	4 2	+ 1.18		1.6 (1.6)
	4 6	− 5.50	+ 75.70	7.3 (6.6)
	6 4	−14.40		19.0 (20.3)
Upper Chords	1 3	− 8.75	− 65.72	13.3 (9.2)
	3 1	−10.70		16.3 (15.2)
	3 5	+ 6.86	− 74.00	9.2 (8.8)
	5 3	− 6.52		8.8 (10.6)
Main Diagonal	1 4	+ 6.52	+ 74.77	8.7 (8.5)
	4 1	+ 5.45		7.3 (8.5)
Second Diagonal	3 6	+ 6.97	+ 64.14	10.8 (8.7)
	6 3	+ 3.40		5.3 (3.8)
Hip Vertical	2 1	+11.90	+ 62.50	19.1 (19.5)
	1 2	+11.22		18.0 (17.3)
Second Vertical	4 3	+15.76	+ 37.76	41.8 (40.1)
	3 4	+16.85		44.7 (42.7)
Middle Vertical	6 5	+ 9.60	0	
	5 6	+ 8.85		

2. *Numerical Example II.* Fig. 36 shows a deck Warren truss with eight panels of a certain Japanese railway bridge. The upper chord joints are loaded with 100,000 lbs. each and the lower chord joints are loaded with 10,000 lbs. each. It is required to find the secondary stresses.

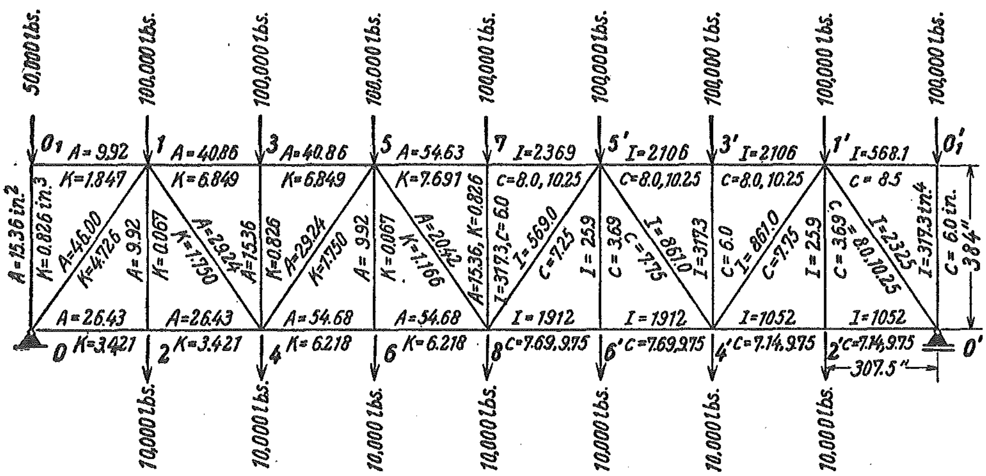

Fig. 36.

The various elements needed in the calculations are written in Fig. 36. In Fig. 37 are shown the stiffness ratio and sectional area-ratio of this truss and in Fig. 38 is shown the standard deck Warren truss with eight panels which is obtained from Fig. 3.

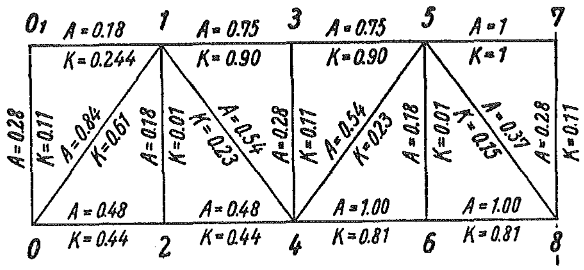

Fig. 37.

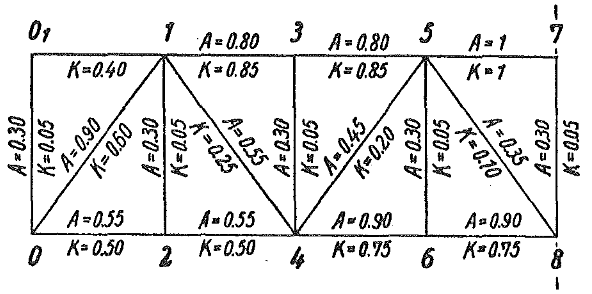

Fig. 38.

In this example, n, P, a_t, a_c, $\frac{a_{\max.}}{A_{\max.}}$ and $\frac{K_{mn}}{k_{mn}}$ become as follows:

$$n = 8$$

$P = 100{,}000$ lbs. for upper joint loads excepting 0_1 and $0'_1$

$= 50{,}000$ lbs. for joint loads on 0_1 and $0'_1$

$= 10{,}000$ lbs. for lower joint loads.

$$a_t = \frac{A_t \cdot a_{\max.}}{A_{\max.}} = \frac{(9.92)(1)}{54.63} = 0.18$$

$$a_c = \frac{A_c \cdot a_{\max.}}{A_{\max.}} = \frac{(15.36)(1)}{54.63} = 0.28$$

$$\frac{a_{\max.}}{A_{\max.}} = \frac{1}{54.63} = 0.0183$$

$$\frac{K_{13}}{k_{13}},\ \frac{K_{35}}{k_{35}},\ \frac{K_{57}}{k_{57}},\ \frac{K_{01}}{k_{01}},\ \frac{K_{02}}{k_{02}},\ \frac{K_{24}}{k_{24}},\ \frac{K_{46}}{k_{46}},\ \frac{K_{68}}{k_{68}} \fallingdotseq \frac{K_{\max.}}{k_{\max.}}$$

$$= \frac{7.691}{1} \fallingdotseq 7.70$$

$$\frac{K_{14}}{k_{14}} = 7.00\,, \qquad \frac{K_{45}}{k_{45}} = 8.75\,, \qquad \frac{K_{58}}{k_{58}} = 11.66$$

$$\frac{K_{12}}{k_{12}},\ \frac{K_{56}}{k_{56}} = 1.34\,, \qquad \frac{K_{00_1}}{k_{00_1}},\ \frac{K_{34}}{k_{34}}.\ \frac{K_{78}}{k_{78}} = 16.5$$

Substituting the above values into the numerical moment formulae for the deck Warren truss with parallel chords, the bending moments at the ends of any of the members can be obtained as follows:

A. Moments due to upper joint loads excepting the loads on joints 0_1 and $0'_1$:

$M_{02} = -141{,}600$,	$M_{20} = -\ 88{,}100$,	$M_{24} = \quad 32{,}800$
$M_{42} = -176{,}400$,	$M_{46} = \quad 5{,}200$,	$M_{64} = -204{,}700$
$M_{68} = \quad 177{,}400$,	$M_{86} = -190{,}300$	
$M_{01_1} = -\ 21{,}400$,	$M_{10_1} = -\ 38{,}200$,	$M_{13} = -274{,}700$
$M_{31} = -502{,}000$,	$M_{35} = \quad 460{,}100$,	$M_{53} = \quad 121{,}700$
$M_{57} = -202{,}300$,	$M_{75} = -660{,}900$	

$M_{01} =$ 111,700 ,	$M_{10} =$ 145,800 ,	$M_{14} =$ 128,100
$M_{41} =$ 44,400 ,	$M_{45} =$ 96,000 ,	$M_{54} =$ 22,900
$M_{58} =$ 53,100 ,	$M_{85} = -$ 18,300	
$M_{0_1 0} =$ 76,400 ,	$M_{00_1} =$ 64,300 ,	$M_{12} =$ 9,000
$M_{21} =$ 9,600 ,	$M_{34} =$ 90,300 ,	$M_{43} =$ 80,900
$M_{56} =$ 4,200 ,	$M_{65} =$ 4,600 ,	$M_{78} =$ 0
$M_{87} =$ 0		

B. Moments due to lower joint loads:

$M_{02} = -$ 29,200 ,	$M_{20} = -$ 34,600 ,	$M_{24} =$ 30,100
$M_{42} =$ 14,500 ,	$M_{46} = -$ 34,300 ,	$M_{64} = -$ 61,300
$M_{68} =$ 58,600 ,	$M_{86} =$ 23,500	
$M_{01_1} = -$ 2,900 ,	$M_{10_1} = -$ 7,900 ,	$M_{13} = -$ 12,100
$M_{31} = -$ 23,000 ,	$M_{35} =$ 18,300 ,	$M_{53} = -$ 21,300
$M_{57} =$ 12,900 ,	$M_{75} = -$ 29,200 ,	
$M_{01} =$ 24,400 ,	$M_{10} =$ 11,500 ,	$M_{14} =$ 9,000
$M_{41} =$ 4,500 ,	$M_{45} =$ 11,900 ,	$M_{54} =$ 2,900
$M_{58} =$ 4,900 ,	$M_{85} = -$ 1,500	
$M_{0_1 0} =$ 10,400 ,	$M_{00_1} =$ 10,300 ,	$M_{12} =$ 700
$M_{21} =$ 800 ,	$M_{34} =$ 10,000 ,	$M_{43} =$ 9,400
$M_{56} =$ 400 ,	$M_{65} =$ 500 ,	$M_{78} =$ 0
$M_{87} =$ 0		

C. Moments due to loads at joints 0_1 and $0_1'$:

$M_{02} =$ 4,200 ,	$M_{20} =$ 1,300 ,	$M_{24} = -$ 700
$M_{42} =$ 500 ,	$M_{46} =$ 1,300 ,	$M_{64} =$ 400
$M_{01_1} =$ 4,500 ,	$M_{10_1} =$ 20,800 ,	$M_{13} = -$ 17,100
$M_{31} = -$ 4,800 ,	$M_{35} =$ 4,500 ,	$M_{53} =$ 1,500
$M_{57} =$ 1,000		
$M_{01} = -$ 500 ,	$M_{10} = -$ 6,500 ,	$M_{14} = -$ 5,000
$M_{41} = -$ 2,000		
$M_{0_1 0} = -$ 16,200 ,	$M_{00_1} = -$ 7,500 ,	$M_{12} = -$ 200
$M_{21} = -$ 100		

Therefore the total moments are obtained as follows:

$M_{02} = -166{,}600$,	$M_{20} = -121{,}400$,	$M_{24} = 62{,}200$
$M_{42} = -161{,}400$,	$M_{46} = -\ 27{,}800$,	$M_{64} = -265{,}600$
$M_{68} = 236{,}000$,	$M_{86} = -166{,}800$	
$M_{0_11} = -\ 19{,}800$,	$M_{10_1} = -\ 29{,}500$,	$M_{13} = -303{,}900$
$M_{31} = -529{,}800$,	$M_{35} = 483{,}800$,	$M_{53} = 101{,}900$
$M_{57} = -188{,}400$,	$M_{75} = -690{,}100$	
$M_{01} = 135{,}600$,	$M_{10} = 150{,}800$,	$M_{14} = 132{,}100$
$M_{41} = 46{,}900$,	$M_{45} = 107{,}900$,	$M_{54} = 25{,}800$
$M_{58} = 58{,}000$,	$M_{85} = -\ 19{,}800$	
$M_{0_10} = 70{,}600$,	$M_{00_1} = 67{,}100$,	$M_{12} = 9{,}500$
$M_{21} = 10{,}300$,	$M_{34} = 100{,}300$,	$M_{43} = 90{,}300$
$M_{56} = 4{,}600$,	$M_{65} = 5{,}100$,	$M_{78} = 0$
$M_{87} = 0$		

The total sum of the moments around joint 0_1 is $-19{,}800+70{,}600 = 50{,}800$. Therefore distributing this discrepancy among the members around joint 0_1 proportionally to their stiffness values, corrected moments of M_{0_11} and M_{0_10} are obtained as follows:

$$M_{0_11} = -54{,}900\ , \qquad M_{0_10} = 54{,}900$$

For all other joints such a correction can be neglected, as the amount of the discrepancy is negligible small for all practical purposes.

All the above values of moments are given in in.–lbs.

Using the above values of moments, the secondary stresses can be obtained from equation (q). In Table 16, the desired secondary stresses are tabulated. These are given in pounds per sq. in. and also the percentages of the mean allowable stress of 13,000 lbs. In regard to the signs, they are the same as in the preceding example. For one end of a member, only one value which has the same property as of that the primary stress is shown.

The values in brackets show the values calculated by the Slope Distribution Method as follows:

The calculation procedure of the Slope Distribution Method is omitted and only the results of the calculation will be written here.

Table 16.

Member		Most dangerous Fibre	Secondary Stress	Percent Secondary of Allowable Stress of 13,000 lbs./sq. in
Lower Chords	0 2	Top	−1131	8.7 (9.0)
	2 0	Bottom	−1125	8.6 (8.1)
	2 4	Bottom	576	4.4 (6.0)
	4 2	Bottom	−1456	11.2 (12.9)
	4 6	Top	− 112	0.8 (0.4)
	6 4	Bottom	−1354	10.4 (9.6)
	6 8	Bottom	1204	9.4 (9.3)
	8 6	Bottom	850	6.5 (7.0)
Upper Chords	0 1	—	− 821	6.3 (6.2)
	1 0	—	− 441	3.4 (4.1)
	1 3	Bottom	−1479	11.4 (10.6)
	3 1	Top	−2012	15.5 (18.3)
	3 5	Top	1837	14.1 (15.8)
	5 3	Bottom	496	3.8 (5.6)
	5 7	Bottom	− 816	6.3 (7.6)
	7 5	Top	−2330	18.0 (18.2)
Diagonals	0 1	Top	466	3.6 (3.2)
	1 0	Bottom	665	5.1 (5.6)
	1 4	—	1189	9.2 (10.5)
	4 1	—	422	3.2 (1.9)
	4 5	—	971	7.5 (7.6)
	5 4	—	232	1.8 (1.5)
	5 8	—	739	5.7 (4.9)
	8 5	—	− 252	1.9 (2.0)
Sub-verticals	0_1 0	—	1038	8.0 (7.8)
	0 0_1	—	1268	9.7 (7.6)
	1 2	—	1354	10.5 (11.9)
	2 1	—	1468	11.3 (12.7)
	3 4	—	1895	14.6 (12.1)
	4 3	—	1707	13.1 (10.3)
	5 6	—	655	5.0 (5.1)
	6 5	—	727	5.6 (5.5)
	7 8	—	0	0 (0)
	8 7	—	0	0 (0)

For this truss, the member-rotation angles became as follows by the use of Williot-Mohr's Displacement Diagram:

$\Psi_{02} = -549,000$,	$\Psi_{24} = -441,000$,	$\Psi_{46} = -276,000$
$\Psi_{68} = -\ 96,000$,	$\Psi_{0_11} = -516,600$,	$\Psi_{13} = -497,400$
$\Psi_{.5} = -219,600$,	$\Psi_{57} = -152,400$,	$\Psi_{01} = -461,400$
$\Psi_{14} = -360,000$,	$\Psi_{45} = -213,600$,	$\Psi_{58} = -\ 78,600$
$\Psi_{00_1} = -413,400$,	$\Psi_{12} = -357,600$,	$\Psi_{34} = -240,000$
$\Psi_{56} = -120,000$,	$\Psi_{78} = 0$	

where Ψ denotes the member-rotation angle multiplied by $-6E$.

By the Slope Distribution Method, the joint rotation angles were obtained as follows:

$\varphi_0 = 158,900$,	$\varphi_2 = 181,060$,	$\varphi_4 = 103,480$
$\varphi_6 = 66,910$,	$\varphi_8 = 0$	
$\varphi_{0_1} = 159,500$,	$\varphi_1 = 168,740$,	$\varphi_3 = 118,680$
$\varphi_5 = 61,380$,	$\varphi_7 = 0$	

where φ denotes the joint-rotation angle multiplied by $2E$

Substituting the above values into equation (a) the bending moments at the ends of every one of members were obtained as follows:

$M_{02} = -172,000$,	$M_{20} = -\ 96,000$,	$M_{24} = 84,000$
$M_{42} = -181,000$,	$M_{46} = -\ 13,200$,	$M_{64} = -241,000$
$M_{68} = 235,000$,	$M_{86} = -181,000$	
$M_{0_11} = -\ 53,300$,	$M_{10_1} = -\ 36,200$,	$M_{13} = -282,000$
$M_{31} = -625,000$,	$M_{35} = 542,000$,	$M_{53} = 150,000$
$M_{57} = -228,000$,	$M_{75} = -700,000$	
$M_{01} = 119,000$,	$M_{10} = 166,000$,	$M_{14} = 142,000$
$M_{41} = 27,500$,	$M_{45} = 95,600$,	$M_{54} = 22,100$
$M_{58} = 51,000$,	$M_{85} = -\ 19,900$	
$M_{0_10} = 53,300$,	$M_{00_1} = 52,800$,	$M_{12} = 10,800$
$M_{21} = 11,600$,	$M_{34} = 83,300$,	$M_{43} = 70,700$
$M_{56} = 4,700$,	$M_{65} = 5,000$,	$M_{78} = 0$
$M_{87} = 0$		

The secondary stresses due to the above obtained moments are shown in brackets in Table 20 and these are given in the percentages of the mean allowable stress of 13,000 lbs./sq. in.

3. *Numerical Example III.* Fig. 39 shows a through Warren truss with six panels which is quoted from "Secondary Stresses in Bridge Trusses" by C. R. Grimm*. It is required to find the secondary stresses due to dead load and live load as indicated in Fig. 39.

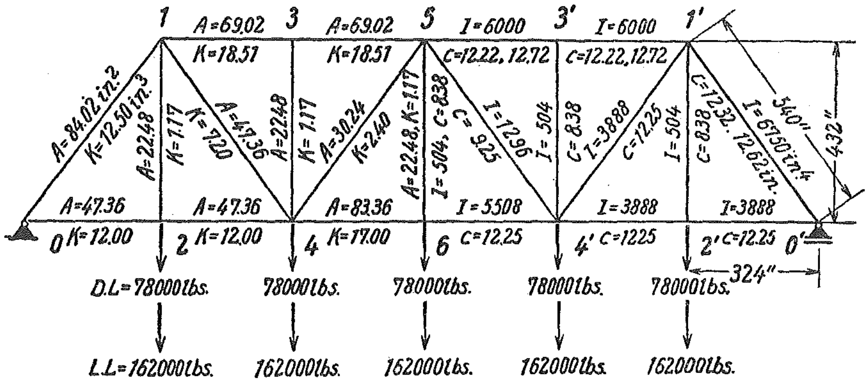

Fig. 39.

The various elements needed in the calculations are written in Fig. 39. In Fig. 40 are shown the stiffness ratio and sectional area-ratio of this truss and in Fig. 41 is shown the standard through Warren truss with six panels which is obtained from Fig. 2.

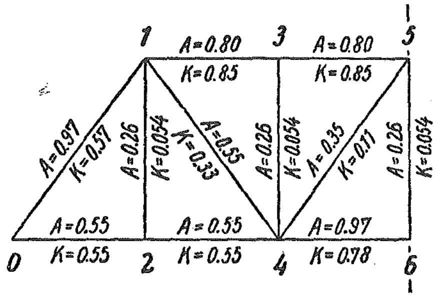

Fig. 40.

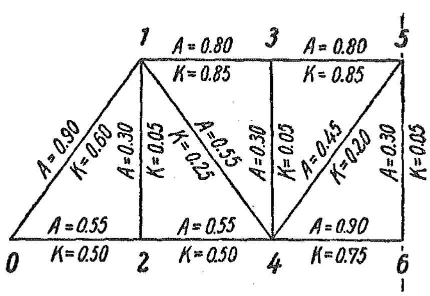

Fig. 41.

In this example, n, P, a_t, $\frac{a_{\text{max.}}}{A_{\text{max.}}}$ and $\frac{K_{mn}}{k_{mn}}$ become as follows:

$$n = 6$$

$$P = 78{,}000+162{,}000 = 240{,}000 \text{ lbs.}$$

$$a_t = \frac{A_t \cdot a_{\text{max.}}}{A_{\text{max.}}} = \frac{(22.48)(0.80)}{69.02} = 0.260$$

* P. 104, John Wiley & Sons, New York, 1908.

In this truss, the ratio of the height of truss and the length of panel is 432/324 = 1.33 which value is comparatively greater than that of the standard truss in which the corresponding ratio is 1.25. The effect of a sub-vertical is proportional to its length and therefore the value of a_t shall be corrected as follows:

$$a_t = \frac{A_t \cdot a_{\text{max.}}}{A_{\text{max.}}} \cdot \frac{1.25}{1.33} = (0.260)(0.94) = 0.244$$

$$\frac{a_{\text{max.}}}{A_{\text{max.}}} = \frac{0.8}{69.02} = 0.0116$$

$$\frac{K_{01}}{k_{01}} = 20.83\,, \quad \frac{K_{02}}{k_{02}}\,, \quad \frac{K_{24}}{k_{24}} = 24.00\,, \quad \frac{K_{46}}{k_{46}} = 22.66$$

$$\frac{K_{13}}{k_{13}}\,, \quad \frac{K_{35}}{k_{35}} = 21.77\,, \quad \frac{K_{14}}{k_{14}} = 28.80\,, \quad \frac{K_{45}}{k_{44}} = 12.00\,, \quad \frac{K_{12}}{k_{12}}\,,$$

$$\frac{K_{34}}{k_{34}}\,, \quad \frac{K_{56}}{k_{56}} = 23.40$$

Substituting the above values into the numerical moment formulae for the through Warren truss with parallel chords, the bending moments of every member can be obtained as follows:

$M_{01} = 796{,}200$, $M_{10} = 211{,}700$
$M_{02} = -917{,}400$, $M_{20} = -1{,}186{,}600$, $M_{24} = 1{,}056{,}400$
$M_{42} = 515{,}100$, $M_{46} = -928{,}000$, $M_{64} = -2{,}085{,}600$
$M_{13} = -524{,}800$, $M_{31} = -883{,}700$, $M_{35} = 775{,}200$
$M_{53} = -603{,}600$
$M_{14} = 277{,}400$, $M_{41} = 88{,}200$, $M_{45} = 140{,}600$
$M_{54} = -13{,}000$
$M_{12} = 100{,}000$, $M_{21} = 127{,}000$, $M_{34} = 118{,}000$
$M_{43} = 111{,}000$, $M_{56} = 0$, $M_{65} = 0$

The total sums of the moments around the joints 0, 1 and 4 are respectively $-121{,}200$, $64{,}300$ and $-73{,}200$. Hence, distributing these discrepancies among the members around their joints proportionally to their stiffness values so as to satisfy the equilibrium condition, final moments are obtained as follows:

$M_{01} = 858{,}200$, $M_{10} = 191{,}300$
$M_{02} = -858{,}200$, $M_{20} = -1{,}186{,}600$, $M_{24} = 1{,}056{,}400$
$M_{42} = 537{,}200$, $M_{46} = -896{,}700$, $M_{64} = -2{,}085{,}600$
$M_{13} = -555{,}000$, $M_{31} = -883{,}700$, $M_{35} = 775{,}200$
$M_{53} = -603{,}600$

$M_{14} = 265{,}600$, $M_{41} = 101{,}400$, $M_{45} = 145{,}000$
$M_{54} = -13{,}000$,
$M_{12} = 98{,}100$, $M_{21} = 127{,}000$, $M_{34} = 118{,}000$
$M_{43} = 113{,}200$, $M_{56} = 0$, $M_{65} = 0$

All the above moments are given in in.-lbs.

Using the above values of moments, the secondary stresses can be obtained from equation (q). In Table 17, the desired secondary stresses are tabulated. These are given in pounds per sq. in. and also the percentages of the primary stresses. In the table, the values of primary stresses and the percentages in brackets are those calculated by Grimm.

In regard to the signs, they are the same as in the preceding examples.

Table 17.

Member		Most Dangerous Fibre	Secondary Stress	Primary Stress	Percent Secondary of Primary
Portal	0 1	Top	1570	−8900	18 (18)
	1 0	Bottom	350		4
Lower Chords	0 2	—	−2700	11500	24
	2 0	—	−3720		32 (32)
	2 4	—	3320	11500	29 (29)
	4 2	—	1700		15
	4 6	—	−2000	12000	17
	6 4	—	−4650		39 (43)
Upper Chords	1 3	Bottom	−1180	−10400	11
	3 1	Top	−1800		17 (20)
	3 5	Top	1580	−10400	15 (17)
	5 3	Top	−1230		12
Diagonals	1 4	—	840	11500	7 (10)
	4 1	—	320		3
	4 5	—	1040	− 5000	21 (22)
	5 4	—	−93		2
Sub-Verticals	1 2	—	1630	11200	15
	2 1	—	2120		19 (19)
	3 4	—	1960	—	—
	4 3	—	1880		—
	5 6	—	0	11200	0
	6 5	—	0		0

VII. Accuracy of the Results by Numerical Moment Formulae.

When the truss has the same form, stiffness ratio and sectional area ratio as the standard truss proposed in this paper, the grade of accuracy of the results can be considered to be the same as by the Slope Deflection Method, because, although the constants in the numerical moment formulae were calculated by the Slope Distribution Method the results by two methods are exactly the same.

Even in the case when the truss has a quite different stiffness ratio of web members and the sectional area-ratio of sub-verticals from that of the standard truss, the results by the numerical moment formulae are also sufficiently accurate for all practical purposes.

When the truss has rather widely different stiffness ratio of chord members and sectional area-ratio of chord and web members excepting sub-verticals the results will give also comparatively approximate values but a great many examples of trusses actually already constructed show that the variations both of stiffness and sectional area-ratio in these members are limited within a small range.

The trusses in the numerical examples were all those of actual bridges taken at random, and their stiffness ratios and sectional area-ratios are not the same as the standard trusses. Yet, the results show a sufficient reliability for all practical purposes, that is, in Tables 15 to 17 the differences of percent secondary stresses of primary or allowable stress in each of the proposed method and exact one are, in most values, less than two or three percent.

Summary and Conclusions.

The general conclusions to be drawn from the investigations described in this paper are included in the resumé below:

The proposed nearly eight hundred numerical moment formulae are applicable for the secondary stress computation of the most commonly adopted types of bridge truss, that is, of through and deck Warren trusses with parallel chords, through Pratt trusses with parallel chords, and through Warren trusses with polygonal chords.

The special feature of the new method consists in the rapidity of the calculation for bending moments, by which secondary stresses are caused, at the ends of any member. It is worthy of special attention that the labour and time necessary to work out the bending moments at the ends of members are nearly the same as for the primary stress computation, because the required bending moments are to be directly calculated from the simple equation,

$$M_{mn} = \left\{(a \cdot n + b) + c \cdot \frac{1}{a_t}\right\} P \cdot \frac{a_{\max.}}{A_{\max.}} \frac{K_{mn}}{k_{mn}}$$

or

$$M_{mn} = \left\{(a \cdot n + b) + c \cdot \frac{1}{a_c}\right\} P \cdot \frac{a_{\max.}}{A_{\max.}} \frac{K_{mn}}{k_{mn}}$$

where

M_{mn} = bending moment at the end m of member m–n,
a, b, c = constant dependent on the kind of trusses,
n = number of panels in a span,
P = intensity of joint loads,
$a_t = \dfrac{A_t \cdot a_{\max.}}{A_{\max.}}$,
$a_c = \dfrac{A_c \cdot a_{\max.}}{A_{\max.}}$,
$a_{\max.}$ = the maximum sectional area in upper chord members of the standard truss,
$A_{\max.}$ = the maximum sectional area in upper chord members of the truss in question,
A_t, A_c = sectional areas respectively of tensile sub-verticals and compressive sub-verticals,
K_{mn} = stiffness of member m–n in the truss in question,
k_{mn} = stiffness of member m–n in the standard truss.

In this method there is no labour to calculate the primary stresses and member-rotation angles and there is also no labour to solve many simultaneous equations.

The merit of the method consists also in the possibility for free, selective calculation of required moments independently of the other moments, selecting any desired ones and yet the calculation time estimated is given in minutes.

The results by the proposed method have sufficient reliability for all practical purposes.

Finally, the author wishes to offer most grateful thanks to Prof. F. Takabeya for providing facilities and for guidance in the completion of the present work.