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Author(s)	Takabeya, Fukuhei; Sakai, Tadaaki
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A New Method of Rapid Computation of Secondary Stresses of Simple Bridge Trusses.

By

Fukuhei TAKABEYA and Tadaaki SAKAI.

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Introduction.

The use of rigid joint frames, either of electric welded or machine riveted, has been greatly increased and the practical importance of the proper treatment of secondary stresses has likewise remarkably grown.

The method here proposed lends itself well to an exact and rapid solution of secondary stresses for bridge trusses. The calculation results obtained from some eighty problems for a series of similar trusses of Warren and Pratt types, which have been solved by the new method proposed here, give, within a few minutes, the bending moments, i. e., the secondary stresses at any desired panel point of the trusses with any number of panels. We have computed all the panel-point-moments in both cases of load unity at every panel point and of the full uniform loading, for trusses of theoretical proportion on the sectional dimensioning of each member, the trusses being chosen from some fifty actual examples of highway bridges in about seventeen prefectures, inclusive of twenty examples of railway bridge trusses.

The present paper chiefly deals with the following three items:

(1) There is firstly treated the method of calculation to find the secondary stresses in general, by distributing the unbalanced amount of joint-rotation angles, until it is balanced by the mechanical method, undertaken on the truss-schema itself.

(2) Next, there are treated the determination of standard trusses and making up of stress diagrams of some 50 kinds to be applied to the practical and rapid computation of the secondary stresses for any sizes of Warren and Pratt trusses.

(3) Some characteristics both of primary and secondary stresses are described.

I. Determination of Standard Trusses.

After close investigation of a great many examples of trusses actually already constructed, especially parallel chord ones, it became obvious that the members of the trusses have almost constant ratio, both for stiffness and sectional area, between one member and another.

A few trusses of theoretical proportion of the sectional area were necessary and important in order to determine the characteristics of secondary stresses, increasing the number of panels one by one as well as by shifting the position of the load unity from one panel point to another. The trusses of theoretical proportion of the sectional areas are denoted in this paper as Standard Trusses. Forty-seven bridges contributed to the determination of the Standard Trusses and there are denoted, in this paper, the sectional area of each member by (A), the length of the member (l), moment of inertia of the section (I), distance between the extreme fibre and neutral axis of the member (e), and stiffness value of member ($I/l = K$). From among these trusses, Warren and Pratt types with parallel chord-members were chiefly introduced and tabulated in Appendix, Fig. 1 to Fig. 2, in which Trusses No. 1 to 10 are shown in Fig. 1 and Trusses No. 11 to 30 in Fig. 2. These tabulations seem to show the limit of the variation both for stiffness ratio and sectional area-ratio with respect to the trusses, above mentioned, existing in the present day in our country. As the standard, trusses with ten panels are considered, regarding both the stiffness K and the sectional area A respectively as unity with respect to the upper chord members of the central two panels.

The stiffness values and sectional areas of other members were computed as less than unity, relative to those of the central upper chords. For trusses with eight panels, the computations likewise were made taking unity for the stiffness and sectional areas of the central upper chords with respect to the Warren type trusses and 0.95 for the same with respect to the Pratt type. For Warren truss with six panels, $K = 0.85$ and $A = 0.8$ were used for the central upper chords and for Pratt truss with six panels, $K = 0.90$ and $A = 0.85$ were used respectively.

Considering these stiffness values and sectional area-distributions as well as the effect of the direct stresses, there were chosen the standard types of Warren and Pratt trusses for convenience in the study of secondary stresses due to the load position and to the variation of the number of panels. Fig. 1 shows the standard type of the Warren truss and Fig. 2 that of the Pratt truss.

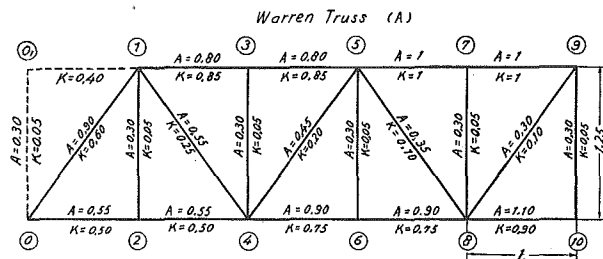


Fig. 1.

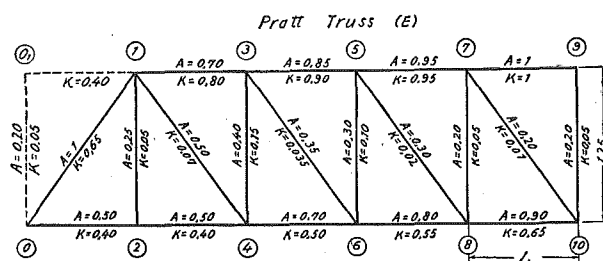


Fig. 2.

Some of the actual examples were different from the standard sections chosen by the authors, especially in respect to web members and subverticals (Figs. 3 and 4). Yet the effect of the difference from the standard sections may be easily corrected by the method of correction proposed below in this paper. The trusses shown in Figs. 5 and 6 have constant stiffness and sections in all the end posts, upper chords and lower chords; these special trusses were also important to the study of secondary stresses in general.

As the ratio between the height and panel length (h/l) 1.25 was chosen as shown in Figs. 1 and 2 which was the case in many actual examples.

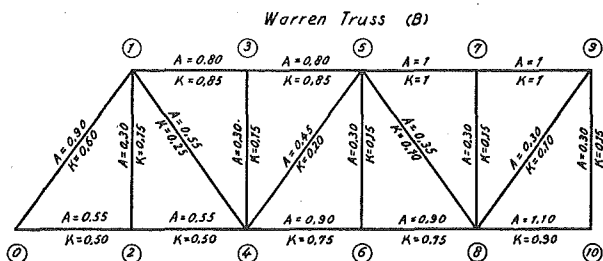


Fig. 3.

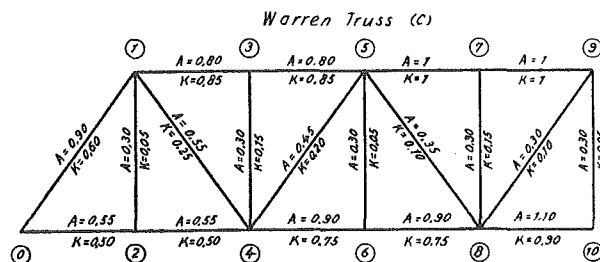


Fig. 4.

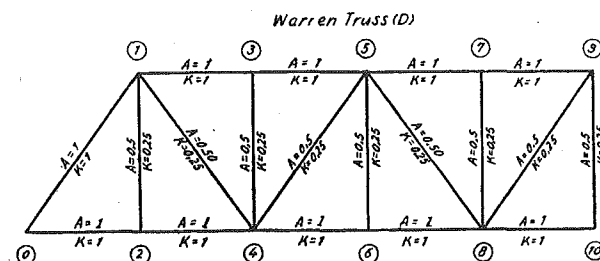


Fig. 5.

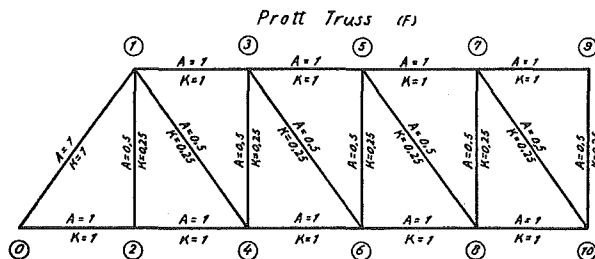


Fig. 6.

II. Relation of Panel-Point-Moments Among Many Trusses with Constant Ratio Both of Stiffness and Section Area.

In ordinary construction of bridge trusses, panel points carry all the loads and the loaded chords carry no intermediate loads; in consequence of the loading condition the moment at the end A of the member AB restrained at the ends is expressed by the well known equation :

$$M_{ab} = K_{ab}(2\varphi_a + \varphi_b + \psi_{ab}) \dots\dots\dots (1)$$

in this equation, we denote by K_{ab} the moment of inertia of the section divided by the length of the member AB , i. e., K_{ab} stiffness value of the member AB ; by φ_a and φ_b the joint-rotation angle of the ends A and B , respectively, multiplied by $2E$; E the modulus of elasticity of the material; and by ψ_{ab} the member revolution angle multiplied by $-6E$. The resisting moment on a section is considered positive when the couple acts in a clockwise direction upon the portion of the member considered; the joint-rotation angle is considered positive when the angle has been turned clockwise, measured from its initial position; the member revolution angle is also positive in case of revolution in clockwise direction from the initial position of the member.

As the value ψ_{ab} for an absolutely stiff connecting joint is but slightly different from that for a pin connecting joint, it is usually found by Williot-Mohr's Displacement Diagram, considering it the same value and it is substituted as a known quantity into the Slope Deflection Equation (1).

For a joint m shown in Fig. 7 (a) the equilibrium condition of the total moment gives:

$$\left. \begin{aligned} \sum M_m &= 0 \\ M_{m1} + M_{m2} + \dots + M_{mn} &= 0 \end{aligned} \right\} \dots\dots\dots (2)$$

in which

$$\left. \begin{aligned} M_{m1} &= K_{m1}(2\varphi_m + \varphi_1 + \psi_{m1}) \\ M_{m2} &= K_{m2}(2\varphi_m + \varphi_2 + \psi_{m2}) \\ \vdots \\ M_{mn} &= K_{mn}(2\varphi_m + \varphi_n + \psi_{mn}) \end{aligned} \right\} \dots\dots\dots (3)$$

Substituting equations (3) in (2) gives:

$$\begin{aligned} &2\varphi_m(K_{m1} + K_{m2} + \dots + K_{mn}) + \varphi_1 K_{m1} + \varphi_2 K_{m2} + \dots + \varphi_n K_{mn} \\ &= -(\psi_{m1} K_{m1} + \psi_{m2} K_{m2} + \dots + \psi_{mn} K_{mn}) \dots\dots\dots (4) \end{aligned}$$

From this condition of equilibrium, each joint gives one equation and m joints give therefore m equations, in which m unknowns φ are contained.

Next, we consider another frame of similar form, having the same stiffness ratio as well as the same section area-ratio, as shown in Fig. 7 (b), while the stiffness values are k times the former frame in Fig. 7 (a) and the section areas α times the same.

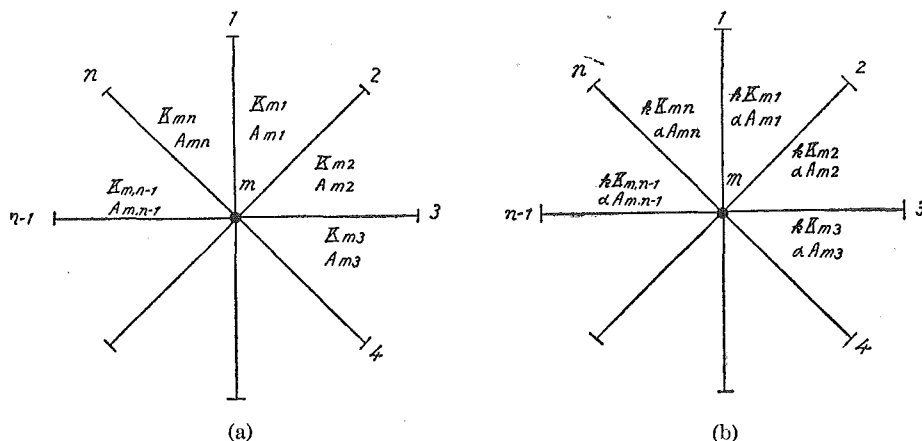


Fig. 7.

For the joint m in Fig. 7 (b), the equilibrium condition gives in a similar way :

$$2\varphi'_m(kK_{m1} + K_{m2} + \dots + K_{mn}) + \varphi'_1 kK_{m1} + \varphi'_2 kK_{m2} + \dots + \varphi'_n kK_{mn} \\ = -k(\psi'_{m1}K_{m1} + \psi'_{m2}K_{m2} + \dots + \psi'_{mn}K_{mn}) \dots\dots\dots (5)$$

Mohr's Displacement Diagram shows generally :

$$\psi' = \frac{\psi}{\alpha} \dots\dots\dots (6)$$

Equation (5) therefore becomes :

$$2\varphi'_m(K_{m1} + K_{m2} + \dots + K_{mn}) + \varphi'_1 K_{m1} + \varphi'_2 K_{m2} + \dots + \varphi'_n K_{mn} \\ = -\frac{1}{\alpha}(\psi_{m1}K_{m1} + \psi_{m2}K_{m2} + \dots + \psi_{mn}K_{mn}) \dots\dots\dots (7)$$

From equations (7) and (4) it is obvious :

$$\varphi'_m = \frac{\varphi_m}{\alpha}, \quad \varphi'_1 = \frac{\varphi_1}{\alpha}, \quad \varphi'_2 = \frac{\varphi_2}{\alpha} \dots\dots \varphi'_n = \frac{\varphi_n}{\alpha} \dots\dots\dots (8)$$

and

$$\begin{aligned} M'_{m1} &= kK_{m1} \left(2 \frac{\mathcal{P}_m}{a} + \frac{\mathcal{P}_1}{a} + \frac{\psi_{m1}}{a} \right) \\ M'_{m2} &= kK_{m2} \left(2 \frac{\mathcal{P}_m}{a} + \frac{\mathcal{P}_2}{a} + \frac{\psi_{m2}}{a} \right) \\ &\vdots \\ M'_{mn} &= kK_{mn} \left(2 \frac{\mathcal{P}_m}{a} + \frac{\mathcal{P}_n}{a} + \frac{\psi_{mn}}{a} \right) \end{aligned}$$

or

$$\left. \begin{aligned} M'_{m1} &= \frac{k}{a} M_{m1} \\ M'_{m2} &= \frac{k}{a} M_{m2} \\ &\vdots \\ M'_{mn} &= \frac{k}{a} M_{mn} \end{aligned} \right\} \dots\dots\dots (9)$$

Equation (9) are found to be very important relation and very practical to be applied in the calculation of secondary stresses, whose calculation procedure will be described later on.

III. The Slope Distribution Method Applied to Secondary Stress-Calculation.

The Slope Distribution Method. The exact calculation of secondary stresses by Manderla, and that of Müller-Breslau, Ritter and Mohr are the current methods of calculation for the secondary stresses induced by joint rigidity; and recently there is sometimes applied the method of distributing end-moment proposed firstly by H. Cross. The Cross Method seems to the present authors to have its merit, as proposed by Cross for the problem of continuous beams, for the treating of problems with a relative minority of members at each joint while the other old methods seem to be in most cases very long and to require considerable labour; on the other hand, the Slope Deflection Equation principally applied by Wilson and by Maney as well as by Gehler, simplifies the redundancies and the Mechanical Tabulation Method proposed about ten years ago by the senior author of this paper made the calculation process quite mechanical, while the simultaneous equations, from which the unknown quantities are to be determined, have

been obtained without considering any equilibrium equations, and these have been solved by the process of iteration.

The Slope Distribution Method proposed by the senior author and here applied*, eliminates completely the first process of writing the simultaneous equations and makes the second process of iteration a simple and mechanical method of calculation on the frame-schama itself. This method treats one unknown at each joint and successive similar calculations are carried out until the unbalanced slopes or joint-rotation-angles are distributed. The calculation of joint moment, using the values of the resulting slopes which have been calculated, is performed only once, and is not repeated. As demonstrated in the previous article, each panel joint of a truss gives :

$$2\varphi_m(K_{m1}+K_{m2}+\cdots+K_{mn})+\varphi_1K_{m1}+\varphi_2K_{m2}+\cdots+\varphi_nK_{mn} \\ = -(\psi_{m1}K_{m1}+\psi_{m2}K_{m2}+\cdots+\psi_{mn}K_{mn}) \dots\dots\dots (10)$$

Putting $j_m = 2(K_{m1}+K_{m2}+\cdots+K_{mn}) = 2 \times (\text{Sum of } K$
which meet at joint $m) \dots (11)$

equation (10) gives :

$$\varphi_m = -\{\varphi_1\gamma_{m1}+\varphi_2\gamma_{m2}+\cdots+\varphi_n\gamma_{mn}+\psi_{m1}\gamma_{m1}+\psi_{m2}\gamma_{m2}+\cdots+\psi_{mn}\gamma_{mn}\} \\ \dots\dots\dots (12)$$

where

$$\left. \begin{aligned} \gamma_{m1} &= \frac{K_{m1}}{j_m}, & \gamma_{m2} &= \frac{K_{m2}}{j_m} \\ \gamma_{mn} &= \frac{K_{mn}}{j_m} \end{aligned} \right\} \dots\dots\dots (13)$$

* TAKABEYA: Practical Solution of High Storied Bents by the Method of Slope Distribution, Journal of the Institute of Japanese Architects, Vol. XLVIII, No. 591, November, 1934.

Method of Slope Distribution Applied to Analysis of Statically Indeterminate Structures, Journal of the Civil Engineering Society, Japan, Vol. XXI, No. 1, January, 1935.

On Secondary Stress-Calculation of Trusses by the Slope Distribution Method, Do. Vol. XXI, No. 2, February, 1935.

Méthode des déformations réparties (Nouvelle méthode de calcul des ossatures à grand nombre d'étages sous l'action du vent et des sollicitations sismiques, L'OSSATURE METALLIQUE, Bruxelles, No. 5, Mai, 1935.

Tall Building Frames Subjected to Horizontal Joint Loads and a New Method of Calculation for Approximate Values of Joint Rotation Angles, Journal of the Institute of Japanese Architects, Vol. XLIX, No. 604, October, 1935.

The quantities γ are called "Slope Distributing Factors" and are given as numerical values, when the sectional areas of the members are assumed. ψ are given by the following expressions:

$$\left. \begin{aligned} \psi_{m1} &= -6E \frac{d_{m1}}{l_{m1}}, & \psi_{m2} &= -6E \frac{d_{m2}}{l_{m2}} \\ \psi_{mn} &= -6E \frac{d_{mn}}{l_{mn}} \end{aligned} \right\} \dots\dots\dots (14)$$

Using equation (14), and from equation (12)

$$\varphi_m = \Delta_m - \{\varphi_1 \gamma_{m1} + \varphi_2 \gamma_{m2} + \dots + \varphi_n \gamma_{mn}\} \dots\dots\dots (15)$$

where

$$\Delta_m = 6 \left\{ \frac{Ed_{m1}}{l_{m1}} \gamma_{m1} + \frac{Ed_{m2}}{l_{m2}} \gamma_{m2} + \dots + \frac{Ed_{mn}}{l_{mn}} \gamma_{mn} \right\} \dots\dots\dots (16)$$

In equation (16), the terms Ed_{m1} , Ed_{m2} , etc. are determined by Mohr's Displacement Diagram and are therefore given as numerical values for a system of given loads.

If all of the slopes of the neighbouring panel joints of the panel joint m in consideration are zero, i. e., if $\varphi_1 = \varphi_2 = \dots = \varphi_n = 0$ equation (15) gives:

$$\varphi_m^{(0)} = \Delta_m \dots\dots\dots (17)$$

In the Slope Distribution Method, we take Δ_m as the assumed value of φ_m and denote it by $\varphi_m^{(0)}$.

Regarding all the slopes of the neighbouring panel joints of joints 1, 2, 3 and 4 respectively (Fig. 8), equation (17) gives:

$$\left. \begin{aligned} \varphi_1^{(0)} &= \Delta_1, & \varphi_2^{(0)} &= \Delta_2 \\ \varphi_3^{(0)} &= \Delta_3, & \varphi_4^{(0)} &= \Delta_4 \end{aligned} \right\} \dots\dots\dots (18)$$

Substituting these $\varphi^{(0)}$ into equation (15) and writing φ_m in the case $\varphi_m^{(1)}$ gives:

$$\varphi_m^{(1)} = \Delta_m - \{\varphi_1^{(0)} \gamma_{m1} + \varphi_2^{(0)} \gamma_{m2} + \dots\} \dots\dots\dots (19)$$

The value $\varphi_m^{(1)}$ can then be numerically determined, when the righthand member of equation (19) is completely given by numerical values.

The relation given by equation (19) shows that one equation (19) determines one unknown $\varphi_m^{(1)}$. In a similar way, n equations given

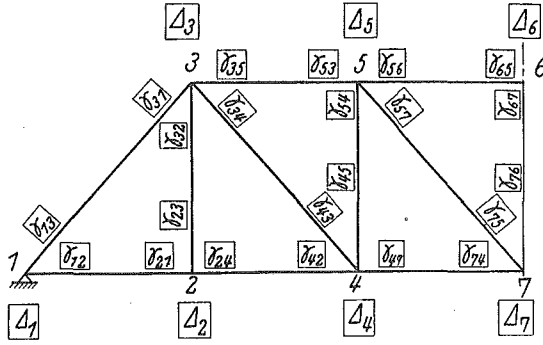


Fig. 8.

in the same form as equation (19) determine n unknowns $\varphi^{(1)}$, without solving them simultaneously, but solving n times one unknown from one equation for different n equations; this method of solution is described in the following. This characteristic makes the proposed method practical and the calculation very short. The more the number of unknowns increases the more the convenience of speciality is promoted.

Now, it is required to find $\varphi_1^{(1)}$ for the truss shown in Fig. 8. From equations (18) and (19) we have:

$$\begin{aligned}\varphi_1^{(1)} &= \Delta_1 - (\varphi_2^{(0)}\gamma_{12} + \varphi_3^{(0)}\gamma_{13}) \\ &= \Delta_1 - (\Delta_2\gamma_{12} + \Delta_3\gamma_{13}) \dots\dots\dots (a)\end{aligned}$$

Next, to find $\varphi_2^{(1)}$, applying the above value of $\varphi_1^{(1)}$, equation (19) gives:

$$\begin{aligned}\varphi_2^{(1)} &= \Delta_2 - (\varphi_1^{(1)}\gamma_{21} + \varphi_3^{(0)}\gamma_{23} + \varphi_4^{(0)}\gamma_{24}) \\ &= \Delta_2 - (\varphi_1^{(1)}\gamma_{21} + \Delta_3\gamma_{23} + \Delta_4\gamma_{24}) \dots\dots\dots (b)\end{aligned}$$

Similarly at joint 3:

$$\varphi_3^{(1)} = \Delta_3 - (\varphi_1^{(1)}\gamma_{31} + \varphi_2^{(1)}\gamma_{32} + \Delta_4\gamma_{34} + \Delta_5\gamma_{35}) \dots\dots\dots (c)$$

In the above equations, all the values Δ and γ are given as numerical values and they are to be written in the definite places marked in Fig. 8.

The calculation of $\varphi_1^{(1)}$ is done, for example, in such a way on the truss-schema as the following numerical example shows.

Look at first joint 1 in Fig. 8, put the numerical value γ_{12} on a slide rule, then multiply Δ_2 by it, give it a minus sign and write the product under the place Δ_1 . Likewise put the numerical value γ_{13} on

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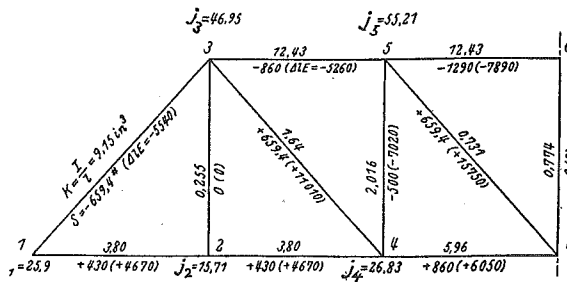


Fig. 10.

Case I. Load of 1,000 lbs. at Joint 7. member (K) or (I/l) and double the value of the sum of K at any joint are first computed. Next, the stress in each member (S) and the deformation of member (ΔLE) or (Sl/A), multiplied by the modulus of elasticity E are computed. These are conveniently shown on the diagram of the truss in Fig. 10.

A) *Calculation of the Member-Rotation Angles.* Using the value ΔLE the displacement of each joint and the rotation angle of member will be easily determined by Williot-Mohr's Displacement Diagram.

Since truss and its loading are symmetrical about the span center, it can readily be seen that the center vertical 6-7 is not subjected to a rotation due to the deformation of the truss members. Member 6-7 will therefore be taken as the reference member and joint 7 will be taken as the reference point. Hence, starting at 7 Williot-Mohr's Diagram can be drawn as shown in Fig. 11. The truss as a whole is not rotated and this diagram shows the correct relative position of the joints after the deformation of members.

Then the displacement of each joint may be found by measuring the distance from point 1 to each corresponding point

at 9 and 3,000 lbs. at 11. For all cases, the primary stresses in the left half of the truss will be the same. The final values of secondary stresses can readily be reduced to the same basis by dividing by the joint load assumed in each case.

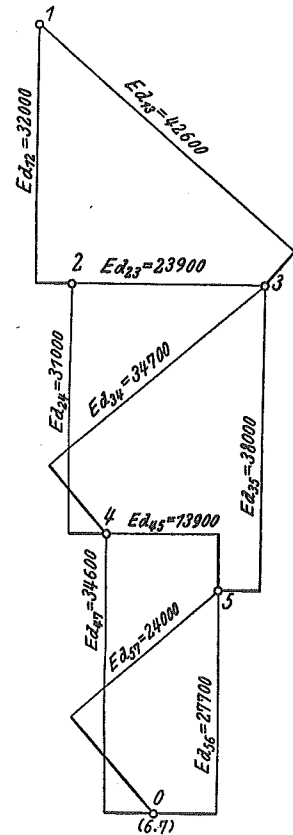


Fig. 11.

and also, in this diagram, the deflection of one end of each member relative to the other is given. These values are as follows:

$$Ed_{12} = 32,000$$

$$Ed_{13} = 42,600$$

$$Ed_{23} = 23,900$$

etc.

The fact that the deflection Ed is directly shown in the diagram is a very convenient aid to the calculation of the secondary stresses.

Next, dividing the value of Ed , obtained from Williot-Mohr's Diagram, by the length of the member, the member-rotation angle is obtained as follows:

$$\frac{Ed_{12}}{l_{12}} = \frac{32,000}{320} = 100$$

$$\frac{Ed_{13}}{l_{13}} = \frac{42,600}{490.7} = 86.8$$

$$\frac{Ed_{23}}{l_{23}} = \frac{23,900}{372} = 64.4$$

etc.

These results are written in Fig. 12 on the corresponding members.

B) Calculation of Distributing Factors (γ). From equation (13), the numerical values of the distributing factor (γ) are obtained as follows, using the values of K and j in Fig. 10:

At joint 1 we get:

$$\gamma_{12} = \frac{3.80}{25.9} = 0.1465, \quad \gamma_{13} = \frac{9.15}{25.9} = 0.353$$

In a similar way, at joint 2 we get:

$$\gamma_{21} = \frac{3.80}{15.71} = 0.242, \quad \gamma_{23} = \frac{0.255}{15.71} = 0.0162$$

$$\gamma_{24} = \frac{3.80}{15.71} = 0.242$$

etc.

In Fig. 12 there are shown the distributing factors of all members on the corresponding ends of members.

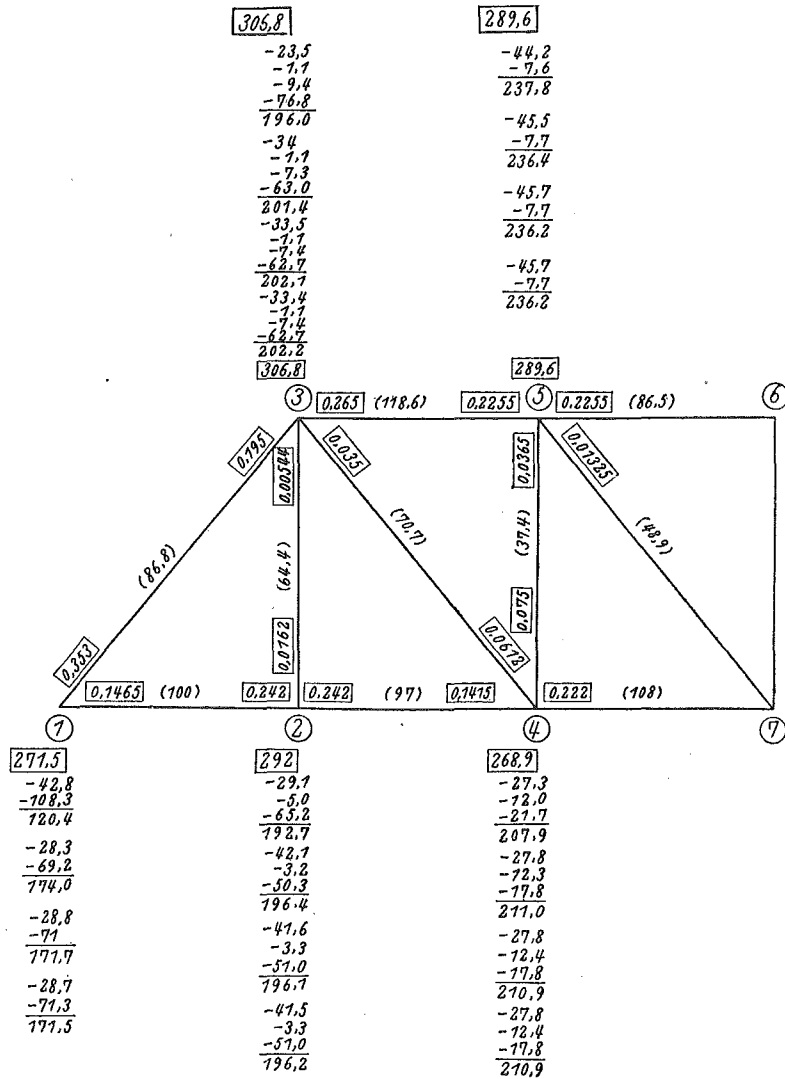


Fig. 12.

C) Calculation of Δ . From equation (16), the numerical values of Δ_m are obtained as follows, using the values of Ed/l and γ above calculated:

$$\begin{aligned} \Delta_1 &= 6 \left\{ \frac{Ed_{12}}{l_{12}} \gamma_{12} + \frac{Ed_{13}}{l_{13}} \gamma_{13} \right\} \\ &= 6 \{ (100)(0.1465) + (86.8)(0.353) \} = 271.5 . \end{aligned}$$

In a similar way,

$$\Delta_2 = 6 \{ (100)(0.242) + (64.4)(0.0162) + (97)(0.242) \} = 292$$

$$\begin{aligned} \Delta_3 &= 6 \{ (86.8)(0.195) + (118.6)(0.265) + (70.7)(0.035) \\ &\quad + (64.4)(0.00544) \} = 306.8 \end{aligned}$$

etc.

These values of Δ_m at all joints are written in Fig. 12 at the corresponding joints.

D) Calculation of Joint-Rotation Angles (φ) on Truss-Schema. Taking the value of Δ_m as the assumed value of the joint-rotation angle φ_m , we get

$$\varphi_1^{(0)} = 271.5$$

$$\varphi_2^{(0)} = 292.0$$

$$\varphi_3^{(0)} = 306.8$$

etc.

Substituting these values of $\varphi_m^{(0)}$ into equation (19), the first approximate values of the joint-rotating angles are obtained as follows:

$$\begin{aligned} \varphi_1^{(1)} &= \Delta_1 - \{ \varphi_2^{(0)} \gamma_{12} + \varphi_3^{(0)} \gamma_{13} \} \\ &= 271.5 - \{ (292)(0.1465) + (306.8)(0.352) \} \\ &= 271.5 - 42.8 - 108.3 = 120.4 \end{aligned}$$

We next shift the calculation to the second joint and the similar computation is continued as follows:

$$\begin{aligned} \varphi_2^{(1)} &= \Delta_2 - \{ \varphi_1^{(1)} \gamma_{21} + \varphi_3^{(0)} \gamma_{23} + \varphi_4^{(0)} \gamma_{24} \} \\ &= 292 - \{ (120.4)(0.242) + (306.8)(0.0162) + (268.9)(0.242) \} \\ &= 192.7 \end{aligned}$$

At this joint, it is advisable to use the value of $\varphi_1^{(1)}$ instead of $\varphi_1^{(0)}$ for the computations of $\varphi_2^{(1)}$.

At the following joints the calculations are the same as at the previous joints and the results are as follows :

$$\varphi_3^{(1)} = 196.0$$

$$\varphi_4^{(1)} = 207.9$$

etc.

Since the values of Δ_m , $\varphi_m^{(0)}$ and γ needed for the calculation of φ are all written in Fig. 12, the above calculations may be made directly on the truss-schema with the help of a slide rule. The above equations are nothing but to explain the order of procedure of calculation.

When the calculation process has been made for all the panel joints, the same calculation is repeated this time with the first approximate values $\varphi_m^{(1)}$ which have been obtained by the previous computation and the second approximate values of φ_m are obtained as follows :

$$\varphi_1^{(2)} = \Delta_1 - \{\varphi_2^{(1)}\gamma_{12} + \varphi_3^{(1)}\gamma_{13}\}$$

$$= 271.5 - 28.3 - 69.2 = 174.0$$

$$\varphi_2^{(2)} = \Delta_2 - \{\varphi_1^{(2)}\gamma_{21} + \varphi_3^{(1)}\gamma_{23} + \varphi_4^{(1)}\gamma_{24}\}$$

$$= 292 - 42.1 - 3.2 - 50.3 = 196.4$$

etc.

For practical purposes, such repetitions 4 or 5 times give the exact values of φ_m . In this example, after repetition 4 times we determined the values of φ_m as follows :

$$\varphi_1 = 171.5 \quad \varphi_2 = 196.2 \quad \varphi_3 = 202.2$$

$$\varphi_4 = 210.9 \quad \varphi_5 = 236.2 \quad \varphi_6 = \varphi_7 = 0$$

E) *Calculation of Secondary Stresses.* The bending moments are found from equation (1), as follows :

$$M_{ab} = K_{ab} \left\{ 2\varphi_a + \varphi_b + \psi_{ab} \right\} = \frac{I_{ab}}{l_{ab}} \left\{ 2\varphi_a + \varphi_b - 6\frac{I_{ab}}{l_{ab}} \right\} \dots\dots\dots (20)$$

From the expression $\sigma_{ab} = M_{ab} \cdot c_{ab} / I_{ab}$, we have from the above equation,

$$\begin{aligned}\sigma_{ab} &= \frac{I_{ab}}{l_{ab}} \left\{ 2\varphi_a + \varphi_b - 6 \frac{Ed_{ab}}{l_{ab}} \right\} \frac{c_{ab}}{I_{ab}} \\ &= \frac{c_{ab}}{l_{ab}} \left\{ 2\varphi_a + \varphi_b - 6 \frac{Ed_{ab}}{l_{ab}} \right\} \dots\dots\dots (21)\end{aligned}$$

In these equations M_{ab} and σ_{ab} are respectively the bending moment and extreme fibre stress at joint A in member AB , and the distance of the extreme fibre from neutral axis of the member AB .

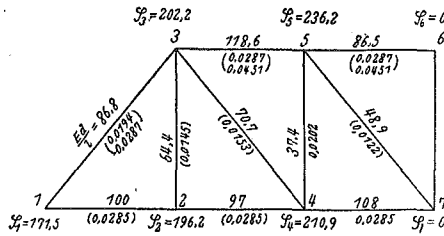


Fig. 13.

In Fig. 13 there are shown the values of c/l , and Ed/l .

Hence, from Fig. 13 we can determine the secondary stresses in all members as follows, using equation (21):

$$\begin{aligned}\sigma_{12} &= \frac{c_{12}}{l_{12}} \left\{ 2\varphi_1 + \varphi_2 - 6 \frac{Ed_{12}}{l_{12}} \right\} \\ &= 0.0285 \{ 2(171.5) + 196.2 - 6(100) \} = -1.7(1.73) \\ \sigma_{21} &= 0.0285 \{ 2(196.2) + 171.5 - 6(100) \} = -1.0(1.01)\end{aligned}$$

In a similar way,

$$\begin{aligned}\sigma_{18}^{(1)} &= 0.7(0.72), & \sigma_{18}^{(2)} &= 0.5(0.49) \\ \sigma_{31}^{(1)} &= 1.6(1.65), & \sigma_{31}^{(2)} &= 1.1(1.12) \\ \sigma_{23} &= 3.0(3.04), & \sigma_{32} &= 3.1(3.14) \\ \sigma_{24} &= 0.6(0.6), & \sigma_{42} &= 1.0(1.05) \\ \sigma_{34} &= 2.9(2.94), & \sigma_{43} &= 3.2(3.07) \\ \sigma_{35}^{(1)} &= -3.2(3.26), & \sigma_{35}^{(2)} &= -2.0(2.07) \\ \sigma_{53}^{(1)} &= -1.7(1.78), & \sigma_{53}^{(2)} &= -1.1(1.14) \\ \sigma_{45} &= 8.8(8.75), & \sigma_{54} &= 9.3(9.24) \\ \sigma_{47} &= -6.4(6.42), & \sigma_{74} &= -12.4(12.44) \\ \sigma_{66}^{(1)} &= -2.1(2.05), & \sigma_{66}^{(2)} &= -1.3(1.3) \\ \sigma_{65}^{(1)} &= -12.7(12.65), & \sigma_{65}^{(2)} &= -8.1(8.06) \\ \sigma_{57} &= 2.2(2.17), & \sigma_{75} &= -0.7(0.7)\end{aligned}$$

But, in point of fact, this reference member 1-2 is not fixed in direction, as assumed, and the rotation angle of the reference member is equal to the vertical movement of the free end of the

truss divided by the span length. The rotation angles for the members include also the rotation angle for the reference member. Hence, to

determine the true rotation angles for the members, correction must be made for Ed'/l and the correction angle, multiplied by E , is $180,000/(6 \times 320) = 93.8$.

Therefore, the true rotation angles of members are as follows:

$$\frac{Ed}{l} = 0 + 93.8 = 93.8$$

$$\frac{Ed}{l} = -12.3 + 93.8 = 81.5$$

$$\frac{Ed}{l} = -34.6 + 93.8 = 59.2$$

etc.

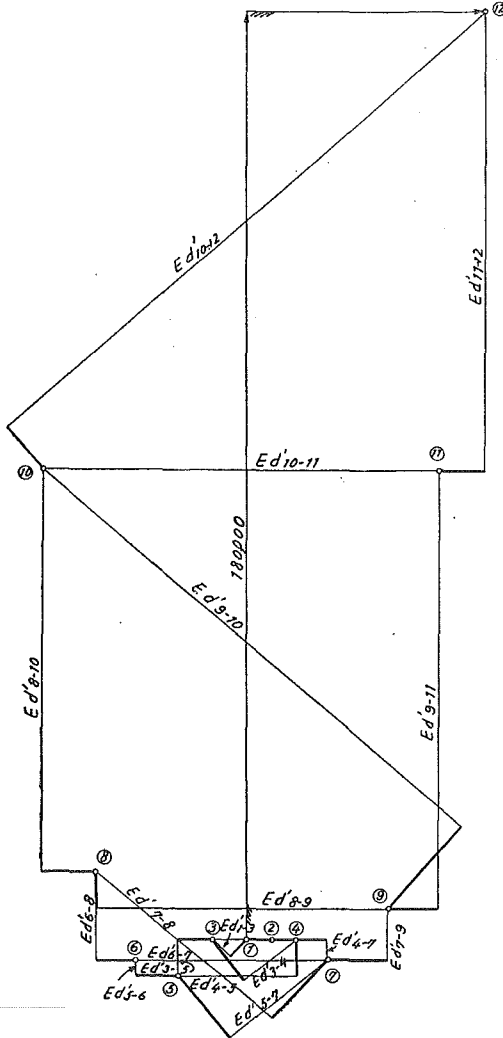
These results are written in Fig. 16 under the corresponding members.

B) *Calculation of Distributing Factors (γ)*. The numerical values of distributing factors are the same as in the previous case, and these values are written in Fig. 16 on the corresponding ends of members.

C) *Calculation of Δ* . From equation (16), the values of Δ_m are obtained

as follows, using the values of Ed/l and γ above calculated:

$$\begin{aligned} \Delta_1 &= 6 \left\{ \frac{Ed_{12}}{l_{12}} + \frac{Ed_{13}}{l_{13}} \gamma_{13} \right\} \\ &= 6 \{ (93.8)(0.1465) + (81.5)(0.353) \} = 255.0 \end{aligned}$$



The above calculations are carried out directly on the truss-schema with the help of a slide rule or calculating machine.

The second approximate values are also obtained in the same way as the first ones and these values are written in Fig. 16 at the corresponding joints.

After repetition 4 times we determined the values of φ_m as follows:

$$\begin{aligned}\varphi_1 &= 153.2, & \varphi_2 &= 197.7, & \varphi_3 &= 206.2, & \varphi_4 &= 161.2, \\ \varphi_5 &= 186.8, & \varphi_6 &= 160.3, & \varphi_7 &= 198.0, & \varphi_8 &= -116.0, \\ \varphi_9 &= -85.3, & \varphi_{10} &= -356.2, & \varphi_{11} &= -428.2, & \varphi_{12} &= -305.6,\end{aligned}$$

E) Calculation of Secondary Stresses (σ). For convenience in calculations of bending moment and fibre stress the values of φ , Ed/l and c/l are shown in Fig. 17.

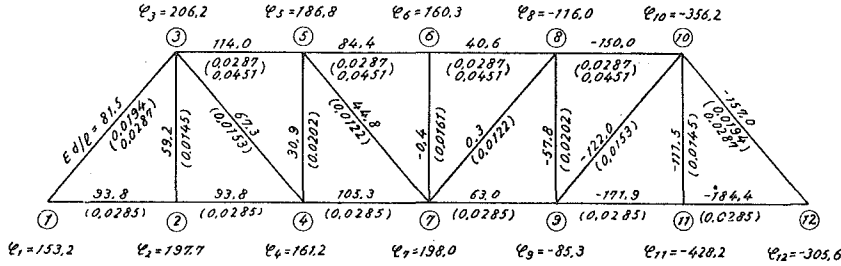


Fig. 17.

From Fig. 17, we can determine the secondary stresses in all members for a load of 1,000 lbs. as follows, using equation (21):

$$\begin{aligned}\sigma_{12} &= \frac{c_{12}}{l_{12}} \left\{ 2\varphi_1 + \varphi_2 - 6 \frac{Ed_{12}}{l_{12}} \right\} \div 1.5 \\ &= 0.0285 \{ 2(153.2) + 197.7 - 6(93.8) \} \div 1.5 = -1.12(1.18) \\ \sigma_{21} &= 0.0285 \{ 2(197.7) + 153.2 - 6(93.8) \} \div 1.5 = -0.27(0.33)\end{aligned}$$

In a similar way,

$$\begin{aligned}\sigma_{13}^{(1)} &= 0.45(0.50), & \sigma_{31}^{(1)} &= 1.46(1.50), & \sigma_{13}^{(2)} &= 0.30(0.33) \\ \sigma_{31}^{(2)} &= 1.00(1.02), & \sigma_{23} &= 2.38(2.39), & \sigma_{22} &= 2.46(2.48) \\ \sigma_{24} &= -0.01(0.02), & \sigma_{34} &= 1.73(1.78), & \sigma_{43} &= 1.27(1.27) \\ \sigma_{42} &= -0.79(0.77), & \sigma_{35}^{(1)} &= -2.56(2.59), & \sigma_{53}^{(1)} &= -3.13(3.27) \\ \sigma_{35}^{(2)} &= -1.65(1.65), & \sigma_{53}^{(2)} &= -1.98(2.09), & \sigma_{45} &= 4.35(4.08)\end{aligned}$$

$$\begin{aligned}
\sigma_{54} &= 4.70(4.44), & \sigma_{47} &= -2.12(2.12), & \sigma_{74} &= -1.43(1.42) \\
\sigma_{69}^{(1)} &= 0.83(1.13), & \sigma_{66}^{(1)} &= 0.03(0.30), & \sigma_{66}^{(2)} &= 0.52(0.72) \\
\sigma_{66}^{(2)} &= 0.02(0.19), & \sigma_{67} &= 2.46(2.43), & \sigma_{76} &= 2.53(2.50) \\
\sigma_{67} &= 5.60(5.76), & \sigma_{76} &= 6.00(6.17), & \sigma_{68}^{(1)} &= -1.17(1.30) \\
\sigma_{86}^{(1)} &= -9.50(9.71), & \sigma_{68}^{(2)} &= -0.74(0.82), & \sigma_{68}^{(2)} &= -6.03(6.18) \\
\sigma_{78} &= 2.26(2.07), & \sigma_{87} &= -0.30(0.50), & \sigma_{79} &= -1.28(1.32) \\
\sigma_{97} &= -6.67(6.77), & \sigma_{89} &= 0.39(0.16), & \sigma_{98} &= 0.81(0.58) \\
\sigma_{810}^{(1)} &= 9.95(9.74), & \sigma_{108}^{(1)} &= 2.15(2.53), & \sigma_{810}^{(2)} &= 6.33(6.20) \\
\sigma_{108}^{(2)} &= 1.37(1.61), & \sigma_{910} &= 2.10(2.08), & \sigma_{109} &= -0.67(0.68) \\
\sigma_{911} &= 8.20(8.51), & \sigma_{119} &= 1.70(2.04), & \sigma_{1011} &= -4.55(4.71) \\
\sigma_{1110} &= -5.26(5.38), & \sigma_{1012}^{(1)} &= -1.46(1.69), & \sigma_{1210}^{(1)} &= -0.48(0.47) \\
\sigma_{1012}^{(2)} &= -1.00(1.14), & \sigma_{1210}^{(2)} &= -0.33(0.33), & \sigma_{1112} &= 1.06(1.33) \\
\sigma_{1211} &= 1.30(1.18).
\end{aligned}$$

The values in bracket are those given by Johnson. The sign of stress indicates the direction of the bending moment and not the stress.

Case III. Load of 3,000 lbs. at joint 11. In Fig. 18 there are shown the values of K , j , S and ΔE . Quite in the same way as mentioned in case II, we can determine the values of member-rotation angle Ed/l , distributing factor γ , Δ and joint-rotation angle φ as shown in Fig. 19.

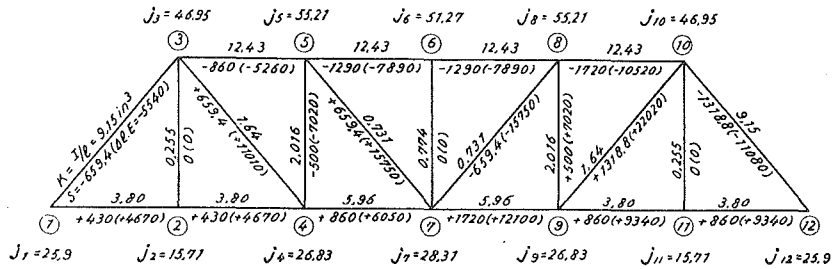


Fig. 18.

From the above calculations, we determine the final values of φ as follows:

$$\begin{array}{lll}
 \varphi_1 = 155.5 & \varphi_2 = 198.7 & \varphi_3 = 211.6 \\
 \varphi_4 = 182.7 & \varphi_5 = 200.8 & \varphi_6 = 126.2 \\
 \varphi_7 = 127.8 & \varphi_8 = 28.3 & \varphi_9 = 226.2 \\
 \varphi_{10} = -178.4 & \varphi_{11} = -373.1 & \varphi_{12} = -912.1
 \end{array}$$

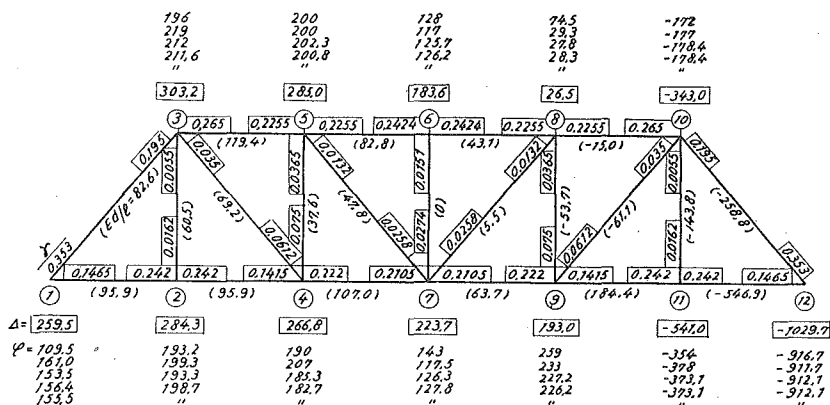


Fig. 19.

The values of c/l are the same as in case II.

Next, substituting the above calculated values in equation (21), and dividing by 3, we can determine the secondary stresses in members for 1,000 lbs. load at joint 11. These values are as follows:

$$\begin{array}{lll}
 \sigma_{12} = -0.62(0.59), & \sigma_{21} = -0.21(0.23), & \sigma_{13}^{(1)} = 0.26(0.25) \\
 \sigma_{31}^{(1)} = 0.79(0.71), & \sigma_{13}^{(2)} = 0.17(0.17), & \sigma_{31}^{(2)} = 0.53(0.48) \\
 \sigma_{23} = 1.19(1.14), & \sigma_{32} = 1.24(1.19), & \sigma_{24} = 0.05(0.08) \\
 \sigma_{42} = -0.11(0.12), & \sigma_{34} = 0.97(0.94), & \sigma_{43} = 0.82(0.78) \\
 \sigma_{35}^{(1)} = -1.39(1.26), & \sigma_{53}^{(1)} = -1.55(1.41), & \sigma_{35}^{(2)} = -0.88(0.80) \\
 \sigma_{53}^{(2)} = -0.99(0.90), & \sigma_{45} = 2.29(2.31), & \sigma_{54} = 2.38(2.45) \\
 \sigma_{47} = -1.41(1.43), & \sigma_{74} = -1.93(1.91), & \sigma_{46}^{(1)} = 0.47(0.31) \\
 \sigma_{65}^{(1)} = -0.65(0.78), & \sigma_{65}^{(2)} = 0.30(0.20), & \sigma_{65}^{(3)} = -0.41(0.50) \\
 \sigma_{57} = 0.99(1.00), & \sigma_{75} = 0.69(0.71), & \sigma_{67} = 2.04(2.11) \\
 \sigma_{76} = 2.05(2.12), & \sigma_{68}^{(1)} = 0.33(0.41), & \sigma_{68}^{(2)} = -0.98(1.13) \\
 \sigma_{68}^{(3)} = 0.21(0.26), & \sigma_{86}^{(2)} = -0.63(0.72), & \sigma_{78} = 1.02(1.04) \\
 \sigma_{87} = 0.62(0.62), & \sigma_{79} = 0.95(0.92), & \sigma_{97} = 1.88(1.81)
 \end{array}$$

$$\begin{aligned}
\sigma_{89} &= 4.08(4.08), & \sigma_{98} &= 5.40(5.40), & \sigma_{810}^{(1)} &= -0.48(0.49) \\
\sigma_{108}^{(1)} &= -3.60(3.61), & \sigma_{810}^{(2)} &= -0.30(0.31), & \sigma_{108}^{(2)} &= -2.26(2.30) \\
\sigma_{910} &= 3.14(3.31), & \sigma_{109} &= 1.05(1.25), & \sigma_{911} &= -9.75(9.55) \\
\sigma_{119} &= -15.40(15.20), & \sigma_{1011} &= 0.65(0.69), & \sigma_{1110} &= -0.30(0.23) \\
\sigma_{1012}^{(1)} &= 2.72(2.67), & \sigma_{1210}^{(1)} &= -4.30(4.25), & \sigma_{1012}^{(2)} &= 1.84(1.80) \\
\sigma_{1210}^{(2)} &= -2.90(2.88), & \sigma_{1112} &= 15.40(15.20), & \sigma_{1211} &= 9.37(10.13).
\end{aligned}$$

The values in brackets are those given by Johnson and the sign of stress is the same in the above two cases.

From the above three cases, the fibre stress in any member, due to 1,000 lbs. at any joint, can be obtained. Thus for member 2-4, load at 2, the stress is the same as in member 11-9, for load at 11, but with the opposite sign, etc.

In Table 1, are brought together the values of the primary and secondary stresses at each end of each member of one-half of the truss, for a load of 1,000 lbs. at each joint. There is also given the total secondary stress which occurs for the same loading as the maximum primary stress. This is given both in pounds per sq. in. and also as percentage of the maximum primary stress.

For the end post and top chord sections both top and bottom fibre stresses are given. For the other members, only one value is given, this value being the stress in the fibre first met in passing around the joint in clockwise rotation. These signs indicate the properties of stress and a plus sign will then signify a tensile stress and a minus sign a compressive stress.

The values in brackets show the values calculated by Johnson.

Comparing the results calculated by the Slope Distribution Method with those given by Johnson, they bear a striking resemblance to each other. And yet the Slope Distribution Method, as illustrated by the preceding problem, saves much trouble of writing and solving the many simultaneous elastic equations and makes the calculation process a simple and quite mechanical one.

Hence the secondary stress whose calculation has been generally considered by engineers a very tedious and laborious operation, can be both easily and accurately determined.

Table 1. Primary and Secondary Stresses for 1,000 lbs. Joint Loads.

Member		Secondary Stresses for 1,000 lbs. at					Max. Primary Stress	Total Secondary Stress for Max. Primary	Per cent Secondary of Max. Primary
		2	4	7	9	11			
End Post	13 T	-2.90	-0.33	-0.50	-0.30	-0.17	} -56.40	-4.20	7.5 (7.5)
	B	+4.30	+0.48	+0.7	+0.45	+0.26		-	
	31 T	-1.84	+1.00	+1.1	+1.00	+0.53		-	
	B	+2.72	-1.46	-1.6	-1.46	-0.78		-2.59	4.6 (5.1)
Top Chords	35 T	-2.26	+1.37	+2.0	+1.66	+0.88	} -65.72	-	
	B	+3.60	-2.15	-3.2	-2.56	-1.39		-5.70	8.7 (9.2)
	53 T	+0.30	-6.33	-1.1	-1.98	-0.99		-10.10	15.4 (15.2)
	B	-0.48	+9.95	+1.7	+3.13	+1.55		-	
	56 T	-0.63	-6.03	+1.3	-0.52	-0.30	} -74.00	-6.18	8.4 (8.8)
	B	+0.98	+9.50	-2.1	+0.83	+0.47		-	
	65 T	-0.31	+0.74	-8.1	-0.02	-0.41		-8.00	10.2 (10.6)
	B	+0.33	-1.17	+12.7	+0.03	+0.65		-	
Bottom Chords	12	+9.37	+1.30	+1.7	+1.12	+0.62	} +73.10	+14.11	19.4 (20.3)
	21	+15.40	-1.06	+1.0	+0.27	+0.21		+15.82	21.6 (21.4)
	24	-15.40	+1.70	-0.6	+0.01	-0.05	} +73.10	-14.34	19.7 (19.0)
	42	-9.75	+8.20	-1.0	+0.79	+0.11		-1.65	2.2 (1.6)
	47	+1.88	-6.67	+6.4	+2.12	+1.41	} +75.70	+5.14	6.8 (6.6)
	74	+00.95	-1.28	+12.4	+1.43	+1.93		+15.43	20.4 (20.3)
Hip Vertical	23	-0.30	-5.26	-3.0	-2.38	-1.19	} +62.50	-12.13	19.4 (19.5)
	32	+0.95	-4.55	-3.1	-2.46	-1.24		-10.70	17.4 (17.3)
Second Vertical	45	+5.40	+0.81	-8.8	-4.35	-2.29	} -37.76	-15.44	40.9 (40.1)
	54	+4.08	+0.39	-9.3	-4.70	-2.38		-16.38	43.3 (42.7)
Middle Vertical	76	+2.05	+6.00	0	-6.00	-2.05	} 0	-8.05	
	67	+2.04	+5.60	0	-5.60	-2.04		-7.64	
Main Diagonal	34	+1.05	-0.67	-2.9	-1.73	-0.97	} +74.77	-6.32	8.5 (8.5)
	43	+3.14	+2.10	-3.1	-1.27	-0.82		-3.09	4.1 (4.1)
Second Diagonal	57	+0.62	+0.30	-2.2	-2.46	-0.99	} +64.14	-5.65	8.8 (8.7)
	75	+1.02	+2.26	+0.7	-2.53	-0.69		-2.52	3.9 (3.8)

T = top fibre B = bottom fibre

IV. Joint Moment Diagrams of Standard Trusses and Method of Their Applications.

Making up of Joint Moment Diagrams of Standard Trusses. For the purpose of making up of joint moment diagrams of standard trusses, the bending moments at the ends of members have been calculated, that is, the joint moments, of the standard Warren and Pratt trusses with any number of panels less than ten for a single load unity placed successively at each joint.

In regard to the standard trusses, these have been already determined above in Section I. The calculations of joint moments have been carried out by the Slope Distribution Method and their results have the same accuracy as the ones obtained by solving the many simultaneous elastic equations such as equation (4).

In the Appendix, Figs. 3 to 29, there are shown these results. That is, Appendix, Figs. 3 to 7 show the joint moments of the standard Warren truss for a single load unity at each joint in lower chord and, Figs. 8 to 12 show respectively the joint moments of the same truss for single load unity at each joint in upper chord. Figs. 13 to 21 show the joint moments of a standard Pratt truss for a single load unity at each joint in lower chord and Figs. 22 to 29 show the same values respectively of the same truss for single load of unity at each joint in the upper chord.

Next, in the Appendix, Figs. 30 to 37, there are shown the joint moments of the special trusses with constant stiffness and sections throughout the end posts, upper chords and lower chords, such as Figs. 5 and 6.

In these figures, joint moments are written on the corresponding ends of members. Thus the joint moment at the end 1 of the member 1-3, denoted by M_{13} , is written at the left end of that member, etc. The value in the first rank expresses the joint moment due to a single load unity at joint 2.

Likewise the values in the second and third ranks express the joint moments due to a simple load unity at joints 4 and 6 respectively and so forth. But for the loads applied at the upper chord joints, the values in the first, second and third ranks express the joint moments due to a single load unity at joints 1, 3, and 5 respectively and so forth.

The value in the third rank from the last expresses the joint moment due to a full load. The values in the second and first ranks

from the last express the positive and negative maximum joint moments respectively.

In regard to signs, these indicate the direction of the moment; the joint moment is considered positive when it acts in a clockwise direction upon the end of the member considered.

In the Pratt truss, the joint moments due to upper-joint loads will be somewhat different from those due to lower-joint loads owing to the difference in stress in the vertical posts. However, their difference is small and it is sufficiently accurate for most purposes to assume that both results are the same. That is especially the case with the truss of a through bridge as the load applied at the upper joints is generally small, compared to the total load.

However, for the joint loads at the two end of a sub-vertical a great difference will occur. Hence, so far as the joint load at the top of a sub-vertical is concerned, the joint moment even in the upper-joint loads were computed, as shown in the Appendix, Figs. 22 to 29.

For the Pratt truss with an even number of panels, counters are not used, but for those with an odd number of panels, a pair of diagonals are used in the center panel to obtain a symmetrical form.

Though the number of panels is even, in practice when the diagonals are not stiffly designed to be capable of taking either tension or compression, counters are to be used in those panels where the maximum and minimum shears are of opposite signs. However, such a case is not treated here.

The Practical and Rapid Computation of Secondary Stresses by Joint Moment Diagrams of Standard Trusses. We consider a truss of similar form, having the same stiffness ratio as well as the same sectional area ratio as the standard truss, while the stiffness values are k times those of the standard truss and the sectional areas α times the same.

Then, from the relation of equation (9) the joint moments of this truss are k/α times those of the standard truss.

Namely

$$M = \frac{k}{\alpha} M_0 \dots\dots\dots (22)$$

As described in Section I, a great many examples of trusses actually already constructed, have the same stiffness ratio as well as the same sectional area-ratio as the standard truss. Hence we can very rapidly estimate the joint moments, i. e., the secondary stresses with any number of panels less than ten for various loading conditions.

It is of course true that some of the examples are different from the standard sections, especially for web members and sub-verticals as shown in Figs. 3 and 4. Yet, the effect of the difference may be easily corrected by the method of correction described later.

Stiffness-Correction of Joint Moment Diagrams. For the same sectional area the web members take different stiffness values according to the form of sections. Some of them are stiffly built up to resist either tension or compression and their stiffness values are large. On the contrary, others are slenderly built to resist only tension and their stiffness values are small.

Hence, some of the actual examples are different from the standard truss in their stiffness ratio.

Fig. 3 shows a Warren truss with stiff sub-verticals whose stiffness values are all equally 0.15. In Fig. 4 is shown a Warren truss with both the stiff and slender sub-verticals according as they are compression or tension members; their stiffness values are 0.15 and 0.05 respectively.

The bending moment at the end A of the member AB is expressed by equation (1), that is,

$$M_{ab} = K_{ab}(2\varphi_a + \varphi_b + \psi_{ab})$$

The stiffness of web members is very small as compared with that of chord members and the effect of the web members is negligible upon the joint-rotation angles, φ_a and φ_b . Also the member-rotation angle ψ_{ab} is considered to be independent of the stiffness of the member for truss and it has relation only to the sectional area of the same.

Hence it may be said that the end moments of the chord members are not effected by the stiffness values of the web members and those end moments of the web members are proportional to their stiffness values under the assumption that the sectional areas of web members are constant for any stiffness value of them.

By this important characteristic, we can compute the joint moment diagram of any truss which has the different stiffness value from the standard truss in web members, from one of the standard truss.

As the equilibrium condition at a joint, namely, $\sum M = 0$, is disturbed from the above correction, this discrepancy of the equilibrium condition should be distributed to each member which meets at this joint, proportionally to their stiffness values.

Now, for example, let a Warren truss be considered with 10 panels and stiff sub-verticals, having the stiffness value of 0.15, while the other members are the same as those of the standard truss.

The joint moments of such a truss due to full load are easily and rapidly determined by the use of the joint moment diagram of the standard truss as follows:

From the joint moment diagram in the Appendix, Fig. 3, joint moments of the standard Warren truss with 10 panels due to full load are obtained as follows:

At joint 0,

$$M_{01} = 15.42, \quad M_{02} = -15.42$$

At joint 1,

$$M_{10} = 5.87, \quad M_{13} = -17.20, \quad M_{14} = 7.77$$

$$M_{12} = 3.56.$$

At joint 2,

$$M_{20} = -16.12, \quad M_{24} = 11.80, \quad M_{21} = 4.31$$

At joint 3,

$$M_{31} = -21.76, \quad M_{35} = 16.80, \quad M_{34} = 4.97$$

Etc.

From the above values, the joint moments of a truss in this example are determined as follows:

At joint 0, all members which meet at this joint are the same as of the standard truss. Hence, the joint moments at this joint are also the same as those of the standard one with values as follows:

$$M_{01} = 15.42(16.04), \quad M_{02} = -15.42(16.04)$$

At joint 1, as the stiffness values of members, 1-0, 1-3, and 1-4 are the same as of the standard truss, the joint moments of these members are the same as of the standard one. However, the stiffness value of member, 1-2, is 3 times that of the standard truss. Hence, the joint moment of this member is also 3 times that of the standard truss. These values are as follows:

$$M_{10} = 5.87, \quad M_{13} = -17.20, \quad M_{14} = 7.77$$

$$M_{12} = 3.56 \times \frac{0.15}{0.05} = 10.68.$$

The discrepancy of the equilibrium condition at joint 1 due to the stiffness-correction becomes as follows :

$$\Sigma M = 5.87 - 17.20 + 7.77 + 10.63 = +7.12 .$$

Distributing this discrepancy to each member in proportion to the stiffness values, the final joint moments at joint 1 can be determined as follows :

$$\begin{aligned} M_{10} &= 5.87 - 7.12 \times \frac{0.60}{0.60 + 0.85 + 0.25 + 0.15} = 5.87 - 7.12 \times \frac{0.60}{1.85} \\ &= 5.87 - 2.31 = 3.56(4.93) . \end{aligned}$$

$$M_{13} = -17.20 - 7.12 \times \frac{0.85}{1.85} = -17.20 - 3.27 = -20.47(21.41)$$

$$M_{14} = 7.77 - 7.12 \times \frac{0.25}{1.85} = 7.77 - 0.96 = 6.81(6.73)$$

$$M_{12} = 10.68 - 7.12 \times \frac{0.15}{1.85} = 10.68 - 0.58 = 10.10(9.75)$$

In the same way, at joint 2 we get,

$$M_{20} = -19.87(19.11) , \quad M_{24} = 8.05(7.50) , \quad M_{21} = 11.81(11.61)$$

At joint 3,

$$M_{35} = 12.30(12.43) , \quad M_{31} = -26.26(26.60) , \quad M_{34} = 14.00(14.17)$$

Etc.

The values in brackets are the ones directly calculated by the Slope Distribution Method as shown in Appendix, Fig. 38.

Area-Correction of Joint Moment Diagram. The variation in the area of a sub-vertical has great effect upon the secondary stress in a chord member and some of the actual examples are different from the area-ratio in the standard truss. Hence, it is necessary to find this effect.

In the standard truss of Appendix, Figs. 3 and 8, let

A_0 = sectional area of member 1-2 ;

M_u = joint moment of any member $m-n$ due to a load at joint 1 ;

M_l = joint moment of member $m-n$ due to a load at joint 2 ;

Then the joint moment of member $m-n$ in the standard truss due to a load at joint 2 is expressed as follows :

$$M_t = M_u + (M_t - M_u) \dots \dots \dots (23)$$

In the above equation, M_u is constant for any value of the sectional area of a sub-vertical, as the stress in this member is always zero for a load at joint 1 and the difference between M_t and M_u , namely, $M_t - M_u$ gives the moment arising from the elastic elongation of the sub-vertical 1-2 alone. This amount of $M_t - M_u$ should be directly proportional to the elastic elongation of the sub-vertical 1-2 and accordingly inversely proportional to the sectional area of that member.

Therefore, when the sectional area of the sub-vertical 1-2 in the standard truss takes the amount A instead A_0 , the joint moment becomes

$$M = M_u + (M_t - M_u) \frac{A_0}{A} \dots \dots \dots (24)$$

In a similar way, when the sectional area of the sub-vertical 3-4 takes the amount A instead of A_0 , the joint moment due to a load at joint 3 becomes

$$M = M_t + (M_u - M_t) \frac{A_0}{A} \dots \dots \dots (25)$$

where M_t and M_u are the moments of any member in the standard truss due to a load at joints 4 and 3 respectively.

Now, for example, applying area-correction above mentioned to the bending moment diagram of the standard truss, we shall find the joint moments of a truss as shown in Appendix, Fig. 39 due to a load unity at joint 2. The sectional areas of sub-verticals in this truss are all equally 0.15 which value is one half of that in a standard truss. There the stiffness values are assumed to be equal to each other.

From the joint moment diagram in the Appendix, Fig. 8, joint moments of the standard truss due to a load unity at joint 1 are as follows :

$$M_{01} = 1.75, \quad M_{02} = -1.75, \quad M_{10} = -3.23, \quad M_{12} = 0.08$$

$$M_{14} = 0.39, \quad M_{13} = 2.76, \quad M_{20} = -4.52, \quad M_{24} = 4.30$$

etc.

From the joint moment diagram in the Appendix, Fig. 3, joint moments of the standard truss due to a load unity at joint 2 are as follows :

$$M_{01} = 8.50, \quad M_{02} = -8.50, \quad M_{10} = -0.60, \quad M_{12} = -0.03$$

$$M_{14} = -0.93, \quad M_{13} = 1.56, \quad M_{20} = -14.55, \quad M_{24} = 14.42$$

etc.

Hence, by equation (24) we get the joint moments of a truss of this example as follows:

$$M_{01} = 1.75 + (8.50 - 1.75) \frac{0.30}{0.15} = 15.25(15.24)$$

$$M_{02} = -1.75 + (-8.50 + 1.75) \frac{0.30}{0.15} = -15.25(15.24)$$

$$M_{10} = -3.23 + (-0.60 + 3.23) \frac{0.30}{0.15} = 2.03(2.04)$$

$$M_{12} = 0.08 + (-0.03 - 0.08) \frac{0.30}{0.15} = -0.14(0.14)$$

$$M_{14} = 0.39 + (-0.93 - 0.39) \frac{0.30}{0.15} = -2.29(2.25)$$

$$M_{13} = 2.76 + (1.56 - 2.76) \frac{0.30}{0.15} = 0.35(0.35)$$

$$M_{20} = -4.52 + (-14.55 + 4.52) \frac{0.30}{0.15} = -24.58(24.58)$$

$$M_{24} = 4.30 + (14.42 - 4.30) \frac{0.30}{0.15} = 24.56(24.55)$$

etc.

In the Appendix, Fig. 39, there are shown the values of joint moments which were calculated directly by the Slope Distribution Method. The above values in brackets are those values.

Computation of Secondary Stresses of Deck truss. The joint moment diagram were made up for a through truss, while these diagrams are applicable to the computation of secondary stresses of a deck truss.

With the exception of the moments around joints 0 and 1, the joint moments of a through truss may be practically taken for those of deck one, as the effect of the members 0-0₁ and 1-0₁ upon their moments are negligibly small. Hence it is sufficient to make up the joint moment diagrams only for members 0-0₁ and 1-0₁. In the Appendix, Figs. 40 and 41, there are shown these joint moment diagrams for trusses with 10 panels.

The moment of members $0-0_1$ and $1-0_1$ in trusses of less than 10 panels may also be approximately estimated from these diagrams as follows:

For a full loading or for one that causes maximum secondary stress, joint moments of trusses with 9, 8, 7, and 6 panels are respectively 0.9, 0.8, 0.7, and 0.6 times that of the truss with 10 panels.

For a single load applied near the member in question and exerting great effect upon this member, joint moments of all trusses of less than 10 panels are equal to those of the truss with 10 panels.

These relation can be easily found from the following, Tables 2, 3, and 4.

Table 2.

Number of Panels Joint Moment	10	8	6	4
M_{13}	-26.37	-21.79	-17.13	-12.33
M_{10}	10.69	9.12	7.64	5.91
M_{12}	4.52	3.50	2.54	1.52
M_{01}	10.16	8.37	6.76	4.79

Table 3.

Number of Panels Joint Moment	10	9	8	7	6
M_{13}	-26.92	-23.71	-18.82	-13.40	-9.91
M_{10}	18.02	16.09	12.71	8.88	6.62
M_{12}	4.79	4.21	3.45	2.70	2.07
M_{01}	16.71	16.29	14.63	13.33	12.18

Table 4.

Number of Panels Joint Moment	10	8	6	4
M_{02}	-8.50	-8.49	-8.49	-8.23
M_{20}	-14.55	-14.64	-14.69	-14.45
M_{24}	14.42	14.55	14.64	14.49
M_{42}	9.25	9.45	9.57	9.61
M_{41}	-2.75	-2.79	-2.93	-3.11

Table 2 shows the maximum joint moments M_{13} , M_{10} , M_{12} , and M_{01} of the standard Warren truss due to upper joint loads for every number of the panels and Table 3 those of the standard Pratt truss due to lower joint loads.

Table 4 shows the joint moments M_{02} , M_{20} , M_{24} , M_{42} , and M_{41} of the standard Warren truss due to a single load at joint 2.

Moments of the members around the joints 0 and 1 are obtained from those of the through truss, distributing the values of $-(M_{001})$ and $-(M_{101})$ to these members except the members 0-0₁ and 1-0₁, proportionally to their stiffness values so as to satisfy the condition that the summation of moments around a joint in question must equal zero for equilibrium.

Numerical Example I. Fig. 20 shows a Pratt truss with six panels, subjected to live joint loads of 1000 lbs. It is required to find the secondary stresses, by the use of the joint moment diagram.

This example is quoted from "The Theory and Practice of Modern Framed Structures" by J. B. Johnson, C. W. Bryan and F. E. Turneure and is the same as that considered in Section III.

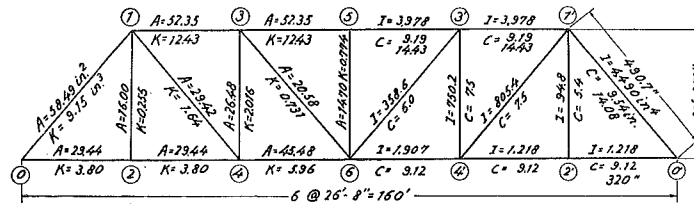


Fig. 20.

It will be seen that for some members the secondary stresses are not a maximum when the primary stresses are a maximum, but inasmuch as the primary stress is generally much larger than the secondary stress, the total is generally a maximum when the primary

stress is a maximum. On this basis, we shall compute the secondary stresses which occur for the same loading as the maximum primary stresses.

The various elements needed in the calculations are written in Fig. 20 and in Fig. 21 there are shown

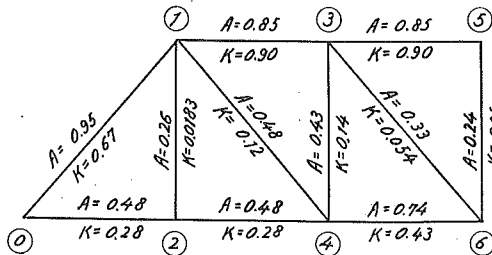


Fig. 21.

the stiffness ratio and sectional area-ratio of this truss on the corresponding members.

By comparing the values of stiffness and sectional area of the central upper chords of this truss to those of the standard truss with 6 panels, we get

$$k = \frac{12.43}{0.90} = 13.81$$

$$\alpha = \frac{52.35}{0.85} = 61.59$$

and

$$\frac{k}{\alpha} = \frac{13.81}{61.59} = 0.224$$

If this truss had the same stiffness ratio as well as the same sectional area-ratio as the standard truss the joint moments of this truss would be 0.224 times those of the standard truss from the relation of equation (9) for the same loading.

However, as shown in Fig. 21, there is some difference between their stiffness ratios, especially in web members. Hence the stiffness correction is necessary.

The primary stress in any chord member is at the maximum under full load and joint moments in chord members of the standard truss due to such a loading, that is, unit loads at all lower joints, are directly obtained from the joint moment diagram in the Appendix, Fig. 17 as follows:

$$\begin{array}{lll} M_{02} = -12.18, & M_{01} = 12.18, & M_{10} = 6.62 \\ M_{13} = -9.91, & M_{20} = -13.70, & M_{24} = 11.40 \\ M_{31} = -19.19, & M_{35} = 13.00, & M_{42} = 1.60 \\ M_{46} = -5.43, & M_{53} = -12.54, & M_{64} = -15.54 \end{array}$$

Since the truss is symmetrical about the span centre, the joint moments for the right half need not be computed here.

Multiplying the above values by 1000×0.224 , joint moments of a truss shown in Fig. 20 due to joint loads of 1000 lbs. at all lower joints are obtained as follows:

$$\begin{array}{l} M_{02} = -12.18 \times 1000 \times 0.224 = -2728 \\ M_{01} = 12.18 \times 1000 \times 0.224 = 2728 \\ M_{13} = -9.91 \times 1000 \times 0.224 = -2220 \end{array}$$

In a similar way

$$\begin{aligned} M_{10} &= 1483, & M_{20} &= -3069, & M_{24} &= 2553 \\ M_{31} &= -4299, & M_{35} &= 2921, & M_{42} &= 358 \\ M_{46} &= -1216, & M_{53} &= -2809, & M_{64} &= -1554 \end{aligned}$$

The stiffness ratios of members 0-2 and 2-4 are equally 0.28. while those of the standard truss are 0.40 and there is a large difference between their values. Hence, for members 0-2 and 2-4, the stiffness correction to the joint moments at both their end is applied as follows:

$$M_{02} = -2728 \times \frac{0.28}{0.40} = -1910$$

$$M_{20} = -3069 \times \frac{0.28}{0.40} = -2150$$

$$M_{24} = 2553 \times \frac{0.28}{0.40} = 1790$$

$$M_{42} = 358 \times \frac{0.28}{0.40} = 252$$

That is to say, the total sum of the moments around the joint 0 is $2728 - 1910 = 818$. Hence by distribution of this discrepancy to members 0-1 and 0-2 proportionally to their stiffness values so as to satisfy the equilibrium condition, final moments are obtained as follows:

$$M_{01} = 2728 - 818 \times \frac{0.67}{0.67 + 0.28} = 2728 - 573 = 2150$$

$$M_{02} = -1910 - 818 \times \frac{0.28}{0.67 + 0.28} = -1910 - 240 = -2150$$

For all other joints such a correction can be neglected, as the amount of discrepancy is not effective for the practical purpose and all the desired joint moments in chord members are therefore as follows:

$$\begin{aligned} M_{01} &= 2150, & M_{02} &= -2150, & M_{10} &= 1483 \\ M_{13} &= -2220, & M_{20} &= -2150, & M_{24} &= 1790 \\ M_{31} &= -4299, & M_{35} &= 2912, & M_{42} &= 252 \\ M_{46} &= -1216, & M_{53} &= -2809, & M_{64} &= -3481 \end{aligned}$$

All the above values of moments are given in in.-lb.

In web members the loading for the maximum primary stress causes nearly the maximum secondary stress. Hence, for the web members, the maximum joint moments are computed.

From the joint moment diagram in Appendix, Fig. 17 the maximum joint moments in the web members of the standard truss due to unit joint loads are directly obtained as follows:

$$\begin{aligned} M_{14} &= 1.83, & M_{41} &= 1.62, & M_{34} &= 7.25 \\ M_{43} &= 7.01, & M_{36} &= 1.09, & M_{63} &= 0.64 \\ M_{12} &= 2.25, & M_{21} &= 2.30, & M_{56} &= 1.42 \\ M_{65} &= 1.54 \end{aligned}$$

Multiplying these values by 1000×0.224 , the maximum joint moments of a truss shown in Fig. 20 due to joint loads of 1000 lbs. are computed as follows:

$$\begin{aligned} M_{14} &= 1.83 \times 1000 \times 0.224 = 410 \\ M_{41} &= 1.62 \times 1000 \times 0.224 = 363 \\ M_{34} &= 7.25 \times 1000 \times 0.224 = -1624 \end{aligned}$$

In a similar way

$$\begin{aligned} M_{43} &= 1570, & M_{36} &= 244, & M_{63} &= 143 \\ M_{12} &= 504, & M_{21} &= 512, & M_{56} &= 318 \\ M_{65} &= 345 \end{aligned}$$

The stiffness ratios of members 1-4, 3-6, and 1-2 are respectively 0.12, 0.054, and 0.0185, while those of the standard truss are respectively 0.07, 0.035, and 0.05. Hence for these members the stiffness correction is applied to the joint moments at both their ends as follows:

$$\begin{aligned} M_{14} &= 410 \times \frac{0.12}{0.07} = 705 & M_{41} &= 363 \times \frac{0.12}{0.07} = 620 \\ M_{36} &= 244 \times \frac{0.054}{0.035} = 377 & M_{63} &= 143 \times \frac{0.054}{0.035} = 234 \\ M_{12} &= 504 \times \frac{0.0185}{0.05} = 187 & M_{21} &= 512 \times \frac{0.0185}{0.05} = 190 \end{aligned}$$

For practical purposes, there is no need of correction of equilibrium, and all desired joint moments in web members are therefore as follows:

$$\begin{aligned}
M_{14} &= 705, & M_{41} &= 620, & M_{34} &= 1624 \\
M_{43} &= 1570, & M_{36} &= 377, & M_{63} &= 234 \\
M_{12} &= 187, & M_{21} &= 190, & M_{56} &= 318 \\
M_{65} &= 345
\end{aligned}$$

All the above values of moments are given in in.-lb.

After the values of the joint moments have been obtained, the secondary stresses can be obtained from the general equation

$$\sigma_{ab} = \frac{M_{ab} \cdot c_{ab}}{I_{ab}} \dots\dots\dots (26)$$

In this equation M_{ab} and σ_{ab} are respectively the joint moment and secondary stress at joint A in member A-B, c_{ab} distance of fibre from neutral axis, and I_{ab} moment of inertia of section of member.

The calculation is as follows :

$$\sigma_{01} = 2150 \times \frac{9.54}{4490} = 4.60 \text{ (compression)}$$

$$\sigma_{10} = 1483 \times \frac{14.08}{4490} = 4.66 \text{ (,,)}$$

$$\sigma_{13} = -2220 \times \frac{14.43}{3978} = -8.06 \text{ (,,)}$$

etc.

In table 5 are tabulated the secondary stresses which occur for the same loading as the maximum primary stresses. For unsymmetrical members, such as the post and upper chord, top and bottom fibre stresses have different values and properties, but in this table only one value which has the same property as of the maximum primary stress is given.

These are given in pounds per sq. in. and also percentages of the maximum primary stresses. In the table the values in brackets are those calculated by Johnson.

In regard to signs, the sign of the primary stress indicates the properties of the stress, so a plus sign will then signify a tensile stress and a minus sign a compressive stress, while the sign of secondary stress indicates the directions of moment, by which secondary stress is caused.

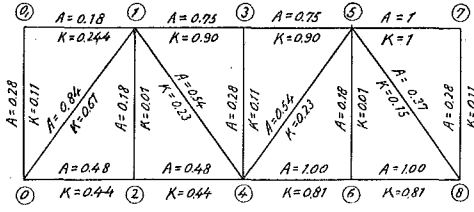


Fig. 23.

The various elements needed in the calculations are written in Fig. 22 and in Fig. 23 there are shown the stiffness ratio and sectional area-ratio of this truss.

The stiffness and sectional area of the central upper chord member 5—7 are respectively 7.70 cub. in. and 54.63 sq. in. and those of the standard truss with eight panels are unity each. Hence one gets

$$k = \frac{7.70}{1} = 7.70$$

$$\alpha = \frac{54.63}{1} = 54.63$$

and

$$\frac{k}{\alpha} = \frac{7.70}{54.63} = 0.141$$

A) *Area Correction of Joint Moment Diagram.* The sectional area-ratios of sub-verticals 1-2, 5-6, 5'-6', and 1'-2' are all equally 0.18, while those of the standard truss are 0.30. Thus there is a large difference between them. Hence for the joint moments due to the loads at joints 2, 6, 6', and 2' it is necessary to apply the area correction.

The total moments of the standard truss due to units loads at joints 2, 6, 6', and 2' are obtained from the joint moment diagram of the Appendix, Fig. 4 as follows:

$M_{01} = 10.77,$	$M_{02} = -10.77,$	$M_{10} = 0.08$
$M_{13} = -3.70,$	$M_{14} = 2.82,$	$M_{12} = 0.80$
$M_{20} = -16.59,$	$M_{21} = 1.11,$	$M_{24} = 15.47$
$M_{31} = -2.68,$	$M_{35} = 0.73,$	$M_{34} = 1.97$
$M_{42} = 11.85,$	$M_{41} = 2.58,$	$M_{43} = 1.86$
$M_{45} = 3.49,$	$M_{46} = -19.77,$	$M_{53} = -16.79$
$M_{57} = 14.90,$	$M_{58} = 1.45,$	$M_{56} = 0.64$
$M_{54} = -0.19,$	$M_{64} = -32.41,$	$M_{65} = 0.74$
$M_{68} = 31.68,$	$M_{75} = 0.25,$	$M_{78} = 0$
$M_{86} = 19.55,$	$M_{85} = -0.02,$	$M_{87} = 0$

Likewise, the total joint moments of the standard truss due to unit loads at joints 1, 5, 5', and 1' are as follows:

$$\begin{array}{lll}
 M_{01} = 3.75, & M_{02} = -3.75, & M_{10} = -2.24 \\
 M_{13} = -1.72, & M_{14} = -2.92, & M_{12} = 1.04 \\
 M_{20} = -5.23, & M_{21} = 1.38, & M_{24} = 3.89 \\
 M_{31} = -2.11, & M_{35} = 0.29, & M_{34} = 1.82 \\
 M_{42} = -1.91, & M_{41} = 1.75, & M_{43} = 1.62 \\
 M_{45} = 2.59, & M_{46} = -4.05, & M_{53} = -16.79 \\
 M_{57} = 15.21, & M_{58} = 1.48, & M_{56} = 0.69 \\
 M_{54} = -0.59, & M_{64} = -14.56, & M_{65} = 0.79 \\
 M_{68} = 13.77, & M_{75} = 0.41, & M_{78} = 0 \\
 M_{86} = 2.22, & M_{85} = 0.00, & M_{87} = 0
 \end{array}$$

Substituting these values into equation (24), we get the corrected total joint moments of the standard truss, whoses sub-verticals have a sectional area of 0.18 instead of 0.30, due to unit loads at joints 2, 4, 4', and 2' as follows:

$$\begin{aligned}
 M_{01} &= M_u + (M_l - M_u) \frac{A_0}{A} \\
 &= 3.75 + (10.77 - 3.75) \frac{0.30}{0.18} = 3.75 + 11.70 = 15.45 \\
 M_{02} &= -3.75 + (-10.77 + 3.75) \frac{0.30}{0.18} = -3.75 - 11.70 = -15.45 \\
 M_{10} &= -2.24 + (0.08 + 2.24) \frac{0.30}{0.18} = -2.24 + 3.87 = 1.63
 \end{aligned}$$

In a similar way

$$\begin{array}{lll}
 M_{13} = -5.03, & M_{14} = 2.76, & M_{12} = 0.64 \\
 M_{20} = -24.17, & M_{21} = 0.93, & M_{24} = 23.23 \\
 M_{31} = -3.06, & M_{35} = 1.02, & M_{34} = 2.07 \\
 M_{42} = 21.07, & M_{41} = 3.14, & M_{43} = 2.03 \\
 M_{45} = 4.09, & M_{46} = -30.30, & M_{53} = -16.79 \\
 M_{57} = 14.68, & M_{58} = 1.43, & M_{56} = 0.61 \\
 M_{54} = 0.08, & M_{64} = -44.36, & M_{65} = 0.71 \\
 M_{68} = 43.68, & M_{75} = 0.15, & M_{78} = 0 \\
 M_{86} = 31.16, & M_{85} = -0.03, & M_{87} = 0
 \end{array}$$

For the unit loads at all upper joints and the lower joints 4, 8, and 4', the joint moment diagram of the standard truss can be used with no correction. Therefore the total moments due to unit loads at all upper joints are directly given from the joint moment diagram of Appendix, Fig. 9 as follows :

$$\begin{array}{lll}
 M_{01} = 8.37, & M_{02} = -8.37, & M_{10} = 9.12 \\
 M_{13} = -21.79, & M_{14} = 9.17, & M_{12} = 3.50 \\
 M_{20} = -5.15, & M_{21} = 3.76, & M_{24} = 1.38 \\
 M_{31} = -35.96, & M_{35} = 32.96, & M_{34} = 3.00 \\
 M_{42} = -13.34, & M_{41} = 3.14, & M_{43} = 2.65 \\
 M_{45} = 6.03, & M_{46} = 1.52, & M_{53} = 6.28 \\
 M_{57} = -11.96, & M_{58} = 2.62, & M_{56} = 1.83 \\
 M_{54} = 1.24, & M_{64} = -14.74, & M_{65} = 1.97 \\
 M_{68} = 12.77, & M_{75} = -44.37, & M_{78} = 0 \\
 M_{86} = -12.30, & M_{85} = -0.62, & M_{87} = 0
 \end{array}$$

Also the total joint moments due to unit loads at lower joints 4, 8, and 4' are obtained from the diagram in Appendix, Fig. 4 as follows :

$$\begin{array}{lll}
 M_{01} = 2.89, & M_{02} = -2.89, & M_{10} = 4.24 \\
 M_{13} = -8.92, & M_{14} = 2.94, & M_{12} = 1.74 \\
 M_{20} = 0.66, & M_{21} = 2.00, & M_{24} = -2.65 \\
 M_{31} = -15.14, & M_{35} = 13.73, & M_{34} = 1.42 \\
 M_{42} = -11.03, & M_{41} = -0.03, & M_{43} = 1.19 \\
 M_{45} = 3.44, & M_{46} = 6.42, & M_{53} = 1.35 \\
 M_{57} = -4.71, & M_{58} = 0.89, & M_{56} = 1.01 \\
 M_{54} = 1.45, & M_{64} = 0.10, & M_{65} = 1.09 \\
 M_{68} = -1.20, & M_{75} = -21.05, & M_{78} = 0 \\
 M_{86} = -14.62, & M_{85} = -0.74, & M_{87} = 0
 \end{array}$$

B) *Computation of Joint Moments.* When the additional members 0-0₁ and 1-0₁ and the stiffness correction are not taken into consideration, the joint moments of a given truss for given loading are calculated by the use of the above obtained values as follows ;

$$\begin{aligned}
 M_{01} &= 8.37 \times 0.141 \times 100,000 + 2.89 \times 0.141 \times 10,000 \\
 &\quad + 15.45 \times 0.141 \times 10,000 \\
 &= 118,000 + 4,070 + 21,780 = 144,000
 \end{aligned}$$

$$\begin{aligned}
 M_{02} &= -8.37 \times 0.141 \times 100,000 - 2.89 \times 0.141 \times 10,000 \\
 &\quad - 15.45 \times 0.141 \times 10,000 \\
 &= -118,000 - 4,070 - 21,780 = -144,000
 \end{aligned}$$

$$\begin{aligned}
 M_{10} &= 9.12 \times 0.141 \times 100,000 + 4.24 \times 0.141 \times 10,000 \\
 &\quad + 1.63 \times 0.141 \times 10,000 = 137,000
 \end{aligned}$$

In a similar way

$$\begin{array}{lll}
 M_{13} = -327,000, & M_{14} = 137,000, & M_{12} = 52,700 \\
 M_{20} = -106,000, & M_{21} = 57,100, & M_{24} = 48,500 \\
 M_{31} = -533,000, & M_{35} = 486,000, & M_{34} = 47,200 \\
 M_{42} = -174,000, & M_{41} = 48,400, & M_{43} = 41,900 \\
 M_{45} = 95,600, & M_{46} = -12,300, & M_{53} = 64,000 \\
 M_{57} = -183,000, & M_{58} = 40,200, & M_{56} = 28,100 \\
 M_{54} = 20,000, & M_{64} = -270,000, & M_{65} = 30,300 \\
 M_{68} = 240,000, & M_{75} = -655,000, & M_{78} = 0 \\
 M_{86} = -150,000, & M_{85} = -9,800, & M_{87} = 0
 \end{array}$$

C) *Stiffness Correction.* The stiffness ratios of tensile and compressive sub-verticals are respectively 0.01 and 0.11, while those of the standard truss are all equally 0.05. The stiffness ratio of the diagonal 5-8 is also different from the standard truss. Hence for the joint moments at both ends of these members the stiffness correction is needed. That correction is as follows:

$$M_{12} = 52,700 \times \frac{0.01}{0.05} = 10,500$$

$$M_{21} = 57,100 \times \frac{0.01}{0.05} = 11,400$$

$$M_{34} = 47,200 \times \frac{0.11}{0.05} = 104,000$$

In a similar way

$$\begin{array}{lll}
 M_{43} = 92,000, & M_{56} = 5,600, & M_{65} = 6,100 \\
 M_{68} = 60,000, & M_{85} = -14,700 &
 \end{array}$$

For practical purposes, the discrepancy of the joint moments at every joint due to the above stiffness correction has no effect so the correction for equilibrium at the joints is neglected.

D) *Computation of Joint Moments in Additional Members.* For the standard truss with ten panels, the joint moments at both ends of additional members 0-0₁ and 1-0₁ due to unit load at all upper joints except 0₁ are directly given from the joint moment diagram of Appendix, Fig. 40 as follows :

$$\begin{aligned} M_{0_0 1} &= 2.67, & M_{1_0 1} &= -5.87, & M_{0_1 1} &= -3.23 \\ M_{0_1 0} &= 3.23 \end{aligned}$$

For unit loads at all lower joints, Appendix, Fig. 40 gives :

$$\begin{aligned} M_{0_0 1} &= 3.56, & M_{1_0 1} &= -10.23, & M_{0_1 1} &= -3.86 \\ M_{0_1 0} &= 3.86 \end{aligned}$$

For a unit load at joint 0₁, Appendix, Fig. 40 gives :

$$\begin{aligned} M_{0_0 1} &= -0.46, & M_{1_0 1} &= 4.59, & M_{0_1 1} &= 1.01 \\ M_{0_1 0} &= -1.01 \end{aligned}$$

Taking 80 percent of the above values the joint moments of the standard truss with eight panels can be determined. Therefore the joint moments of the given truss for the given loading are computed as follows :

$$\begin{aligned} M_{0_1 1} &= 0.8 \times 0.141(-3.23 \times 100,000 - 3.86 \times 10,000 + 1.01 \times 50,000) \\ &= -0.8 \times 43,000 = -35,000 \end{aligned}$$

In a similar way

$$M_{0_1 0} = 35,000, \quad M_{1_0 1} = -51,900, \quad M_{0_0 1} = 31,500$$

Since the stiffness ratios of these additional members 0-0₁ and 1-0₁ are respectively 0.244 and 0.11 which are different from the standard truss, the stiffness correction is applied as follows :

$$M_{0_1 1} = -35,000 \times \frac{0.244}{0.40} = -21,400$$

$$M_{0_1 0} = 35,000 \times \frac{0.11}{0.05} = 77,000$$

$$M_{10_1} = -51,900 \times \frac{0.244}{0.40} = -31,600$$

$$M_{00_1} = 31,500 \times \frac{0.11}{0.05} = 70,000$$

Next the sum of the joint moments at joint 0_1 after the stiffness correction becomes $77,000 - 21,400 = 55,600$. Therefore distributing this discrepancy among the members around joint 0_1 proportionally to their stiffness values,

$$\begin{aligned} M_{0_1 1} &= -21,400 - 55,600 \times \frac{0.244}{0.11 + 0.244} = -21,400 - 38,300 \\ &= -59,700 \end{aligned}$$

$$\begin{aligned} M_{0_1 0} &= 77,000 - 55,600 \times \frac{0.11}{0.11 + 0.244} = 77,000 - 17,300 \\ &= 59,700 \end{aligned}$$

Also, at joints 0 and 1, by the additional moments M_{00_1} and M_{10_1} the sum of the moments around these joints become 70,000 and -31,600 respectively. Distributing these discrepancies among the members around joints 0 and 1 except $0-0_1$ and $1-0_1$,

$$\begin{aligned} M_{01} &= 144,000 - 70,000 \times \frac{0.61}{0.44 + 0.61} = 144,000 - 40,500 \\ &= 103,500 \end{aligned}$$

$$\begin{aligned} M_{02} &= -144,000 - 70,000 \times \frac{0.44}{0.44 + 0.61} = -144,000 - 29,500 \\ &= -173,500 \end{aligned}$$

In a similar way at joint 1 one gets,

$$\begin{aligned} M_{10} &= 148,000, & M_{13} &= -310,700, & M_{14} &= 141,100 \\ K_{12} &= 10,700 \end{aligned}$$

From the above calculations, the final joint moments of all members of the given truss for the given loading are tabulated as follows:

$$\begin{array}{lll} M_{01} = 103,500, & M_{02} = -173,500, & M_{10} = 148,000 \\ M_{13} = -310,700, & M_{14} = 141,000, & M_{12} = 10,700 \\ M_{20} = -106,000, & M_{21} = 11,400, & M_{24} = 48,500 \\ M_{31} = -533,000, & M_{35} = 486,000, & M_{34} = 104,000 \end{array}$$

$$\begin{aligned}
M_{42} &= -174,000, & M_{41} &= 48,400, & M_{43} &= 92,000 \\
M_{45} &= 95,600, & M_{46} &= -12,300, & M_{53} &= 64,000 \\
M_{57} &= -183,000, & M_{58} &= 60,000, & M_{56} &= 5,600 \\
M_{54} &= 20,000, & M_{64} &= -270,000, & M_{65} &= 6,100 \\
M_{68} &= 240,000, & M_{75} &= -655,000, & M_{78} &= 0 \\
M_{86} &= -150,000, & M_{85} &= -14,700, & M_{87} &= 0 \\
M_{01} &= 70,000, & M_{010} &= 59,700, & M_{101} &= -31,600 \\
M_{011} &= -59,700
\end{aligned}$$

All the above values of moments are given in in.-lb.

Table 6.

Member		Secondary Stress	Percent Secondary of Allowable Stress of 13,000 lbs./sq. in.
Upper Chords	01	- 890	6.9 (6.2)
	10	- 470	3.6 (4.1)
	13	-1500	11.5 (10.6)
	31	- 2020	15.6 (18.3)
	35	2370	18.2 (20.2)
	53	310	2.4 (5.9)
	57	- 794	6.1 (7.6)
	75	-2210	17.0 (18.2)
Lower Chords	02	-1180	9.0 (9.0)
	20	- 980	7.5 (8.1)
	24	450	3.5 (6.0)
	42	- 1620	12.5 (12.9)
	46	50	0.4 (0.4)
	64	-1380	10.6 (9.6)
	68	1220	9.4 (9.3)
	86	- 760	5.8 (7.0)
Vertical Members	0,0	1130	8.7 (7.8)
	00,1	1320	10.1 (7.6)
	12	1530	11.8 (11.9)
	21	1630	12.6 (12.7)
	34	1960	15.1 (12.1)
	43	1740	13.3 (10.3)
	56	800	6.1 (5.1)
	65	870	6.7 (5.5)
	78	0	—
	87	0	—
Diagonal Members	01	360	2.7 (3.2)
	10	650	5.0 (5.6)
	14	1270	9.8 (10.5)
	41	430	3.3 (1.9)
	45	860	6.6 (7.6)
	54	180	1.4 (1.5)
	58	760	5.8 (4.9)
	85	- 190	1.5 (2.0)

E) Computation of Secondary Stresses. After the values of moments have been obtained, the secondary stresses can be obtained from the general equation (26) as follows :

$$\sigma_{01} = 103,500 \times 0.00344 = 356$$

$$\sigma_{02} = -173,500 \times 0.0068 = -985$$

etc.

In table 6, the desired secondary stresses are tabulated. These are given in pounds per sq. in. and also the percentages of the mean allowable stress of 13,000 lbs./sq. in., where the values in brackets show the values calculated by the Slope Distribution Method.

In regard to the signs, they are the same as in the preceeding example. For one end of a member, only one value which has the same property as of that the primary stress is shown.

V. Some Characteristics of Secondary and Primary Stresses.

1) *On the Relation between Sectional Area and Joint Moment.* In a great many examples of trusses actually already constructed, the relation between the stiffness K and sectional area A of the central upper chord member, whose sectional area takes the maximum value among the upper chord members, is shown in Fig. 24. In this figure, the values of K and A of a truss with any number of panels are converted from those for a truss with ten panels. For example, K and A of a Warren truss with six panels are divided by 0.85 and 0.80 respectively. For a Pratt truss with six panels, K and A are divided by 0.90 and 0.85 respectively.

The curve in Fig. 24 is expressed by a parabolic equation

$$\left. \begin{array}{l} K = \frac{1}{320} A^2 \quad \text{in in. unit} \\ \text{or} \\ K = \frac{1}{813} A^2 \quad \text{in cm. unit} \end{array} \right\} \dots\dots\dots (27)$$

Therefore, when the joint moment of any member of the standard truss is denoted by M_0 , the joint moment of a member in question in any truss whose largest upper chord member has the converted sectional area and stiffness of A and K is given by the equation

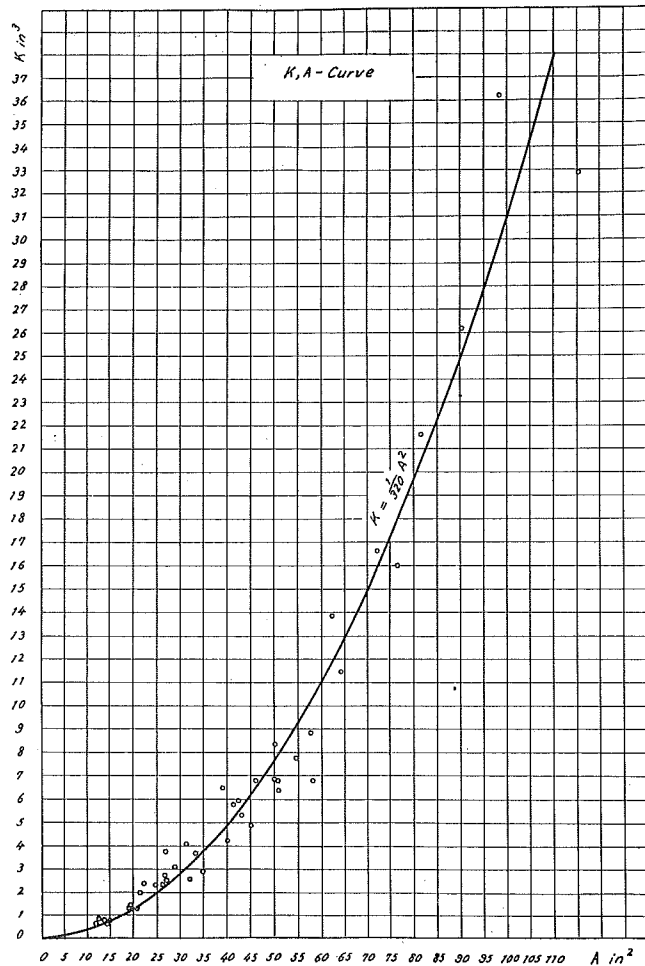


Fig. 24.

$$\left. \begin{aligned} M &= \frac{K}{A} M_0 = \frac{1}{320} A M_0 && \text{in in. unit} \\ \text{or} \\ M &= \frac{1}{813} A M_0 && \text{in cm. unit} \end{aligned} \right\} \dots\dots\dots (28)$$

That is to say, for the actual examples of trusses the amount of the joint moment is directly proportional to the sectional area of the central upper chord member.

2) *On the Relation between Size of Truss and Secondary Stress.* When the sectional area of members is constant and a truss has always similar form, the joint rotation angles φ_a and φ_b and member rotation angle ψ_{ab} are constant for any size of truss. Therefore, from

equation (1), it may be said that the amount of the joint M_{ab} and secondary stress σ_{ab} are directly proportional to K_{ab} , namely, I_{ab}/l_{ab} and consequently that they are inversely proportional to the size of truss.

3) *On the Relation between Moment of Inertia and Joint Moment.* When the sectional area of members and size of truss are constant, the joint moment M_{ab} increases proportionally to the moment of inertia I_{ab} . It is because, under these assumptions, φ_a , φ_b and Ψ_{ab} are constant and K_{ab} is directly proportional to I_{ab} in equation (1).

Since the secondary stress σ_{ab} is determined by the equation

$$\begin{aligned}\sigma_{ab} &= M_{ab} \frac{c_{ab}}{I_{ab}} \\ &= \frac{c_{ab}}{I_{ab}} (2\varphi_b + \varphi_b + \Psi_{ab}),\end{aligned}$$

the amount of the secondary stress is directly proportional to the distance of extreme fibre from the neutral axis of member and consequently to the height of the section of member.

4) *On the Relation between Loading and Secondary Stress.* The characteristics obtained for the standard truss hold true for almost all actual examples of the same kind trusses. By the use of the joint moment diagrams or the influence lines which will be easily drawn from the values in joint moment diagrams the relations can be obtained between the loading and the secondary stresses.

The secondary stresses in the chord members are generally of like sign for all loads, so that these stresses are maximum, or nearly so, for full load. This relation is apt to be disturbed for a load applied near the member in question. Thus for member 4-6 of the standard through Warren truss all secondary stresses are negative except for load at 4, which gives a positive value as shown in Appendix, Figs 3, 4, and 5. Again, in member 2-0, all values are negative except for load at 4.

As the primary stress is generally much larger than the secondary one, the total is generally a maximum when the primary stress is a maximum.

In the web members the loading for maximum primary stress causes nearly the maximum secondary stresses. In diagonal 4-5, for example, the maximum primary stress requires joints 6, 8, ... 4' and 2' to be loaded. This loading gives also the maximum secondary stress, excepting for the small stress for load at 4. In diagonal 5-8 the same is true, excepting for the small stress for load at 8.

In the sub-verticals the maximum secondary stress is produced by the loads at joints on the right side of the member in question. Thus all loads which are placed at 4 and to the right produce the maximum secondary stresses in sub-vertical 3-4 as shown in Appendix, Figs. 3, 4, and 5. Again all loads which are placed at 6 and to the right produce the maximum secondary stress in sub-vertical 5-6.

These relations are true for upper chord joint loads. And also, these relations can be seen for the standard Pratt and special Warren and Pratt trusses such as in Figs. 2, 5 and 6.

For a single load the chord members are most affected by a load placed at the end of the member in question or at the joint right above or under this member. Thus, in the Appendix, Figs. 3, 4, and 5, the secondary stresses σ_{53} and σ_{64} take the maximum values by a single load at joint 6.

In the web-members joint moments are most affected by a load that is placed one panel distant from the member in question. In Appendix, Figs. 3, 4, and 5, for example, secondary stress σ_{54} takes the maximum value by a single load at 2 or 8.

Compared to these maximum values, the secondary stresses due to a single load at other joints are very small and when the position of a single load is over 5 panels distant from the member in question, the amount of the secondary stress decreases nearly proportionally to the distance from the position of a single load. Thus, in Appendix, Fig. 3, joint moments around the joints 0 and 1 take the values as written in Table 7 for a single load that is remote from the members in question.

Table 7.

Load Position Joint Moment	10	8'	6'	4'	2'
M_{01}	0.95	0.75	0.56	0.37	0.18
M_{02}	-0.95	-0.75	-0.56	-0.37	-0.18
M_{10}	—	0.62	0.45	0.30	0.15
M_{13}	—	-1.86	-1.39	-0.92	-0.47
M_{14}	—	0.85	0.64	0.43	0.21
M_{12}	—	0.39	0.29	0.19	0.10
M_{20}	—	-0.11	-0.09	-0.06	-0.03
M_{24}	—	-0.36	-0.26	-0.18	-0.09
M_{21}	—	0.47	0.35	0.23	0.12

Next, in the web members, the signs of joint moments at both ends are the same in almost all cases. Therefore the web members will bend generally in the form of a letter *S* under any kind of loading. In the vertical members, including sub-verticals, the amounts of the joint moments are nearly equal, but in the diagonal members they are quite different. For examples, Tables 8 and 9 show the joint moments of each vertical and diagonal member in the standard Warren and Pratt trusses with ten panels, respectively. The joint moments are for the full load.

Table 8.

End \ Member	1-2	3-4	5-6	7-8	1-4	4-5	5-8	8-9
Upper	3.56	4.97	3.58	2.44	7.77	3.78	4.64	0.93
Lower	4.31	4.49	3.92	2.30	3.84	10.25	0.84	3.51

Table 9.

End \ Member	1-2	3-4	5-6	7-8	1-4	3-6	5-8	7-10
Upper	4.79	20.21	12.20	3.61	4.10	4.15	1.81	0.47
Lower	4.96	17.14	12.81	3.88	2.31	2.84	0.95	-0.09

The same relations can also be seen in the special Warren and Pratt trusses. In regard to the distribution of the secondary stresses in truss, for the special trusses (D) and (F), joint moments and secondary stresses have a tendency to increase from the centre to the support in both chord and web members. But for the standard trusses this tendency can not be seen.

5) *On the Relation between Number of Panels and Secondary Stress.* For the following three cases the relations between the secondary stresses and the variation of the number of panels in span will next be found.

- A) Secondary stress for full load
- B) Maximum and minimum secondary stresses
- C) Secondary stress for a single load.

A) *Secondary Stress for Full Load.* The variation of the joint moment and consequently the secondary stress due to the increase of the number of panels in a span is linear for any kind of trusses, except for the joint moments of the members near the span centre.

In the standard Warren truss, for example, the variation of the joint moment is as indicated in Tables 10 and 11.

Table 10.

Number of Panels Joint Moment	10	8	6	4	2
M_{01}	15.42	13.66	12.03	10.06	8.18
M_{10}	5.87	4.32	2.95	1.84	2.00
M_{02}	-15.42	-13.66	-12.03	-10.06	-8.18
M_{20}	-16.12	-15.93	-15.85	-14.99	-13.33
M_{13}	-17.20	-12.62	-8.09	-3.73	—
M_{31}	-21.76	-17.82	-14.79	-12.25	—
M_{24}	11.80	12.82	13.87	14.10	—
M_{42}	-3.36	0.82	4.81	7.18	—
M_{35}	16.80	14.46	13.07	—	—
M_{53}	-19.45	-15.44	-8.46	—	—
M_{46}	-15.20	-13.35	-11.25	—	—
M_{64}	-35.03	-32.31	-28.20	—	—

Table 11.

Number of Panels Joint Moment	10	8	6	4	2
M_{12}	3.56	2.54	1.59	0.69	0
M_{21}	4.31	3.11	1.98	0.89	0
M_{14}	7.77	5.76	3.55	1.19	—
M_{11}	3.84	2.55	0.90	-1.32	—
M_{34}	4.97	3.39	1.73	0	—
M_{43}	4.49	3.06	1.61	0	—
M_{45}	10.25	6.93	3.94	—	—
M_{51}	3.78	1.26	-0.60	—	—
M_{56}	3.58	1.65	0	—	—
M_{65}	3.92	1.83	0	—	—
M_{58}	4.64	2.34	—	—	—
M_{85}	0.84	-0.76	—	—	—

Again, in the special Warren truss (D), the variation of the joint moment is as shown in Tables 12 and 13.

Table 12.

Number of Panels Joint Moment	10	8	6	4	2
M_{01}	23.35	19.16	16.08	12.84	9.84
M_{10}	15.98	11.52	7.57	4.02	2.40
M_{02}	-22.35	-19.16	-16.08	-12.84	-9.84
M_{20}	-15.60	-16.83	-17.84	-18.24	-15.57
M_{13}	-38.33	-27.68	-17.49	-7.81	—
M_{31}	-38.21	-29.44	-21.77	-15.57	—
M_{24}	-4.83	1.92	8.38	—	—
M_{42}	-25.56	-15.10	-5.39	—	—
M_{35}	14.22	13.07	13.35	—	—
M_{53}	-28.38	-20.06	-9.65	—	—
M_{46}	-11.11	-9.83	-7.05	—	—
M_{64}	-41.16	-35.10	-27.34	—	—

Table 13.

Number of Panels Joint Moment	10	8	6	4	2
M_{12}	17.15	13.39	7.75	3.33	0
M_{21}	20.43	14.88	9.46	4.18	0
M_{14}	5.20	3.77	2.16	0.48	—
M_{41}	3.30	2.01	0.40	-1.48	—
M_{34}	23.39	16.36	8.42	0	—
M_{43}	22.07	15.06	7.73	0	—
M_{45}	11.27	7.82	4.30	—	—
M_{54}	2.54	0.87	-0.79	—	—
M_{56}	12.93	6.46	0	—	—
M_{65}	14.14	7.09	0	—	—
M_{53}	8.06	4.68	—	—	—
M_{65}	-0.94	-2.22	—	—	—

In the Appendix, Fig. 42 there is shown graphically this linear variation of the joint moments for the standard and special Warren

trusses respectively. For the standard and special Pratt trusses, there is the same relation as for the Warren truss. Appendix, Fig. 43 is for the Pratt trusses.

In the figures, the ordinates represent the amount of the joint moment and the abscissae the corresponding number of panels in the span.

B) Maximum and Minimum Secondary Stresses. In the chord members the relation between the maximum secondary stress and the variation of the number of panels in the span is generally like that for full load, as the secondary stress is nearly maximum for full load in general.

In the web members, the positive maximum secondary stress is produced by the loads at the joints on the right side of the member in question and the minimum secondary stress, namely, the negative maximum secondary stress is produced by the loads at the joints on the left side of the member in question.

The variation of the maximum stresses due to the increase of the number of panels in span is also linear, except for the secondary stresses of the members near the centre of the span. In the standard Warren truss, for example, the variation of the maximum joint moments, from which the maximum secondary stresses are produced, is as shown in Table 14.

Table 14.

Number of Panels Joint Moment	10	8	6	4	2
M_{12}	3.59	2.60	1.69	0.86	0
M_{21}	4.31	3.11	1.98	0.93	0
M_{14}	8.70	6.76	4.67	2.52	—
M_{41}	7.45	6.17	4.94	2.61	—
M_{34}	5.34	3.80	2.23	0.63	—
M_{43}	5.06	3.66	2.29	0.81	—
M_{15}	11.95	8.72	5.82	—	—
M_{54}	6.06	3.79	1.83	—	—
M_{56}	4.25	2.51	0.93	—	—
M_{65}	4.55	2.65	0.94	—	—
M_{58}	5.18	3.21	—	—	—
M_{85}	4.11	2.62	—	—	—

Again, in the special Warren truss (D), the variation of the joint moments is as indicated in Table 15.

Table 15.

Number of Panels Joint Moment	10	8	6	4	2
M_{12}	17.18	12.55	8.11	4.06	0
M_{21}	20.43	14.88	9.46	4.41	0
M_{14}	6.11	4.72	3.14	1.58	—
M_{41}	6.09	4.78	3.40	2.05	—
M_{34}	25.47	17.99	10.39	2.73	—
M_{43}	24.38	17.51	10.49	3.46	—
M_{45}	12.96	9.57	6.20	—	—
M_{54}	4.36	2.75	1.43	—	—
M_{56}	15.54	9.48	3.86	—	—
M_{65}	16.69	10.08	3.92	—	—
M_{58}	8.78	5.57	—	—	—
M_{85}	3.99	2.90	—	—	—

In the Appendix, Fig. 44 there are shown graphically these linear variations of the maximum joint moments in the standard and special Warren trusses respectively. In the Pratt trusses, there is also the same relation and in Appendix, Fig. 45 there are shown the linear variations of the maximum joint moments of the standard and special Pratt trusses respectively.

The negative maximum secondary stresses decreases with the increase of the number of panels in the span. This variation deviates from the linear variation and the greater the increase in the number of panels in the span, the less the rate of variation of the secondary stress. In the standard Warren truss, for example, the variation of the negative maximum joint moments, from which the negative maximum secondary stresses are produced, is as shown in Table 16.

Table 16.

Number of Panels Joint Moment	10	8	6	4
M_{14}	-0.93	-1.00	-1.12	-1.33
M_{41}	-3.61	-3.62	-4.04	-3.94
M_{34}	-0.37	-0.41	-0.50	-0.63
M_{43}	-0.57	-0.61	-0.68	-0.81
M_{45}	-1.70	-1.79	-1.88	—

Again, in the special Warren truss (D), the variation of the negative maximum joint moments is as shown in Table 17.

Table 17.

Number of Panels Joint Moment	10	8	6	4
M_{14}	-0.91	-0.95	-0.98	-1.12
M_{41}	-2.79	-2.77	-3.00	-3.53
M_{34}	-1.48	-1.63	-1.97	-2.73
M_{43}	-2.31	-2.45	-2.76	-3.46
M_{45}	-1.69	-1.75	-1.90	—

C) *Secondary Stress for a Single Load.* In chord members, the amount of the maximum joint moment due to a single load is little affected by the variation of the number of panels in span. Thus, in the standard Warren truss, M_{01} , M_{02} , M_{20} , M_{24} , and M_{42} reach the maximum values from the single load at joint 2. These values are nearly constant for any number of panels in span as shown in Table 18.

Table 18.

Number of Panels Joint Moment	10	8	6	4	2
M_{01}	8.50	8.49	8.49	8.23	8.14
M_{02}	- 8.50	- 8.49	- 8.49	- 8.23	- 8.14
M_{20}	-14.55	-14.64	14.69	-14.45	-13.33
M_{24}	14.42	14.55	14.64	14.49	—
M_{42}	9.25	9.47	9.57	9.61	—

Joint moments M_{13} , M_{31} , and M_{35} take the maximum values by the single load at joint 4. These amounts for any number of panels are written in Table 19. They are also nearly constant.

Table 19.

Number of Panels Joint Moment	10	8	6	4
M_{13}	- 5.48	- 5.28	- 4.91	4.20
M_{31}	-11.58	-11.36	-11.37	-10.31
M_{35}	11.16	11.01	11.20	—

Tables 20 and 21 are for the special Warren truss (D). Joint moments in Table 20 are for a single load at joint 2 and those in Table 21 for a single load at joint 4.

Table 20.

Number of Panels Joint Moment	10	8	6	4	2
M_{01}	10.53	10.45	10.23	9.90	9.84
M_{02}	-10.53	-10.45	-10.23	-9.90	-9.84
M_{20}	-19.15	-19.15	-19.07	-18.95	-17.28
M_{24}	18.53	18.68	18.85	19.18	—
M_{42}	10.48	10.78	11.11	11.64	—

Table 21.

Number of Panels Joint Moment	10	8	6	4
M_{13}	-10.86	-10.43	-9.44	-7.87
M_{31}	-15.73	-15.30	-14.41	-13.66
M_{35}	13.22	13.05	13.01	—

For the Pratt truss, the same is true. In the Appendix, Figs. 46 (29-31) and 47 (39-41), there are shown graphically these relations for both standard Warren and Pratt trusses respectively.

In the web members the maximum joint moment due to a single load is slightly affected by the variation of the number of panels in the span. But the greater the number of panels in the span, the less the effect of the variation of the number of panels upon the joint moment. For ordinary joint moments in the chord and web members due to a single load the relation between the joint moment and the number of panels is generally the same as for the maximum joint moment in the web member.

In the standard truss, for example, M_{14} M_{41} M_{34} M_{43} and M_{45} take the values as written in Table 22 for a single load at joint 2.

Table 22.

Number of Panels Joint Moment	10	8	6	4
$M_{1,4}$	-0.93	-1.00	-1.12	-1.33
$M_{4,1}$	-2.75	-2.79	-2.93	-3.13
$M_{3,4}$	-0.37	-0.41	-0.50	-0.63
$M_{4,3}$	-0.57	-0.61	-0.68	-0.81
$M_{4,5}$	-1.70	-1.79	-1.88	—

In Appendix, Figs. 46 and 47 there are shown graphically the relation between the joint moments produced by a single load comparatively near and comparatively far from the member in question and the number of panels in the span.

6) *On the Relation between the Sectional Area of a Sub-Vertical and the Secondary Stress.* The effect of the variation of the sectional area of sub-vertical 1-2 on the joint moment is produced only by a single load at joint 2 and from other joint loads there is no effect. Likewise those of sub-verticals 3-4, 5-6, 7-8, ..., are effected respectively only by the load at joints 3, 6, 7, ... These effects can be easily obtained by equations (24) and (25). Thus, substituting the values obtained from Appendix, Figs. 3 and 8 into equation (24) the variation of the joint moments in the standard Warren truss with ten panels due to the variation of the sectional area of sub-vertical 1-2 can be accurately obtained as follows:

$$M_{01} = 1.75 + (8.50 - 1.75) \frac{0.30}{A} = 1.75 + (6.75) \frac{0.30}{A}$$

$$M_{02} = -1.75 + 6(8.50 - 1.75) \frac{0.30}{A} = -1.75 - (6.75) \frac{0.30}{A}$$

$$M_{10} = -3.23 + (-0.60 + 3.23) \frac{0.30}{A} = -3.23 + (2.63) \frac{0.30}{A}$$

In a similar way

$$M_{13} = 2.76 - (1.20) \frac{0.30}{A}, \quad M_{14} = 0.39 - (1.32) \frac{0.30}{A}$$

$$M_{20} = -4.52 - (10.03) \frac{0.30}{A}, \quad M_{24} = 4.30 + (10.12) \frac{0.30}{A}$$

$$M_{31} = -0.86 - (0.32) \frac{0.30}{A}, \quad M_{35} = 1.07 + (0.48) \frac{0.30}{A}$$

$$M_{34} = -0.21 - (0.16) \frac{0.30}{A}, \quad M_{42} = 0.67 + (8.58) \frac{0.30}{A}$$

$$M_{41} = -0.73 - (2.02) \frac{0.30}{A}, \quad M_{43} = -0.21 - (0.36) \frac{0.30}{A}$$

$$M_{45} = -0.27 - (1.43) \frac{0.30}{A}, \quad M_{46} = 0.54 - (4.77) \frac{0.30}{A}$$

etc.

The above values are all for a single load at joint 2. It is seen that the variation of the sectional area A of a sub-vertical exerts a very large effect upon the secondary stress under such a single load as produces the primary stress in this sub-vertical.

7) *On the Relation between the Number of Panels and Primary Stress.* For facility in computation of primary stress, namely, the direct stress and the ratio of secondary stress, to the direct stress, we have derived the equations which represent the relation between the number of panels in a span and the direct stress as follows:

A) *For a full Load.* Load is considered to be applied at each joint and the magnitude of a joint load is unity.

Let

x = number of panels in the span;

n = number of panels to the left of the opposite vertex of chord member in question;

and θ = angle between diagonal and vertical members.

Then the direct stress in any chord member of a truss with any number of panels is computed from the following equation.

$$\begin{aligned} S &= \left[\frac{1}{2} n(x-1) - \{1+2+3+\dots+(n-1)\} \right] \tan \theta \\ &= \frac{1}{2} (nx - n^2) \tan \theta \dots\dots\dots (29) \end{aligned}$$

For members 0-2-4 in Warren and Pratt trusses,

$$S = \frac{1}{2} (x-1) \tan \theta \quad (n=1)$$

For members 1-3 and 4-6 in Pratt truss and 1-3-5 in Warren truss,

$$S = \frac{1}{2}(2x-4) \tan \theta \quad (n = 2)$$

For members 3-5 and 6-8 in Pratt truss and 4-6-8 in Warren truss,

$$S = \frac{1}{2}(3x-9) \tan \theta \quad (n = 3)$$

etc.

In the above equations the sign of the stresses is omitted. The upper chord stresses should be compression and the lower chord stresses tension.

These equations are in the family of straight lines with a parameter of n . Therefore the direct stress in any chord member varies linearly by the variation of the number of panels in span. In Appendix, Fig. 48 there are shown graphically this family of straight lines, where the ordinate represents the direct stress S and the abscissa the number of panels x . On every line, is written the corresponding member, where W and P represent the kind of trusses, that is, W denotes the Warren truss and P the Pratt. In regard to signs, a plus sign indicates the tension and negative sign the compression stress.

This family of straight lines has an envelope expressed by the following parabolic equation;

$$S = \frac{1}{8}x^2 \tan \theta \dots\dots\dots (30)$$

and this family of straight lines is a series of tangent lines to this parabola.

It is an interesting fact that every intersecting point of these straight lines represents one of the direct stresses in the chord members. Thus the value of the interesting point of two straight lines

$$S = \frac{1}{2}(2x-4) \tan \theta$$

and

$$S = \frac{1}{2}(6x-36) \tan \theta$$

is $S = 6 \tan \theta$ and this value is the same as for the direct stress in chord member 1-3-5 of the Warren truss with eight panels or in chord members 1-3 and 4-6 of the Pratt truss with the same number of panels. Again, the value of the intersecting point of two straight lines

$$S = \frac{1}{2}(2x-4) \tan \theta$$

and

$$S = \frac{1}{2}(8x-64) \tan \theta$$

is $S = 8 \tan \theta$ and this value is the same as for the direct stress in the chord member 1-3-5 of the Warren truss with ten panels or in chord members 1-3 and 4-6 of the Pratt truss with the same number of panels.

For upper joint loads, equation (29) may be used as it is.

Next, let n = the number of panels to the left of the web member in question, then the direct stresses in web members are computed from the following equations:

For vertical members

$$S = \frac{1}{2}(x-1)-n \dots\dots\dots (31)$$

For diagonal members

$$S = \left\{ \frac{1}{2}(x-1)-n \right\} \sec \theta \dots\dots\dots (32)$$

In vertical members, sub-verticals are not included. When the loaded chord is upper, we should take $n-1$ instead of n in the above equations for the stress in vertical member of the Pratt truss. These equations represent the family of parallel straight lines with a parameter of n as shown in Appendix, Fig. 48. These lines have the vertical intervals of 1 or $1 \cdot \sec \theta$ between them.

B) Maximum and Minimum Direct Stresses in Web Member. In the chord members the maximum direct stress is produced by the full load, while in the web members it is produced by a partical load. The maximum and minimum stresses in web members are computed from the following equations:

For vertical members

$$\begin{aligned} S_{\max.} &= \frac{1}{x} \{1+2+3+\dots+(x-n-)\} \\ &= \frac{1}{2x} (x-n-1)(x-n) \dots\dots\dots (33) \end{aligned}$$

$$\begin{aligned}
 S_{\min.} &= \frac{1}{x}(1+2+3+\cdots+n) \\
 &= \frac{1}{2x}n(n+1) \dots\dots\dots (34)
 \end{aligned}$$

For diagonal members

$$S_{\max.} = \frac{1}{2x}(x-n-1)(x-n) \sec \theta \dots\dots\dots (35)$$

$$S_{\min.} = \frac{1}{2x}n(n+1) \sec \theta \dots\dots\dots (36)$$

In these equations n = number of panels to the left of the web member in question and sub-verticals are not included here. When the loaded chord is an upper one, we should take $n-1$ instead of n for the stress in vertical member of the Pratt truss.

The maximum and minimum direct stresses in web members are represented by two families of curves each with a parameter of n . In Appendix, Fig. 49 there are shown graphically these two families of curves. The family which represents the maximum direct stress is observed nearly to be composed of straight lines. The family of curves for the minimum stress is the family of hyperbola with an asymptote of $S = 0$.

Every intersecting point of these two families of curves represents one of the direct stresses in the web members. Thus the value of the interesting point of two curves

$$S = \frac{1}{2x}(x-2)(x-1) \sec \theta \quad (n = 1)$$

and

$$S = \frac{21}{x} \sec \theta \quad (n = 6)$$

is $2.625 \sec \theta$. This value is the same as for the maximum direct stress in diagonal member 1-4 in the Warren and Pratt trusses with eight panels. Again, the value of the interesting point of two curves

$$S = \frac{6}{x} \sec \theta \quad (n = 3)$$

and

$$S = \frac{1}{2x}(x-5)(x-4) \sec \theta \quad (n = 4)$$

is $0.75 \sec \theta$. This value is the same as for the stress in diagonal 5-8 of the Warren and Pratt with eight panels.

C) For a Single Load.

Let

x = number of panels in span ;

n = number of panels to the left of the opposite vertex of the chord member in question ;

m = number of panels to the left of a loaded joint.

Then the direct stress in a chord member due to a single load is computed from the following equations :

$$S = \left\{ \left(\frac{x-m}{x} \right) n - (n-m) \right\} \tan \theta$$

$$= \left(m - \frac{mn}{x} \right) \tan \theta \quad \text{in the case } n \geq m \dots\dots\dots (37)$$

or

$$S = \left\{ \frac{(x-m)}{x} n \right\} \tan \theta$$

$$= \left\{ n - \frac{nm}{x} \right\} \tan \theta \quad \text{in the case } n < m \dots\dots\dots (38)$$

Equation (37) represents the family of hyperbolas with two parameters m and n . For a single load at joint 2, the value of m is 1 and the direct stress in any chord member is represented by the family of hyperbolas

$$S = \left(1 - \frac{n}{x} \right) \tan \theta$$

In a similar way,
for a single load at 4

$$S = 2 \left(1 - \frac{n}{x} \right) \tan \theta \quad (m = 2)$$

for a single load at 6

$$S = 3 \left(1 - \frac{n}{x} \right) \tan \theta \quad (m = 3)$$

for a single load at 8

$$S = 4 \left(1 - \frac{n}{x} \right) \tan \theta \quad (m = 4)$$

etc.

These families of hyperbolas are shown graphically in Appendix, Fig. 50.

In this Figure the families of hyperbolas denoted by (I), (II), (III), etc. are respectively for $m = 1$, $m = 2$, $m = 3$, etc. and these families of hyperbolas have the asymptotes which are represented by $S = 1$, $S = 2$, $S = 3$, etc. respectively. On every line, there are written the corresponding load position and member. Load position is denoted by arabic numerals written in the circles. P and W represent the kind of truss.

For example, in the Warren truss, the direct stress in chord member 1-3-5 due to a single load at joint 2 is represented by the equation

$$S = \left(1 - \frac{2}{x}\right) \tan \theta$$

because, in equation (37) $m = 1$ and $m = 2$. This curve is shown in the family of hyperbolas denoted by (I) and the notations ② W_{1-3-5} are written on this curve.

The curve represented by equation (38) coincides with one of the curves which are represented by equation (37). Therefore, for any chord member which is on the left of the loaded joint, the direct stress may be represented by one of the curves of equation (37). Thus, for a single load at 4, the direct stress in member 0-2-4 is expressed by the equation

$$S = \left(1 - \frac{2}{x}\right) \tan \theta$$

because, in equation (38) $m = 2$ and $n = 1$. This curve is the same as for the stress in member 1-3 due to a single load at joint 2.

For a single load to the right of the span center, we can also obtain the stress from the curve which is represented by equation (37). Thus, for a single load at joint 2', $m = x - 1$ and then from equation (38)

$$\begin{aligned} S &= \left\{n - \frac{n(x-1)}{x}\right\} \tan \theta \\ &= \frac{n}{x} \tan \theta \end{aligned}$$

The value of this equation is obtained from the curve $S = \left(1 - \frac{n}{x}\right) \tan \theta$, taking the distance from its asymptote. Therefore

in Appendix, Fig. 50 for the stress due to a single load to the right of the center of the span, the distance from the asymptote should be taken. In the Warren truss, for example, direct stress in chord member 1-3-5 due to a single load at joint 2' is obtained, by taking the distance between the asymptote $S = 1$ and a curve which is denoted by the notation ②' W_{1-3-5} in the family of hyperbola (I). For the truss with eight panels this distance is $0.25 \tan \theta$.

For web members the direct stresses due to a single load are computed from the following equations:

For vertical members

$$S = \frac{m}{x} \quad \text{in the case } m \leq n \dots\dots\dots (39)$$

$$S = 1 - \frac{m}{x} \quad \text{in the case } m > n \dots\dots\dots (40)$$

For diagonal members

$$S = \frac{m}{x} \sec \theta \quad \text{in the case } m \leq n \dots\dots\dots (41)$$

$$S = \left(1 - \frac{m}{x}\right) \sec \theta \quad \text{in the case } m > n \dots\dots\dots (42)$$

In these equations, n = number of panels to the left of the web member in question and the sub-verticals are not considered. When the loaded chord is an upper one, $n = 1$ should be taken instead of n for a vertical member of Pratt truss.

These equations represent also two families of hyperbolas and in the Appendix, Fig. 51 these curves are shown. Every direct stress is at an intersecting point of these families of hyperbolas.

8) *On the Ratio of Secondary Stress to Primary Stress.* By the direct stress diagrams in the Appendix, Figs. 48 to 51, direct stress, namely, primary stress in any member of a truss with any number of panels for any kind of loading can be directly computed and also the variation of the direct stresses due to the increase of the number of panels in a span is clearly shown.

Therefore the ratio of the secondary stress to the direct stress may be easily determined by the use of joint moment and direct stress diagrams.

In the case where both direct and secondary stresses increase to an unlimited extent with the increase of the number of panels in a

span, how does the ratio of the secondary stress vary due to the increase of the number of panels?

For the full load, secondary stress S and direct stress S_0 vary both linearly by the increase of the number of panels as mentioned in the preceding paragraphs. These variations are represented by the following equations:

Secondary stress

$$S = k(ax+b) \dots\dots\dots (43)$$

Direct stress

$$S_0 = \frac{1}{2}k_0(nx-n^2)\dots\text{for chord member} \dots\dots (44)$$

$$S_0 = k_0\left\{\frac{1}{2}(x-1)-n\right\}\dots\text{for web member} \dots\dots (45)$$

In these equations K and K_0 are constant.

In the ratios of the secondary stresses to the direct stresses, some increase with the number of panels, some decrease with the increase of the number of panels and some remain constant for any number of panels. However, in all cases, the ratios have always maximum or minimum limit and increase or decrease toward the limit.

For the chord members, this limit is

$$\lim_{x \rightarrow \infty} \frac{S(x)}{S_0(x)} = \lim_{x \rightarrow \infty} \frac{S'(x)}{S'_0(x)} = \frac{2a}{n} \frac{k}{k_0} \dots\dots\dots (46)$$

and for the web members, it is

$$\lim_{x \rightarrow \infty} \frac{S(x)}{S_0(x)} = 2a \frac{k}{k_0} \dots\dots\dots (47)$$

For example take the Warren truss. From Appendix, Fig. 42 (4), the secondary stress S in the lower end of the web member 1-4 is represented by

$$S = k(0.75x-3)$$

and the direct stress S_0 in the same member, by

$$\begin{aligned} S_0 &= k_0\left\{\frac{1}{2}(x-1)-1\right\} \\ &= k_0(0.5x-1.5) \end{aligned}$$

Therefore,

$$\lim_{x \rightarrow \infty} \frac{S(x)}{S_0(x)} = 2a \frac{k}{k_0} = 2 \times 0.75 \frac{k}{k_0} = 1.5 \frac{k}{k_0}$$

Hence the ratio of secondary stress to the direct stress increases with the number of panels, but however much the number of panels increases, the ratio does not exceed the value of $1.5 \frac{k}{k_0}$. The state of the variation of this ratio is as follows:

Table 23.

coefficient of ratio = $\frac{k}{k_0}$

x	6	8	10	12	14	16	—	∞
$\frac{S}{S_0}$	1	1.20	1.29	1.33	1.36	1.38	—	1.5

For the lower end of the web member 4-5, the secondary stress is represented by

$$S = k(1.57x - 5.5)$$

and the direct stress, by

$$\begin{aligned} S_0 &= k_0 \left\{ \frac{1}{2}(x-1) - 2 \right\} \\ &= k_0(0.5x - 2.5) \end{aligned}$$

Hence,

$$\lim_{x \rightarrow \infty} \frac{S(x)}{S_0(x)} = 2 \times 1.57 \frac{k}{k_0} = 3.14 \frac{k}{k_0}$$

Therefore the ratio decreases with the increase of the number of panels, but it is always greater than the value of $3.14 \frac{k}{k_0}$. The state of the variation of this ratio is as follows:

Table 24.

coefficient of ratio = $\frac{k}{k_0}$

x	6	8	10	12	14	16	—	∞
$\frac{S}{S_0}$	7.84	4.70	4.10	3.81	3.66	3.56	—	3.14

In the web members, excepting the sub-verticals, the maximum secondary and direct stresses S and S_0 are represented by the equations

$$S = k(ax+b) \dots\dots\dots (48)$$

and

$$S_0 = k_0 \frac{1}{2x}(x-n-1)(x-n) \dots\dots\dots (49)$$

Therefore, the maximum or minimum limit of the ratio, S/S_0 , is

$$\lim_{x \rightarrow \infty} \frac{S(x)}{S_0(x)} = 2a \frac{k}{k_0} \dots\dots\dots (50)$$

In the sub-verticals, the direct stress has no relation to the number of panels. The sub-vertical acts in case it supports or suspends a load and the direct stress is equal to $1 \cdot k_0$. For all other loads direct stress is always zero. While the maximum secondary stress increases with the number of panels to an unlimited extent.

Therefore the variation of the ratio of the maximum secondary stress to the direct stress is the same as for the maximum secondary stress.

Summary and Conclusions

The general conclusions to be drawn from the investigations described in this paper may be summarized as follows;

I. After close investigation of great many examples of bridge trusses actually already constructed, it became obvious that the members of the trusses have almost constant ratio, both for stiffness and sectional area, between one member and another one. Therefore, for the Warren and Pratt trusses the junior author could propose the standard trusses, having nearly the same stiffness ratio as well as the same sectional area as the many actual examples. The standard trusses are as shown in Figs. 1 and 2.

II. Take two different trusses of similar form in each other, having the same stiffness ratio as well as the same sectional area ratio, while the stiffness values are k times the other one and the sectional areas α times the same. Then, between their joint moments M_{mn} and M'_{mn} there exists the following important relation:

$$M'_{mn} = \frac{k}{\alpha} M_{mn}$$

III. The Slope Distribution Method proposed by the senior author is applicable and lends itself well to an exact and rapid solution of the secondary stresses for the bridge trusses. This Slope Distribution Method eliminates completely the first process of writing the simultaneous equations and makes the second process of iteration a simple and mechanical method of calculation on the frame-schema itself. This method treats one unknown at each joint and successive similar calculations are carried out until the unbalanced slopes or joint rotation angles are distributed. The calculation of joint moment, using the values of the resulting slopes which have been calculated, is performed only once, and is not repeated.

IV. By the use of the Slope Distribution Method, the junior author calculated the joint moments of the standard trusses with any number of panels less than ten for an unit load placed successively at each joint and made up the joint moment diagrams as shown in Appendix, Fig. 3 to Fig. 29. For the study of the secondary stresses, there were also drawn up the joint moment diagrams of special Warren and Pratt trusses, having constant stiffness and sections throughout the end posts, upper chords and lower chords. From the relation of $M'_{mn} = k/\alpha M_{mn}$, anyone can compute, within a few minutes, the joint moments and accordingly the secondary stresses at any desired member of the trusses with any number of panels by the use of the joint moment diagrams. By the use of the results which have been already calculated for certain trusses, the secondary stresses of other trusses with similar form can also be computed from the above relation.

It is of course true that some of the examples are different from the standard section, especially in respect to web members and sub-verticals. But for such trusses, the joint moments are easily computed by the simple corrections such as the stiffness correction, sectional area correction etc.

For the deck trusses the joint moment diagrams of the standard truss are applicable. The results obtained by joint moment diagrams are very reliable for practical purposes.

V. With regard to the characteristics of the secondary and primary stresses, they have been concluded by the junior author, to be as follows:

1. Between the joint moment M and the greatest converted sectional area A among the upper chord members in a great many examples of actual trusses, there occurs the following relation:

$$M = \frac{1}{320} A M_0 \quad \text{in in. unit}$$

or

$$M = \frac{1}{813} A \cdot M_0 \quad \text{in cm. unit}$$

where M_0 = joint moment of the member in question for the standard truss.

2. When the sectional area of members is constant and the truss has always similar form, the joint moment and the secondary stress in any member are directly proportional to it stiffness and inversely proportional to the size of the truss.

3. When the sectional area of members and the size of truss are constant, the amount of the secondary stress is directly proportional to the distance of extreme fibre from the neutral axis of the member and also to the height of the section of member.

4. The secondary stresses in chord members are maximum, or nearly so, for full load in general.

In the web members the loading for maximum primary stress causes nearly the maximum secondary stress.

In the sub-verticals the maximum secondary stress is produced by the load at joints on the right side of the member in question.

For a single load the chord members are most affected by a load placed at the end of the member in question or at the joint right above or under this member. In the web members joint moments are most affected by a load that is placed one panel distant from the member in question.

In the vertical members, including the sub-verticals, the amounts of the joint moments are nearly equal, but in the diagonals they are quite different.

5. In any member, the secondary stress due to full load varies linearly with the increase of the number of panels in the span, except for the members near the center of the span.

The variation of the maximum secondary stress due to the increase of the number of panels is also linear, except for the members near the span center. The negative maximum joint moment decreases with the increase of the number of panels and this variation deviates from the linear variation. The larger the number of panels in the span, the less becomes the rate of the variation of the joint moment.

The amount of the maximum secondary stress in chord members due to a single load is little affected by the variation of the number of panels. By the use of these characteristics, joint moments in a truss with any number of panels greater than ten can also be computed by the Appendix, Figs. 42 to 47.

6. The variation of the sectional area in a sub-vertical exerts a very large effect upon the secondary stresses under such a single load as produces the primary stress in this member. When the sectional area takes the value of A instead of A_0 this effect can be represented by the equations

$$M = M_u + (M_l - M_u) \frac{A_0}{A} \quad \text{for tensile sub-vertical}$$

$$M = M_l + (M_u - M_l) \frac{A_0}{A} \quad \text{for compressive sub-vertical}$$

where M_u = joint moment in any member due to a single load at the upper joint of the sub-vertical in question, and M_l = joint moment in any member due to a single load at the lower joint of the same member. These amount can be obtained from the joint moment diagrams.

7. Between the primary stresses, namely, the direct stresses for any kind of load and the number of panels in a span, these are very interesting and regular relations.

These relations are clearly shown in the Appendix, Figs. 48 to 51. These direct stress diagrams lend themselves well to a rapid computation of the direct stresses for a bridge truss with any number of panels.

Direct stresses due to a full load increase linearly with the number of panels in the span. The maximum direct stresses in web members increase with the number of panels. This variation slightly deviates from the linear, but it may be considered as a linear variation. The minimum direct stresses decrease with the increase of the number of panels and this variation is represented by the family of hyperbolas with asymptote of $S = 0$. Direct stresses due to a single load increase or decrease toward the finite limit. This variation is also represented by hyperbolas.

8. For a full load, the secondary and direct stresses both vary linearly to an unlimited extent with the increase of the number of

panels in a span, while the ratio of the secondary stress to the direct stress always has a maximum or minimum limit and increases or decreases toward this limit.

For the maximum secondary stress, it is the same as in the above case.

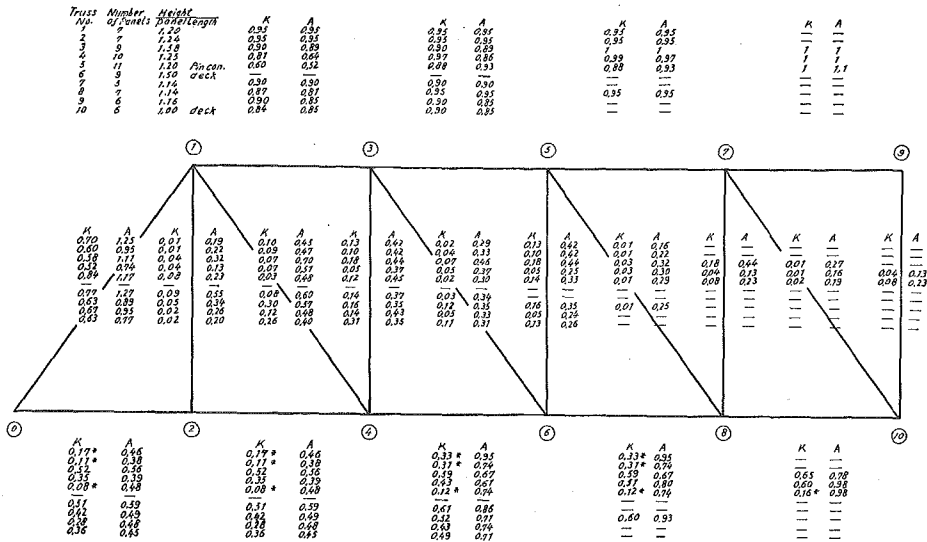
For the sub-verticals, the ratio of the maximum secondary stress to the direct stress increases linearly to an unlimited extent with the number of panels in the span.

For all the calculations, the junior author is responsible.

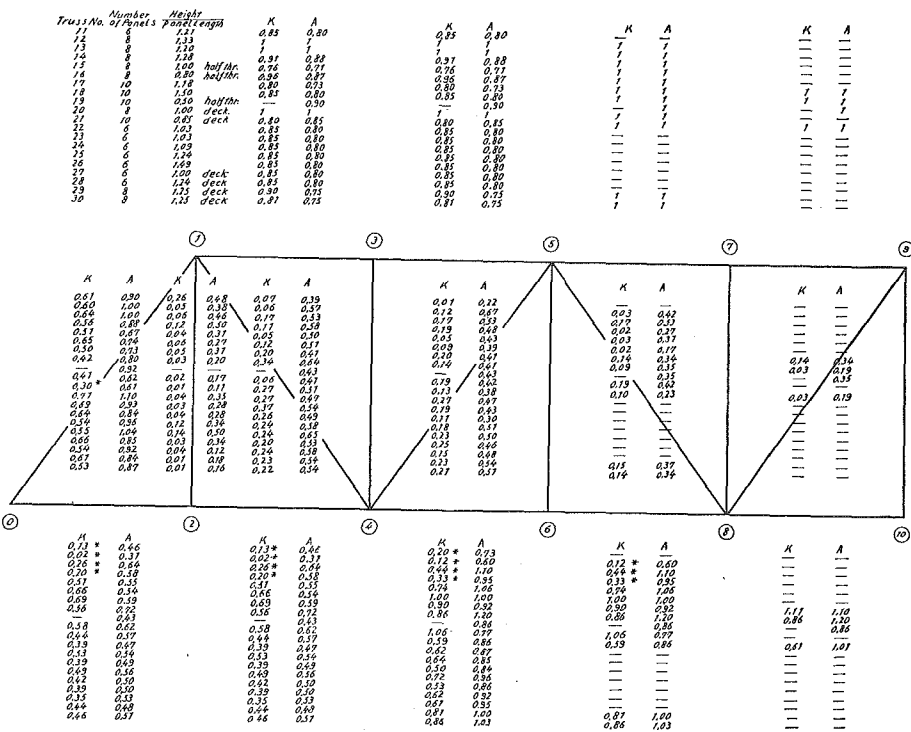
With regard to the joint moment diagrams for the Warren and Pratt trusses with curved chord, they will be reserved for some future paper.

The authors cordially thank the authorities who have offered to them the examples of actual bridge trusses.

New Method of Rapid Computation of Secondary Stresses of Simple Bridge Trusses. 187

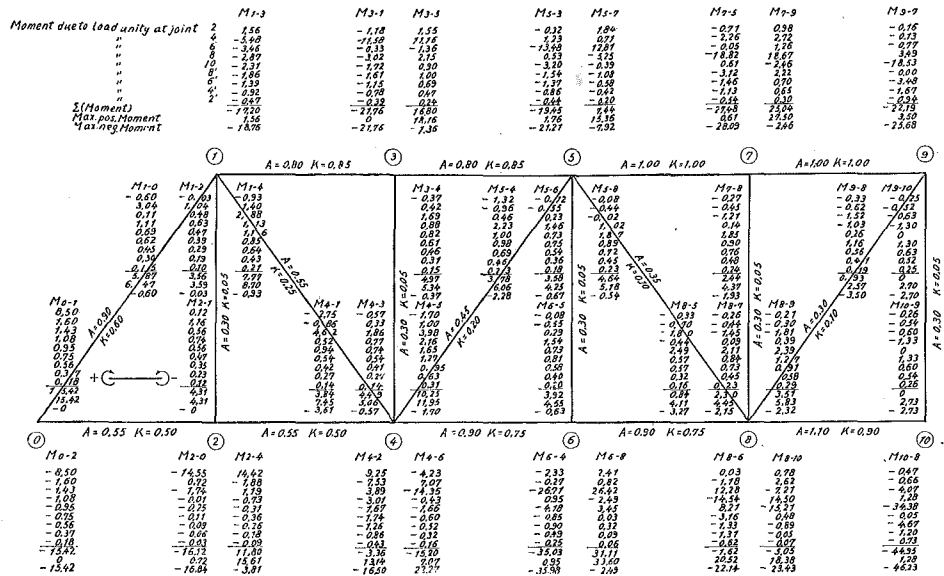


Stiffness and Sectional Area Ratios in Actual Examples of Pratt Truss.
Appendix, Fig. 1.

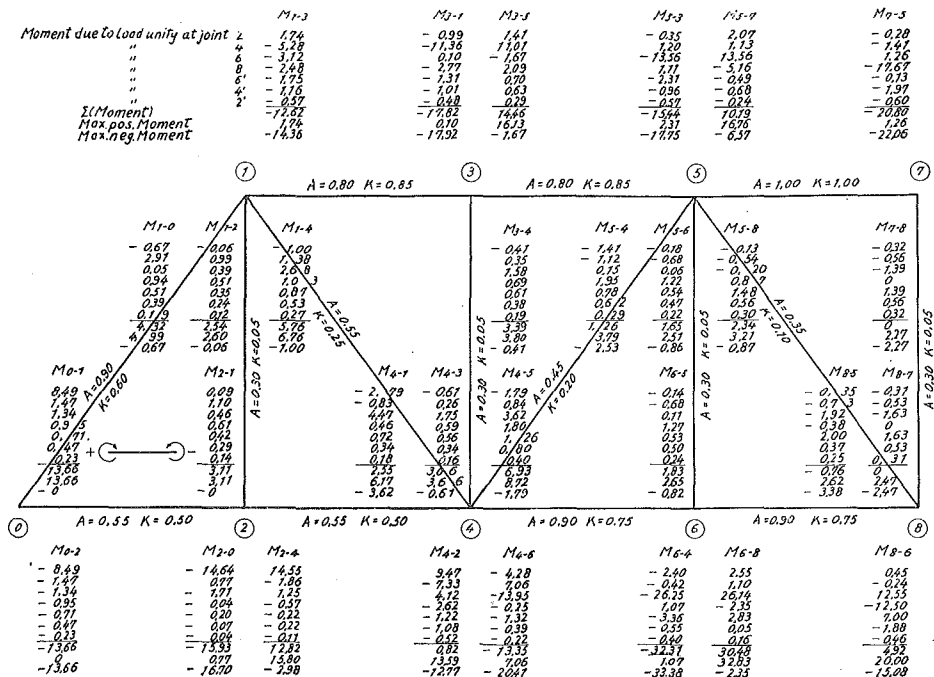


Stiffness and Sectional Area Ratios in Actual Examples of Warren Truss.
Appendix, Fig. 2.

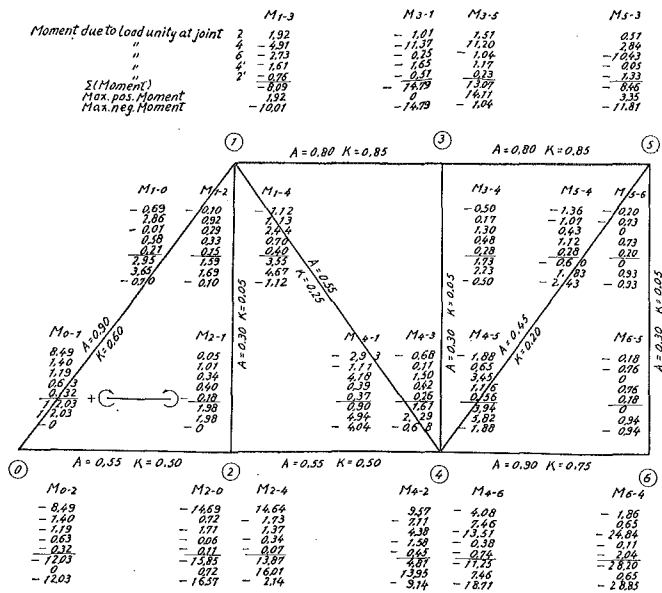
Joint Moment in Warren Truss (A) with 10 panels.



Joint Moment in Warren Truss (A) with 8 panels.

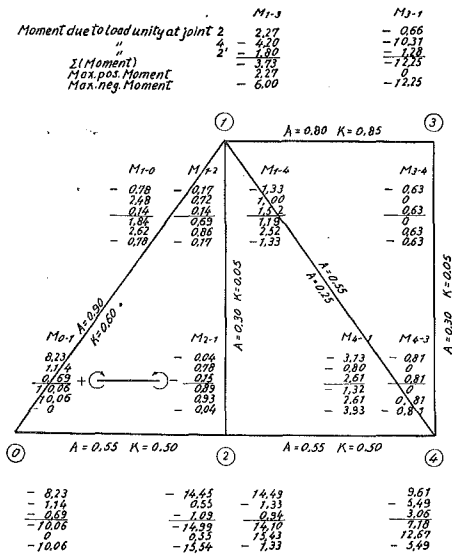


Joint Moment in Warren Truss (A) with 6 panels.



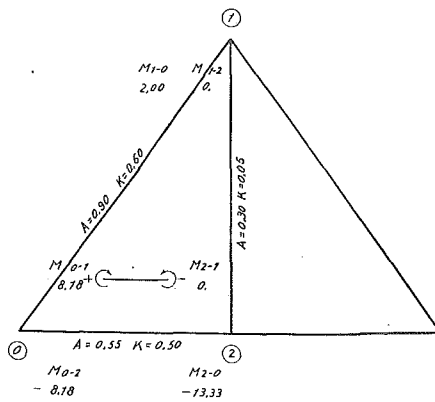
Appendix, Fig. 5.

Joint Moment in Warren Truss (A)
with 4 panels.



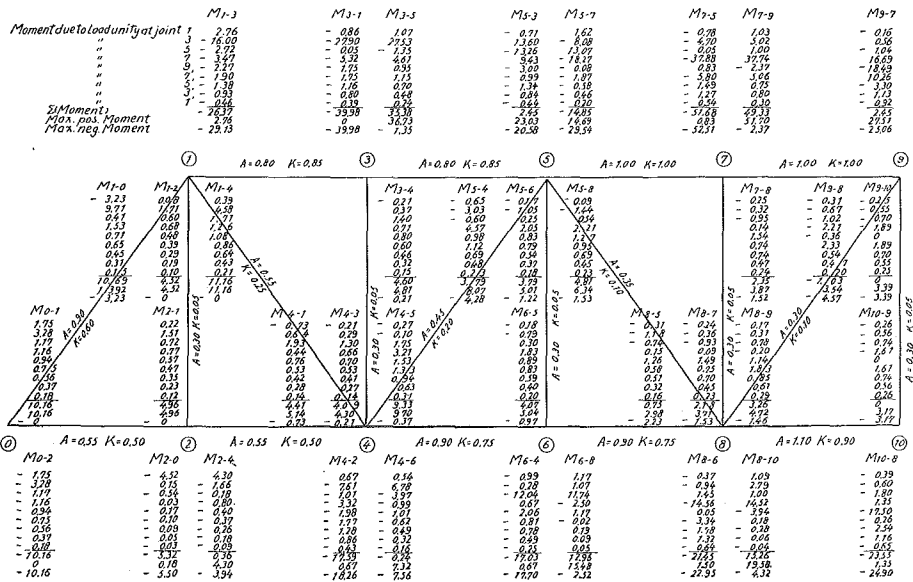
Appendix, Fig. 6.

Joint Moment in Warren Truss (A)
with 2 panels.



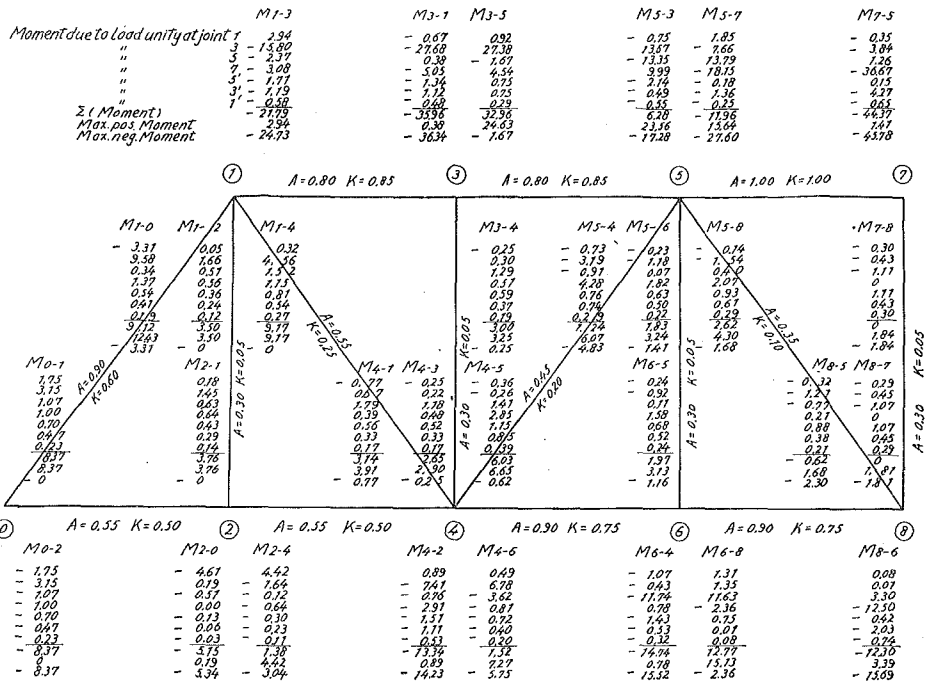
Appendix, Fig. 7.

Joint Moment in Warren Truss (A) with 10 panels.



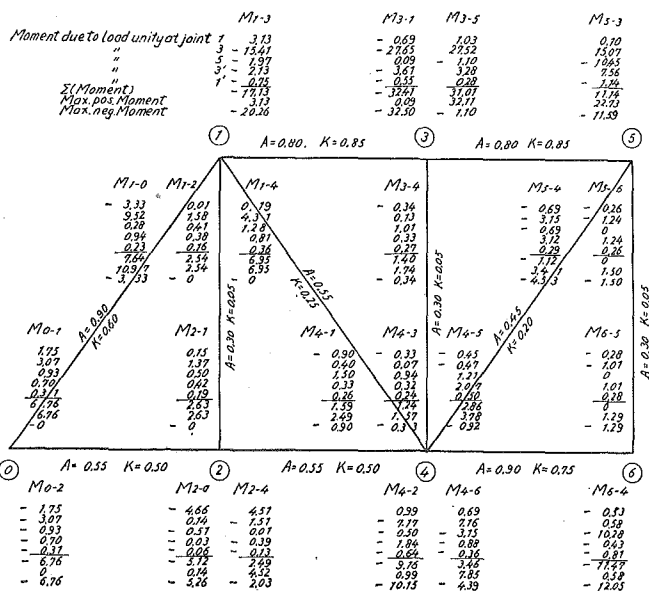
Appendix, Fig. 8.

Joint Moment in Warren Truss (A) with 8 panels.



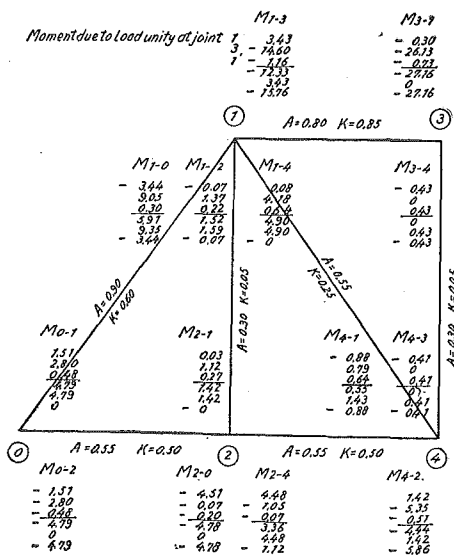
Appendix, Fig. 9.

Joint Moment in Warren Truss (A) with 6 panels.



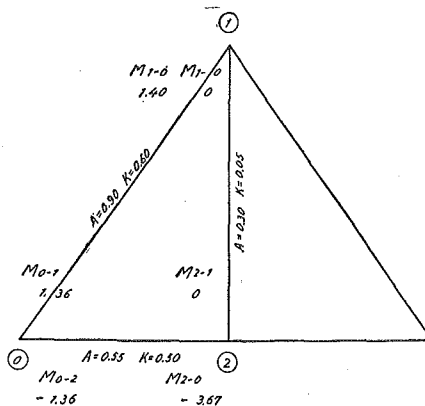
Appendix, Fig. 10.

Joint Moment in Warren Truss (A) with 4 panels.



Appendix, Fig. 11.

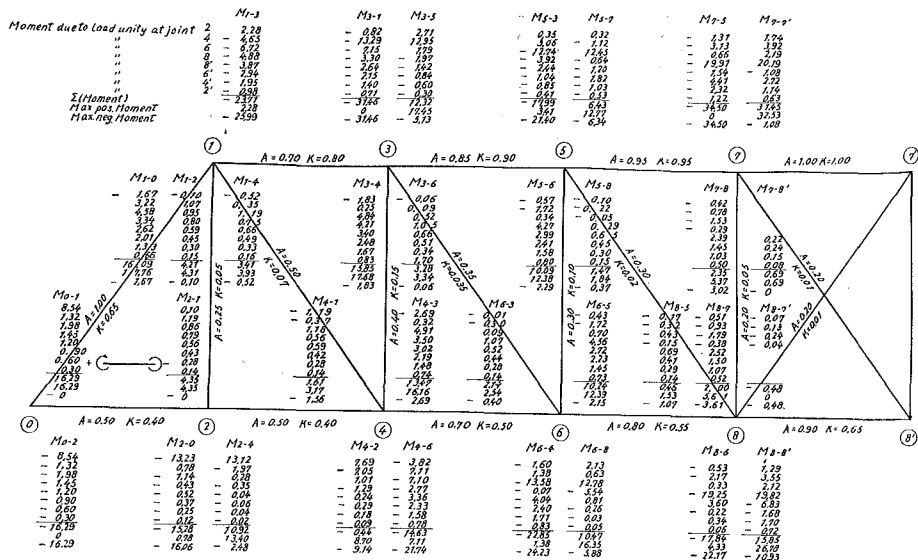
Joint Moment in Warren Truss (A) with 2 panels.



Appendix, Fig. 12.

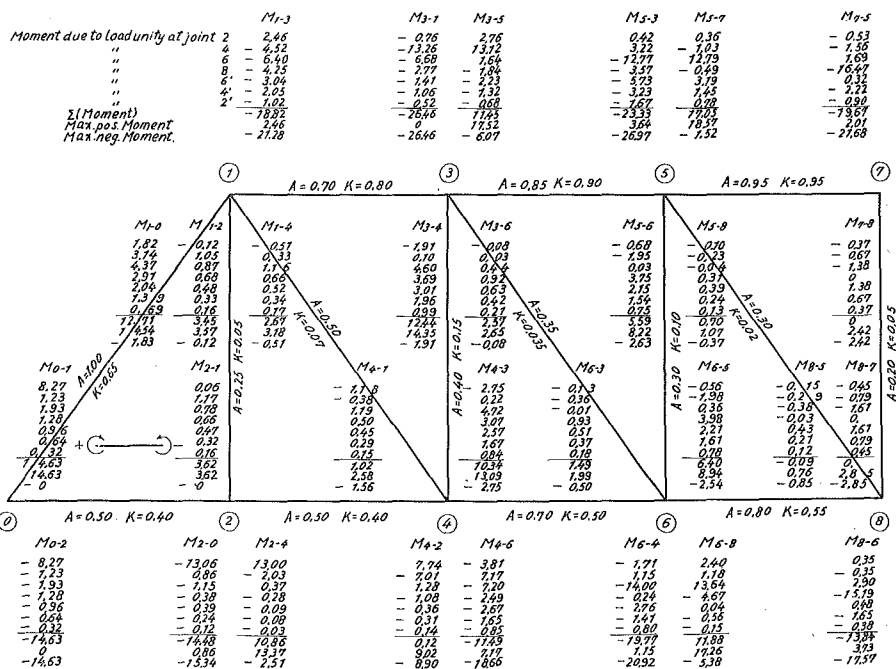
[illegible]

Joint Moment in Pratt Truss (E) with 9 panels.



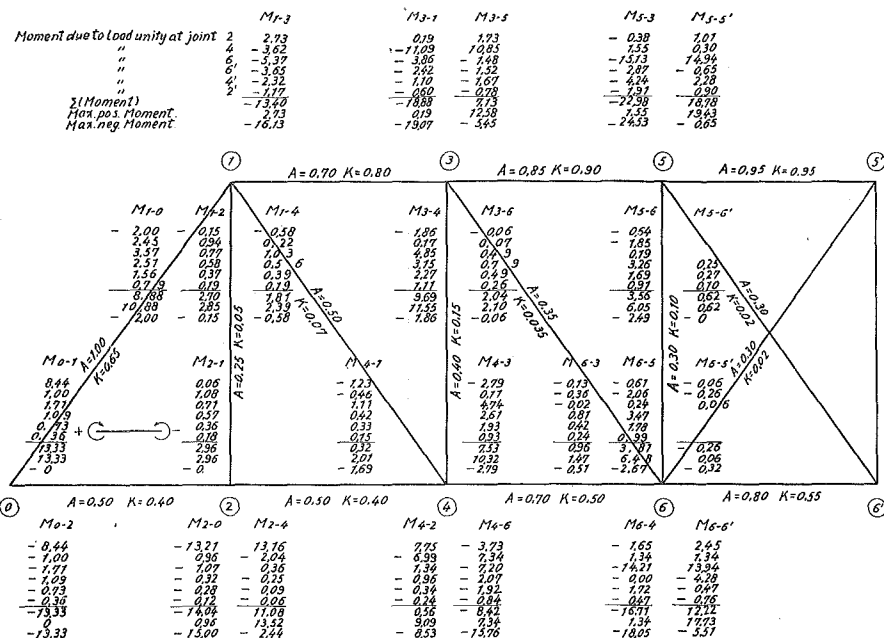
Appendix, Fig. 14.

Joint Moment in Pratt Truss (E) with 8 panels.

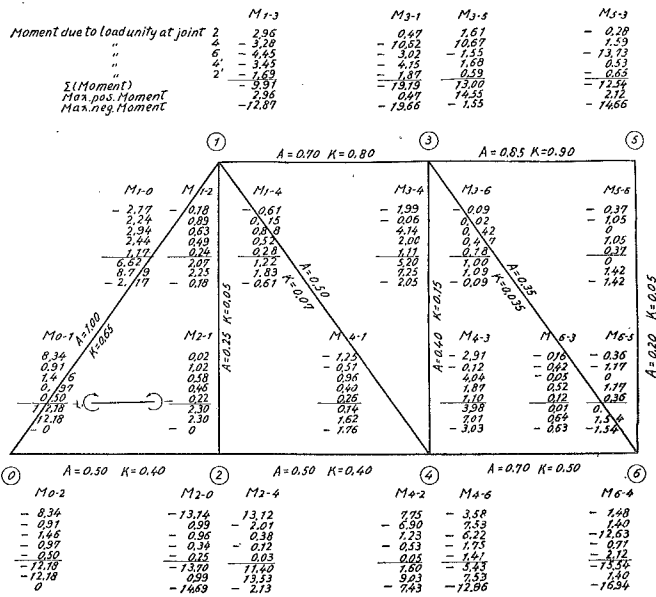


Appendix, Fig. 15.

Joint Moment in Pratt Truss (E) with 7 panels.

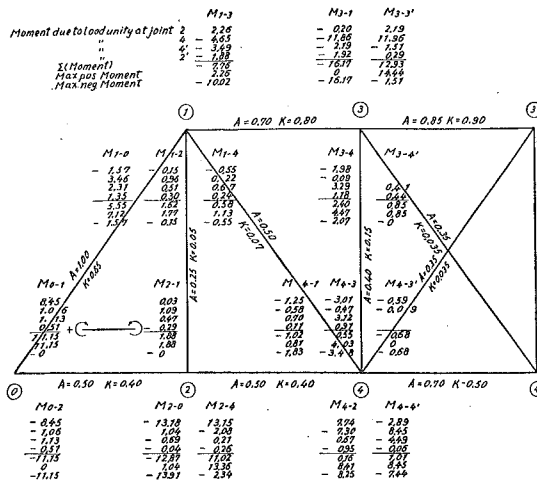


Joint Moment in Pratt Truss (E) with 6 panels.



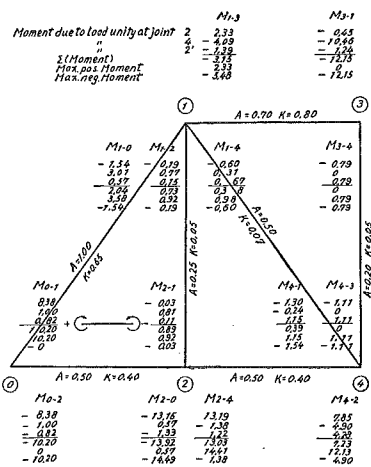
Appendix, Fig. 17.

Joint Moment in Pratt Truss (E) with 5 panels.



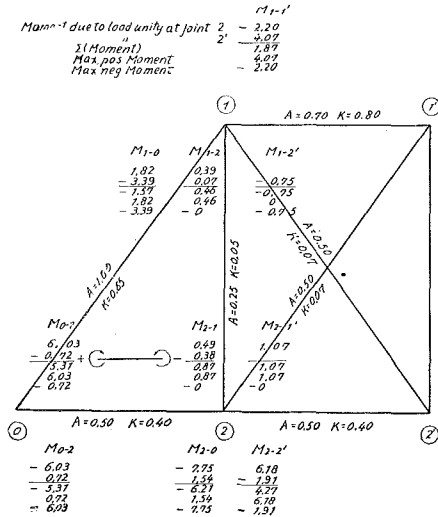
Appendix, Fig. 18.

Joint Moment in Pratt Truss (E) with 4 panels.



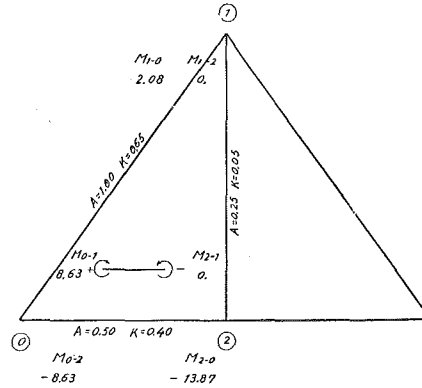
Appendix, Fig. 19.

Joint Moment in Pratt Truss (E)
with 3 panels.



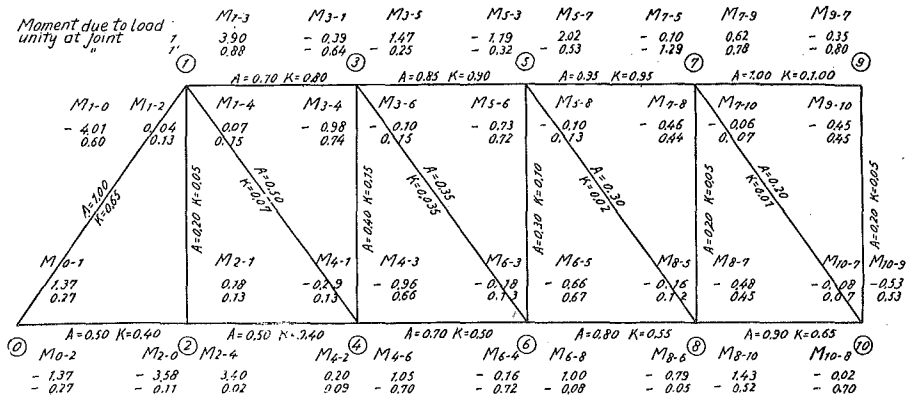
Appendix, Fig. 20.

Joint Moment in Pratt Truss (E)
with 2 panels.



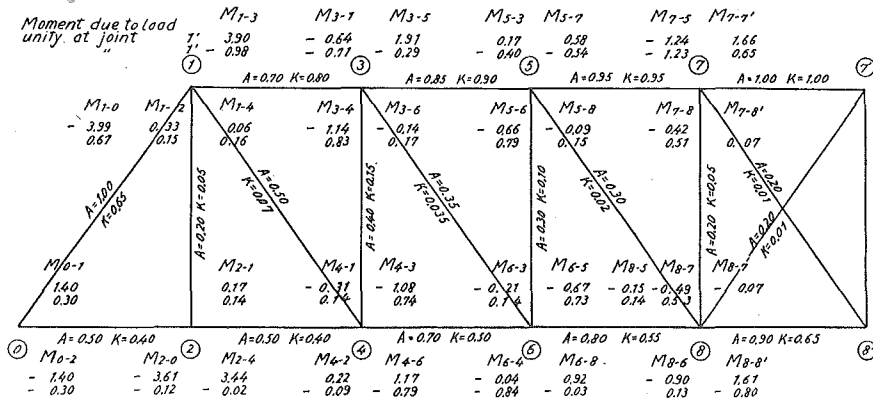
Appendix, Fig. 21.

Joint Moment in Pratt Truss (E) with 10 panels.



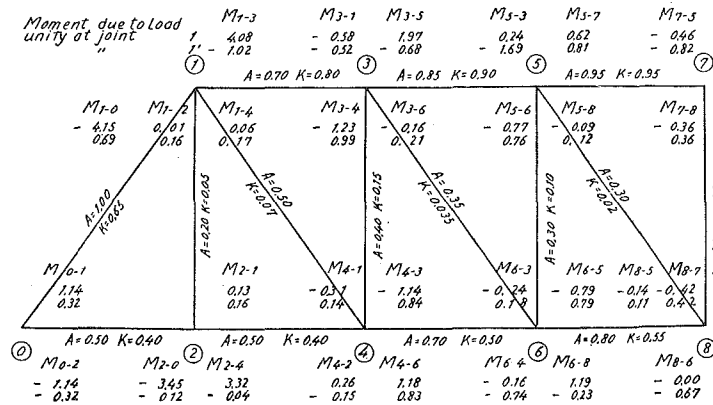
Appendix, Fig. 22.

Joint Moment in Pratt Truss (E) with 9 panels.



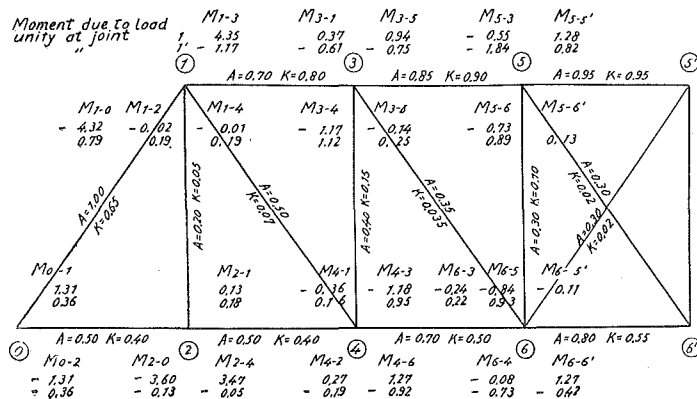
Appendix, Fig. 23.

Joint Moment in Pratt Truss (E) with 8 panels.



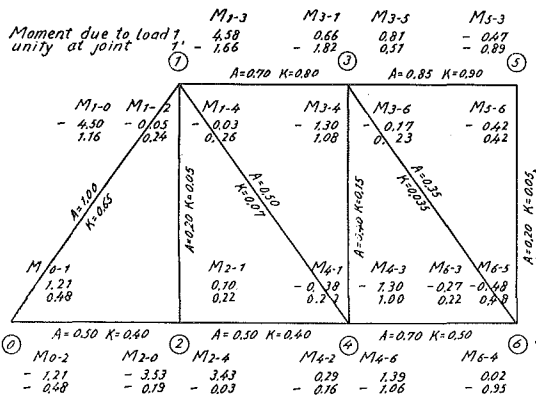
Appendix, Fig. 24.

Joint Moment in Pratt Truss (E) with 7 panels.



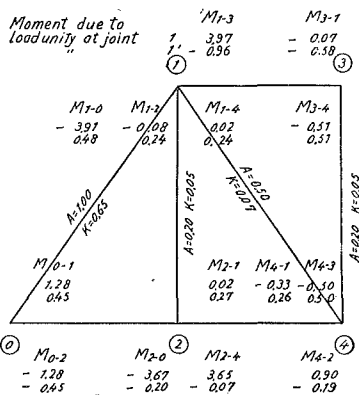
Appendix, Fig. 25.

Joint Moment in Pratt Truss (E) with 6 panels.



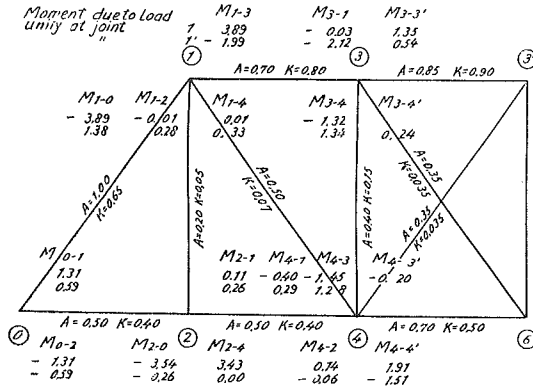
Appendix, Fig. 26.

Joint Moment in Pratt Truss (E) with 4 panels.



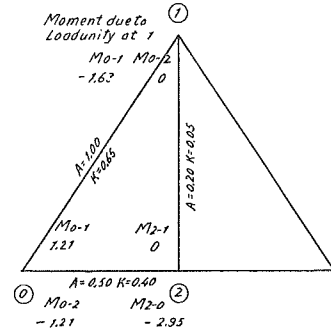
Appendix, Fig. 27.

Joint Moment in Pratt Truss (E) with 5 panels.



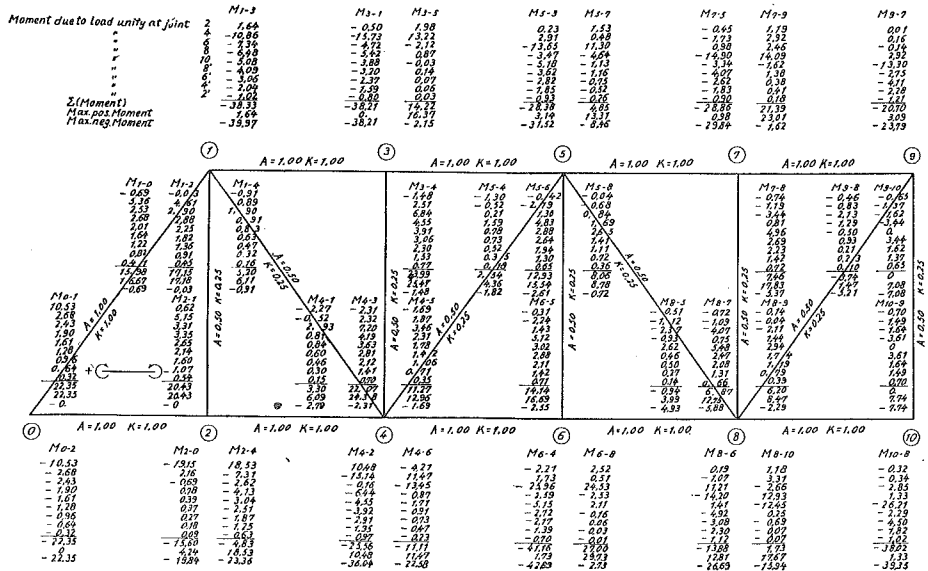
Appendix, Fig. 28.

Joint Moment in Pratt Truss (E) with 2 panels.



Appendix, Fig. 29.

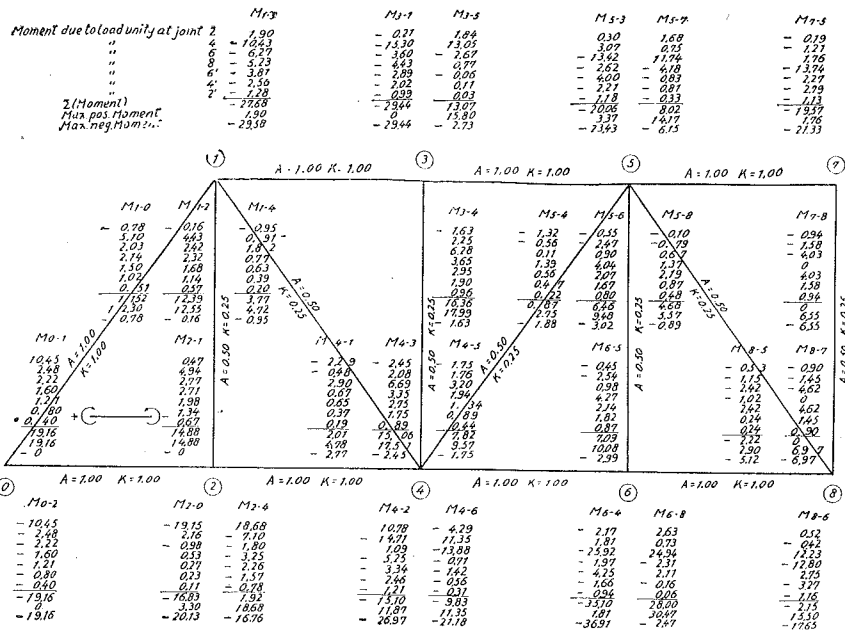
Joint Moment in Warren Truss (D) with 10 panels.



Appendix, Fig. 30.

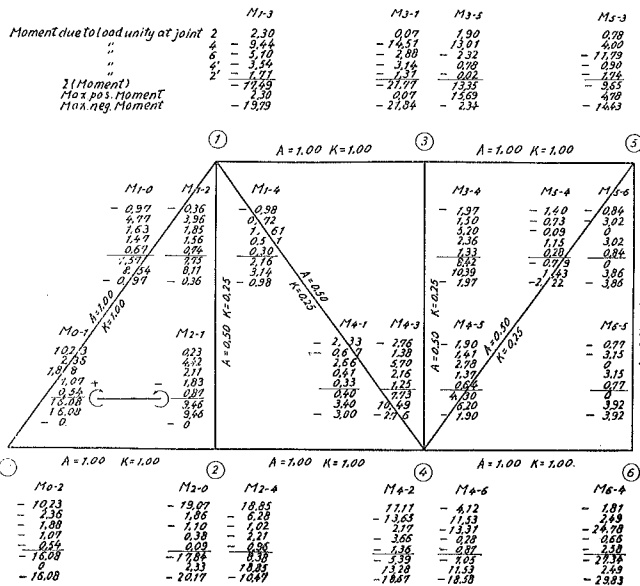
Joint Moment in Warren Truss (D) with 8 panels.

Joint Moment in Warren Truss (D) with 8 panels



Appendix, Fig. 31.

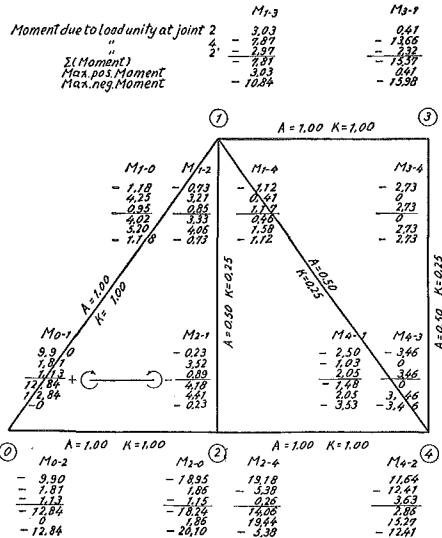
Joint Moment in Warren Truss (D) with 6 panels.



Appendix, Fig. 32.

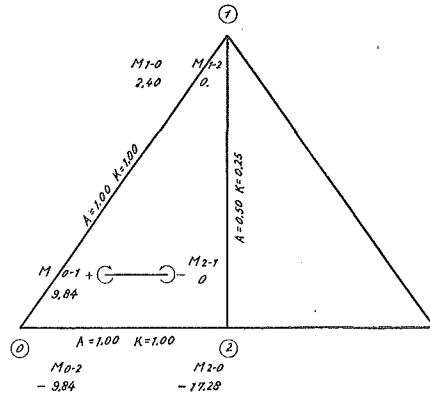


Joint Moment in Warren Truss (D)
with 4 panels.



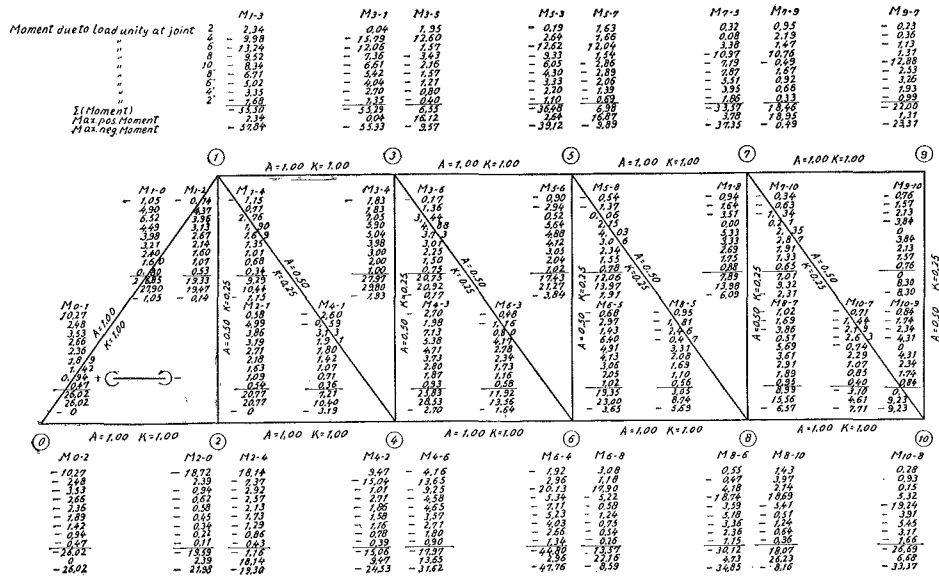
Appendix, Fig. 33.

Joint Moment in Warren Truss (D)
with 2 panels.



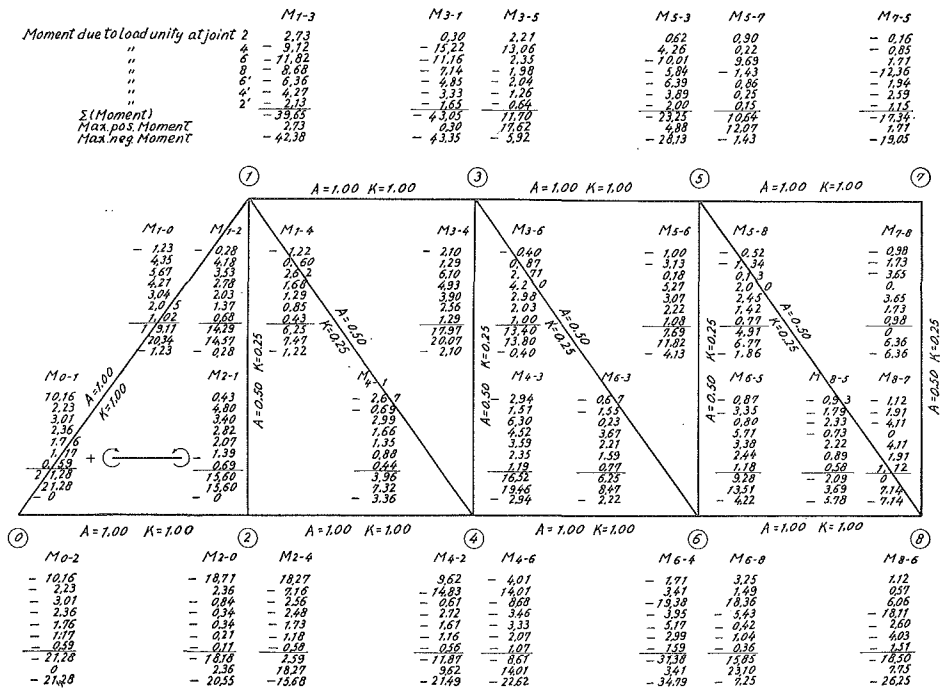
Appendix, Fig. 34.

Joint Moment in Pratt Truss (F) with 10 panels.



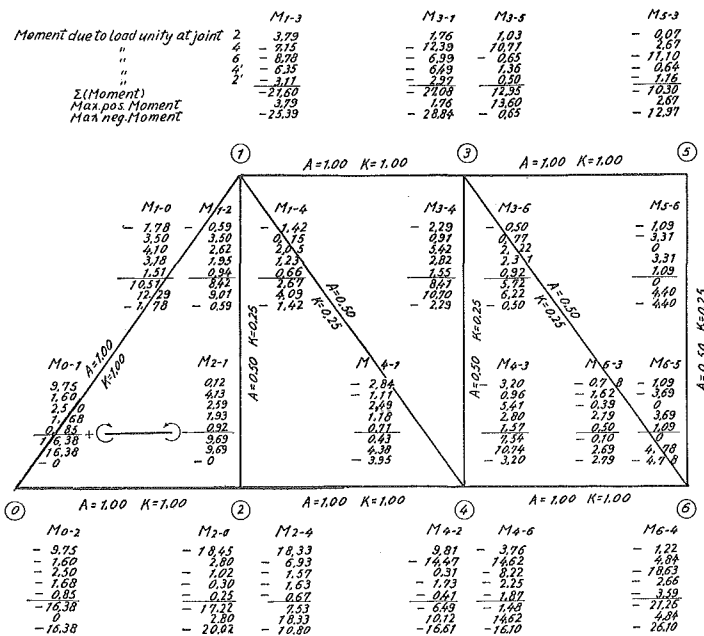
Appendix, Fig. 35.

Joint Moment in Pratt Truss (F) with 8 panels.



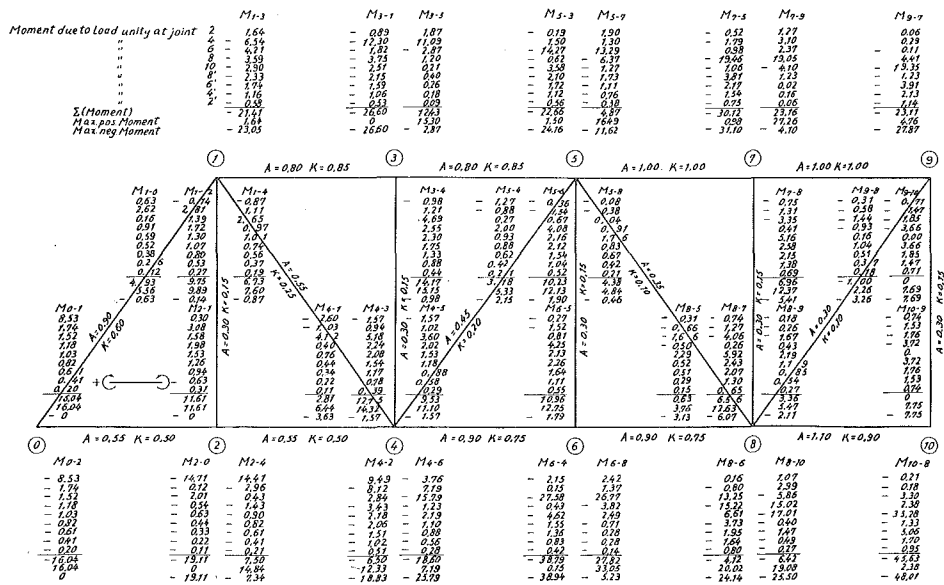
Appendix, Fig. 36.

Joint Moment in Pratt Truss (F) with 6 panels.



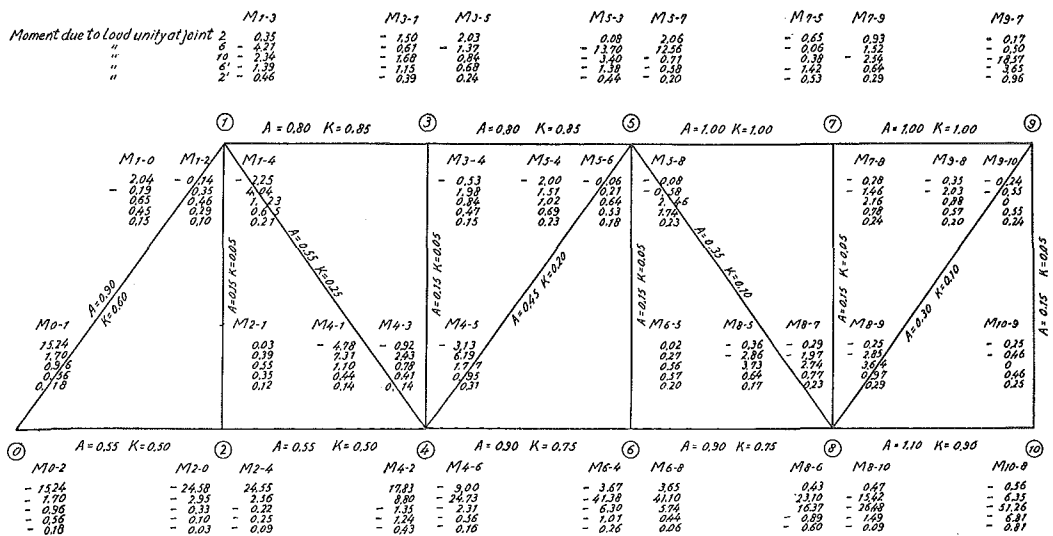
Appendix, Fig. 37.

Joint Moment in Warren Truss (B) with 10 panels.



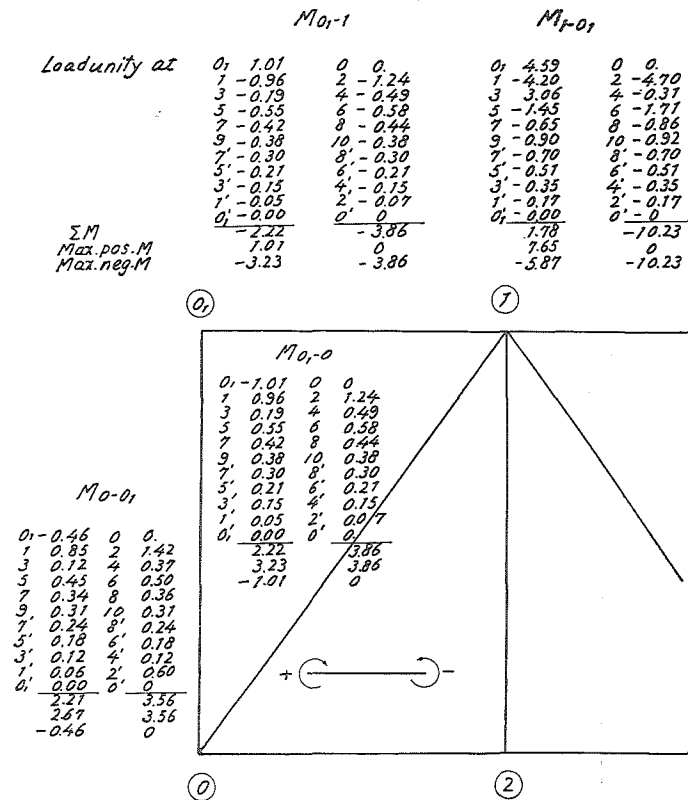
Appendix, Fig. 38.

Joint Moment in Warren Truss with 10 panels.



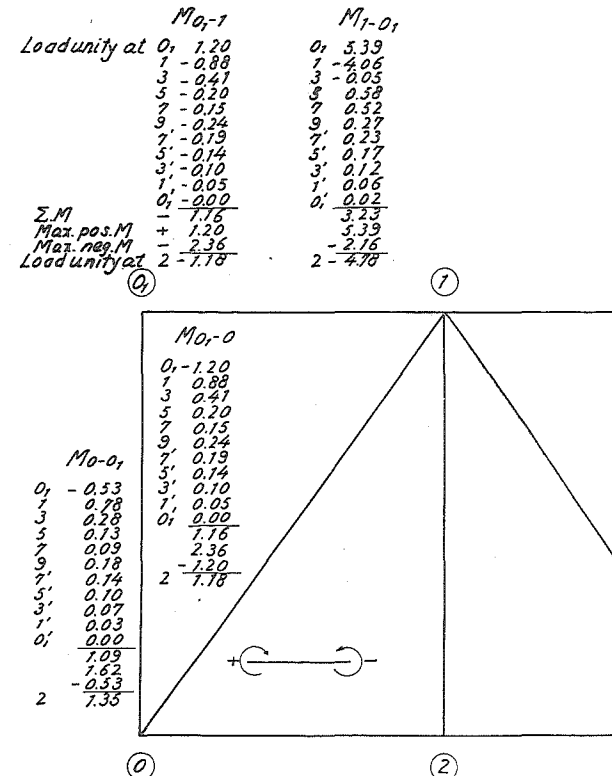
Appendix, Fig. 39.

Joint Moment in Standard Deck Warren Truss with 10 Panels.



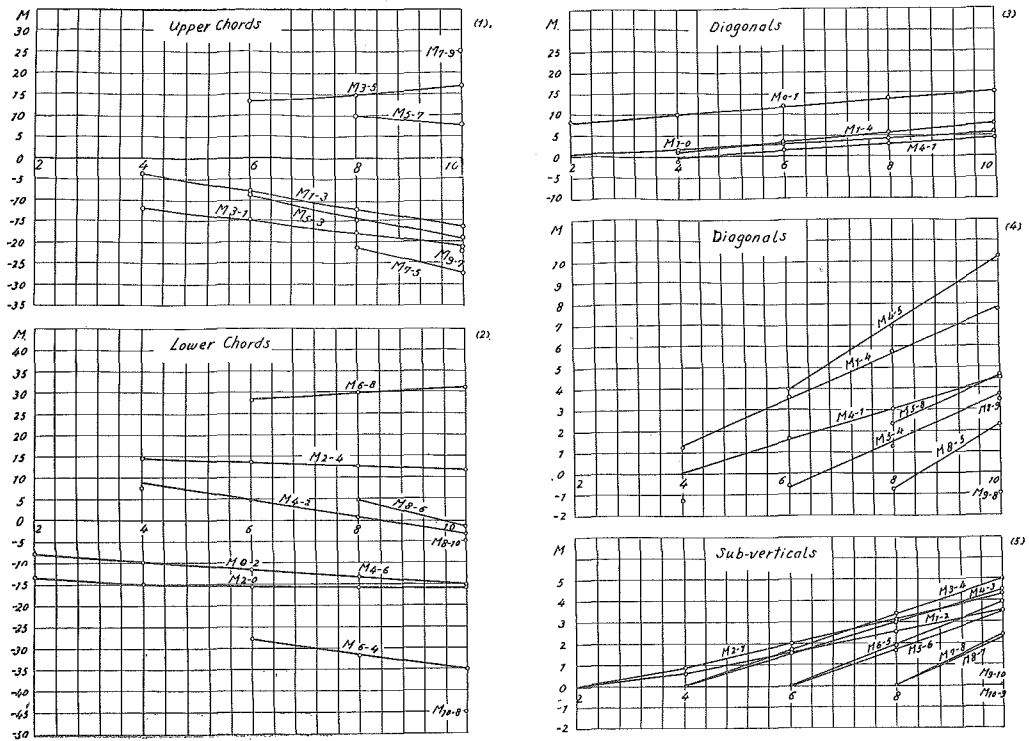
Appendix, Fig. 40.

Joint Moment in Standard Deck Pratt Truss with 10 Panels.

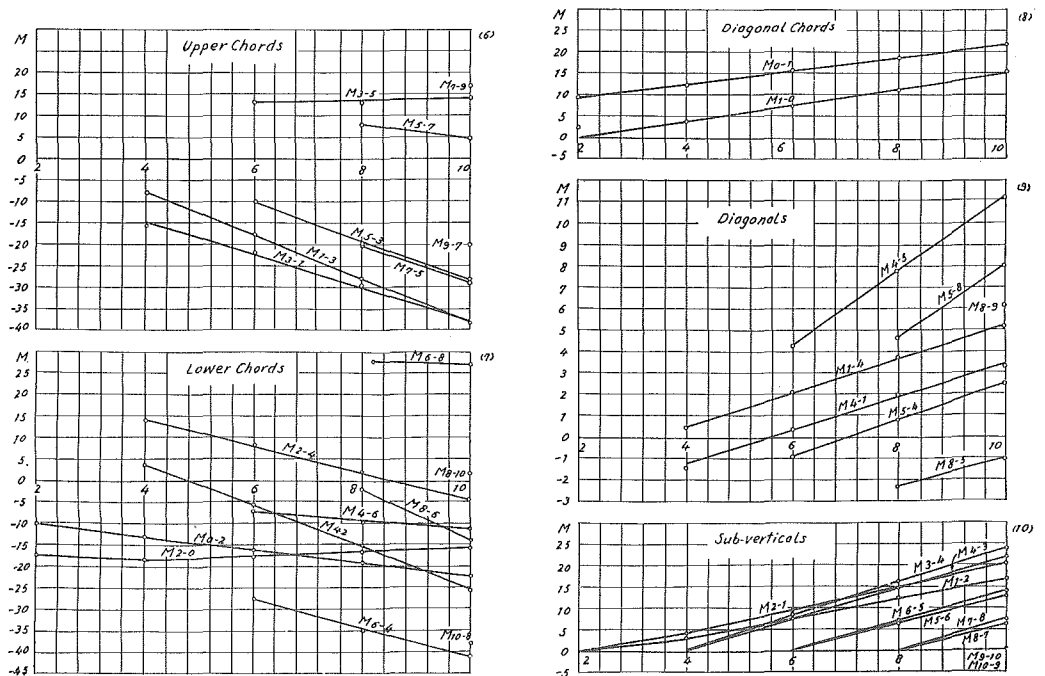


Appendix, Fig. 41.

(Standard Warren Truss A)

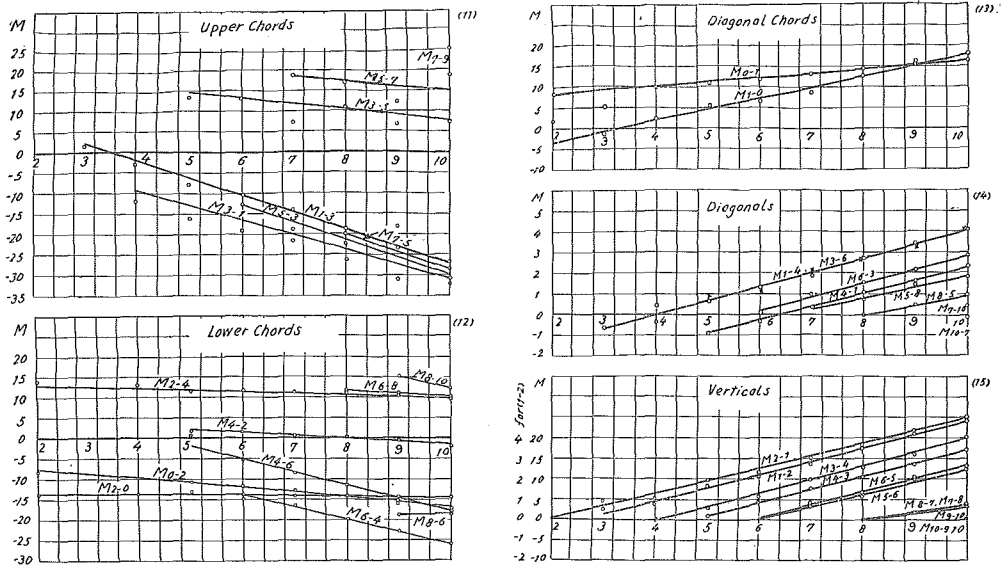


(Special Warren Truss D)

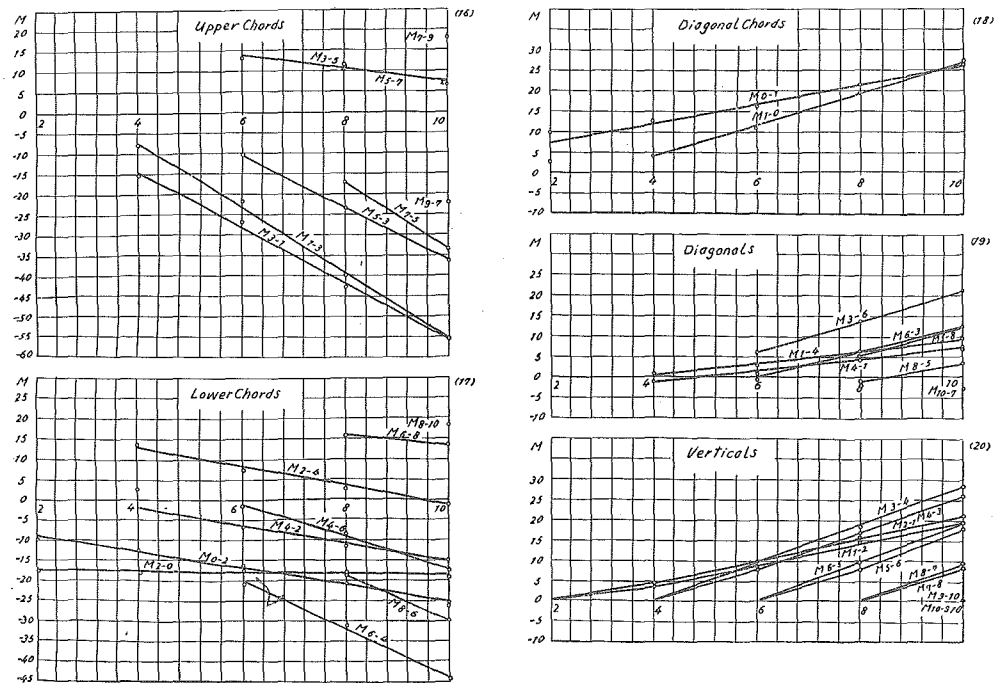


Relation between Joint Moment due to Full Load and Number of Panels in Span.

(Standard Pratt Truss E)



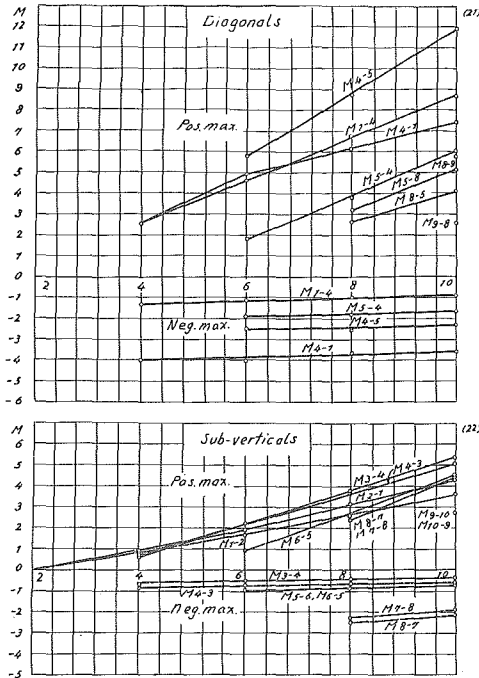
(Special Pratt Truss F)



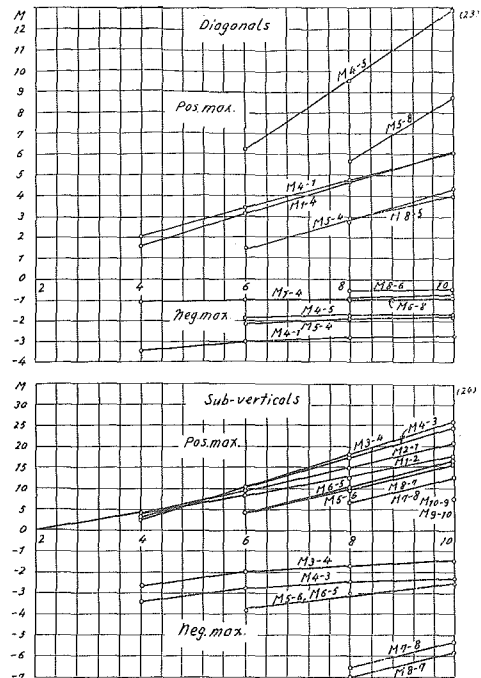
Relation between Joint Moment due to Full load and Number of Panels in Span.

Appendix, Fig. 43.

(Standard Warren truss A)

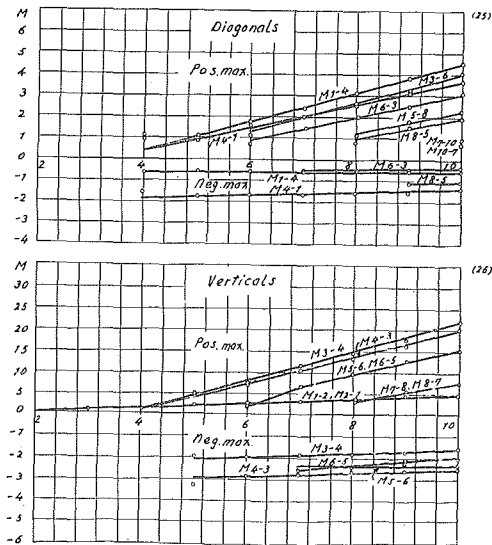


(Special Warren Truss D)

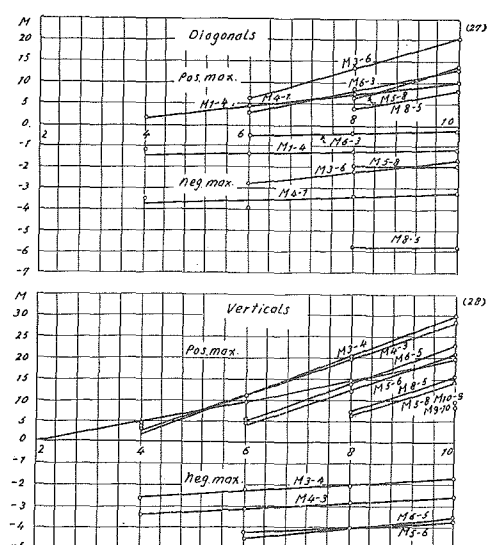


Relation between Max. Joint Moment in Web Member and Number of Panels in Span.
Appendix, Fig. 44.

(Standard Warren Truss E)

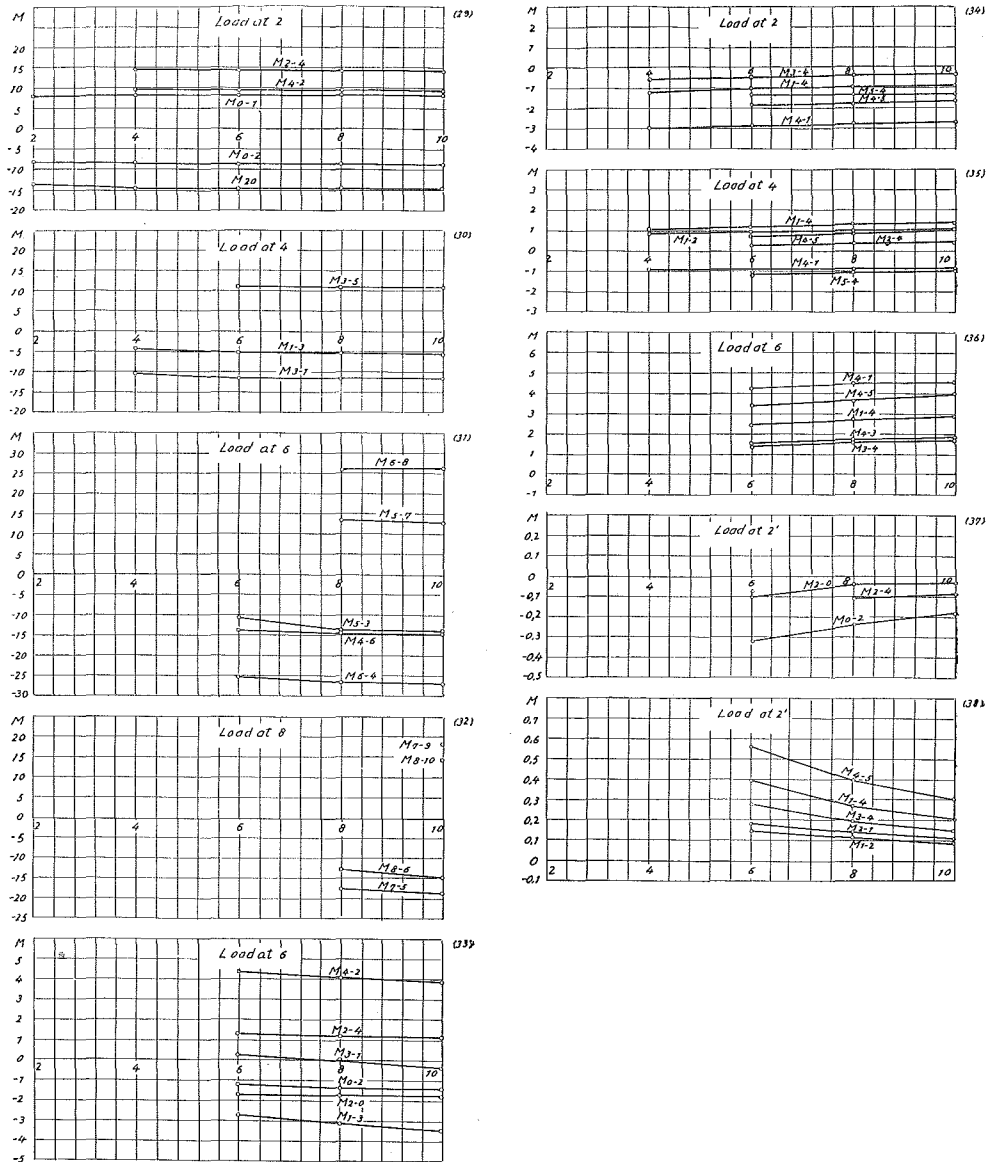


(Special Pratt Truss F)

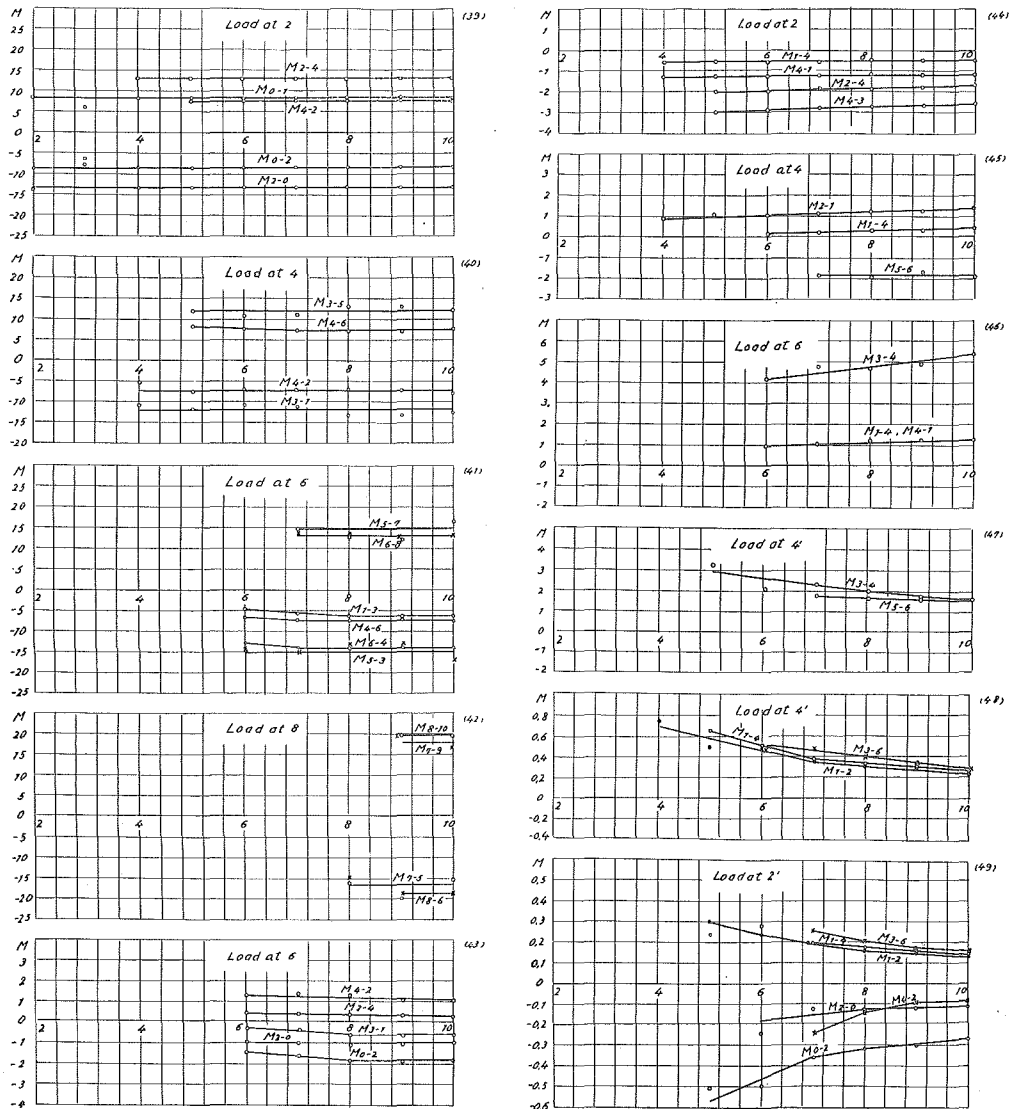


Relation between Max. Joint Moment in Web Member and Number of Panels in Span.
Appendix, Fig. 45.

(Standard Warren Truss A)

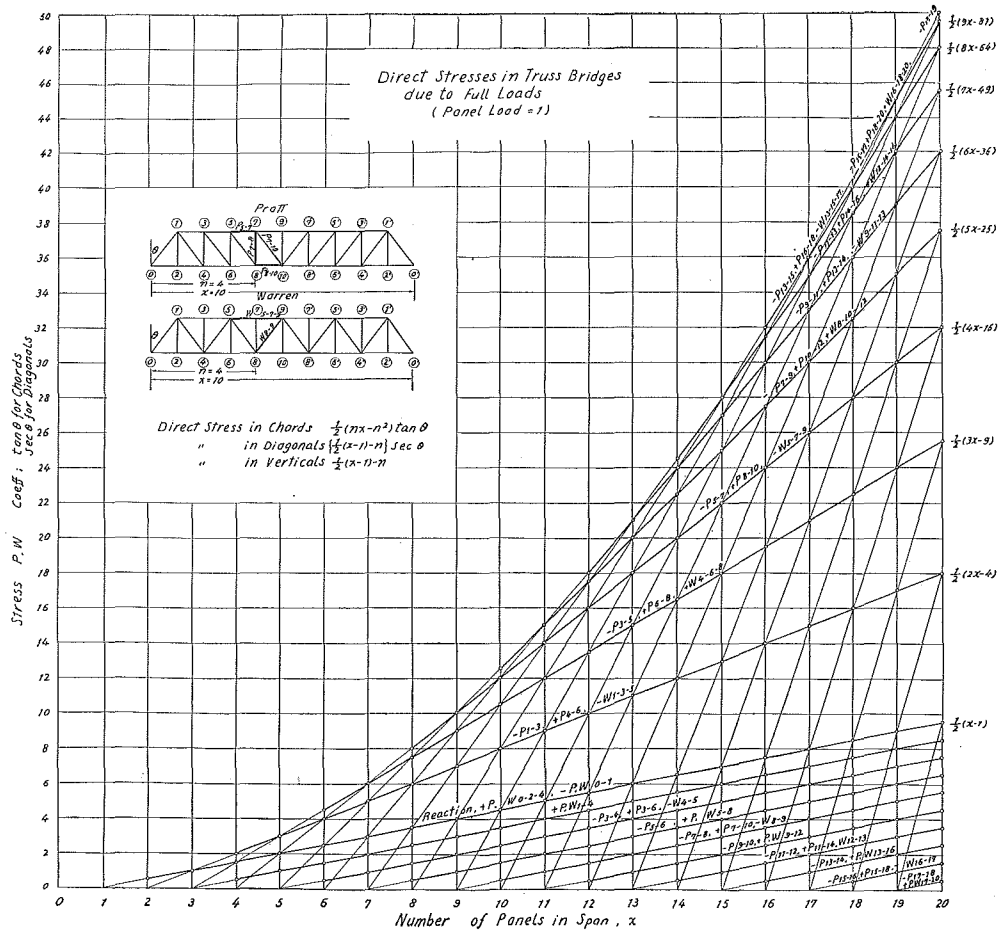


(Standard Pratt Truss E)

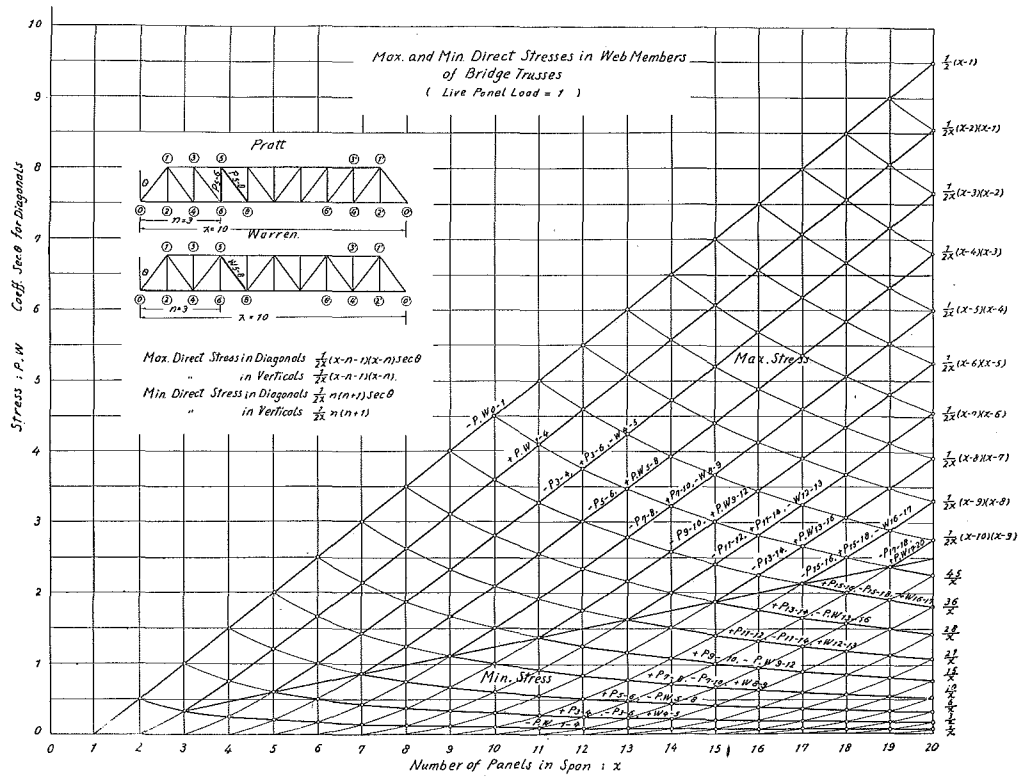


Relation between Joint Moment due to a Load Unity and Number of Panels in Span.

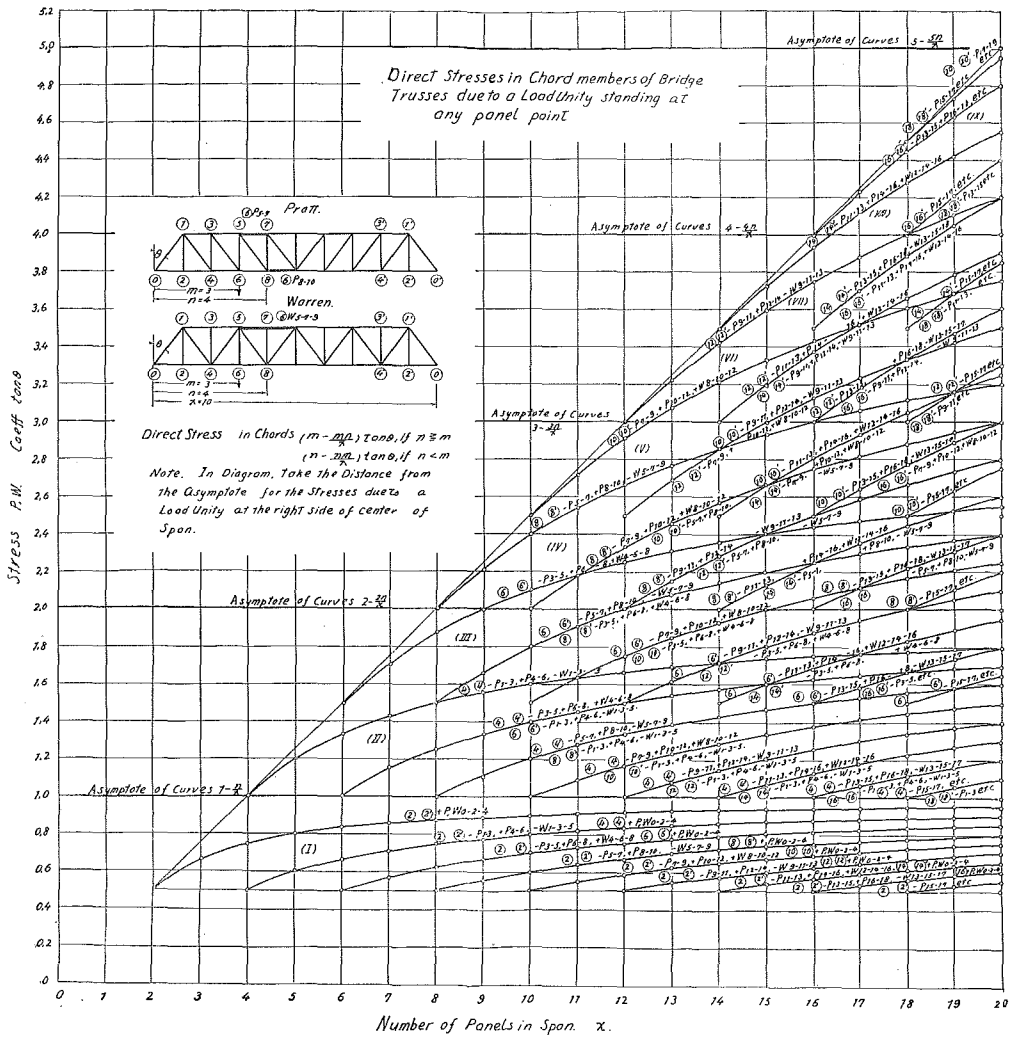
Appendix, Fig. 47.



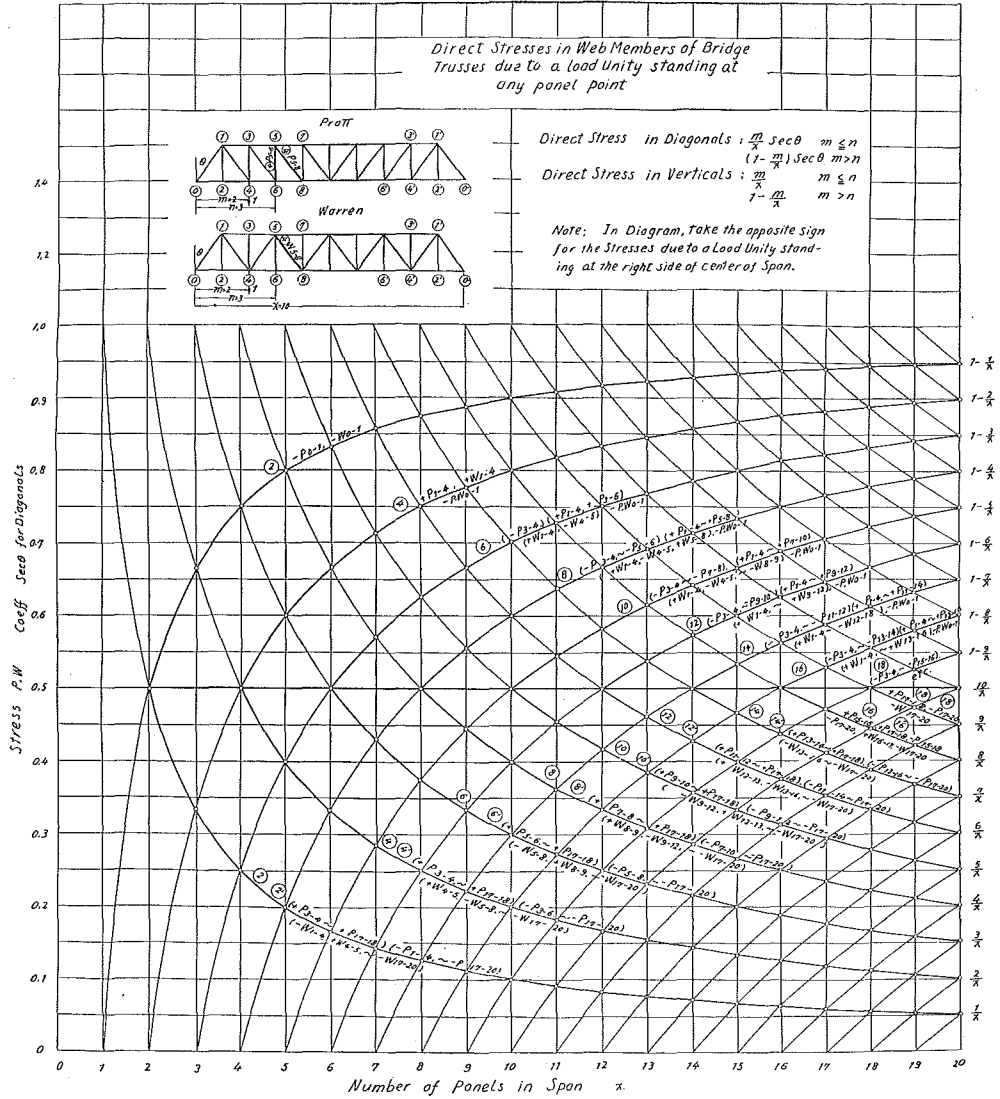
Appendix, Fig. 48.



Appendix, Fig. 49.



Appendix, Fig. 50.



Appendix, Fig. 51.