Rapid and Rigorous Calculation for Adjustment of Fundamental Triangulation Nets by "Mechanical Sketch Method."

By

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Contents.

INTRODUCTION.

In general, as widely noted, Balancing a Survey is accomplished by applying the principle of Least Squares and for that purpose the "Correlate Method" is convenient. If acculate results in triangulation are desired, conditions in the observations are increased and this produces an increase in number of condition equations. In accordance with an increase of condition equations and a serieses of individual triangles, the computation of correlate equations becomes more and more complicated and even mistakes getting in during the course of computation accidentally can perhaps not be avoided and even their existence is difficult to discover.

For these computations, hitherto, there have been several available methods: Determinant Method as described in references (1), (2) and (3) in the foot note below, Elimination Method as in (4), (5), (6), (7) and (8), Successive Approximation as in (9), etc. For elementary triangulation nets, i.e., quadrilateral and centralpolygons, compact forms of results are obtained by the methods proposed in (10) and (11).

In this paper, is described a new method of making correlate equations and their computation for fundamental triangulation nets, viz., central polygins, a single row triangulation net and a quadrilateral

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(1) Beiträge zur direkten Auflösung der Normalgleichungen unter Besonderer Berücksichtigung der Geodätischen Netzausgleichung. von Konrad Friedrich. Z. f. V. 1930 S. 461-469, 525-539, 671-697.

(2) Whittaker & Robinson; The Calculus of Observation.

(3) Jordan-Eggert; Handbuch der Vermessungskunde, Bd. 1, § 77, S. 279.

(4) Einige Bemerkungen über die gemeinsame Ausgleichung von Winkeln und Längen. Chr. Ganeff. Z. f. V. 1930 S. 157-180.

(5) Ein vereinfaches Rechnungsschema über die gemeinsame Ausgleichung für Rechnenmachiene. Dr. Otto Gruber. Z. f. V. 1925 S. 133-140.

(6) Die Auflösung von Normalgleichungen mit Hilfe der Nova-Brunsviga Rechnenmachine. P. Werkmeister. Z. f. V. 1928 S. 437-441.

(7) Über die Bildung von Polygonbedingungsgleichungen mit Hilfe fingierten Beobachtungen. W. Jenne. Z. f. V. 1932 S. 273-283.

(8) Die Auflösung eines Systems linearer Gleichungen mit mehreren Unbekanten und grossen Koefficienten und Absolutgliedern mittels der Gaussischen Additions-bezw. Subtraktionslogarithmen. Karl Effinger. Z. f. V. 1934 S. 353-355.

(9) Auflösung der Normalgleichungen von Dreiecksnetzen durch Schrittweiseannährung. Dr. Pinkwart. Z. f. V. 1927 S. 257-270.

(10) Die Ausgleichung elementarer Dreiecksgruppen. Th. Galachow. Z. f. V. 1935 S. 449-458.

(11) Zur Ausgleichung von Zentralsystem. W. Dumke. Z. f. V. 1937 S. 689-694.

by means of sketching a figure, accompanied by the method of iteration under more complicated conditions, and is treated the case, in which all angles are observed with the equal weight. This paper consists of three Chapters:

Chapter 1 contains reductions of correlate equations in general and values of correlates in special cases obtained by method of elimination.

Chapter 2 contains the principle of the Mechanical Sketch Method with some examples and is the main part of this paper.

Chapter 3 contains values of correlates in compact forms obtained by method of elimination in general case.

This new method is one application of the solution of structural frames, proposed by Prof. F. Takabeya and T. Sakai in references (12), (13) and (14), to the solution of many simultaneous equations of correlates in adjustment of triangulation nets.

The present author offers grateful thanks to the authors of these references, and especially to Prof. F. Takabeya for his kind guidance and encouragement during the preparation of this paper.

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(12) F. Takabeya; Rahmentafeln.

(13) Practical Method of Calculation for Vierendeel Truss Bridges. F. Takebaya. Memoirs of the Faculty of Eng., Hokkaido Imp. Univ. Vol. 4, No. 2, Dec. 1937.

(14) A New Method of Rapid Computation of Secondary Stresses of Simple Bridge Trusses. F. Takabeya and T. Sakai. Memoirs of the Faculty of Eng., Hokkaido Imp. Univ. Vol. 4, No. 2, Dec. 1937.

CHAPTER 1.

REDUCTIONS AND RESULTS BY METHOD OF ELIMINATION.

§ 1. GENERAL DESCRIPTION.

In general, there are three kinds of condition equations neccessary in adjustment of plane triangulation nets, and they are point equations at every station, angle equations at each individual triangle and a side equation in a set of triangulation net which is between two base lines or in a closed figure.

The number of these condition equations are expressed in well known formulae as follows:

point equations ; $N_p = m-(s-1)$.
angle equations ; $N_a = S_1-(T_1-1)$.
side equations ; $N_s = S_2-(2T_2-3)$.

where

N_p = the number of point equations
m = the number of angles observed at a station
s = the number of stations sighted at a station
N_a = the number of angle equations
S_1 = the number of sides at whose ends angles are observed
T_1 = the number of transit stations
N_s = the number of side equations
S_2 = the total number of sides
T_2 = the total number of stations.

The process of calculation for adjustment can be performed conveniently and systematically by applying the Correlate Method. In this paper, a correlate method is also applied.

In the explanation of this new method, the writer calls correlates which are related to point equations, angle equations and side equations by the names "Point Correlates," "Angle Correlates" and "Side Correlates" respectively, and expresses them by the following notations;

point correlates, K_0, K_p, K_q, $K_{(i)}$, $(i = 1, 2, \cdots, n, n+1)$
each suffix 0, p, q, (i) denotes the station where the point equation is composed,

angle correlates, K_i, $(i = 1, 2, \cdots, n)$
each suffix i denotes each individual triangle where the angle equation is composed,

a side correlate, K_s.

Other notations;

$\boldsymbol{P}$, $\boldsymbol{Q}$, $(\mathbf{1})$, $(\mathbf{2})$, $\cdots$, $(\boldsymbol{n})$, $(\boldsymbol{n}+\mathbf{1})$, $\mathbf{1}$, $\mathbf{2}$, $\cdots$, $\boldsymbol{n}$, $\mathbf{3}\boldsymbol{i}-\mathbf{2}$, $\mathbf{3}\boldsymbol{i}-\mathbf{1}$, $\mathbf{3}\boldsymbol{i}$, $\boldsymbol{\gamma}$, & etc.,
= observed values of each angle respectively,

v_p, v_q, $v_{(1)}$, $v_{(2)}$, $\cdots$, $v_{(n)}$, $v_{(n+1)}$, v_{3i-2}, v_{3i-1}, v_{3i}, v_γ,
= correction of each observed angle in seconds,

Δ_{3i-2}, Δ_{3i-1}, $(i = 1, 2, \cdots, n)$
= tabular difference of log. sin. $\mathbf{3}\boldsymbol{i}-\mathbf{2}$ or log. sin. $\mathbf{3}\boldsymbol{i}-\mathbf{1}$ per $1''$ in the unit of the last place of logarithmic table,
or $(\log. e/\rho'')\cdot\cot. \mathbf{3}\boldsymbol{i}-\mathbf{2}$ or $(\log. e/\rho'')\cdot\cot. \mathbf{3}\boldsymbol{i}-\mathbf{1}$,

where $\log. e/\rho'' = (4.7494\times 10^5)^{-1}$.

§2. CLOSED CENTRAL POLYGON.

In a Closed Central Polygon as in Fig. 1, all inner and outer

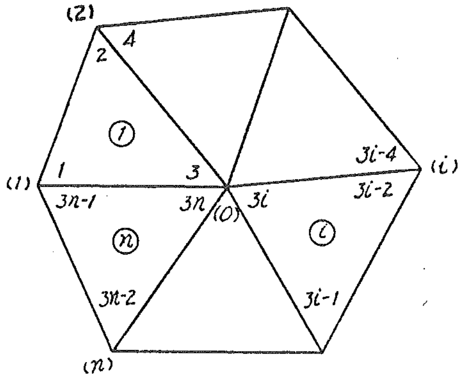

Fig. 1.

angles are assumed to be observed, then the number of condition equations is as follows;

$N_p = n-(n-1) = 1$ at the central station (0),

and $3-(3-1) = 1$ at each corner station,

$$N_a = 2n-(n+1-1) = n$$

$$N_s = 2n-\{2(n+1)-3\} = 1.$$

The total number of condition equations is $2(n+1)$, and they are as follows:

$$\left.\begin{array}{l}
\text{A point equation at the central station (0);} \\
\qquad\qquad \sum_{i=1}^{n} v_{3i} = \omega_0 \,. \\
n \text{ point equations at each corner station;} \\
\quad v_{(i)}+v_{3i-2}+v_{3i-4} = \omega_{(i)} \qquad (i = 1, 2, \cdots, n)\,. \\
n \text{ angle equations at each individual triangle;} \\
\quad v_{3i-2}+v_{3i-1}+v_{3i} = \omega_i \qquad (i = 1, 2, \cdots, n)\,. \\
\text{A side equation of the closed figure;} \\
\quad \sum_{i=1}^{n} (\Delta_{3i-2} \cdot v_{3i-2} - \Delta_{3i-1} \cdot v_{3i-1}) = \omega_s \,.
\end{array}\right\} \cdots\cdots (1)$$

where

$$\left.\begin{array}{lcl}
\omega_0 & = & 360^\circ - \sum_{i=1}^{n} \mathbf{3i} \\
\omega_{(i)} & = & 360^\circ - (\,(\boldsymbol{i}) + \mathbf{3i-2} + \mathbf{3i-4}\,) \\
\omega_i & = & 180^\circ - (\,\mathbf{3i-2} + \mathbf{3i-1} + \mathbf{3i}) \\
\omega_s & = & \sum_{i=1}^{n} (\log.\sin.\, \mathbf{3i-1} - \log.\sin.\, \mathbf{3i-2})\,.
\end{array}\right\} \cdots\cdots (2)$$

Applying the principle of the method of least squares, we make $M = \min.$, instead of $\sum v \cdot v = \min.$,
where

$$\begin{aligned}
M = \sum v \cdot v - 2K_0\left(\sum_{i=1}^{n} v_{3i} - \omega_0\right) & \\
-2\sum_{i=1}^{n} K_{(i)}\,(v_{(i)}+v_{3i-2}+v_{3i-4}-\omega_{(i)}) & \\
-2\sum_{i=1}^{n} K_i\,(v_{3i-2}+v_{3i-1}+v_{3i}-\omega_i) & \\
-2K_s\left\{\sum_{i=1}^{n} (\Delta_{3i-2} \cdot v_{3i-2} - \Delta_{3i-1} \cdot v_{3i-1}) - \omega_s\right\} & .
\end{aligned}$$

From $M = \min.$, viz., $\partial M/\partial v = 0$, the following relations are obtained;

$$\left.\begin{aligned} v_{(i)} &= K_{(i)} \\ v_{3i-4} &= K_{(i)}+K_{i-1}-\Delta_{3i-4}\cdot K_s \\ v_{3i-2} &= K_{(i)}+K_i+\Delta_{3i-2}\cdot K_s \\ v_{3i-1} &= K_{(i+1)}+K_i-\Delta_{3i-1}\cdot K_s \\ v_{3i} &= K_0+K_i \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots (3)$$

Putting relations (3) into (1), one obtains correlate equations as follows;

$$\left.\begin{aligned} nK_0 + \sum_{i=1}^{n} K_i &= \omega_0 \\ 3K_{(i)}+K_{i-1}+K_i + d_{(i)}\cdot K_s &= \omega_{(i)} \quad (i=1,\,2,\,\cdots,\,n) \\ K_0+K_{(i)}+K_{(i+1)}+3\,K_i+d_i\cdot K_s &= \omega_i \quad (i=1,\,2,\,\cdots,\,n) \\ \sum_{i=1}^{n} d_{(i)}\cdot K_{(i)}+\sum_{i=1}^{n} d_i\cdot K_i+d_s\cdot K_s &= \omega_s \end{aligned}\right\}\cdot (4)$$

where

$$\left.\begin{aligned} d_{(i)} &= \Delta_{3i-2}-\Delta_{3i-4} \\ d_i &= \Delta_{3i-2}-\Delta_{3i-1} \\ d_s &= \sum \Delta\cdot\Delta \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (5)$$

If observed angles around corner stations are separately corrected, or simply, outer angles are not observed, point equations at each corner station are omitted in equation (1) and eqnation (3) or (4) becomes as follows;

$$\left.\begin{aligned} v_{3i-4} &= K_{i-1}-\Delta_{3i-4}\cdot K_s \\ v_{3i-2} &= K_i \;\; +\Delta_{3i-2}\cdot K_s \\ v_{3i-1} &= K_i \;\; -\Delta_{3i-1}\cdot K_s \\ v_{3i} &= K_0+K_i \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots\cdots (6)$$

and

$$\left.\begin{aligned} nK_0 + \sum_{i=1}^{n} K_i &= \omega_0 \\ K_0 + 3K_i+d_i\cdot K_s &= \omega_i \quad (i=1,\,2,\,\cdots,\,n) \\ \sum_{i=1}^{n} d_i\cdot K_i+d_s\cdot K_s &= \omega_s\,. \end{aligned}\right\} \cdots\cdots\cdots (7)$$

Solving equation (7), correlates and corrections of angles are obtained as follows;

$$\left.\begin{aligned} K_0 &= 3/2n \cdot (\omega_0 - \omega/3) + d/2n \cdot K_s \\ K_i &= 1/3 \cdot (\omega_i - \omega/2n) - \omega_0/2n - 1/3 \cdot (d_i + d/2n) \cdot K_s \\ K_s &= \mathfrak{N}/\mathfrak{D}\ , \end{aligned}\right\} \cdots\cdots (8)$$

where

$$\left.\begin{aligned} \mathfrak{N} &= \omega_s - 1/3 \cdot \left(\sum_{i=1}^{n} d_i \cdot \omega_i + d \cdot \omega/2n \right) + 1/2n \cdot d\omega_0 \\ \mathfrak{D} &= d_s - 1/3 \cdot \left(\sum_{i=1}^{n} d_i^2 + d^2/2n \right) \\ \omega &= \sum_{i=1}^{n} \omega_i \qquad\qquad d = \sum_{i=1}^{n} d_i\ . \end{aligned}\right\} \cdots\cdots (9)$$

And

$$\left.\begin{aligned} v_{3i-2} &= 1/3 \cdot (\omega_i + \omega/2n) - \omega_0/2n + \left\{ \Delta_{3i-2} - 1/3 \cdot (d_i + d/2n) \right\} \cdot K_s \\ v_{3i-1} &= 1/3 \cdot (\omega_i + \omega/2n) - \omega_0/2n - \left\{ \Delta_{3i-1} + 1/3 \cdot (d_i + d/2n) \right\} \cdot K_s \\ v_{3i} &= 1/3 \cdot (\omega_i - \omega/n) + \omega_0/n - 1/3 \cdot (d_i - d/n) \cdot K_s \end{aligned}\right\} \cdot\ (10)$$

Relations (6) and (7) coincide with those obtained in references (10) and (11).

§ 3. UNCLOSED CENTRAL POLYGON.

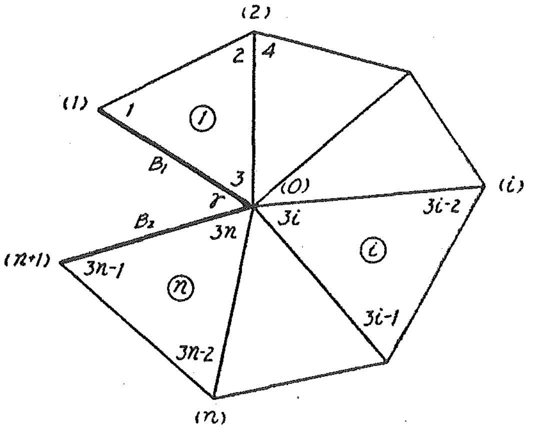

Fig. 2.

In an Unclosed Central Polygon as in Fig. 2, all inner and outer angles are assumed to be observed and length of sides at both ends B_1 and B_2 also to be known, then the number of condition equations is as follows;

	$N_p = n+1-(n+1-1) = 1$	at the central station (0),
and	$2-(2-1) = 1$	at the corner stations (1) and $(n+1)$,
	$3-(3-1) = 1$	at each corner station except (1) and $(n+1)$,
	$N_a = 2n+1-(n+2-1) = n$	
	$N_s = 1.$	

The total number of condition equations is $2n+3$, and they are as follows:

$$\left.\begin{array}{l}
\text{A point equation at the central station (0);} \\
\qquad \sum_{i=1}^{n} v_{3i} + v_{\gamma} = \omega_0 \,. \\
n+1 \text{ point equations at each corner station;} \\
\qquad v_{(1)} + v_1 = \omega_{(1)} \\
\qquad v_{(i)} + v_{3i-2} + v_{3i-4} = \omega_{(i)} \qquad (i = 2, 3, \cdots, n) \\
\qquad v_{(n+1)} + v_{3n-1} = \omega_{(n+1)} \,. \\
n \text{ angle equations at each individual triangle;} \\
\qquad v_{3i-2} + v_{3i-1} + v_{3i} = \omega_i \qquad (i = 1, 2, \cdots, n) \,. \\
\text{A side equation of the figure;} \\
\qquad \sum_{i=1}^{n} (\varDelta_{3i-2} \cdot v_{3i-2} - \varDelta_{3i-1} \cdot v_{3i-1}) = \omega_s \,.
\end{array}\right\} \cdots\cdots (11)$$

where

$$\left.\begin{array}{ll}
\omega_0 & = 360^\circ - \left(\sum_{i=1}^{n} \mathbf{3i} + \boldsymbol{\gamma} \right) \\
\omega_{(1)} & = 360^\circ - (\mathbf{(1)} + \mathbf{1}) \\
\omega_{(i)} & = 360^\circ - (\mathbf{(i)} + \mathbf{3i-2} + \mathbf{3i-4}) \qquad (i = 2, 3, \cdots, n) \\
\omega_{(n+1)} & = 360^\circ - (\mathbf{(n+1)} + \mathbf{3n-1}) \\
\omega_i & = 180^\circ - (\mathbf{3i-2} + \mathbf{3i-1} + \mathbf{3i}) \qquad (i = 1, 2, \cdots, n) \\
\omega_s & = \sum_{i=1}^{n} (\log.\sin.\mathbf{3i-1} - \log.\sin.\mathbf{3i-2}) + \log.B_2 - \log.B_1 \,.
\end{array}\right\} (12)$$

Applying the principle of the method of least squares, we make $M=\text{min.}$, instead of $\sum v\cdot v=\text{min.}$,

where

$$\begin{aligned} M=\sum v\cdot v-2K_0\left(\sum_{i=1}^{n} v_{3i}+v_{\Upsilon}-\omega_0\right) & \\ -2K_{(1)}\left(v_{(1)}+v_1-\omega_{(1)}\right) & \\ -2\sum_{i=2}^{n} K_{(i)}\left(v_{(i)}+v_{3i-2}+v_{3i-4}-\omega_{(i)}\right) & \\ -2K_{(n+1)}\left(v_{(n+1)}+v_{3n-1}-\omega_{(n+1)}\right) & \\ -2\sum_{i=1}^{n} K_i\left(v_{3i-2}+v_{3i-1}+v_{3i}-\omega_i\right) & \\ -2K_s\left\{\sum_{i=1}^{n}\left(\varDelta_{3i-2}\cdot v_{3i-2}-\varDelta_{3i-1}\cdot v_{3i-1}\right)-\omega_s\right\} & . \end{aligned}$$

From $M=\text{min.}$, viz., $\partial M/\partial v=0$, we obtain

$$\left.\begin{aligned} & v_{\Upsilon}=K_0 \\ & v_{(i)}=K_{(i)} \qquad (i=1,2,\cdots,n+1) \\ & \left.\begin{aligned} & v_{3i-4}=K_{(i)}+K_{i-1}-\varDelta_{3i-4}\cdot K_s \\ & v_{3i-2}=K_{(i)}+K_i+\varDelta_{3i-2}\cdot K_s \\ & v_{3i-1}=K_{(i+1)}+K_i-\varDelta_{3i-1}\cdot K_s \\ & v_{3i}=K_0+K_i\,. \end{aligned}\right\}(i=1,2,\cdots,n) \end{aligned}\right\}\cdots(13)$$

Putting relation (13) into (11), correlate equations are obtained as follows:

$$\left.\begin{aligned} & (n+1)K_0+\sum_{i=1}^{n}K_i=\omega_0 \\ & 2K_{(1)}+K_1+d_{(1)}\cdot K_s=\omega_{(1)} \\ & 3K_{(i)}+K_{i-1}+K_i+d_{(i)}\cdot K_s=\omega_{(i)}\quad (i=2,3,\cdots,n) \\ & 2K_{(n+1)}+K_n+d_{(n+1)}\cdot K_s=\omega_{(n+1)} \\ & K_0+K_{(i)}+K_{(i+1)}+3K_i+d_i\cdot K_s=\omega_i\quad (i=2,3,\cdots,n) \\ & \sum_{i=1}^{n+1}d_{(i)}\cdot K_{(i)}+\sum_{i=1}^{n}d_i\cdot K_i+d_s\cdot K_s=\omega_s \end{aligned}\right\}(14)$$

where

$$\left.\begin{aligned} d_{(i)} &= \Delta_1 \\ d_{(i)} &= \Delta_{3i-2}-\Delta_{3i-4} \\ d_{(n+1)} &= -\Delta_{3n-1} \\ d_i &= \Delta_{3i-2}-\Delta_{3i-1} \\ d_s &= \sum \Delta\cdot\Delta\ . \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots\cdots\cdots \quad (15)$$

If observed angles around corner stations are independently corrected, or simply, outer angles are assumed to be not observed, point equations at each corner station are omitted in equation (11) and equation (13) or (14) becomes as follow;

$$\left.\begin{aligned} v_{3i-4} &= K_{i-1}-\Delta_{3i-4}\cdot K_s \\ v_{3i-2} &= K_i \quad +\Delta_{3i-2}\cdot K_s \\ v_{3i-1} &= K_i \quad -\Delta_{3i-1}\cdot K_s \\ v_{3i} &= K_0 \quad +K_i \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots\cdots\cdots \quad (16)$$

and

$$\left.\begin{aligned} (n+1)\,K_0 + \sum_{i=1}^{n} K_i \qquad &= \omega_0 \\ K_0 + 3K_i+d_i\cdot K_s &= \omega_i \qquad (i=1,\ 2,\ \cdots,\ n) \\ \sum_{i=1}^{n} d_i\cdot K_i+d_s\cdot K_s &= \omega_s \end{aligned}\right\} \cdot \ (17)$$

Solving equation (17), correlates and corrections of angles are obtained as follows;

$$\left.\begin{aligned} K_0 &= 3/(2n+3)\ \cdot(\omega_0-\omega/3)\ +d/(2n+3)\cdot K_s \\ K_i &= 1/3\cdot\left\{\omega_i+\omega/(2n+3)\right\}\ -\omega_0/(2n+3)\ -1/3\cdot\left\{d_i+d/(2n+3)\right\}\cdot K_s \\ K_s &= \mathfrak{N}/\mathfrak{D}\ , \end{aligned}\right\} \ (18)$$

where

$$\left.\begin{aligned} \mathfrak{N} &= \omega_s -1/3\cdot\left(\sum_{i=1}^{n} d_i\cdot\omega_i+d\cdot\omega/(2n+3)\right)\ +d\cdot\omega_0/(2n+3) \\ \mathfrak{D} &= d_s\ -1/3\cdot\left(\sum_{i=1}^{n} d_i^2+d^2/(2n+3)\right) \\ \omega &= \sum_{i=1}^{n}\omega_i \\ d &= \sum_{i=1}^{n} d_i\ . \end{aligned}\right\} \cdot \ (19)$$

And

$$\left.\begin{aligned} v_{3i-2} &= 1/3\cdot\left\{\omega_i+\omega/(2n+3)\right\}-\omega_0/(2n+3)+\left[\varDelta_{3i-2}-1/3\cdot\left\{d_i+d/(2n+3)\right\}\right]\cdot K_s \\ v_{3i-1} &= 1/3\cdot\left\{\omega_i+\omega/(2n+3)\right\}-\omega_0/(2n+3)-\left[\varDelta_{3i-1}+1/3\cdot\left\{d_i+d/(2n+3)\right\}\right]\cdot K_s \\ v_{3i} &= 1/3\cdot\left\{\omega_i-2\omega/(2n+3)\right\}+2\omega_0/(2n+3)-1/3\cdot\left\{d_i-2/(2n+3)\cdot d\right\}\cdot K_s \end{aligned}\right\} \quad (20)$$

§4. A SINGLE ROW TRIANGULATION NET.

In a Single Row Triangulation Net as in Fig. 3, all inner and outer angles are assumed to be observed and the length of sides at

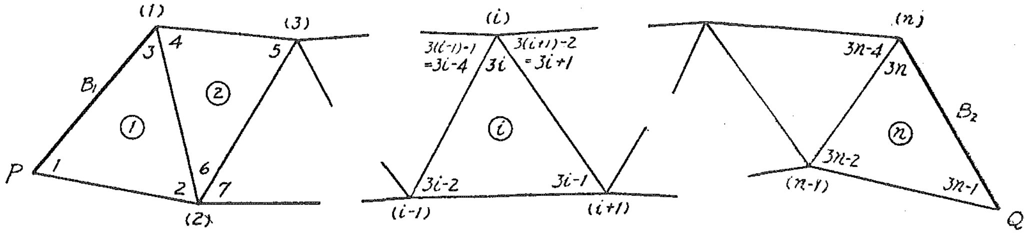

Fig. 3.

both ends B_1 and B_2 also to be known, then the number of condition equations is as follows;

$$\begin{aligned} N_p &= 2-(2-1)=1 && \text{at station } P \text{ or } Q, \\ \text{and} \quad & 3-(3-1)=1 && \text{at station (1) or } (n), \\ & 4-(4-1)=1 && \text{at each station except } P,\ Q,\ (1) \text{ or } (n), \\ N_a &= 2n+1-(n+2-1)=n \\ N_s &= 1. \end{aligned}$$

The total number of condition equations is $2n+3$, and they are as follows;

$n+2$ point equations at each station;

$$\begin{aligned} &v_p \quad +v_1 &&= \omega_p \\ &v_{(1)} \quad +v_3+v_4 &&= \omega_{(1)} \\ &v_{(i)} \quad +v_{3i-4}+v_{3i}+v_{3i+1} &&= \omega_{(i)} \qquad (i=2,\ 3,\ \cdots\cdots,\ n-1) \end{aligned}$$

$$\left.\begin{array}{l} v_{(n)} + v_{3n-4} + v_{3n} = \omega_{(n)} \\ v_q + v_{3n-1} = \omega_q . \\ n \text{ angle equations at each individual triangle;} \\ \quad v_{3i-2} + v_{3i-1} + v_{3i} = \omega_i \qquad (i = 1, 2, \cdots, n) . \\ \text{A side equation of the figure;} \\ \quad \sum_{i=1}^{n} (\varDelta_{3i-2} \cdot v_{3i-2} - \varDelta_{3i-1} \cdot v_{3i-1}) = \omega_s . \end{array}\right\} . \quad (21)$$

where

$$\left.\begin{array}{ll} \omega_p & = 360^\circ - (\boldsymbol{P} + \mathbf{1}) \\ \omega_{(1)} & = 360^\circ - ((\mathbf{1}) + \mathbf{3} + \mathbf{4}) \\ \omega_{(i)} & = 360^\circ - ((\boldsymbol{i}) + \mathbf{3}\boldsymbol{i}-\mathbf{4} + \mathbf{3}\boldsymbol{i} + \mathbf{3}\boldsymbol{i}+\mathbf{1}) \quad (i = 2, 3, \cdots, n-1) \\ \omega_{(n)} & = 360^\circ - ((\boldsymbol{n}) + \mathbf{3}\boldsymbol{n}-\mathbf{4} + \mathbf{3}\boldsymbol{n}) \\ \omega_q & = 360^\circ - (\mathbf{3}\boldsymbol{n}-\mathbf{1} + \boldsymbol{Q}) \\ \omega_i & = 180^\circ - (\mathbf{3}\boldsymbol{i}-\mathbf{2} + \mathbf{3}\boldsymbol{i}-\mathbf{1} + \mathbf{3}\boldsymbol{i}) \qquad (i = 1, 2, \cdots, n) \\ \omega_s & = \sum_{i=1}^{n} (\log. \sin. \mathbf{3}\boldsymbol{i}-\mathbf{1} - \log. \sin. \mathbf{3}\boldsymbol{i}-\mathbf{2}) + \log. B_2 - \log. B_1 . \end{array}\right\} \quad (22)$$

Applying the principle of the method of least squares, we make $M = \min.$, instead of $\sum v \cdot v = \min.$,

where

$$\begin{aligned} M = \sum v \cdot v & - 2K_p (v_p + v_1 - \omega_p) \\ & - 2K_{(1)} (v_{(1)} + v_3 + v_4 - \omega_{(1)}) \\ & - 2 \sum_{i=2}^{n-1} K_{(i)} (v_{3i-4} + v_{3i} + v_{3i+1} - \omega_{(i)}) \\ & - 2K_{(n)} (v_{(n)} + v_{3n-4} + v_{3n} - \omega_{(n)}) \\ & - 2K_q (v_q + v_{3n-1} - \omega_q) \\ & - 2 \sum_{i=1}^{n} K_i (v_{3i-2} + v_{3i-1} + v_{3i} - \omega_i) \\ & - 2K_s \left\{ \sum_{i=1}^{n} (\varDelta_{3i-2} \cdot v_{3i-2} - \varDelta_{3i-1} \cdot v_{3i-1}) - \omega_s \right\} . \end{aligned}$$

From $M = \min.$, viz., $\partial M / \partial v = 0$, we obtain

$$\left.\begin{aligned}
v_p &= K_p \\
v_{(i)} &= K_{(i)} && (i=1,\,2,\,\cdots,\,n) \\
v_q &= K_q \\
v_1 &= K_p \quad + K_1 + \varDelta_1 \cdot K_s \\
v_2 &= K_{(2)} \quad + K_1 - \varDelta_2 \cdot K_s \\
v_4 &= K_{(1)} \quad + K_1 \\
v_{3i-2} &= K_{(i-1)} + K_i + \varDelta_{3i-2} \cdot K_s && (i=2,\,3,\,\cdots,\,n-1) \\
v_{3i-1} &= K_{(i+1)} + K_i - \varDelta_{3i-1} \cdot K_s && (i=2,\,3,\,\cdots,\,n-1) \\
v_{3i} &= K_{(i)} \quad + K_i && (i=2,\,3,\,\cdots,\,n-1) \\
v_{3n-2} &= K_{(n-1)} + K_n + \varDelta_{3n-2} \cdot K_s \\
v_{3n-1} &= K_q \quad + K_n - \varDelta_{3n-1} \cdot K_s \\
v_{3n} &= K_{(n)} \quad + K_n \,.
\end{aligned}\right\} \cdot (23)$$

Putting these relations (23) into (21), correlate equations are obtained as follows;

$$\left.\begin{aligned}
&2K_p \quad + K_1 \qquad\qquad + \quad \varDelta_1 \cdot K_s = \omega_p \\
&3K_{(1)} \quad + K_1 \quad + K_2 \qquad + \quad \varDelta_4 \cdot K_s = \omega_{(1)} \\
&4K_{(i)} \quad + K_{i-1} + K_i + K_{i+1} + \quad d_{(i)} \cdot K_s = \omega_{(i)} \quad (i=2,3,\,\cdots,\,n-1) \\
&3K_{(n)} \quad + K_{n-1} + K_n \qquad - \varDelta_{3n-4} \cdot K_s = \omega_{(n)} \\
&2K_q \quad + K_n \qquad\qquad - \varDelta_{3n-1} \cdot K_s = \omega_q \\
&\ K_{(i-1)} + K_{(i)} \ + K_{(i+1)} + 3K_i + d_i \cdot K_s = \omega_i \quad (i=1,2,\,\cdots,\,n) \\
&\varDelta_1 \cdot K_p + \varDelta_4 \cdot K_{(1)} + \sum_{i=2}^{n-1} d_{(i)} \cdot K_{(i)} - \varDelta_{3n-4} \cdot K_{(n)} \\
&\qquad - \varDelta_{3n-1} \cdot K_q + \sum_{i=1}^{n} d_i \cdot K_i + d_s \cdot K_s = \omega_s \,,
\end{aligned}\right\} \quad (24)$$

where

$$\left.\begin{aligned}
d_{(i)} &= \varDelta_{3i-2} - \varDelta_{3i-4} \\
d_i &= \varDelta_{3i-2} - \varDelta_{3i-1} \\
d_s &= \sum \varDelta \cdot \varDelta \,.
\end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (25)$$

If observed angles around each station are independently corrected, or simply, outer angles are not observed, point equations at each station are omitted in equation (21) and equation (23) or (24) becomes as follows ;

$$\left.\begin{aligned} v_{3i-2} &= K_i + \Delta_{3i-2} \cdot K_s \\ v_{3i-1} &= K_i - \Delta_{3i-1} \cdot K_s \\ v_{3i} &= K_i \qquad (i = 1, 2, \cdots, n), \end{aligned}\right\} \cdots\cdots\cdots\cdots (26)$$

and

$$\left.\begin{aligned} 3K_i + d_i \cdot K_s &= \omega_i \qquad (i = 1, 2, \cdots, n) \\ \sum_{i=1}^{n} d_i \cdot K_i + d_s \cdot K_s &= \omega_s . \end{aligned}\right\} \cdots\cdots (27)$$

Solving equation (27), correlates and corrections of angles are obtained as follows ;

$$\left.\begin{aligned} K_i &= 1/3 \cdot (\omega_i - d_i \cdot K_s) \\ K_s &= \mathfrak{N}/\mathfrak{D} , \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots\cdots (28)$$

where

$$\left.\begin{aligned} \mathfrak{N} &= \omega_s - 1/3 \cdot \sum_{i=1}^{n} d_i \cdot \omega_i \\ \mathfrak{D} &= d_s - 1/3 \cdot \sum_{i=1}^{n} d_1^2 . \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots\cdots (29)$$

$$\left.\begin{aligned} v_{3i-2} &= 1/3 \cdot \omega_i + (\Delta_{3i-2} - 1/3 \cdot d_i) \cdot K_s \\ v_{3i-1} &= 1/3 \cdot \omega_i - (\Delta_{3i-1} + 1/3 \cdot d_i) \cdot K_s \\ v_{3i} &= 1/3 \cdot \omega_i - 1/3 \cdot d_i \cdot K_s . \end{aligned}\right\} \cdots\cdots\cdots\cdots (30)$$

§5. A QUADRILATERAL.

In a quadrilateral as in Fig. 4, all inner and outer angles are assumed to be observed, then the number of condition equations are as follows ;

$N_p = 3-(3-1) = 1$ at each station

$N_a = 6-(4-1) = 3$

$N_s = 6-(8-3) = 1.$

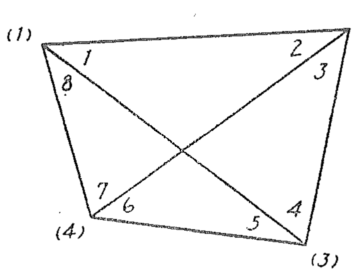

Fig. 4.

The total number of condition equations is eight and they are as follows:

$$\left.\begin{aligned}
&\text{4 point equations at each station;}\\
&\qquad v_{(1)}+v_1+v_8=\omega_{(1)}\\
&\qquad v_{(2)}+v_2+v_3=\omega_{(2)}\\
&\qquad v_{(3)}+v_4+v_5=\omega_{(3)}\\
&\qquad v_{(4)}+v_6+v_7=\omega_{(4)}\,.\\
&\text{3 angle equations;}\\
&\qquad v_1+v_2+v_3+v_4=\omega_1\\
&\qquad v_3+v_4+v_5+v_6=\omega_2\\
&\qquad v_5+v_6+v_7+v_8=\omega_3\,.\\
&\text{A side equation;}\\
&\qquad \sum_{i=1}^{8}(-1)^{i+1}\Delta_i\cdot v_i=\omega_s\,.
\end{aligned}\right\}\cdots\cdots\cdots\cdots (31)$$

where

$$\left.\begin{aligned}
\omega_{(1)}&=360^\circ-(\,(\mathbf{1})+\mathbf{1}+\mathbf{8}\,)\\
\omega_{(2)}&=360^\circ-(\,(\mathbf{2})+\mathbf{2}+\mathbf{3}\,)\\
\omega_{(3)}&=360^\circ-(\,(\mathbf{3})+\mathbf{4}+\mathbf{5}\,)\\
\omega_{(4)}&=360^\circ-(\,(\mathbf{4})+\mathbf{6}+\mathbf{7}\,)\\
\omega_1&=180^\circ-(\,\mathbf{1}+\mathbf{2}+\mathbf{3}+\mathbf{4}\,)\\
\omega_2&=180^\circ-(\,\mathbf{3}+\mathbf{4}+\mathbf{5}+\mathbf{6}\,)\\
\omega_3&=180^\circ-(\,\mathbf{5}+\mathbf{6}+\mathbf{7}+\mathbf{8}\,)\\
\omega_s&=\sum_{i=1}^{8}(-1)^i\log\cdot\sin\boldsymbol{i}
\end{aligned}\right\}\cdots\cdots\cdots\cdots (32)$$

Applying the method of least squares, we make $M=\text{min.}$, instead of $\sum v\cdot v=\text{min.}$,

where,

$$\begin{aligned}
M=\sum v\cdot v&-2K_{(1)}(v_{(1)}+v_1+v_8-\omega_{(1)})\\
&-2K_{(2)}(v_{(2)}+v_2+v_3-\omega_{(2)})
\end{aligned}$$

$$
\begin{aligned}
&-2K_{(3)}\left(v_{(3)}+v_4+v_5-\omega_{(3)}\right)\\
&-2K_{(4)}\left(v_{(4)}+v_6+v_7-\omega_{(4)}\right)\\
&-2K_1\left(v_1+v_2+v_3+v_4-\omega_1\right)\\
&-2K_2\left(v_3+v_4+v_5+v_6-\omega_2\right)\\
&-2K_3\left(v_5+v_6+v_7+v_8-\omega_3\right)\\
&-2K_s\left(\sum_{i=1}^{8}(-1)^{i+1}v_i\cdot v_i-\omega_s\right).
\end{aligned}
$$

From $M=\min.$, viz., $\partial M/\partial v=0$, one obtains

$$
\left.\begin{aligned}
v_{(i)} &= K_{(i)} && (i=1,\ 2,\ 3 \text{ and } 4)\\
v_1 &= K_{(1)}+K_1 + \varDelta_1\cdot K_s\\
v_2 &= K_{(2)}+K_1 - \varDelta_2\cdot K_s\\
v_3 &= K_{(2)}+K_1+K_2+\varDelta_3\cdot K_s\\
v_4 &= K_{(3)}+K_1+K_2-\varDelta_4\cdot K_s\\
v_5 &= K_{(3)}+K_2+K_3+\varDelta_5\cdot K_s\\
v_6 &= K_{(4)}+K_2+K_3-\varDelta_6\cdot K_s\\
v_7 &= K_{(4)}+K_3+\varDelta_7\cdot K_s\\
v_8 &= K_{(1)}+K_3-\varDelta_8\cdot K_s
\end{aligned}\right\}\cdots(33)
$$

Putting the relation (33) into (31), correlate equations are obtained as follows;

$$
\left.\begin{aligned}
&3K_{(1)}+K_1+K_3+d_{(1)}\cdot K_s=\omega_{(1)}\\
&3K_{(2)}+2K_1+K_2+d_{(2)}\cdot K_s=\omega_{(2)}\\
&3K_{(3)}+K_1+2K_2+K_3+d_{(3)}\cdot K_s=\omega_{(3)}\\
&3K_{(4)}+K_2+2K_3+d_{(4)}\cdot K_s=\omega_{(4)}\\
&K_{(1)}+2K_{(2)}+K_{(3)}+4K_1+2K_2+d_1\cdot K_s=\omega_1\\
&K_{(2)}+2K_{(3)}+K_{(4)}+2K_1+4K_2+2K_3+d_2\cdot K_s=\omega_2\\
&K_{(1)}+K_{(3)}+2K_{(4)}+2K_2+4K_3+d_3\cdot K_s=\omega_3\\
&\sum_{i=1}^{4}d_{(i)}\cdot K_{(i)}+\sum_{i=1}^{3}d_i\cdot K_i+d_s\cdot K_s=\omega_s
\end{aligned}\right\}\cdots(34)
$$

where,

$$\left.\begin{array}{ll} d_{(1)}=\varDelta_1-\varDelta_8 & d_{(2)}=\varDelta_3-\varDelta_2 \\ d_{(3)}=\varDelta_5-\varDelta_4 & d_{(4)}=\varDelta_7-\varDelta_6 \\ d_1=\sum_{i=1}^{4}(-1)^{i+1}\varDelta_i & d_2=\sum_{i=3}^{6}(-1)^{i+1}\varDelta_i \\ d_3=\sum_{i=5}^{8}(-1)^{i+1}\varDelta_i & d_s=\sum \varDelta\cdot\varDelta\,. \end{array}\right\} \cdots\cdots\cdots (35)$$

If observed angles around each station are independently corrected, or simply outer angles are not observed, point equations at each station are omitted in equation (31), and equation (33) or (34) becomes as follows;

$$\left.\begin{array}{l} v_1=K_1\qquad\qquad\quad +\varDelta_1\cdot K_s \\ v_2=K_1\qquad\qquad\quad -\varDelta_2\cdot K_s \\ v_3=K_1+K_2\qquad +\varDelta_3\cdot K_s \\ v_4=K_1+K_2\qquad -\varDelta_4\cdot K_s \\ v_5=\qquad K_2+K_3+\varDelta_5\cdot K_s \\ v_6=\qquad K_2+K_3-\varDelta_6\cdot K_s \\ v_7=\qquad\qquad K_3+\varDelta_7\cdot K_s \\ v_8=\qquad\qquad K_3-\varDelta_8\cdot K_s \end{array}\right\} \cdots\cdots\cdots\cdots\cdots (36)$$

$$\left.\begin{array}{l} 2K_1+K_2\qquad\quad +d_1\cdot K_s=\omega_1 \\ K_1+2K_2+K_3+d_2\cdot K_s=\omega_2 \\ \qquad K_2+2K_3+d_3\cdot K_s=\omega_3 \\ \sum_{i=1}^{3}d_i\cdot K_i\qquad +d_s\cdot K_s=\omega_s \end{array}\right\} \cdots\cdots\cdots\cdots\cdots (37)$$

Solving equation (37), values of correlates and corrections of angles are obtained as follows:

$$\begin{array}{l} K_1=1/4\cdot(3\omega_1-2\omega_2+\omega_3)-1/4\cdot(3d_1-2d_2+d_3)\cdot K_s \\ K_2=-1/2\cdot(\omega_1-2\omega_2+\omega_3)+1/2\cdot(d_1-2d_2+d_3)\cdot K_s \\ K_3=1/4\cdot(\omega_1-2\omega_2+3\omega_3)-1/4\cdot(d_1-2d_2+3d_3)\cdot K_s \\ K_s=\mathfrak{N}/\mathfrak{D} \end{array}$$

$$\left.\begin{aligned}\mathfrak{N} &= \omega_s - 1/4 \cdot \Big\{d_1(3\omega_1 - 2\omega_2 + \omega_3) - 2d_2(\omega_1 - 2\omega_2 + \omega_3)\\ &\qquad\qquad + d_3(\omega_1 - 2\omega_2 + 3\omega_3)\Big\}\\ \mathfrak{D} &= d_s - 1/4 \cdot \Big\{d_1(3d_1 - 2d_2 + d_3) - 2d_2(d_1 - 2d_2 + d_3)\\ &\qquad\qquad + d_3(d_1 - 2d_2 + 3d_3)\Big\}\end{aligned}\right\} \cdot (38)$$

$$\left.\begin{aligned}v_1 &= 1/4 \cdot (3\omega_1 - 2\omega_2 + \omega_3) - 1/4 \cdot \Big\{(3d_1 - 2d_2 + d_3) - 4\Delta_1\Big\} \cdot K_s\\ v_2 &= 1/4 \cdot (3\omega_1 - 2\omega_2 + \omega_3) - 1/4 \cdot \Big\{(3d_1 - 2d_2 + d_3) + 4\Delta_2\Big\} \cdot K_s\\ v_3 &= 1/4 \cdot (\omega_1 + 2\omega_2 - \omega_3) - 1/4 \cdot \Big\{(d_1 + 2d_2 - d_3) - 4\Delta_3\Big\} \cdot K_s\\ v_4 &= 1/4 \cdot (\omega_1 + 2\omega_2 - \omega_3) - 1/4 \cdot \Big\{(d_1 + 2d_2 - d_3) + 4\Delta_4\Big\} \cdot K_s\\ v_5 &= 1/4 \cdot (-\omega_1 + 2\omega_2 + \omega_3) - 1/4 \cdot \Big\{(-d_1 + 2d_2 + d_3) - 4\Delta_5\Big\} \cdot K_s\\ v_6 &= 1/4 \cdot (-\omega_1 + 2\omega_2 + \omega_3) - 1/4 \cdot \Big\{(-d_1 + 2d_2 + d_3) + 4\Delta_6\Big\} \cdot K_s\\ v_7 &= 1/4 \cdot (\omega_1 - 2\omega_2 + 3\omega_3) - 1/4 \cdot \Big\{(d_1 - 2d_2 + 3d_3) - 4\Delta_7\Big\} \cdot K_s\\ v_8 &= 1/4 \cdot (\omega_1 - 2\omega_2 + 3\omega_3) - 1/4 \cdot \Big\{(d_1 - 2d_2 + 3d_3) + 4\Delta_8\Big\} \cdot K_s\end{aligned}\right\} (39)$$

where,

$$\left.\begin{aligned}\omega_1 &= 1/2 \cdot \Big\{180^\circ - (\mathbf{1} + \mathbf{2} + \mathbf{3} + \mathbf{4})\Big\}\\ \omega_2 &= 1/2 \cdot \Big\{180^\circ - (\mathbf{3} + \mathbf{4} + \mathbf{5} + \mathbf{6})\Big\}\\ \omega_3 &= 1/2 \cdot \Big\{180^\circ - (\mathbf{5} + \mathbf{6} + \mathbf{7} + \mathbf{8})\Big\}\\ \omega_s &= 1/2 \cdot \sum_{i=1}^{8} (-1)^i \log \cdot \sin . \boldsymbol{i}\\ d_1 &= 1/2 \cdot \sum_{i=1}^{4} (-1)^{i+1} \Delta_i\\ d_2 &= 1/2 \cdot \sum_{i=3}^{6} (-1)^{i+1} \Delta_i\\ d_3 &= 1/2 \cdot \sum_{i=5}^{8} (-1)^{i+1} \Delta_i\\ d_s &= 1/2 \cdot \sum_{i=1}^{8} \Delta_i^2\end{aligned}\right\} \cdots\cdots\cdots (40)$$

CHAPTER 2.

MECHANICAL SKETCH METHOD.

§1. GENERAL DESCRIPTION.

By calculating equations expressed in the preceeding chapter, substituting the values d and ω, values of all correlates and corrections of angles can be gotten. But these calculations are very complex and one can not get rid of various mistakes. As the mistakes are only found out at the final course of the calculation, the time required in calculation and wasted in finding out these mistakes can not be estimated. Therefore, in ordinary practise, especially in civil engineering works, the process may be divided into two :

First ; point equations and angle equations are solved independently and observed angles are corrected.

Second ; with these values, a side equation is solved and then, for the second time, already corrected angles are recorrected.

In the present new computation, many simultaneous equations in chapter 1 are solved at the same time quite regularly on the figure sketched and if there occur any mistakes in any course of calculation quite smoothly, naturally and unconsciously. This is the most advantageous characteristic of our "Mechanical Sketch Method."

In the following paragraphs, are explained the principles and procedures for central polygons, single row triangulation net and a quadrilateral with some numerical examples.

§2. CLOSED CENTRAL POLYGON.

Equations (4) and (7) in §2 of chapter 1 can be expressed in the following Table 1 and 2.

In these tables, the letters or figures in the column of Sign of Equation denote stations, number of triangles where the condition or correlate equations are composed, or kinds of equations. A bracket means the number of station or a point equation, a figure without bracket is the number of an individual triangle or an angle equation, and S means a side equation of the polygon. Figures in the column of Left Side of Equation are coefficients of corresponding correlates above expressed. Letters in the column of Right Side of Equation are absolute terms corresponding to the rows of Sign of Equation.

Table 1.

Sign of equation	Left side of equation																Right side of equation
	K_0	$K_{(1)}$	$K_{(2)}$	→	→	→	$K_{(n-1)}$	$K_{(n)}$	K_1	K_2	→	→	→	K_{n-1}	K_n	K_s	
(0)	n								1	1	→	→	→	1	1		ω_0
(1)		3							1						1	$d_{(1)}$	$\omega_{(1)}$
(2)			3						1	1						$d_{(2)}$	$\omega_{(2)}$
$(n-1)$							3						1	1		$d_{(n-1)}$	$\omega_{(n-1)}$
n								3						1	1	$d_{(n)}$	$\omega_{(n)}$
1	1	1	1						3							d_1	ω_1
2	1		1	1						3						d_2	ω_2
$n-1$	1						1	1						3		d_{n-1}	ω_{n-1}
n	1	1						1							3	d_n	ω_n
S		$d_{(1)}$	$d_{(2)}$	→	→	→	$d_{(n-1)}$	$d_{(n)}$	d_1	d_2	→	→	→	d_{n-1}	d_n	d_s	ω_s

Table 2.

Sign of equation	Left side of equation									Right side of equation
	K_0	K_1	K_2	→	→	→	K_{n-1}	K_n	K_s	
(0)	n	1	1	→	→	→	1	1		ω_0
1	1	3							d_1	ω_1
2	1		3						d_2	ω_2
$n-1$	1						3		d_{n-1}	ω_{n-1}
n	1							3	d_n	ω_n
S		d_1	d_2	→	→	→	d_{n-1}	d_n	d_s	ω_s

Glancing at these tables, it is interesting that the large values of coefficients are arranged in diagonal, from upper left to lower right, and other coefficients are situated regularly and systematically to this diagonal axis. This is the "Key" to this new method of calculation. It may be considered that the correlate which has a large coefficient plays a chief part in that equation, so it is a short cut to obtain the value of the correlate from that equation. The process based upon this principle will be explained by the method of iteration.

Correlates of each kind are expressed in the following forms:
The point correlate $K_{(i)}$;

$$K_{(i)} = 1/3 \cdot (\omega_{(i)} - K_{i-1} - K_i - d_{(i)} \cdot K_s) \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (41)$$

In this formula, angle correlates K_{i-1} and K_i belong to triangles which meet at the station (i), and the denominator is the number of angles around station (i), that is 3.
The angle correlate K_i;

$$K_i = 1/3 \cdot (\omega_i - K_{(i)} - K_{(i+1)} - K_0 - d_i \cdot K_s) \quad \cdots\cdots\cdots\cdots\cdots \quad (42)$$

In this formula, point correlates $K_{(i)}$, $K_{(i+1)}$ and K_0 are three stations of triangle where this angle equation is composed, and the denominator is the number of angles in a triangle, always 3.
The point correlate K_0;

$$K_0 = 1/n \cdot \left(\omega_0 - \sum_{i=1}^{n} K_i\right) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (43)$$

The angle correlates with relation to K_0 are around the centre station (0), viz., all angle correlates, and the denominator is the number of angles around the central station (0), that is n.
The side correlate K_s;

$$K_s = 1/d_s \cdot \left(\omega_s - \sum_{i=1}^{n} d_{(i)} \cdot K_{(i)} - \sum_{i=1}^{n} d_i \cdot K_i\right) \quad \cdots\cdots\cdots\cdots\cdots \quad (44)$$

This has a relation to all point and angle correlates except K_0.

After these relations are understood, tables are not needed and the figure of this polygon becomes convenient (Fig. 5). This will be called a "**Correlate Sketch.**"

First; the figure is sketched which is now going to be adjusted.

Second; double circles are drawn at every station and in each individual triangle. In the inner circles, the number or sign of the

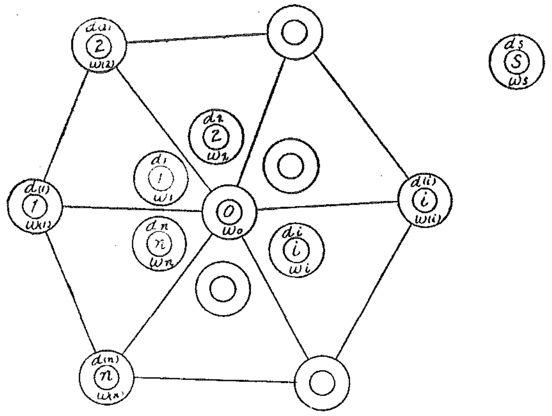

Fig. 5.

corresponding stations and triangles are written, and in the upper and lower spaces between the inner and outer circles d and ω respectively are written. Blank spaces around these double circles serve for the entry of values successively obtained.

Double circles of (0) and S can be conveniently drawn outside of the figure, in order to avoid confusion.

With these, the preparation is over and the calculation is begun.

1. Let first value of K_s be calculated thus; first values of point and angle correlates are multiplied by the upper figures d written in each circle (they are taken up in neighbourhood) and these products are added, with the opposite sign, to the lower figure ω_s in circle S, and this value is divided by the upper figure d_s in circle S. The result is written near circle S.

This process shows that equation (44) is calculated.

2. Let second values of point correlates $K_{(i)}$ be calculated thus; first values of angle correlates in triangles around point (i) and the product of the upper figure in circle (i) and the first value of K_s are added, with the opposite sign, to the lower figure in circle (i), and this value is divided by the number of angles around point (i), namely 3. This result is written under the first value.

This process means that equation (41) is solved.

3. Let second values of angle correlates K_i be calculated thus; second values of point correlates $K_{(i)}$ at three points of this triangle i whose angle correlate is now going to be calculated and the product of the upper figure in circle i and the first value of K_s are added,

with the opposite sign, to the lower figure in circle i and this value is divided by the number of observed angles in the triangle, namely 3. These values are written under the first values.

This process means that equation (42) is solved.

4. Let the second value of the point correlate K_0 be calculated with the second values of angle correlates K_i above obtained and written in each triangle; the second values of angle correlates K_i are added, with the opposite sign, to the lower figure in circle (0) and divided by the number of angles around the station (0), namely n.

In this case, there enters no term of K_s, as there is no figure at the upper portion in circle (0). This value is written under the first value of it.

This calculation means that equation (43) is solved.

5. With the second values of point and angle correlates above obtained and by the same process as in 1, the second value of the side correlate K_s is obtained, and is written under the first value.

These processes 2, 3, 4 and 5 are the first round of calculation. After the first round is over, the second and the third rounds are carried out. After several rounds, the values thus obtained converge to the correct values.

In these calculations, for the addition of ω and the values of correlates and the final simple division, a Japanese "Soroban" is very conveniently used, and for the multiplication of d and K, a calculating machine or a slide rule is convenient, but one is skilled on the Soroban, multiplication and division are also easily carried out with it.

However circuitous this process may appear, it is not really so, but of direct judgment and more convenient. Numerical values which are required for any correlate are always situated near it in the figure and are never separated from it, so mistakes can not occur. And if any mistake occurs in any round, it is immediately and unconsciously corrected in the course of the next round, and moreover, calculation can be stopped at the required place of the results.

And the calculation can be carried entirely on the sketch mechanically, leaving from the consideration of solving many simultaneous equations.

The more complex the figure, the more regularly, systematically and rapidly the calculation can be carried on. These are the most significant and advantageous characteristics of this new method.

The **first values** of K_0, $K_{(i)}$ and K_i are good enough with any values, but, to accelerate the convergence, it is convenient to use the following formulae:

The point correlate K_0;

$$\mathbf{K}_0 = 5/2n \cdot \omega_0 + 1/n \cdot \sum_{i=1}^{n} (\omega_{(i)} - \omega_i) \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (45)$$

where n is the number of triangles which compose the polygon. The first values of other correlates $K_{(i)}$ and K_i are expressed as follows;

$$n = 3,$$

$$\left.\begin{aligned} K_{(i)} &= 0.58\,\omega_{(i)} + 0.33\,\omega_0 + 0.21(\omega_{(i-1)} + \omega_{(i+1)}) \\ &\quad - 0.38(\omega_{i-1} + \omega_i) - 0.25\,\omega_{i+1} \\ K_i &= 0.75\,\omega_i + 0.38(\omega_{i-1} + \omega_{i+1}) - 0.5\,\omega_0 \\ &\quad - 0.38(\omega_{(i)} + \omega_{(i+1)}) - 0.25\,\omega_{(i-1)} \end{aligned}\right\} \cdots\cdots\cdots \quad (46)$$

$$n = 4,$$

$$\left.\begin{aligned} K_{(i)} &= 0.55\,\omega_{(i)} + 0.25\,\omega_0 + 0.17(\omega_{(i-1)} + \omega_{(i+1)}) + 0.12\,\omega_{(i-2)} \\ &\quad - 0.32(\omega_{i-1} + \omega_i) - 0.18(\omega_{i+1} + \omega_{i+2}) \\ K_i &= 0.67\,\omega_i + 0.29(\omega_{i-1} + \omega_{i+1}) + 0.24\,\omega_{i-2} - 0.38\,\omega_0 \\ &\quad - 0.32(\omega_{(i)} + \omega_{(i+1)}) - 0.18(\omega_{(i-1)} + \omega_{(i+2)}) \end{aligned}\right\} \cdots\cdots \quad (47)$$

$$n = 5,$$

$$\left.\begin{aligned} K_{(i)} &= 0.53\,\omega_{(i)} + 0.2\,\omega_0 + 0.15(\omega_{(i-1)} + \omega_{(i+1)}) \\ &\quad + 0.09(\omega_{(i-2)} + \omega_{(i+2)}) - 0.29(\omega_{i-1} + \omega_i) \\ &\quad - 0.15(\omega_{i+1} + \omega_{i-2}) - 0.13\,\omega_{i+2} \\ K_i &= 0.63\,\omega_i + 0.25(\omega_{i-1} + \omega_{i+1}) + 0.19(\omega_{i-2} + \omega_{i+2}) \\ &\quad - 0.3\,\omega_0 - 0.29(\omega_{(i)} + \omega_{(i+1)}) + 0.15(\omega_{(i-1)} + \omega_{(i+2)}) \\ &\quad - 0.13\,\omega_{(i-2)} \end{aligned}\right\} \cdots\cdots \quad (48)$$

$n=6$,

$$\left.\begin{aligned}
K_{(i)} &= 0.51\,\omega_{(i)}+0.17\,\omega_0+0.13\,(\omega_{(i-1)}+\omega_{(i+1)})\\
&\quad+0.08(\omega_{(i-2)}+\omega_{(i+2)})+0.07\,\omega_{(i-3)}-0.27(\omega_{i-1}+\omega_i)\\
&\quad-0.13(\omega_{i-2}+\omega_{i+1})-0.10(\omega_{i-3}+\omega_{i+2})\\
K_i &= 0.60\,\omega_i+0.22(\omega_{i-1}+\omega_{i+1})+0.16(\omega_{i-2}+\omega_{i+2})\\
&\quad+0.15\,\omega_{i-3}-0.25\,\omega_0-0.27(\omega_{(i)}+\omega_{(i+1)})\\
&\quad-0.13(\omega_{(i-1)}+\omega_{(i+2)})-0.10(\omega_{(i-2)}+\omega_{(i+3)})
\end{aligned}\right\}\cdots\cdots(49)$$

$n=7$,

$$\left.\begin{aligned}
K_{(i)} &= 0.50\,\omega_{(i)}+0.14\,\omega_0+0.12(\omega_{(i-1)}+\omega_{(i+1)})\\
&\quad+0.07(\omega_{(i-2)}+\omega_{(i+2)})+0.06(\omega_{(i-3)}+\omega_{(i+3)})\\
&\quad-0.26(\omega_{i-1}+\omega_i)-0.11(\omega_{i-2}+\omega_{i+1})\\
&\quad-0.09(\omega_{i-3}+\omega_{i+2}+\omega_{i+3})\\
K_i &= 0.58\,\omega_i+0.19(\omega_{i-1}+\omega_{i+1})+0.14(\omega_{i-2}+\omega_{i+2})\\
&\quad+0.13\,(\omega_{i-3}+\omega_{i+3})-0.21\,\omega_0-0.26(\omega_{(i+1)}+\omega_{(i)})\\
&\quad-0.11(\omega_{(i+2)}+\omega_{(i-1)})-0.09(\omega_{(i-3)}+\omega_{(i-2)}+\omega_{(i+3)})\,.
\end{aligned}\right\}\cdots\cdots(50)$$

These formulae are expressed conveniently in **Fig. 6.** In **Fig. 6,** numerical values denote coefficients of ω in that position and a circle denotes the coefficient whose correlate it is desired to obtain. These values are obtained by taking terms of ω only and neglecting the

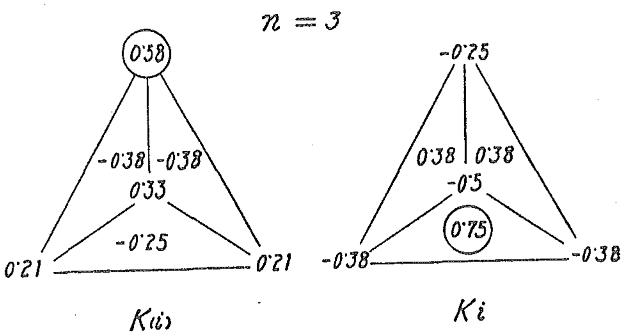

Fig. 6.

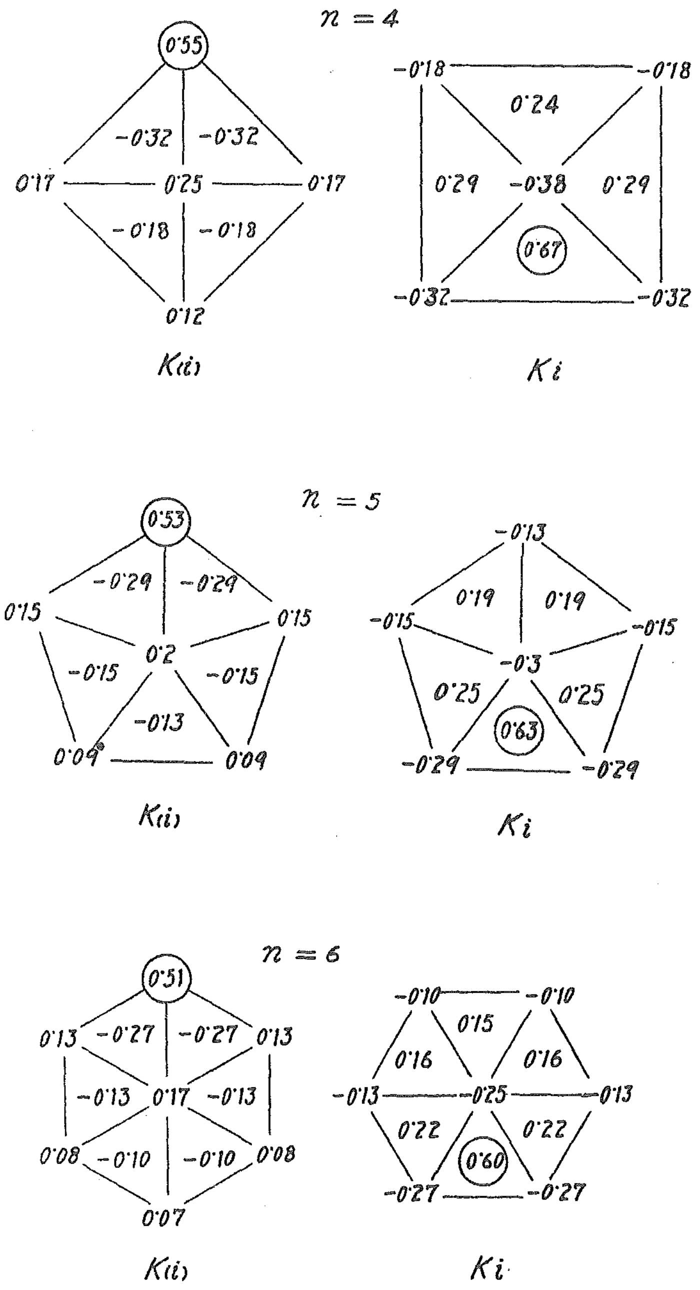
n = 4
0·55
-0·32
-0·32
0·17
0·25
0·17
-0·18
-0·18
0·12
K(i)
-0·18
-0·18
0·24
0·29
-0·38
0·29
0·67
-0·32
-0·32
Ki
n = 5
0·53
-0·29
-0·29
0·15
0·15
0·2
-0·15
-0·15
-0·13
0·09
0·09
K(i)
-0·13
0·19
0·19
-0·15
-0·15
-0·3
0·25
0·25
0·63
-0·29
-0·29
Ki
n = 6
0·51
0·13
-0·27
-0·27
0·13
-0·13
0·17
-0·13
0·08
-0·10
-0·10
0·08
0·07
K(i)
-0·10
-0·10
0·15
0·16
0·16
-0·13
-0·25
-0·13
0·22
0·22
0·60
-0·27
-0·27
Ki

Fig. 6.

$n = 7$

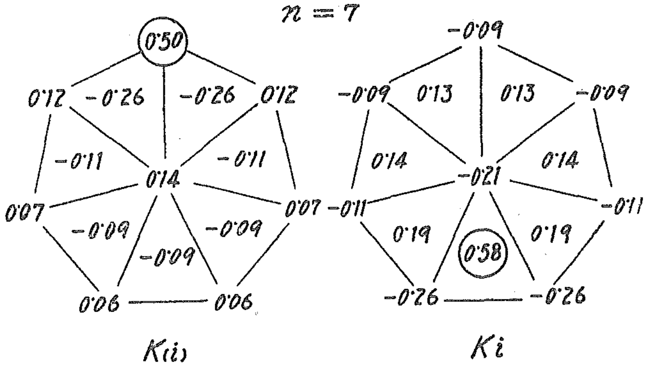

Fig. 6.

term of K_s in equations (72) to (78) in §1 in Chapter 3. The calculation of these first values is sufficiently accurate up to two places with a slide rule.

In the case of Table 2, the method of calculation is the same as above described, but terms $K_{(i)}$, $d_{(i)}$ and $\omega_{(i)}$ do not exist, so the process can be continued more simply, that is, process 2 above mentioned is omitted. Then the formulae are limited to (42), (43) and (44), and the **first values** of the correlates are able to be obtained from equations (8) and (9), by omitting the term K_s.

$$\left.\begin{aligned} K_0 &= 3/2n\cdot(\omega_0-\omega/3) \\ K_i &= 1/3\cdot(\omega_i+\omega/2n)-\omega_0/2n \\ \text{where}\quad \omega &= \sum_{i=1}^{n}\omega_i\,. \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (51)$$

After the values of all correlates are obtained, the values of corrections of the observed angles are gotton from equation (3) or (6) in §2 in Chapter 1.

Numerical Example 1. A Closed Central Hexagon.

All inner and outer angles are observed; the problem is to adjust them. Figures obtained by calculation are shown in Table 3. Numbers in the above are orders of calculation.

(1) ; number of angles,

(2) ; observed values of angles and sum of three angles in each individual triangle,

(3) ; values of log.sin. of observed angles $3i-2$ and $3i-1$, and values of ω_i, ω_0, ω_s, and $\omega_{(i)}$,

(4) : tabular difference of log.sin. per 1″,

(5) ; d_i and $d_{(i)}$, for example,

$$d_1 = \varDelta_1 - \varDelta_2 = 9.05 - 19.43 = -10.38$$
$$d_{(1)} = \varDelta_1 - \varDelta_{18} = 9.05 - 1.38 = 7.67 \text{ \& etc.,}$$

(6) ; $\varDelta \cdot \varDelta$ for angles $3i-2$ and $3i-1$,

(7) ; **Correlate Sketch,** this is the main point of this paper, values of all correlates are obtained (this process will be explained later),

(8) ; $\varDelta \cdot K_s$, K_s obtained from (7),

(9) ; corrections v from K_0, $K_{(i)}$ and K_i in (7) and $\varDelta \cdot K_s$ in (8),

(12) ; corrected values of angles by (9),

(13) ; corrected log. sin. for calculation of side length of triangulation nets, with $\varDelta \cdot v$ in (10),

(11) ; $v \cdot v$ for the calculation of probable error.

Now **Correlate Sketch** in (7) will be explained. (**Fig. 7**). Figures in the double circles are d and ω which are shown in (5) in Table 3, for example, $d_1 = -10.38$, $\omega_1 = -5.3$ in circle 1,
$d_{(1)} = 7.67$, $\omega_{(1)} = 6.3$ in circle (1).

The first values of correlates are obtained from (49) or **Fig. 6** ($n = 6$) and (**45**) as follows:

The point correlate K_0 from (**45**);

$$\begin{aligned} K_0 &= 0.42 \times 6.6 + 0.17\,(6.3 - 7.2 - 5.1 + 3.1 + 7.4 - 4.8) \\ &\qquad - 0.25\,(-5.3 - 7.9 - 9.3 + 9.5 + 3.1 - 9.9) \\ &= 7.7, \end{aligned}$$

Point correlates $K_{(i)}$ from **Fig. 6** ;

$$\begin{aligned} K_{(1)} = {} & 0.17 \times 6.6 + 0.51 \times 6.3 + 0.13(-7.2 - 4.8) + 0.08(-5.1 + 7.4) \\ & + 0.07 \times 3.1 - 0.27(-5.3 - 9.9) - 0.13(-7.9 + 3.1) \\ & - 0.10(-9.3 + 9.5) = 7.9, \end{aligned}$$

similarly, $K_{(2)} = 2.8$, $K_{(3)} = 3.5$, $K_{(4)} = 4.6$, $K_{(5)} = 4.7$, $K_{(6)} = 2.9$.

Angle correlates K_i from **Fig. 6** ;

$$\begin{aligned} K_1 = {} & -0.25 \times 6.6 - 0.27(6.3 - 7.2) - 0.13(-5.1 - 4.8) \\ & - 0.10(7.4 + 3.1) + 0.60 \times -5.3 + 0.22(-7.9 - 9.9) \\ & + 0.16(-9.3 + 3.1) + 0.15 \times 9.5 = -7.8, \end{aligned}$$

similarly, $K_2 = -7.4$, $K_3 = -8.4$, $K_4 = -2.4$, $K_5 = -4.0$,
$K_6 = -9.5$.

These first values are written in the spaces near their double circles.

The side correlate K_s.

First values of point and angle correlates above obtained and written on the sketch are multiplied by the upper figures in each circle, they are situated in neighbourhood, and these products are added. Then the sum is added, with the opposite sign, to the lower figure in circle S ($\omega_s = -326$) and divided by the upper figure in circle S ($d_s = 2{,}336.68$), thus the first value of the side correlate K_s is obtained. That is,

$$K_s = \underbrace{1/2{,}336.68}_{\substack{\text{upper}\\ \text{figure}\\ \text{in circle } S}} \cdot \Big\{ \underbrace{-\,326}_{\substack{\text{lower}\\ \text{figure}\\ \text{in circle } S}} - \Big(\underbrace{\underbrace{7.67}_{\substack{\text{upper}\\ \text{figure}\\ \text{in circle (1)}}} \times \underbrace{7.9}_{\substack{\text{1st. value}\\ \text{of } K_{(1)}}}}_{\text{at point (1)}} - \underbrace{\underbrace{2.35}_{\substack{\text{upper}\\ \text{figure}\\ \text{in circle (2)}}} \times \underbrace{2.8}_{\substack{\text{1st. value}\\ \text{of } K_{(2)}}}}_{\text{at point (2)}} - \underbrace{\cdots\cdots}_{\cdots\cdots}$$

$$- \underbrace{\underbrace{10.38}_{\substack{\text{upper}\\ \text{figure}\\ \text{in circle 1}}} \times \underbrace{-7.8}_{\substack{\text{1st. value}\\ \text{of } K_1}}}_{\text{in triangle 1}} + \underbrace{\underbrace{4.15}_{\substack{\text{upper}\\ \text{figure}\\ \text{in circle 2}}} \times \underbrace{-7.4}_{\substack{\text{1st. value}\\ \text{of } K_2}}}_{\text{in triangle 2}} + \underbrace{\cdots\cdots}_{\cdots\cdots} \Big) \Big\}$$

$$= -0.11\,.$$

This value is written under circle S.

This process means that equation (44) has been calculated.

Second values of correlates are obtained as follows:

Point correlates $K_{(i)}$.

First values of angle correlates in the triangles around point (i) and the product of the upper figure in circle (i) and the first value of K_s are added, with the opposite sign, to the lower figure in circle (i) and then this value is divided by the number of angles around point (i), namely 3.

$$K_{(1)} = \underbrace{1/3}_{\substack{\text{no. of}\\ \text{angles}\\ \text{around}\\ \text{point}\\ (1)}} \cdot \big(\underbrace{6.3}_{\substack{\text{lower}\\ \text{figure}\\ \text{in circle (1)}}} + \underbrace{\underbrace{9.5 + 7.8}_{\substack{\text{1st. values in}\\ \text{triangles 6}\\ \text{and 1 around}\\ \text{point (1)}}} + \underbrace{7.67}_{\substack{\text{upper}\\ \text{figure}\\ \text{in circle (1)}}} \times \underbrace{0.11}_{\substack{\text{1st.}\\ \text{value}\\ \text{of } K_s}}}_{\text{added with the opposite sign}} \big) = 8.1\,.$$

This value 8.1 is written under the first value 7.9.

$$K_{(2)} = 1/3 \cdot (-7.2 + \underbrace{7.8 + 7.4} - 2.35 \times 0.11) = 2.6\,.$$

no. of angles around point (2)	lower figure in circle (2)	**1st.** values in triangles 1 and 2 around point (2)	upper figure in circle (2)	**1st.** value of K_s

(1st. values in triangles 1 and 2 around point (2), upper figure in circle (2) and 1st. value of K_s:) added with the opposite sign

This value 2.6 is written under the first value 2.8.

similarly $K_{(3)} = 3.4$, $K_{(4)} = 4.5$, $K_{(5)} = 4.3$, $K_{(6)} = 3.2$.

These calculations show that equation (41) has been solved.

The point correlate K_0.

Similarly to the case of point correlates $K_{(i)}$ above, but the only difference is such that the number of angles around the point (0) is 6 and there is no term K_s.

$$K_0 = 1/6 \cdot (6.6 + \underbrace{7.8 + 7.4 + 8.4 + 2.4 + 4.0 + 9.5}) = 7.6\,.$$

no. of angles around point (0)	lower figure in circle (0)	**1st.** values written in all triangles around the central point (0), added with the opposite sign

This value 7.6 is written under its first value 7.7. This process means that equation (43) has been solved.

Angle correlates K_i.

Second values of point correlates at three points in the triangle i, whose angle correlate is now going to be obtained, and the product of the upper figure in circle i and the first value of K_s are added, with the opposite sign, to the lower figure in circle i, and this value is divided by the number of observed angles in a triangle, namely 3.

$$K_1 = 1/3 \cdot (-5.3 - \underbrace{8.1 - 2.6 - 7.6} - 10.35 \times 0.11) = -8.3\,.$$

no. of angles in triangle 1	lower figure in circle 1	**2nd.** values written at three vertices (1), (2) and (0) in triangle 1	upper figure in circle 1	**1st.** value of K_s

(2nd. values written at three vertices (1), (2) and (0) in triangle 1, upper figure in circle 1 and 1st. value of K_s:) added with the opposite sign

This value -8.3 is written under the first value -7.8.

$$K_2 = \underset{\substack{\text{no. of}\\ \text{angles}\\ \text{in tri-}\\ \text{angle 2}}}{1/3}\cdot(\underset{\substack{\text{lower}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 2}}}{-7.9}\; \underbrace{\underset{\substack{\text{2nd. values written}\\ \text{at three vertices (2),}\\ \text{(3) and (0) in}\\ \text{triangle 2}}}{\underbrace{-2.6-3.4-7.6}} + \underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 1}}}{4.15} \times \underset{\substack{\text{1st.}\\ \text{value}\\ \text{of}\\ K_s}}{0.11}}_{\text{added with the opposite sign}}) = -7.0\,.$$

This value -7.0 is written under the first value -7.4.

Similarly $K_3 = -8.5$, $K_4 = -2.3$, $K_5 = -4.3$, $K_6 = -8.8$.

This process means that equation (42) is calculated.

The side correlate K_s is obtained similarly to the case of its first value, but second values of point correlates written at each point and angle correlates written in each triangle are used.

$$K_s = 1/\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle } S}}{2{,}336.68}\cdot\Big\{\underset{\substack{\text{lower}\\ \text{figure}\\ \text{in cir-}\\ \text{cle } S}}{-326} - (\underbrace{\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle (1)}}}{7.67} \times \underset{\substack{\text{2nd.}\\ \text{value}\\ \text{of}\\ K_{(1)}}}{8.1}}_{\text{at point (1)}} - \underbrace{\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle (2)}}}{2.35} \times \underset{\substack{\text{2nd.}\\ \text{value}\\ \text{of}\\ K_{(2)}}}{2.6}}_{\text{at point (2)}} + \underset{\cdots\cdots}{\cdots\cdots}$$

$$- \underbrace{\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 1}}}{10.38} \times \underset{\substack{\text{2nd.}\\ \text{value}\\ \text{of}\\ K_1}}{-8.3}}_{\text{in triangle 1}} + \underbrace{\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 2}}}{4.15} \times \underset{\substack{\text{2nd.}\\ \text{value}\\ \text{of}\\ K_2}}{-7.0}}_{\text{in triangle 2}} + \underset{\cdots\cdots}{\cdots\cdots})\Big\}$$

$$= -0.134\,.$$

This value -0.134 is written under the first value -0.11.

This process means that equation (44) is solved.

Thus the first round of calculation is over, and the next rounds are proceeded with. In this case, the calculation reaches its end at the fifth round and corrections of observed angles can be obtained to the second grade below zero from equation (3) in § 2 in Chapter 1. Time required in this calculation is only three hours. If ordinary rigorous process of solution is applied, the time required may be three or four times as great.

Table 3.

Angles											
(1)	(2)	(12)	(3)	(13)	(4)	(5)	(8)	(9)	(10)	(11)	(6)
No.	Observed Values	Corrected Values	Initial Log. sin.	Corrected Log. sin.	Tabu. Diff. per 1″ Δ	d	$\Delta\cdot K_s$	Correction v	$\Delta\cdot v$	$v\cdot v$	$\Delta\cdot\Delta$
	° ′ ″	° ′ ″									
1	66 44 31.7	66 44 30.23	9.9631912	9.9631899	9.05	−10.38	−1.233	−1.47	− 13.3	2.1609	81.9025
2	47 17 6.8	47 17 3.69	9.8661335	9.8661275	19.43		2.646	−3.11	− 60.4	9.6721	377.5249
3	65 58 26.8	65 58 26.08						−0.72		0.5184	
	180 00 5.3	180 00 00.00	$\omega_1 = -5.3$								
4	50 57 34.0	50 57 27.31	9.8902535	9.8902421	17.08	4.15	−2.326	−6.69	−114.3	44.7561	291.7264
5	58 26 16.4	58 26 14.52	9.9304769	9.9304745	12.93		1.761	−1.88	− 24.3	3.5344	167.1849
6	70 36 17.5	70 36 18.17						0.67		0.4489	
	180 00 7.9	180 00 00.00	$\omega_2 = -7.9$								
7	65 36 12.8	65 36 6.26	9.9593797	9 9593735	9.55	−6.37	−1.301	−6.54	− 62.5	42.7716	91.2025
8	52 55 19.0	52 55 17.16	9.9019021	9.9018992	15.92		2.168	−1.84	− 29.3	3 3856	253.4464
9	61 28 37.5	61 28 36.58						−0.92		0.8464	
	180 00 9.3	180 00 00.00	$\omega_3 = -9.3$								
10	57 17 50.7	57 17 51.09	9.9250471	9.9250476	13.52	0.65	−1.841	0.39	5.3	0.1521	182.7904
11	58 35 10.7	58 35 14.50	9.9311660	9.9311709	12 87		1.753	3.80	48.9	14.4400	165.6369
12	64 6 49.1	64 6 54.41						5.31		28.1661	
	179 59 50.5	180 00 00.00	$\omega_4 = 9.5$								
13	75 24 46.2	75 24 45.44	9.9857702	9.9857698	5.47	−7.83	−0.745	−0.76	− 4.2	0.5776	29 9209
14	57 42 32.7	57 42 33.30	9.9270348	9.9270356	13.30		1.811	0.60	8.0	0.3600	176.8900
15	46 52 38.0	46 52 41.26						3.26			
	179 59 56.9	180 00 00.00	$\omega_5 = 3.1$								
16	42 48 9.2	42 48 0.64	9.8321728	9.8321533	22.73	21.35	−3.096	−8.56	−194.6	73.2736	516.6529
17	86 14 56.2	86 14 55.86	9.9990686	9.9990685	1.38		0.188	−0.34	− 0.5	0.1156	1.9044
18	50 57 4.5	50 57 3.50						−1.00		1.0000	
	180 00 9.9	180 00 00.00	$\omega_6 = -9.9$								
			$\omega_0 = 6.6$						$\Sigma\,\Delta\cdot v =$		$d_s =$
			$\omega_s = -326$						−326.0		2,336.6831

Table 3.—(*Continued*)

Angles											
(1)	(2)	(12)	(3)	(13)	(4)	(5)	(8)	(9)	(10)	(11)	(6)
No.	Observed Values	Corrected Values	Initial Log. sin.	Corrected Log. sin.	Tabu. Diff. per 1″ Δ	d	$\Delta \cdot K_s$	Correction v	$\Delta \cdot v$	$v \cdot v$	$\Delta \cdot \Delta$
(1)	207° 00′ 25.8″	207° 00′ 33.91″	$\omega_{(1)} = 6.3$			7.67		8.11		65.7721	
(2)	261 45 26.4	261 45 29.00	$\omega_{(2)} = -7.2$			−2.35		2.60		6.7600	
(3)	235 57 35.9	235 57 39.22	$\omega_{(3)} = -5.1$			−3.38		3.32		11.0224	
(4)	249 46 47.2	249 46 51.75	$\omega_{(4)} = 3.1$			−2 40		4.55		20.7025	
(5)	225 59 55.7	226 00 0.06	$\omega_{(5)} = 7.4$			−7.40		4.36		19.0096	
(6)	259 29 22.9	259 29 26.06	$\omega_{(6)} = -4.8$			9.43		3.16		9.9856	
										$\Sigma v \cdot v =$ 370.0592	

Probable Error $= \pm 0.6745\sqrt{370.0592/14} = \pm 3.''46$

(7)

Correlate Sketch.

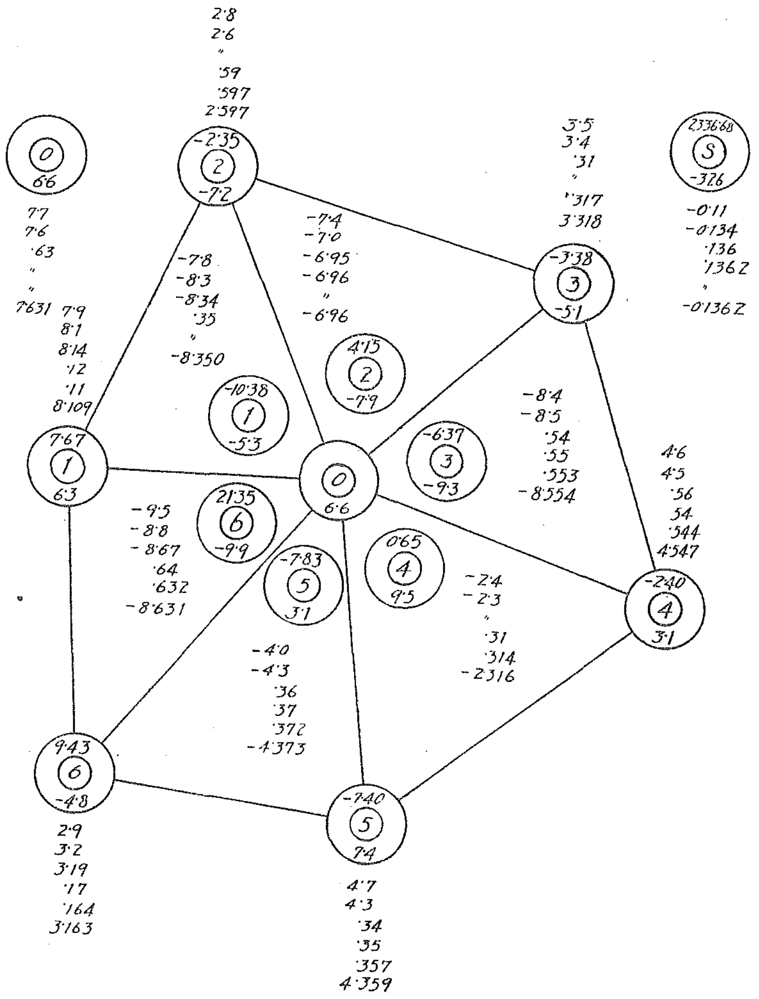

Fig. 7.

Numerical Example 2. A Closed Central Hexagon.

Only inner angles of the Hexagon in **Example 1** above are assumed to be observed; the problem is to adjust them. The following Table 4 and process are all similar to those in **Example 1**, and the process becomes simpler. Only the **Correlate Sketch** will be explained. (Fig. 8).

First values of correlates are obtained from (**51**) as follows:

$$\omega = -5.3-7.9-9.3+9.5+3.1-9.9 = -19.8,$$
$$1/3 \cdot \omega = -19.8/3 = -6.6, \quad 1/2n \cdot \omega = -19.8/12 = -1.7.$$

With these values, one obtains;

$$K_0 = 1/4 \cdot (6.6+6.6) = 3.3,$$
$$K_1 = 1/3 \cdot (-7.9-1.7) = 6.6/12 = -2.9,$$

similarly $K_2 = -3.7$, $K_3 = -4.2$, $K_4 = 2.1$, $K_5 = -0.1$, $K_6 = -4.4$, these values are written in the spaces near their circles.

The side correlate K_s.

First values of angle correlates above obtained and written in each triangle are multiplied by the upper figures in each circle and these products are added, with the opposite sign, to the lower figure -326 in circle S. This value is divided by the upper figure 2,336.68 in circle S.

$$K_s = 1/\underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle } S}}{2{,}336.68} \cdot \Big\{\underset{\substack{\text{lower}\\\text{figure}\\\text{in cir-}\\\text{cle } S}}{-326} - \Big(\underbrace{\underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle 1}}}{-10.38} \times \underset{\substack{\textbf{1st.}\\\text{value}\\\text{of}\\K_1}}{-2.9}}_{\text{in triangle 1}} + \underbrace{\underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle 2}}}{4.15} \times \underset{\substack{\textbf{1st.}\\\text{value}\\\text{of}\\K_2}}{-3.7}}_{\text{in triangle 2}} + \underbrace{\cdots\cdots}_{\cdots\cdots}\Big)\Big\}$$
$$= -0.12\,.$$

This value -0.12 is written under circle S.

This process shows that equation (44) has been solved.

Second values of correlates are obtained as follows:

Angle correlates K_i.

First value of the point correlate K_0 and the product of the upper figure in circle i and the first value of K_s are added, with the opposite sign, to the lower figure in circle i. This value is divided by the number of the observed angles in the triangle i, namely 3.

$$K_1 = 1/\underset{\substack{\text{no. of}\\\text{angles}\\\text{in tri-}\\\text{angle 1}}}{3} \cdot \Big(\underset{\substack{\text{lower}\\\text{figure}\\\text{in cir-}\\\text{cle 1}}}{-\,5.3} \underbrace{-\underset{\substack{\textbf{1st.}\\\text{value}\\\text{of}\\K_0}}{3.3} - \underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle 1}}}{10.38} \times \underset{\substack{\textbf{1st.}\\\text{value}\\\text{of}\\K_s}}{0.12}}_{\text{added with the opposite sign}}\Big) = -3.3\,,$$

This value -3.3 is written under the first value -2.9.

$$K_2 = \underset{\substack{\text{no. of}\\\text{angles}\\\text{in tri-}\\\text{angle 2}}}{1/3} \cdot (\underset{\substack{\text{lower}\\\text{figure}\\\text{in cir-}\\\text{cle 2}}}{-\,7.9} \underbrace{\underset{\substack{\textbf{1st.}\\\text{value}\\\text{of}\\K_0}}{-\,3.3} + \underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle 2}}}{4.15} \times \underset{\substack{\textbf{1st.}\\\text{value}\\\text{of}\\K_s}}{0.12}}_{\text{added with the opposite sign}}) = -3.6\,,$$

this value -3.6 is written under the first value -3.7.

Similarly $K_3 = -4.5$, $K_4 = 2.1$, $K_5 = -0.4$, $K_6 = -3.5$.

This process means that equation (42) is solved.

The point correlate K_0.

Second values of all angle correlates K_i around the central point (0) are added, with the opposite sign, to the lower figure in circle (0) and this value is divided by the number of angles around the central point (0), namely 6.

$$K_0 = \underset{\substack{\text{no. of}\\\text{angles}\\\text{around}\\\text{point}\\(0)}}{1/6} \cdot (\underset{\substack{\text{lower}\\\text{figure}\\\text{in cir-}\\\text{cle (0)}}}{6.6} + \underbrace{3.3 + 3.6 + 4.5 + 2.1 + 0.4 + 3.5}_{\substack{\textbf{2nd.}\text{ values of all angle correlates}\\\text{written in triangles around point}\\\text{(0), added with the opposite sign}}}) = 3.27\,,$$

this value 3.27 is written under the first value 3.3.

This process means that equation (43) is solved.

The side correlate K_s.

Second values of angle correlates above obtained and written in each triangle are multiplied by the upper figures in each circle and these products are added, with the opposite sign, to the lower figure -326 in circle S. This value is divided by the upper figure 2,336.68 in circle S.

$$K_s = 1/\underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle }S}}{2{,}336.68} \cdot \Big\{\underset{\substack{\text{lower}\\\text{figure}\\\text{in cir-}\\\text{cle }S}}{-\,326} - (\underbrace{\underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle 1}}}{-\,10.38} \times \underset{\substack{\textbf{2nd.}\\\text{value}\\\text{of}\\K_1}}{-3.3}}_{\text{in triangle 1}} + \underbrace{\underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle 2}}}{4.15} \times \underset{\substack{\textbf{2nd.}\\\text{value}\\\text{of}\\K_2}}{-3.6}}_{\text{in triangle 2}} + \underset{\cdots\cdots}{\cdots\cdot})\Big\}$$

$$= -0.130\,.$$

This value -0.130 is written under the first value -0.12 near circle S. This process means that equation (44) is calculated for the second time.

Table 4.

Angles											
(1)	(2)	(12)	(3)	(13)	(4)	(5)	(8)	(9)	(10)	(11)	(6)
No.	Observed Values	Corrected Values	Initial Log. sin.	Corrected Log. sin.	Tabu. Diff. per 1″ Δ	d	$\Delta \cdot K_s$	Correction v	$\Delta \cdot v$	$v \cdot v$	$\Delta \cdot \Delta$
	° ′ ″	° ′ ″									
1	66 44 31.7	66 44 27.21	9.9631912	9.9631871	9.05	−10.38	−1.180	−4.49	− 40.6	20.1601	81.9025
2	47 17 6.8	47 17 6.02	9.8661335	9.8661320	19.43		2.533	−0.78	− 15.2	0.6084	377.5249
3	65 58 26.8	65 58 26.77						−0.03		0.0009	
	180 00 5.3	180 00 00.00	$\omega_1 = -5.3$								
4	50 57 34.0	50 57 28.23	9.8902535	9.8902436	17.08	4.15	−2.227	−5.77	− 98.6	33.2929	291.7264
5	58 26 16.4	58 26 14.54	9.9304769	9.9304745	12.93		1.686	−1.86	− 24.0	3.4596	167.1849
6	70 36 17.5	70 36 17.23						−0.27		0.0729	
	180 00 7.9	180 00 00.00	$\omega_2 = -7.9$								
7	65 36 12.8	65 36 7.08	9.9593797	9.9593742	9.55	− 6.37	−1.245	−5.72	− 54.6	32.7184	91.2025
8	52 55 19.0	52 55 16.60	9.9019021	9.9018983	15.92		2.075	−2.40	− 38.2	5.7600	253.4464
9	61 28 37.5	61 28 36.32						−1.18		1.3924	
	180 00 9.3	180 00 00.00	$\omega_3 = -9.3$								
10	57 17 50.7	57 17 51.04	9.9250471	9.9250476	13.52	0.65	−1.763	0.34	4.6	0 1156	182.7904
11	58 35 10.7	58 35 14.48	9.9311660	9.9311709	12.87		1.678	3.78	48.6	14.2884	165.6369
12	64 6 49.1	64 6 54.48						5.38		28.9444	
	179 59 50.5	180 00 00.00	$\omega_4 = 9.5$								
13	75 24 46.2	75 24 45.09	9.9857702	9.9857696	5.47	− 7.83	−0.713	−1.11	− 6.1	1.2321	29.9209
14	57 42 32.7	57 42 34.03	9.9270348	9.9270366	13.30		1.734	1.33	17.7	1.7689	176.8900
15	46 52 38.0	46 52 40.88						2.88		8.2944	
	179 59 56.9	180 00 00.00	$\omega_5 = 3.1$								
16	42 48 9.2	42 48 2.77	9.8321728	9.8321582	22.73	21.35	−2.963	−6.43	−146.1	41.3449	516.6529
17	86 14 56.2	86 14 52.91	9.9990686	9.9990682	1.38		0.180	−3.29	− 4 5	10.8241	1.9044
18	50 57 4.5	50 57 4.32						−0.18		0.0324	
	180 00 9.9	180 00 00.00	$\omega_6 = -9.9$								
			$\omega_0 = 6.6$						$\Sigma \Delta \cdot v =$	$\Sigma v \cdot v =$	$d_s =$
			$\omega_s = -326$						−325.8	204.3108	2,336.6831

$$\text{Probable Error} = \pm 0.6745\sqrt{204.3108/8} = \pm 3.''41$$

Thus the first round of calculation is over, and the next rounds are carried on.

In this case, the calculation reaches its end at the fourth round and corrections of observed angles are obtained up to the second grade below zero from equation (6) in §2 in Chapter 1. The time required in this process of calculation is only one hour. If an ordinary exact process of calculation is employed, three or more hours may be wasted.

(7)

Correlate Sketch.

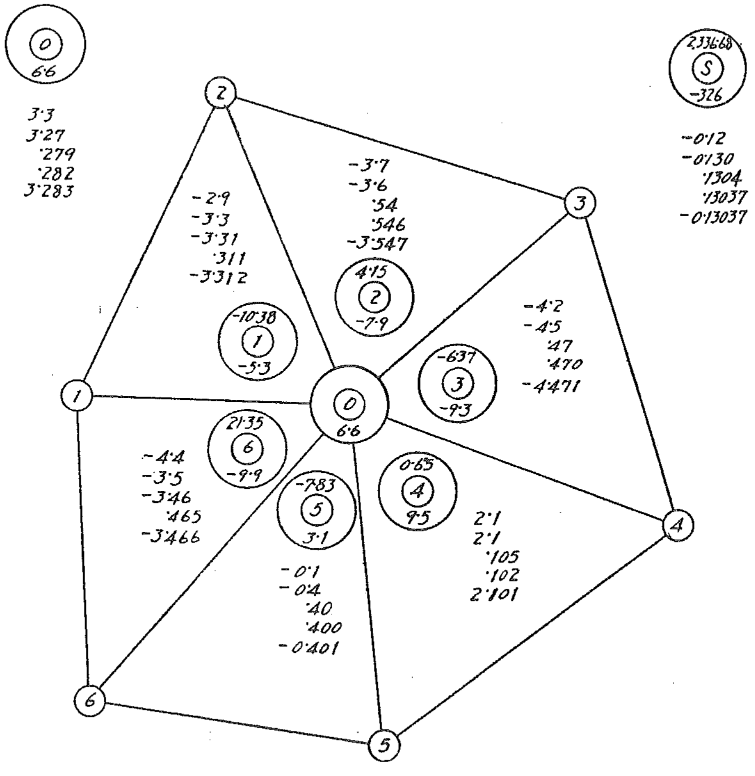

Fig. 8.

§3. UNCLOSED CENTRAL POLYGON.

Equations (14) and (17) in §3 of Chapter 1 can be expressed as in the following Tables 5 and 6 respectively.

Table 5.

Sign of equation	Left Side of equation															Right Side of equation
	K_0	$K_{(1)}$	$K_{(2)}$			→	$K_{(n)}$	$K_{(n+1)}$	K_1	K_2		→	K_{n-1}	K_n	K_s	
(0)	$n+1$								1	1			1	1		ω_0
(1)		2							1						$d_{(1)}$	$\omega_{(1)}$
(2)			3						1	1					$d_{(2)}$	$\omega_{(2)}$
(n)							3						1	1	$d_{(n)}$	$\omega_{(n)}$
(n+1)								2						1	$d_{(n+1)}$	$\omega_{(n+1)}$
1	1	1	1						3						d_1	ω_1
2	1		1	1						3					d_2	ω_2
$n-1$	1					1	1						3		d_{n-1}	ω_{n-1}
n	1						1	1						3	d_n	ω_n
S		$d_{(1)}$	$d_{(2)}$			→	$d_{(n)}$	$d_{(n+1)}$	d_1	d_2		→	d_{n-1}	d_n	d_s	ω_s

Table 6.

Sign of equation	Left Side of equation									Right Side of equation
	K_0	K_1	K_2			→	K_{n-1}	K_n	K_s	
(0)	$n+1$	1	1				1	1		ω_0
1	1	3							d_1	ω_1
2	1		3						d_2	ω_2
$n-1$	1						3		d_{n-1}	ω_{n-1}
n	1							3	d_n	ω_n
S		d_1	d_2			→	d_{n-1}	d_n	d_s	ω_s

These tables are the same in the case of the Closed Central Polygon in §2, the only difference being in values of coefficients of point correlates K_0, $K_{(1)}$ and $K_{(n+1)}$. But this is easily understood by sketching a figure, because angles around station 0 are $n+1$, and those around station (1) or $(n+1)$ are two respectively.

The **Correlate Sketch** is shown below (Fig. 9).

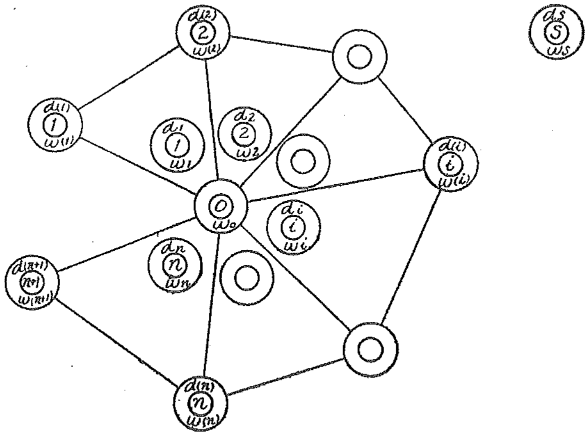

Fig. 9.

The formulae to obtain values of the correlates in this case are as follows:

Point correlates $K_{(i)}$;

$$\left.\begin{aligned} K_{(1)} &= 1/2 \cdot (\omega_{(1)} - K_1 - d_{(1)} \cdot K_s) \\ K_{(i)} &= 1/3 \cdot (\omega_{(i)} - K_{i-1} - K_i - d_{(i)} \cdot K_s) \\ &\qquad\qquad (i = 2, 3, \cdots, n) \\ K_{(n+1)} &= 1/2 \cdot (\omega_{(n+1)} - K_n - d_{(n+1)} \cdot K_s) . \end{aligned}\right\} \cdots (52)$$

Angle correlates K_i;

$$K_i = 1/3 \cdot (\omega_i - K_{(i)} - K_{(i+1)} - K_0 - d_i \cdot K_s) \qquad (i = 1, 2, \cdots, n) \quad \cdots\cdots (53)$$

The point correlate K_0;

$$K_0 = 1/(n+1) \cdot \left(\omega_0 - \sum_{i=1}^{n} K_i\right) \quad \cdots\cdots (54)$$

The side correlate K_s;

$$K_s = 1/d_s \cdot \left(\omega_s - \sum_{i=1}^{n+1} d_{(i)} \cdot K_{(i)} - \sum_{i=1}^{n} d_i \cdot K_i\right) \quad \cdots\cdots (55)$$

The process of calculation is also the same as described above in the preceeding paragraph; **first values** of correlates become a little different and are as follows:

$$\left.\begin{aligned}
&n=2, \\
&K_{(1)} = 0.66\,\omega_{(1)} + 0.14(\omega_{(2)}+\omega_0) + 0.06\,\omega_{(3)} - 0.30\,\omega_1 - 0.10\,\omega_2 \\
&K_{(2)} = 0.52\,\omega_{(2)} + 0.19\,\omega_0 + 0.14(\omega_{(1)}+\omega_{(3)}) - 0.29(\omega_1+\omega_2) \\
&K_{(3)} = 0.66\,\omega_{(3)} + 0.14(\omega_{(2)}+\omega_0) + 0.06\,\omega_{(1)} - 0.30\,\omega_2 - 0.10\,\omega_1 \\
&K_1 = 0.63\,\omega_1 + 0.23\,\omega_2 - 0.31\,\omega_{(1)} - 0.29(\omega_{(2)}+\omega_0) - 0.11\,\omega_{(3)} \\
&K_2 = 0.63\,\omega_2 + 0.23\,\omega_1 - 0.31\,\omega_{(3)} - 0.29(\omega_{(2)}+\omega_0) - 0.11\,\omega_{(1)}\,.
\end{aligned}\right\} \quad (56)$$

$$\left.\begin{aligned}
&n=3, \\
&K_{(1)} = 0.65\,\omega_{(1)} + 0.12\,\omega_0 + 0.14\,\omega_{(2)} + 0.06\,\omega_{(3)} + 0.04\,\omega_{(4)} \\
&\qquad -0.30\,\omega_1 - 0.10\,\omega_2 - 0.07\,\omega_3 \\
&K_{(2)} = 0.51\,\omega_{(2)} + 0.16\,\omega_0 + 0.14\,\omega_{(1)} + 0.13\,\omega_{(3)} + 0.03\,\omega_{(4)} \\
&\qquad -0.27\,(\omega_1+\omega_2) - 0.12\,\omega_3 \\
&K_{(3)} = 0.51\,\omega_{(3)} + 0.16\,\omega_0 + 0.14\,\omega_{(4)} + 0.13\,\omega_{(2)} + 0.06\,\omega_{(1)} \\
&\qquad -0.27\,(\omega_2+\omega_3) - 0.12\,\omega_1 \\
&K_{(4)} = 0.65\,\omega_{(4)} + 0.12\,\omega_0 + 0.14\,\omega_{(3)} + 0.06\,\omega_{(2)} + 0.04\,\omega_{(1)} \\
&\qquad -0.30\,\omega_3 - 0.10\,\omega_2 - 0.07\,\omega_1 \\
&K_1 = 0.60\,\omega_1 + 0.21\,\omega_2 + 0.14\,\omega_3 - 0.24\,\omega_0 - 0.30\,\omega_{(1)} \\
&\qquad -0.27\,\omega_{(2)} - 0.12\,\omega_{(3)} - 0.07\,\omega_{(4)} \\
&K_2 = 0.60\,\omega_2 + 0.21(\omega_1+\omega_3) - 0.25\,\omega_0 - 0.27(\omega_{(2)}+\omega_{(3)}) \\
&\qquad -0.10(\omega_{(1)}+\omega_{(4)}) \\
&K_3 = 0.60\,\omega_3 + 0.21\,\omega_2 + 0.14\,\omega_1 - 0.24\,\omega_0 - 0.30\,\omega_{(4)} \\
&\qquad -0.27\,\omega_{(3)} - 0.12\,\omega_2 - 0.07\,\omega_{(1)}\,.
\end{aligned}\right\} \quad (57)$$

$$
\left.
\begin{aligned}
&n = 4,\\
&K_{(1)} = 0.65\,\omega_{(1)} + 0.10\,\omega_0 + 0.13\,\omega_{(2)} + 0.05\,\omega_{(3)} + 0.03(\omega_{(4)}\\
&\qquad + \omega_{(5)}) - 0.29\,\omega_1 - 0.09\,\omega_2 - 0.07\,\omega_3 - 0.06\,\omega_4\\
&K_{(2)} = 0.51\,\omega_{(2)} + 0.14\,\omega_0 + 0.13\,\omega_{(1)} + 0.12\,\omega_{(3)} + 0.06\,\omega_{(4)}\\
&\qquad + 0.04\,\omega_{(5)} - 0.26(\omega_1 + \omega_2) - 0.11\,\omega_3 - 0.08\,\omega_4\\
&K_{(3)} = 0.51\,\omega_{(3)} + 0.15\,\omega_0 + 0.12(\omega_{(2)} + \omega_{(4)}) + 0.05(\omega_{(1)} + \omega_{(5)})\\
&\qquad - 0.26(\omega_2 + \omega_3) - 0.11(\omega_1 + \omega_4)\\
&K_{(4)} = 0.51\,\omega_{(4)} + 0.14\,\omega_0 + 0.13\,\omega_{(5)} + 0.12\,\omega_{(3)} + 0.06\,\omega_{(2)}\\
&\qquad + 0.04\,\omega_{(1)} - 0.26(\omega_3 + \omega_4) - 0.11\,\omega_2 - 0.08\,\omega_1\\
&K_{(5)} = 0.65\,\omega_{(5)} + 0.10\,\omega_0 + 0.13\,\omega_{(4)} + 0.05\,\omega_{(3)} + 0.03(\omega_{(2)}\\
&\qquad + \omega_{(1)}) - 0.29\,\omega_4 - 0.09\,\omega_3 - 0.07\,\omega_2 - 0.06\,\omega_1\\
&K_1 = 0.59\,\omega_1 + 0.19\,\omega_2 + 0.13\,\omega_3 + 0.11\,\omega_4 - 0.32\,\omega_0 - 0.29\,\omega_{(1)}\\
&\qquad - 0.26\,\omega_{(2)} - 0.11\,\omega_{(3)} - 0.08\,\omega_{(4)} - 0.06\,\omega_{(5)}\\
&K_2 = 0.57\,\omega_2 + 0.20\,\omega_3 + 0.19\,\omega_1 + 0.13\,\omega_4 - 0.22\,\omega_0 - 0.26(\omega_{(2)}\\
&\qquad + \omega_{(3)}) - 0.11\,\omega_{(4)} - 0.13\,\omega_{(5)} - 0.09\,\omega_{(1)}\\
&K_3 = 0.57\,\omega_3 + 0.20\,\omega_2 + 0.19\,\omega_4 + 0.13\,\omega_1 - 0.22\,\omega_0 - 0.26(\omega_{(3)}\\
&\qquad + \omega_{(4)}) - 0.11\,\omega_{(2)} - 0.13\,\omega_{(1)} - 0.09\,\omega_{(5)}\\
&K_4 = 0.59\,\omega_4 + 0.19\,\omega_3 + 0.13\,\omega_2 + 0.11\,\omega_1 - 0.32\,\omega_0 - 0.29\,\omega_{(5)}\\
&\qquad - 0.26\,\omega_{(4)} - 0.11\,\omega_{(3)} - 0.08\,\omega_{(2)} - 0.06\,\omega_{(1)}\,.
\end{aligned}
\right\} \quad (58)
$$

$$
\begin{aligned}
&n = 5,\\
&K_{(1)} = 0.64\,\omega_{(1)} + 0.09\,\omega_0 + 0.12\,\omega_{(2)} + 0.05\,\omega_{(3)} + 0.04\,\omega_{(4)} + 0.02\,\omega_{(5)}\\
&\qquad + 0.03\,\omega_{(6)} - 0.29\,\omega_1 - 0.09\,\omega_2 - 0.06\,\omega_3 - 0.05\,(\omega_4 + \omega_5)\\
&K_{(2)} = 0.50\,\omega_{(2)} + 0.12\,(\omega_0 + \omega_{(1)}) + 0.11\,\omega_{(3)} + 0.06\,\omega_{(4)} + 0.05\,\omega_{(5)}\\
&\qquad + 0.03\,\omega_{(6)} - 0.25\,(\omega_1 + \omega_2) - 0.10\,\omega_3 - 0.08\,\omega_4 - 0.07\,\omega_5\\
&K_{(3)} = 0.50\,\omega_{(3)} + 0.13\,\omega_0 + 0.12\,\omega_{(4)} + 0.11\,\omega_{(2)} + 0.06\,\omega_{(5)} + 0.05\,\omega_{(1)}\\
&\qquad + 0.04\,\omega_{(6)} - 0.25\,(\omega_2 + \omega_3) - 0.10\,(\omega_1 + \omega_4) - 0.07\,\omega_5
\end{aligned}
$$

$$\left.\begin{aligned}
K_{(4)} &= 0.50\,\omega_{(4)}+0.13\,\omega_0+0.12\,\omega_{(3)}+0.11\,\omega_{(5)}+0.06\,\omega_{(2)}+0.05\,\omega_{(6)}\\
&\quad +0.04\,\omega_{(1)}-0.25\,(\omega_3+\omega_4)-0.10\,(\omega_2+\omega_5)-0.07\,\omega_1\\
K_{(5)} &= 0.50\,\omega_{(5)}+0.12\,(\omega_0+\omega_{(6)})+0.11\,\omega_{(4)}+0.06\,\omega_{(3)}+0.05\,\omega_{(2)}\\
&\quad +0.03\,\omega_{(1)}-0.25\,(\omega_4+\omega_5)-0.10\,\omega_3-0.08\,\omega_2-0.07\,\omega_1\\
K_{(6)} &= 0.64\,\omega_{(6)}+0.09\,\omega_0+0.12\,\omega_{(5)}+0.05\,\omega_{(4)}+0.04\,\omega_{(3)}+0.02\,\omega_{(2)}\\
&\quad +0.03\,\omega_{(1)}-0.29\,\omega_5-0.09\,\omega_4-0.06\,\omega_3-0.05\,\omega_2-0.05\,\omega_1\\
K_1 &= 0.57\,\omega_1+0.17\,\omega_2+0.12\,\omega_3+0.11\,\omega_4+0.10\,\omega_5-0.18\,\omega_0\\
&\quad -0.29\,\omega_{(1)}-0.25\,\omega_{(2)}-0.10\,\omega_{(3)}-0.08\,\omega_{(4)}-0.07\,{}_{(5)}-0.05\,\omega_{(6)}\\
K_2 &= 0.56\,\omega_2+0.18\,\omega_3+0.17\,\omega_1+0.12\,\omega_4+0.11\,\omega_5-0.19\,\omega_0\\
&\quad -0.25\,(\omega_{(2)}+\omega_{(3)})-0.10\,\omega_{(4)}-0.09\,\omega_{(1)}-0.08\,\omega_{(5)}-0.05\,\omega_{(6)}\\
K_3 &= 0.56\,\omega_3+0.18\,(\omega_2+\omega_4)+0.12\,(\omega_1+\omega_5)-0.19\,\omega_0\\
&\quad -0.25\,(\omega_{(3)}+\omega_{(4)})-0.10\,(\omega_{(2)}+\omega_{(5)})-0.06\,(\omega_{(1)}+\omega_{(6)})\\
K_4 &= 0.56\,\omega_4+0.18\,\omega_3+0.17\,\omega_5+0.12\,\omega_2+0.11\,\omega_1-0.19\,\omega_0\\
&\quad -0.25\,(\omega_{(4)}+\omega_{(5)})-0.10\,\omega_{(3)}-0.09\,\omega_{(6)}-0.08\,\omega_{(2)}-0.05\,\omega_{(1)}\\
K_5 &= 0.57\,\omega_5+0.17\,\omega_4+0.12\,\omega_3+0.11\,\omega_2+0.10\,\omega_1-0.18\,\omega_0\\
&\quad -0.29\,\omega_{(6)}-0.25\,\omega_{(5)}-0.10\,\omega_{(4)}-0.08\,\omega_{(3)}-0.07\,\omega_{(2)}-0.05\,\omega_{(1)}\,.
\end{aligned}\right\}\quad(59)$$

$$\left.\begin{aligned}
&n = 6,\\
K_{(1)} &= 0.64\,\omega_{(1)}+0.08\,\omega_0+0.12\,\omega_{(2)}+0.04\,\omega_{(3)}+0.03\,(\omega_{(4)}+\omega_{(5)}+\omega_{(6)})\\
&\quad +0.02\,\omega_{(7)}-0.28\,\omega_1-0.08\,\omega_2-0.05\,(\omega_3+\omega_4+\omega_5)-0.04\,\omega_6\\
K_{(2)} &= 0.49\,\omega_{(2)}+0.11\,\omega_0+0.12\,\omega_{(1)}+0.11\,\omega_{(3)}+0.05\,(\omega_{(4)}+\omega_{(5)})\\
&\quad +0.04\,\omega_{(6)}+0.03\,\omega_{(7)}-0.24\,(\omega_1+\omega_2)-0.09\,\omega_3-0.07\,(\omega_4\\
&\quad +\omega_5)-0.06\,\omega_6\\
K_{(3)} &= 0.49\,\omega_{(3)}+0.11\,(\omega_0+\omega_{(2)}+\omega_{(4)})+0.05\,(\omega_{(5)}+\omega_{(6)})+0.04\,\omega_{(1)}\\
&\quad +0.03\,\omega_{(7)}-0.24\,(\omega_2+\omega_3)-0.09\,(\omega_1+\omega_4)-0.07\,\omega_5-0.06\,\omega_6\\
K_{(4)} &= 0.49\,\omega_{(4)}+0.11\,(\omega_{(3)}+\omega_{(5)}+\omega_0)+0.05\,(\omega_{(2)}+\omega_{(6)})+0.03\,(\omega_{(1)}\\
&\quad +\omega_{(7)})-0.24\,(\omega_3+\omega_4)-0.09\,(\omega_2+\omega_5)-0.07\,(\omega_1+\omega_6)
\end{aligned}\right|$$

$$\left.\begin{aligned}
K_{(5)} = {} & 0.49\,\omega_{(5)}+0.11\,(\omega_{(4)}+\omega_{(6)})+0.05\,(\omega_{(2)}+\omega_{(3)})+0.04\,\omega_{(7)} \\
& +0.03\,\omega_{(1)}+0.11\,\omega_0-0.24\,(\omega_4+\omega_5)-0.09\,(\omega_3+\omega_6) \\
& -0.07\,\omega_2-0.06\,\omega_1 \\
K_{(6)} = {} & 0.49\,\omega_{(6)}+0.11\,(\omega_0+\omega_{(5)})+0.12\,\omega_{(7)}+0.05\,(\omega_{(3)}+\omega_{(4)}) \\
& +0.04\,\omega_{(2)}+0.03\,\omega_{(1)}-0.24(\omega_5+\omega_6)-0.09\,\omega_4-0.07(\omega_2+\omega_3) \\
& -0.06\,\omega_1 \\
K_{(7)} = {} & 0.64\,\omega_{(7)}+0.08\,\omega_0+0.12\,\omega_{(6)}+0.04\,\omega_{(5)}+0.03\,(\omega_{(2)}+\omega_{(3)} \\
& +\omega_{(4)})+0.02\,\omega_{(1)}-0.28\,\omega_6-0.08\,\omega_5-0.05\,(\omega_2+\omega_3+\omega_4) \\
& -0.04\,\omega_1 \\
K_1 = {} & 0.56\,\omega_1+0.16\,\omega_2+0.10\,(\omega_3+\omega_4)+0.09\,(\omega_5+\omega_6)-0.16\,\omega_0 \\
& -0.28\,\omega_{(1)}-0.24\,\omega_{(2)}-0.09\,\omega_{(3)}-0.07\,\omega_{(4)}-0.06\,(\omega_{(5)}+\omega_{(6)}) \\
& -0.04\,\omega_{(7)} \\
K_2 = {} & 0.55\,\omega_2+0.17\,\omega_3+0.16\,\omega_1+0.11\,\omega_4+0.10\,\omega_5+0.09\,\omega_6 \\
& -0.17\,\omega_0-0.24\,(\omega_{(2)}+\omega_{(3)})-0.09\,\omega_{(4)}-0.08\,\omega_{(1)}-0.07\,(\omega_{(5)} \\
& +\omega_{(6)})-0.05\,\omega_{(7)} \\
K_3 = {} & 0.55\,\omega_3+0.17\,(\omega_2+\omega_4)+0.11\,\omega_5+0.10\,(\omega_1+\omega_6)-0.17\,\omega_0 \\
& -0.24\,(\omega_{(3)}+\omega_{(4)})-0.09\,(\omega_{(2)}+\omega_{(5)})-0.07\,\omega_{(6)}-0.05\,(\omega_{(7)} \\
& +\omega_{(1)}) \\
K_4 = {} & 0.55\,\omega_4+0.17\,(\omega_3+\omega_5)+0.11\,\omega_2+0.10\,(\omega_1+\omega_6)-0.17\,\omega_0 \\
& -0.24\,(\omega_{(4)}+\omega_{(5)})-0.09\,(\omega_{(3)}+\omega_{(6)})-0.07\,\omega_{(2)}-0.05\,(\omega_{(1)} \\
& +\omega_{(7)}) \\
K_5 = {} & 0.55\,\omega_5+0.17\,\omega_4+0.16\,\omega_6+0.11\,\omega_3+0.10\,\omega_2+0.09\,\omega_1 \\
& -0.17\,\omega_0-0.24\,(\omega_{(5)}+\omega_{(6)})-0.09\,\omega_{(4)}-0.08\,\omega_{(7)}-0.07\,(\omega_{(2)} \\
& +\omega_{(3)})-0.05\,\omega_{(1)} \\
K_6 = {} & 0.56\,\omega_6+0.16\,\omega_5+0.10\,(\omega_4+\omega_3)+0.09\,(\omega_1+\omega_2)-0.16\,\omega_0 \\
& 0.28\,\omega_{(7)}-0.24\,\omega_{(6)}-0.09\,\omega_{(5)}-0.07\,\omega_{(4)}-0.06\,(\omega_{(3)}+\omega_{(2)}) \\
& -0.04\,\omega_{(1)}\ .
\end{aligned}\right\} \quad (60)$$

These values are expressed conveniently in the following **Fig. 10.** In this figure, numerical values denote coefficients of ω in those positions and a circle denotes the coefficient whose correlate is about to be obtained. These values are obtained by taking terms of ω only and neglecting the term of K_s in equations (79) to (84) in § 2 in Chapter 3.

The calculation of these **first values** is sufficiently accurate up to two places with a slide rule.

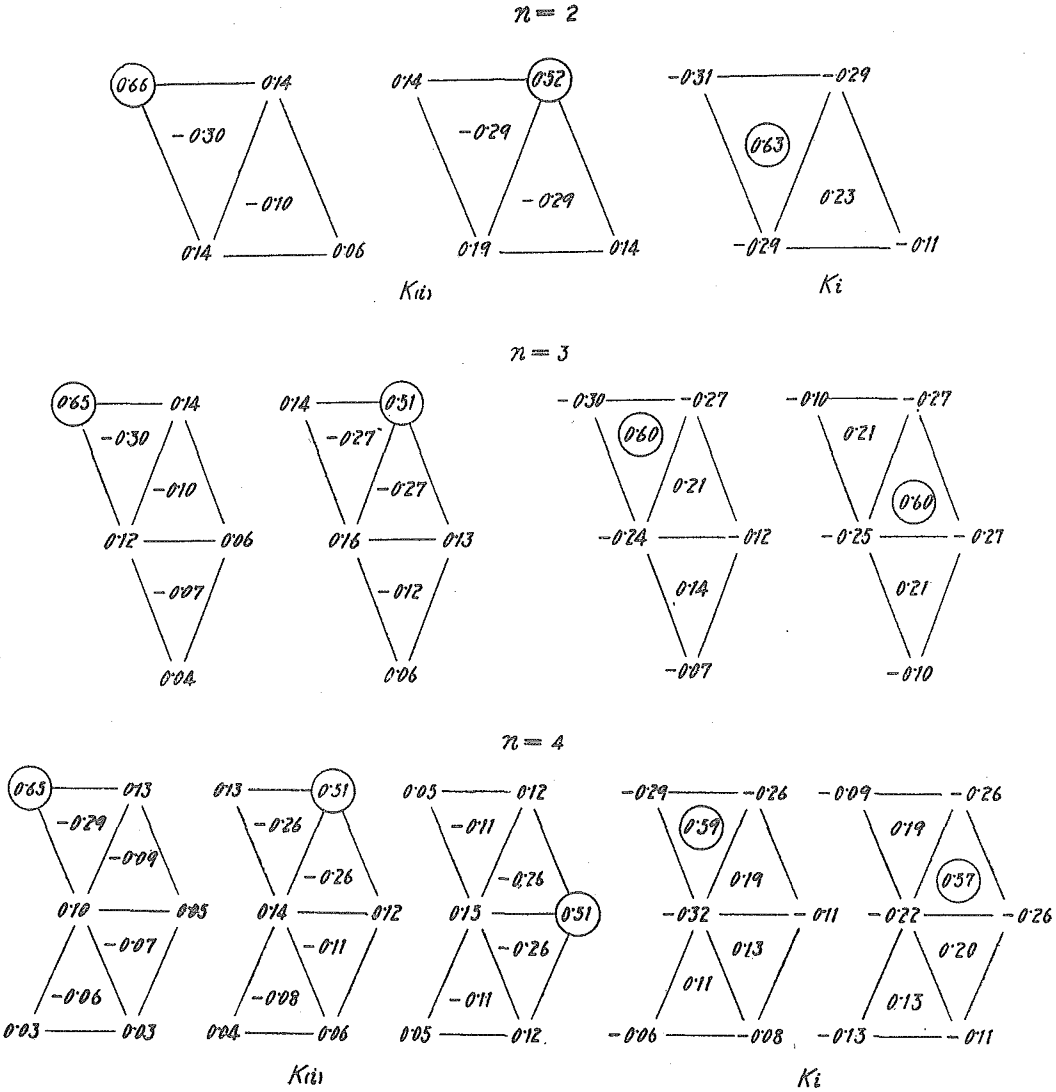

Fig 10.

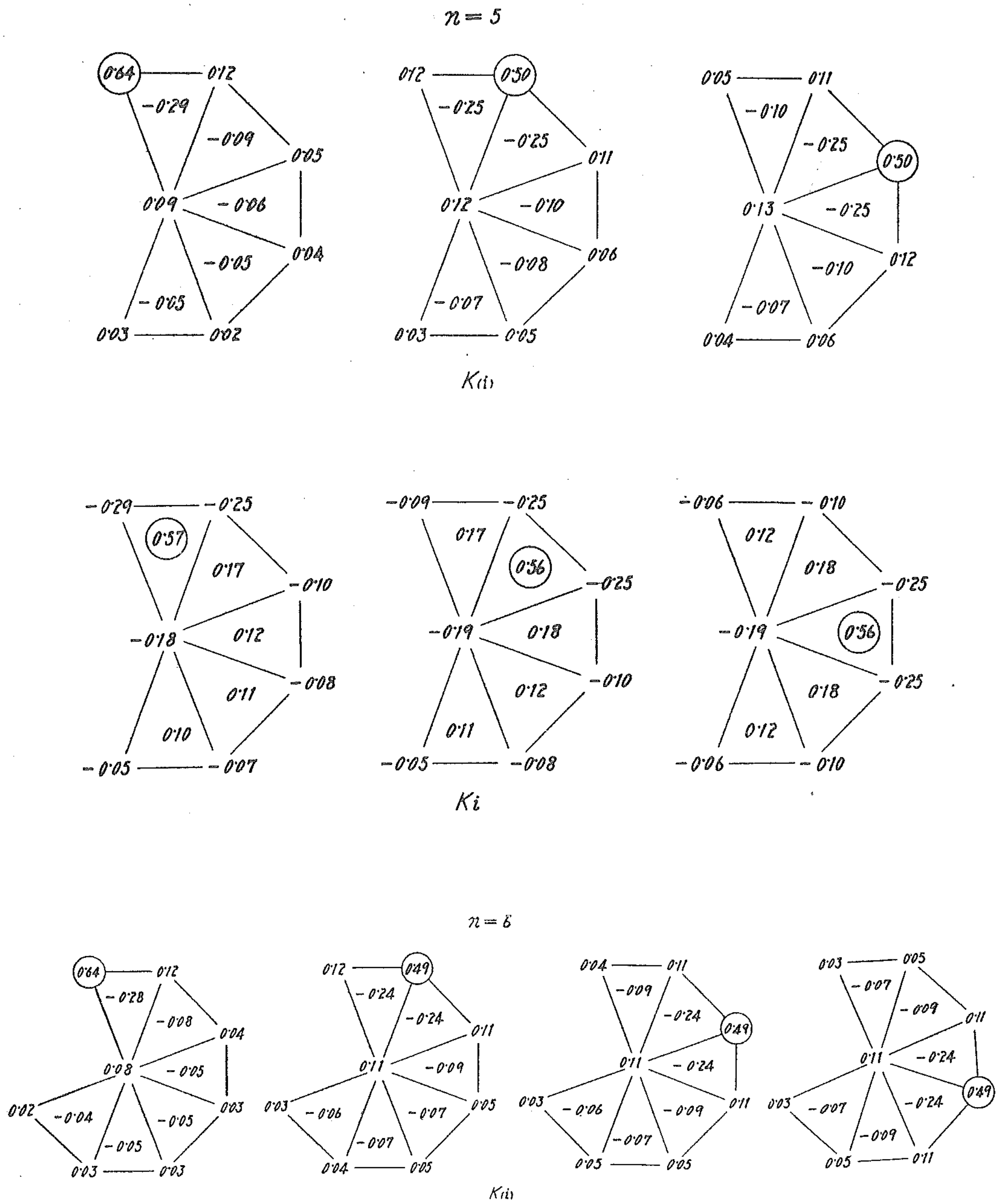

Fig. 10.

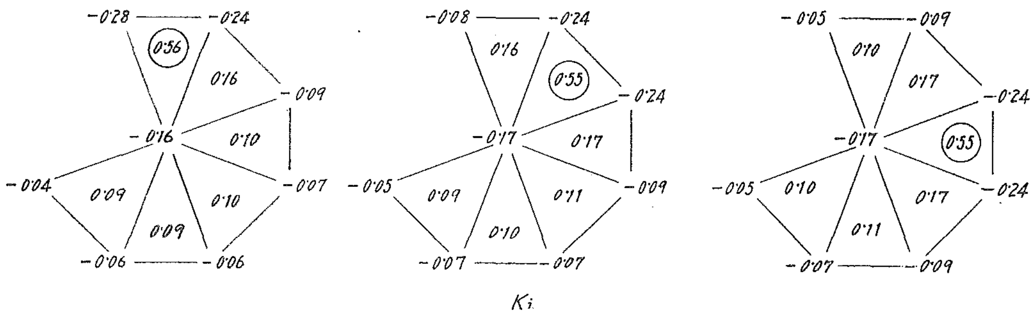

Fig. 10.

In the case of Table 6, the method of calculation is the same as above described, but terms $K_{(i)}$, $d_{(i)}$ and $\omega_{(i)}$ do not exist, so the process can be continued more simply, that is to say, the formulae are limited by (53), (54) and (55). **First values** may be obtained from equations (18) and (19), by omitting the term K_s.

$$\left.\begin{aligned} &K_0 = 3/(2n+3) \cdot (\omega_0 - \omega/3) \\ &K_i = 1/3 \cdot \left\{\omega_i + \omega/(2n+3)\right\} - \omega_0/(2n+3), \\ &\text{where} \\ &\omega = \sum_{i=1}^{n} \omega_i \,. \end{aligned}\right\} \cdots\cdots\cdots\cdots (61)$$

After the values of all correlates are obtained, the values of corrections of the observed angles are gotton from equation (13) or (16) in §3 in Chapter 1.

Numerical Example 3.

An Unclosed Central Polygon consisting of three triangles.

All inner and outer angles are observed; the problem is to adjust them. Here are shown values observed and obtained by calculation in Table 7. Numbers in the above are in order of writing and calculation.

(1) ; number of angles or sign of sides,

(2) ; observed values of angles, length of known side and sum of three angles in each individual triangle,

(3) ; values of log.sin. of observed angles $3i-2$ and $3i-1$, and values of ω,

(4) ; tabular difference of long.sin. per $1''$,

(5) ; d_i and $d_{(i)}$, for example $d_1 = \Delta_1 - \Delta_2 = 1.19 - 0.56 = 0.63$,
$d_{(1)} = \Delta_1 = 1.19$,
$d_{(2)} = \Delta_1 - \Delta_2 = 1.05 - 0.56 = 0.49$
& etc.,

(6) ; $\Delta \cdot \Delta$ for angles $3i-2$ and $3i-1$,

(7) ; **Correlate Sketch**, this is the main point of this paper, values of all correlates are obtained by this (the process is explained in detail later),

(8) ; $\Delta \cdot K_s$, K_s are obtained from (7),

(9) ; corrections v from $K_{(i)}$, K_0 and K_i in (7), and $\Delta \cdot K_s$ in (8),

(12) ; corrected values of angles by (9),

(13) ; corrected log. sin. for calculation of side length of triangulation nets, with $\Delta \cdot v$ in (10),

(11) ; $v \cdot v$ for calculation of probable error.

Correlate Sketch in (7) will be explained (Fig. 10). Figures in the double circles are d and ω which are shown in column (5) in the table, for example $d_1 = 0.63$, $\omega_1 = -11$ circle 1,
$d_{(1)} = 1.19$, $\omega_{(1)} = 8$ in circle (1), & etc.,

The **first values** of correlates are obtained from (56) or **Fig. 10** ($n = 3$) as follows:

Point correlates $K_{(i)}$;

$$K_{(1)} = 0.65 \times 8 + 0.14 \times -12 + 0.12 \times 8 + 0.06 \times -7 + 0.04 \times 14 \\ -0.30 \times -11 - 0.10 \times 15 - 0.07 \times 10 = 5.7,$$

similarly $K_{(2)} = -6.1$, $K_{(3)} = -6.9$, $K_{(4)} = -4.9$.

Angle correlates K_i;

$$K_1 = 0.60 \times -11 + 0.21 \times 15 + 0.14 \times 10 - 0.30 \times 8 - 0.27 \times -12 \\ -0.24 \times 8 - 0.12 \times -7 - 0.07 \times 14 = -3.3,$$

similarly $K_2 = 9.6$, $K_3 = 4.3$.

The point correlate K_0.

First values of angle correlates K_i above obtained and written in each triangle are added, with the opposite sign, to the lower figure in circle (0) and this value is divided by the number of observed angles around point (0).

$$K_0 = \underset{\substack{\text{no. of}\\\text{angles}\\\text{around}\\\text{point}\\\text{(0)}}}{1/4} \cdot (\underset{\substack{\text{lower}\\\text{figure}\\\text{in cir-}\\\text{cle (0)}}}{8} + \underbrace{3.3 - 9.6 - 4.3}_{\substack{\text{1st. values written}\\\text{in triangles around}\\\text{point (0), added}\\\text{with the opposite}\\\text{sign}}}) = -0.7,$$

this value -0.7 is written near circle (0).
This process means that equation (54) is calculated.

The side correlate K_s.

First values of four point correlates written at each point and three angle correlates written in each triangle are multiplied by the upper figures in each circle and these products are added, with the opposite sign, to the lower figure 20 in circle S, and then this value is divided by the upper figure 8.81 in circle S.

$$K_s = 1/\underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle } S}}{8.81}\cdot\Big\{\underset{\substack{\text{lower}\\\text{figure}\\\text{in cir-}\\\text{cle } S}}{20} - (\underbrace{\underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle (1)}}}{1.19} \times \underset{\substack{\text{1st.}\\\text{value}\\\text{of}\\K_{(1)}}}{5.7}}_{\text{at point (1)}} + \underbrace{\underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle (2)}}}{0.49} \times \underset{\substack{\text{1st.}\\\text{value}\\\text{of}\\K_{(2)}}}{-6.1}}_{\text{at point (2)}} + \underset{\cdots\cdots}{\cdots\cdots}$$

$$+ \underbrace{\underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle 1}}}{0.63} \times \underset{\substack{\text{1st.}\\\text{value}\\\text{of}\\K_1}}{-3.3}}_{\text{in triangle 1}} - \underbrace{\underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle 2}}}{0.29} \times \underset{\substack{\text{1st.}\\\text{value}\\\text{of}\\K_2}}{9.6}}_{\text{in triangle 2}} - \underbrace{\underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle 3}}}{0.49} \times \underset{\substack{\text{1st.}\\\text{value}\\\text{of}\\K_3}}{4.3}}_{\text{in triamgle 3}})\Big\}$$

$$= 3.4\,.$$

This value 3.4 is written near circle S.
This process shows that equation (55) is solved on the sketch.

Second values of correlates are obtained as follows:

Point correlates $K_{(i)}$.

First values of angle correlates in triangles around the point (i) and the product of the upper figure in circle (i) and the first value of K_s are added, with the opposite sign, to the lower figure in circle (i), and divided by the number of angles around point (i).

$$K_{(1)} = 1/\underset{\substack{\text{no. of}\\\text{angles}\\\text{around}\\\text{point}\\\text{(1)}}}{2}\cdot(\underset{\substack{\text{lower}\\\text{figure}\\\text{in cir-}\\\text{cle (1)}}}{8} \underbrace{+ \underset{\substack{\text{1st.}\\\text{value}\\\text{in tri-}\\\text{angle}\\1}}{3.3} - \underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle (1)}}}{11.9} \times \underset{\substack{\text{1st.}\\\text{value}\\\text{of}\\K_s}}{3.4}}_{\text{added with the opposite sign}}) = 3.6\,,$$

this value 3.6 is written under the first value 5.7.

$$K_{(2)} = 1/3 \cdot (-\,12 + \underbrace{\underbrace{3.3 - 9.6} - 0.49 \times 3.4}_{\text{added with the opposite sign}}) = -6.7\,,$$

no. of angles around point (2)	lower figure in circle (2)	**1st.** values in triangles 1 and 2 around point (2)	upper figure in circle (2)	**1st.** value of K_s

this value -6.7 is written under the first value -6.1.
Similarly $K_{(3)} = -6.8$, $K_{(4)} = 7.7$.
This process means that equation (52) is solved on the sketch.

Angle correlates K_i.

Second values of point correlates above obtained and written at each point in triangle i and the product of the upper figure in circle i and the first value of K_s are added, with the opposite sign, to the lower figure in circle i, and this value is divided by the number of observed angles in the triangle, namely 3.

$$K_1 = 1/3 \cdot (-\,11 \underbrace{\underbrace{-\,3.6 + 6.7 + 0.7} - 0.63 \times 3.4}_{\text{added with the opposite sign}}) = -3.1\,,$$

no. of angles in triangle 1	lower figure in circle 1	**2nd.** values written at three points in triangle 1	upper figure in circle 1	**1st.** value of K_s

this value -3.1 is written under the first value -3.3.
Similarly $K_2 = 10.1$, $K_3 = 3.8$.
This process means that equation (53) is solved on the sketch.

The point correlate K_0.

Second values of all angle correlates K_i above obtained and written in triangles around point (0) are added, with the opposite sign, to the lower figure in circle (0) and divided by the number of angles around point (0), namely 4.

$$K_0 = 1/4 \cdot (\ 8 \underbrace{+\,3.1 - 10.1 - 3.8}) = -0.7\,,$$

no. of angles around point (0)	lower figure in circle (0)	**2nd.** values written in all triangles around point (0), added with the opposite sign

this value -0.7 is written under its first value -0.7.
This process means that equation (54) is calculated on the sketch.

The side correlate K_s.

Using second values of four point correlates written at each point and three angle correlates in each triangle, and by the same process as in the case of obtaining the first value of K_s, the second value is gotten.

$$K_s = 1/8.81 \cdot \Big\{20 - (1.19 \times 3.6 + 0.46 \times -6.7 + \cdots$$

upper figure in circle S	lower figure in circle S	upper figure in circle (1)	2nd. value of $K_{(1)}$	upper figure in circle (2)	2nd. value of $K_{(2)}$	$\cdots$
		at point (1)		at point (2)		$\cdots$

$$+ 0.63 \times -3.1 - 0.29 \times 10.1 - 0.49 \times 3.8)\Big\}$$

upper figure in circle 1	2nd. value of K_1	upper figure in circle 2	2nd. value of K_2	upper figure in circle 3	2nd. value of K_3
in triangle 1		in triangle 2		in triangle 3	

$$= 4.3\,.$$

This value 4.3 is written under the first value 3.4.

This process means that equation (55) is solved on the sketch for the second time.

Thus the first round of calculation is over, and the second and the third rounds are carried on.

In this case, the calculation is stopped at the seventh round; the time required is only one and a half hours. If the simultaneous equations are solved by the rigorous method ordinarily, at least three times as long would be required.

After the values of all correlates are determined, the corrections of the observed angles are calculated from equation (13) in §3 in Chapter 1.

Table 7.

Angles (1) No.	(2) Observed Values ° ′ ″	(12) Corrected Values ° ′ ″	(3) Initial Log. sin.	(13) Corrected Log. sin.	(4) Tabu. Diff. per 1″ Δ	(5) d	(8) $\Delta \cdot K_s$	(9) Correction v	(10) $\Delta \cdot v$	(11) $v \cdot v$	(6) $\Delta \cdot \Delta$
1	60 20 58	60 21 3.3	9.939049	9.939055	1.19	0.63	5.55	5.3	6.3	28.09	1.4161
2	75 6 45	75 6 32.3	9.985171	9.985164	0.56		−2.61	−12.7	−7.1	50.28	0.3136
3	44 32 28	44 32 24.4						− 3.6		12.96	
	180 00 11	180 00 00.0	$\omega_1 = -11$								
4	63 27 30	63 27 38.0	9.951634	9.951642	1.05	−0.29	4.90	8.0	8.4	70.56	1.1025
5	57 33 55	57 33 52.4	9.926344	9.926341	1.34		−6.25	− 2.6	−3.5	12.25	1.7956
6	58 58 20	58 58 29.6						9.6		92.16	
	179 59 45	180 00 00.0	$\omega_2 = 15$								
7	60 44 55	60 44 57.3	9.940758	9.940761	1.18	−0.49	5.50	2.3	2.7	7.29	1.3924
8	51 34 30	51 34 34 9	9.893996	9.894004	1.67		−7.80	4.9	8.2	67.24	2.7889
9	67 40 25	67 40 27.8						2.8			
	179 59 50	180 00 00.0	$\omega_3 = 10$								
γ	188 48 39	188 48 38.2						− 0.8	$\Sigma\,\Delta \cdot v =$	0.64	$d_s =$
			$\omega_0 = 8$						19.8		8.8091
B_1	1,656.3		3.21914								
B_2	1,759.2		3.24509								
			$\omega_s = 20$								
(1)	299 38 54	299 38 56.7				1.19		2.7		7.89	
			$\omega_{(1)} = 8$								
(2)	221 25 57	221 25 49.7				0.49		− 7.3		53.29	
			$\omega_{(2)} = -12$								
(3)	241 41 17	241 41 10.3				−0.16		− 6.7		44.89	
			$\omega_{(3)} = -7$								
(4)	308 25 16	308 25 25.1				−1.67		9.1		82.81	
			$\omega_{(4)} = 14$								
										$\Sigma\, v \cdot v =$ 537.72	

Probable Error $= \pm 0.6745\sqrt{537.72/9} = \pm 5.''21$

(7)

Correlate Sketch.

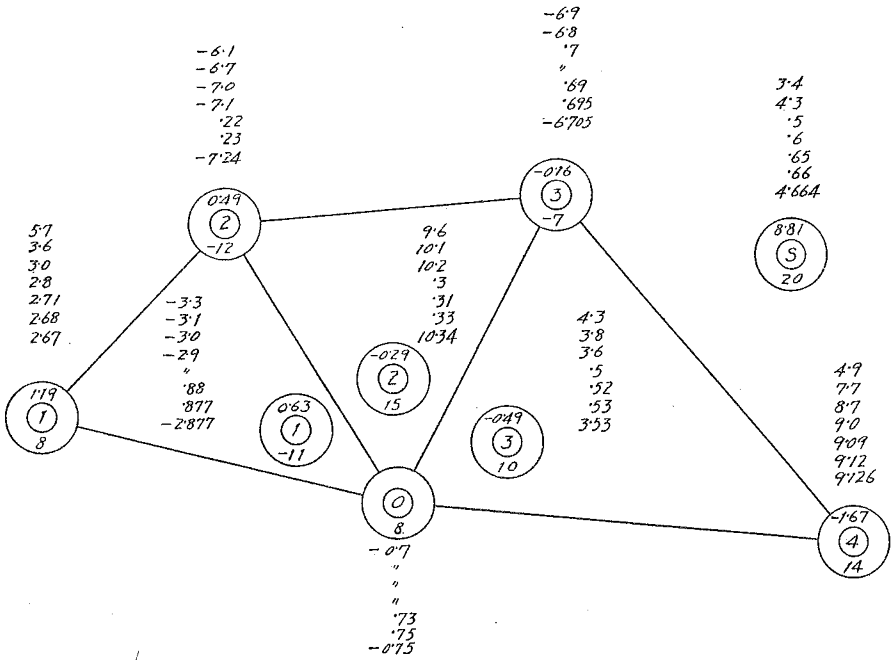

Fig. 11.

Numerical Example 4.

An Unclosed Central Polygon consisting of three triangles. In **Example 3** above, outer angles are assumed to be adjusted at each station independently, or to be not observed; it is purposed to adjust the angles at (0) and inner angles only. The following Table 8 and the process are all similar to those in **Example 3**, but the process becomes simpler.

Now only the **Correlate Sketch** will be explained (Fig. 12).

First values of correlates are obtained from **(61)** as follows:

$$\omega = -11+15+10 = 14, \quad \omega/3 = 4.67, \quad \omega/(2n+3) = 14/9 = 1.56.$$

The point correlate K_0;

$$K_0 = 3/9 \cdot (8-4.67) = 1.11.$$

Angle correlates K_i;

$$K_1 = 1/3 \cdot (-11+1.56) - 8/9 = -4.04,$$

similarly $K_2 = 4.64$, $K_3 = 2.97$.

The side correlate K_s ,

First values of angle correlates K_i above obtained and written in each circle are multiplied by the upper figures in each circle and these products are added, with the opposite sign, to the lower figure 20 in circle S, and this value is divided by the upper figure 8.81 in circle S.

$$K_s = \underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle } S}}{1/8.81} \cdot \Big\{ \underset{\substack{\text{lower}\\ \text{figure}\\ \text{in cir-}\\ \text{cle } S}}{20} - (\underbrace{\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 1}}}{0.63} \times \underset{\substack{\textbf{1st.}\\ \text{value}\\ \text{of}\\ K_1}}{-4.04}}_{\text{in triangle 1}} - \underbrace{\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 2}}}{0.29} \times \underset{\substack{\textbf{1st.}\\ \text{value}\\ \text{of}\\ K_2}}{4.64}}_{\text{in triangle 2}} - \underbrace{\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 3}}}{0.49} \times \underset{\substack{\textbf{1st.}\\ \text{value}\\ \text{of}\\ K_3}}{2.97}}_{\text{in triangle 3}}) \Big\}$$

$$= 2.88 \, .$$

This value is written near circle S.

This process means that equation (55) is solved on the sketch.

Second values of correlates are obtained as follows:

Angle correlates K_i .

First value at the centre point (0) and the product of the upper figure in circle i and the first value of K_s are added, with the opposite sign, to the lower figure in circle i, and this result is divided by the number of angles in the triangle, namely 3.

$$K_1 = \underset{\substack{\text{no. of}\\ \text{angles}\\ \text{in tri-}\\ \text{angle 1}}}{1/3} \cdot \big(- \underset{\substack{\text{lower}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 1}}}{11} \underbrace{- \underset{\substack{\textbf{1st.}\\ \text{value}\\ \text{at}\\ \text{point}\\ (0)}}{1.11} - \underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 1}}}{0.63} \times \underset{\substack{\textbf{1st.}\\ \text{value}\\ \text{of}\\ K_s}}{2.88}}_{\text{added with the opposite sign}}\big) = -4.64 \, ,$$

this value is written under the first value -4.04.

Similarly $K_2 = 4.91$, $K_3 = 3.43$.

This process means that equatiou (53) is calculated on the sketch.

The point correlate K_0 .

Second values of all angle correlates written in the triangles around the central point (0) are added, with the opposite sign, to the lower figure in circle (0) and this value is divided by the number of angles around point (0), namely 4.

$$K_0 = \underset{\substack{\text{no. of}\\ \text{angles}\\ \text{around}\\ \text{point}\\ (0)}}{1/4} \cdot \big(\underset{\substack{\text{lower}\\ \text{figure}\\ \text{in cir-}\\ \text{cle (0)}}}{8} \underbrace{+ 4.64 - 4.91 - 3.43}_{\substack{\textbf{2nd.}\text{ values written in}\\ \text{triangles around point}\\ \text{(0), added with the}\\ \text{opposite sign}}} \big) = 1.07 \, ,$$

this value is written under its first value 1.11.

This process means that equation (54) is calculated on the sketch.

Table 8.

Angles											
(1)	(2)	(12)	(3)	(13)	(4)	(5)	(8)	(9)	(10)	(11)	(6)
No.	Observed Values	Corrected Values	Initial Log. sin.	Corrected Log. sin.	Tabu. Diff. per 1″ Δ	d	$\Delta\cdot K_s$	Correction v	$\Delta\cdot v$	$v\cdot v$	$\Delta\cdot\Delta$
	° ′ ″	° ′ ″									
1	60 20 58	60 20 56.88	9.939049	9.939048	1.19	0.63	3.519	−1.12	−1.3	1.2544	1.4161
2	75 6 45	75 6 38.70	9.985171	9.985168	0.56		−1.656	−6.30	−3.5	39.6900	0.3136
3	44 32 28	44 32 24.42						−3.58		11.8164	
	180 00 11	180 00 00.00	$\omega_1 = -11$								
4	63 27 30	63 27 38.04	9.951634	9.951642	1 05	−0.29	3.105	8.04	8.4	64.6416	1.1025
5	57 33 55	57 33 55.97	9.926344	9.926345	1.34		−3.962	0.97	1.3	0.9409	1.7956
6	58 58 20	58 58 25.99						5.99		35.8801	
	179 59 45	180 00 00.00	$\omega_2 = 15$								
7	60 44 55	60 45 1.95	9.940758	9.940766	1.18	−0.49	3.490	6.95	8.2	48.3025	1.3924
8	51 34 30	51 34 28.52	9.893996	9.893994	1.67		−4.938	−1.48	−2.5	2.1904	2.7889
9	67 40 25	67 40 29.53						4 53		20.5209	
	179 59 50	180 00 00.00	$\omega_3 = 10$								
γ	188 48 39	188 48 40.06	$\omega_0 = 8$					1.06	$\Sigma\,\Delta\cdot v =$ 20.0	1.1236	$d_s =$ 8.8091
B_1	1,656.3		3.21914								
B_2	1,759.2		3.24509							$\Sigma\, v\cdot v =$ 227.3608	
			$\omega_s = 20$								

$$\text{Probable Error} = \pm 0.6745\sqrt{227.3608/5} = \pm 5.09''$$

The side correlate K_s.

Similarly to the case of the **first value**, the second value is obtained, but the second values of angle correlates written in each triangle are used.

$$K_s = 1/\underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle } S}}{8.84} \cdot \Big\{\underset{\substack{\text{lower}\\\text{figure}\\\text{in cir-}\\\text{cle } S}}{20} - \big(\underbrace{\underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle 1}}}{0.63} \times \underset{\substack{\textbf{2nd.}\\\text{value}\\\text{of}\\K_1}}{-4.64}}_{\text{in triangle 1}} - \underbrace{\underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle 2}}}{0.29} \times \underset{\substack{\textbf{2nd.}\\\text{value}\\\text{of}\\K_2}}{4.91}}_{\text{in triangle 2}} - \underbrace{\underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle 3}}}{0.49} \times \underset{\substack{\textbf{2nd.}\\\text{value}\\\text{of}\\K_3}}{3.43}}_{\text{in triangle 3}}\big)\Big\}$$

$$= 2.957\,.$$

(12)

Correlate Sketch.

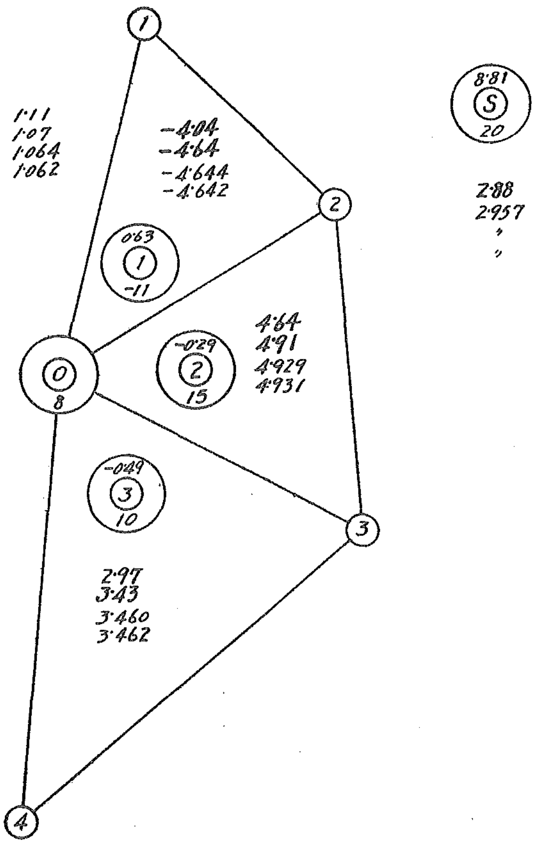

Fig. 12.

This value is written under the first value 2.88, and this process shows that equation (55) is solved on the sketch for the second time.

Thus the first round of calculation is over, and the next rounds are carried on. In this case, the calculation is sufficient at the fourth round and the time required is less than one hour.

The values of corrections of observed angles are obtained from equation (16) in §3 in Chapter 1.

This example is adopted from references (10) and (11).

§4. A SINGLE ROW TRIANGULATION NET.

Equations (24) and (27) in §4 in Chapter 1 can be expressed as in the following Tables 9 and 10 respectively.

Table 9.

Sign of equation	Left side of equation															Right Side of equation
	K_P	$K_{(1)}$	$K_{(2)}$	→	→	$K_{(n-1)}$	$K_{(n)}$	K_Q	K_1	K_2	→	→	K_{n-1}	K_n	K_s	
P	2								1						Δ_1	ω_P
(1)		3							1	1					Δ_4	$\omega_{(1)}$
(2)			4						1	1	1				$d_{(2)}$	$\omega_{(2)}$
↓			↘	↘						↘	↘	↘			↓	↓
↓					↘						↘	↘	↘		↓	↓
$(n-1)$						4						1	1	1	$d_{(n-1)}$	$\omega_{(n-1)}$
(n)							3						1	1	$-\Delta_{3n-4}$	ω_n
Q								2						1	$-\Delta_{3n-1}$	ω_Q
1	1	1	1						3						d_1	ω_1
2		1	1	1						3					d_2	ω_2
↓			↘	↘	↘						↘				↓	↓
↓				↘	↘	↘						↘			↓	↓
$n-1$					1	1	1						3		d_{n-1}	ω_{n-1}
n						1	1	1						3	d_n	ω_n
S	Δ_1	Δ_4	$d_{(2)}$	→	→	$d_{(n-1)}$	$-\Delta_{3n-4}$	$-\Delta_{3n-1}$	d_1	d_2	→	→	d_{n-1}	d_n	d_s	ω_s

Table 10.

Sign of equation	Left Side of equation						Right Side of equation
	K_1	K_2	$\longrightarrow$	K_{n-1}	K_n	K_s	
1	3					d_1	ω_1
2		3				d_2	ω_2
$\downarrow$			$\searrow$			$\downarrow$	$\downarrow$
$n-1$				3		d_{n-1}	ω_{n-1}
n					3	d_n	ω_n
s	d_1	d_2	$\longrightarrow$	d_{n-1}	d_n	d_s	ω_s

These tables are similar to the cases previously mentioned in §2 and 3 of this Chapter, but the correlate K_0 does not exist, because there is no central station in the figure. Coefficients of K_p and K_q are 2, that of $K_{(1)}$ and $K_{(n)}$ are 3, and that of other correlates are all 4, because the angles around them are two, three and four respectively.

The **Correlate Sketch** is as follows (Fig. 13).

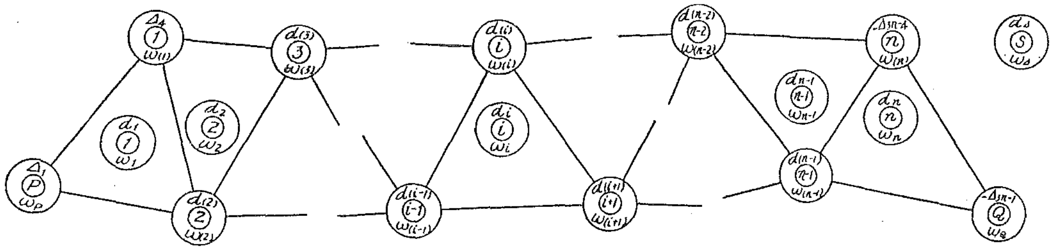

Fig. 13.

Formulae to obtain correlates are as follows:

Point correlates;

$$\left.\begin{aligned}
K_p &= 1/2\cdot(\omega_p-K_1-\Delta_1\cdot K_s)\\
K_{(1)} &= 1/3\cdot(\omega_{(1)}-K_1-K_2-\Delta_4\cdot K_s)\\
K_{(i)} &= 1/4\cdot(\omega_{(i)}-K_{i-1}-K_i-K_{i+1}-d_{(i)}\cdot K_s)\\
&\qquad (i=2,\ 3,\ \cdots,\ n-1)\\
K_{(n)} &= 1/3\cdot(\omega_{(n)}-K_{n-1}-K_n+\Delta_{3n-4}\cdot K_s)\\
K_q &= 1/2\cdot(\omega_q-K_n+\Delta_{3n-1}\cdot K_s)\ .
\end{aligned}\right\}\quad\cdots\cdots\cdot\ (62)$$

Angle correlates;

$$\left.\begin{aligned} K_1 &= 1/3\cdot(\omega_1 - K_p - K_{(1)} - K_{(2)} - d_1\cdot K_s) \\ K_i &= 1/3\cdot(\omega_i - K_{(i-1)} - K_{(i)} - K_{(i-1)} - d_i\cdot K_s) \\ &\qquad\qquad (i = 2,\ 3,\ \cdots,\ n-1) \\ K_n &= 1/3\cdot(\omega_n - K_{(n-1)} - K_{(n)} - K_q - d_n\cdot K_s)\ . \end{aligned}\right\} \quad \cdots\cdots\cdots (63)$$

The side correlate;

$$K_s = 1/d_s\cdot\Big(\omega_s - \varDelta_1\cdot K_p - \varDelta_4\cdot K_{(1)} - \sum_{i=2}^{n-1} d_{(i)}\cdot K_{(i)} + \varDelta_{3-4}\cdot K_{(n)} + \varDelta_{3n-1}\cdot K_q - \sum_{i=1}^{n} d_i\cdot K_i\Big)\ . \quad \cdots\cdots (64)$$

Formulae to obtain **first values** are as follows:
Angle correlates;

$$\left.\begin{aligned} \mathbf{K}_1 &= 0.57\omega_1 + 0.15\omega_2 - (0.28\omega_p + 0.24\omega_{(1)} + 0.19\omega_{(2)}) \\ \mathbf{K}_i &= 0.44\omega_i - 0.11(\omega_{(i-1)} + \omega_{(i)} + \omega_{(i+1)}) \\ &\qquad\qquad (i = 2,\ 3,\ \cdots,\ n-1) \\ \mathbf{K}_n &= 0.57\omega_n + 0.15\omega_{n-1} - (0.28\omega_q + 0.24\omega_{(n)} + 0.19\omega_{(n-1)})\ . \end{aligned}\right\} (65)$$

These values are obtained by taking terms of ω only and neglecting the term of K_s in equations (85) and (86) in § 3 in Chapter 3. Other point and side correlates are obtained from formulae (62) and (64).

In the case of Table 6, formulae are (63) and (64), and the **first value** is only for K_s as follows;

$$\mathbf{K}_s = \omega_s/d_s\ . \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (66)$$

The process of calculation is similar to two cases above described and will be explained in the following examples in detail.

Numerical Example 5.

A Single Row Triangulation Net consisting of six triangles.

All inner and outer angles are assumed to be observed; the problem is to adjust them. In Table 11 are tabulated figures which are observed and obtained by calculation. Numbers in the above are in order of writing and calculation.

(1) ; number of angles or sign of sides,
(2) ; observed values of angles, length of known sides and sum of three angles in each individual triangle,
(3) ; values of log. sin. of observed angles $3i-2$, $3i-1$ and values of ω.
(4) ; tabular difference of log.sin. per 1″,
(5) ; d_i and $d_{(i)}$, for example
$d_1 = \varDelta_1 - \varDelta_2 = 8.68 - 8.78 = -0.10$,
$d_{(1)} = \varDelta_4 = 11.38$, $d_{(2)} = \varDelta_7 - \varDelta_2 = 17.37 - 8.78 = 8.59$, & etc.,
(6) ; $\varDelta \cdot \varDelta$ for angles $3i-2$ and $3i-1$,
(7) ; **Correlate Sketch** by means of which values of all correlates are obtained,
(8) ; $\varDelta \cdot K_s$, K_s is obtained from (7),
(9) ; correction v from $K_{(i)}$ and K_i in (7) and $\varDelta \cdot K_s$ in (8),
(10) ; corrected values of angles by (9),
(12) ; corrected log.sin. for calculation of side length of triangulation nets, with $\varDelta \cdot v$ in (11),
(13) ; $v \cdot v$ for calculation of probable error.

Correlate Sketch in (7) will now be explained (Fig. 13). Figures in the double circles d and ω which are shown in (5) and (3) in Table 11, for example $d_1 = -0.10$, $\omega_1 = -5$ in circle 1,
$d_{(1)} = 11.38$, $\omega_{(1)} = -5$ in circle (1), & etc.,

First values of correlates are obtained from (**65**) as follows;

$$K_1 \doteq 0.57 \times -5 + 0.15 \times -12 - (0.28 \times 7 + 0.24 \times -5 + 0.19 \times 15) = -8,$$

$$K_2 = 0.44 \times -12 - 0.11(-5 + 15 + 6) = -7,$$

similarly $K_3 = 3$, $K_4 = 4$, $K_5 = -3$, $K_6 = 5$,

The side correlate K_s is assumed roughly from (**66**);

$$K_s = 319/2{,}361.51 = 0.14 .$$

Point correlates $K_{(i)}$.

First values of angle correlates obtained above and written in the triangles around point (i) and the product of the upper figure in circle (i) and the first value of K_s are added, with the opposite sign, to the lower figure in circle (i); this value is divided by the number of angles around point (i), namely 2 at end points, 3 at the next points and 4 at the other intermediate points.

$$K_p = \underset{\substack{\text{no. of}\\\text{angles}\\\text{around}\\\text{point}\\(P)}}{1/2} \cdot (\underset{\substack{\text{lower}\\\text{figure}\\\text{in cir-}\\\text{cle }(P)}}{7} + \underbrace{\underset{\substack{\text{1st.}\\\text{value}\\\text{in tri-}\\\text{angle 1}}}{8} - \underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle }(P)}}{8.68} \times \underset{\substack{\text{1st.}\\\text{value}\\\text{of}\\K_s}}{0.14}}_{\text{added with the opposite sign}}) = 7 ,$$

this value is written near circle (P).

$$K_{(1)} = 1/3 \cdot (-5 + \underbrace{8 + 7} - 11.38 \times 0.14) = 3\,,$$

no. of angles around point (1)	lower figure in circle (1)	1st. values in triangles 1 and 2 around point (1)	upper figure in circle (1)	1st. value of K_s

added with the opposite sign

this value is written near circle (1).

$$K_{(2)} = 1/4 \cdot (15 + \underbrace{8 + 7 - 3} - 8.59 \times 0.14) = 6\,,$$

no. of angles around point (2)	lower figure in circle (2)	1st. values in triangles 1, 2 and 3 around point (2)	upper figure in circle (2)	1st. value of K_s

added with the opposite sign

this value is written near circle (2).

Similarly $K_{(3)} = 2$, $K_{(4)} = -4$, $K_{(5)} = -3$, $K_{(6)} = 2$, $K_q = 4$.
This calculation means that equation (62) is solved on the sketch.

Second values of correlates are obtained as follows;

Angle correlates K_i.

First values of point correlates written at three points of triangle i above obtained and the product of the upper figure in circle i and the first value of K_s are added, with the opposite sign, to the lower figure in circle i and divided by the number of observed angles in the triangle, namely 3.

$$K_1 = 1/3 \cdot (-5 - \underbrace{7 - 3 - 6} + 0.10 \times 0.14) = -7\,,$$

no. of angles in triangle 1	lower figure in circle 1	1st. values at three vertices in triangle 1	upper figure in circle 1	1st. value of K_s

added with the opposite sign

this value is written under the first value -8.

$$K_2 = 1/3 \cdot (-12 - \underbrace{3 - 6 - 2} + 7.15 \times 0.14) = -7.4\,,$$

no. of angles in triangle 2	lower figure in circle 2	1st. values at three vertices in triangle 2	upper figure in circle 2	1st. value of K_s

added with the opposite sign

this value is written under the first value -7.

Similarly $K_3 = 1.6$, $K_4 = 4.2$, $K_5 = -1.6$, $K_6 = 5$.
This process means that equation (63) is calculated on the sketch.

Point correlates $K_{(i)}$.

With a similar process to the case of the first values, the second values are obtained, but the second values of angle correlates written near each circle i are taken.

$$K_p = \underset{\substack{\text{no. of}\\ \text{angles}\\ \text{around}\\ \text{point}\\ (P)}}{1/2} \cdot \Big(\underset{\substack{\text{lower}\\ \text{figure}\\ \text{in circle }(P)}}{7} + \underbrace{\underset{\substack{\textbf{2nd.}\text{ value}\\ \text{in triangle}\\ 1}}{7} - \underset{\substack{\text{upper}\\ \text{figure}\\ \text{in circle }(P)}}{8.68} \times \underset{\substack{\textbf{1st.}\text{ value}\\ \text{of}\\ K_s}}{0.14}}_{\text{added with the opposite sign}} \Big) = 6.4,$$

this value is written under the first value 7.

$$K_{(1)} = \underset{\substack{\text{no. of}\\ \text{angles}\\ \text{around}\\ \text{point}\\ (1)}}{1/3} \cdot \Big(\underset{\substack{\text{lower}\\ \text{figure}\\ \text{in circle (1)}}}{-5} + \underbrace{\underset{\substack{\textbf{2nd.}\text{ values}\\ \text{in triangle}\\ \text{1 and 2}\\ \text{around}\\ \text{point (1)}}}{\underbrace{7 + 7.4}} - \underset{\substack{\text{upper}\\ \text{figure}\\ \text{in circle (1)}}}{11.38} \times \underset{\substack{\textbf{1st.}\text{ value}\\ \text{of}\\ K_s}}{0.14}}_{\text{added with the opposite sign}} \Big) = 2.6,$$

this value is written under the first value 3.

$$K_{(2)} = \underset{\substack{\text{no. of}\\ \text{angles}\\ \text{around}\\ \text{point}\\ (2)}}{1/4} \cdot \Big(\underset{\substack{\text{lower}\\ \text{figure}\\ \text{in circle (2)}}}{15} + \underbrace{\underset{\substack{\textbf{2nd.}\text{ values in tri-}\\ \text{angles 1, 2 and 3}\\ \text{around point (2)}}}{\underbrace{7 + 7.4 - 1.6}} - \underset{\substack{\text{upper}\\ \text{figure}\\ \text{in circle (2)}}}{8.59} \times \underset{\substack{\textbf{1st.}\text{ value}\\ \text{of}\\ K_s}}{0.14}}_{\text{added with the opposite sign}} \Big) = 6.7,$$

this value is written under the first value 6.

Similarly $K_{(3)} = 2.1$, $K_{(4)} = -3.7$, $K_{(5)} = -3.6$, $K_{(6)} = 1.6$, $K_q = 3.5$.

This process means that equation (62) is calculated on the sketch for the second time.

The side correlate K_s.

Second values of point correlates written at each point and angle correlates written in each triangle are multiplied by the upper figures in each circle and these are added, with the opposite sign, to the

Table 11.

Angles											
(1)	(1)	(10)	(3)	(12)	(4)	(5)	(8)	(9)	(11)	(13)	(6)
No.	Observed Values	Corrected Values	Initial Log. sin.	Corrected Log. sin.	Tabu. Diff. per 1″ Δ	d	$\Delta\cdot K_s$	Correction v	$\Delta\cdot v$	$v\cdot v$	$\Delta\cdot\Delta$
	° ′ ″	° ′ ″									
1	67 35 43	67 35 43.53	9.9659137	9.9659142	8.68	−0.10	1.069	0.53	4.6004	0.2809	75.3424
2	67 21 7	67 21 5.74	6.9651487	9.9651476	8.78		−1.081	−1.26	−11.0628	1.5876	77.0884
3	45 3 15	45 3 10.73						−4.27		18.2329	
	180 00 5	180 00 00.00	$\omega_1 = -5$								
4	61 36 10	61 36 6.52	9.9443206	9.9443166	11.38	−7.15	1.401	−3.48	−39.6024	12.1104	129.5044
5	48 38 10	48 38 2.27	9.8753667	9 8753524	18.53		−2.281	−7.73	−143.2369	59.7529	343.3609
6	69 45 52	69 45 51.21						−0.79		0.6241	
	180 00 12	180 00 00.00	$\omega_2 = -12$								
7	50 29 3	50 29 13.23	9.8873071	9.8873249	17.37	8 30	2.138	10.23	177.6951	104.6529	301.7169
8	66 41 17	66 41 13.34	9.9630148	9.9630115	9.07		−1.117	−3.66	−33.1962	13.3956	82.2649
9	62 49 30	62 49 33.43						3.43		11.7649	
	179 59 50	180 00 00.00	$\omega_3 = 10$								
10	58 12 47	58 12 55.12	9.9294254	9.9294360	13.05	−3 97	1.607	8.12	105.9660	65.9344	170.3025
11	51 3 00	51 2 58.34	9.8908092	9.8908064	17.02		−2.095	−1.66	− 28.2532	2.7556	289.6804
12	70 44 6	70 44 6.54						0.54		0 2916	
	179 59 53	180 00 00.00	$\omega_4 = 7$								
13	54 33 43	54 33 39.93	9.9110205	9.9110159	14.98	6.33	1.844	− 3.07	−45.9886	9.4249	224.4004
14	67 39 3	67 39 2.09	9.9660872	9.9660864	8.65		−1.065	−0.91	− 7.8715	0.8281	74.8225
15	57 47 23	57 47 17.98						−5.02		25.2004	
	180 00 9	180 00 00.00	$\omega_5 = -9$								
16	68 35 17	68 35 19.58	9.9626874	9.9626898	9.12	−13.46	1.123	2.58	23.5296	6.6564	83.1744
17	43 00 00	43 00 5.79	9 8337833	9.8337964	22.58		−2.780	5.79	130.7382	33.5241	509.8564
18	68 24 28	68 24 34.63						6.63		43.9569	
	179 59 45	180 00 00.00	$\omega_6 = 15$								
B_1	270.418		2.4320356						$\Sigma\,\Delta\cdot v =$		$d_s =$
B_2	345.567		2.5385323						319.0825		2,361.5145
			$\omega_s = 319$								

Table 11.—(*Continued*)

Angles											
(1)	(2)	(10)	(3)	(12)	(4)	(5)	(8)	(9)	(11)	(13)	(6)
No.	Observed Values	Corrected Values	Initial Log. sin.	Corrected Log. sin.	Tabu. Diff. per 1″ Δ	d	$\Delta \cdot K_s$	Correction v	$\Delta \cdot v$	$v\ v$	$\Delta \cdot \Delta$
	° ′ ″	° ′ ″									
(P)	292 24 8	292 24 14.47	$\omega_p =$ 7			8.68		6.47		41.8609	
(1)	253 20 40	253 20 42.75	$\omega_{(1)} =$ −5			11.38		2.75		7.5625	
(2)	172 23 43	172 23 49.82	$\omega_{(2)} =$ 15			8.59		6 82		46.5124	
(3)	190 19 27	190 19 29.18	$\omega_{(3)} =$ 6			− .48		2.18		4.7524	
(4)	168 1 4	168 1 0.22	$\omega_{(4)} =$ −10			5.91		−3.78		14.2884	
(5)	182 34 28	182 34 24.10	$\omega_{(5)} =$ −8			−7.90		−3.90		15.2100	
(6)	223 56 22	223 56 23.28	$\omega_{(6)} =$ 7			−8.65		1.28		1.6384	
(Q)	316 59 51	316 59 54.21	$\omega_q =$ 9			−22.58		3.21		10.3041	
										$\Sigma v \cdot v =$ 559.7601	

Probable Error $= \pm 0.6745\sqrt{559.7601/15} = \pm 4.''12$

(7)

Correlate Sketch.

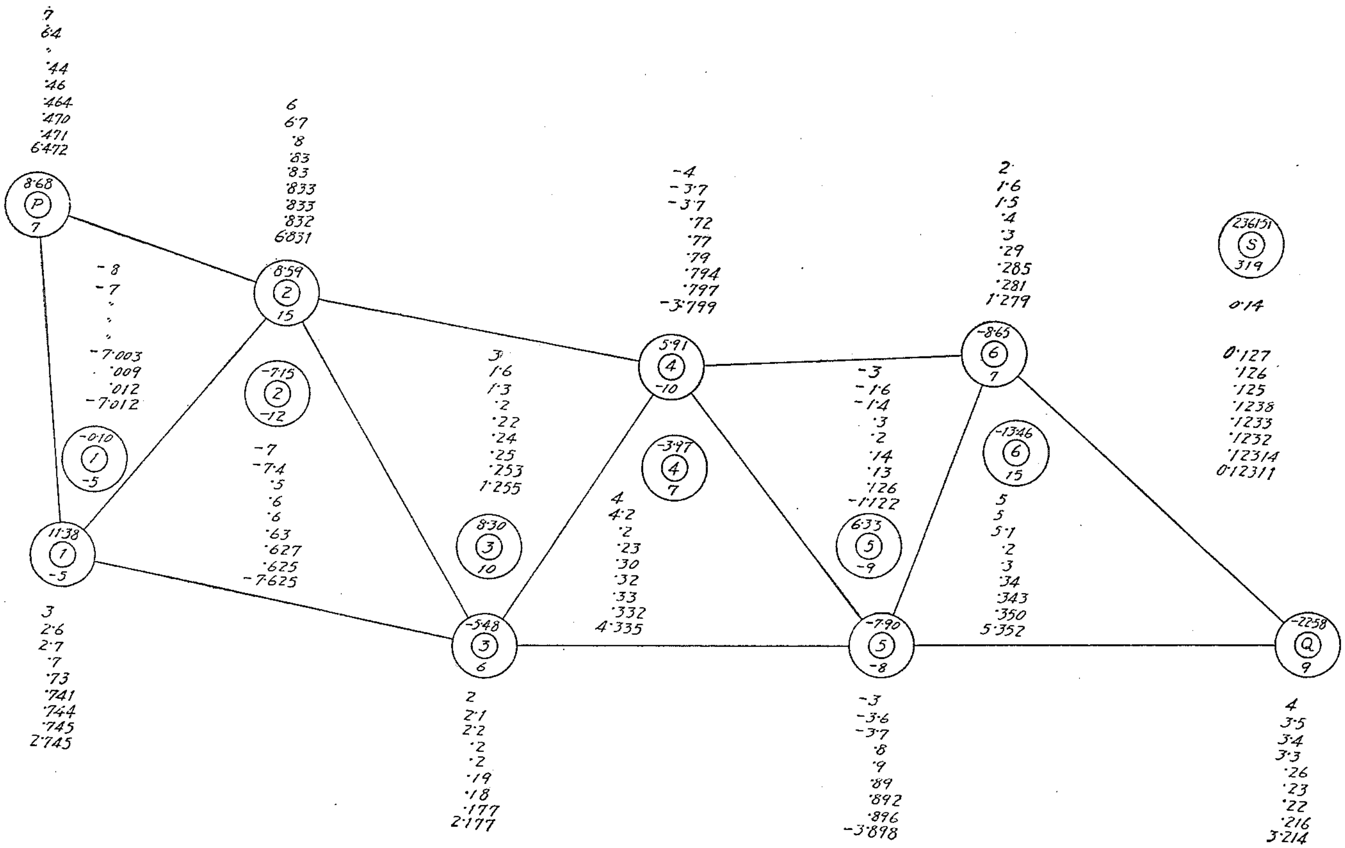

Fig. 14.

lower figure 319 in circle S, and then this value is divided by the upper figure 2,361.51 in circle S.

$$K_s = \underset{\substack{\text{upper}\\ \text{figure}\\ \text{in circle } S}}{1/2{,}361.51} \cdot \Big\{ \underset{\substack{\text{lower}\\ \text{figure}\\ \text{in circle } S}}{319} - \Big(\underbrace{\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in circle } (P)}}{8.68} \times \underset{\substack{\textbf{2nd.}\\ \text{value}\\ \text{of}\\ K_p}}{6.4}}_{\text{at point } (P)} + \underbrace{\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in circle } (1)}}{11.38} \times \underset{\substack{\textbf{2nd.}\\ \text{value}\\ \text{of}\\ K_{(1)}}}{2.6}}_{\text{at point } (1)} + \underset{\cdots\cdot}{\cdots\cdot}$$

$$- \underbrace{\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in circle } 1}}{0.10} \times \underset{\substack{\textbf{2nd.}\\ \text{value}\\ \text{of}\\ K_1}}{-7}}_{\text{in triangle } 1} - \underbrace{\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in circle } 2}}{7.15} \times \underset{\substack{\textbf{2nd.}\\ \text{value}\\ \text{of}\\ K_2}}{-7.4}}_{\text{in triangle } 2} + \cdots\cdot \Big) \Big\}$$

$$= 0.127 .$$

This value is written under the first value 0.14.

This process means that equation (64) is calculated on the sketch for the second time.

Thus the first round of calculation is over, and the next rounds are carried out. In this case, the calculation is stopped at the eighth round; the time required for this calculation is four hours.

After these values of all correlates are determined, the values of corrections of observed angles are obtained from equation (**23**) in §3 in Chapter 1.

Numerical Example 6.

A Single Row Triangulation Net consisting of six triangles.

In **Example 5** above, the outer angles are assumed to be adjusted at each station independently, or to be not observed; it is proposed to adjust inner angles only. The following Table 12 and the process are all similar to **Example 5**, but the process becomes simpler. Now only the **Correlate Sketch** will be explained (Fig. 15). In this case, the **first value** is only that of K_s from (**66**) as follows;

$$K_s = 319/2{,}361.51 = 0.14.$$

With this value, the first round of calculation is begun as follows:

Angles correlates K_i.

The product of the upper figure in circle i multiplied by the first value of K_s is added, with the opposite sign, to the lower figure in circle i, and this value is divided by the number of the observed angles in the triangle, namely constant 3.

$$K_1 = \underbrace{1/3}_{\text{no. of angles in triangle 1}} \cdot \Big(\underbrace{-5}_{\text{lower figure in circle 1}} + \underbrace{\underbrace{0.10}_{\text{upper figure in circle 1}} \times \underbrace{0.14}_{\text{1st. value of } K_s}}_{\text{added with the opposite sign}}\Big) = -1.66\,,$$

this value is written near its circle 1.

$$K_2 = \underbrace{1/3}_{\text{no. of angles in triangle 2}} \cdot \Big(\underbrace{-12}_{\text{lower figure in circle 2}} + \underbrace{\underbrace{7.15}_{\text{upper figure in circle 2}} \times \underbrace{0.14}_{\text{1st. value of } K_s}}_{\text{added with the opposite sign}}\Big) = -3.67\,,$$

this value is written near the circle 2.

Similarly $K_3 = 2.95$, $K_4 = 2.52$, $K_5 = -3.30$, $K_6 = 5.63$.
This process shows that equation (63) is calculated on the sketch.

The side correlate K_s.

The products of the upper figures in each circle and second values of angle correlates in the corresponding triangles are added, with the opposite sign, to the lower figure 319 in circle S, and this value is divided by the upper figure 2,361.51 in circle S.

$$K_s = \underbrace{1/2{,}361.51}_{\text{upper figure in circle } S} \cdot \Big\{\underbrace{319}_{\text{lower figure in circle } S} - \Big(\underbrace{-\underbrace{0.10}_{\text{upper figure in circle 1}} \times \underbrace{-1.66}_{\text{1st. value of } K_1}}_{\text{in triangle 1}} \underbrace{-\underbrace{7.15}_{\text{upper figure in circle 2}} \times \underbrace{-3.67}_{\text{1st. value of } K_2}}_{\text{in triangle 2}} + \underbrace{\cdots\cdot}_{\cdots\cdot}\Big)\Big\}$$

$$= 0.159\,.$$

This value is written under the first value 0.14, and this process means that equation (64) is calculated on the sketch.

The first round is over, then their succeeding rounds are continued. In this case, the convergency is very rapid, so the calculation can be stopped at the third round. The time required is less than one hour.

After the values of all correlates are determined, the corrections of observed angles are obtained from equation (26) in § 4 in Chapter I.

Table 12.

Angles											
(1)	(2)	(10)	(3)	(12)	(4)	(5)	(8)	(9)	(11)	(13)	(6)
No.	Observed Values	Corrected Values	Initial Log. sin.	Corrected Log. sin.	Tabu. Diff. per 1″ Δ	d	$\Delta \cdot K_s$	Correction v	$\Delta \cdot v$	$v\ v$	$\Delta \cdot \Delta$
	° ′ ″	° ′ ″									
1	67 35 43	67 35 42.72	9.9659137	9.9659135	8.68	−0.10	1 386	−0.28	− 2.4304	0.0784	75.3424
2	67 21 7	67 21 3.94	9.9651487	9.9651460	8.78		1.402	−3.06	−26.8668	9.3636	77.0884
3	45 3 15	45 3 13.34						−1.66		2.7556	
	180 00 5	180 00 00.00	$\omega_1 = -5$								
4	61 36 10	61 36 8.20	9.9443206	9.9443186	11.38	−7.15	1.817	−1.80	−20.4840	3.2400	129.5044
5	48 38 10	48 38 3.42	9.8753667	9.8753545	18.53		2.959	−6.58	−121.9274	43 2964	343.3609
6	69 45 52	69 45 48.38						−3.62		13.1044	
	180 00 12	180 00 00.00	$\omega_2 = -12$								
7	50 29 3	50 29 8.67	9.8873071	9.8873170	17.37	8.30	2.774	5.67	98.4879	32.1489	301.7169
8	66 41 17	66 41 18.44	9.9630148	9 9630161	9.07		1.448	1.44	13.0608	2.0736	82.2649
9	62 49 30	62 49 32.89						2.89		8.3521	
	179 59 50	180 00 00.00	$\omega_3 = 10$								
10	58 12 47	58 12 51.63	9.9294254	9.9294314	13.05	−3.97	2.084	4.63	60.4215	21.4369	170.3025
11	51 3 00	51 2 59.83	9.8908092	9.8908089	17.02		2.718	−0.17	−2.8934	0.0289	289.6804
12	70 44 6	70 44 8.54						2.54		6.4516	
	179 59 53	180 00 00.00	$\omega_4 = 7$								
13	54 33 43	54 33 42.06	9.9110205	9.9110191	14.98	6.33	2.392	−0.94	−14.0812	0.8836	224.4004
14	67 39 3	67 38 58.28	9.9660872	9.9660831	8.65		1.381	−4.72	−40.8280	22.2784	74.8225
15	57 47 23	57 47 19.66						−3.34		11.1556	
	180 00 9	180 00 00.00	$\omega_5 = -9$								
16	68 35 17	68 35 24.17	9.9626874	9.9626939	9.12	−13.46	1.456	7.17	65.3904	51.4089	83.1744
17	43 00 00	43 00 2.11	9.8337833	9.8337881	22.58		3.606	2.11	47.6438	4.4521	509.8564
18	68 24 28	68 24 33.72						5.72		32.7184	
	179 59 45	180 00 00.00	$\omega_6 = 15$								
B_1	270.418		2.4320356						$\Sigma \Delta \cdot v =$	$\Sigma v \cdot v =$	$d_s =$
B_2	345.567		2.5385323						319.1152	265.2274	2,361.5145
			$\omega_s = 319$								

$$\text{Probable Error} = \pm 0.6745\sqrt{265.2274/7} = \pm 4.15''$$

(7)

Correlate Sketch.

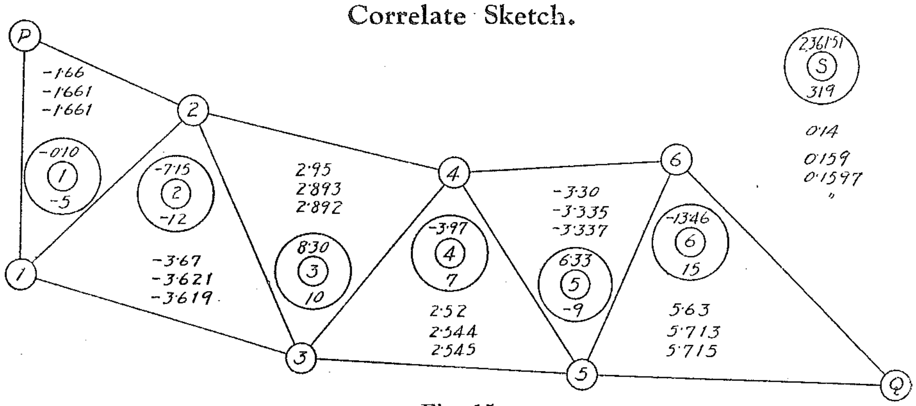

Fig. 15.

§5. A QUADRILATERAL.

Equations (34) and (37) in §5 in Chapter 1 can be expressed in the following Tables 13 and 14 respectively.

Table 13.

Sign of equation	Left Side of equation								Right Side of equation
	$K_{(1)}$	$K_{(2)}$	$K_{(3)}$	$K_{(4)}$	K_1	K_2	K_3	K_s	
(1)	3				1		1	$d_{(1)}$	$\omega_{(1)}$
(2)		3			2	1		$d_{(2)}$	$\omega_{(2)}$
(3)			3		1	2	1	$d_{(3)}$	$\omega_{(3)}$
(4)				3		1	2	$d_{(4)}$	$\omega_{(4)}$
1	1	2	1		4	2		d_1	ω_1
2		1	2	1	2	4	2	d_2	ω_2
3	1		1	2		2	4	d_3	ω_3
S	$d_{(1)}$	$d_{(2)}$	$d_{(3)}$	$d_{(4)}$	d_1	d_2	d_3	d_s	ω_s

Table 14.

Sign of equation	Left Side of equation				Right Side of equation
	K_1	K_2	K_3	K_s	
1	2	1		d_1	ω_1
2	1	2	1	d_2	ω_2
3		1	2	d_3	ω_3
S	d_1	d_2	d_3	d_s	ω_s

In this case, the peculiarity is that the triangles composing the quadrilateral all intersect, so this appears in the Tables.

Inquiring Table 13, there are the following dispositions.

The point equations:

1. The coefficients of the point correlates are the number of angles around each station, namely 3.

2. The angle correlates are those of triangles with touching at the station where the point equation is composed, and their coefficients are the number of angles in the triangles which are concentrated at the station.

For example in point equation (3), the angle correlates are K_1, K_2 and K_3 and their coefficients are 1, 2 and 1 respectively, because the angles at station (3) are *4* in triangle 1, *4* and *5* in triangle 2 and *5* in triangle 3.

The angle equations:

3. The coefficient of the angle correlate of the triangle at which the angle equation is composed is the number of the observed angles in the triangle, that is always 4 in this kind of a quadrilateral.

4. In this case, there are the other angle correlates in the angle equation. They are those of the triangles intersecting with the triangle and their coefficients are the number of angles which are common in the intersected triangles.

5. The point correlates are those at the stations in the triangle and their coefficients are the number of angles concentrated at each station.

The other points are all the same as described in the other former cases.

Table 14 represents only the angle equations, the lower right one quarter from Table 13 and in it *all coefficients and absolute terms are devided by 2.*

The special points known from Table 14 are as follows.

6. The large coefficients in diagonal are 2 and the small ones are all 1.

7. The correlate whose coefficient is 2 is the angle correlate of the triangle intersected with the triangle in which the angle equation is composed.

Point, angle and side correlates can be conveniently obtained from the following equations (67), (68) and (69) respectively.

$$\left.\begin{aligned}
K_{(1)} &= 1/3\cdot(\omega_{(1)}-K_1-K_3-d_{(1)}\cdot K_s)\\
K_{(2)} &= 1/3\cdot(\omega_{(2)}-2K_1-K_2-d_{(2)}\cdot K_s)\\
K_{(3)} &= 1/3\cdot(\omega_{(3)}-K_1-2K_2-K_3-d_{(3)}\cdot K_s\\
K_{(4)} &= 1/3\cdot(\omega_{(4)}-K_2-2K_3-d_{(4)}\cdot K_s)
\end{aligned}\right\}\cdots\cdots\cdots (67)$$

$$\left.\begin{aligned}
K_1 &= 1/4\cdot(\omega_1-K_{(1)}-2K_{(2)}-K_{(3)}-2K_2-d_1\cdot K_s)\\
K_2 &= 1/4\cdot(\omega_2-K_{(2)}-2K_{(3)}-K_{(4)}-2K_1-2K_3-d_2\cdot K_s)\\
K_3 &= 1/4\cdot(\omega_3-K_{(3)}-2K_{(4)}-K_{(1)}-2K_2-d_3\cdot K_s)
\end{aligned}\right\}\cdots (68)$$

$$K_s = 1/d_s\cdot\left(\omega_s-\sum_{i=1}^{4} d_{(i)}\cdot K_{(i)}-\sum_{i=1}^{3} d_i\cdot K_i\right).\quad\cdots\cdots\cdots\cdots (69)$$

The **Correlate Sketch** is as in Fig. 16 or Fig. 17.
Fig. 16 is used when all inner and outer angles are observed, Fig. 17 is used when the inner angles only are to be adjusted.

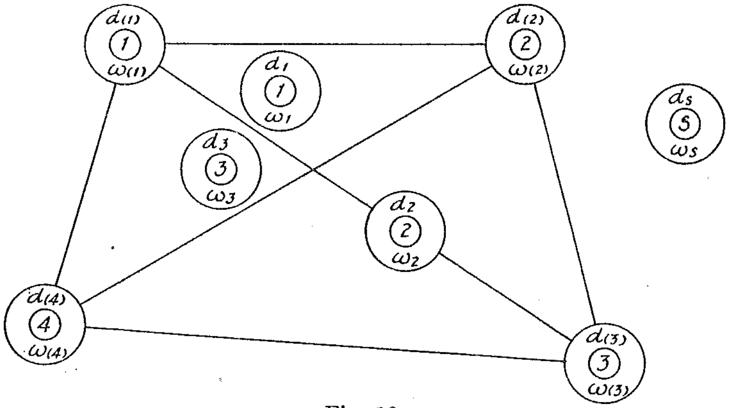

Fig. 16.

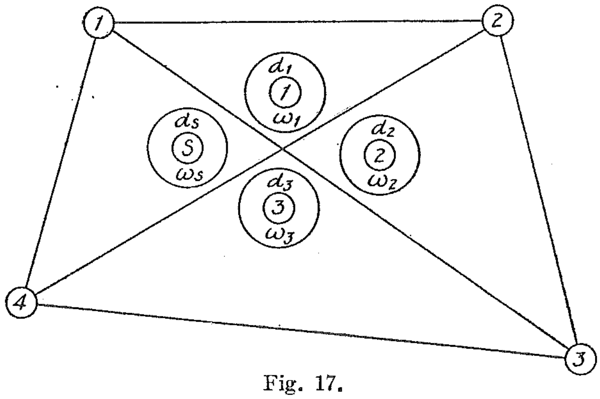

Fig. 17.

The process of calculation is a little complex than the former cases and is as follows :

1. First values of point and angle correlates are calculated and written near their own double circles on the sketch (first values will be explained later).

2. The side correlate K_s.

The products of the upper figures in each circle and the first values of corresponding point and angle correlates are added, with the opposite sign, to the lower figure in circle S, and this value is divided by the upper figure in circle S. Thus the first value of K_s is obtained and written near circle S.

3. The point correlates $K_{(i)}$.

First values of the angle correlates of the triangles whose angles meet at point (i) and the product of the upper figure in circle (i) and the first value of K_s above obtained are added, with the opposite sign, to the lower figure in circle (i), and this value is divided by the number of angles around point (i), namely 3. Thus the second values of the point correlates are obtained and written under the first values.

In this case, the values of angle correlates are added once if one angle and doubled if two angles in the triangle are concentrated at the point. For example, at point (2), the angle correlate K_1 is added two times, as the angles *2* and *3* in triangle 1 meet at the point. The other correlate is K_2 and the angle is only *3* in triangle 2, so K_2 is added once only.

4. Angle correlates K_i.

Second values of point correlates obtained just above and written at the points of which the triangle i is composed, the values of angle correlates obtained just before of the triangles which are intersected with triangle i, and the product of (the upper figure in circle i)$\times$(the first value of K_s) are given the opposite sign and then added to the lower figure in circle i. Then this value is divided by the number of observed angles in the triangle, namely 4. The number of times of addition of point correlates is the number of angles in the triangle concentrated to the point, and that of angle correlates is the number of common angles in the intersected triangle, namely always 2.

5. The second value of the side correlate K_s is obtained as in 1, but the second values of all correlates are taken.

These processes 2, 3, 4 and 5 are such that equations (69), (67), (68) and (69) are respectively calculated entirely on the sketch and this is the first round of calculation. After the first round of calculation is over, the second and the third rounds are continued.

In the case of Table 14, the equations are limited to (70) and (71) as follows;

$$\left.\begin{aligned} K_1 &= 1/2\cdot(\omega_1 - K_2 - d_1\cdot K_s) \\ K_2 &= 1/2\cdot(\omega_2 - K_1 - K_3 - d_2\cdot K_s) \\ K_3 &= 1/2\cdot(\omega_3 - K_2 - d_3\cdot K_s) \end{aligned}\right\} \quad \cdots\cdots\cdots \quad (70)$$

$$K_s = 1/d_s\cdot\left(\omega_s - \sum_{i=1}^{3} d_i\cdot K_i\right), \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (71)$$

where the values of ω and d are given in (40) in §5 in Chapter 1 and are **a half of those** in (67), (68), and (69).

The process of obtaining the value of K_s is the same as 2. above, but that of obtaining the angle correlates is simpler and as follows. The values of angle correlates obtained just before in the intersected triangle and the product of the upper figure in each circle by the first value of K_s are added, with the opposite sign, to the lower figure in each circle, and this value is divided by the costant 2. Thus the values of the angle correlates are gotten and written near their own circles.

The **first values** of point and angle correlates are conveniently obtained from the following **Table 15** and **16** for the cases of Tables 13 and 14 respectively.

Table 15.

	$\omega_{(1)}$	$\omega_{(2)}$	$\omega_{(3)}$	$\omega_{(4)}$	ω_1	ω_2	ω_3
$K_{(1)}$	7	2	1	2	-4.5	3	-4.5
$K_{(2)}$	2	7	2	1	-4.5		-1.5
$K_{(3)}$	1	2	7	2	-1.5	-3	-1.5
$K_{(4)}$	2	1	2	7	-1.5		-4.5
K_1	-4.5	-4.5	-1.5	-1.5	9	-4.5	4.5
K_2	3		-3		-4.5	9	-4.5
K_3	-4.5	-1.5	-1.5	-4.5	4.5	-4.5	9

factor 1/12

Table 16.

	ω_1	ω_2	ω_3
K_1	3	-2	1
K_2	-1	2	-1
K_3	1	-2	3

factor 1/4

Table 15 is obtained from equations (87) and (89) in §4 in Chapter 3 by taking the terms of ω only and omitting the terms of K_s, **Table 16** is obtained from equation (38) in §5 in Chapter 1.

Numerical example 7. A quadrilateral.

All inner and outer angles are assumed to be observed; the problem is to adjust them. Figures obtained by calculation are shown in Table 17. Numbers in the above are orders of writing and calculation.

(1) ; number of angles,

(2) ; observed values of angles and sum of four angles in each triangle,

(3) ; values of log.sin. of observed inner angles, and values of ω_1, ω_s and $\omega_{(i)}$,

(4) ; tabular difference of log.sin. per 1'',

(5) ; d_i and $d_{(i)}$, for example
$d_1 = \Delta_1 - \Delta_2 + \Delta_3 - \Delta_4 = 21.9 - 34.0 + 8.1 - 29.8 = -33.8$,
$d_{(1)} = \Delta_1 - \Delta_8 = 21.9 - 44.8 = -22.9$, & etc.,

(6) ; $\Delta \cdot \Delta$ for inner angles,

(7) ; **Correlate Sketch**, this is the main point of this paper, by means of which values of all correlates are obtained (this process will be explained later),

(8) ; $\Delta \cdot K_s$; K_s is obtained from (7),

(9) ; corrections v from $K_{(i)}$ and K_i in (7) and $\Delta \cdot K_s$ in (8),

(12) ; corrected values of observed angles from (9),

(13) ; corrected log. sin. for calculation of side length of the quadrilateral, with $\Delta \cdot v$ in (10),

(11) ; $v \cdot v$ for the calculation of probable error.

Now **Correlate Sketch** in (7) will be explained (Fig. 18). Figures in the double circles are d and ω which are shown in (5) in Table 17, for example,

$d_1 = -33.8$, $\omega_1 = -9.4$, in circle 1,
$d_{(1)} = -22.9$, $\omega_{(1)} = 5.1$, in circle (1), & etc.

First values are obtained from **Table 15** as follows:
This calculation can be carried out by writing the numerical values of ω on the top column in **Table 15**, performing the multiplication mentally or with a slide rule and the addition with a pencil or with a soroban.

Point correlates $K_{(1)}$.

$$\begin{aligned}
K_{(1)} &= 1/12 \cdot (7 \times 5.1 + 2 \times -6.4 - 1 \times 7.7 + 2 \times -6.1 - 4.5 \times -9.4 \\
&\qquad + 3 \times -11.5 - 4.5 \times -6.3) \\
&= 1/12 \cdot (35.7 - 12.8 + 7.7 - 12.2 + 42.3 - 34.5 + 28.4) \\
&= 54.6/12 = 4.6, \\
K_{(2)} &= 1/12 \cdot (2 \times 5.1 + 7 \times -6.4 + 2 \times 7.7 + 1 \times -6.1 \\
&\qquad - 4.5 \times -9.4 \qquad - 1.5 \times -6.3) \\
&= 1/12 \cdot (10.2 - 44.8 + 15.4 - 6.1 + 42.3 \qquad + 9.5) \\
&= 26.5/12 = 2.2,
\end{aligned}$$

similarly $K_{(3)} = 7.7, \quad K_{(4)} = 1.6.$

Angle correlates K_i.

$$\begin{aligned}
K_1 &= 1/12 \cdot (-4.5 \times 5.1 - 4.5 \times -6.4 - 1.5 \times 7.7 - 1.5 \times -6.1 \\
&\qquad + 9 \times -9.4 - 4.5 \times -11.5 + 4.5 \times -6.3) \\
&= 1/12 \cdot (-23.0 + 28.8 - 11.5 + 9.2 - 84.6 + 51.7 - 28.4) \\
&= -57.8/12 = -4.8,
\end{aligned}$$

similarly $K_2 = -3.4, \quad K_3 = -3.7.$

These first values are written near their own circles.

The side correlate K_s.

First values of each correlate are multiplied by the upper figures in each circle and these products are added, with the opposite sign, to the lower figure $-1,224$ in circle S, and this value is divided by the upper figure $17,297$ in circle S.

$$K_s = \underbrace{1/17,297}_{\substack{\text{upper figure}\\ \text{in circle } S}} \cdot \Big\{ \underbrace{-1,224}_{\substack{\text{lower figure}\\ \text{in circle } S}} - \Big(\underbrace{\underbrace{-22.9}_{\substack{\text{upper figure}\\ \text{in circle (1)}}} \times \underbrace{4.6}_{\substack{\text{1st. value}\\ \text{of } K_{(1)}}}}_{\text{at point (1)}} \underbrace{\underbrace{-25.9}_{\substack{\text{upper figure}\\ \text{in circle (2)}}} \times \underbrace{2.2}_{\substack{\text{1st. value}\\ \text{of } K_{(2)}}}}_{\text{at point (2)}} + \cdots\cdots$$

$$+ \underbrace{\underbrace{33.8}_{\substack{\text{upper figure}\\ \text{in circle 1}}} \times \underbrace{4.8}_{\substack{\text{1st. value}\\ \text{of } K_1}}}_{\text{in triangle 1}} \underbrace{\underbrace{-80.7}_{\substack{\text{upper figure}\\ \text{in circle 2}}} \times \underbrace{3.4}_{\substack{\text{1st. value}\\ \text{of } K_2}}}_{\text{in triangle 2}} \underbrace{\underbrace{-61.6}_{\substack{\text{upper figure}\\ \text{in circle 3}}} \times \underbrace{3.7}_{\substack{\text{1st. value}\\ \text{of } K_3}}}_{\text{in triangle 3}} \Big) \Big\}$$

$$= -0.1 .$$

This value is written near circle S, and the process means that equation (69) is solved on the sketch.

Second values of correlates are obtained as follows:

Point correlates $K_{(i)}$.

First values of angle correlates in triangles whose vertex is point (i) and the product of the upper figure in circle (i) and the first value of K_s near circle S are added, with the opposite sign, to the lower figure in circle (i), and then this value is divided by the number of angles around point (i), namely 3.
Number of times of addition of angle correlates of the triangle are the number of angles concentrated at point (i) and in that triangle. That is,

$$K_{(1)} = \underbrace{1/3}_{\substack{\text{no. of}\\ \text{angles}\\ \text{around}\\ \text{point}\\ \text{(1)}}} \cdot (\underbrace{5.1}_{\substack{\text{lower}\\ \text{figure}\\ \text{in cir-}\\ \text{cle (1)}}} + \underbrace{\underbrace{\underbrace{3.7}_{\substack{\text{in tri-}\\ \text{angle 3}}} + \underbrace{4.8}_{\substack{\text{in tri-}\\ \text{angle 1}}}}_{\substack{\text{1st. values of}\\ \text{angle correlates}}} - \underbrace{22.9}_{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle (1)}}} \times \underbrace{0.1}_{\substack{\text{1st.}\\ \text{value}\\ \text{of}\\ K_s}}}_{\text{added with the opposite sign}}) = 4\,,$$

This value 4 is written under the first value 4.6.

$$K_{(2)} = \underbrace{1/3}_{\substack{\text{no. of}\\ \text{angles}\\ \text{around}\\ \text{point}\\ \text{(2)}}} \cdot (\underbrace{-6.4}_{\substack{\text{lower}\\ \text{figure}\\ \text{in cir-}\\ \text{cle (2)}}} + \underbrace{\underbrace{\underbrace{4.8 + 4.8}_{\substack{\text{2 times, in}\\ \text{triangle 1}}} + \underbrace{3.4}_{\substack{\text{in tri-}\\ \text{angle 2}}}}_{\substack{\text{1st. value of angle}\\ \text{correlates}}} - \underbrace{25.9}_{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle (2)}}} \times \underbrace{0.1}_{\substack{\text{1st.}\\ \text{value}\\ \text{of}\\ K_s}}}_{\text{added with the opposite sign}}) = 2\,,$$

This value 2 is written under the first value 2.2.

$$K_{(3)} = \underbrace{1/3}_{\substack{\text{no. of}\\ \text{angles}\\ \text{around}\\ \text{point}\\ \text{(3)}}} \cdot (\underbrace{7.7}_{\substack{\text{lower}\\ \text{figure}\\ \text{in cir-}\\ \text{cle (3)}}} + \underbrace{\underbrace{\underbrace{4.8}_{\substack{\text{in tri-}\\ \text{angle 1}}} + \underbrace{3.4 + 3.4}_{\substack{\text{2 times, in}\\ \text{triangle 2}}} + \underbrace{3.7}_{\substack{\text{in tri-}\\ \text{angle 3}}}}_{\text{1st. value of angle correlates}} + \underbrace{82.4}_{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle (3)}}} \times \underbrace{0.1}_{\substack{\text{1st.}\\ \text{value}\\ \text{of}\\ K_s}}}_{\text{added with the opposite sign}})$$

$$= 10.0\,.$$

This value 10.0 is written under the value 7.7.

$$K_{(4)} = \underset{\substack{\text{no. of}\\\text{angles}\\\text{around}\\\text{point}\\\text{(4)}}}{1/3}\cdot(\underset{\substack{\text{lower}\\\text{figure}\\\text{in cir-}\\\text{cle (4)}}}{-6.1} \underbrace{\underbrace{+\underset{\substack{\text{in tri-}\\\text{angle 2}}}{3.4} + \underbrace{3.7+3.7}_{\substack{\text{2 times, in}\\\text{triangle 3}}}}_{\substack{\text{1st. values of angle}\\\text{correlates}}} - \underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle (4)}}}{5.8} \times \underset{\substack{\text{1st.}\\\text{value}\\\text{of}\\K_s}}{0.1}}_{\text{added with the opposite sign}}) = 1.0\,.$$

This value 1.0 is written under the first value 1.6.

This process means that equation (67) is solved on the sketch.

Angle correlates K_i.

Second values of point correlates at three points of the triangle i, values of the angle correlates obtained just before and written in the triangles which intersect triangle i and the product of the upper figure in circle i and the first value of K_s are added, with the opposite sign, to the lower figure in circle i, and this value is divided by the number of observed angles in the triangle, namely 4.

Number of addition of point correlates is the number of observed angles concentrated at the points, and that of other angle correlates is the common observed angles, namely 2.

$$K_1 = \underset{\substack{\text{no. of}\\\text{angles}\\\text{in tri-}\\\text{angle 1}}}{1/4}\cdot(\underset{\substack{\text{lower}\\\text{figure}\\\text{in cir-}\\\text{cle 1}}}{-9.4} \underbrace{\underbrace{-\underset{\substack{\text{at}\\\text{point}\\\text{(1)}}}{4} - \underbrace{2-2}_{\substack{\text{2 times,}\\\text{at point}\\\text{(2)}}} - \underset{\substack{\text{at}\\\text{point}\\\text{(3)}}}{10}}_{\text{2nd. values}} + \underbrace{\underbrace{3.4+3.4}_{\substack{\text{2 times, in}\\\text{triangle 2}}}}_{\text{1st. value}} - \underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle 1}}}{33.8} \times \underset{\substack{\text{1st.}\\\text{value}\\\text{of}\\K_s}}{0.1}}_{\text{added with the opposite sign}})$$

$$= -6.0\,.$$

This value −3.0 is written under the first value −4.8.

$$K_2 = \underset{\substack{\text{no. of}\\\text{angles}\\\text{in tri-}\\\text{angle 2}}}{1/4}\cdot(\underset{\substack{\text{lower}\\\text{figure}\\\text{in cir-}\\\text{cle 2}}}{-11.5} \underbrace{\underbrace{-\underset{\substack{\text{at}\\\text{point}\\\text{(2)}}}{2} - \underbrace{10-10}_{\substack{\text{2 times,}\\\text{at}\\\text{point (3)}}} - \underset{\substack{\text{at}\\\text{point}\\\text{(4)}}}{1.0} + \underbrace{6+6}_{\substack{\text{2 times,}\\\text{in tri-}\\\text{angle 1}}}}_{\substack{\text{2nd. values of point and}\\\text{angle correlates}}} + \underbrace{\underbrace{3.7+3.7}_{\substack{\text{2 times, in}\\\text{triangle 3}}}}_{\substack{\text{1st. value of}\\\text{angle correlate}}} + \underset{\substack{\text{upper}\\\text{figure}\\\text{in cir-}\\\text{cle 1}}}{80.7} \times \underset{\substack{\text{1st.}\\\text{value}\\\text{of}\\K_s}}{0.1}}_{\text{added with the opposite sign}})$$

$$= -2\,.$$

This value −2 is written under the first value −3.4.

$$K_3 = 1/\underset{\substack{\text{no. of}\\ \text{angles}\\ \text{in tri-}\\ \text{angle 3}}}{4}\cdot(\underset{\substack{\text{lower}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 3}}}{-\,6.3}\ \underbrace{\underbrace{\underset{\substack{\text{at}\\ \text{point}\\ \text{(3)}}}{-\,10}\ \underset{\substack{\text{2 times,}\\ \text{at point}\\ \text{(4)}}}{\underbrace{-\,1.0-1.0}}\ \underset{\substack{\text{at}\\ \text{point}\\ \text{(1)}}}{-\,4}\ \underset{\substack{\text{2 times,}\\ \text{in tri-}\\ \text{angle 2}}}{\underbrace{+\,2+2}}}_{\substack{\text{2nd. values of point and}\\ \text{angle correlates}}}\ +\ \underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 3}}}{61.6}\times\underset{\substack{\text{1st.}\\ \text{value}\\ \text{of}\\ K_s}}{0.1}}_{\text{added with the opposite sign}})$$

$$= -3\,.$$

This value -3 is written under the first value -3.7.

These processes mean that equation (68) is solved on the sketch.

The *side correlate* K_s.

The same process as in the case of the first value of K_s is performed. But the second values of all correlates are taken.

$$K_s = 1/\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle } S}}{17{,}297}\cdot\Big\{-\underset{\substack{\text{lower}\\ \text{figure}\\ \text{in cir-}\\ \text{cle } S}}{1{,}224}\Big(\underbrace{-\,\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle (1)}}}{22.9}\times\underset{\substack{\text{2nd.}\\ \text{value}\\ \text{of}\\ K_{(1)}}}{4}}_{\text{at point (1)}}\ \underbrace{-\,\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle (2)}}}{25.9}\times\underset{\substack{\text{2nd.}\\ \text{value}\\ \text{of}\\ K_{(2)}}}{2}}_{\text{at point (2)}}+\cdots\cdots$$

$$\underbrace{+\,\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 1}}}{33.8}\times\underset{\substack{\text{2nd.}\\ \text{value}\\ \text{of}\\ K_1}}{6}}_{\text{in triangle 1}}\ \underbrace{-\,\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 2}}}{80.7}\times\underset{\substack{\text{2nd.}\\ \text{value}\\ \text{of}\\ K_2}}{2}}_{\text{in triangle 2}}\ \underbrace{-\,\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 3}}}{61.6}\times\underset{\substack{\text{2nd.}\\ \text{value}\\ \text{of}\\ K_3}}{3}}_{\text{in triangle 3}}\Big)\Big\}$$

$$= -0.101\,.$$

This value -0.101 is written under its first value -0.1.

This process means that equation (69) is solved for the second time.

Thus the first round of calculation is over, and the next rounds are continued. In this case, the calculation is stopped at the seventh round; the time required for this calculation is two and a half hours. After these values of all correlates are determined, the values of corrections of angles are obtained from (33) in §5 in Chapter 1.

Table 17.

Angles											
(1)	(2)	(12)	(3)	(13)	(4)	(5)	(8)	(9)	(10)	(11)	(6)
No.	Observed Values	Corrected Values	Initial Log. sin.	Corrected Log. sin.	Tabu. Diff. per 1″ Δ	d	$\Delta\cdot K_s$	Correction v	$\Delta\cdot v$	$v\ v$	$\Delta\cdot\Delta$
	° ′ ″	° ′ ″									
1	43 52 4.0	43 51 58.6	9.8407311	9.8407193	21.9		−2.34	−5.4	−118.26	29.16	479.61
2	31 48 21 3	31 48 19.7	9.7218465	9.7218411	34.0		3.63	−1.6	− 54.40	2.56	1,156.00
3	69 6 15.1	69 6 8.2	9.9704541	9.9704485	8.1		−0.87	−6.9	− 55.89	47.61	65.61
4	35 13 29.0	35 13 33.5	9.7610139	9.7610273	29.8		3.19	4.5	134.10	20.25	888.04
	180 00 9.4	180 00 00.0				−33.8					
			$\omega_1 = -\ 9.4$								
5	10 37 34.0	10 37 27.7	9.2657591	9.2656884	112.2		−11.99	−6.3	−706.86	39.69	12,588.84
6	65 2 53.4	65 2 50.6	9.9574457	9.9574430	9.8		1.05	−2.8	− 27.44	7.84	96.04
	180 00 11.5	180 00 00.0				80.7					
			$\omega_2 = -11.5$								
7	79 9 28.6	79 9 25.2	9.9921775	9.9921761	4.0		−0 43	−3.4	− 13.60	11.56	16.00
8	25 10 10.3	25 10 16.5	9.6286933	9.6287211	44.8		4.79	6.2	277.76	38.44	2,007.04
	180 00 6.3	180 00 00.0				61.6					
			$\omega_3 = -\ 6.3$						$\Sigma\,\Delta\cdot v =$		$d_s =$
			$\omega_s = -1224$						−1224.63		17,297.18
(1)	290 57 40.6	290 57 44.9						4.3		18.49	
			$\omega_{(1)} =\ \ 5.1$			−22.9					
(2)	259 5 30.0	259 5 32.1						2.1		4.41	
			$\omega_{(2)} = -\ 6.4$			−25.9					
(3)	314 8 49.3	314 8 58.8						9.5		90.25	
			$\omega_{(3)} =\ \ 7.7$			82.4					
(4)	215 47 44.1	215 47 44.2						0.1		0.01	
			$\omega_{(4)} = -\ 6.1$			−5.8					
										$\Sigma\, v\cdot v =$	
										310.27	

Probable Error $= \pm 0.6745\sqrt{310.27/8} = \pm 4.''20$

(7)

Correlate Sketch.

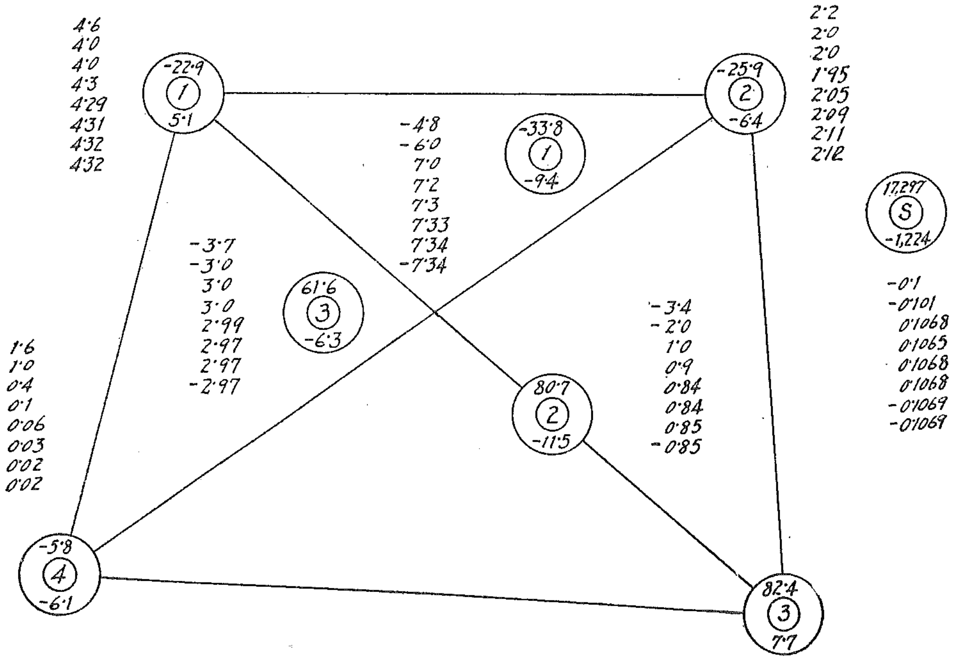

Fig. 18.

Numerical example 8. A quadrilateral.

Only the inner angles of a quadrilateral in **Example 7** above are assumed to be observed; the problem is to adjust them. The values in the following Table 18 are all similar to those in **Example 7**, but the values of d and ω are *a half of* those in **Example 7.**

Now only the **Correlate Sketch** (Fig. 19) will be explained.

First values of angle correlates are obtained from Table 16 as follows:

$$K_1 = 1/4 \cdot (3 \times -4.7 - 2 \times -5.75 + 1 \times -3.15) = -1.4,$$

$$K_2 = 1/4 \cdot (-1 \times -4.7 + 2 \times -5.75 - 1 \times -3.15) = -0.9,$$

$$K_3 = 1/4 \cdot (1 \times -4.7 - 2 \times -5.75 + 3 \times -3.15) = -0.7.$$

These values are written near their own circles.

The first value of the side correlate K_s.

The products of the upper figures in each circle and the corresponding first values of the angle correlates above obtained and written in each triangle are added, with the opposite sign, to the lower figure -612 in circle S, and this value is divided by the upper figure $8{,}648.5$ in circle S.

$$K_s = 1/\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle } S}}{8{,}648.5} \cdot \Big\{\underset{\substack{\text{lower}\\ \text{figure}\\ \text{in cir-}\\ \text{cle } S}}{-612} - (\underbrace{\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 1}}}{-16.9} \times \underset{\substack{\textbf{1st.}\\ \text{value}\\ \text{of}\\ K_1}}{-1.4}}_{\text{in triangle 1}} + \underbrace{\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 2}}}{40.35} \times \underset{\substack{\textbf{1st.}\\ \text{value}\\ \text{of}\\ K_2}}{-0.9}}_{\text{in triangle 2}} + \underbrace{\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 3}}}{30.8} \times \underset{\substack{\textbf{1st.}\\ \text{value}\\ \text{of}\\ K_3}}{-0.7}}_{\text{in triangle 3}})\Big\}$$

$$= -0.07\,.$$

This value is written near circle S.

This process means that equation (71) is solved.

Second values of correlates are obtained as follows:

Angle correlates K_i.

First values of the angle correlates in the triangles which intersect the triangle i and the product of the upper figure in circle i and the first value of K_s are added, with the opposite sign, to the lower figure in circle i, and this value is divided by constant two.

$$K_1 = \underset{\text{const.}}{1/2} \cdot (\underset{\substack{\text{lower}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 1}}}{-4.7} \underbrace{+ \underset{\substack{\textbf{1st.}\\ \text{value}\\ \text{of}\\ K_2}}{0.9} - \underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 1}}}{16.9} \times \underset{\substack{\textbf{1st.}\\ \text{value}\\ \text{of}\\ K_s}}{0.07}}_{\text{added with the opposite sign}}) = -2.5\,.$$

This value -2.5 is written under the first value -1.4.

$$K_2 = \underset{\text{const.}}{1/2} \cdot (\underset{\substack{\text{lower}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 2}}}{-5.75} \underbrace{+ \underset{\substack{\textbf{2nd.}\\ \text{value}\\ \text{of}\\ K_1}}{2.5} + \underset{\substack{\textbf{1st.}\\ \text{value}\\ \text{of}\\ K_3}}{0.7} + \underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 2}}}{40.35} \times \underset{\substack{\textbf{1st.}\\ \text{value}\\ \text{of}\\ K_s}}{0{,}07}}_{\text{anded with the opposite sign}}) = 0.14\,.$$

This value 0.14 is written under the first value -0.9.

$$K_3 = \underset{\text{const.}}{1/2}\cdot\Big(\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 3}}}{-3.15}\ \underbrace{-\underset{\substack{\textbf{2nd.}\\ \text{value}\\ \text{of}\\ K_2}}{0.14} + \underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 3}}}{30.8} \times \underset{\substack{\textbf{1st.}\\ \text{value}\\ \text{of}\\ K_s}}{0.07}}_{\text{added with the apposite sign}}\Big) = -0.6\,.$$

This value -0.6 is written under the first value -0.7.

These processes mean that equation (70) is solved.

The side correlate K_s.

By the same process as the case of the first value, the second value is obtained, but the second values of angle correlates obtained above and written on the sketch are used in this case.

$$K_s = 1/\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle } S}}{8{,}648.5}\cdot\Big\{\underset{\substack{\text{lower}\\ \text{figure}\\ \text{in cir-}\\ \text{cle } S}}{-612} - \Big(\underbrace{\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 1}}}{-16.9} \times \underset{\substack{\textbf{2nd.}\\ \text{value}\\ \text{of}\\ K_1}}{-2.5}}_{\text{in triangle 1}} + \underbrace{\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 2}}}{40.35} \times \underset{\substack{\textbf{2nd.}\\ \text{value}\\ \text{of}\\ K_2}}{0.14}}_{\text{in triangle 2}} + \underbrace{\underset{\substack{\text{upper}\\ \text{figure}\\ \text{in cir-}\\ \text{cle 3}}}{30.8} \times \underset{\substack{\textbf{2nd.}\\ \text{value}\\ \text{of}\\ K_3}}{-0.6}}_{\text{in triangle 3}}\Big)\Big\}$$

$$= -0.075\,.$$

This value -0.075 is written under the first value -0.07.

This process means that equation (71) is calculated for the second time.

Thus the first round of calculation is over, and the succeeding rounds are carried out. In this case, the calculation is stopped at the eighth round and the corrections of angles can be obtained accurately to the second grade below zero from equation (36) in §5 in Chapter 1. Time required for the calculation of correlates is only **forty minutes.**

This example is adopted from reference (10).

Table 18.

(1)	(2)	(12)	(3)	(13)	(4)	(5)	(8)	(9)	(10)	(11)	(6)
No.	Angles: Observed Values	Angles: Corrected Values	Initial Log. sin.	Corrected Log. sin.	Tabu. Diff. per 1″ Δ	d	$\Delta \cdot K_s$	Correction v	$\Delta \cdot v$	$v \cdot v$	$\Delta \cdot \Delta$
	° ′ ″	° ′ ″									
1	43 52 4.0	43 51 58.87	9.8407311	9.8407199	21.9		−1.717	−5.13	−112.347	26.3169	479.61
2	31 48 21.3	31 48 20.56	9.7218465	9.7218440	34.0		2.666	−0.74	−25.160	0.5476	1,156.00
3	69 6 15.1	69 6 11.85	9.9704541	9.9704515	8.1		−0.635	−3.25	−26.325	10.5625	65.61
4	35 13 29.0	35 13 28.72	9.7610139	9.7610131	29.8		2.336	−0.28	− 8.344	0.0784	888.04
	180 00 9.4	180 00 00.00	$\omega_1 = -\ 9.4/2 = -4.7$			−33.8/2 =−16.9					
5	10 37 34.0	10 37 25.23	9.2657591	9.2656607	122.2		−8.796	−8.77	−983.994	76.9129	12,588.84
6	65 2 53.4	65 2 54.20	9.9574457	9.9574465	9.8		0.768	0.80	7.840	0.6400	96.04
	180 00 11.5	180 00 00.00	$\omega_2 = -11.5/2 = -5.75$			80.7/2 =40.35					
7	79 9 28.6	79 9 27.52	9.9921775	9.9921771	4.0		−0.314	−1.08	− 4.320	1.1664	16.00
8	25 10 10.3	25 10 13.05	9.6286933	9.6287056	44.8		3.512	2.75	123.200	7.5625	2,007.04
	180 00 6.3	180 00 00.00	$\omega_3 = -\ 6.3/2 = -3.15$			61.6/2 = 30.8			$\Sigma\, \Delta \cdot v =$	$\Sigma\, v \cdot v =$	$d_s =$
			$\omega_S = -1{,}224/2 = -612$						−1,224.522	123.7872	$\dfrac{17{,}297.18}{2} = 8{,}648.59$

$$\text{Probable Error} = \pm 0.6745 \sqrt{123.7872/4} = \pm 3.''75$$

(7)

Correlate Sketch.

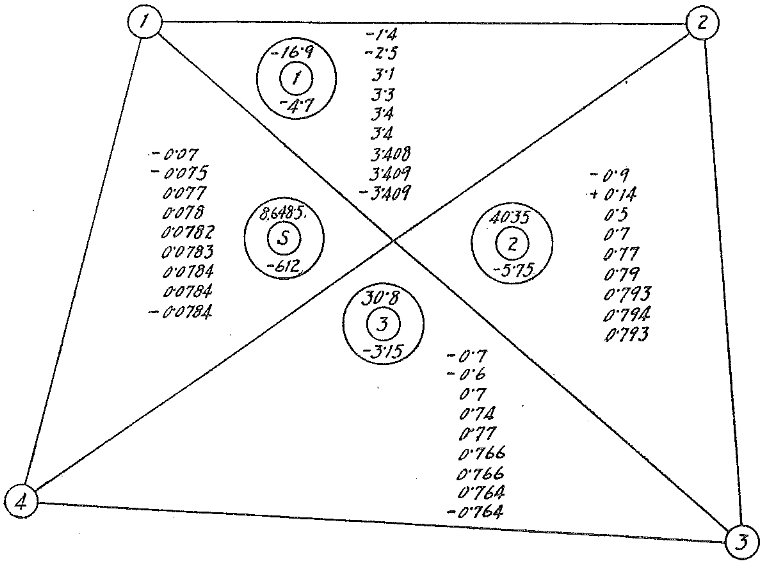

Fig. 19.

CHAPTER 3.

COMPACT FORMS OF VALUES OF CORRELATES.

§1. CLOSED CENTRAL POLYGON.

Eliminating point correlates K_0 and $K_{(i)}$ in equation (4) in §2 in Chapter 1, equations are obtained as in the following Table 19.

Table 19.

Sign of equation	Left side of equation									Right side of equation
	K_1	K_2	K_3	→	→	K_{n-2}	K_{n-1}	K_n	K_s	
1	$3-7n$	$n+3$	3	→	→	3	3	$n+3$	D_1	η_1
2	$n+3$	$3-7n$	$n+3$	3	→	→	3	3	D_2	η_2
3	3	$n+3$	↘	↘	↘	↘	↘	3	D_3	η_3
↓	↓	↘	↘	↘	↘	↘	↘	↓	↓	↓
↓	↓	↘	↘	↘	↘	↘	↘	↓	↓	↓
$n-2$	3	↘	↘	↘	↘	↘	↘	3	D_{n-2}	η_{n-2}
$n-1$	3	→	→	→	3	$n+3$	$3-7n$	$n+3$	D_{n-1}	η_{n-1}
n	$n+3$	3	→	→	→	3	$n+3$	$3-7n$	D_n	η_n
s	D_1	D_2	D_3	→	→	D_{n-2}	D_{n-1}	D_n	D_s	η_s

where

$$\left.\begin{aligned}
D_i &= n(d_{(i)}+d_{(i-1)}-3\cdot d_i) \qquad (i=1,\,2,\,\cdots,\,n)\\
\eta_i &= n(\omega_{(i)}+\omega_{(i-1)}-3\cdot\omega_i)+3\cdot\omega_0\\
&\qquad\qquad\qquad\qquad\qquad (i=1,\,2,\,\cdots,\,n)\\
D_s &= n\sum_{i=1}^{n} d_{(i)}^2-3nd_s\\
\eta_s &= n\sum_{i=1}^{n} d_{(i)}\cdot\omega_{(i)}-3n\omega_s\,.
\end{aligned}\right\}\quad \cdots\cdots (72)$$

Solving these equations for $n=3,4,5,6$ and 7, values of correlates are as follows;

$$\left.\begin{aligned} K_0 &= \omega_0/n+1/2n^2 \cdot \sum_{i=1}^{n}\eta_i - 1/2n^2 \cdot \sum_{i=1}^{n} D_i \cdot K_s \\ \text{or}\quad & 5/2n \cdot \omega_0 + 1/2n \cdot \left(2\sum_{i=1}^{n}\omega_{(i)} - 3\sum_{i=1}^{n}\omega_i\right) \\ & -1/2n \cdot \left(2\sum_{i=1}^{n} d_{(i)} - 3\sum_{i=1}^{n} d_i\right) \cdot K_s \,. \end{aligned}\right\} \cdots\cdots (73)$$

$$\left.\begin{aligned} & n=3\,, \\ & K_{(i)} = 1/3 \cdot \omega_{(i)} + 1/72 \cdot (3\eta_i + 2\eta_{i+1} + 3\eta_{i+2}) \\ & \qquad -\left\{1/3 \cdot d_{(i)} + 1/72 \cdot (3D_i + 2D_{i+1} + 3D_{i+2})\right\} \cdot K_s \\ & K_i = -1/24 \cdot (2\eta_i + \eta_{i+1} + \eta_{i+2}) \\ & \qquad + 1/24 \cdot (2D_i + D_{i+1} + D_{i+2}) \cdot K_s \\ & K_s = \mathfrak{N}/\mathfrak{D} \\ & \mathfrak{N} = 216\,\eta_s + \sum_{i=1}^{3} D_i \cdot (2\eta_i + \eta_{i+1} + \eta_{i+2}) \\ & \mathfrak{D} = 216\,D_s + \sum_{i=1}^{3} D_i \cdot (2D_i + D_{i+1} + D_{i+2}) \,. \end{aligned}\right\} \cdots\cdots (74)$$

$$\left.\begin{aligned} & n=4\,, \\ & K_{(i)} = 1/3 \cdot \omega_{(i)} + 1/336 \cdot (9\eta_i + 5\eta_{i+1} + 5\eta_{i+2} + 9\eta_{i+3}) \\ & \qquad -\left\{1/3 \cdot d_{(i)} + 1/336 \cdot (9D_i + 5D_{i+1} + 5D_{i+2} + 9D_{i+3})\right\} \cdot K_s \\ & K_i = -1/2{,}016 \cdot (113\,\eta_i + 49\,\eta_{i+1} + 41\,\eta_{i+2} + 49\,\eta_{i+3}) \\ & \qquad + 1/2{,}016 \cdot (113\,D_i + 49\,D_{i+1} + 41\,D_{i+2} + 49\,D_{i+3}) \cdot K_s \\ & K_s = \mathfrak{N}/\mathfrak{D} \\ & \mathfrak{N} = 24{,}192\,\eta_s + \sum_{i=1}^{4} D_i \cdot (113\,\eta_i + 49\,\eta_{i+1} + 41\,\eta_{i+2} + 49\,\eta_{i+3}) \\ & \mathfrak{D} = 24{,}192 D_s + \sum_{i=1}^{4} D_i \cdot (113D_i + 49D_{i+1} + 41D_{i+2} + 49D_{i+3}) \,. \end{aligned}\right\} \cdots (75)$$

$$\begin{aligned} & n=5\,, \\ & K_{(i)} = 1/3 \cdot \omega_{(i)} + 1/825 \cdot (16\eta_i + 8\eta_{i+1} + 7\eta_{i+2} + 8\eta_{i+3} + 16\eta_{i+4}) \\ & \qquad -\left\{1/3 \cdot d_{(i)} + 1/825 \cdot (16D_i + 8D_{i+1} + 7D_{i+2} + 8D_{i+3} \right. \\ & \qquad\qquad \left. + 16D_{i+4})\right\} \cdot K_s \end{aligned}$$

$$\left.\begin{aligned}
K_i &= -1/550\cdot(23\eta_i+9\eta_{i+1}+7\eta_{i+2}+7\eta_{i+3}+9\eta_{i+4})\\
&\quad +1/550\cdot(23D_i+9D_{i+1}+7D_{i+2}+7D_{i+3}+9D_{i+4})\cdot K_s\\
K_s &= \mathfrak{N}/\mathfrak{D}\\
\mathfrak{N} &= 8{,}250\eta_s+\sum_{i=1}^{5}D_i\cdot(23\eta_i+9\eta_{i+1}+7\eta_{i+2}+7\eta_{i+3}+9\eta_{i+4})\\
\mathfrak{D} &= 8{,}250D_s+\sum_{i=1}^{5}D_i\cdot(23D_i+9D_{i+1}+7D_{i+2}+7D_{i+3}+9D_{i+4})\,.
\end{aligned}\right\} \quad (76)$$

$$\left.\begin{aligned}
n &= 6\,,\\
K_{(i)} &= 1/3\cdot\omega_{(i)}+1/864\cdot(13\eta_i+6\eta_{i+1}+5\eta_{i+2}+5\eta_{i+3}\\
&\qquad +6\eta_{i+4}+13\eta_{i+5})\\
&\quad -\Big\{1/3\cdot d_{(i)}+1/864\cdot(13D_i+6D_{i+1}+5D_{i+2}+5D_{i+3}\\
&\qquad +6D_{i+4}+13D_{i+5})\Big\}\cdot K_s\\
K_i &= -1/2{,}592\cdot(86\eta_i+31\eta_{i+1}+23\eta_{i+2}+22\eta_{i+3}+23\eta_{i+4}\\
&\qquad +31\eta_{i+5})\\
&\quad +1/2{,}592\cdot(86D_i+31D_{i+1}+23D_{i+2}+22D_{i+3}+23D_{i+4}\\
&\qquad +31D_{i+5})\cdot K_s\\
K_s &= \mathfrak{N}/\mathfrak{D}\\
\mathfrak{N} &= 15{,}552\,\eta_s+\sum_{i=1}^{6}D_i\cdot(86\eta_i+31\eta_{i+1}+23\eta_{i+2}+22\eta_{i+3}\\
&\qquad +23\eta_{i+4}+31\eta_{i+5})\\
\mathfrak{D} &= 15{,}552D_s+\sum_{i=1}^{6}D_i\cdot(86D_i+31D_{i+1}+23D_{i+2}+22D_{i+3}\\
&\qquad +23D_{i+4}+31D_{i+5})\,.
\end{aligned}\right\} \quad (77)$$

$$\left.\begin{aligned}
n &= 7\,,\\
K_{(i)} &= 1/3\cdot\omega_{(i)}+1/55{,}419\cdot(677\eta_i+292\eta_{i+1}+236\eta_{i+2}+229\eta_{i+3}\\
&\qquad +236\eta_{i+4}+292\eta_{i+5}+677\eta_{i+6})\\
&\quad -\Big\{1/3\cdot d_{(i)}+1/55{,}419\cdot(677D_i+292D_{i+1}+236D_{i+2}\\
&\qquad +229D_{i+3}+236D_{i+4}+292D_{i+5}\\
&\qquad +677D_{i+6})\Big\}\cdot K_s\\
K_i &= -1/36{,}946\cdot(1{,}013\eta_i+341\eta_{i+1}+243\eta_{i+2}+229\eta_{i+3}\\
&\qquad +229\eta_{i+4}+243\eta_{i+5}+341\eta_{i+6})\\
&\quad +1/36{,}946\cdot(1{,}013D_i+341D_{i+1}+243D_{i+2}+229D_{i+3}\\
&\qquad +229D_{i+4}+243D_{i+5}+341D_{i+6})\cdot K_s
\end{aligned}\right\} \quad (78)$$

$$\left.\begin{aligned}
K_s &= \mathfrak{N}/\mathfrak{D} \\
\mathfrak{N} &= 775{,}866\,\eta_s + \sum_{i=1}^{7} D_i \cdot (1{,}013\eta_i + 341\eta_{i+1} + 243\eta_{i+2} + 229\eta_{i+3} \\
&\qquad + 229\eta_{i+4} + 243\eta_{i+5} + 341\eta_{i+6}) \\
\mathfrak{D} &= 775{,}866 D_s + \sum_{i=1}^{7} D_i \cdot (1{,}013 D_i + 341 D_{i+1} + 243 D_{i+2} \\
&\qquad + 229 D_{i+3} + 229 D_{i+4} + 243 D_{i+5} \\
&\qquad + 341 D_{i+6}) \,.
\end{aligned}\right\}$$

§ 2. UNCLOSED CENTRAL POLYGON.

Eliminating point correlates K_0 and $K_{(i)}$ in equation (14) in § 3 in Chapter 2, equations are obtained as in the following Table 20.

Table 20.

Sign of equation	Left Side of equation							Right Side of equation
	K_1	K_2	→	→	K_{n-1}	K_n	K_s	
1	$-\frac{1}{2}(7+13n)$	$n+4$	3	→	3	3	D_1	η_1
2	$n+4$	$-(4+7n)$	$n+4$	↘	→	3	D_2	η_2
↓	3	↘	↘	↘	↘	↓	↓	↓
↓	↓	↘	↘	↘	↘	3	↓	↓
$n-1$	3	→	↘	$n+4$	$-(4+7n)$	$n+4$	D_{n-1}	η_{n-1}
n	3	→	→	3	$n+4$	$-\frac{1}{2}(7+13n)$	D_n	η_n
s	D_1	D_2	→	→	D_{n-1}	D_n	D_s	η_s

where

$$\left.\begin{aligned}
D_1 &= (n+1)/2 \cdot (3d_{(1)} + 2d_{(2)} - 6d_1) \\
D_i &= (n+1) \cdot (d_{(i)} + d_{(i+1)} - 3d_i) \qquad (i = 2,\ 3,\ \cdots,\ n-1) \\
D_n &= (n+1)/2 \cdot (3d_{(n+1)} + 2d_{(n)} - 6d_n) \\
\eta_1 &= 3\omega_0 + (n+1)/2 \cdot (3\omega_{(1)} + 2\omega_{(2)} - 6\omega_1) \\
\eta_i &= 3\omega_0 + (n+1) \cdot (\omega_{(i)} + \omega_{(i+1)} - 3\omega_i) \quad (i = 2,\ 3,\ \cdots,\ n-1)
\end{aligned}\right\} \quad (79)$$

$$\left.\begin{aligned}
\eta_n &= 3\omega_0 + (n+1)/2 \cdot (3\omega_{(n+1)} + \omega_{(n)} - 6\omega_n) \\
D_s &= (n+1)/2 \cdot (3d^2_{(1)} + 2\sum_{i=2}^{n} d^2_{(i)} + 3d^2_{(n+1)} - 6d_s) \\
\eta_s &= (n+1)/2 \cdot (3d_{(1)} \cdot \omega_{(1)} + 2\sum_{i=2}^{n} d_{(i)} \cdot \omega_{(i)} + 3d_{(n+1)} \cdot \omega_{(n+1)} - 6\omega_s .
\end{aligned}\right\}$$

Solving these equations for $n = 2, 3, 4, 5$ and 6, values of correlates are as follows;

$$\left.\begin{aligned}
&n = 2\,, \\
&\quad K_0 = 1/3 \cdot \omega_0 + 2/63 \cdot (\eta_1 + \eta_2) - 2/63 \cdot (D_1 + D_2) \cdot K_s \\
&\quad K_{(1)} = 1/2 \cdot \omega_{(1)} + 1/315 \cdot (11\eta_1 + 4\eta_2) \\
&\qquad\qquad - \left\{1/2 \cdot d_{(1)} + 1/315 \cdot (11D_1 + 4D_2)\right\} \cdot K_s \\
&\quad K_{(2)} = 1/3 \cdot \omega_{(2)} + 2/63 \cdot (\eta_1 + \eta_2) \\
&\qquad\qquad - \left\{1/3 \cdot d_{(2)} + 2/63 \cdot (D_1 + D_2)\right\} \cdot K_s \\
&\quad K_{(3)} = 1/2 \cdot \omega_{(3)} + 1/315 \cdot (4\eta_1 + 11\eta_2) \\
&\qquad\qquad - \left\{1/2 \cdot d_{(3)} + 1/315 \cdot (4D_1 + 11D_2)\right\} \cdot K_s \\
&\quad K_1 = -1/315 \cdot (22\eta_1 + 8\eta_2) + 1/315 \cdot (22D_1 + 8D_2) \cdot K_s \\
&\quad K_2 = -1/315 \cdot (8\eta_1 + 22\eta_2) + 1/315 \cdot (8D_1 + 22D_2) \cdot K_s \\
&K_s = \mathfrak{N}/\mathfrak{D} \\
&\quad \mathfrak{N} = 315\eta_s + D_1 \cdot (22\eta_1 + 8\eta_2) + D_2 \cdot (8\eta_1 + 22\eta_2) \\
&\quad \mathfrak{D} = 315D_s + D_1 \cdot (22D_1 + 8D_2) + D_2 \cdot (8D_1 + 22D_2) .
\end{aligned}\right\} \quad (80)$$

$$\begin{aligned}
&n = 3\,, \\
&\quad K_0 = 1/4 \cdot \omega_0 + 1/10{,}452 \cdot (208\eta_1 + 221\eta_2 + 208\eta_3) \\
&\qquad\qquad - 1/10{,}452 \cdot (208D_1 + 221D_2 + 208D_3) \cdot K_s \\
&\quad K_{(1)} = 1/2 \cdot \omega_{(1)} + 1/10{,}452 \cdot (263\eta_1 + 91\eta_2 + 62\eta_3) \\
&\qquad\qquad - \left\{1/2 \cdot d_{(1)} + 1/10{,}452 \cdot (263D_1 + 91D_2 + 62D_3)\right\} \cdot K_s \\
&\quad K_{(2)} = 1/3 \cdot \omega_{(2)} + 1/5{,}226 \cdot (118\eta_1 + 117\eta_2 + 51\eta_3) \\
&\qquad\qquad - \left\{1/3 \cdot d_{(2)} + 1/5{,}226 \cdot (118D_1 + 117D_2 + 51D_3)\right\} \cdot K_s
\end{aligned}$$

$$\left.\begin{aligned}
K_{(3)} &= 1/3 \cdot \omega_{(3)} + 1/5{,}226 \cdot (51\eta_1 + 117\eta_2 + 118\eta_3) \\
&\qquad -\left\{1/3 \cdot d_{(3)} + 1/5{,}226 \cdot (51D_1 + 117D_2 + 118D_3)\right\} \cdot K_s \\
K_{(4)} &= 1/2 \cdot \omega_{(4)} + 1/10{,}452 \cdot (62\eta_1 + 91\eta_2 + 263\eta_3) \\
&\qquad -\left\{1/2 \cdot d_{(4)} + 1/10{,}452 \cdot (62D_1 + 91D_2 + 263D_3)\right\} \cdot K_s \\
K_1 &= -1/5{,}226 \cdot (263\eta_1 + 91\eta_2 + 62\eta_3) \\
&\qquad\qquad + 1/5{,}226 \cdot (263D_1 + 91D_2 + 62D_3) \cdot K_s \\
K_2 &= -1/402 \cdot (7\eta_1 + 20\eta_2 + 7\eta_3) \\
&\qquad\qquad + 1/402 \cdot (7D_1 + 20D_2 + 7D_3) \cdot K_s \\
K_3 &= -1/5{,}226 \cdot (62\eta_1 + 91\eta_2 + 263\eta_3) \\
&\qquad\qquad + 1/5{,}226 \cdot (62D_1 + 91D_2 + 263D_3) \cdot K_s \\
K_s &= \mathfrak{N}/\mathfrak{D} \\
\mathfrak{N} &= \eta_s + 1/5{,}226 \cdot \left\{D_1(263\eta_1 + 91\eta_2 + 62\eta_3) + D_2(91\eta_1 + 260\eta_2\right. \\
&\qquad \left. + 91\eta_3) + D_3(62\eta_1 + 91\eta_2 + 263\eta_3)\right\} \\
\mathfrak{D} &= D_s + 1/5{,}226 \cdot \left\{D_1(263D_1 + 91D_2 + 62D_3)\right. \\
&\qquad + D_2(91D_1 + 260D_2 + 91D_3) \\
&\qquad \left. + D_3(62D_1 + 91D_2 + 263D_3)\right\} .
\end{aligned}\right\} \quad (81)$$

$n = 4$,

$$\left.\begin{aligned}
K_0 &= 1/5 \cdot \omega_0 + 1/1{,}030 \cdot (14\eta_1 + 17\eta_2 + 17_3 + 14\eta_4) \\
&\qquad -1/1{,}030 \cdot (14D_1 + 17D_2 + 17D_3 + 14D_4) \cdot K_s \\
K_{(1)} &= 1/2 \cdot \omega_{(1)} + 1/52{,}530 \cdot (1{,}024\eta_1 + 332\eta_2 + 229\eta_3 + 200\eta_4) \\
&\qquad -\left\{1/2 \cdot d_{(1)} + 1/52{,}530 \cdot (1{,}024D_1 + 332D_2 + 229D_3\right. \\
&\qquad\qquad \left. + 200D_4)\right\} \cdot K_s \\
K_{(2)} &= 1/3 \cdot \omega_{(2)} + 1/52{,}530 \cdot (904\eta_1 + 895\eta_2 + 380\eta_3 + 286\eta_4) \\
&\qquad -\left\{1/3 \cdot d_{(2)} + 1/52{,}530 \cdot (904D_1 + 895D_2 + 380D_3\right. \\
&\qquad\qquad \left. + 286D_4)\right\} \cdot K_s
\end{aligned}\right.$$

$$
\left.\begin{aligned}
K_{(3)} &= 1/3\cdot\omega_{(3)} + 1/52{,}530\cdot(374\eta_1+901\eta_2+901\eta_3+374\eta_4)\\
&\quad -\Big\{1/3\cdot d_{(3)} + 1/52{,}530\cdot(374D_1+901D_2+901D_3\\
&\qquad\qquad +374D_4)\Big\}\cdot K_s\\
K_{(4)} &= 1/3\cdot\omega_{(4)} + 1/52{,}530\cdot(286\eta_1+380\eta_2+895\eta_3+904\eta_4)\\
&\quad -\Big\{1/3\cdot d_{(4)} + 1/52{,}530\cdot(286D_1+380D_2+895D_3\\
&\qquad\qquad +904D_4)\Big\}\cdot K_s\\
K_{(5)} &= 1/2\cdot\omega_{(5)} + 1/52{,}530\cdot(200\eta_1+229\eta_2+332\eta_3+1{,}024\eta_4)\\
&\quad -\Big\{1/2\cdot d_{(5)} + 1/52{,}530\cdot(200D_1+229D_2+332D_3\\
&\qquad\qquad +1{,}024D_4)\Big\}\cdot K_s\\
K_1 &= -1/26{,}265\cdot(1{,}024\eta_1+332\eta_2+229\eta_3+200\eta_4)\\
&\quad +1/26{,}265\cdot(1{,}024D_1+332D_2+229D_3+200D_4)\cdot K_s\\
K_2 &= -1/52{,}530\cdot(664\eta_1+2{,}021\eta_2+682\eta_3+458\eta_4)\\
&\quad +1/52{,}530\cdot(664D_1+2{,}021D_2+682D_3+458D_4)\cdot K_s\\
K_3 &= -1/52{,}530\cdot(458\eta_1+682\eta_2+2{,}021\eta_3+664\eta_4)\\
&\quad +1/52{,}530\cdot(458D_1+682D_2+2{,}021D_3+664D_4)\cdot K_s\\
K_4 &= -1/26{,}265\cdot(200\eta_1+229\eta_2+332\eta_3+1{,}024\eta_4)\\
&\quad +1/26{,}265\cdot(200D_1+229D_2+332D_3+1{,}024D_4)\cdot K_s\\
K_s &= \mathfrak{N}/\mathfrak{D}\\
\mathfrak{N} &= \eta_s + 1/52{,}530\cdot\Big\{2D_1(1{,}024\eta_1+332\eta_2+229\eta_3+200\eta_4)\\
&\qquad +D_2(664\eta_1+2{,}021\eta_2+682\eta_3+458\eta_4)\\
&\qquad +D_3(458\eta_1+682\eta_2+2{,}021\eta_3+664\eta_4)\\
&\qquad +2D_4(200\eta_1+229\eta_2+332\eta_3+1{,}024\eta_4)\Big\}\\
\mathfrak{D} &= D_s + 1/52{,}530\cdot\Big\{2D_1(1{,}024D_1+332D_2+229D_3+200D_4)\\
&\qquad +D_2(664D_1+2{,}021D_2+682D_3+458D_4)\\
&\qquad +D_3(458D_1+682D_2+1{,}021D_3+664D_4)\\
&\qquad +D_4(200D_1+229D_2+332D_3+1{,}024D_4)\Big\}.
\end{aligned}\right\}\quad (82)
$$

$n=5$,

$$
\left.
\begin{aligned}
K_0 &= 1/6 \cdot \omega_0 + 1/11{,}142 \cdot (110\eta_1 + 118\eta_2 + 119\eta_3 + 118\eta_4 + 110\eta_5) \\
&\quad -1/11{,}142 \cdot (110D_1 + 118D_2 + 119D_3 + 118D_4 + 110D_5) \cdot K_s \\
K_{(1)} &= 1/2 \cdot \omega_{(1)} + 1/330{,}546 \cdot (5{,}237\eta_1 + 1{,}600\eta_2 + 1{,}068\eta_3 \\
&\qquad + 981\eta_4 + 904\eta_5) \\
&\quad -\Big\{1/2 \cdot d_{(1)} + 1/330{,}546 \cdot (5{,}237D_1 + 1{,}600D_2 + 1{,}068D_3 \\
&\qquad + 981D_4 + 904D_5)\Big\} \cdot K_s \\
K_{(2)} &= 1/3 \cdot \omega_{(2)} + 1/495{,}819 \cdot (6{,}837\eta_1 + 6{,}749\eta_2 + 2{,}714.5\eta_3 \\
&\qquad + 2{,}106.5\eta_4 + 1{,}885\eta_5) \\
&\quad -\Big\{1/3 \cdot d_{(2)} + 1/495{,}819 \cdot (6{,}837D_1 + 6{,}749D_2 + 2{,}714.5D_3 \\
&\qquad + 2{,}106.5D_4 + 1{,}885D_5)\Big\} \cdot K_s \\
K_{(3)} &= 1/3 \cdot \omega_{(3)} + 1/495{,}819 \cdot (2{,}668\eta_1 + 6{,}795.5\eta_2 + 6{,}808.5\eta_3 \\
&\qquad + 2{,}772\eta_4 + 2{,}049\eta_5) \\
&\quad -\Big\{1/3 \cdot d_{(3)} + 1/495{,}819 \cdot (2{,}668D_1 + 6{,}795.5D_2 \\
&\qquad + 6{,}808.5D_3 + 2{,}772D_4 \\
&\qquad + 2{,}049D_5)\Big\} \cdot K_s \\
K_{(4)} &= 1/3 \cdot \omega_{(4)} + 1/495{,}819 \cdot (2{,}049\eta_1 + 2{,}772\eta_2 + 6{,}808.5\eta_3 \\
&\qquad + 6{,}795.5\eta_4 + 2{,}668\eta_5) \\
&\quad -\Big\{1/3 \cdot d_{(4)} + 1/495{,}819 \cdot (2{,}049D_1 + 2{,}772D_2 \\
&\qquad + 6{,}808.5D_3 + 6{,}795.5D_4 \\
&\qquad + 2{,}668D_5)\Big\} \cdot K_s \\
K_{(5)} &= 1/3 \cdot \omega_{(5)} + 1/495{,}819 \cdot (1{,}885\eta_1 + 2{,}106.5\eta_2 + 2{,}714.5\eta_3 \\
&\qquad + 6{,}749\eta_4 + 6{,}837\eta_5) \\
&\quad -\Big\{1/3 \cdot d_{(5)} + 1/495{,}819 \cdot (1{,}885D_1 + 2{,}106.5D_2 \\
&\qquad + 2{,}714.5D_3 + 6{,}749D_4 \\
&\qquad + 6{,}837D_5)\Big\} \cdot K_s
\end{aligned}
\right|
$$

$$\left.\begin{aligned}
K_{(6)} &= 1/2\cdot\omega_{(6)} + 1/330{,}346\cdot(904\eta_1+981\eta_2+1{,}068\eta_3+1{,}600\eta_4\\
&\qquad +5{,}237\eta_5)\\
&\quad -\Big\{1/2\cdot d_{(6)} + 1/330{,}346\cdot(904D_1+981D_2+1{,}068D_3\\
&\qquad +1{,}600D_4+5{,}237D_5)\Big\}\cdot K_s\\
K_1 &= -1/165{,}273\cdot(5{,}237\eta_1+1{,}600\eta_2+1{,}068\eta_3+981\eta_4\\
&\qquad +904\eta_5)\\
&\quad +1/165{,}273\cdot(5{,}237D_1+1{,}600D_2+1{,}068D_3+981D_4\\
&\qquad +904D_5)\cdot K_s\\
K_2 &= -1/165{,}273\cdot(1{,}600\eta_1+5{,}149\eta_2+1{,}646.5\eta_3+1{,}125.5\eta_4\\
&\qquad +981\eta_5)\\
&\quad +1/165{,}273\cdot(1{,}600D_1+5{,}149D_2+1{,}646.5D_3\\
&\qquad +1{,}125.5D_4+981D_5)\cdot K_s\\
K_3 &= -1/165{,}273\cdot(1{,}068\eta_1+1{,}646.5\eta_2+5{,}162\eta_3+1{,}646.5\eta_4\\
&\qquad +1{,}068\eta_5)\\
&\quad +1/165{,}273\cdot(1{,}068D_1+1{,}646.5D_2+5{,}162D_3\\
&\qquad +1{,}646.5D_4+1{,}068D_5)\cdot K_s\\
K_4 &= -1/165{,}273\cdot(981\eta_1+1{,}125.5\eta_2+1{,}646.5\eta_3+5{,}149\eta_4\\
&\qquad +1{,}600\eta_5)\\
&\quad +1/165{,}273\cdot(981D_1+1{,}125.5D_2+1{,}646.5D_3\\
&\qquad +5{,}149D_4+1{,}600D_5)\cdot K_s\\
K_5 &= -1/165{,}273\cdot(904\eta_1+981\eta_2+1{,}068\eta_3+1{,}600\eta_4\\
&\qquad +5{,}237\eta_5)\\
&\quad +1/165{,}273\cdot(904D_1+981D_2+1{,}068D_3+1{,}600D_4\\
&\qquad +5{,}237D_5)\cdot K_s\\
K_s &= \mathfrak{N}/\mathfrak{D}\\
\mathfrak{N} &= \eta_s + 1/165{,}273\cdot\Big\{D_1(5{,}237\eta_1+1{,}600\eta_2+1{,}068\eta_3+981\eta_4\\
&\qquad +904\eta_5)
\end{aligned}\right\}\quad (83)$$

$$
\begin{aligned}
&+D_2(1{,}600\eta_1+5{,}149\eta_2+1{,}646.5\eta_3+1{,}125.5\eta_4+981\eta_5)\\
&+D_3(1{,}068\eta_1+1{,}646.5\eta_2+5{,}162\eta_3+1{,}646.5\eta_4+1{,}068\eta_5)\\
&+D_4(981\eta_1+1{,}125.5\eta_2+1{,}646.5\eta_3+5{,}149\eta_4+1{,}600\eta_5)\\
&+D_5(904\eta_1+981\eta_2+1{,}068\eta_3+1{,}600\eta_4+5{,}237\eta_5)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{D} = D_s &+1/165{,}273 \cdot\Big\{D_1(5{,}237D_1+1{,}600D_2+1{,}068D_3+981D_4\\
&\qquad +904D_5)\\
&+D_2(1{,}600D_1+5{,}149D_2+1{,}646.5D_3+1{,}125.5D_4\\
&\qquad +981D_5)\\
&+D_3(1{,}068D_1+1{,}646.5D_2+5{,}162D_3+1{,}646.5D_4\\
&\qquad +1{,}068D_5)\\
&+D_4(981D_1+1{,}125.5D_2+1{,}646.5D_3+5{,}149D_4\\
&\qquad +1{,}600D_5)\\
&+D_5(904D_1+981D_2+1{,}068D_3+1{,}600D_4+5{,}237D_5)\Big\}\ .
\end{aligned}
$$

$n=6$,

$$
\begin{aligned}
K_0 &= 1/7\cdot\omega_0 +1/38{,}409 \cdot(288\eta_1+309\eta_2+312\eta_3+312\eta_4\\
&\qquad +309\eta_5+288\eta_6)\\
&\quad -1/38{,}409 \cdot(288D_1+309D_2+312D_3+312D_4+309D_5\\
&\qquad +288D_6)\cdot K_s\\
K_{(1)} &= 1/2\cdot\omega_{(1)} +1/8{,}949{,}297 \cdot(119{,}149\eta_1+34{,}577\eta_2\\
&\qquad +22{,}234\eta_3+20{,}405\eta_4\\
&\qquad +19{,}945\eta_5+18{,}554\eta_6)\\
&\quad -\Big\{1/2\cdot d_{(1)} +1/8{,}949{,}297 \cdot(119{,}149D_1+34{,}577D_2\\
&\qquad +22{,}234D_3+20{,}405D_4\\
&\qquad +19{,}945D_5+18{,}554D_6)\Big\}\cdot K_s\\
K_{(2)} &= 1/3\cdot\omega_{(2)} +1/8{,}949{,}297 \cdot(102{,}484\eta_1+100{,}888\eta_2\\
&\qquad +38{,}474\eta_3+29{,}329\eta_4\\
&\qquad +27{,}728\eta_5+25{,}666\eta_6)\\
&\quad -\Big\{1/3\cdot d_{(2)} +1/8{,}949{,}297 \cdot(102{,}484D_1+100{,}888D_2\\
&\qquad +38{,}474D_3+29{,}329D_4\\
&\qquad +27{,}728D_5+25{,}666D_6)\Big\}\cdot K_s
\end{aligned}
$$

$$K_{(3)} = 1/3\cdot\omega_{(3)} + 1/8{,}949{,}297\cdot(37{,}874\eta_1 + 101{,}488\eta_2 + 101{,}692\eta_3 + 39{,}506\eta_4 + 30{,}157\eta_5 + 26{,}900\eta_6)$$

$$-\left\{1/3\cdot d_{(3)} + 1/8{,}949{,}297\cdot(37{,}874D_1 + 101{,}488D_2 + 101{,}692D_3 + 39{,}506D_4 + 30{,}157D_5 + 26{,}900D_6)\right\}\cdot K_s$$

$$K_{(4)} = 1/3\cdot\omega_{(4)} + 1/8{,}949{,}297\cdot(28{,}426\eta_1 + 39{,}377\eta_2 + 101{,}821\eta_3 + 101{,}821\eta_4 + 39{,}377\eta_5 + 28{,}426\eta_6)$$

$$-\left\{1/3\cdot d_{(4)} + 1/8{,}949{,}297\cdot(28{,}426D_1 + 39{,}377D_2 + 101{,}821D_3 + 101{,}821D_4 + 39{,}377D_5 + 28{,}426D_6)\right\}\cdot K_s$$

$$K_{(5)} = 1/3\cdot\omega_{(5)} + 1/8{,}949{,}297\cdot(26{,}900\eta_1 + 30{,}157\eta_2 + 39{,}506\eta_3 + 101{,}692\eta_4 + 101{,}488\eta_5 + 37{,}874\eta_6)$$

$$-\left\{1/3\cdot\omega_{(5)} + 1/8{,}949{,}297\cdot(26{,}900\eta_1 + 30{,}157D_2 + 39{,}506D_3 + 101{,}692D_4 + 101{,}488D_5 + 37{,}874D_6)\right\}\cdot K_s$$

$$K_{(6)} = 1/3\cdot\omega_{(6)} + 1/8{,}949{,}297\cdot(25{,}666\eta_1 + 27{,}728\eta_2 + 29{,}329\eta_3 + 38{,}474\eta_4 + 100{,}488\eta_5 + 102{,}484\eta_6)$$

$$-\left\{1/3\cdot d_{(6)} + 1/8{,}949{,}297\cdot(25{,}666D_1 + 27{,}728D_2 + 29{,}329D_3 + 38{,}474D_4 + 100{,}488D_5 + 102{,}484D_6)\right\}\cdot K_s$$

$$K_{(7)} = 1/2\,\omega_{(7)} + 1/8{,}949{,}297\cdot(18{,}554\eta_1 + 19{,}945\eta_2 + 20{,}405\eta_3 + 22{,}234\eta_4 + 34{,}577\eta_5 + 119{,}149\eta_6)$$

$$-\left\{1/2\cdot d_{(7)} + 1/8{,}949{,}297\cdot(18{,}554D_1 + 19{,}945D_2 + 20{,}405D_3 + 22{,}234D_4 + 34{,}577D_5 + 119{,}149D_6)\right\}\cdot K_s$$

$$
\left.
\begin{aligned}
K_1 &= -1/8{,}949{,}297 \cdot (238{,}298\eta_1+69{,}154\eta_2+44{,}468\eta_3\\
&\qquad +40{,}810\eta_4+39{,}890\eta_5+37{,}108\eta_6)\\
&\quad +1/8{,}949{,}297 \cdot (238{,}298D_1+69{,}154D_2+44{,}468D_3\\
&\qquad +40{,}810D_4+39{,}890D_5+37{,}108D_6)\cdot K_s\\
K_2 &= -1/8{,}949{,}297 \cdot (69{,}154\eta_1+233{,}510\eta_2+70{,}954\eta_3\\
&\qquad +47{,}177\eta_4+43{,}294\eta_5+39{,}890\eta_6)\\
&\quad +1/8{,}949{,}297 \cdot (69{,}154D_1+233{,}510D_2+70{,}954D_3\\
&\qquad +47{,}177D_4+43{,}294D_5+39{,}890D_6)\cdot K_s\\
K_3 &= -1/8{,}949{,}297 \cdot (44{,}468\eta_1+70{,}954\eta_2+234{,}122\eta_3\\
&\qquad +71{,}341\eta_4+47{,}177\eta_5+40{,}810\eta_6)\\
&\quad +1/8{,}949{,}297 \cdot (44{,}468D_1+70{,}954D_2+234{,}122D_3\\
&\qquad +71{,}341D_4+47{,}177D_5+40{,}810D_6)\cdot K_s\\
K_4 &= -1/8{,}949{,}297 \cdot (40{,}810\eta_1+47{,}177\eta_2+71{,}341\eta_3\\
&\qquad +234{,}122\eta_4+70{,}954\eta_5+44{,}468\eta_6)\\
&\quad +1/8{,}949{,}297 \cdot (40{,}810D_1+47{,}177D_2+71{,}341D_3\\
&\qquad +234{,}122D_4+70{,}954D_5+44{,}468D_6)\cdot K_s\\
K_5 &= -1/8{,}949{,}297 \cdot (39{,}890\eta_1+43{,}294\eta_2+47{,}177\eta_3\\
&\qquad +70{,}954\eta_4+233{,}510\eta_5+69{,}154\eta_6)\\
&\quad +1/8{,}949{,}297 \cdot (39{,}890D_1+43{,}294D_2+47{,}177D_3\\
&\qquad +70{,}954D_4+233{,}510D_5+69{,}154D_6)\cdot K_s\\
K_6 &= -1/8{,}949{,}297 \cdot (37{,}108\eta_1+39{,}890\eta_2+40{,}810\eta_3\\
&\qquad +44{,}468\eta_4+69{,}154\eta_5+238{,}298\eta_6)\\
&\quad +1/8{,}949{,}297 \cdot (37{,}108D_1+39{,}890D_2+40{,}810D_3\\
&\qquad +44{,}468D_4+69{,}154D_5+238{,}298D_6)\cdot K_s\\
K_s &= \mathfrak{N}/\mathfrak{D}\\
\mathfrak{N} &= \eta_s+1/8{,}949{,}297 \cdot \{D_1(238{,}298\eta_1+69{,}154\eta_2+44{,}468\eta_3\\
&\qquad +40{,}810\eta_4+39{,}890\eta_5+37{,}108\eta_6)\\
&\quad +D_2(69{,}154\eta_1+233{,}510\eta_2+70{,}954\eta_3+47{,}177\eta_4\\
&\qquad +43{,}294\eta_5+39{,}890\eta_6)
\end{aligned}
\right\} \quad (84)
$$

$$+D_3(44{,}468\eta_1+70{,}954\eta_2+234{,}122\eta_3+71{,}341\eta_4$$
$$+47{,}177\eta_5+40{,}810\eta_6)$$
$$+D_4(40{,}810\eta_1+47{,}177\eta_2+71{,}341\eta_3+234{,}122\eta_4$$
$$+70{,}954\eta_5+44{,}468\eta_6)$$
$$+D_5(39{,}890\eta_1+43{,}294\eta_2+47{,}177\eta_3+70{,}954\eta_4$$
$$+233{,}510\eta_5+69{,}154\eta_6)$$
$$+D_6(37{,}108\eta_1+39{,}890\eta_2+40{,}810\eta_3+44{,}468\eta_4$$
$$+69{,}154\eta_5+238{,}298\eta_6)\Big\}$$

$$\mathfrak{D}=D_s+1/8{,}949{,}297\cdot\Big\{D_1(238{,}298D_1+69{,}154D_2+44{,}468D_3$$
$$+40{,}810D_4+39{,}890D_5+37{,}108D_6)$$
$$+D_2(69{,}154D_1+233{,}510D_2+70{,}954D_3+47{,}177D_4$$
$$+43{,}294D_5+39{,}890D_6)$$
$$+D_3(44{,}468D_1+70{,}954D_2+234{,}122D_3+71{,}341D_4$$
$$+47{,}177D_5+40{,}810D_6)$$
$$+D_4(40{,}810D_1+47{,}177D_2+71{,}341D_3+234{,}122D_4$$
$$+70{,}954D_5+44{,}468D_6)$$
$$+D_5(39{,}890D_1+43{,}294D_2+47{,}177D_3+70{,}954D_4$$
$$+233{,}510D_5+69{,}154D_6)$$
$$+D_6(37{,}108D_1+39{,}890D_2+40{,}810D_3+44{,}468D_4$$
$$+69{,}154D_5+238{,}298D_6)\Big\}\,.$$

§ 3. A SINGLE ROW TRIANGULATION NET.

Eliminating point correlates K_p, K_q and $K_{(i)}$ in equation (24) in § 4 in Chapter 1, equations are obtained as in the following Table 21.

Table 21.

Sign of equation	Left side of equation													Right Side of equation
	K_1	K_2	K_3	K_4	$\longrightarrow$				K_{n-3}	K_{n-2}	K_{n-1}	K_n	K_s	
1	-23	7	3										D_1	η_1
2	7	-26	6	3									D_2	η_2
3	1	2	-9	2	1								D_3	η_3
4		1	2	-9	2	1							D_4	η_4
$n-3$							1	2	-9	2	1		D_{n-3}	η_{n-3}
$n-2$								1	2	-9	2	1	D_{n-2}	η_{n-2}
$n-1$									3	6	-26	7	D_{n-1}	η_{n-1}
n										3	7	-23	D_n	η_n
s	D_1	D_2	$3D_3$	$3D_4$	$\longrightarrow$				$3D_{n-3}$	$3D_{n-2}$	D_{n-1}	D_n	D_s	η_s

where

$$\left.\begin{aligned}
D_1 &= 6\Delta_1 + 4\Delta_4 + 3\,d_{(2)} - 12\,d_1 \\
D_2 &= 4\Delta_4 + 3\,d_{(2)} + 3\,d_{(3)} - 12\,d_2 \\
D_i &= d_{(i-1)} + d_{(i)} + d_{(i+1)} - 4\,d_i \qquad (i = 3,\ 4,\ \cdots\cdots,\ n-2) \\
D_{n-1} &= 3\,d_{(n-2)} + 3\,d_{(n-1)} - 4\Delta_{2n-4} - 12\,d_{n-1} \\
D_n &= 3\,d_{(n-1)} - 4\Delta_{3n-4} - 6\Delta_{3n-1} - 12\,d_n \\
D_s &= 12\,d_s - \left(6\Delta_1^2 + 4\Delta_4^2 + 3\sum_{i=2}^{n-1} d_{(i)}^2 - 4\Delta_{3n-4}^2 - 6\Delta_{3n-1}^2\right) \\
\eta_1 &= 6\,\omega_p + 4\,\omega_{(1)} + 3\,\omega_{(2)} - 12\,\omega_1 \\
\eta_2 &= 4\,\omega_{(1)} + 3\,\omega_{(2)} + 3\,\omega_{(3)} - 12\,\omega_2 \\
\eta_i &= \omega_{(i-1)} + \omega_{(i)} + \omega_{(i+1)} - 4\,\omega_i \qquad (i = 3,\ 4,\ 5,\ \cdots\cdots,\ n-2) \\
\eta_{n-1} &= 3\,\omega_{(n-2)} + 3\,\omega_{(n-1)} + 4\,\omega_{(n)} - 12\,\omega_{n-1} \\
\eta_n &= 3\,\omega_{(n-1)} + 4\,\omega_{(n)} + 6\,\omega_q - 12\,\omega_n \\
\eta_s &= 12\,\omega_s - \Big(6\Delta_1 \cdot \omega_p + 4\Delta_4 \cdot \omega_{(1)} + 3\sum_{i=2}^{n-1} d_{(i)} \cdot \omega_{(i)} \\
&\qquad\qquad - 4\Delta_{3n-4} \cdot \omega_{(n)} - 6\Delta_{3n-1} \cdot \omega_q\Big)\,.
\end{aligned}\right\} \quad (85)$$

Solving these equations quite approximately, that is, neglecting influences of correlates whose coefficients are less than three and side correlate, then following values are obtained:

$$
\left.
\begin{aligned}
K_1 &= -1/549 \cdot (26\,\eta_1 + 7\,\eta_2 + 14/3 \cdot \eta_3) \\
\text{or} \quad & -1/549 \cdot (156\,\omega_p + 132\,\omega_{(1)} + 103.7\,\omega_{(2)} + 25.7\,\omega_{(3)} \\
& \qquad + 4.7\,\omega_{(4)} - 312\,\omega_1 - 84\,\omega_2 - 18.8\,\omega_3)\,, \\
K_2 &= -1/549 \cdot (7\,\eta_1 + 23\,\eta_2 + 46/3 \cdot \eta_3) \\
\text{or} \quad & -1/549 \cdot (42\,\omega_p + 120\,\omega_{(1)} + 105.3\,\omega_{(2)} + 84.3\,\omega_{(3)} \\
& \qquad + 15.3\,\omega_{(4)} - 84\,\omega_1 - 276\,\omega_2 - 61.2\,\omega_3)\,, \\
K_i &= -1/9 \cdot \eta_i \\
\text{or} \quad & 4/9 \cdot \omega_i - 1/9 \cdot (\omega_{(i-1)} + \omega_{(i)} + \omega_{(i+1)}) \\
& \qquad (i = 3,\ 4,\ 5,\ \cdots,\ n-2)\,, \\
K_{n-1} &= -1/549 \cdot (46/3 \cdot \eta_{n-2} + 23\,\eta_{n-1} + 7\,\eta_n) \\
\text{or} \quad & -1/549 \cdot (15.3\,\omega_{(n-3)} + 84.3\,\omega_{(n-2)} + 105.3\,\omega_{(n-1)} \\
& \qquad + 120\,\omega_{(n)} + 42\,\omega_q - 61.2\,\omega_{n-2} - 276\,\omega_{n-1} \\
& \qquad - 84\,\omega_n)\,, \\
K_n &= -1/549 \cdot (14/3 \cdot \eta_{n-2} + 7\,\eta_{n-1} + 26\,\eta_n) \\
\text{or} \quad & -1/549 \cdot (4.7\,\omega_{(n-3)} + 25.7\,\omega_{(n-2)} + 103.7\,\omega_{(n-1)} + 132\,\omega_{(n)} \\
& \qquad + 156\,\omega_q - 18.8\,\omega_{n-2} - 84\,\omega_{n-1} - 312\,\omega_n)\,.
\end{aligned}
\right\} \quad (86)
$$

§4. A QUADRILATERAL.

Eliminating point correlates $K_{(i)}$ ($i = 1$, 2, 3 and 4) in equation (34) in §5 in Chapter 1, equations are obtained as in the following Table 22.

Table 22.

Sign of equation	Left Side of equation				Right Side of equation
	K_1	K_2	K_3	K_S	
1	3	1	-1	D_1	η_1
2	1	3	1	D_2	η_2
3	-1	1	3	D_3	η_3
S	D_1	D_2	D_3	D_S	η_S

Solving these equations, values of correlates and corrections of observed angles are obtained as follows:

$$\left.\begin{aligned}
K_1 &= 1/4\cdot(2\eta_1-\eta_2+\eta_3)-1/4\cdot(2D_1-D_2+D_3)\cdot K_s \\
K_2 &= 1/4\cdot(-\eta_1+2\eta_2-\eta_3)-1/4\cdot(-D_1+2D_2-D_3)\cdot K_s \\
K_3 &= 1/4\cdot(\eta_1-\eta_2+2\eta_3)-1/4\cdot(D_1-D_2+2D_3)\cdot K_s \\
K_{(1)} &= \omega_{(1)}/3-1/12\cdot(3\eta_1-2\eta_2+3\eta_3) \\
&\qquad -\left\{d_{(1)}/3-1/12\cdot(3D_1-2D_2+3D_3)\right\}\cdot K_s \\
K_{(2)} &= \omega_{(2)}/3-1/12\cdot(3\eta_1+\eta_3) \\
&\qquad -\left\{d_{(2)}/3-1/12\cdot(3D_1+D_3)\right\}\cdot K_s \\
K_{(3)} &= \omega_{(3)}/3-1/12\cdot(\eta_1+2\eta_2+\eta_3) \\
&\qquad -\left\{d_{(3)}/3-1/12\cdot(D_1+2D_2+D_3)\right\}\cdot K_s \\
K_{(4)} &= \omega_{(4)}/3-1/12\cdot(\eta_1+3\eta_3) \\
&\qquad -\left\{d_{(4)}/3-1/12\cdot(D_1+3D_3)\right\}\cdot K_s \\
K_s &= \mathfrak{N}/\mathfrak{D} \\
\mathfrak{N} &= 4\eta_s-\Big\{D_1(2\eta_1-\eta_2+\eta_3)+D_2(-\eta_1+2\eta_2-\eta_3) \\
&\qquad\qquad +D_3(\eta_1-\eta_2+2\eta_3)\Big\} \\
\mathfrak{D} &= 4D_s-\Big\{D_1(2D_1-D_2+D_3)+D_2(-D_1+2D_2-D_3) \\
&\qquad\qquad +D_3(D_1-D_2+2D_3)\Big\},
\end{aligned}\right\} \quad (87)$$

and

$$\left.\begin{aligned}
v_{(i)} &= K_{(i)} \qquad\qquad (i=1,\ 2,\ 3 \text{ and } 4) \\
v_1 &= \omega_{(1)}/3-1/12\cdot(3\eta_1-\eta_2) \\
&\qquad -\left\{1/3\cdot(d_{(1)}-3\Delta_1)-1/12\cdot(3D_1-D_2)\right\}\cdot K_s \\
v_2 &= \omega_{(2)}/3-1/12\cdot(3\eta_1-3\eta_2+2\eta_3) \\
&\qquad -\left\{1/3\cdot(d_{(2)}+3\Delta_2)-1/12\cdot(3D_1-3D_2+2D_3)\right\}\cdot K_s \\
v_3 &= \omega_{(2)}/3-1/12\cdot(3\eta_2-\eta_3) \\
&\qquad -\left\{1/3\cdot(d_{(3)}-3\Delta_3)--1/12\cdot(3D_2-D_3)\right\}\cdot K_s
\end{aligned}\right|$$

$$\left.\begin{aligned}
v_4 &= \omega_{(3)}/3 - 1/12 \cdot (2\eta_1+\eta_2-\eta_3) \\
&\qquad - \left\{1/3 \cdot (d_{(3)}+3\varDelta_4) - 1/12 \cdot (2D_1+D_2-D_3)\right\} \cdot K_s \\
v_5 &= \omega_{(3)}/\,3 - 1/12 \cdot (-\eta_1+\eta_2+2\eta_3) \\
&\qquad - \left\{1/3 \cdot (d_{(3)}-3\varDelta_5) - 1/12 \cdot (-D_1+D_2+2D_3)\right\} \cdot K_s \\
v_6 &= \omega_{(4)}/3 - 1/12 \cdot (-\eta_1+3\eta_3) \\
&\qquad - \left\{1/3 \cdot (d_{(4)}+3\varDelta_6) - 1/12 \cdot (-D_1+3D_3)\right\} \cdot K_s \\
v_7 &= \omega_{(4)}/3 - 1/12 \cdot (2\eta_1-3\eta_2+3\eta_3) \\
&\qquad - \left\{1/3 \cdot (d_{(4)}-3\varDelta_7) - 1/12 \cdot (2D_1-3D_2+3D_3)\right\} \cdot K_s \\
v_8 &= \omega_{(1)}/3 - 1/12 \cdot (-\eta_2+3\eta_3) \\
&\qquad - \left\{1/3 \cdot (d_{(1)}+3\varDelta_8) - 1/12 \cdot (-D_2+3D_3)\right\} \cdot K_s\,,
\end{aligned}\right\} \quad (88)$$

where

$$\left.\begin{aligned}
\eta_1 &= 1/2 \cdot \left\{3\omega_1-(\omega_{(1)}+2\omega_{(2)}+\omega_{(3)})\right\} \\
\eta_2 &= 1/2 \cdot \left\{3\omega_2-(\omega_{(2)}+2\omega_{(3)}+\omega_{(4)})\right\} \\
\eta_2 &= 1/2 \cdot \left\{3\omega_3-(\omega_{(3)}+2\omega_{(4)}+\omega_{(1)})\right\} \\
\eta_s &= 1/2 \cdot \left\{3\omega_s-\sum_{i=1}^{4} d_{(i)} \cdot \omega_{(i)}\right\} \\
D_1 &= 1/2 \cdot \left\{3d_1-(d_{(1)}+2d_{(2)}+d_{(3)})\right\} \\
D_2 &= 1/2 \cdot \left\{3d_2-(d_{(2)}+2d_{(3)}+d_{(4)})\right\} \\
D_3 &= 1/2 \cdot \left\{3d_3-(d_{(3)}+2d_{(4)}+d_{(1)})\right\} \\
D_s &= 1/2 \cdot \left\{3d_s-\sum_{i=1}^{4} d_{(i)}^2\right\}.
\end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots \quad (89)$$

§5. ANOTHER COMPACT FORM IN A QUADRILATERAL.

In a quadrilateral as in Fig. 4 in §5 in Chapter 1, if another way of making the angle equation, proposed by Prof. Galachow in the reference (10), is employed, the following results are obtained.

Condition equations:

$$\left.\begin{aligned}
&\text{4 Point equations at each station;}\\
&\qquad v_{(1)}+v_1+v_8=\omega_{(1)}\\
&\qquad v_{(2)}+v_2+v_3=\omega_{(2)}\\
&\qquad v_{(3)}+v_4+v_5=\omega_{(3)}\\
&\qquad v_{(4)}+v_6+v_7=\omega_{(4)}\,.\\
&\text{3 Angle equations;}\\
&\qquad v_1+v_2+v_3+v_4=\omega_1\\
&\qquad v_5+v_6+v_7+v_8=\omega_2\\
&\qquad v_1+v_2+v_7+v_8-(v_3+v_4+v_5+v_6)=\omega_3\\
&\text{A side equation;}\\
&\qquad \sum_{i=1}^{8}(-1)^{i+1}\Delta_i\cdot v_i=\omega_s\,,
\end{aligned}\right\}\cdots\cdots\cdots (90)$$

where,

$$\left.\begin{aligned}
\omega_{(1)}&=360^\circ-((\mathbf{1})+\mathbf{1}+\mathbf{8})\\
\omega_{(2)}&=360^\circ-((\mathbf{2})+\mathbf{2}+\mathbf{3})\\
\omega_{(3)}&=360^\circ-((\mathbf{3})+\mathbf{4}+\mathbf{5})\\
\omega_{(4)}&=360^\circ-((\mathbf{4})+\mathbf{6}+\mathbf{7})\\
\omega_1&=180^\circ-(\mathbf{1}+\mathbf{2}+\mathbf{3}+\mathbf{4})\\
\omega_2&=180^\circ-(\mathbf{5}+\mathbf{6}+\mathbf{7}+\mathbf{8})\\
\omega_3&=\mathbf{3}+\mathbf{4}+\mathbf{5}+\mathbf{6}-(\mathbf{1}+\mathbf{2}+\mathbf{7}+\mathbf{8})\\
\omega_s&=\sum_{i=1}^{8}(-1)^i\log.\sin.\boldsymbol{i}\,.
\end{aligned}\right\}\cdots\cdots\cdots\cdots (91)$$

Applying the principle of the method of least squares, we make $M=min.$, instead of $\sum v\cdot v=min.$,

where,

$$\begin{aligned}
M=\sum v\cdot v&-2K_{(1)}(v_{(1)}+v_1+v_8-\omega_{(1)})\\
&-2K_{(2)}(v_{(2)}+v_2+v_3-\omega_{(2)})\\
&-2K_{(3)}(v_{(3)}+v_4+v_5-\omega_{(3)})\\
&-2K_{(4)}(v_{(4)}+v_6+v_7-\omega_{(4)})
\end{aligned}$$

$$
\begin{aligned}
&-2K_1(v_1+v_2+v_3+v_4-\omega_1)\\
&-2K_2(v_5+v_6+v_7+v_8-\omega_2)\\
&-2K_3(v_1+v_2+v_7+v_8-v_3-v_4-v_5-v_6-\omega_3)\\
&-2K_s\left(\sum_{i=1}^{8}(-1)^{i+1}\varDelta_i\cdot v_i-\omega_s\right).
\end{aligned}
$$

From $M=min.$, viz., $\partial M/\partial v=0$, we obtain,

$$
\left.\begin{aligned}
v_{(i)} &= K_{(i)} \qquad (i=1,\ 2,\ 3,\ \text{and}\ 4)\\
v_1 &= K_{(1)}+K_1+K_3+\varDelta_1\cdot K_s\\
v_2 &= K_{(2)}+K_1+K_3-\varDelta_2\cdot K_s\\
v_3 &= K_{(2)}+K_1-K_3+\varDelta_3\cdot K_s\\
v_4 &= K_{(3)}+K_1-K_3-\varDelta_4\cdot K_s\\
v_5 &= K_{(3)}+K_2-K_3+\varDelta_5\cdot K_s\\
v_6 &= K_{(4)}+K_2-K_3-\varDelta_6\cdot K_s\\
v_7 &= K_{(4)}+K_2+K_3+\varDelta_7\cdot K_s\\
v_8 &= K_{(1)}+K_2+K_3-\varDelta_8\cdot K_s\,.
\end{aligned}\right\} \cdots\cdots\cdots\cdots (92)
$$

Putting the relation (92) into (90), correlate equations are obtained as follows;

$$
\left.\begin{aligned}
3K_{(1)}+K_1+K_2+2K_3+d_{(1)}\cdot K_s &= \omega_{(1)}\\
3K_{(2)}+2K_1+d_{(2)}\cdot K_s &= \omega_{(2)}\\
3K_{(3)}+K_1+K_2-2K_3+d_{(3)}\cdot K_s &= \omega_{(3)}\\
3K_{(4)}+2K_2+d_{(4)}\cdot K_s &= \omega_{(4)}\\
K_{(1)}+2K_{(2)}+K_{(3)}+4K_1+d_1\cdot K_s &= \omega_1\\
K_{(1)}+K_{(3)}+2K_{(4)}+4K_2+d_2\cdot K_s &= \omega_2\\
2K_{(1)}-2K_{(3)}+8K_3+d_3\cdot K_s &= \omega_3\\
\sum_{i=1}^{4}d_{(i)}\cdot K_{(i)}+\sum_{i=1}^{3}d_i\cdot K_i+d_s\cdot K_s &= \omega_s\,,
\end{aligned}\right\} \cdots\cdots\cdots (93)
$$

where

$$
\begin{aligned}
d_{(1)} &= \varDelta_1-\varDelta_8\\
d_{(2)} &= \varDelta_3-\varDelta_2\\
d_{(3)} &= \varDelta_5-\varDelta_4.
\end{aligned}
$$

$$\left.\begin{aligned} d_{(4)} &= \Delta_7 - \Delta_6 \\ d_1 &= \sum_{i=1}^{4}(-1)^{i+1}\Delta_i \\ d_2 &= \sum_{i=5}^{8}(-1)^{i+1}\Delta_i \\ d_3 &= \Delta_1+\Delta_4+\Delta_6+\Delta_7-\Delta_2-\Delta_3-\Delta_5-\Delta_8 \\ d_s &= \sum \Delta\cdot\Delta\,. \end{aligned}\right\} \quad \cdots\cdots\cdots \text{(94)}$$

Eliminating point correlates $K_{(i)}(i=1, 2, 3 \text{ and } 4)$ in equation (93), the following equations are obtained.

$$\left.\begin{aligned} 6K_1-2K_2 \qquad\qquad\qquad +D_1\cdot K_s &= \eta_1 \\ -2K_1+6K_2 \qquad\qquad\qquad +D_2\cdot K_s &= \eta_2 \\ 16K_3+D_3\cdot K_s &= \eta_3 \\ D_1\cdot K_1+D_2\cdot K_2+D_3\cdot K_3+D_s\cdot K_s &= \eta_s\,. \end{aligned}\right\} \quad \cdots\cdots\cdots \text{(95)}$$

Solving equation (95), values of correlates and corrections of observed angles are obtained as follows;

$$\left.\begin{aligned} K_1 &= (3\eta_1+\eta_2)/16 - (3D_1+D_2)/16\cdot K_s \\ K_2 &= (\eta_1+3\eta_2)/16 - (D_1+3D_2)/16\cdot K_s \\ K_3 &= \eta_3/16 - D_3/16\cdot K_s \\ K_{(1)} &= \omega_{(1)}/3 - (2\eta_1+2\eta_2+\eta_3)/24 - \left\{d_{(1)}/3 - (2D_1+2D_2+2D_3)/24\right\}\cdot K_s \\ K_{(2)} &= \omega_{(2)}/3 - (3\eta_1+\eta_2)/24 - \left\{d_{(2)}/3 - (3D_1+D_2)/24\right\}\cdot K_s \\ K_{(3)} &= \omega_{(3)}/3 - (2\eta_1+2\eta_2-\eta_3)/24 - \left\{d_{(3)}/3 - (2D_1+2D_2-D_3)/24\right\}\cdot K_s \\ K_{(4)} &= \omega_{(4)}/3 - (\eta_1+3\eta_2)/24 - \left\{d_{(4)}/3 - (D_1+3D_2)/24\right\}\cdot K_s \\ K_s &= \mathfrak{N}/\mathfrak{D}\,, \\ \text{where} \qquad \mathfrak{N} &= \omega_s - (3\eta_1^2+2\eta_1\cdot\eta_2+3\eta_2^2+\eta_3^2)/16 \\ \mathfrak{D} &= D_s - (3D_1^2+2D_1\cdot D_2+3D_2^2+D_3^2)/16\,. \end{aligned}\right\} \quad \text{(96)}$$

$$\left.\begin{aligned}
v_{(i)} &= K_{(i)} \qquad\qquad (i = 1,\ 2,\ 3 \text{ and } 4)\\
v_1 &= \omega_{(1)}/3 + (5\eta_1 - \eta_2 + \eta_3)/48\\
&\quad - \left\{d_{(1)}/3 - \varDelta_1 + (5D_1 - D_2 + D_3)/48\right\} \cdot K_s\\
v_2 &= \omega_{(2)}/3 + (3\eta_1 + \eta_2 + 3\eta_3)/48\\
&\quad - \left\{d_{(2)}/3 + \varDelta_2 + (3D_1 + D_2 + 3D_3)/48\right\} \cdot K_s\\
v_3 &= \omega_{(2)}/3 + (3\eta_1 + \eta_2 - 3\eta_3)/48\\
&\quad - \left\{d_{(2)}/3 - \varDelta_3 + (3D_1 + D_2 - 3D_3)/48\right\} \cdot K_s\\
v_4 &= \omega_{(3)}/3 + (5\eta_1 - \eta_2 + 5\eta_3)/48\\
&\quad - \left\{d_{(3)}/3 + \varDelta_4 + (5D_1 - D_2 + 5D_3)/48\right\} \cdot K_s\\
v_5 &= \omega_{(3)}/3 + (-\eta_1 + 5\eta_2 + 5\eta_3)/48\\
&\quad - \left\{d_{(3)}/3 - \varDelta_5 + (-D_1 + 5D_2 + 5D_3)/48\right\} \cdot K_s\\
v_6 &= \omega_{(4)}/3 + (\eta_1 + 3\eta_2 - 3\eta_3)/48\\
&\quad - \left\{d_{(4)}/3 + \varDelta_6 + (D_1 + 3D_2 - 3D_3)/48\right\} \cdot K_s\\
v_7 &= \omega_{(4)}/3 + (\eta_1 + 3\eta_2 + 3\eta_3)/48\\
&\quad - \left\{d_{(4)}/3 - \varDelta_7 + (D_1 + 3D_2 + 3D_3)/48\right\} \cdot K_s\\
v_8 &= \omega_{(1)}/3 + (-\eta_1 + 5\eta_2 + \eta_3)/48\\
&\quad - \left\{d_{(1)}/3 + \varDelta_8 + (-D_1 + 5D_2 + D_3)/48\right\} \cdot K_s,
\end{aligned}\right\} \quad \cdots\cdots (97)$$

where

$$\left.\begin{aligned}
\eta_1 &= 3\omega_1 - (\omega_{(1)} + 2\omega_{(2)} + \omega_{(3)})\\
\eta_2 &= 3\omega_2 - (\omega_{(1)} + \omega_{(3)} + 2\omega_{(4)})\\
\eta_3 &= 3\omega_3 - (2\omega_{(1)} - 2\omega_{(3)})\\
\eta_s &= 3\omega_s - \sum_{i=1}^{4} d_{(i)} \cdot \omega_{(i)}\\
D_1 &= 3d_1 - (d_{(1)} + 2d_{(2)} + d_{(3)})\\
D_2 &= 3d_2 - (d_{(1)} + d_{(3)} + 2d_{(4)})\\
D_3 &= 3d_3 - (2d_{(1)} - 2d_{(3)})\\
D_s &= 3d_s - \sum_{i=1}^{4} d_{(i)}^2 .
\end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (98)$$

If observed angles around a station are independently corrected, or simply, outer angles are not observed, point equations at each station are omitted in equation (90), and equation (92) or (93) becomes as follows;

$$\left.\begin{aligned}
v_1 &= K_1+K_3+\varDelta_1\cdot K_s\\
v_2 &= K_1+K_3-\varDelta_2\cdot K_s\\
v_3 &= K_1-K_3+\varDelta_3\cdot K_s\\
v_4 &= K_1-K_3-\varDelta_4\cdot K_s\\
v_5 &= K_2-K_3+\varDelta_5\cdot K_s\\
v_6 &= K_2-K_3-\varDelta_6\cdot K_s\\
v_7 &= K_2+K_3+\varDelta_7\cdot K_s\\
v_8 &= K_2+K_3-\varDelta_8\cdot K_s\,,
\end{aligned}\right\}\quad\cdots\cdots\cdots\cdots\cdots\ (99)$$

and

$$\left.\begin{aligned}
4K_1+d_1\cdot K_s &= \omega_1\\
4K_2+d_2\cdot K_s &= \omega_2\\
8K_3+d_3\cdot K_s &= \omega_3\\
\sum_{i=1}^{3} d_i\cdot K_i+d_s\cdot K_s &= \omega_s\,.
\end{aligned}\right\}\quad\cdots\cdots\cdots\cdots\cdots\ (100)$$

Solving equation (100), correlates and corrections of angles are obtained as follows;

$$\left.\begin{aligned}
K_1 &= 1/4\cdot(\omega_1-d_1\cdot K_s)\\
K_2 &= 1/4\cdot(\omega_2-d_2\cdot K_s)\\
K_3 &= 1/8\cdot(\omega_3-d_3\cdot K_s)\\
K_s &= \mathfrak{N}/\mathfrak{D}\,,\\
&\text{where}\\
\mathfrak{N} &= \omega_s-1/4\cdot(d_1\cdot\omega_1+d_2\cdot\omega_2+1/2\cdot d_3\cdot\omega_3)\\
\mathfrak{D} &= d_s-1/4\cdot(d_1^2+d_2^2+1/2\cdot d_3^2)\,.
\end{aligned}\right\}\quad\cdots\cdots\cdots\ (101)$$

$$\begin{aligned}
v_1 &= 1/4\cdot(\omega_1+1/2\cdot\omega_3)+\left\{\varDelta_1-1/4\cdot(d_1+1/2\cdot d_3)\right\}\cdot K_s\\
v_2 &= 1/4\cdot(\omega_1+1/2\cdot\omega_3)-\left\{\varDelta_2+1/4\cdot(d_1+1/2\cdot d_3)\right\}\cdot K_s
\end{aligned}$$

$$\left.\begin{aligned}
v_3 &= 1/4\cdot(\omega_1-1/2\cdot\omega_3)+\left\{\Delta_3-1/4\cdot(d_1-1/2\cdot d_3)\right\}\cdot K_s\\
v_4 &= 1/4\cdot(\omega_1-1/2\cdot\omega_3)-\left\{\Delta_4+1/4\cdot(d_1-1/2\cdot d_3)\right\}\cdot K_s\\
v_5 &= 1/4\cdot(\omega_2-1/2\cdot\omega_3)+\left\{\Delta_5-1/4\cdot(d_2-1/2\cdot d_3)\right\}\cdot K_s\\
v_6 &= 1/4\cdot(\omega_2-1/2\cdot\omega_3)-\left\{\Delta_6+1/4\cdot(d_2-1/2\cdot d_3)\right\}\cdot K_s\\
v_7 &= 1/4\cdot(\omega_2+1/2\cdot\omega_3)+\left\{\Delta_7-1/4\cdot(d_2+1/2\cdot d_3)\right\}\cdot K_s\\
v_8 &= 1/4\cdot(\omega_2+1/2\cdot\omega_3)+\left\{\Delta_8+1/4\cdot(d_2+1/2\cdot d_3)\right\}\cdot K_s
\end{aligned}\right\}\cdots(102)$$

Relations (99) to (102) coincide with those obtained in reference (10).

Equations (93), (95) and (100) are able to be expressed in the following Table 23, 24 and 25 respectively.

Table 23.

Sign of equation	Left Side of equation								Right Side of equation
	$K_{(1)}$	$K_{(2)}$	$K_{(3)}$	$K_{(4)}$	K_1	K_2	K_3	K_s	
(1)	3				1	1	2	$d_{(1)}$	$\omega_{(1)}$
(2)		3			2			$d_{(2)}$	$\omega_{(2)}$
(3)			3		1	1	-2	$d_{(3)}$	$\omega_{(3)}$
(4)				3		2		$d_{(4)}$	$\omega_{(4)}$
1	1	2	1		4			d_1	ω_1
2	1		1	2		4		d_2	ω_2
3	2		-2				8	d_3	ω_3
S	$d_{(1)}$	$d_{(2)}$	$d_{(3)}$	$d_{(4)}$	d_1	d_2	d_3	d_s	ω_s

Table 24.

Sign of equation	Left side of equation				Right Side of equation
	K_1	K_2	K_3	K_s	
1	6	-2		D_1	η_1
2	-2	6		D_2	η_2
3			16	D_3	η_3
S	D_1	D_2	D_3	D_s	η_s

Table 25.

Sign of equation	Left Side of equation				Right side of equation
	K_1	K_2	K_3	K_s	
1	4			d_1	ω_1
2		4		d_2	ω_2
3			8	d_3	ω_3
S	d_1	d_2	d_3	d_s	ω_s

SUMMARY AND CONCLUSIONS.

The general conclusions to be drawn from the investigations described in this paper may be summarised as follows:

1. The process of the calculation by "Mechanical Sketch Method" can be carried out quite mechanically and easily, in place of solving many simultaneous equations.

2. The calculation consists of simple addition, multiplication and division.

3. On the Correlate Sketch, the numerical values used in obtaining the value of any correlate are always written near it and also near each other, and never isolated.

4. Numerical values for the addition and multiplication can be taken up immediately from the drawing, and the divisor is the number of angles with relation to the correlate, so it is a simple round number and can be known directly from the sketch, with one exception of the case of the side correlate K_s, whose divisor is d_s, but this is also easily taken from the double circle in the sketch.

5. So the calculation can be carried out easily with a slide rule and conveniently with soroban even in the field, where a calculating machine can not be used.

6. Thus the process is carried out mechanically and simply, so chances of mistakes occurring are minimized. If any mistake should creep in any round, it is soon recognized and corrected immediately and unconsciously in the course of the next round. So there is no need especially to correct the mistakes, if they are not very large.

7. The more complex a triangulation net is and the larger the number of condition equations become, the more regularly, systematically and rapidly the calculation can be carried out which some has hitherto been so complex and troublesome that various approximate methods have been adopted in some quarters for many years.

8. The time required to solve many simultaneous equations rigorously by this new method is incomparably less than by methods of elimination or determinant ordinarily used hitherto, and may be even less than by approximate methods.

9. Figures and Tables which give the first values of correlates redouble the merits of this new method and this calculation is also very simple.

10. Only four kinds of elementary triangulation nets are treated in this paper, but these are all fundamental forms. However complex a triangulation net is, it may be a combination of these four kinds of fundamental forms, so this method is applicable to any compound higher or geodetic triangulation nets.

11. This new method is one use of the solution for structural frames, proposed by Prof. F. Takabeya, applied to the adjustment of triangulation nets, and shows the wide range of application proposed by that author.

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