# On a Seawall.

By Yoshiro IKEDA

(Received, Jan. 22, 1952.)

A mathematical investigation on the outside form of a seawall is described. In order to avoid the sedimentation of sand along the seawall it is natural to build such a construction that the magnitude of the velocity of the flow along the construction should either increase or remain constant. On the other hand in order to avoid the erosion of the foundation of the seawall it is natural to build such a construction that the magnitude of the velocity of the flow along it should either gradually decrease or remain constant. Therefore it is preferable to build the construction along which the magnitude of the velocity of the flow is constant, in order to satisfy the both conditions. The velocity of the littoral current must be reduced near the construction at the beach and it is impossible to keep the magnitude of the velocity of the flow constant along all the parts of the construction. But it is possible to build the construction along which the magnitude of the velocity of the flow gradually increases up to a point at some distance from the beach and then is kept constant at all farther points along it from the beach.

Now we assume that the curve in Figure 1 is the one which will satisfy these conditions.

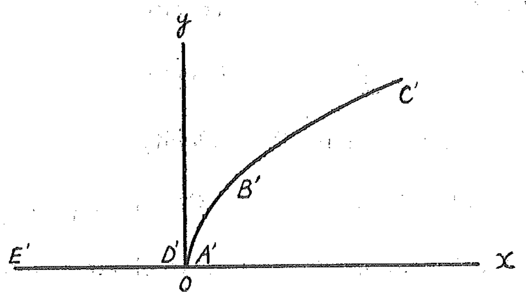


Fig. 1.

$E'\ D'$: beach line

$A'\ B'$: uniform curve along which the magnitude of the velocity of the flow gradually increasses.

$B'\ C'$: uniform curve along which the magnitude of the velocity of the flow keeps constant.

As we showed in the preceding article, if we set

$$\varsigma = \frac{d \log v}{d\varphi} - i\frac{d\alpha}{d\varphi}, \qquad (1)$$

in which $v$ is the velocity, $\varphi$ potential and $\alpha$ the angle between the flow and $x$ axis, then we have

$$\frac{v_x}{v^2} + i\,\frac{v_y}{v^2} = \frac{1}{v_1}\, e^{-\int_{W_1}^{W} \zeta dW} \quad , \qquad (2)$$

in which $W = \varphi + i\psi$ and $\psi$ is the stream function, and

$$x + iy = \frac{1}{v_1}\int_{W_1}^{W} e^{-\int_{W_1}^{W} \zeta dW} dW + x_1 + iy_1 \quad , \qquad (3)$$

where $v_1$ is the magnitude of the velocity which corresponds to $W_1$ and $x_1$ and $y_1$ the positions which correspond to $W_2$.

The curve in Figure 1 can be transformed into the $\zeta$ -plane by equation (1) as shown in Figure 2. $A$, $B$, $C$, $D$ and $E$ correspond to $A'$, $B'$, $C'$, $D'$ and $E'$, respectively and $C$ coincides with $E$. A polygone can be transformed into a half plane by Schwarz- Christoffel transformation. The polygone in Figure 2 can be represented by the real axis in the half plane by the following formula

$$\zeta = -\frac{\sqrt{w}\,\sqrt{w+1}}{2} + \frac{1}{2}\log\left(\sqrt{w} + \sqrt{w+1}\right) \quad . \qquad (4)$$

Again the real axis in $w$-plane is transformed in the real axis of $W$ -plane by the following formula

$$w = \frac{1}{e^{-\pi i}\, W} \qquad (5)$$

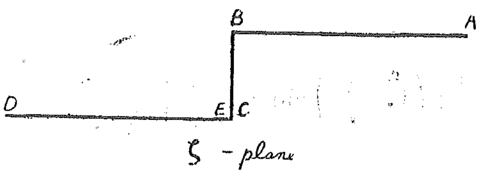

Fig. 2.

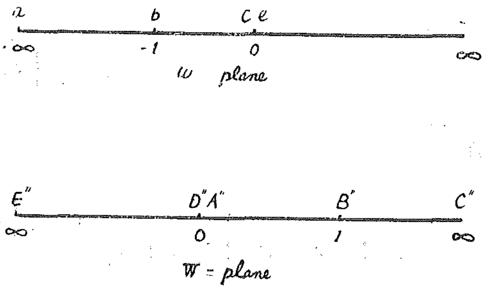

Fig. 3.

$A$, $B$, $C$, $D$ and $E$ correspond to $a$, $b$, $c$, $d$ and $e$ respectively. We place $b = -1$ and $c = d = 0$ without losing generality. Again $a$, $b$, $c$, $d$ and $e$ in $w$-plane correspond to $A''$, $B''$, $C''$, $D''$ and $E''$ in $W$-plane. The real axis in $W$-plane corresponds to the curve in Figure 1 which satisfies the required conditions. From (4) and (5) it follows

$$\zeta = -\frac{\sqrt{1-W}}{e^{-i\pi}\,W} + \frac{1}{2}\log\left(1 + \sqrt{1+e^{-i\pi}\,W}\right) - \frac{1}{4}\log\left(e^{-i\pi}\,W\right) \quad . \qquad (6)$$

Hence

$$-\int_{\infty}^{w}\zeta\, dW = -\frac{\sqrt{1-W}}{2} + \left(1-\frac{W}{2}\right)\log\frac{1+\sqrt{1-W}}{\sqrt{e^{-i\pi}W}}. \quad (7)$$

According to (2) and (3) we obtain from $C'$ to $D'$,

$$\frac{1}{v} = \frac{1}{v_\infty} e^{-\frac{\sqrt{1+|\varphi|}}{2}} \left(\frac{1+\sqrt{1-\varphi}}{\sqrt{\varphi}}\right)^{1+\frac{|\varphi|}{2}} \quad [0<\varphi<1],$$

and

$$x = -\frac{1}{v_\infty}\int_0^{-\varphi} e^{-\frac{\sqrt{1+|\varphi|}}{2}} \left(\frac{1+\sqrt{1+|\varphi|}}{\sqrt{|\varphi|}}\right)^{1+\frac{|\varphi|}{2}} d\varphi$$

From $A'$ to $B'$

$$\frac{v_x}{v^2} + i\frac{v_y}{v^2} = \frac{1}{v_\infty} e^{-\frac{\sqrt{1-\varphi}}{2}} \left(\frac{1+\sqrt{1-\varphi}}{\sqrt{\varphi}}\right)^{1-\frac{\varphi}{2}} \left(\sin\frac{\pi}{4}\varphi + i\cos\frac{\pi}{4}\varphi\right),$$

and

$$x + iy = \frac{1}{v_\infty}\int_0^{\varphi} e^{-\frac{\sqrt{1-\varphi}}{2}} \left(\frac{1+\sqrt{1-\varphi}}{\sqrt{\varphi}}\right)^{1-\frac{\varphi}{2}} \left(\sin\frac{\pi}{4}\varphi + i\ \mathrm{ces}\frac{\pi}{4}\varphi\right) d\varphi .$$

From $B'$ to $C'$

$$\frac{v_x}{v^2} + i\frac{v_y}{v^2} = \frac{1}{v_\infty} e^{i\left\{\frac{\sqrt{\varphi-1}}{2} + \left(1-\frac{\varphi}{2}\right)\sin^{-1}\sqrt{\frac{1}{\varphi}}\right\}},$$

and

$$x + iy = x_1 + iy_1 + \int_0^{\varphi}\frac{1}{v_\infty} e^{i\left\{\frac{\sqrt{\varphi-1}}{2} + \left(1-\frac{\varphi}{2}\right)\sin^{-1}\sqrt{\frac{1}{\varphi}}\right\}} d\varphi .$$

The calculations are carried out by series expansion and by numerical integration and the results are shown in Table 1.

In order to obtain the stream lines near the seawall, the intersecting points of the stream lines $\psi = 0.1$, $0.2$, $0.3$, $0.4$ and $0.5$ with the equipotential lines $\varphi = -1, 0$ and $1$ are calculated by series expansion with the results as tabulated in Table 2.

When $\varphi$ is large, the coordinates $(x, y)$ of the seawall can be developed by the descending power series of $\varphi$ in the following form;

$$x = \varphi - 0.2222\log\varphi + 0.0362\frac{1}{\varphi} + 0.0063\frac{1}{\varphi^2} + 0.0004\frac{1}{\varphi^3} - 0.555,$$

$$y = \sqrt{\varphi}\left(\frac{4}{3} - 0.0346\frac{1}{\varphi} - 0.0051\frac{1}{\varphi^2} - 0.0032\frac{1}{\varphi^3}\right) + 0.7044 \quad .$$

Table 1.

| $\varphi$ | $x$ | $\varphi$ | $x$ | $y$ | $\varphi$ | $x$ | $y$ |
|---|---|---|---|---|---|---|---|
| −0.1 | 0.793 | 0.1 | 0.019 | 0.740 | 1.6 | 0.966 | 2.361 |
| −0.2 | 1.159 | 0.2 | 0.051 | 1.020 | 1.7 | 1.051 | 2.414 |
| −0.3 | 1.449 | 0.3 | 0.091 | 1.223 | 1.8 | 1.137 | 2.466 |
| −0.4 | 1.706 | 0.4 | 0.137 | 1.385 | 1.9 | 1.223 | 2.514 |
| −0.5 | 1.941 | 0.5 | 0.187 | 1.521 | 2.0 | 1.311 | 2.564 |
| −0.6 | 2.162 | 0.6 | 0.240 | 1.642 | 2.1 | 1.398 | 2.611 |
| −0.7 | 2.371 | 0.7 | 0.298 | 1.748 | 2.2 | 1.488 | 2.657 |
| −0.8 | 2.571 | 0.8 | 0.359 | 1.840 | 2.3 | 1.577 | 2.702 |
| −0.9 | 2.764 | 0.9 | 0.424 | 1.922 | 2.4 | 1.666 | 2.746 |
| −1.0 | 2.951 | 1.0 | 0.492 | 1.996 | 2.5 | 1.757 | 2.784 |
| −1.1 | 3.133 | 1.1 | 0.565 | 2.064 | 2.6 | 1.847 | 2.831 |
| −1.2 | 3.312 | 1.2 | 0.641 | 2.129 | 2.7 | 1.938 | 2.873 |
| −1.3 | 3.486 | 1.3 | 0.720 | 2.190 | 2.8 | 2.030 | 2.913 |
| −1.4 | 3.658 | 1.4 | 0.801 | 2.250 | 2.9 | 2.121 | 2.952 |
| −1.5 | 3.826 | 1.5 | 0.883 | 2.306 | 3.0 | 2.214 | 2.993 |

Table 2.

| | $w = 1 + i\psi$ | | $w = 0 + i\psi$ | | $w = -1 + i\psi$ | |
|---|---|---|---|---|---|---|
| $\psi$ | $x$ | $y$ | $x$ | $y$ | $x$ | $y$ |
| 0.1 | 0.420 | 2.068 | −0.560 | 0.602 | −2.951 −0.002 | 0.185 |
| 0.2 | 0.352 | 2.149 | −0.780 | 0.882 | 〃 −0.009 | 0.369 |
| 0.3 | 0.269 | 2.233 | −0.966 | 1.114 | 〃 −0.020 | 0.552 |
| 0.4 | 0.208 | 2.335 | −1.075 | 1.302 | 〃 −0.035 | 0.732 |
| 0.5 | 0.105 | 2.443 | −1.190 | 1.514 | 〃 −0.053 | 0.909 |

From the formulae, the values of $x$ and $y$ are calculated as shown in Table 3. By using suitable unit, the value of $x$ at the point ($\varphi = 1$, $\psi = 0$) can be taken to be equal to 1. Then from Table 2 and 3 it will be written as in Table 4. The values of the coordinates of the seawall shown in Table 4 may be expressed approximately in the interval $x = 0$ and $x = 20$ by the following formula

$$y = 4\sqrt[4]{x + 0.0617\,x^2} \quad .$$

Table 3

| $\varphi$ | $x$ | $y$ |
|---|---|---|
| 4 | 3.037 | 3.353 |
| 5 | 4.088 | 3.670 |
| 6 | 5.053 | 3.955 |
| 7 | 6.023 | 4.218 |
| 8 | 6.987 | 4.462 |
| 9 | 7.960 | 4.693 |
| 10 | 8.937 | 4.909 |

Table 4

| $\varphi$ | $x$ | $y$ |
|---|---|---|
| 1 | 1.000 | 4.057 |
| 2 | 2.665 | 5.211 |
| 3 | 4.500 | 6.083 |
| 4 | 6.173 | 6.815 |
| 5 | 8.309 | 7.459 |
| 6 | 10.269 | 8.039 |
| 7 | 12.243 | 8.574 |
| 8 | 14.201 | 9.029 |
| 9 | 16.180 | 9.544 |
| 10 | 18.164 | 9.978 |

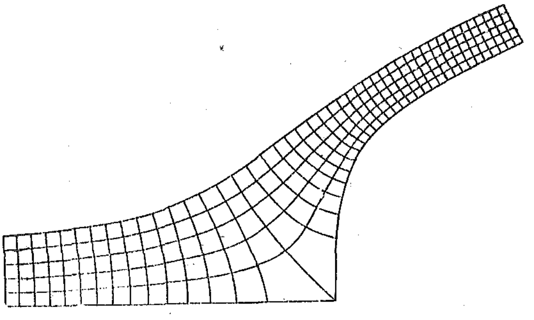

Plate 1.

There are many small ports built on the sandy beaches of Hokkaido, Japan, but only a few of them* are in a tolerable state to avoid the sedimentation of sand or erosion of the foundation of the seawall. outside forms of seawalls resemble somehow the curve shown in Plate 1. Though the stream lines near the coast are too complicated to be calculated from mathematical consideration alone, it might be advised at least that the construction on sand beach should have the outside form of this type.

* Kitami Monbetsu is an excellent example.