Tripping Characteristics of Protective Relays on Transmission Network, Expressed by Circle Diagram Method. (III)

(The End)

(With APPENDIX; a Stability Calculation

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§ 3. Reactance, Susceptance and Power Factor Circles to Express the Tripping Area of Reactance Relay and Mho Relay.

Fig. 16 (a) shows tripping area of impedance relay, reactance line of ohm unit, X, mho circle of mho unit or starting element of reactance relay, M, directional line of impedance relay, D, and probable range of short circuit fault, A on the $R+jX$ impedance plane. Their inverted figures on the $g+jb$ admittance plane are shown in Fig. 16 (b).

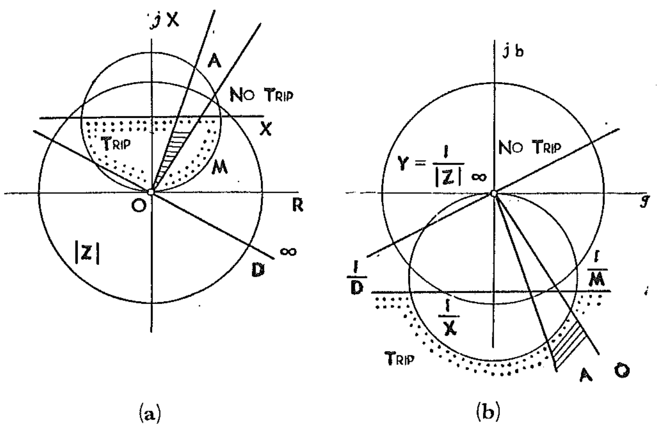


Fig. 16. Tripping characteristic of impedance relays.
(a) on impedance plane.
(b) on admittance plane.

It is assumed that the relay setting point is at the same point as the actual power receiving end, through a transmission line $ABCD$, and that there is a single power generator at the sending end.

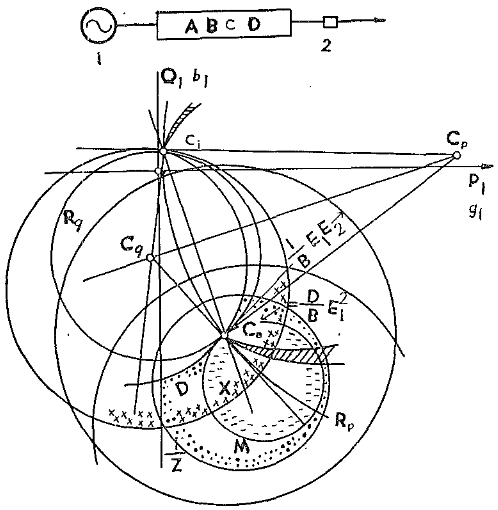


Fig. 17. Tripping characteristics of distance relay on power circle diagram co-ordinate plane.

The sending power circle diagram co-ordinate plane, P_1+jQ_1 may be also used as its admittance co-ordinate plane, because the sending voltage E_1 is constant in the relation,

$$g_1 - jb = (P_1 + jQ_1) \div E_1^2$$

The tripping characteristics of distance relay in Fig. 16 (b) are transfigured on the sending power co-ordinate plane as shown in Fig. 17. As explained in Appendix, Part II, the co-ordinate axes g and b are transfigured to the fundamental circles R_p and R_q, while changing ∞ point to $\frac{D}{B}E_1^2$ and O point to $\frac{C}{A}E_1^2$ point on the sending power co-ordinate plane. The admittance circles have their centers on $\overline{C_iC_e}$ line. The reactance circles are always through the C_i point, having their centers on $\overline{C_iC_q}$ line.

The susceptance circles are always through the C_e point, having their centers on $\overline{C_eC_q}$ line. The power factor constant circles are those passing through C_e, C_i points. C_p and C_q are the centers of the fundamental circles, R_p and R_q. Numerical calculation of these curves will be illustrated in the next example.

The advantage of this expression is that one may directly see the normal operating characteristics of transmission power as well as the distance relay operation, by drawing the power circle diagram which has its center at $\frac{\boldsymbol{D}}{\boldsymbol{B}} E_1^2$, with its radius as $\frac{E_1 E_2}{\boldsymbol{B}}$, on the same graph.

Futhermore, it is obvious that the actual power receiving end the relay setting point may be at different points, or that different values of $\frac{\boldsymbol{D}}{\boldsymbol{B}} E_1^2$ for them may be taken.

Example 3.

A transmission line 30 MVA, 10 KV has line constants $\boldsymbol{A}=\boldsymbol{D}=1$, $\boldsymbol{C}=0$, $1/\boldsymbol{B}=5-j\,20$ p.u. 10 MVA base and $E_1=E_2=1$ p.u..

Draw the power circle diagram and the tripping areas of impedance, mho, ohm unit of distance relay at the same receiving end.

Solution:

Center and radius of power circle diagram:

$$\boldsymbol{C}_e = \frac{\boldsymbol{D}}{\boldsymbol{B}} E_1^2 = (5-j\,20)\times 1 = 5-j\,20 \text{ p.u. } (=50 \text{ MW}-j\,200 \text{ Mvar})$$

$$R = \frac{1}{|\boldsymbol{B}|} E_1 E_2 = |5-j\,20| = 20.3 \quad \text{p. u.}$$

Power factor 0 circle:

$$R_p = \frac{1}{\boldsymbol{B}_k + \boldsymbol{B}} = \frac{1}{\dfrac{1}{5+j20} + \dfrac{1}{5-j\,20}} = 42.5 \quad \text{p. u.}$$

$$\boldsymbol{C}_p = 42.5+j\,0 \quad \text{p.u.}$$

Power factor 1 circle:

$$R_q = \frac{1}{\boldsymbol{B}_k - \boldsymbol{B}} = 10.6 \quad \text{p. u.}$$

$$\boldsymbol{C}_q = 0-j\,10.6 \quad \text{p. u.}$$

The intersecting points of R_p, R_q circles are $\boldsymbol{C}_e=5-j20$ and $\boldsymbol{C}_i=0$ points.

$$\boldsymbol{C}_i = \frac{\boldsymbol{C}}{\boldsymbol{A}} E_1^2 = 0$$

Admittance circles to show the tripping area of impedance element have their centers on C_iC_e line. And the admittance,

$$|Y| = \frac{1}{|Z|} = \frac{\overline{C_iX}}{\overline{C_eX}}\left|\frac{A}{B}\right|$$

where X is the intersecting point of $\overline{C_iC_e}$ line and admittance circle. Therefore, the bisecting line has

$$|Y| = 1 \times \left|\frac{A}{B}\right| = \sqrt{5^2+20^2} = 20.3 \quad \text{p. u.}$$

1/3 of $\overline{C_iC_e}$ point circle

$$|Y| = 3 \times \sqrt{5^2+20^2} = 60.9 \quad \text{p. u.}$$

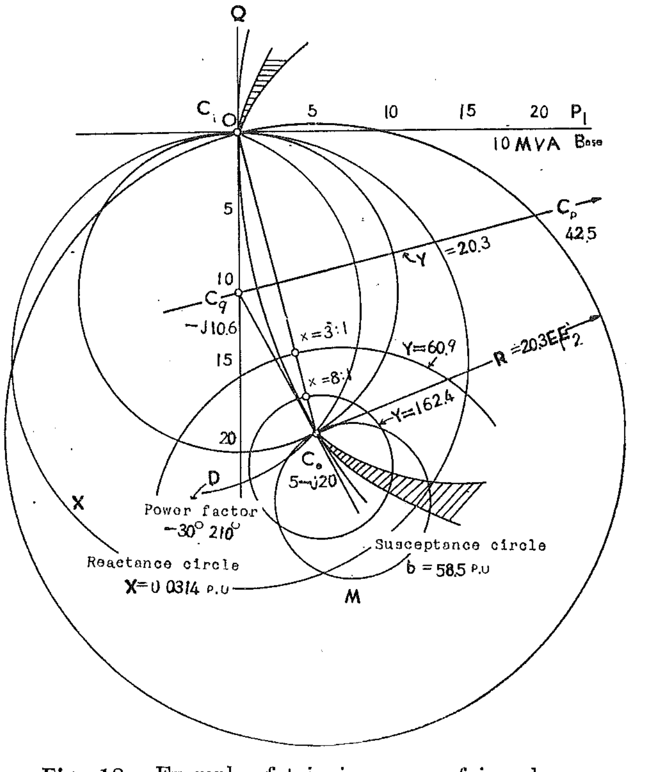


Fig. 18. Example of tripping areas of impedance, Mho and Ohm unit of a distance aelay.

C_e : Center of power circle diagram. M : Mho unit.
Y : Impedance (Admittance) unit. X : Reactance unit.
D : Directional unit. ////// area : Probable fault locus.

Reactance circle to show the tripping area of ohm unit:

$$\boldsymbol{X}=\frac{E_1^2}{\boldsymbol{A}}\left(\frac{1}{2R_q}-\frac{1}{2r_x}\right).$$

For example $r_x=3/2\ R_q$ circle,

$$\boldsymbol{X}=1\times\left(\frac{1}{2\times 10.6}-\frac{1}{2\times\frac{3}{2}10.6}\right)$$

$$=1/31.8=0.0314\ \text{p.u.}$$

Susceptance circle for mho unit:

$$\boldsymbol{M}=\frac{E}{\boldsymbol{B}}\left(\frac{1}{2R_q}+\frac{1}{2r_b}\right)$$

For example $r_b=1/2\,R_q$

$$\boldsymbol{M}=(20.3)^2\times 3/(2\times 10.6)=58.5\ \text{p.u.}$$

Power factor circles for the directional element or the probable fault locus may be obtained from the circles through $\boldsymbol{C}_e$ and $\boldsymbol{C}_i$ point, measuring their power factor angle at the above points, as $\boldsymbol{C}_p$ circle has zero angle. The results are shown in Fig. 18.

§ 4. Frequency Swing Locus and Relay Tripping area.

The frequency swing of a sound transmission system with parallel running generators is a troublesome problem in respect to relay protection. The opening of the sound line is nearly always undesirable, as the system can recover in ordinary operation. But, in some cases the opening is desirable to prevent out-of-step condition.

Swing locus for a transmission line with synchronous machines at its two ends is expressed on the sending power co-ordinate plane, P_1+jQ_1, as shown in Fig. 19.

It is the usual power circle diagram itself.

At first, one may consider the matter of 3-phase short circuit fault. assuming that the relay is set at the middle point, m of the line and also at the same point short circuit takes place.

Then, the short circuit point on the P_1+jQ_1 plane is at the point $\frac{2\boldsymbol{D}}{\boldsymbol{B}}E_1^2$, or on the power circle, as it is the middle point. The short circuit current is obtained from the vector $\overline{OS}$. Various relay charac-

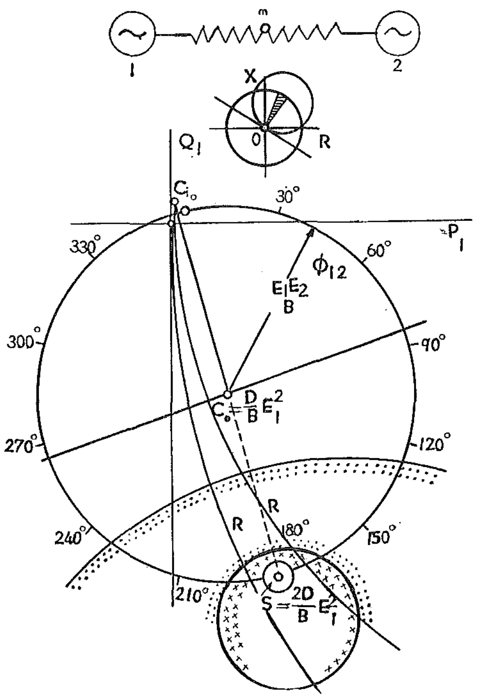


Fig. 19. Frequency swing locus and relay tripping.
S: Center of power circle diagsam, considered the middle point, m as a receiving end.
R: Resistance circles to limit the tripping area during frequency swing.

teristics are shown in Fig. 19. The short circuit points are in general at further points from the center C_e, if the faults take place on the line between the two ends.

Nextly, one may consider frequency swinging. When small frequency difference between the parallel running machines exists, the power angle ϕ_{12} traverses 0° to 360°. At about 180° angle, the operating point on the power circle diagram may agree with the above short circuit fault point, S, flowing large current.

Furthermore, the voltage of the middle point of the transmission line in the case of frequency swinging is zero, because the new power circle diagram, considering the receiving end at the middle point of the line, has its center at S, $\frac{2\boldsymbol{D}}{\boldsymbol{B}}E_1^2$ and its radius $\frac{2}{|\boldsymbol{B}|}E_1E_2$ is obviously zero. The same result as a shirt circuit fault occurs on a line. However, this frequency swing recovers at ordinary operation without occurrence of out-of-step. There is no need to open the circuit breaker.

To prevent the misoperation of impedance relay due to frequency swing, out-of-step blocking is used nearly always in conjunction with high speed relays.

This blocking is accomplished due to the fact that the short circuit fault occurs suddenly, while the frequency swing traverses on the power circle locus more gradually.

The impedance relay has a blocking secondary zone having larger impedance round the first tripping zone, as shown in Fig. 19. At frequency swing the relay at once enters in the blocking zone to stop the relay tripping. However, when short circuit fault occurs, by abrupt change of the impedance the relay goes to the tripping area without time to block.

The resistance circle with its centers on C_iC_p line, passing always through C_i point, may be available to limit the tripping area of $|Z|$ circle, as shown above in circle R, in Fig. 19. To prevent misoperation of the relay, the resistance circle element as well as the directional element or reactance element is useful to make the tripping area as small as possible, enclosing the fault locus.

Carrier-pilot system:

Furthermore, the tripping area may be limited by using Carrier-pilot type impedance relays similar to the above explained methods. The tripping occurs only when two relay indication are given simultaneously at both ends of the line. This characteristic may easily be obtained by superposing the tripping characteristics at the two setting points.

Frequency swing locus on the admittance plane of the relay setting point:

From Equation (11) in Appendix, Part II, the frequency swing equation is

$$y_p = g+jb = \frac{-A_1+D_2 n\varepsilon^{j\phi_{12}}}{-B_1+B_2 n\varepsilon^{j\phi_{12}}} \tag{11}$$

$$n = \frac{E_1}{E_2}, \quad \phi_{12}: \ \text{power angle},$$

or, transforming the above equation.

$$n\varepsilon^{j\phi_{12}} = \frac{-\dfrac{A_1}{B_1}-y_p}{-\dfrac{D_2}{B_2}+y_p} \cdot \frac{B_1}{B_2}. \tag{14}$$

By the method of conformal transformation of the complex function of the first order, frequency swing loci may be obtained for ϕ_{12} or $n=\dfrac{E_1}{E_2}$. The relay tripping characteristics are shown in the preceding section 4, Fig. 16(b). A numerical example will be shown below.

Example 4.

A transmission line with 20 MVA machine at its two ends. Line constant $A=D=1$, $C=0$, $B=1/(5-j20)$ p.u. 10 MVA base, $E_1=1$ p.u.

Draw the swing curves and the impedance relay tripping areas, assuming their setting point at the middle point of the line.

Solution :

At the middle point, $\frac{1}{B_1}=\frac{1}{B_2}=10-j40$, $A_1=D_1=A_2=D_2=1$.
From Equations (17) (11),

$$\frac{E_1}{E_2}\varepsilon^{j\phi_{12}}=\frac{-\frac{A_1}{B_1}-y_p}{-\frac{D_2}{B_2}+y_p}\,\frac{B_1}{B_2}=\frac{-10+j40-y_p}{-10+j40+y_p}$$

$$y_p=\frac{-A_1+D_2\frac{E_1}{E_2}\varepsilon^{j\phi_{12}}}{-B_1+B_2\frac{E_1}{E_2}\varepsilon^{j\phi_{12}}}=(10-j40)\frac{-1+1\frac{E_1}{E_2}\varepsilon^{j\phi_{12}}}{-1+1\frac{E_1}{E_2}\varepsilon^{j\phi_{12}}}\,.$$

The curves for the frequency swing, taking ϕ_{12} as a variable and for the voltage variation, taking the voltage ratio, $\frac{E_1}{E_2}$ as a variable, are shown in Fig. 20.

At the points $y_p=10-j40$ or $=-10+j40$, the voltage E_2 or E_1

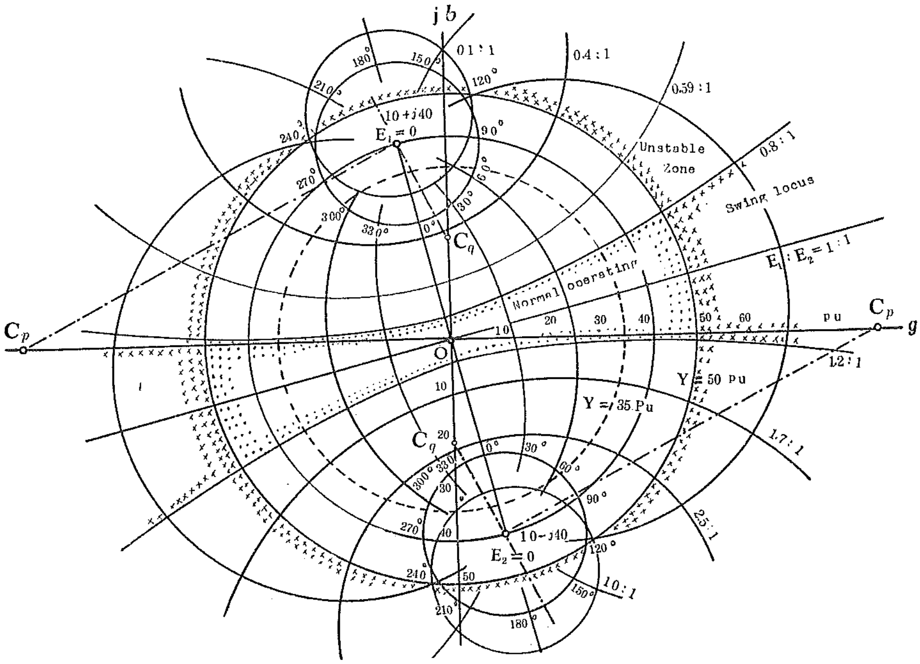


Fig. 2. Tripping area of the impedance relay on the admittance plane at the relay setting point.

respectively become zero.

The tripping area of an impedance relay with $|Y|=50$ p.u. is outside of its circle as shown in Fig. 20.

The normal operating zone is shown by the dotted area, which is enclosed by the relay circle and the voltage variation curve, if 20% voltage variation is allowable.

The unstable zone, where large frequency swing occurs, is at the outer side of the co-ordinate plane, and the phase angle between E_1 and E_2 are about $120°\sim240°$.

The voltage and power at the middle point of the line is obtained from the fundamental circles,

$$C_p = \frac{1}{B_k+B} = 85 \text{ p.u.}$$

$$C_q = \frac{1}{B_k-B} = -j21.65 \text{ p.u.}$$

taking terminal 1 as sending end or terminal 2 as receiving end with respect to the middle point.

By the above two difference methods, however one obtains the same results.

The receiving power from terminal 1 at the middle point is of course equal to the sending power from the middle point to the terminal 2, as it is a sound line in the case of frequency swing.

The graphical expression is not shown in Fig. 20.

The process of securing this expression is as follows.

The watt powers at the middle point are obtained from the circles, passing through $C_e=\pm10\pm j40$ and having their centers at $\overline{C_eC_p}$ line.
(2) The middle point voltage E_m is obtained from the distance between the point, C_e and a certain point, X on the co-ordinate plane, by using the relation $E_m = \dfrac{E_1 \text{ or } E_2}{|B_1| \times \overline{C_eX}}$.

Therefore, at the remote point from C_e the middle point voltage may become zero.

Conclusion:

On the admittance co-ordinate plane, the tripping characteristics of relays, which are at a certain place of the line may be expressed as well as the transmission characteristics at normal operation. In this expression, various circle loci, which are introduced from the fundamental circles in Chapter 2 may be available.

APPENDIX

TRANSIENT STABILITY OF TRANSMISSION NETWORK, WHEN THE EFFECTS OF SALIENT POLE MACHINES ARE CONSIDERED

§ 1. Outline of Applying the Power Circle Diagram Method for the System Stability Criterion.

The power circle diagram method may also availed for stability criterion. Some simple examples will be shown for the purpose of explaining how this method can be applied.

In Fig. 1, the power circle diagram for no loss line, connecting two-synchronous machines, has the radius of $\frac{E_2E_2}{\dot{j}x}$, and its center is located at $\frac{E_1^2}{\dot{j}x}$. The power limit is at the point, $\phi=90°$ as shown in the above figures of the circle and the sine curves.

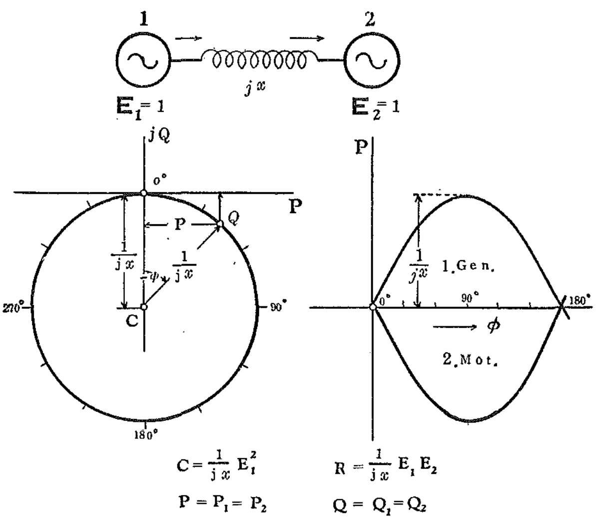


Fig. 1. Power circle diagram for no loss line, indicating sending and receiving powers.

If the series reactance suddenly increased to $j2x$ by disconnecting one line of the double circuits, the radius and the position of center

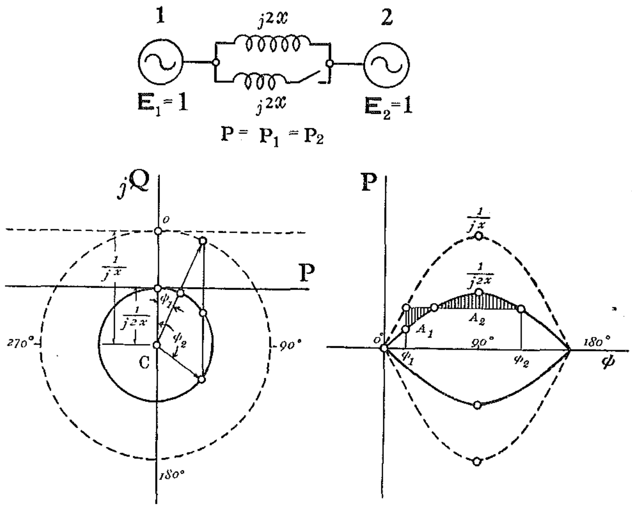


Fig. 2. Equal area method for the stability criterion.

are changed into $\dfrac{E_1E_2}{j\,2x}$ & $\dfrac{E_1^2}{j\,2x}$, respectively, as shown in Fig. 2.

The equal area method may be applied to sine curves as shown in Fig. 2. That is the integration of the accelerating power P_a or $\int_{\phi_1}^{\phi_2} P_a=0$. And the condition, $A_1=A_2$ is graphically obtained.

When shunt inductance loads, that is $-jy$, are connected at the

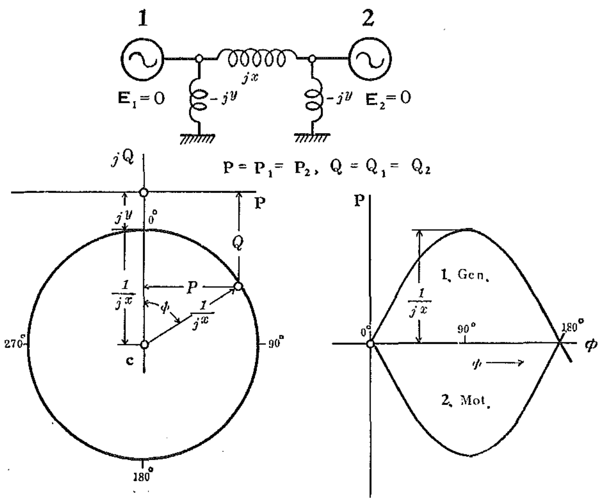


Fig. 3. Circle diagram and inductance loads.

two ends of the line, the circle diagram moves as much as $-jy$ on the vertical axis as shown in Fig. 3. For simplicity, $E_1=E_2=1$ is assumed. It is remembered that the power limit is independent on the shunt loads.

For a phase ground or a phase short circuit unsymmetrical fault, it is known that the equivalent positive sequence circuit can be expressed by an equivalent intermediate loaded line as shown in Fig. 4 (a) or (b). For a phase disconnecting fault, its equivalent positive sequence line can be obtained by inserting a series reactance as shown in Fig. 4 (c). In these cases, stability limit may be determined by the method explained in Figs. 2 and 3.

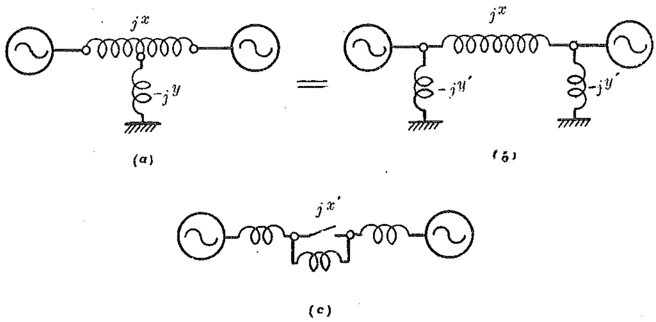


Fig. 4. Equivalent positive sequence circuits,
(a), (b): Phase grounding or a phase short circuit,
(c): Disconnecting fault.

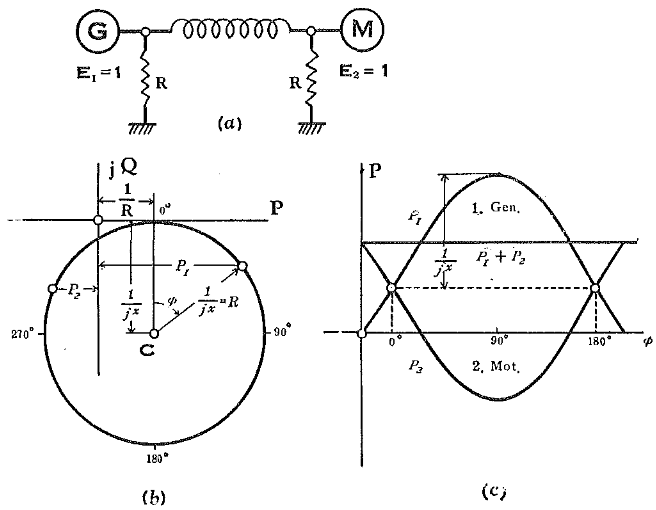


Fig. 5. Circle diagram and resistance load.

For shunt resistance loads, the center of the circle moves in the horizontal direction as much as $\frac{1}{R}$ in distance. These sine curves and the circle locus for power and power angle are shown in Fig. 4 (b)(c). It is remembered that the accelerating power is independent on the shunt load.

For a transmission line with a series of resistance and reactance, the circle diagram may have its center at $\frac{1}{R+jX}$. Therefore, the relations between the power and power angle are shown by the sine curves or the circle in Fig. 5 (b) (c). The sum, P_1+P_2 is the total transmission loss.

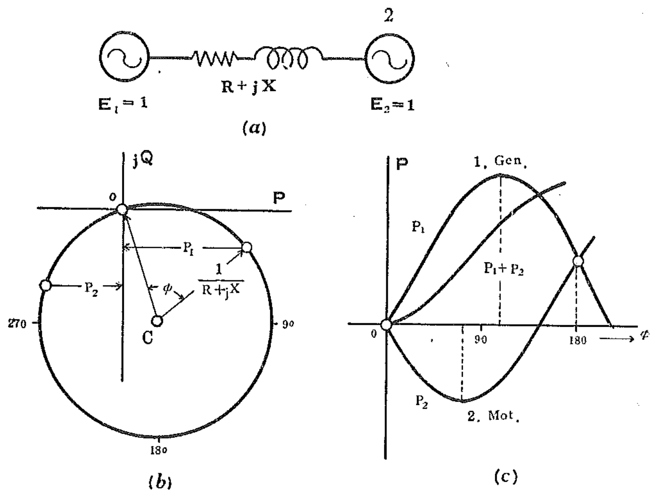


Fig. 6. Circle diagram with a series resistance.

The T-circuit with an intermediate load may be transformed into an equivalent π-circuit as shown in Fig. 6 (a) (b). The relations between the power circle or power curves and the power angle are shown in Fig. 6 (c) and (d), which are easily obtained by the same way as the previous examples.

Lastly, in the case of salient pole machines, power circle diagram is not applicable. However, vector locus of power and power angle as shown in Fig. 7 and 8 may be used. In this case, the stability limit is not at $\phi=90°$. These kinds of cases will be explained in the following sections.

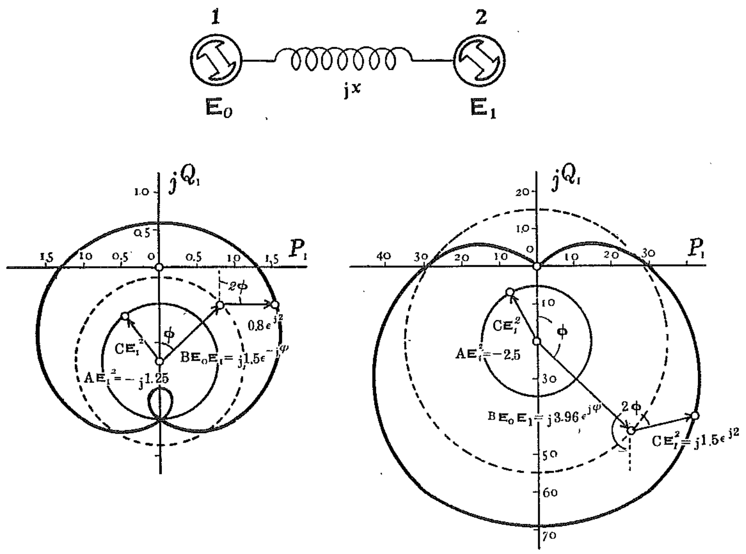


Fig. 7. Salient Phle machine.

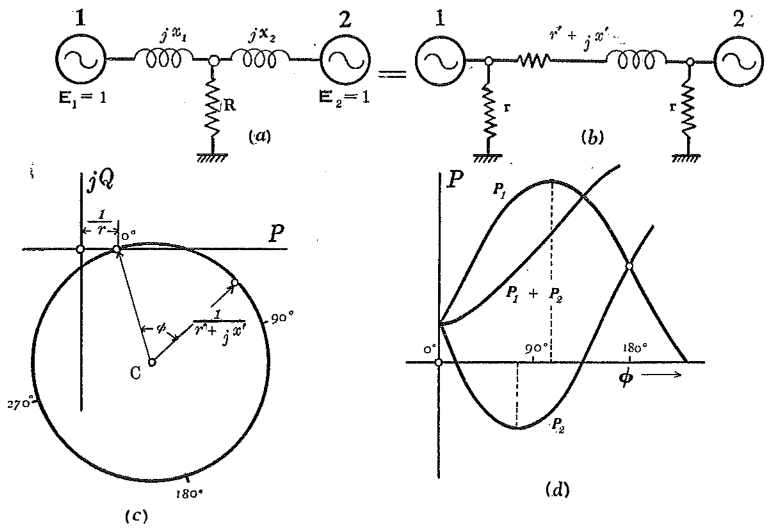


Fig. 8. (a)

Fig. 8. Examples of current vector loci for Salient Pole machine.

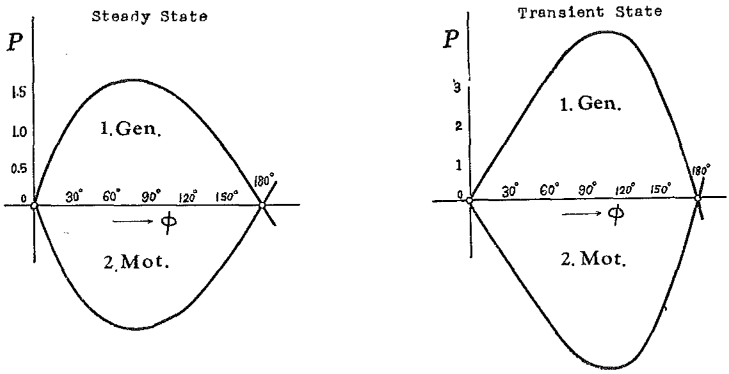


Fig. 8. (b)

§ 2. Introduction to Salient Pole Machines with Network.

This section is intended to explain how the deformation of the usual power circle diagrams is caused by the salient pole machines. The effects are not negligibly small. However, the calculation requires a considerable labor.

The characteristics of the salient pole machine are expressed by the following formulas.

Excitation voltages are,

$$\begin{aligned} E_0 &= \boldsymbol{E}_1 + x_d I_1 \sin\theta + jX_q I_1 \cos\theta \\ &= \boldsymbol{E}_1 + j\frac{1}{2}(x_d + x_q)\,\boldsymbol{I} - j\frac{1}{2}(x_d - x_q)\,\boldsymbol{I}_{1k} \end{aligned}$$

$\cdots\cdots\cdots\cdots\cdots$ for steady state.

$$E_0' = \boldsymbol{E}_1 + j\frac{1}{2}(x_d' + x_q)\,\boldsymbol{I}_1 - j\frac{1}{2}(x_d' - x_q)\,\boldsymbol{I}_{1k} \qquad (1)$$

$\cdots\cdots\cdots\cdots\cdots$ for transient state.

$E_{1g} = \boldsymbol{E}_1 + jx_d'\boldsymbol{I}_1$ $\cdots\cdots\cdots\cdots\cdots$ for approximate calculation.

In the above relations,

E_0: excitation voltage
E_0': excitation voltage at transient state
$\boldsymbol{E}_1$: terminal voltage
$\boldsymbol{I}_1$ & $\boldsymbol{I}_{1k}$: current and its conjugate value
x_d & x_q: direct and quadrature axis reactance

x_d' : transient reactance at transient state
E_{1g} : voltage behind the transient reactance
E_p : voltage stands for Potier reactance
x_p : Potier reactance.

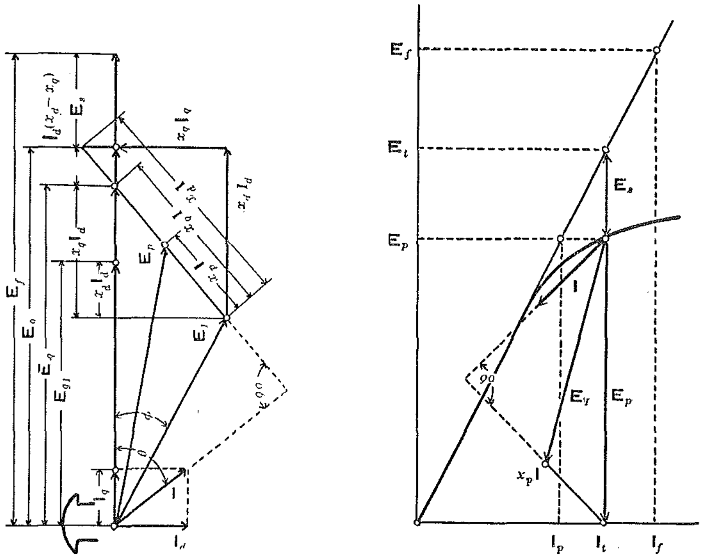


Fig. 9. Vector diagram for salient pole machine.

The vector diagram is as shown in Fig. 9. The output current under a constant terminal voltage and excitation is expressed by

$$I_1 = \frac{j\frac{1}{2}(x_d+x_q)}{\left\{\frac{1}{2}(x_d+x_q)\right\}^2-\left\{\frac{1}{2}(x_d-x_q)\right\}^2} E_1 - j\frac{1}{x_d}E_0\varepsilon^{+j\phi}$$
$$-\frac{j\frac{1}{2}(x_d-x_q)}{\left\{\frac{1}{2}(x_d+x_q)\right\}^2-\left\{\frac{1}{2}(x_d-x_q)\right\}^2} E_1\varepsilon^{+j2\phi} \qquad (3)$$

when E_1 is taken as the standard vector. ϕ is the power angle between E_0 and E_1.

The watt and wattless power at the machine terminal is

$$P_1+jQ_1 = E_{1k}I_1 \qquad (4a)$$

in which I_1 is the value of equation (3). In order to obtain a similar diagram as the power circle diagram, one obtains the following equation after changing the sign of current I_1,

$$P_1+jQ_1=\frac{-j\frac{1}{2}(x_d+x_q)}{\left\{\frac{1}{2}(x_d+x_q)\right\}^2-\left\{\frac{1}{2}(x_d-x_q)\right\}^2}|E_1|^2\varepsilon^{j2\phi}+j\frac{1}{x_d}E_1E_0\varepsilon^{j\phi}$$
$$+\frac{j\frac{1}{2}x_d-x_q)}{\left\{\frac{1}{2}(x_d+x_q)\right\}^2-\left\{\frac{1}{2}(x_d-x_q)\right\}^2}|E_1|^2\varepsilon^{j2\phi} \qquad (4)$$

This locus, as well as the current expressed by Eq. (3), is a so called Snake curve as shown in Fig. 8.

For the calculation of stability at the transient state, one may take transient reactances x_d', x_q' and transient excitation voltage E_0' in Eq. (4).

The voltage back to the quadrature axis reactance is

$$E_Q=E_1+jx_qI_1=E_0-i_d(x_d-x_q)$$
$$i_d=I_1\sin\theta \qquad (5)$$

The terminal voltage is

$$E_1=E_Q-jx_qI_1$$
$$=E_0-i_d(x_d-x_q)-jx_qI_1 \qquad (6)$$

Example 1.

Draw the output power locus of the salient polemachine at steady state and transient state, when its terminal and excitation voltages are constant. The data given are as follows:

$E_1=1$	p. u.	$x_d=1.0$
$E_0=1.5$	p. u.	$x_d'=0.3$
$E_0'=1.2$	p. u.	$x_q=x_q=0.6$

Solution:

At steady state, taking E_1 as the reference vector, one has

$$P_1+jQ_1=-j1.25+j1.5\,\varepsilon^{j\phi}+j0.25\,\varepsilon^{j2\phi}\,.$$

At transient state,

$$P_1+jQ=-j2.5+j3.95\,\varepsilon^{j\phi}-j0.835\,\varepsilon^{j2\phi}\,.$$

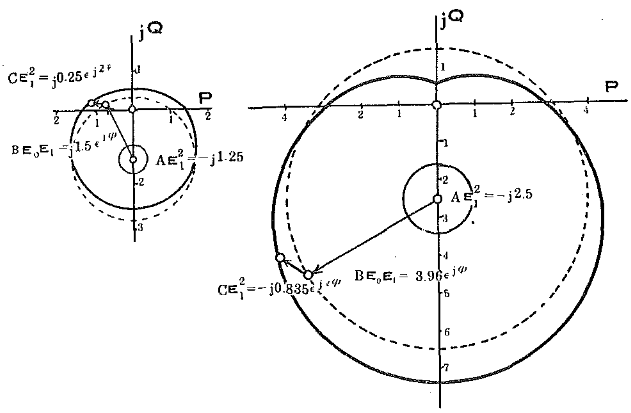


Fig. 10. Power loci for salient pole.

In the above example, the effect of magnetic saturation is neglected. The magnetic saturation effect is shown in Fig. 9. In the case of constant excitation field current, E_0 changes due to the direct axis component i_d. The exact solution is very complicated.

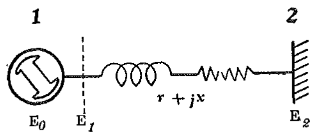


Fig. 11. Salient pole mechine with a series impedance.

A salient pole machine is connected to an infinite bus through a series impedance $Z=r+jx$, as is shown in Fig. 11. Equation (3) takes the following form, as will easily be proved:

$$\begin{aligned} I_1 = & -\frac{r-jx-j\frac{1}{2}(x_d+x_q)}{r^2+\left\{x+\frac{1}{2}(x_d+x_q)\right\}^2-\left\{\frac{1}{2}(x_d-x_q)\right\}^2}E_2 \\ & +\frac{r-jx-jx_{0q}}{r^2+\left\{x+\frac{1}{2}(x_d+x_q)\right\}^3-\left\{\frac{1}{2}(x_d-x_q)\right\}^2}E_0\varepsilon^{j\phi} \\ & -\frac{j\frac{1}{2}(x_d-x_q)}{r^2+\left\{x+\frac{1}{2}(x_d+x_q)\right\}^2-\left\{\frac{1}{2}(x_d-x_q)\right\}^2}E_2\varepsilon^{j2\phi} \end{aligned} \qquad (7)$$

Terminal voltage E_1 at the salient machine is

$$\boldsymbol{E}_1 = E_2+(r+jx)\,\boldsymbol{I} \tag{8}$$

Power of the machine is

$$P_1+jQ_1 = \boldsymbol{E}_{1k}\boldsymbol{I}_{2k} = E_{2k}\boldsymbol{I}_1+r\boldsymbol{I}_1\boldsymbol{I}_{1k}-jx\boldsymbol{I}_1\boldsymbol{I}_{1k}$$

Power at ∞ bus is

$$P_2+jQ_2 = \boldsymbol{E}_{2d}\boldsymbol{I}_1$$

$$P_1+jQ_1 = P_2+r|\boldsymbol{I}_1|^2+j(Q_2+x|\boldsymbol{I}_1|^2) \tag{9}$$

Nextly, a salient pole machine is connected to an ∞ bus through a general transmission line as is shown in Fig. 12.

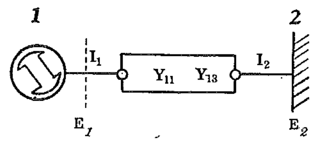


Fig. 12. A salient pole machine with a general transmission line.

The general matrix Equation (3) gives

$$\boldsymbol{I}_1 = \boldsymbol{Y}_{11}\boldsymbol{E}_1-\boldsymbol{y}_{12}\boldsymbol{E}_2$$

consequently,

$$\boldsymbol{E}_1 = \frac{\boldsymbol{I}_1+\boldsymbol{y}_{12}\boldsymbol{E}_2}{\boldsymbol{Y}_{11}} \tag{10}$$

Putting short circuit admittances as $\dfrac{1}{\boldsymbol{Y}_{11}}=R+jX$ and $\dfrac{\boldsymbol{y}_{12}}{\boldsymbol{Y}_{11}}=\boldsymbol{D}$, one obtains

$$\boldsymbol{E}_1 = \frac{1}{\boldsymbol{D}}\boldsymbol{E}_2+(R+jX)\,\boldsymbol{I}_1 \tag{11}$$

When this equation is compared with Equation (8), the current and power may be obtained in the same way as was employed to derive Equations (7), (8) and (9).

Two salient pole machines are connected through a line impedance $r+jx$, as is shown in Fig. 13.

$$E_{01} = E_1+j\frac{1}{2}(x_{d1}+x_{q1})\,\boldsymbol{I}-j\frac{1}{2}(x_{d1}-x_{q1})\,\boldsymbol{I}_k$$

$$E_1 = E_2+(r+jx)\,\boldsymbol{I}$$

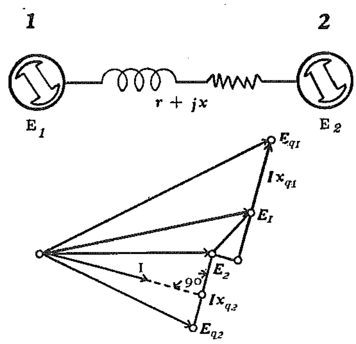


Fig. 13. Two salient pole machine problem.

$$E_{92} = E_2 - j\frac{1}{2}(x_{d2}+x_{q2})\boldsymbol{I} + j\frac{1}{2}(x_{d2}-x_{q2})\boldsymbol{I}_k)$$

Therefore,

$$E_{61} = E_{92} + (r+jx)\boldsymbol{I} + j\frac{1}{2}(x_{d1}+x_{q1})\boldsymbol{I}$$

$$+ j\frac{1}{2}(x_{d2}+x_{q2})\boldsymbol{I} - j\frac{1}{2}(x_{d1}-x_{q1})\boldsymbol{I} - j\frac{1}{2}(x_{d2}-x_{q2})\boldsymbol{I}$$

or neglecting small value of r, and putting

$$\left.\begin{aligned} X_d &= x_{d1}+x_{d2}+x \\ X_q &= x_{q1}+x_{q2}+x \end{aligned}\right\} \tag{12}$$

one obtains

$$E_{91} = E_{92} + j\frac{1}{2}(X_d+X_q)\boldsymbol{I} - j\frac{1}{2}(X_d-X_q)\boldsymbol{I}_k\,. \tag{13}$$

This equation is the same type as Equation (1) explained above. Therefore, two salient pole machines may be treated as an equivalent single machine.

§ 3. Transmission Network System Containing Salient Pole Machines, and their Power Distribution.

The current matrix for multi-terminal network, expressed by Equation (3), takes the following form when the terminal voltages of the salient pole machines are taken, and when all static load terminals are eliminated by the method of short circuit matrix.

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_n \end{pmatrix} = \begin{pmatrix} Y_{11}E_1, & -y_{12}E_2, & -y_{13}E_3\cdots\cdots & \\ -y_{12}E_1, & Y_{22}E_2, & -y_{23}E_3\cdots\cdots & \\ & -y_{23}E_2, & Y_{33}E_3\cdots\cdots & \\ & & \ddots & \\ & & & Y_{nn}E_n \end{pmatrix} \tag{14}$$

The terminal voltages of the salient pole machines are given by Equation (1):

$$\left.\begin{aligned} E_1 &= E_{01} - j\frac{1}{2}(x_{d1}+x_{q1})I_1 + j\frac{1}{2}(x_{d1}-x_{q1})I_1 \\ E_2 &= E_{02} - j\frac{1}{2}(x_{d2}+x_{q2})I_2 + j\frac{1}{2}(x_{d2}-x_{q2})I_2 \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{aligned}\right\} \tag{15}$$

Introducing the above equation in Matrix (14), one can obtain all the currents between any two machines, from which the network system currents are composed. Their solution are explained in the preceding section. However, the solution requires an enormous labor. In the following section, the case of two machines and three machines will be explained.

§ 4. General Process of Transient Stability Calculation, by using the Symmetrical Component Admittance Matrix.

(1) Stability of the system without salient pole machine. Practical method of stability calculation.

By using the symmetrical component admittance matrix equation, the positive sequence currents and the voltages for stability calculation may easily be obtained at any sort of faults. For the transient state calculation, the transient reactance x'_d and the voltage behind the transient reactance E'_0 of each machine must be taken. The transient power is E'_0 times positive sequence current.

The initial conditions of the system are obtained by the power matrix Equation (3), in which the machine reactances are not included, by using the machine terminal voltages. Therefore, the air gap voltage E_{1g} and the power angles, including the machine transient reactance, must be obtained from the terminal voltages and the currents of machines at steady state.

The relations between power angle and time for stability determination are calculated by the step-by-step method or by using a. c. calculating boads.

As a practical calculation, all resistances of the network are neglected, and the transient reactance are used for steady state calculation.

(2) Stability of the system, containing salient pole machines.

As the matrix equation can not include the salient pole machine constants, one constructs at first the current matrix of symmetrical admittance components by introducing the symmetrical voltage and currents at machine terminals—E^1, E^2, E^0, I^1, I^2, I^0.

The machine constants—x_d, x_q, Z^0 and Z^2—are not included in the above matrix. These constants satisfy the following relations:

$$
\begin{aligned}
E^1 &= E_0^1 - j\frac{1}{2}(x_d + x_q)I^1 + j\frac{1}{2}(x_d - x_q)I_k^1 \\
E^2 &= (E_0^2 = 0) - Z^2 I^2 \qquad (16) \\
E^0 &= (E_0^0 = 0) - Z^0 I^0
\end{aligned}
$$

By using the latter two relations in this equation, one can eliminate the negative and zero sequence component. In practice, zero components at machine terminals are usually zero, because the high tension side of the transformer neutral is grounded.

When only the positive sequence components are taken into consideration, the power of the salient pole machines at any faulted condition are obtained by the method explained in Section (2).

At steady state, the excitation voltages, powers and power angles of the machines may be obtained from their terminal voltages and currents by using steady state direct and quadrature axis reactance. However, these values are not used for transient stability.

At transient state, the transient direct axis reactance and the transient excitation voltage must be used.

The effect of the saliency of the machine on the system stability is not small. Moreover, there are more complicated effect of the magnetic saturation, which is non-linear.

To understand the above explanation, a simple case of stability problem may be shown in the following section.

§ 5. Transient Stability of a Single or Two Machine System at a Ground Fault, Calculated by using Symmetrical Sequence Admittance Matrix.

A transmission system is shown in Fig. 14.

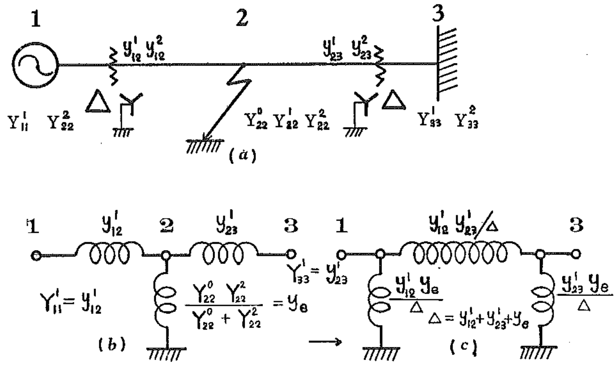


Fig. 14. A phase ground fault.

At terminal 1, a cylindrical pole generator is connected to an infinite bus 3. Terminal 2 is considered as a ground faulted point, where a phase is grounded. Then, the current matrix expressed with symmetrical admittance components is as follows;

$$\begin{pmatrix} (I_1=0)+I_1^1+I_1^2 \\ (I_1'=0)+a^2I_1^1+aI_1^2 \\ (I_1^0=0)+aI_1^1+a^2I_1^2 \\ I_2^0+I_2^1+I_2^2 \\ I_2^0+a^2I_2^1+aI_2^2 \\ I_2^0+aI_2^1+a^2I_2^2 \\ I_2^0=I_2^1=I_2^2 \\ (I_3^0=0)+I_3^1+I_3^2 \\ 0+a^2I_3^1+aI_3^2 \\ 0+aI_3^1+a^2I_3^2 \end{pmatrix} = \begin{pmatrix} 0 & Y_{11}^1 & Y_{11}^2 & 0 & -y_{12}^1 & -y_{12}^2 & 0 & 0 & 0 \\ 0 & a^2Y_{11}^1 & aY_{11}^2 & 0 & -a^2y_{12}^1 & -ay_{12}^2 & 0 & 0 & 0 \\ 0 & aY_{11}^1 & a^2Y_{11}^2 & 0 & -ay_{12}^1 & -a^2y_{12}^2 & 0 & 0 & 0 \\ 0 & -y_{12}^1 & -y_{12}^2 & Y_{22}^0 & Y_{22}^1 & Y_{22}^2 & 0 & -y_{23}^1 & -y_{23}^2 \\ 0 & -a^2y_{12}^1 & -ay_{12}^2 & Y_{22}^0 & a^2Y_{22}^1 & aY_{22}^2 & 0 & -a^2y_{23}^1 & -ay_{23}^2 \\ 0 & -ay_{22}^1 & -a^2y_{12}^2 & Y_{22}^0 & aY_{22}^1 & a^2Y_{22}^2 & 0 & -ay_{23}^1 & -a^2y_{23}^2 \\ 0 & 0 & 0 & 0 & -y_{23}^1 & -y_{23}^2 & 0 & Y_{33}^1 & Y_{33}^2 \\ 0 & 0 & 0 & 0 & -a^2y_{23}^1 & -ay_{23}^2 & 0 & a^2Y_{33}^1 & aY_{33}^2 \\ 0 & 0 & 0 & 0 & -ay_{23}^1 & -a^2y_{23}^2 & 0 & aY_{33}^1 & a^2Y_{33}^2 \end{pmatrix} \begin{pmatrix} 0 \\ E_1^1 \\ 0 \\ E_2^0= \\ -E_2^1-E_2^2 \\ E_2^1 \\ E_2^2 \\ 0 \\ E_3^1 \\ 0 \end{pmatrix} \qquad \cdots\cdots (17)$$

The short circuit admittances at the generator terminal 1 and the infinite bus 3 have no zero sequence component, as the high tension sides of the transformers are grounded. Also, by the same reason, there is no zero sequence driving admittances between the terminals 1 and 2, 2 and 3. Only the short circuit admittances at the ground fault point 2 exist as the zero sequence.

Taking out only the positive sequence current from the matrix (17), one obtains

$$\begin{pmatrix} I_1^1 \\ I_2^1 \\ I_3^1 \end{pmatrix} = \begin{pmatrix} Y_{11}^1 & -y_{12}^1 & 0 \\ -y_{12}^1 & Y_{22}^1 & -y_{23}^1 \\ 0 & -y_{23}^1 & Y_{33}^1 \end{pmatrix} \begin{pmatrix} E_1^1 \\ E_2^1 \\ E_3^1 \end{pmatrix}$$

From the relations $I_2^0 = I_2^2 = I_2^1$ and $-Y_{22}^0 E_2^1 - Y_{22}^0 E_2^2 = Y_{22}^2 E_2^2 = -y_{12} E_1^1 + Y_{22} E_2^1 - y_{23}^1 E_3^1$, one has

$$E_2^2 = \frac{-Y_{22}^0}{Y_{22}^0 + Y_{22}^2} E_2^1 \tag{18}$$

$$E_2^1 = \frac{y_{12}^1 E_1^1 + y_{23}^1 E_3^1}{Y_{22}^1 + \dfrac{1}{Z^0 + Z^2}} \tag{19}$$

$$I_2^1 = \frac{-Y_{22}^0 Y_{22}^2}{Y_{22}^0 + Y_{22}^2} E_2^1 = \frac{-E_2^1}{Z^0 + Z^2}, \qquad \text{form (18)}$$

$$\begin{pmatrix} I_1^1 \\ -\dfrac{E_2^1}{Z^0+Z^2} \\ I_3^1 \end{pmatrix} = \begin{pmatrix} Y_{11}^1 & -y_{12}^1 & 0 \\ -y_{12}^1 & Y_{22}^1 & -y_{23}^1 \\ 0 & -y_{23}^1 & Y_{33}^1 \end{pmatrix} \begin{pmatrix} E_1^1 \\ E_2^1 \\ E_3^1 \end{pmatrix} \tag{20}$$

Equation (20) expresses that a phase grounding is equivalent to the load of $Z_{22}^0 + Z_{22}^2$ at the ground point 2.

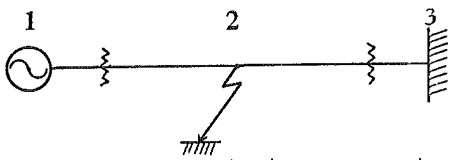
$$\begin{pmatrix} I_1 \\ 0 \\ I_3 \end{pmatrix} = \begin{pmatrix} Y_{11}^1 E_1^1, & -y_{12}^1 E_2^1, & 0 \\ -y_{12}^1 E_1^1, & Y_{22}^1 + \dfrac{1}{Z_{22}^0 + Z_{22}^2} E_2^1, & -y_{23}^1 E_3^1 \\ 0\,, & -y_{23}^1 E_2^1\,, & Y_{33}^1 E_3^1 \end{pmatrix} \tag{21}$$

in which, $y_e = \dfrac{1}{Z_{22}^0 + Z_{22}^2}$ and $Y_{22}^1 + y_e = y_{12}^1 + y_{23}^1 + y_e = \Delta$. When Matrix (21) is multiplied by $\begin{vmatrix} 1 & 0 & 0 \\ \dfrac{y_{12}}{\Delta} & 0 & \dfrac{y_{23}}{\Delta} \\ 0 & 0 & 1 \end{vmatrix}$, one obtains

$$\begin{pmatrix} I_1 \\ \\ I_3 \end{pmatrix} = \begin{pmatrix} \left\{ Y^1_{11} - \dfrac{(y^1_{12})^2}{\Delta} \right\} E^1_1 , & \dfrac{y^1_{23} y^1_{12}}{\Delta} E^1_3 \\ \\ -\dfrac{y^1_{12} y^1_{23}}{\Delta} , & \left\{ Y^1_{33} - \dfrac{(y^1_{23})^2}{\Delta} \right\} E^1_3 \end{pmatrix} \qquad (22)$$

When this relation is expressed with power unit, the result is,

$$\begin{pmatrix} P_1 + jQ_1 \\ \\ P_3 + jQ_3 \end{pmatrix} = \begin{pmatrix} \dfrac{y_{12} y_{23} + y_{12} y_e}{y_{12} + y_{23} + y_e} E^2_1 , & \dfrac{-y_{12} y_{23}}{\Delta} E_3 E_{1k} \\ \\ \dfrac{-y_{12} y_{23}}{\Delta} E_1 E_{3k} , & \dfrac{y_{12} y_{23} + y_{23} y_e}{\Delta} E^2_3 \end{pmatrix} \qquad (23)$$



Ground Fault $-Y^1_{11} = y^1_{12} = j3.03$ $\quad Y^1_{22} = -j7.38$
$-Y^2_{22} = y^2_{12} = j4.35$ $\quad Y^3_{22} = -j11.05$
$-Y^1_{33} = y^1_{23} = j4.35$ $\quad Y^0_{22} = -j15.6$
$-Y^2_{33} = y^2_{23} = j6.7$

Normal State $Y^1_{11} = y^1_{13} = Y^1_{33} = -j1.78$ p.u.

Fig. 15.

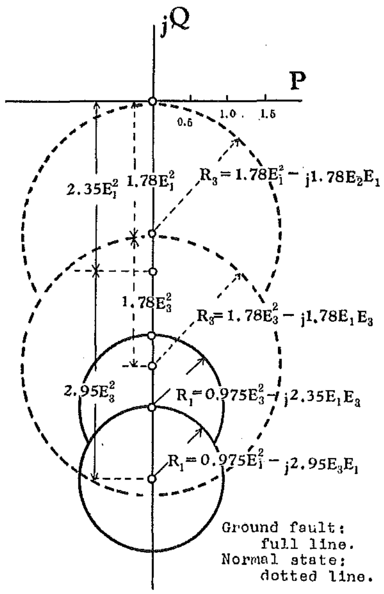


Fig. 16.

Example 2.

Obtain the power circle diagrams for a ground fault in the transmission line shown in Fig. 15.

Solution:

At the normal condition,

$$\begin{pmatrix} P_1 + jQ_1 \\ P_3 + jQ_3 \end{pmatrix} = \begin{pmatrix} -j\,1.78 E^2_1 & +j\,1.78 E_3 E_{1k} \\ j\,1.78 E_1 E_{3k} & -j\,1.78 E^2_3 \end{pmatrix}$$

which is shown in Fig. 16 in dotted lines.

At a ground fault, one obtains the following power matrix from Equation (23):

$$\begin{pmatrix} P_1 + jQ_1 \\ P_3 + jQ_3 \end{pmatrix} = \begin{pmatrix} -j\,2.35 E^2_1 & +j\,0.97 E_3 E_{1k} \\ -j\,0.97 E_1 E_{3k} & -j\,2.95 E^2_3 \end{pmatrix}$$

This is shown in Fig. 16 in full line circles.

For the salient pole machine, one employs the machine terminal voltages E_1^1, E_1^2 and E_1^0 in the above matrix Equation (17), excluding the machine reactances. E_1^0 may be taken as zero, since the transformer neutral is grounded.

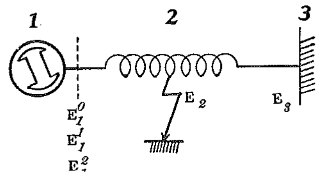


Fig. 17.

$$\begin{pmatrix} I_1^1 + I_1^2 \\ a^2 I_1^1 + a I_1^2 \\ a I_1^1 + a^2 I_1^2 \\ I_2^0 + I_2^1 + I_2^2 \\ 0 \\ 0 \\ I_3^1 + I_3^2 \\ a^2 I_3^1 + a I_3^2 \\ a I_3^1 + a^2 I_3^2 \end{pmatrix} = \begin{pmatrix} 0 & Y_{11}^1 & Y_{11}^2 & 0 & -y_{12}^1 & -y_{12}^2 & 0 & 0 & 0 \\ 0 & a^2 Y_{11}^1 & a Y_{11}^2 & 0 & -a^2 y_{12}^1 & -a y_{12}^2 & 0 & 0 & 0 \\ 0 & a Y_{11}^1 & a^2 Y_{11}^2 & 0 & -a y_{12}^1 & -a^2 y_{12}^2 & 0 & 0 & 0 \\ 0 & -y_{12}^1 & -y_{12}^2 & Y_{22}^0 & Y_{22}^1 & Y_{22}^2 & & -y_{23}^1 & -y_{23}^2 \\ 0 & -a^2 y_{12}^1 & -a y_{12}^2 & Y_{22}^0 & a^2 Y_{22}^1 & a Y_{22}^2 & & -a^2 y_{23}^2 & -a y_{23}^2 \\ 0 & -a y_{12}^1 & -a^2 y_{12}^2 & Y_{22}^0 & a Y_{22}^1 & a^2 Y_{22}^2 & & -a y_{23}^1 & -a^2 y_{23}^2 \\ 0 & 0 & 0 & 0 & -y_{23}^1 & -y_{23}^2 & 0 & Y_{33}^1 & Y_{33}^2 \\ 0 & 0 & 0 & 0 & -a^2 y_{23}^1 & -a y_{23}^2 & 0 & a^2 Y_{33}^1 & a Y_{33}^2 \\ 0 & 0 & 0 & 0 & -a y_{23}^1 & -a^2 y_{23}^2 & 0 & a Y_{33}^1 & a^2 Y_{33}^2 \end{pmatrix} \begin{pmatrix} 0 \\ E_1^1 \\ E_2^1 \\ E_2^0 = -(E_2^1 + E_2^2) \\ E_2^1 \\ E_2^1 \\ 0 \\ E_3^1 \\ 0 \end{pmatrix} \quad \cdots\cdots\cdots (24)$$

Since $I_2^0 = I_2^1 = I_2^2$,

$$-Y_{22}^0 E_2^1 - Y_{22}^0 E_2^2 = -y_{12}^2 E_1^2 + Y_{22}^2 E_2^2 = -y_{12}^1 E_1^1 + Y_{12}^1 E_2^1 - y_{23}^1 E_3^1$$

However, $I_1^2 = Y_{11}^2 E_1^2 - y_{12}^2 E_2^2 = \dfrac{-E_1^2}{Z^2}$, and Z^2 is the machine negative impedance.

$$E_1^2 = \frac{y_{12}^2}{Y_{22}^0 + \dfrac{1}{Z^2}} E_2^2$$

$$\therefore \quad -Y_{22}^0 E_2^1 - Y_{22}^0 E_2^2 = -y_{12}^2 \left(\frac{y_2^2}{Y_{11}^2 + \dfrac{1}{Z^2}} \right) E_2^2 + Y_{22}^2 E_2^2 = Y_{22}^{2\prime} E_2^2$$

$$E_2^2 = \frac{-Y_{22}^0}{Y_{22}^0 + Y_{22}^{2\prime}} E_2^1 \qquad Y_{22}^{2\prime} = -y_{12}^2 \frac{y_{12}^2}{Y_{11}^2 + \dfrac{1}{Z^2}} + Y_{22}^2 = \frac{1}{Z^{2\prime}} \tag{25}$$

Therefore, $I_2^1 = \dfrac{Y_{22}^0 Y_{22}^{2'}}{Y_{22}^0 + Y_{22}^{2'}} E_2^1$

Similar relation obtained in Equation (20) is

$$\begin{pmatrix} I_1^1 \\ \dfrac{-E_2^1}{Z+Z^{2'}} \\ I_3^1 \end{pmatrix} = \begin{pmatrix} Y_{11}^1 & -y_{12}^1 & 0 \\ -y_{12}^1 & Y_{22}^1 & -y_{23}^1 \\ 0 & -y_{33}^1 & Y_{33}^1 \end{pmatrix} \begin{pmatrix} E_1^1 \\ E_2^1 \\ E_3^1 \end{pmatrix} \tag{26}$$

Example 3.

Obtain the positive phase current of the salient pole machine at a ground fault. The line constants are given in Fig. 18.

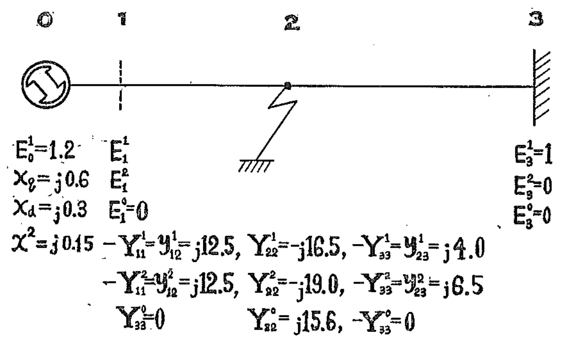


Fig. 18. (a) A salient pole machine at a ground fault.

Solution:

By Equation (25), one has

$$Y_{22}^{2'} = -y_{12}^2 \frac{y_{12}^2}{Y_{11}^2 + \dfrac{1}{Z^2}} + Y_{22}^2$$

$$= \frac{12.5^2}{-j12.5 - j6.66} - j19.0 = -j10.85$$

$$Z^{2'} = \frac{1}{Y^{2'}} = j0.092$$

$$Z^0 + Z^{2'} = j0.064 + j0.092 = j0.156$$

$$y_e = \frac{1}{Z^0 + Z^2} = -j6.4$$

By Equation (26),

$$\begin{pmatrix} I_1^1 \\ j6.4E_2^1 \\ I_2^1 \end{pmatrix} = \begin{pmatrix} -j12.5, & j12.5, & 0 \\ j12.5, & -j16.5, & j4 \\ 0, & -j4, & -j4 \end{pmatrix} \begin{pmatrix} E_1^1 \\ E_2^1 \\ E_3^1 \end{pmatrix}$$

Elimination of E_2^1 gives,

$$\begin{bmatrix} I_1^1 \\ I_3^1 \end{bmatrix} = \begin{bmatrix} -j5.69, & j2.19 \\ j2.19, & -j3.31 \end{bmatrix} \begin{bmatrix} E_1^1 \\ E_3^1 \end{bmatrix}$$

The terminal voltage E_1 and the current I_1 of the salient pole machine given by Equation (11) is

$$E_1 = \frac{1}{D} E_3 + (R+jX) I_1 = \frac{y_{12}}{Y_{11}} E_3 + \frac{1}{Y_{11}} I_1$$
$$= 0.385 E_3 + j0.176 I_1 .$$

Therefore, the air-gap voltage of the machine is

$$E_0 = 0.385 E_3 + j0.176 I_1 + j\frac{1}{2}(x_d' + x_q) I_1 - j\frac{1}{2}(x_d' - x_q) I_{1k}$$

Taking E_3 as the standard vector, one has the following relation by Equation (7).

$$-I_1^1 = -\frac{-jx - j\frac{1}{2}(x_d + x_q)}{\left\{x + \frac{1}{2}(x_d + x_q)\right\}^2 - \left\{\frac{1}{2}(x_d - x_q)\right\}^2} E_2$$
$$+ \frac{-jx - jx_q}{\left\{x + \frac{1}{2}(x_d + x_q)\right\}^2 - \left\{\frac{1}{2}(x_d - x_q)\right\}^2} E_0 \varepsilon^{j\phi}$$
$$- \frac{j\frac{1}{2}(x_d - x_q)}{\left\{x + \frac{1}{2}(x_d + x_q)\right\}^2 - \left\{\frac{1}{2}(x_d - x_q)\right\}^2} E_2 \varepsilon^{j2\delta}$$

Substitution of

$$jx = 0.176 \qquad x_d = 0.3 \qquad x_q = 0.6$$
$$E_0 = 1.2 \qquad E_0 = 0.385 \qquad E_3 = 0.385$$

gives

$$I_1^1 = -j0.65 + j2.5\,\varepsilon^{j\phi} - j0.155\,\varepsilon^{j2\phi}$$

The results this final form is shown in Fig. 18 (b).

The positive sequence power of the machine is

$$
\begin{aligned}
P_0+jQ_0 &= E^1_{1k} I^1_1 \\
&= (0.385\,E_{3k} - j\,0.176\,I^1_{1k})\,I^1_1 \\
&= 0.385 \times I^1_1 - j\,0.176\,|I^1_1|^2
\end{aligned}
$$

Therefore, the locus is the same as Fig. 18 (b), except the position of the curve moves down as much as $-j0.176\,|I^1_1|^2$ and the scale for power changes to the value multiplied by the factor 0.385.

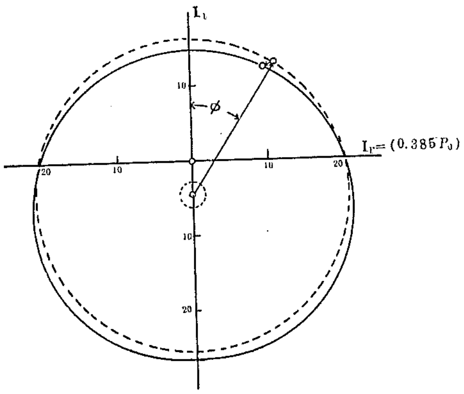


Fig. 18 (b).

§ 6. Transient Stability Matrix of Two Machine System at Single Phase Short Circuit Faults.

$$
\begin{pmatrix}
\boldsymbol{I}^0_1 + \boldsymbol{I}^1_1 + \boldsymbol{I}^2_1 \\
\boldsymbol{I}^0_1 + a^2\boldsymbol{I}^1_1 + a\boldsymbol{I}^2_1 \\
\boldsymbol{I}_1 + a\boldsymbol{I}^1_1 + a^2\boldsymbol{I}^2_1 \\
(\boldsymbol{I}^0_2 = 0) + \boldsymbol{I}^1_2 + \boldsymbol{I}^2_2 \\
(\boldsymbol{I}^0_2 = 0) + a^2\boldsymbol{I}^1_2 + a\boldsymbol{I}^2_2 \\
(\boldsymbol{I}^0_2 = 0) + a\boldsymbol{I}^1_2 + a^2\boldsymbol{I}^2_2 \\
\boldsymbol{I}_1 = \boldsymbol{I}_2 \quad \therefore \boldsymbol{I}_b = -\boldsymbol{I}_c \\
\boldsymbol{I}^0_3 + \boldsymbol{I}^1_3 + \boldsymbol{I}^2_3 \\
\boldsymbol{I}^0_3 + a^2\boldsymbol{I}^1_3 + a\boldsymbol{I}^2_3 \\
\boldsymbol{I}^0_3 + a\boldsymbol{I}^1_3 + a^2\boldsymbol{I}^2_3
\end{pmatrix}
=
\begin{pmatrix}
\boldsymbol{Y}^0_{11} & \boldsymbol{Y}^1_{11} & \boldsymbol{Y}^2_{11} & -y^0_{12} & -y^1_{12} & -y^2_{12} & 0 & 0 & 0 \\
\boldsymbol{Y}^0_{11} & a^2\boldsymbol{Y}^1_{11} & a\boldsymbol{Y}^2_{11} & -y^0_{12} & -a^2y^1_{12} & -ay^2_{12} & 0 & 0 & 0 \\
\boldsymbol{Y}^0_{11} & a\boldsymbol{Y}^1_{11} & a^2\boldsymbol{Y}^2_{11} & -y^0_{12} & -ay^1_{12} & -a^2y^2_{12} & 0 & 0 & 0 \\
-y^0_{12} & -y^1_{12} & -y^2_{11} & \boldsymbol{Y}^0_{22} & \boldsymbol{Y}^1_{22} & \boldsymbol{Y}^2_{22} & -y^0_{23} & -y^1_{23} & -y^2_{23} \\
-y^0_{12} & -a^2y^1_{12} & -ay^2_{11} & \boldsymbol{Y}^0_{22} & a^2\boldsymbol{Y}^1_{22} & a\boldsymbol{Y}^2_{22} & -y^0_{23} & -a^2y^1_{23} & -ay^2_{23} \\
-y^0_{12} & -ay^1_{12} & -a^2y^2_{11} & \boldsymbol{Y}^0_{22} & a\boldsymbol{Y}^1_{22} & a^2\boldsymbol{Y}^2_{22} & -y^0_{23} & -ay^1_{23} & -ay^{22}_{23} \\
 & & & & & & & & \\
0 & 0 & 0 & -y^0_{23} & -y^1_{23} & -y^2_{23} & \boldsymbol{Y}^0_{33} & \boldsymbol{Y}^1_{33} & \boldsymbol{Y}^2_{33} \\
0 & 0 & 0 & -y^0_{23} & -a^2y^1_{23} & -ay^2_{23} & \boldsymbol{Y}^0_{33} & a^2\boldsymbol{Y}^1_{33} & a\boldsymbol{Y}^2_{33} \\
0 & 0 & 0 & -y^0_{23} & -ay^1_{23} & -a^2y^2_{23} & \boldsymbol{Y}^0_{33} & a\boldsymbol{Y}^1_{33} & a^2\boldsymbol{Y}^2_{33}
\end{pmatrix}
\begin{pmatrix}
0 \\
E^1_1 \\
0 \\
E^0_2 = 0 \\
E^1_2 \\
E^2_2 = E^1_2 \\
 \\
0 \\
E^1_3 \\

\end{pmatrix}
\qquad \cdots\cdots\cdots\cdots (27)
$$

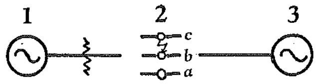


Fig. 19.

Refering to the unsymmetrical matrix equation and terminal condition in equation, one obtains the above equation for a single phase short circuit. Taking out only the positive sequence, one has

$$\begin{pmatrix} I_1^1 \\ I_2^1 \\ I_3^1 \end{pmatrix} = \begin{pmatrix} Y_{11}^1 & -y_{12}^1 & 0 \\ -y_{12}^1 & Y_{22}^1 & -y_{23}^1 \\ 0 & -y_{23}^1 & Y_{33}^1 \end{pmatrix} \begin{pmatrix} E_1^1 \\ E_2^1 \\ E_3^1 \end{pmatrix}$$

For positive sequence and negative sequence, there exist the following relations: $E_2^1 = E_2^2$, $I_2^1 = I_2^2 = Y_{22}^1 E_2^1 = Y_{22}^2 E_2^2$, where $\dfrac{1}{Y_{22}^2} = Z^2$, and Z^2 is negative impedance.

Therefore,

$$\begin{pmatrix} I_1^1 \\ Y_{22}^2 E_2^1 \\ I_3^1 \end{pmatrix} = \begin{pmatrix} Y_{11}^1 & -y_{12}^1 & 0 \\ -y_{12}^1 & Y_{22}^1 & -y_{23}^1 \\ 0 & -y_{23}^1 & Y_{33}^1 \end{pmatrix} \begin{pmatrix} E_1^1 \\ E_2^1 \\ E_3^1 \end{pmatrix}$$

By elimination of the faulted terminal 2, in the same way as explained in Section 5, one obtains the result as follows:

$$\begin{pmatrix} P_1 + jQ_1 \\ \\ P_3 + jQ_3 \end{pmatrix} = \begin{pmatrix} \left(Y_{11}^1 - \dfrac{(y_{12}^1)^2}{Y_{22}^1 + Y_{22}^2}\right) |E_1^1|^2, & -\dfrac{y_{12}^1 y_{23}^1}{Y_{22}^1 + Y_{22}^2} E_3^1 E_{1k}^1 \\ -\dfrac{y_{12}^1 y_{23}^1}{Y_{22}^1 + Y_{22}^2} E_1^1 E_{3k}^1, & \left(Y_{33}^1 - \dfrac{(y_{23}^1)^2}{Y_{22}^1 + Y_{22}^2}\right) |E_3^1|^2 \end{pmatrix} \tag{28}$$

Thus, one can treat this problem as a two machine problem.

§ 7. Transient Stability Matrix of Two Machine System at Two Phase Ground Fault.

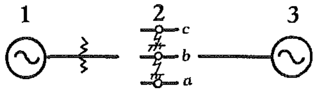


Fig. 20.

The admittance matrix is

$$
\begin{pmatrix}
I_1^0+I_1^1+I_1^2 \\
I_1^0+a^2I_1^1+aI_1^2 \\
I_1^0+aI_1^1+a^2I_1^2 \\
I_2^0+I_2^1+I_2^2=0 \\
I_2^1=-(I_2^0+I_2^2) \\
I_2^0+a^2I_2^1+aI_2^2 \\
I_2^0+aI_2^1+a^2I_2^2 \\
I_3^0+I_3^1+I_3^2 \\
I_3^0+a^2I_3^1+aI_3^2 \\
I_3^0+aI_3^1+a^2I_3^2
\end{pmatrix}
=
\begin{pmatrix}
Y_{11}^0 & Y_{11}^1 & Y_{11}^2 & -y_{12}^0 & -y_{12}^1 & -y_{12}^2 & 0 & 0 & 0 \\
Y_{11}^0 & a^2Y_{11}^1 & aY_{11}^2 & -y_{12}^0 & -a^2y_{12}^1 & -ay_{12}^2 & 0 & 0 & 0 \\
Y_{11}^0 & aY_{11}^1 & a^2Y_{11}^2 & -y_{12}^0 & -ay_{12}^1 & -a^2y_{12}^2 & 0 & 0 & 0 \\
-y_{12}^0 & -y_{12}^1 & -y_{12}^2 & Y_{22}^0 & Y_{22}^1 & Y_{22}^2 & -y_{23}^0 & -y_{23}^1 & -y_{23}^2 \\
 & & & & & & & & \\
-y_{12}^0 & -a^2y_{12}^1 & -ay_{12}^2 & Y_{22}^0 & a^2Y_{22}^1 & aY_{22}^2 & -y_{23}^0 & -a^2y_{23}^1 & -ay_{23}^2 \\
-y_{12}^0 & -ay_{12}^1 & -a^2y_{12}^2 & Y_{22}^0 & aY_{22}^1 & a^2Y_{22}^2 & -y_{23}^0 & -ay_{23}^1 & -a^2y_{23}^2 \\
0 & 0 & 0 & -y_{23}^0 & -y_{23}^1 & -y_{23}^2 & Y_{33}^0 & Y_{33}^1 & Y_{33}^2 \\
0 & 0 & 0 & -y_{23}^0 & -a^2y_{23}^1 & -ay_{23}^2 & Y_{33}^0 & a^2Y_{33}^1 & aY_{33}^2 \\
0 & 0 & 0 & -y_{23}^0 & -ay_{23}^1 & -a^2y_{23}^2 & Y_{33}^0 & aY_{33}^1 & a^2Y_{33}^2
\end{pmatrix}
\begin{pmatrix}
0 \\ E_1^1 \\ 0 \\ E_2^0=E_2^1 \\ \\ E_2^1 \\ E_2^2=E_2^1 \\ 0 \\ E_3^1 \\ 0
\end{pmatrix}
\qquad \cdots\cdots\cdots\cdots (29)
$$

Since $I_2^1=-(I_2^0+I_2^2)=-(Y_{22}^2+Y_{22}^0)E_2^1$,

$$
\begin{pmatrix}
I_1^1 \\
-(Y_{22}^0+Y_{22}^2)E_2^1 \\
I_3^1
\end{pmatrix}
=
\begin{pmatrix}
Y_{11}^1 & -y_{12}^1 & 0 \\
-y_{12}^1 & Y_{22}^1 & -y_{23}^1 \\
0 & -y_{23}^1 & Y_{33}^1
\end{pmatrix}
\begin{pmatrix}
E_1^1 \\ E_2^1 \\ E_3^1
\end{pmatrix}
$$

The two machine matrix for two phase ground is

$$
\begin{pmatrix}
P_1+jQ_1 \\
P_3+jQ_3
\end{pmatrix}
=
\begin{pmatrix}
\left(Y_{11}^1-\dfrac{(y_{12}^1)^2}{Y_{22}^0+Y_{22}^1+Y_{22}^2}\right)|E_1|^2, & \dfrac{-y_{12}^1y_{23}^1}{Y_{22}^0+Y_{22}^1+Y_{22}^2}E_3^1E_{1k}^1 \\
\dfrac{-y_{12}^1y_{23}^1}{Y_{22}^0+Y_{22}^1+Y_{22}^2}E_1^1E_{3k}^1, & \left(Y_{33}^1-\dfrac{(y_{23}^1)^2}{Y_{33}^0+Y_{22}^1+Y_{22}^2}\right)|E_3|^2
\end{pmatrix}
\qquad \cdots\cdots\cdots\cdots (30)
$$

§ 8. Transient Stability Matrix of Two Machine System at a Disconnecting Fault.

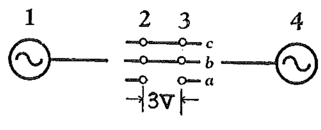


Fig. 21.

Admittance matrix is as follows.

$$\begin{pmatrix} I^0_1+ I^1_1+ I^2_1 \\ I^0_1+a^2I^1_1+aI^2_1 \\ I^0_1+aI^1_1+a^2I^2_1 \\ I^0_2+I^1_2+I^2_2=0 \\ -(I^0_2+a^2I^1_2+aI^2_2)=I_2 \\ -(I^0_2+aI^1_2+a^2I^2_2)=I_{c2} \\ -(I^0_3+I^1_3+I^2_3)=0 \\ I^0_3+a^2I^1_3+aI^2_3=I_{b2} \\ I^0_3+aI^1_3+a^2I^2_3=I_{c2} \\ I^0_4+I^1_4+I^2_4 \\ I^0_4+a^2I^1_4+aI^2_4 \\ I^0_4+aI^1_4+a^2I^2_4 \end{pmatrix} = \begin{pmatrix} Y^0_{11} & Y^1_{11} & Y^2_{11} & -y^0_{12} & -y^1_{12} & -y^2_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ Y^0_{11} & a^2Y^1_{11} & aY^2_{11} & -y^0_{12} & -a^2y^1_{12} & -ay^2_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ Y^0_{11} & aY^1_{11} & a^2Y^2_{11} & -y^0_{12} & -ay^1_{12} & -a^2y^2_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ -y^0_{12} & -y^1_{12} & -y^2_{12} & Y^0_{22} & Y^1_{22} & Y^2_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ -y^0_{12} & -a^2y^1_{12} & -ay^2_{12} & Y^0_{22} & a^2Y^1_{22} & aY^2_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ -y^0_{12} & -ay^1_{12} & -a^2y^2_{12} & Y^0_{22} & aY^1_{22} & a^2Y^2_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Y^0_{33} & Y^1_{33} & Y^2_{33} & -y^0_{34} & -y^1_{34} & -y^2_{34} \\ 0 & 0 & 0 & 0 & 0 & 0 & Y^0_{33} & a^2Y^1_{33} & aY^2_{33} & -y^0_{34} & -a^2y^1_{34} & -ay^2_{34} \\ 0 & 0 & 0 & 0 & 0 & 0 & Y^0_{33} & aY^1_{33} & a^2Y^2_{33} & -y^0_{34} & -ay^1_{34} & -ay^2_{34} \\ 0 & 0 & 0 & 0 & 0 & 0 & -y^0_{34} & -y^1_{34} & -y^2_{34} & Y^0_{34} & Y^1_{34} & Y^2_{34} \\ 0 & 0 & 0 & 0 & 0 & 0 & -y^0_{34} & -a^2y^2_{34} & -ay^2_{34} & Y^0_{34} & a^2Y^1_{34} & aY^2_{34} \\ 0 & 0 & 0 & 0 & 0 & 0 & -y^0_{34} & -ay^2_{34} & -a^2y^2_{34} & Y^0_{34} & aY^1_{34} & a^2Y^2_{34} \end{pmatrix} \begin{pmatrix} 0 \\ E^1_1 \\ 0 \\ E^0_2+V \\ E^1_2+V \\ E^2_2+V \\ E^0_2 \\ E^1_2 \\ E^2_2 \\ 0 \\ E^2_4 \\ 0 \end{pmatrix} \quad \cdots\cdots\cdots\cdots\cdots (31)$$

From second and third row of this matrix, one obtains

zero sequence ; $Y^0_{22}(E^0_2+V)+Y^0_{33}E^0_2=0 \quad \therefore \quad E^0_2=\dfrac{-Y^0_{22}}{Y^0_{22}+Y^0_{33}}V$

neg. sequence ; $Y^2_{22}(E^2_2+V)+Y^2_{33}E^2_2=0 \quad \therefore \quad E^2_2=\dfrac{-Y^2_{22}}{Y^2_{22}+Y^2_{33}}V \qquad (32)$

post. sequence ; $-y^1_{12}E^1_1+Y^1_{22}(E^1_2+V)+Y^1_{33}E^1_2-y^1_{34}E^1_4=0$

$$\therefore \quad E^1_2=\frac{-Y^1_{22}V+y^1_{12}E^1_1+y^1_{34}E^1_4}{Y^1_{22}+Y^1_{33}}$$

And also,

$$Y^0_{33}E^0_2+Y^1_{33}E^1_2+Y^2_{33}E^2_2-y^1_{34}E^1_4=0$$

therefore,

$$-\frac{Y^0_{22}Y^0_{33}}{Y^0_{22}+Y^0_{33}}V-\frac{Y^1_{22}Y^1_{33}}{Y^1_{22}+Y^1_{33}}V-\frac{Y^2_{22}Y^2_{33}}{Y^2_{22}+Y^2_{33}}V+\frac{Y^1_{33}y^1_{12}}{Y^1_{22}+Y^1_{33}}E^1_1$$
$$+\frac{Y^1_{33}y^1_{34}}{Y^1_{22}+Y^1_{33}}E^1_4-\frac{(Y^1_{22}+Y^1_{33})y^1_{34}}{Y^1_{22}+Y^1_{33}}E^1_4=0$$

and

$$V=\frac{\dfrac{y^1_{12}Y^1_{33}}{Y^1_{22}+Y^1_{33}}E^1_1-\dfrac{y^1_{34}Y^1_{22}}{Y^1_{22}+Y^1_{33}}E^1_4}{\dfrac{Y^0_{22}Y^0_{33}}{Y^0_{22}+Y^0_{33}}+\dfrac{Y^1_{22}Y^1_{33}}{Y^1_{22}+Y^1_{33}}+\dfrac{Y^2_{22}Y^2_{33}}{Y^2_{22}+Y^2_{33}}}$$

$$E_2^1+V=\frac{Y_{33}^1}{Y_{22}^1+Y_{33}^1}V+\frac{y_{12}^1}{Y_{22}^1+Y_{33}^1}E_1^1+\frac{y_{34}^1}{Y_{22}^1+Y_{33}^1}E_4^1 \tag{33}$$

Taking out only the positive component from the matrix (31), one has

$$\begin{pmatrix} I_1^1 \\ I_2^1 \\ I_3^1 \\ I_4^1 \end{pmatrix}=\begin{pmatrix} Y_{11}^1E_1^1 & -y_{12}^1(E_2^1+V) & 0 & 0 \\ -y_{12}^1E_1^1 & Y_{22}^1(E_2^1+V) & 0 & 0 \\ 0 & 0 & Y_{33}^1E_2^1 & -y_{12}^1E_4^1 \\ 0 & 0 & -y_{34}^1E_2^1 & Y_{44}^1E_4^1 \end{pmatrix} \tag{34}$$

However, there are following relations in the symmetrical sequence;

$$I_2^1+I_2^2+I_2^0=I_{a2}=0$$

$$I_2^1=-I_3^1, \quad I_2^2=-I_3^2, \quad I_2^0=-I_3^0 .$$

Therefore,

$$I_2^1=-I_2^0-I_2^2=+I_3^0+I_3^2=Y_{33}^0E_2^0+Y_{33}^2E_2^2$$

When it is assumed that there are no shunt loads at terminals 1 and 4,

$$Y_{11}^1=y_{12}^1=Y_{22}^1$$

$$Y_{44}^1=y_{34}^1=Y_{33}^1$$

$$\begin{pmatrix} I_1^1 \\ 0 \\ 0 \\ I_4^1 \end{pmatrix}=\begin{pmatrix} y_{12}^1E_1^1 & -y_{12}^1(E_2^1+V) & 0 & 0 \\ -y_{12}^1E_1^1 & y_{12}^1(E_2^1+V) & -I_2^1 & 0 \\ 0 & -I_3^1 & y_{34}^1E_2^1 & -y_{34}^1E_4^1 \\ 0 & 0 & -y_{34}^1E_2^1 & y_{34}^1E_4^1 \end{pmatrix} \tag{34 a}$$

Also, the following relation holds;

$$\begin{aligned}-I_2^1=I_3^1&=+I_2^0+I_2^2=y_{34}^0\frac{-y_{22}^0}{y_{22}^0+y_{33}^0}V-y_{34}^2\frac{y_{22}^2}{y_{22}^2+y_{33}^2}V \\ &=\left(\frac{1}{Z_{12}^0+Z_{34}^0}+\frac{1}{Z_{12}^2+Z_{34}^2}\right)V=\left(\frac{1}{Z_{14}^0}+\frac{1}{Z_{14}^2}\right)V \\ &=\frac{V}{\dfrac{Z_{14}^0\,Z_{14}^2}{Z_{14}^0+Z_{14}^2}}=\frac{V}{Z_d},\end{aligned}$$

$$\begin{pmatrix} I_1^1 \\ 0 \\ 0 \\ I_4^1 \end{pmatrix} = \begin{pmatrix} y_{12}^1, & -y_{12}^1, & 0 & 0 \\ -y_{12}^1, & \left(y_{12}^1+\dfrac{1}{Z_d}\right), & \dfrac{-1}{Z_d}, & 0 \\ 0 & -\dfrac{1}{Z_d}, & y_{34}^1+\dfrac{1}{Z_d}, & -y_{34}^1 \\ 0 & 0 & -y_{34}^1, & y_{34}^1 \end{pmatrix} \begin{pmatrix} E_1^1 \\ E_2^1+V \\ E_2^1 \\ E_4^1 \end{pmatrix} \tag{35}$$

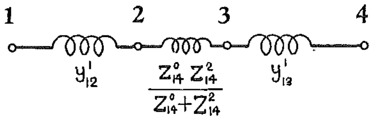


Fig. 22.

Matrix equation (35) means that a disconnection fault is equivalent to a series connectoon of a impedance Z_d as is shown in Fig. 22.

By eliminating E_2^1 and V, one obtatns,

$$\begin{pmatrix} I_1^1 \\ I_4^1 \end{pmatrix} = \begin{pmatrix} \left\{y_{12}^1+\dfrac{(y_{12}^1)^2(1+y_{34}^1Z_d)}{y_{12}^1+y_{34}^1+y_{12}^1y_{34}^1Z_d}\right\}E_1^1, & \left\{-\dfrac{y_{12}^1\,y_{34}^1}{y_{12}^1+y_{34}^1+y_{12}^1y_{34}^1Z_d}\right\}E_4^1 \\ \left\{-\dfrac{y_{12}^1\,y_{34}^1}{y_{12}^1+y_{34}^1+y_{12}^1y_{34}^1Z_d}\right\}E_1^1, & \left\{y_{34}^1-\dfrac{(y_{34}^1)^2(1+y_{12}^1Z_d)}{y_{12}^1+y_{34}^1+y_{12}^1y_{34}^1Z_d}\right\}E_4^1 \end{pmatrix} \qquad \cdots\cdots\cdots\cdots\cdots (36)$$

The power matrix is

$$\begin{pmatrix} P_1+jQ_1 \\ P_4+jQ_4 \end{pmatrix} = \begin{pmatrix} \left\{y_{12}^1+\dfrac{(-y_{12}^1)^2(1+y_{34}^1Z_d)}{y_{12}^1+y_{34}^1+y_{12}^1y_{34}^1Z_d}\right\}|E_1^1|^2, & \left\{-\dfrac{y_{12}^1\,y_{34}^1}{y_{12}^1+y_{34}^1+y_{12}^1y_{34}^1Z_d}\right\}E_1^1E_{4k} \\ \left\{-\dfrac{y_{12}^1\,y_{34}^1}{y_{12}^1+y_{34}^1+y_{12}^1y_{34}^1Z_d}\right\}E_1^1E_{4k}, & \left\{y_{34}^1+\dfrac{(y_{34}^1)^2(1+y_{12}^1Z_d)}{y_{12}^1+y_{34}^1+y_{12}^1y_{34}^1Z_d}\right\}|E_4^1|^2 \end{pmatrix} \qquad \cdots\cdots\cdots\cdots\cdots (37)$$

§9. Matrix for Two Machine Transient Stability Calculation, when Multiple Faults Occured.

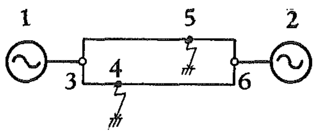


Fig. 23. Multiple faults.

In the case of multiple faults, the power matrix, which is necessary for stability calculation, may easily be obtained by increasing the number of terminal. For example, if a single phase and a ground fault are occured, at the point 4 and 5 in Fig. 9, then the positive phase matrix can be obtained from Equations (9.17) and 9.27;

$$\begin{pmatrix} I_1 \\ I_3=0 \\ I_4 \\ I_6=0 \\ I_5 \\ I_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & -y^1_{13} & 0 & 0 & 0 & 0 \\ -y^1_{31} & Y_{33} & -y^1_{34} & 0 & -y^1_{35} & 0 \\ 0 & -y^1_{34} & Y_{44} & -y^1_{46} & 0 & 0 \\ 0 & 0 & -y^1_{46} & Y_{66} & -\psi^1_{65} & -y^1_{62} \\ 0 & -y^1_{35} & 0 & -y^1_{56} & Y_{55} & 0 \\ 0 & 0 & 0 & -y^1_{26} & 0 & Y_{22} \end{pmatrix} \begin{pmatrix} E_1 \\ E_3 \\ E_4 \\ E_6 \\ E_5 \\ E_2 \end{pmatrix} \qquad (38)$$

Substitution of the fault condition into I_4 and I_5 gives the required results after eliminating E_3, E_4, E_5 and E_6.

§ 10. Stability of a Multi-machine System.

Formal solution of simulteneous differential equations for the multi-machine system,

$$\begin{cases} M_1 \dfrac{d^2\phi_{12}}{dt^2} = P_{a1} \\ M_2 \dfrac{d^2\phi_{23}}{dt^2} = P_{a2} \\ M_3 \dfrac{d^2\phi_{34}}{dt^2} = P_{a3} \\ \cdots\cdots\cdots\cdots\cdots\cdots \end{cases}$$

are not feasible. Although the less accurate, the following step-by-step method is known, when it is assumed that accerelating powers P_a are constant in a small interval Δt, (page 59).

$$\omega_n = \frac{d\phi_n}{dt} = \omega_{n-1} + \frac{\Delta t}{M} P_{a(n-1)}$$

$$\phi_n = \phi_{n-1} + \Delta t\, \omega_{n-1} + \frac{(\Delta t)^2}{2M} P_{a(n-1)}$$

The positive sequence powers P_a may by calculated from the above explained methods at any faults. One may calculate the stability of a three-machine system, when a ground fault occurs. Its simplified circuit in which the sequence component admittances are expressed by 100 MVA base is as shown in Fig. 24.

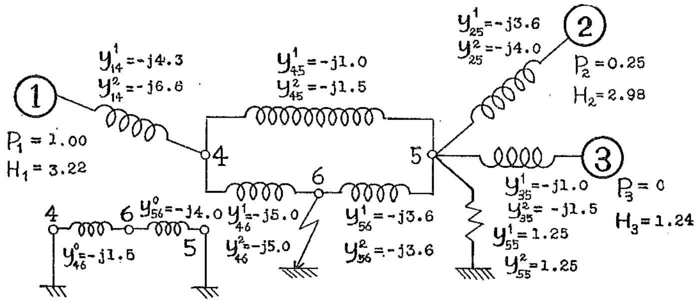


Fig. 24. Three-machine system.
1, 2, 3 : Synchronous machine, air gap terminal.
45, 465 : Duplicate line.
6 : The point of ground fault.

			1	2	3	
	I_{a1}		$0\ \ -j4.3\ \ -j6.6$			
1	I_{b1}		$0\ \ -a^2j4.3\ \ -aj6.6$		Continued ······	···
	I_{c1}		$0\ \ -aj4.3\ \ -a^2j6.6$			
	I_{a2}			$0\ \ -j3.6\ \ -j4.0$		
2	I_{b2}			$0\ \ -a^2j3.6\ \ -aj4.0$		
	I_{c2}			$0\ \ -aj3.6\ \ -a^2j4.0$		
	I_{a3}				$0\ \ -j1.0\ \ -j1.5$	
3	I_{b3}				$0\ \ a^2j1.0\ \ aj1.5$	
	I_{c3}	=			$0\ \ aj1.0\ \ a^2j1.5$	
	0		$0\ \ j4.3\ \ j6.6$			
4	**0**		$0\ \ a^2j4.3\ \ aj6.6$			
	0		$0\ \ aj4.3\ \ a^2j6.6$			
	0			$0\ \ j3.6\ \ j4.0$	$0\ \ j1.0\ \ j1.5$	
5	**0**			$0\ \ a^2j3.6\ \ aj4.0$	$0\ \ a^2j1.0\ \ aj1.5$	
	0			$0\ \ aj3.6\ \ a^2j4.0$	$0\ \ aj1.0\ \ a^2j1.5$	
	I_{a6}					
6	**0**					
	0					

At initial state, the system operated with a load $y_{55}=1.25$ at the terminal 5 of which voltage is keeping $\boldsymbol{E}_5=1$. A ground fault is occurred at the point 6. The zero sequence are ended at point 4 and 5 grounding the transformer neutral.

The fully described current matrix at the grounding is a as follows:

Zero sequence reactance between 4 and 5 is not considered, because it has no effect for the ground fault 6 by the transformer neutral grounding.

4			5			6			
0	$j4.3$	$j6.6$							0
0	$a^2j4.3$	$aj6.6$							$\boldsymbol{E}_1^1$
0	$aj^24.3$	$a^2j6.6$							0
			0	$j3.6$	$j4.0$				0
			0	$a^2j3.6$	$aj4.0$				$\boldsymbol{E}_2^1$
			0	$aj3.6$	$a^2j4.0$				0
			0	$j1.0$	$j1.5$				0
			0	$a^2j1.0$	$aj1.5$				$\boldsymbol{E}_3^1$
			0	$aj1.0$	$a^2j1.5$				0
0	$j12.9$	$j15.2$	0	$j3.6$	$j3.6$	0	$j5.0$	$j5.0$	0
0	$a^2j12.9$	$aj1.52$	0	$a^2j3.6$	$aj3.6$	0	$a^2j5.0$	$aj5.0$	$\boldsymbol{E}_4^1$
0	$aj12.9$	$a^2j15.2$	0	$aj3.6$	$a^2j3.6$	0	$aj5.0$	$a^2j5.0$	$\boldsymbol{E}_4^2$
0	$j3.6$	$j3.5$	0	$1.25-j20.2$	$1.25-j21.1$	0	$j12.0$	$j12.0$	0
0	$a^2j3.6$	$aj3.5$	0	a^2 ”	a ”	0	$a^2j12.0$	$aj12.0$	$\boldsymbol{E}_5^1$
0	$aj3.6$	$a^2j3.5$	0	a ”	a^2 ”	0	$aj12.0$	$a^2j12.0$	$\boldsymbol{E}_5^2$
0	$j5.0$	$j5.0$	0	$j12.0$	$j12.0$	$-j5.5$	$-j17.0$	$-j17.0$	$\boldsymbol{E}_6^0$
0	$a^2j5.0$	$aj5.0$	0	$a^2j12.0$	$aj12.0$	$-j5.5$	a^2 ”	a ”	$\boldsymbol{E}_6^1$
0	$aj5.0$	$a^2j5.0$	0	$aj12.0$	$a^2j12.0$	$-j5.5$	a ”	a^2 ”	$\boldsymbol{E}_6^2$

The positive sequence matrix:

$$\begin{pmatrix} \boldsymbol{I}_1^1 \\ \boldsymbol{I}_2^1 \\ \boldsymbol{I}_3^1 \\ \mathbf{0} \\ \mathbf{0} \\ \boldsymbol{I}_6^1 \end{pmatrix} = \begin{pmatrix} -j4.3 & 0 & 0 & j4.3 & 0 & 0 \\ 0 & -j3.6 & 0 & 0 & j3.6 & 0 \\ 0 & 0 & -j1.0 & 0 & j1.0 & 0 \\ j4.3 & 0 & 0 & -j12.9 & j3.6 & j5.0 \\ 0 & j3.6 & j1.0 & j3.6 & 1.25-j20.2 & j12.0 \\ 0 & 0 & 0 & j5.0 & j12.0 & -j17.0 \end{pmatrix} \begin{pmatrix} \boldsymbol{E}_1^1 \\ \boldsymbol{E}_2^1 \\ \boldsymbol{E}_3^1 \\ \boldsymbol{E}_4^1 \\ \boldsymbol{E}_5^1 \\ \boldsymbol{E}_6^1 \end{pmatrix} \qquad \cdots\cdots\cdots (40)$$

The negative sequence matrix:

$$\begin{pmatrix} \boldsymbol{I}_1^2 \\ \boldsymbol{I}_2^2 \\ \boldsymbol{I}_3^2 \\ \mathbf{0} \\ \mathbf{0} \\ \boldsymbol{I}_6^2 \end{pmatrix} = \begin{pmatrix} -j6.6 & 0 & 0 & j6.6 & 0 & 0 \\ 0 & -j4.0 & 0 & 0 & j4.0 & 0 \\ 0 & 0 & -j1.5 & 0 & j1.5 & 0 \\ j6.6 & 0 & 0 & -j15.2 & j3.6 & j5.0 \\ 0 & j4.0 & j1.5 & j3.5 & 1.25-j21.1 & j12.0 \\ 0 & 0 & 0 & j5.0 & j12.0 & -j17.0 \end{pmatrix} \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \boldsymbol{E}_4^2 \\ \boldsymbol{E}_5^2 \\ \boldsymbol{E}_6^2 \end{pmatrix} \qquad \cdots\cdots\cdots (40\text{ a})$$

The zero sequence:

$$[\boldsymbol{I}_6^0] = [-j5.5\boldsymbol{E}_6^0] \qquad (40\text{ b})$$

At first, one may eliminate the terminal 5, by the method of the star-mesh transformation or the short circuit matrix transformation

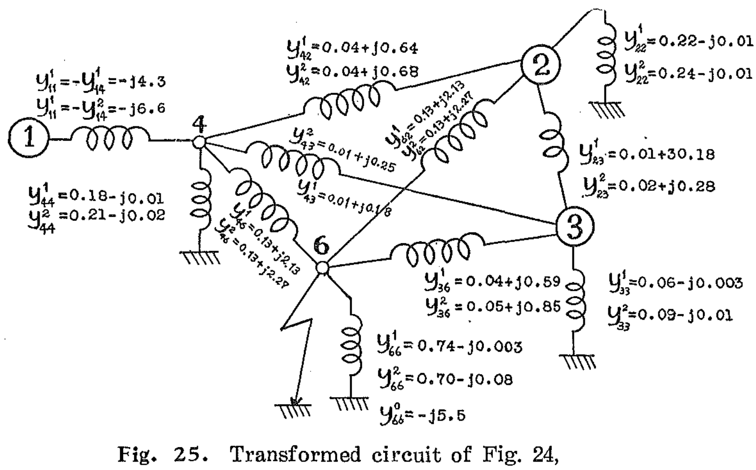


Fig. 25. Transformed circuit of Fig. 24, by eliminating the terminal 5.

for the respective sequence component matrix.

However, for the zero sequence component, the elimination is merely parallel connection of y_{46} and y_{56}. The results are as shown in Fig. 25 and the matrix.

The positive sequence matrix:

$$\begin{pmatrix} I_1^1 \\ I_2^1 \\ I_3^1 \\ I_6^2 \\ 0 \end{pmatrix} = \begin{pmatrix} -j4.3 & 0 & 0 & 0 & j4.3 \\ 0 & 0.04-j2.96 & 0.01-j0.18 & 0.13-j2.13 & 0.04-j0.64 \\ 0 & 0.01-j0.18 & 0.003-j0.95 & 0.04-j0.59 & 0.01-j0.18 \\ 0 & 0.13-j2.13 & 0.04-j0.59 & 0.44-j9.90 & 0{\cdot}13-j7.13 \\ j4.3 & 0.04-j0.64 & 0.01-j0.18 & 0.13-j7.13 & 0.04-j12.26 \end{pmatrix} \begin{pmatrix} E_1^1 \\ E_2^1 \\ E_3^1 \\ E_6^1 \\ E_4^1 \end{pmatrix} \quad \cdots\cdots\cdots\cdots\cdots (41)$$

The negative sequence matrix:

$$\begin{pmatrix} I_1^2 \\ I_2^2 \\ I_3^2 \\ I_6^2 \\ 0 \end{pmatrix} = \begin{pmatrix} -j6.6 & 0 & 0 & 0 & j6.6 \\ 0 & 0.04-j3.24 & 0.02+j0.28 & 0.13+j2.27 & 0.04+j0.68 \\ 0 & 0.02+j0.28 & 0.01-j1.39 & 0.05+j0.85 & 0.01+j0.25 \\ 0 & 0.13+j2.27 & 0.05+j0.85 & 0.40-j10.2 & 0.12+j7.04 \\ j6.6 & 0.04+j0.68 & 0.01+j0.25 & 0.12+j7.04 & 0.04-j14.6 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ E_6^2 \\ E_4^2 \end{pmatrix} \quad \cdots\cdots\cdots\cdots\cdots (41\text{ a})$$

The zero sequence:

$$[I_6^0] = [-j5.5]\,[E_6^0]$$

In the second place, one may eliminate the terminal 4 similarly.

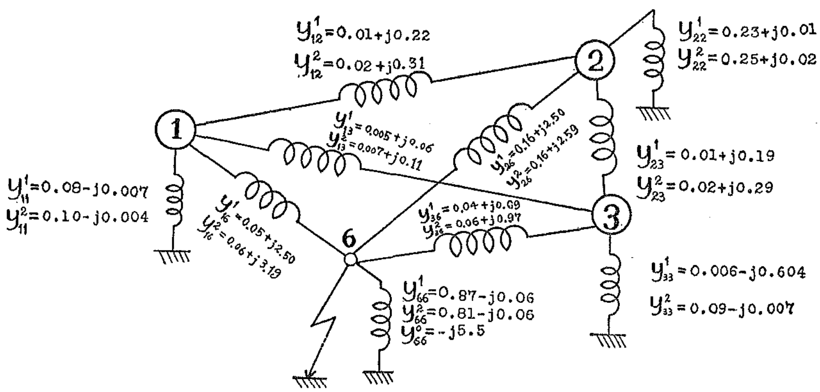


Fig. 26. Transformed circuit of Fig. 24, by eliminating the terminals 4 and 5.

The positive sequence matrix:

$$\begin{pmatrix} \boldsymbol{I}_1^1 \\ \boldsymbol{I}_2^1 \\ \boldsymbol{I}_3^1 \\ \boldsymbol{I}_6^1 \end{pmatrix} = \begin{pmatrix} 0.005-j2.79 & 0.01+j0.22 & 0.004+j0.06 & 0.05+j2.50 \\ 0.01+j0.22 & 0.04-j2.93 & 0.01+j0.19 & 0.16+j2.50 \\ 0.004+j0.06 & 0.01+j0.19 & 0.003-j0.95 & 0.04+j0.69 \\ 0.05+j2.50 & 0.16+j2.50 & 0.04+j0.69 & 0.61-j5.76 \end{pmatrix} \begin{pmatrix} \boldsymbol{E}_1^1 \\ \boldsymbol{E}_2^1 \\ \boldsymbol{E}_3^1 \\ \boldsymbol{E}_6^1 \end{pmatrix} \quad \cdots\cdots\cdots\cdots\cdots (42)$$

The negative sequence matrix:

$$\begin{pmatrix} \boldsymbol{I}_1^2 \\ \boldsymbol{I}_2^2 \\ \boldsymbol{I}_3^2 \\ \boldsymbol{I}_6^2 \end{pmatrix} = \begin{pmatrix} 0.01-j3.61 & 0.02+j0.31 & 0.01+j0.11 & 0.06+j3.19 \\ 0.02+j0.31 & 0.05-j3.21 & 0.02+j0.29 & 0.16+j2.59 \\ 0.01+j0.11 & 0.02+j0.29 & 0.01-j1.39 & 0.06+j0.97 \\ 0.06+j3.19 & 0.16+j2.59 & 0.06+j0.97 & 0.53-j6.82 \end{pmatrix} \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \boldsymbol{E}_6^2 \end{pmatrix} \quad \cdots\cdots\cdots\cdots\cdots (42\text{ a})$$

The zero sequence:

$$[\boldsymbol{I}_6^0] = [-j5.5][\boldsymbol{E}_6^0]$$

From the above shown matrices and grounding conditions, i. e. $\boldsymbol{I}_6^1 = \boldsymbol{I}_6^2 = \boldsymbol{I}_6^0$ and $\boldsymbol{E}_6^0 + \boldsymbol{E}_6^1 + \boldsymbol{E}_6^2 = \mathbf{0}$, the following relations are obtained.

$$[\boldsymbol{I}_6^1] = [0.05+j2.50\boldsymbol{E}_1^1 \; 0.16+j2.50\,\boldsymbol{E}_2^1 \; 0.04+j0.69\boldsymbol{E}_3^1 \; 0.61-j5.76\boldsymbol{E}_6^1]$$

$$[\boldsymbol{I}_6^2] = [0.53-j6.81]\boldsymbol{E}_6^2$$

$$[\boldsymbol{I}_6^0] = [-j5.5][-\boldsymbol{E}_6^1 - \boldsymbol{E}_6^2]$$

Therefore, the unknown voltage $\boldsymbol{E}_6^1$ at the faulted terminal, positive sequence voltage, is as follows.

$$\begin{aligned} \boldsymbol{E}_6^1 = & -(0.009+j0.11)\Big\{(0.05+j2.50)\boldsymbol{E}_1^1 + (0.16+j2.50)\boldsymbol{E}_2^1 \\ & + (0.045-j0.69)\boldsymbol{E}_3^1\Big\} \end{aligned} \quad (43)$$

By substituting this value into the above shown positive sequence matrix, one may get the fault state positive sequence currents on respective power generating terminals.

$$\begin{pmatrix} \boldsymbol{I}_1^1 \\ \boldsymbol{I}_2^1 \\ \boldsymbol{I}_3^1 \end{pmatrix} = \begin{pmatrix} 0.09-j2.09 & 0.13+j0.92 & 0.04+j0.26 \\ 0.13+j0.92 & 0.19-j2.33 & 0.05+j0.38 \\ 0.04+j0.26 & 0.05+j0.38 & 0.01-j0.89 \end{pmatrix} \begin{pmatrix} \boldsymbol{E}_1^1 \\ \boldsymbol{E}_2^1 \\ \boldsymbol{E}_3^1 \end{pmatrix} \quad \cdots\cdots\cdots\cdots (44)$$

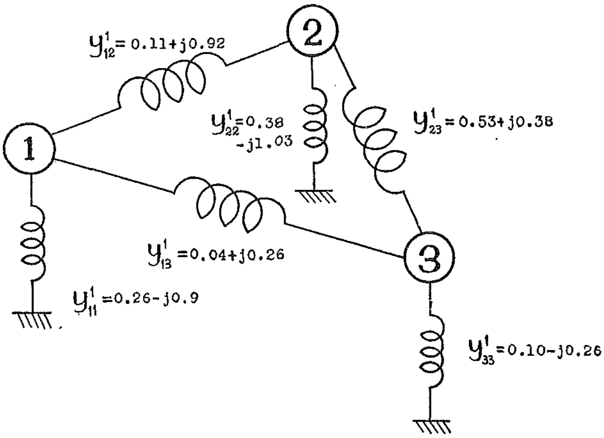


Fig. 27. The positive sequence circuit, when a phase, at the point of 6, is grounding.

At the initial normal state with duplicate lines, the power matrix is as follows, from the above shown matrix (40).

$$\begin{pmatrix} \boldsymbol{P}_1+j\boldsymbol{Q}_1 \\ \boldsymbol{P}_2+j\boldsymbol{Q}_2 \\ \boldsymbol{P}_3+j\boldsymbol{Q}_3 \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} -j14.3\boldsymbol{E}_1^2 & 0 & 0 & -j4.3\boldsymbol{E}_4\boldsymbol{E}_{1k} & 0 & 0 \\ 0 & -j3.6\boldsymbol{E}_2^2 & 0 & 0 & j3.6\boldsymbol{E}_5\boldsymbol{E}_{2k} & 0 \\ 0 & 0 & -j1.0\boldsymbol{E}_3^2 & 0 & j1.0\boldsymbol{E}_5\boldsymbol{E}_{3k} & 0 \\ j4.3\boldsymbol{E}_4 & 0 & 0 & -j12.9\boldsymbol{E}_4^2 & j3.6\boldsymbol{E}_4 & j5.0\boldsymbol{E}_4 \\ 0 & j3.6\boldsymbol{E}_{5k} & j1.0\boldsymbol{E}_{5k} & j3.6\boldsymbol{E}_5 & (1.25-j20.2)\boldsymbol{E}_5^2 & j12.0\boldsymbol{E}_4 \\ 0 & 0 & 0 & j5.0\boldsymbol{E}_6 & j12.0\boldsymbol{E}_6 & -j17.0\boldsymbol{E}_6^2 \end{pmatrix} \qquad \cdots\cdots\cdots\cdots (45)$$

The unknown air-gap voltages $\boldsymbol{E}_1$, $\boldsymbol{E}_2$ and $\boldsymbol{E}_3$ in the above matrix are obtained by introducing the given conditions: $P_1=1$ p.u., $P_2=0.25$ p.u., $P_3=0$, $\boldsymbol{E}_5=1$ p.u. ($=187$ KV), $|\boldsymbol{E}_4|=1.02$ p.u. ($=191$ KV), $\boldsymbol{E}_3=0.862$ p.u., and by elimination of the voltage, $\boldsymbol{E}_6$, as shown in below.

$$\begin{aligned} \boldsymbol{E}_1 &= 1.096 \ \underline{/19°53'} \ \text{p.u.} \\ \boldsymbol{E}_2 &= 1.017 \ \underline{/\ 3°55'} \ \text{p.u.} \\ \boldsymbol{E}_3 &= 0.862 \ \underline{/\ 0°} \ \text{p.u.} \end{aligned} \qquad (46)$$

The matrix (45) may be simplified to the 3-machine system as is shown in Fig. 28 and Matrix (47).

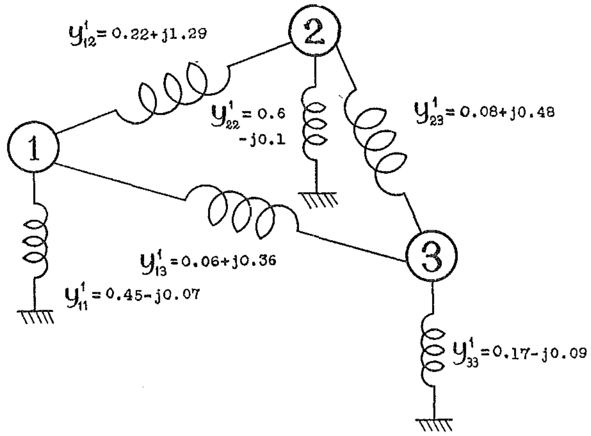


Fig. 28. Normal condition with duplicate lines.

$$\begin{pmatrix} P_1+jQ_1 \\ P_2+jQ_2 \\ P_3+jQ_3 \end{pmatrix} = \begin{pmatrix} 0.16-j1.72E_1^2 & 0.22-j1.29E_2E_{1k} & 00.6-j0.36E_3E_{1k} \\ 0.22-j1.29E_1E_{2k} & 0.30-j1.87E_2^2 & 0.08-j0.48E_3E_{2k} \\ 0.06-j0.36E_1E_{3k} & 0.08-j0.48E_2E_{3k} & 0.02-j0.93E_3^2 \end{pmatrix} \qquad \cdots\cdots\cdots (47)$$

The watt powers are

$$\begin{aligned} P_1 &= 0.20+1.46\sin(9°43'+\phi_{12})+0.34\sin(9°43'+\phi_{13}) \\ P_2 &= 0.31+1.46\sin(9°43'-\phi_{12})+0.43\sin(9°45'+\phi_{23}) \qquad (47\text{ a}) \\ P_3 &= 0.02+0.34\sin(9°43'-\phi_{13})-0.43\sin(9°45'-\phi_{23}) \end{aligned}$$

When a ground fault occurs at the terminal, 6, one may use the following positive sequence powers matrix with 3-machine system from the matrix (44).

$$\begin{pmatrix} P_1+jQ_1 \\ P_2+jQ_2 \\ P_3+jQ_3 \end{pmatrix} = \begin{pmatrix} 0.09-j2.09E_1^2 & 0.13+j0.92E_2E_{1k} & 0.04+j0.26E_3E_{1k} \\ 0.13+j0.92E_1E_{2k} & 0.19-j2.33E_2^2 & 0.05+j0.38E_3E_{2k} \\ 0.04+j0.26E_1E_{3k} & 0.05+j0.38E_2E_{3k} & 0.02-j0.89E_3^2 \end{pmatrix} \qquad \cdots\cdots\cdots (44\text{ a})$$

The watt powers of the above matrix are

$$\begin{aligned} P_1 &= 0.11+1.04\sin(8°11'+\phi_{12})+0.25\sin(8°10'+\phi_{13}) \\ P_2 &= 0.20+1.04\sin(8°11'-\phi_{12})+0.34\sin(8°01'+\phi_{23}) \qquad (44\text{ b}) \\ P_3 &= 0.01+0.25\sin(8°10'-\phi_{13})+0.34\sin(8°01'-\phi_{23}) \end{aligned}$$

By this ground fault, the output powers or the phase angles of the each three machines will be changed from the values of Fquation (47a) to those of Equation (44b). So as to satisfy differential equations of the 3-machine system as above shown.

The results of calculation of the power angles and powers by means of step-by-step method are as shown in the following Table 1 and the swing curves in Fig. 30.

TABLE 1

t	ϕ_1	ϕ_2	ϕ_3	ϕ_{12}	ϕ_{13}	ϕ_{23}	P_1	P_2	P_3
0.0	19°53′	3°55′	0°00′	15°58′	19°53′	3°55′	0.652	0.129	− 0.015
0.1	22°55′	6°58′	1°20′	15°57′	21°35′	5°38′	0.658	0.139	− 0.032
0.2	31°49′	15°36′	8°27′	16°13′	23°22′	7°09′	0.669	0.143	− 0.048
0.3	46°33′	29°37	24°20′	16°56′	22°13′	5°17′	0.677	0.128	− 0.034
0.4	66°54′	49°45′	46°19′	17°09′	19°35′	3°26′	0.671	0.105	− 0.011
0.5	92°59′	77°11′	70°14′	15°48′	22°45′	6°57′	0.660	0.150	− 0.045
0.6	125°00′	109°40′	102°10′	15°20′	22°50′	7°30′			

For the step-by-step calculation in this problem, one takes the inertia constant of the machines.

$$\begin{aligned} \boldsymbol{M}_1 &= 3.22 \\ \boldsymbol{M}_2 &= 2.98 \\ \boldsymbol{M}_3 &= 1.24 \end{aligned} \tag{48}$$

It is assumed that the faulted line is cut off after 0.2 second from the occurrence of the fault to make a single transmission line. The power matrix for this system with a single transmission line may easily be obtained from the matrix (39). The equivalent circuit and power matrix in the single line system are as shown in Fig. 29 and Matrix (49).

$$\begin{pmatrix} \boldsymbol{P}_1+j\boldsymbol{Q}_1 \\ \boldsymbol{P}_2+j\boldsymbol{Q}_2 \\ \boldsymbol{P}_3+j\boldsymbol{Q}_3 \end{pmatrix} = \begin{pmatrix} 0.11-j1.40\boldsymbol{E}_1^2 & 0.20+j1.03\boldsymbol{E}_2\boldsymbol{E}_{1k} & 0.05+j0.29\boldsymbol{E}_3\boldsymbol{E}_{1k} \\ 0.20+j1.03\boldsymbol{E}_1\boldsymbol{E}_{2k} & 0.34-j1.69\boldsymbol{E}_2^2 & 0.09+j0.53\boldsymbol{E}_3\boldsymbol{E}_{2k} \\ 0.05+j0.29\boldsymbol{E}_1\boldsymbol{E}_{3k} & 0.09+j0.53\boldsymbol{E}_2\boldsymbol{E}_{3k} & 0.03-j0.85\boldsymbol{E}_3^2 \end{pmatrix} \quad \cdots\cdots\cdots\cdots\cdots (49)$$

The watt powers are,

$$P_1 = 0.13+1.16\sin(10°53'+\phi_{12})+0.28\sin(10\ 51'+\phi_{13})$$
$$P_2 = 0.35+1.16\sin(10°53'-\phi_{12})+0.47\sin(9°59'+\phi_{23})$$
$$P_3 = 0.02+0.28\sin(10°51'-\phi_{13})+0.47\sin(9°59'-\phi_{23})$$

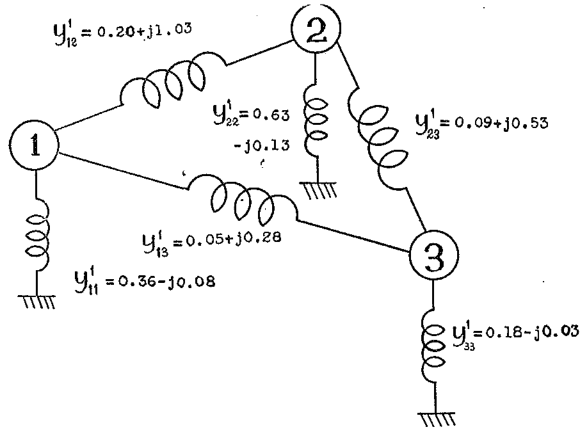


Fig. 29. Equivalent circuit to Fig. 24, when a line 4, 5, 6 is cut off.

The power angle changes in this case are shown in Table 2 and Fig. 30 by step-by-step calculation.

TABLE 2 Calculation of power angles, when the grounded line is cut off after 0.2 second.

t	ϕ_1	ϕ_2	ϕ_3	ϕ_{12}	ϕ_{13}	ϕ_{23}	P_1	P_2	P_3
0.2	31°49′	15°36′	8°27′	16°13′	23°22′	7°09′	0.815	0.381	− 0.016
0.3	45°17′	23°37′	21°20′	21°40′	23°57′	2°17′	0.914	0.232	0.022
0.4	60°15′	32°32′	30°16′	27°43′	29°59′	2°16′	1.037	0.126	− 0.006
0.5	74°35′	47°40′	40°15′	26°55′	34°20′	7°25′	1.039	0.170	− 0.068
0.6	88°14′	66°48′	62°31′	21°26′	25°43′	4°17′	0.917	0.203	− 0.002
0.7	103°20′	85°46′	84°49′	17°34′	18°31′	0°57′	0.820	0.304	− 0.111
0.8	121°34′	102°00′	105°07′	19°43′	16°27′				

One asumes that the ground fault is vanished after the further elapse of 0.4 second, and the other line is reclosed to come back again to the initial state. The power matrix for this case is given by matrix (47). The power angle changes are as shown in Table 3 and Fig. 30.

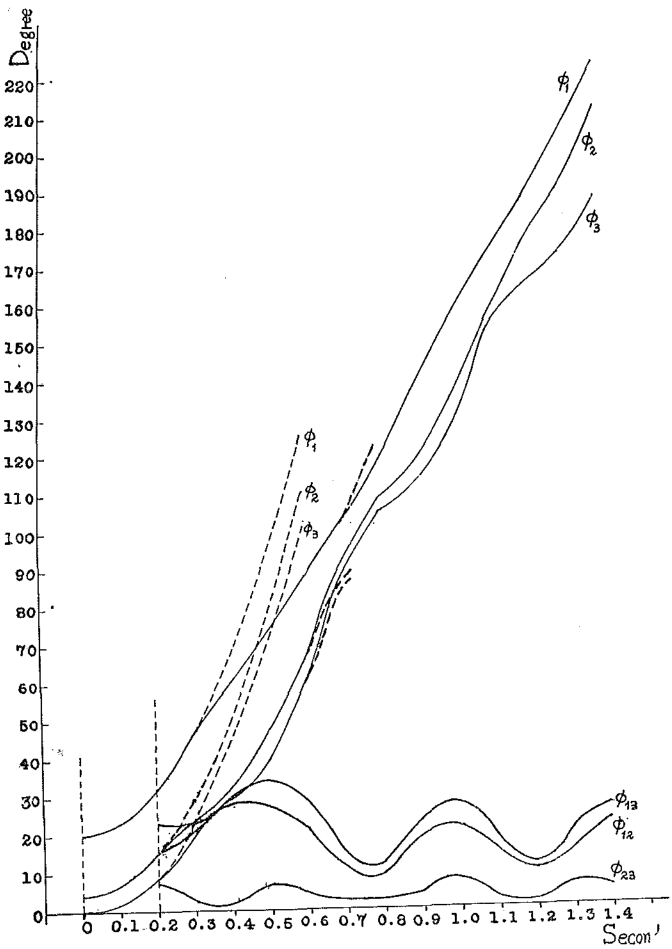


Fig. 30. Time—power angle curve of 3-machine system at ground fault, clearing fault, and reclosing.

TABLE 3 Calculation of power angles, when the system is recovered.

t	ϕ_1	ϕ_2	ϕ_3	ϕ_{12}	ϕ_{13}	ϕ_{23}	P_1	P_2	P_3
0.6	88°14′	66°50′	62°31′	21°26′	25°43′	4°17′	1.15	0.115	− 0.037
0.7	101°20′	90°30′	88°20′	10°48′	13°	2°12′	0.843	0.369	0.054
0.8	117°	108°	104°	8°53′	12°41′	3°40′	0.795	0.429	0.044
0.9	136°	117°	112°	19°30′	23°45′	4°16′	1.10	0.101	− 0.025
1.0	154°	133°	125°	21°	28°30′	7°36′	1.15	0.153	− 0.074
1.1	169°	154°	152°	14°40′	17°	2°22′	0.957	0.271	0.028
1.2	184°	174°	173°	10°20′	11°35′	1°15′	0.823	0.374	0.069
1.3	203°	188°	181°	15°14′	21°40′	6°23′	0.993	0.286	− 0.027
1.4	222°	200°	195°	22°	26°40′	4°30′			

As is shown on the above power angle curves, the three machines are similarly speed down at the ground fault. The relative powers between machines are comparatively small, because they has resistance loads. The system does not loss synchronism. This calculation may be obtained more easily and rigorously by an analogue computer.