Measurement of Small Displacement by using Newton's Rings and an Objective Micrometer

By

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Abstract

Optical method is quite handy in measuring a small displacement such as an extension caused by temperature variation. However, the optical system sometimes brings complicated errors, and the accuracy of measurement is not so good compared to other methods. It may be one of the important reasons of such errors that the points being compared with each other are not able to exist on an optically equivalent surface. By using Newton's rings one may be able to eliminate the difficulty of the optical method, and a very high accuracy can be expected.

Introduction

Displacement has a dimension of length. The measurement of displacement, however, is not so simple. As a displacement is a changing value of length, there must exist a reason for the change in the length — for example, time or temperature. Then, a displacement can be expressed by the form $X=f(t)$. Therefore, when the displacement is measured, a measurement of the element which composes the displacement in such a functional relation must carefully be carried out simultaneously.

The method described in this paper was devised for the investigation of the aging of a concrete piece. The values of the displacement are about 1/100,000 of the total length of the specimen and the time interval employed for the measurement is very long: it sometimes extends over several years. Accordingly, the following two requirements must be premised in the measurement under these circumstances. The first is a requirement for a strict standard of length in each measurement. Secondly, the measuring tool must not be changed by any condition throughout the whole measuring period. If these conditions are not satisfactorily held, the displacements may be involved in experimental errors, and the measuring apparatus must be fixed under the same circumstance extending over a long time for the purpose of eliminating any effect of its movement. If something occurs resulting in a movement of this apparatus, the experiment will become discontinous at that time.

In this experiment, the first condition is satisfied by using a pipe of fused quartz of which coefficient of thermal expansion is about 0.026×10^{-5}. The corresponding method for the second condition is realized by using Newton's rings that are produced by a lens fixed on the surface of the test piece of concrete and a plate of objective micrometer fixed on the standard quartz pipe.

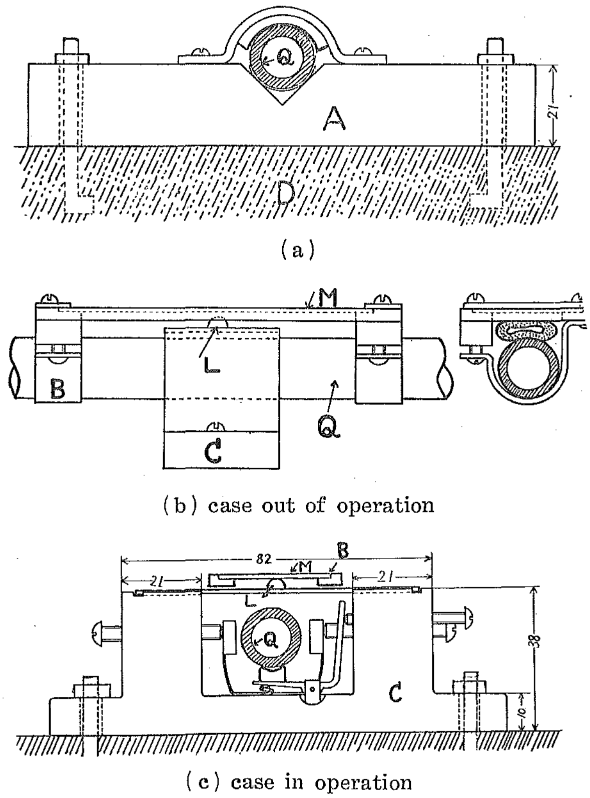

Fig. 1. Three holders.

Q : Quartz pipe (used as the standard length). M : objective micrometer.
L : lens (for the production of Newton's rings). D : specimen.

Apparatus

There are three holders made of cast iron. One of them fixed an end of the standard pipy measure on an end of the test piece, (see A in Fig. 1a & PLATE 1) and the other two holders are separately fixed on the other ends of the measure and piece (See B & C in Figs. 1b and 1c). An objective micrometer with a hundred scale-lines dividing 1 mm into equal parts of 0.01 mm, is stuck on the holder of the measure. On the other hand, a small lens, which has a large curvature and the diameter of about 5 millimetres, is stuck on the top of the holder of the specimen. And these latter two holders are slightly in contact only when the measurement is in operation.

The another set of the experiment is the optical one, that is a metallurgical microscope which was reconstructed for the purpose of having a plenty of parallel light striking the scale and the lens perpendicularly. For the light source, a natrium lamp is used effectively, but when only the accuracy of measurement is concerned, it may be sufficient to use a conbination of 6-volt lamp (for microscopic use) and a thin red filter. It is necessary to decide a mark point on the sample. One can use a diamond needle for hardness tester use as a mark fixed on the test piece. The radius of curvature of the point of the needle is about one micron. But by this method the position of the point is difficult to be found. The weak points of any ocular method can not be avoided. Moreover, the cost of the needle is so high that this method is not practical.

On the other hand, in the method of Newton's rings the center of the rings is not read directly, but many concentric circles appear on the glass surface in contact with the lens. Therefore, two important advantages accrue here: (1) the position of the center of these rings can be obtained with sufficiently high accuracy statistically, (2) those rings, i. e. interference fringes appear on the surface on the micrometer. Accordingly, the main difficulty in using an ocular microscope may be eliminated.

In order to obtain the readings of the positions of the scale lines and the rings, a 35 mm camera and a microphotometer are employed. The magnifying power of the combination of the camera and the microscope is about 33, and that of the microphotometer is 1,000. If the recording data of the microphotometer are required, the movement of the photographed film in the microphotometer is checked compared with the length marks on the sensitive paper of the oscillogram in every 0.1 mm. A part of the light flux which is given from the lamp lighted by a battery penetrates the film and the other part of the light is absorbed by the shade on the film. And then the penetrated flux is enlarged 10 times by an objective microscopic lens and projected on a slit plate. The width of the slit is variable and has an accuracy of 0.01 mm. The light flux passing through the slit strikes a phototube and is converted into electric current. Then, the oscillogram is employed to record the current (see PLATE 3). In this recording method, one must guess one-tenth of the least interval of the marks in the sensitive paper of the oscillogram in order to maintain the magnifying power of 100. Then the total magnifying power becomes 3,300, and it may not be difficult to distinguish the difference of 1

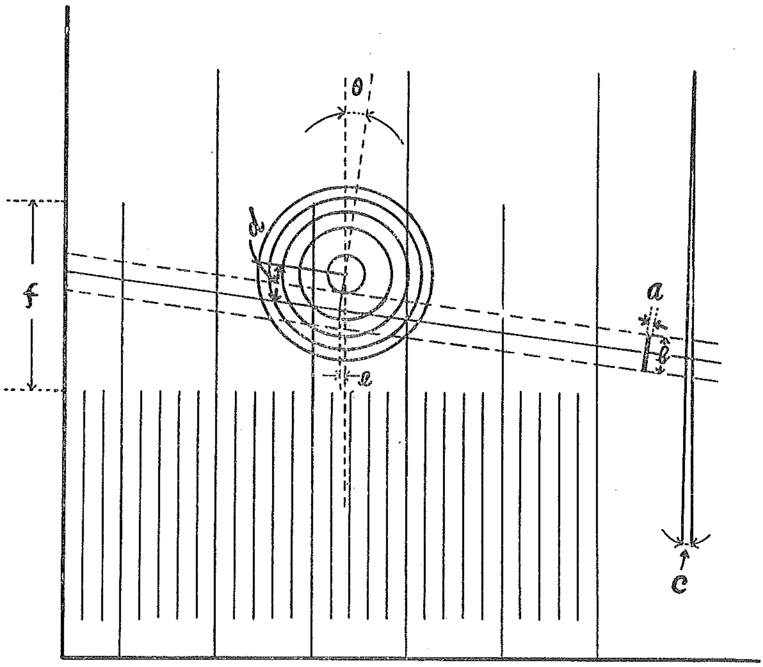

Fig. 2. Diagram for the analysis of errors.

a: width of slit on the surface of micrometer.
b: length of slit on the surface of micrometer.
c: angle=1/50 radian.
d: distance between the center of rings and the center line of slit-band.
θ: inclination of slit-band.
e: resultant error.

micron on the surface of micrometer. Namely, an interval of the scale lines, i. e. 0.01 mm, corresponds to 3.3 units of the marks on the sensitive paper. But actually, only the scale lines of 0.05 mm in distance are used, because it is difficult to read the position of the rings when they overlap with the scale lines (See Fig. 2).

Experimental results

The examples of the Newton's rings photographed by the apparatus described above and some of the recordings are shown on PLATES 2 and 3, respectively. The experiments are divided into three parts: the measurement by microphotometer, photographing, and the production of Newton's rings. All the calculations were strictly performed by the method of least square root.

I. Experiments on the use of microphotometer

1. Effect of the width of the slit

The width of the slit used in the present experiment was from 0.02 mm to 0.05 mm on the film, i. e. from $0.6\,\mu$ to $1.5\,\mu$ on the surface of the micrometer scale. As is shown on PLATE 3, the width of the peaks caused by the scale lines was about 0.3 mm on the slit plate, and the width of the slit of about 0.03 mm on the film surface was considered to be suitable. This width is almost equal to $1\,\mu$ on the surface of the micrometer (see Fig. 2 in regard to the width and the length of slit). A–1 and A–2 in TABLE 1 are the examples obtained in the experiment. The difference of the results can be understood within the range of accidental errors.

2. Effect of the location of the sampling band

Strictly speaking, the slit is not tangential to the rings unless the center-line of the band of film does not pass the center of rings, when the location of the band is changed in the direction normal to the movement of the film. The effect of this change was, however, small enough as will be seen when one compares the results of A–2 and A–3 in TABLE 1, and the small deviations can be neglected.

3. Perpendicularity of the scale lines to the direction of the film-movement

As is shown in Fig. 2, the error e is equal to $d \sin\theta$. It is not difficult to minimize d to a value less than about 0.02 mm which

TABLE 1 Examples of the difference of introduced by microphotometer

Number of Film & sensitive paper	Readings of the scale-lines			Readings of the centers of Newton's rings		Converted results onto the scale	reference	Comparison
	n	α	β	n'	r	$\times 10^{-3}$ mm		
A 1	7	-0.577 ± 0.0055	1.6277 ± 0.0009	11	8.509 ± 0.0026	229.1 ± 0.3	$\alpha=0.05$ mm $\beta=0.05$ mm	Slit width $\alpha=0.35$ mm $\alpha=0.30$ mm
A 2	7	-0.577 ± 0.0055	1.6277 ± 0.0009	11	8.505 ± 0.0047	229.0 ± 0.3	do.	parallel transition
A 3	7	-0.577 ± 0.0055	1.6277 ± 0.0009	11	8.508 ± 0.0032	229.1 ± 0.3	do.	
D 1	9	-0.915 ± 0.0056	1.6292 ± 0.0009	10	8.453 ± 0.0020	237.5 ± 0.3	$\alpha=-0.05$ mm $\beta=0.05$ mm	parallel transition
D 2	10	-0.913 ± 0.0064	1.6284 ± 0.0026	9	8.453 ± 0.0022	237.6 ± 0.5	do.	perpendicularity to direction of film movement
D 3	10	-0.773 ± 0.0082	1.6282 ± 0.0013	9	8.599 ± 0.0024	237.8 ± 0.4	do.	(D-2) good, (D-3) θ : 1/50
D 4	8	-0.883 ± 0.0062	1.6263 ± 0.0010	10	8.473 ± 0.0023	237.6 ± 0.6	do.	
D 5	10	-0.925 ± 0.0064	$1.6268\pm.00010$	10	8.434 ± 0.0023	237.7 ± 0.6	do.	
D 6	9	-1.186 ± 0.0124	1.6293 ± 0.0020	9	8.996 ± 0.0016	237.8 ± 0.6	$\alpha=0.55$ mm $\beta=-0.05$ mm	random setting
D 7	8	-0.174 ± 0.0148	1.6285 ± 0.0024	9	10.000 ± 0.0016	237.6 ± 0.7	do.	
D 8	11	-1.164 ± 0.0096	1.6234 ± 0.0014	9	9.017 ± 0.0015	237.4 ± 0.4	do.	Perpendicularity, good
D 9	9	-1.154 ± 0.0096	1.6295 ± 0.0016	9	9.024 ± 0.0016	237.7 ± 0.4	do.	θ : 1/50
D 10	9	-0.515 ± 0.0117	1.6273 ± 0.0018	9	8.018 ± 0.0015	237.8 ± 0.5	$\alpha=0.50$ mm $\beta=-0.05$ mm	θ : 1/25
D 11	11	-1.160 ± 0.0055	1.6291 ± 0.0009	7	9.019 ± 0.0018	237.6 ± 0.3	$\alpha=0.55$ mm $\beta=-0.05$ mm	good

Note: n and n' are the numbers of the scale-lines and the rings measured, respectively. α and β are the symbols in the least square root method. Observed value is $t=\alpha+\beta t$, where α is the optional scale-line that can be selected as the standard point, and β is the interval of any two adjacent scale-lines.

is comparable to the diameter of the smallest rings. Also, the condition of $\theta < 1/50$ can easily be attained when the photometer is used. Then, $e = d \sin \theta < 0.0004\,\text{mm} = 0.4\,\mu$. This error is again within the region of an accidental error. The data of the examples D–8, 9, 10, 11 and D–2, 3 are shown in TABLE 1.

4. Results of random settings of the film in the photometer

All the recorded data for the series of D in TABLE 1 were obtained from the same film, and D–4, 5, 6 and 7 are the cases of random settings in the manner explained above.

Therefore, it can be concluded that one recorded paper of oscillogram is sufficient for the analysis of a film. One can clearly see this fact in Fig. 3, and the deviations of the observed values are less than $0.4\,\mu$.

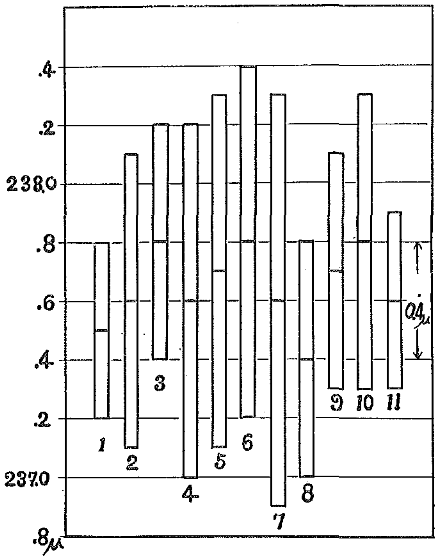

Fig. 3. Deviation of the measured values of a same film with microphotometer.

II. Problems of microscopic photographing

1. Change of the field of vision

The distance of microscopic image of two points which are not perfectly on the same surface is influenced by the movement of the center of reflection (center of the field of vision) even if the states of the objects are identical. On the other hand, the distance of any two

TABLE 2. Examples of the difference introduced by photographing

Number of film & sensitive paper	Readings of the scale-lines			Readings of the centers of Newton's rings		Converted results onto the scale	Reference	Comparison
	n	α	β	n'	γ	$\times 10^{-3}$ mm		
A—4	7	−1.001±0.0047	1.6282±0.0007	11	7.826±0.0033	228.9±0.2	α=0.05 mm β=0.05 mm	transition of micro-scopic field of vision
B—1	7	−0.369±0.0064	1.6280±0.0011	11	8.465±0.0040	228.7±0.3	α=0.50 mm β=−0.05 mm	
C—2	9	−1.348±0.0061	1.6289±0.0009	10	9.115±0.0022	228.8±0.3	α=0.55 mm β=−0.05 mm	
D—8	11	−1.164±0.0096	1.6284±0.0014	9	9.017±0.0015	237.4±0.4	do.	error in focusing
E—1	6	−1.496±0.0015	1.6238±0.0023	10	6.950±0.0014	239.7±0.6	α=0.50 mm β=−0.05 mm	
F—1	9	−0.757±0.0064	1.6317±0.0010	8	7.643±0.0025	242.4±0.3	do.	error in focusing
G—1	8	−1.236±0.0088	1.6329±0.012	7	7.144±0.0032	243.4±0.4	do.	

TABLE 3. Examples under a complete fixing

Number of Film & sensitive paper	Readings of the scale-lines			Readings of the centers of Newton's rings		Converted results onto the scale	reference	Comparison
	n	α	β	n'	γ	$\times 10^{-3}$ mm		
H	9	−0.767±0.0065	1.6353±0.0008	5	10.556±0.0024	203.8±0.3	α=0.55 mm β=−0.05 mm	variation of contact pressure
I	9	−0.684±0.0068	1.6332±0.0012	10	7.371±0.0031	203.4±0.3	α=0.45 mm β=−0.05 mm	variation of contact area
J	10	−1.527±0.0094	1.6350±0.0015	8	8.151±0.0048	204.0±0.4	α=0.50 mm β=−0.05 mm	same state, focus corrected
K	9	−0.654±0.0072	1.6333±0.0012	8	9.015±0.0059	204.0±0.4	α=0.50 mm β=−0.05 mm	
L	10	−1.380±0.0035	1.6335±0.0014	8	9.941±0.0027	203.5±0.3	α=0.55 mm β=−0.05 mm	same state, variation of the inclination of mirror
M	7	−0.918±0.0056	1.6309±0.0009	10	5.474±0.0021	204.0±0.2	α=0.40 mm β=−0.05 mm	

points which are perfectly on the same surface are not influenced by the change of the field of vision. As will be seen in the examples A, B, C in PLATE 2 and TABLE 2. The experimental results show some deviation, which is still within an accidental error. This deviation might have been caused by the adjustment of the focus. The consideration about the deviation owing to the change of the field of vision would not be necessary.

2. Errors with the adjustment of the focus

This effect is most important. In PLATE 2, E is in the same state as D. But the scale lines in E have some width which is not symmetrical, and the peaks in the recorded sensitive paper do not give the centers of the widths of the scale lines. As a result, E and D give different values. F and G in PLATE 2 do not show good results for the same reason.

3. Effect of the inclination of the mirror

The angle of incidence of light on the reflect surface of the micrometer should be small. In fact, the difference of the experimental results originating from by different angles of incidence was negligibly small so far as the numbers of fringe were less than 100 when a microscope was used, because the field of vision was very narrow and the light flux which strikes there is almost parallel.

On the other hand, the angle of incidence has exerted a very large influence on the scale lines of the micrometer. The widths of the lines are about 2μ, and the depths of the grooves of the lines are guessed to be 1μ. If the direction of light flux is not perpendicular to the grooves, the distributions of the reflected rays from the inner parts of the grooves are not symmetrical. From this reason, the error caused by different angles of incidence seems to become about 0.2μ.

Some of the examples are given by K, L and M in TABLE 3. They are similar to A, B and C, which are the examples of the case when the field of vision was moved. They are also similar to I and J in which a movement of the field of vision was accompanied by a change of contact point of lens with scale surface.

III. Effects of the contact of the lens with the scale surface

1. Movement of the contact point in the direction perpendicular to the displacements

In this experiment, the contact point could be put in the region

f which is shown in Fig. 2. The magnitude of the movement of the contact point in this region may be less than 0.05 mm. One can easily derive the relation $\sin\delta = 0.05/l$, where

l: standard length of quartz, which is the distance between the contact point and the center of the holder A,

δ: angle between the direction of displacement and the standard pipy measure.

Since the value of δ is extremely small, the difference of the length caused by this reason can be expressed by $\Delta l = l(1-\cos\delta) \doteqdot \frac{1}{2}\delta^2 l$

Therefore, if $l = 10\,\mathrm{cm}$, then $\Delta l = 0.01\,\mu$, and Δl is negligible. In the examples I and J, however, errors previously mentioned must be involved.

2. Difference of the contact pressure

In order to investigate the effect of contact pressure at the contact point, some light loads were applied. The result is shown by E, F, and H, I. It seems that the errors of these data are introduced by the change of adjustment of focusing.

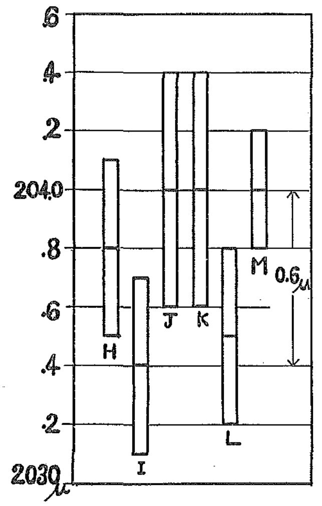

Fig. 4.
Deviation of measured values under a same state by changing the manners of photographing and contact.

In the experiments, A, B, C, D, E, F and G were measured without fixing the scale to the quartz pipe. Therefore, the comparisons among these different films in TABLES 1 & 2 have no meaning, excluding the comparison indicated in the tables. On the contrary, H, I, J, K, L and M were measured under a state of complete fixing and contact. Accordingly, the comparisons among any of them are useful for the present purpose. The results are shown in Fig. 4.

In the process of the experiments, the lens used for the detection of the displacement developed so many flaws that the Newton's rings were not clear, and the rings from 3rd to 16th were used for the determination of the center.

After these experiments, the author could have a new lens which is the first lens of the objective microscopic lens having the

TABLE 4. Examples obtained by using a new lens

Number of Film	Readings of the scale-lines			Readings of the centers of Newton's rings		Converted results onto the scale	Reference	Comparison
	n	α	β	n'	r			
1	6 (7)	− 0.765	1.6257	14 (15)	5.802	151.9 μ	$\alpha=-0.05$ mm $\beta=0.05$ mm	variation of contact pressure & inclinations of mirror
2	7 (7)	− 1.009	1.6275	14 (15)	5.544	151.3	do.	transitions of Newton's rings
3	6 (7)	− 1.050	1.6282	14 (15)	5.505	151.3	do.	transition of microscopic field of vision
4	6 (7)	− 1.265	1.6257	14 (15)	5.279	151.3	do.	
5	5 (7)	− 1.204	1.6212	14 (15)	5.324	151.3	do.	inclination of mirror
6	7 (7)	− 1.599	1.6268	10 (12)	4.956	151.5	do.	
1′	8 (9)	− 0.783	1.6318	20 (23)	5.802	151.8±0.3	do.	same as 1, all available rings are used.

Note: n and n' are the numbers of the scale-lines and the rings measured, and the numbers in ( ) represent those existing in the same interval.

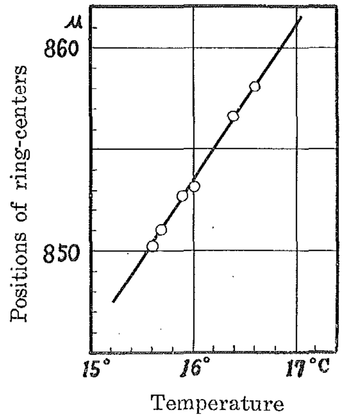

Fig. 5. Magnitude of extension due to the temperature variation.

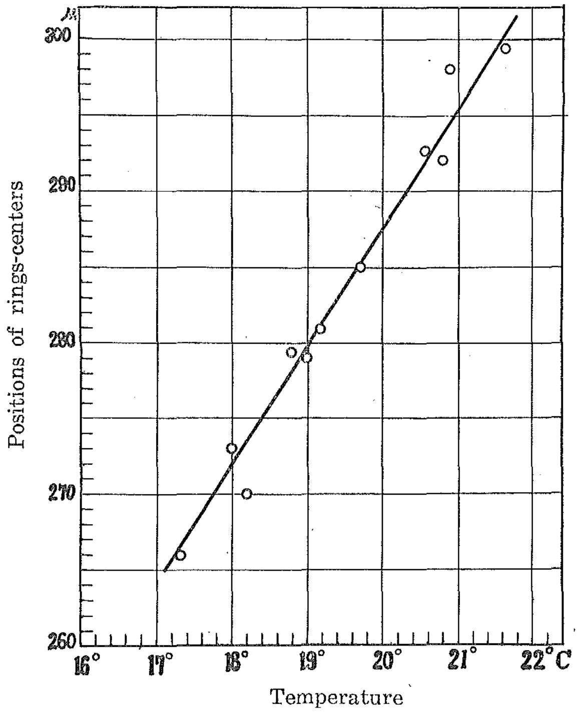

Fig. 6. Magnitude of extension due to the temperature variation. In this case the centers of Newton's rings were directly read on the film.

magnifying power of about 40. Another series of experiments were once again undertaken with the new lens by fixing it to the specimen. At the same time, a pipe of fused quartz was employed as the standard length of 69.8 cm. The result is shown in PLATES 4 & 6 and TABLE 4. The reason of the large error of "1" in TABLE 4 seems to have come from the failure of focus adjustment.

Fig. 5 and PLATES 5 & 6 show the magnitude of extension of a concrete test piece due to the variation of temperature. The variation of temperature, in this case, was quasi-static, and the temperatuer of the concrete was estimated by the temperature of the surrounding circumstances. The coefficient of expansion of the concrete was 1.11×10^{-5}.

Fig. 6 shows the result of another setting of the same piece. In this case, the measurement was not always static in respect to the variation of temperature. In the cases shown in Fig. 5 & Fig. 6, a natrium lamp was employed as the light source. The numbers of the rings on the photographed film reached to a hundred. However, as far as a usual micrometer is used, about 20 of rings are sufficient. On the contrary, if an accurate micrometer is used, the measurement of displacement will be able to attain higher accuracy.

As will be seen in all the results presented in this paper, the author's device of measuring a small displacement can easily attain an accuracy of 1μ. If an accuracy of measuring a displacement is required to be within 0.002 mm one can have the reading directly on the microscopic film which is shown in PLATE 7 as an example without going into the detail of least square root method.

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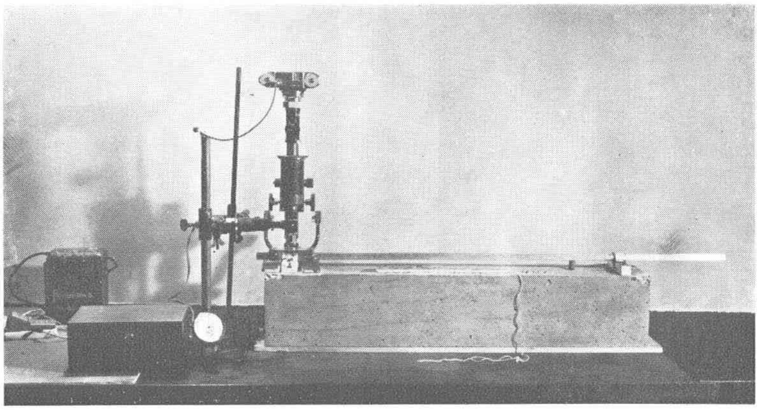
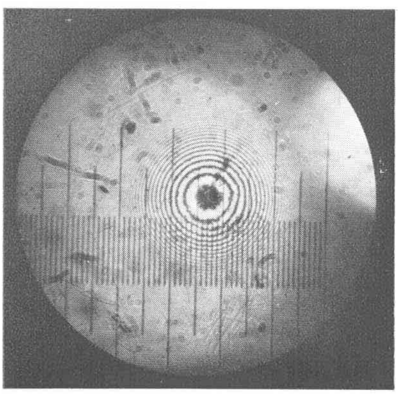
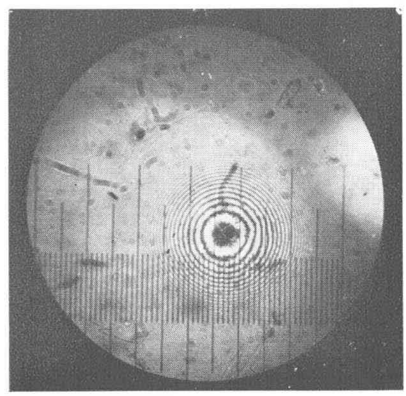


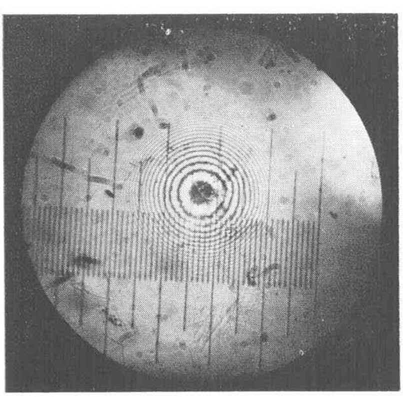
PLATE 1. General view of the experiment.



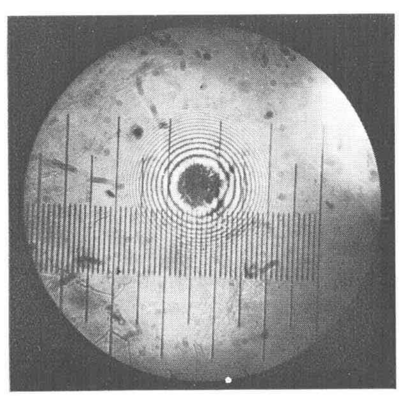
(A)



(B)

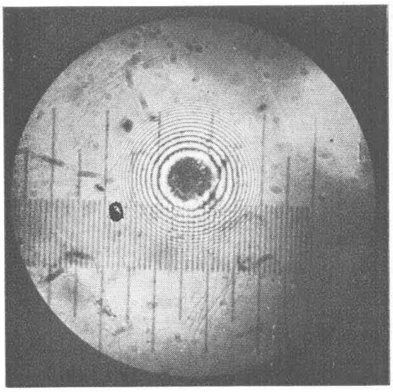


(C)

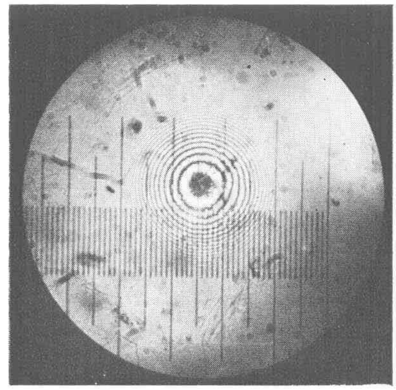


(D)

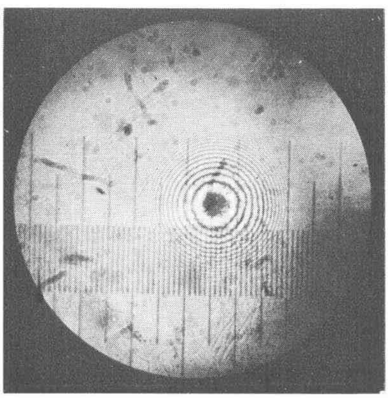
PLATE 2. Examples of microscopic photographs.



(E)



(F)



(G)

PLATE 2. (continued)

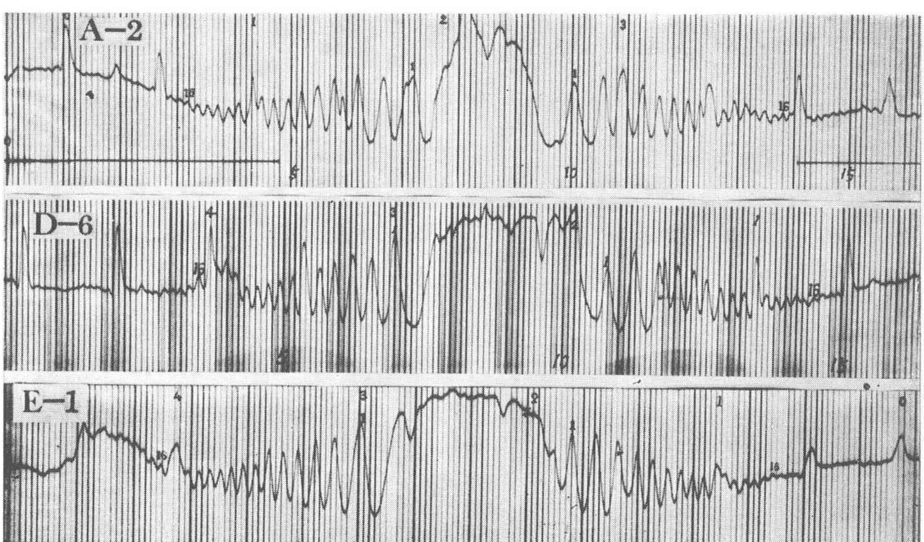


PLATE 3. Examples of sensitive paper.
A, D & E correspond to (A), (D) & (E) in TABLES 1, 2 & PLATE 2. respectively.

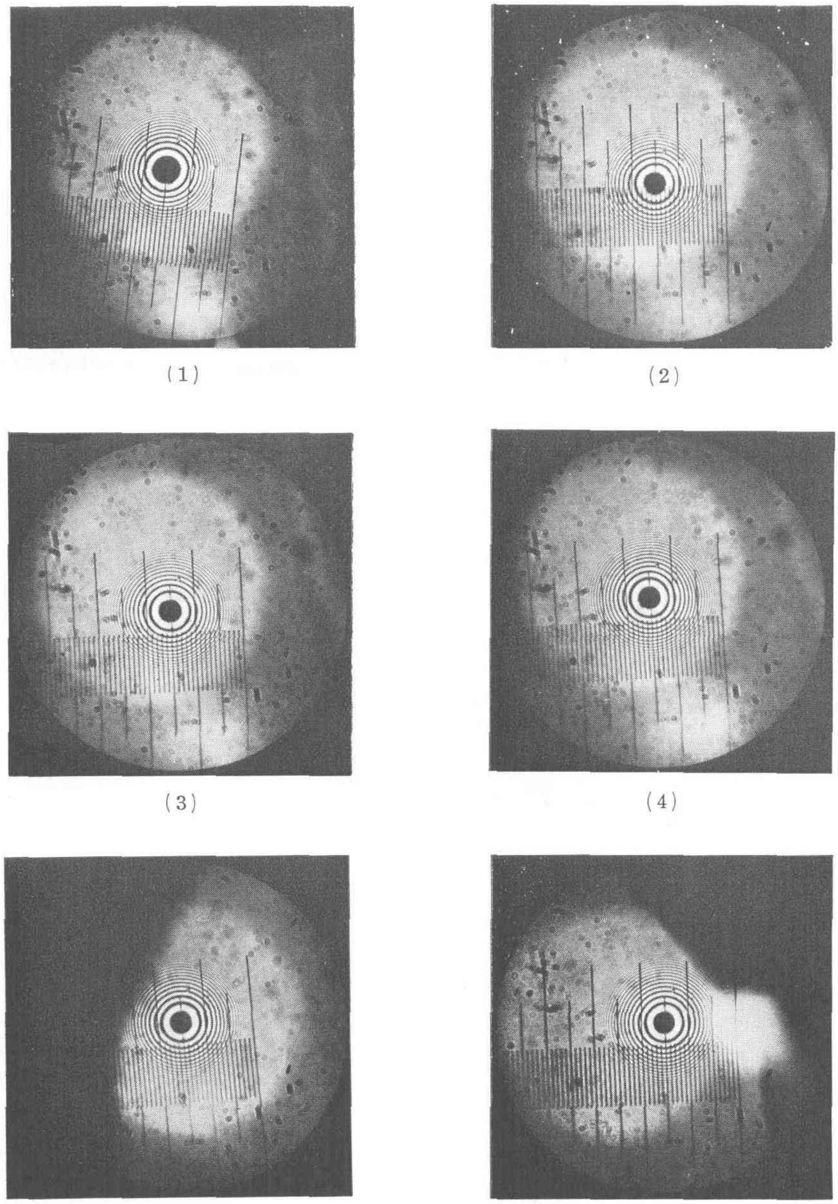
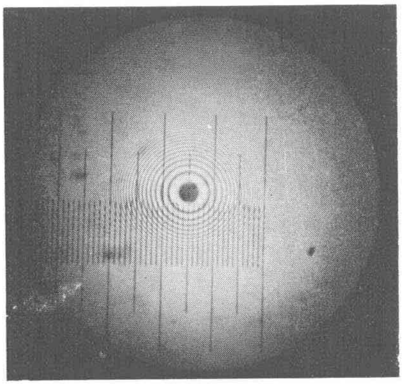
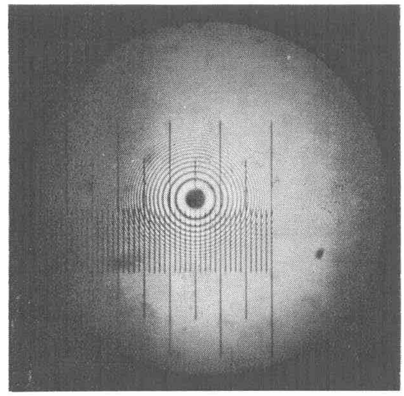


PLATE 4. Examples of microscopic photographs taken by using a new lens and 6-volt lamp.

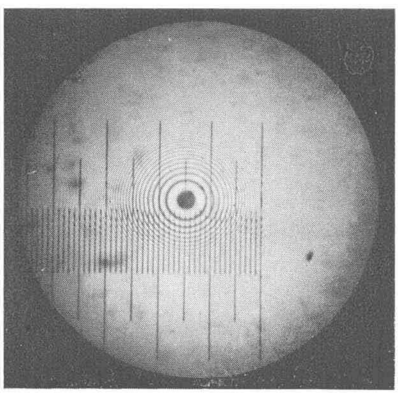
The number of each plate corresponds to the same number in TABLE 4.



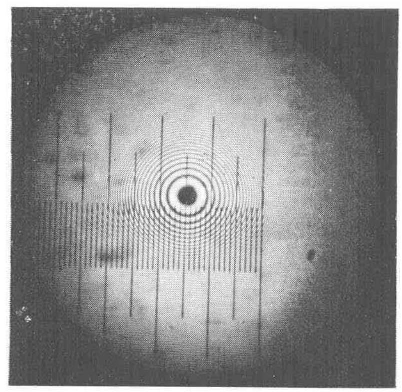
(1) 15.6°C



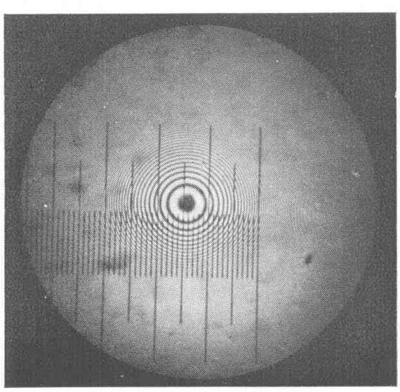
(2) 15.7°C



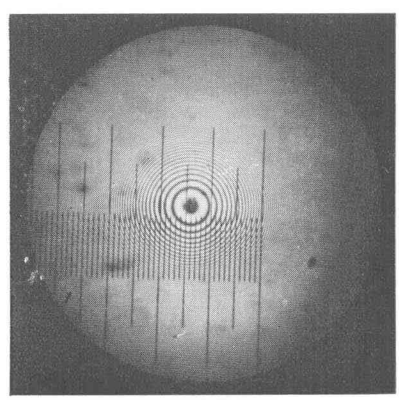
(3) 15.9°C



(4) 16.0°C



(5) 16.4°C



(6) 16.6°C

PLATE 5. Examples of microscopic photographs taken by using a new lens and natrium lamp.

Each plate corresponds to the point in Fig. 5.

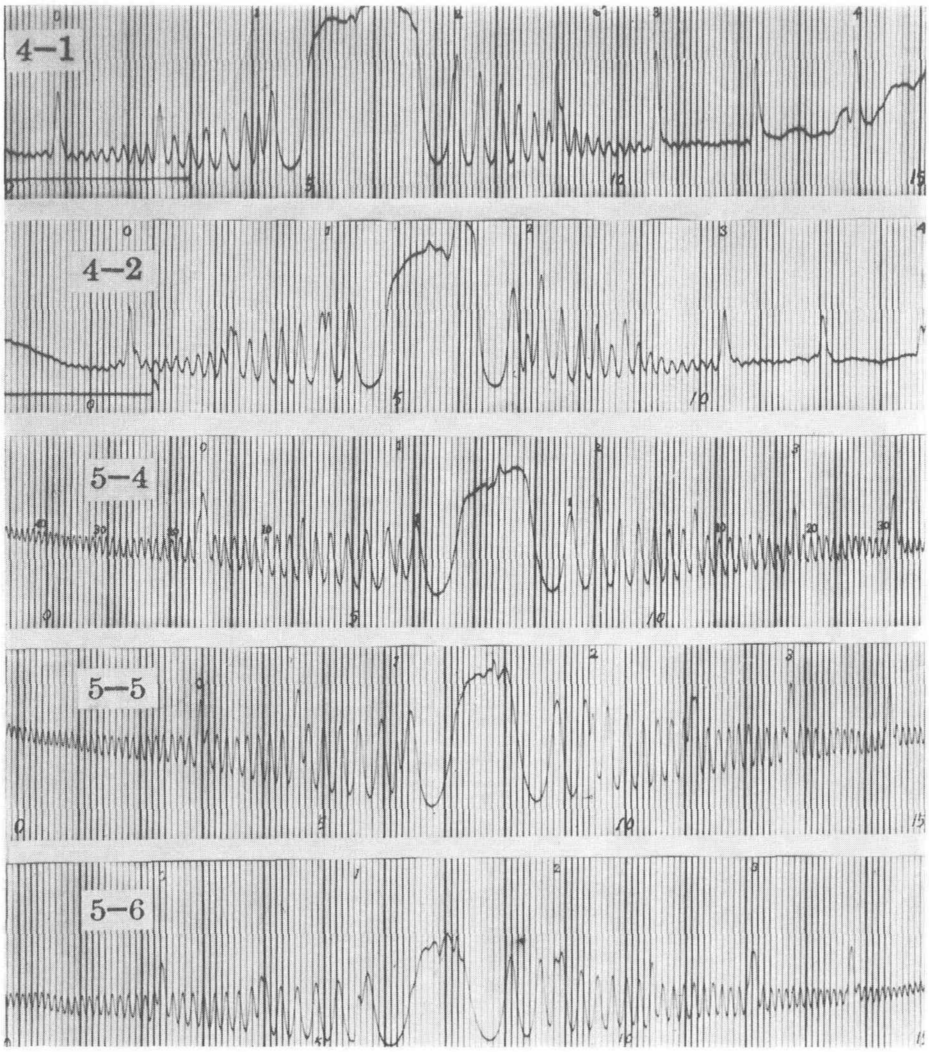


PLATE 6. Examples of Sensitive paper.

The first number refers to the PLATE number, and the following number corresponds to the number in the PLATE.

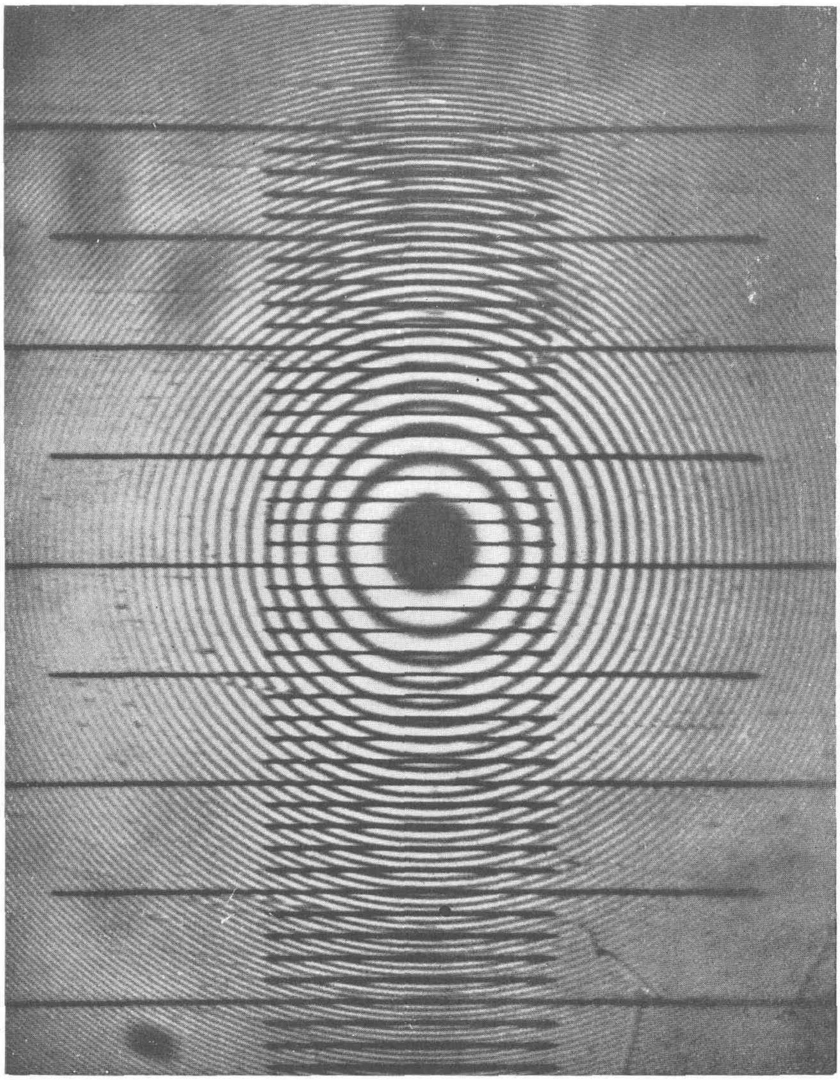


PLATE 7. Magnified photograph of a microscopic film for the purpose of reading the lens-center.

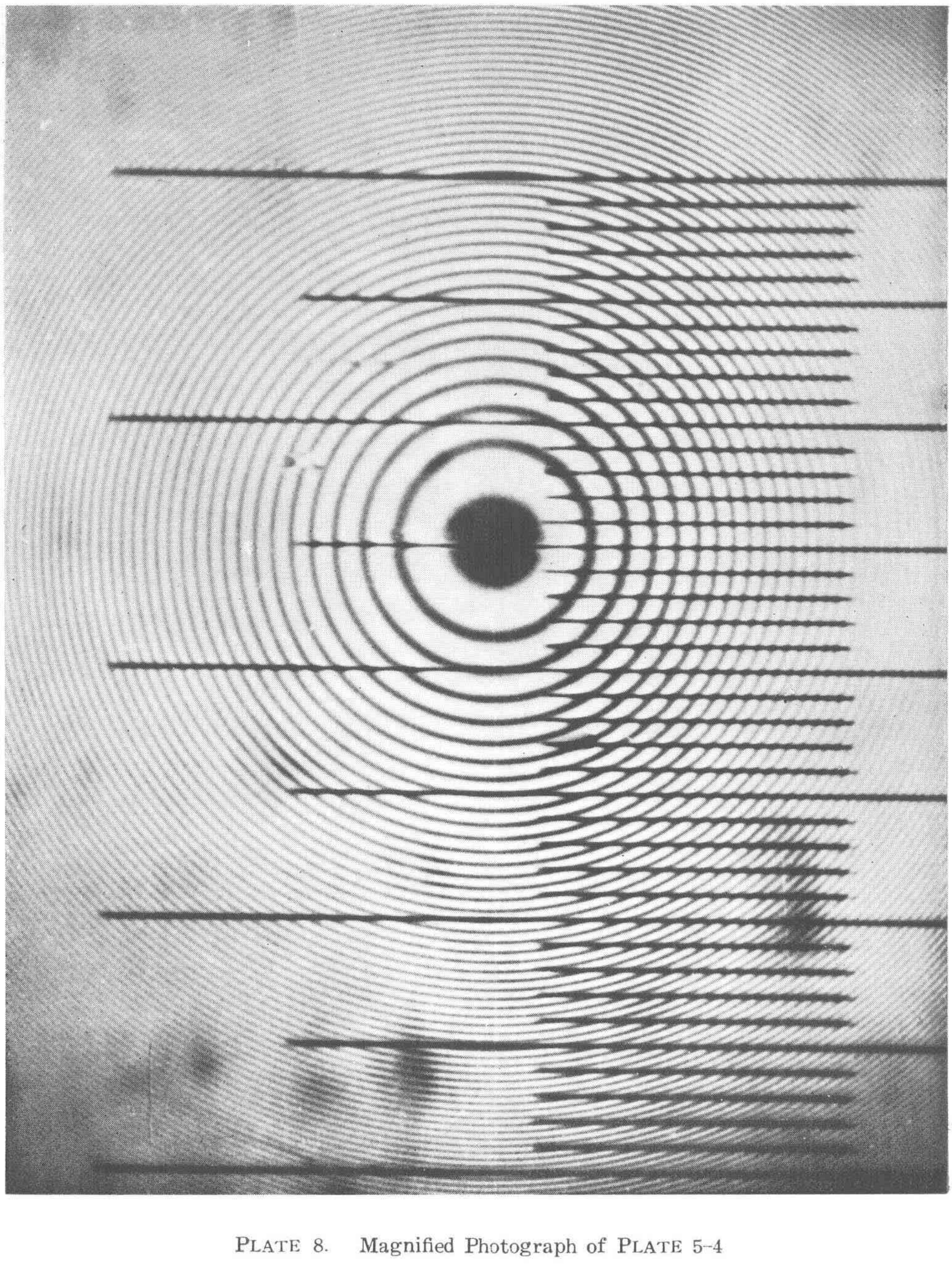


PLATE 8. Magnified Photograph of PLATE 5-4