Formula for Calculating the Value of Free Shrinkage of Mortar

(A partial abstract from "Dry Shrinkage and Cracking of Mortar and Concrete*")

By

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Synopsis

The original paper in Japanese* includes the results of the following four series of laboratory tests on free dry shrinkage of mortar and concrete.

(1) Shrinkage of rich mortar during the first day after mixing.
(2) Quantitative study on expansion of mortar in water and dry shrinkage in room air.
(3) Effect of silt on dry shrinkage.
(4) Shrinkage of concrete of dry and medium consistencies.

This paper is abstracted on the present article from the items (2) and (4).

The author derived a semi-logical formula between the dry shrinkage of mortar and the dry shrinkage of its components:

$$\varepsilon_m = \frac{\varepsilon_p - \varepsilon_a}{1 + \dfrac{E_a A_a}{E_p A_p}} + \varepsilon_a$$

where ε_m, ε_p and ε_a are the dry shrinkage of mixture, neat cement mortar and fine aggregate, E_p and E_a are the secant moduli of neat cement mortar, and aggregate, A_p and A_a are the cross sectional areas of imaginary prisms of neat cement mortar part and aggregate of absolute volume.

The proposed formula shows a good agreement not only with the experimental results of mortar test, but also with concrete tests after some amendment of the calculated data.

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* The original paper was reported by the author in the Bullettin No. 9 of the Faculty of Engineering of Hokkaido University, in 1953

1. Derivation of the formula

A mortar is composed of sand and cement compounds, each of which contains a certain volume of free water. The preliminary assumption is that a mortar prism can be divided into two parts, as will be seen in Fig. II-7. One part is a prism of absolute volume of sand and the other part is a prism of neat cement mortar.

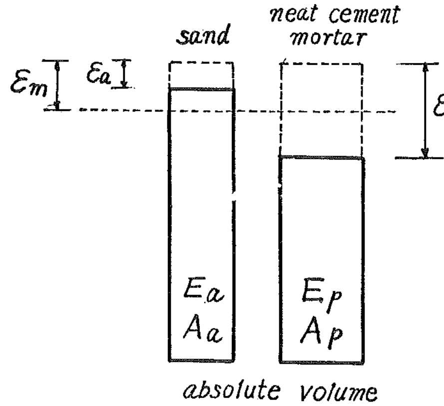

Fig. II-7.

If they were mutually free, they would show different shrinkages ε_a and ε_p. But they must shrink together and as the result the mortar shows the shrinkage of ε_m. Therefore, the sand prism must be compressed and the prism of neat cement mortar must be expanded. The value of compressive strain is $(\varepsilon_m-\varepsilon_a)$ and the tensile strain is $(\varepsilon_p-\varepsilon_m)$.

Assuming that the cross sectional areas of the imaginary prisms of sand and neat cement mortar are A_a and A_p, and that the secant moduli are E_a and E_p when the time effects on their strains are taken into consideration, one can derive the following formula.

$$(\varepsilon_m-\varepsilon_a)\,E_aA_a=(\varepsilon_p-\varepsilon_a)\,E_pA_p$$

$$\therefore\quad \varepsilon_m=\frac{\varepsilon_pE_pA_p+\varepsilon_aE_aA_a}{E_aA_a+E_pA_p}$$

$$\varepsilon_m=\frac{\varepsilon_p-\varepsilon_a}{1+\dfrac{E_aA_a}{E_pA_p}}+\varepsilon_a \qquad (1)$$

If the proportion of mixture is given as $1:m$ by weight, the value of A_a/A_p can be determined by

$$\frac{A_a}{A_p}=\frac{\dfrac{m}{\gamma_s}}{\dfrac{1}{\gamma_c}+w}$$

where,

γ_s : the specific gravity of sand
γ_c : the specific gravity of cement
w : the water-cement ratio.

2. Compatility of the formula on mortar

In order to examine the compatibility of the formula (1), some practical calculations were performed. The value of ε_a could be obtained by measuring the dry shrinkage of prism of the mother-rock of sand, but the author assumed this value as 20×10^{-5} in round number, taking the experimental data of Schumann (D. Bztg. Zementbeilage 1921) for reference.

(a) Calculation based on the true water-cement ratio

The value of E_a/E_p will be different depending upon the water-cement ratio of mortar and the intensity of internal stress. (The poorer the mixture, the larger the tensile strain of neat cement mortar and the smaller the compressive strain of sand). However, it is very difficult to obtain this value from the result of usual short-time test, because the actual value of E_a/E_p must be affected by the plastic deformation of the parts of sand and neat cement mortar. Therefore, the value was calculated, as is shown in Table II–7, from the experimental data of author's shrinkage test* undertaken on 1 : 0 and 1 : 1 mixtures, for the true water-cement ratio (calculated by eliminating the free water after setting of mortar from initial mixing water) of 0.50, 0.55 and 0.60. The effect of the intensity of internal stress was neglected.

TABLE II–7 Derivation of E_a/E_p (unit of shrinkage$=10^{-5}$)

	w/c	0.50	0.55	0.60**
1)	ε_p	152	175	200
2)	$\varepsilon_p-\varepsilon_a$	132	155	180
3)	$\varepsilon_{1:1}$	85	96	108
4)	$\varepsilon_{1:1}-\varepsilon_a$	65	76	88
5)	2)/4)	2.03	2.04	2.05
6)	5)-1	1.03	1.04	1.05
7)	A_a/A_p	0.46	0.43	0.41
8)	$\frac{E_a}{E_p}=\frac{6)}{7)}$	2.34	2.40	2.56

* The test was performed in 1937–1938. Size of speciment: $4\times4\times20$ cm³. Curing: 2 days sealed in the form, 31 days in water of 20°C, or no water curing, then 30 weeks in room air of 20°C and 80% R. H.

** The data was extrapolated from the test results.

Table II–8 shows the results of calculation on the other mortars by the formula (1), when the derived values of E_a/E_p are applied. The calculated values show a good coincidence with the author's experimental data.

TABLE II–8 Calculation (unit of shrinkage $=10^{-5}$)

w/c		0.50		0.55		0.60	
period of curing in water (days)		31	0	31	0	31	0
ε_p		152	97	175	109	200	121
$\varepsilon_p-\varepsilon_a$		132	77	155	89	180	101
E_a/E_p		2.34	2.34	2.40	2.40	2.56	2.56
1 : 1 mortar	A_a/A_p		0.460		0.434		0.411
	$1+\frac{E_aA_a}{E_pA_p}$		2.03		2.04		2.05
	ε_m calculated		58		64		69
	experimental	85	58	96	63	108	68
1 : 2 mortar	A_a/A_p	0.915	0.915	0.868	0.868	0.820	0.820
	$1+\frac{E_aA_a}{E_pA_p}$	3.14	3.14	3.08	3.08	2.90	2.90
	ε_m cal.	62	45	70	49	78	53
	experimental	64	44	71	47	78	49
1 : 3 mortar	A_a/A_p	1.38	1.38	1.30	1.30	1.22	1.22
	$1+\frac{E_aA_a}{E_pA_p}$	4.24	4.24	4.12	4.12	4.13	4.13
	ε_m cal.	51	38	58	42	64	44
	experimental	52	40	56	42	60	44
1 : 4 mortar	A_a/A_p	1.84	1.84	1.73	1.73	1.63	1.63
	$1+\frac{E_aA_a}{E_pA_p}$	5.31	5.31	5.15	5.15	5.18	5.18
	ε_m cal.	45	35	50	37	55	40
	experimental	—	—	—	—	50	40

(b) Calculation based on the initial water-cement ratio.

Another practical calculation was also attempted to simplify the process. In this case, the water-cement ratio of the projected mortar was adopted in place of the true water-cement ratio after setting. The calculated values are shown in Tables II-9 and II-10, and the results are in a good coincidence with the tested data, again.

TABLE II-9 Derivation of E_a/E_p

	w/c	0.40	0.50	0.60	0.70
1)	ε_p	105	132	152	170
2)	$\varepsilon_p-\varepsilon_a$	85	112	132	150
3)	$\varepsilon_{1:1}$	63	79	91	103
4)	$\varepsilon_{1:1}-\varepsilon_a$	43	59	71	83
5)	2)/4)	1.98	1.90	1.85	1.81
6)	5)-1	0.98	0.90	0.85	0.81
7)	A_a/A_p	0.524	0.460	0.411	0.370
8)	$\dfrac{E_a}{E_p}=\dfrac{6)}{7)}$	1.87	1.96	2.06	2.19

TABLE II-10 Comparison between calculated value and experimental data

w/c		0.40		0.50		0.60		0.70	
period of curing in water (days)		31	0	31	0	31	0	31	0
ε_p		105	72	132	85	152	97	170	106
E_a/E_p		1.87	1.87	1.96	1.96	2.06	2.06	2.19	2.19
1 : 1 mortar	A_a/A_p	0.524	0.524	0.460	0.460	0.411	0.411	0.370	0.370
	ε_m cal.		46		57		62		68
	exper.	63	48	79	55	91	60	103	66
1 : 2 mortar	A_a/A_p	1.05	1.05	0.915	0.915	0.820	0.820	0.740	0.740
	ε_m cal.	49	38	60	43	69	49	77	53
	exper.	(50)	(41)	61	44	71	47	81	50
1 : 3 mortar	A_a/A_p	1.58	1.58	1.38	1.38	1.22	1.22	1.10	1.10
	ε_m cal.	41	33	50	38	58	42	64	45
	exper.	—	—	(50)	(40)	58	43	63	45
1 : 4 mortar	A_a/A_p	2.09	2.09	1.84	1.84	1.63	1.63	1.47	1.47
	ε_m cal.	37	31	44	34	50	38	56	40
	exper.	—	—	—	—	(50)	(39)	54	41

(Figures in parentheses were extrapolated from the test results.)

3. Application of the formula on concrete

The formula (1) can also be applied to the calculation of shrinkage of plastic concrete, by a reduction of 10~15% from the calculated values. The basis of this reduction comes from the comparison of the shrinkages of fine sand mortar and coarse sand mortar, which are shown in Fig. II–8.

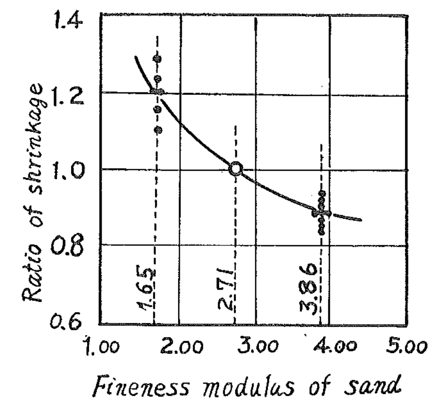

Fig. II–8. Relation between the fineness modulus and the shrinkage of mortar.

Table IV–7 shows the free dry shrinkages of neat cement mortars in the series of tests* on concrete. The shrinkages are taken as the values of ε_p in formula (1) and the ratios of E_a/E_p are assumed to be equal to those already obtained in Table II–9. By taking ε_a as $20\times10,^{-5}$ the values of ε_m can be computed as will be seen in Table IV–8.

The computed results show a considerably good agreement with the experimental results.

TABLE IV–7 Shrinkage of neat cement mortar

designed water-cement ratio	0.30	0.40	0.50	0.60	0.70
true water-cement ratio	0.30	0.40	0.46	0.50	0.54
shrinkage	99	141	162	175	191

* The test was performed in 1951~1952. Size of specimen: 6×6×30 cm³, Curing: 1 day sealed in the form, 1 week in the water, then in the room air of 23°C, 76% R. H.

TABLE IV-8 Calculation and experimental data

designed water-cement ratio	0.40			0.50			0.60			0.70		
$\varepsilon_p \times 10^{-5}$	141			162			175			191		
E_a/E_p	1.87			1.96			2.06			2.49		
mixture*	1 : 3.5	1 : 4.5	1 : 5.5	1 : 4.5	1 : 5.5	1 : 6.5	1 : 5.0	1 : 6.0	1 : 7.0	1 : 6.0	1 : 7.0	1 : 8.0
A_a/A_p	1.85	2.36	2.87	2.06	2.52	2.98	2.05	2.47	2.86	2.22	2.59	2.96
denomination of formula (1)	4.46	5.42	6.38	5.04	5.94	6.84	5.23	6.10	6.90	5.86	6.68	7.48
$\varepsilon_m \times 10^{-5}$	47.1	42.3	38.9	48.2	43.9	40.7	49.6	45.4	42.5	49.1	45.6	42.9
$0.9\ \varepsilon_m$	42.4	38.0	35.0	43.4	39.5	36.6	44.6	40.8	38.2	44.2	41.0	38.5
$0.85\ \varepsilon_m$	40.0	35.9	33.0	41.1	37.3	34.6	42.2	38.6	36.1	41.7	38.7	36.4
experimental	42.1	36.9	31.3	41.7	38.3	35.1	43.9	39.8	34.8	43.3	40.7	35.5

* The proportion of mixture is expressed by weight in the ratio of (cement): (total aggregates).