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ESTIMATION OF INCIPIENT ICE COVER FORMATION DATE OF RESERVOIRS IN HOKKAIDO BY USE OF A TIME SERIES OF DAILY ACCUMULATED AIR TEMPERATURE

Isao Yamaoka
Professor of Civil Engineering
Faculty of Engineering
Hokkaido University
Sapporo
Japan

SYNOPSIS

As a first step towards understanding the annual variation in date of incipient ice cover formation in relatively large reservoirs in Hokkaido, the author presents an estimation method of annual date above mentioned by use of a discrete time series of daily accumulated air temperature Da and the linearly correlated relation of surface water temperature Tw to Da.

The relation $Tw = \alpha Da + \beta$, was derived in an elementary manner based on the convection boundary condition of heat transfer problems. The constant α was obtained as $0.031 \left[hr^{-1}\right]$ by using five years data in 1964-1968 observed in two reservoirs in the middle part of Hokkaido. The constant α represents the quantity $Ah/mc\rho$ where h is the heat transfer coefficient.

RÉSUMÉ

Pour commencer à comprendre la variation annuelle de la date de la formation des champs de glace dans les réservoirs relativement larges en Hokkaido, Japon, l'auteur présente une méthode pour l'estimation de la date mentionée ci-dessus, employant la série du temps de la température atmosphérique accumulée dans un jour Da et la relation linéaire de la température superficielle de l'eau Tw à Da.

La relation $Tw = \alpha Da + \beta$ est dérivée de la manière élémentaire qui se base sur la condition à la frontière pour la convection dans le problème du transfert de la chaleur. La valeur de la constante α obtenue par l'observation dans les deux réservoirs au milieux de Hokkaido, de 1964 à 1968, est $0.031 \left[hr^{-1}\right]$ et représente la quantité $Ah/mc\rho$ où h est la coefficient du transfert de la chaleur.

INTRODUCTION

Hokkaido is the northern-most island of Japan and the northern limiting line for unfreezing lakes passes through the island as shown in Fig. 1. The western part of the island is relatively low land.

Then, most reservoirs created by multipurpose dams have ice covers in every winter (see Fig. 6) and have rather large annual variations in incipient ice cover formation date as an example of the Katsurazawa reservoir shown in Table 1. As the storage water level of most reservoirs in Hokkaido is operated to fall gradually in this season of water shortage, the understanding of annual variation in incipient ice cover formation is needed for designing dams and utilizing reservoirs. However, fully long-period data for the incipient ice cover formation are not available in most cases at present.

As a first step towards statistic studies in this field, the author derived an estimation method of the date in each winter by use of temperature data of the air which are available in most sites for sufficiently long periods. In this study, observed data on the date of ice cover formation and surface air and water temperatures in the Katsurazawa and Kanayama reservoirs (see Fig. 1) were very helpful.

MECHANISM OF ICE COVER FORMATION [1)]

After the reservoir water reaches an unstable isothermal condition a few degrees below the 4°C temperature (about at the end of November in these two reservoirs), the ice cover formation of a reservoir begins on a cold day or night, freezing begins in the surface. And there are two factors in the formation of ice cover. One is the freezing of the upper water layer itself and this produces a smooth homogeneous sheet ice. The other is the fusion of individual ice masses produced by the breakup and refreezing of an ice sheet in its early stage of development or by the snow in the surface water. And in this conditions agglomeritic ice which is rough on the surface and nontransparent is formed.

In the Katsurzawa reservoir, the surface ice formation begins from near the shore, then the complete ice cover surface is formed from seven to ten days later without any ice-free surface. The term incipient ice cover formation in this study means the date when the entire surface is covered with ice sheet or agglomeritic ice. And the last stage of ice cover formation is begun near the center of reservoir. Now, a very complicated mechanism of cover formation by the combination of two factors is considered, however, in this study only an elementary heat transfer model of sheet ice which is produced by rapid freezing of the surface-water film is adopted.

HEAT BUDGET [1)]

Analytical heat budget treats the rates of heat transfer of the several forms of radiant and thermal energy. The equation for the storage of heat Qt in the reservoir for practical use becomes

$$Q_t = Q_B \pm Q_s \pm Q_i - Q_E \tag{1}$$

Where Q_B is the net radiation surplus and Q_s is conduction of heat from the air when it is warmer than the water and transfer of heat from water to air, and Q_i is heat carried in by influent water, and Q_E is energy used in the evaporation process. The solution of an analytical heat budget requires data on the radiation flux, sufficiently detailed temperature series within the reservoir and in the air above the reservoir, and temperature of inflows and outflows in flow-through is significant. However, only heat budget near the reservoir surface is considered in this study and negligible terms are entirely neglected for simplicity. Then Eq. (1) becomes

$$Q_t = Q_s \tag{2}$$

HEAT TRANSFER FORMULATION [2)]

Heat energy is transfered between the air over the reservoir surface and the surface water which are at different temperatures T_a and T_w, respectively. In this case convection are considered as a mode of heat transfer and the convection boundary condition is used for this study. The heat flux across the reservoir surface may be taken as proportional to the difference between the surface water temperature T_w and the surface air temperature T_a. Eq. (2) then takes the form

$$\dot{Q}_s = hA\ (T_a - T_w) \tag{3}$$

where h is termed the (surface) heat transfer coefficient $[\mathrm{Kcal/m^2 hr\,^\circ C}]$.

On the other hand, when the thin layer portion of water (depth: d) near the surface of reservoir is considered and the temperature of this water mass decreases from $(T_w)_1$ to $(T_w)_2$ after time t(hr), the rate of change of heat energy is to be

$$\dot{Q}_s = mc\rho \frac{dT_w}{dt} \tag{4}$$

where m = Ad is mass $[m^3]$, C is the specific heat of water: 1 $[\mathrm{Kal/Kg\,^\circ C}]$, ρ is the density of the water: 1000 $[\mathrm{Kg/m^3}]$ and t is time (hr). From Eq. (3) and Eq. (4)

$$mc\rho \frac{dT_w}{dt} = Ah\ (T_a - T_w)$$

By integrating

$$mc\rho\ \left[T_w(t)\right]_0^{24} = hA \int_0^{24} T_a(t)dt - hA \int_0^{24} T_w(t)dt \tag{5}$$

Then

where

$$\left.\begin{aligned} &T_{w_{24}} = \alpha D_a + \beta \\ &\alpha = hA/mc\rho \qquad D_a = \int_0^{24} T_a(t)dt, \\ &\beta = \left[T_w(t)\right]_{t=0} - \alpha \int_0^{24} T_w(t)dt \end{aligned}\right\} \tag{6}$$

DETERMINATION OF α AND β BY OBSERVED DATA

To obtain the values of α and β in Eq. (6), records of surface water temperature (at the depth of 0.1 m) and water temperature of inflow and a discrete time series of daily accumulated surface air temperature (at the Dam Control Office) were studied in each year (1964-1968). One example of them is shown in Fig. 2. The daily accumulated temperature is an algebraic sum of hourly temperatures in degrees obtained from self-recording continuous series of the surface air temperature. Two results of the crosscorrelation analysis of $D_a(t)$ and $T_w(t)$ are shown in Fig. 3, representing that a couple of time series without shifting has better correlation.

The results of computing α and β in Eq. (6) by using data above-mentioned were shown in Table 2 with sufficiently satisfactory correlation coefficients. And α was found to be almost a constant value in these two reservoirs and in each year, including the similar h. β was also found to have almost a constant value for each year in the same reservoir. However, β has a little different values for two reservoirs, since β depends on decreasing features of the surface water temperature and so on. The relation of T_w to D_a in Eq. (6) in 1964 is shown in Fig. 4 as an example. Data from July 1 to the incipient date of ice cover formation are used. Thus, the linearity of Eq. (6) obtained by observed data gave practical values to this simplest heat transfer model Eq. (6).

D_a AND T_1 REQUIRED FOR ICE COVER FORMATION

Considering the transfer of heat energy required to change the surface water of T_1°C of the depth d(0.1 m, for example) and unit area, to the water of 0°C of the same volume and to convert a part of the water (d_i in depth) at the freezing temperature into ice at the same temperature, the following equation is derived. In this case 79.67 Kcal/Kg of latent heat of conversion must be deducted in Eq. (6)

$$-mc\rho T_1 - 80\, m'\rho = hA(D_a - \int_0^{24} T_w dt) \tag{7}$$

where $m = 1 \times 1 \times d\ [m^3]$: mass of water,

$m' = 1 \times 1 \times d_i\ [m^3]$: mass of water to be converted to ice,

$c = 1\ [Kcal/Kg°C]$: specific heat,

$\rho = 1000\ [Kg/m^3]$: density,

$$T_w = -\frac{T_1}{24} t + T_1$$

Hence $$D_w = \int_0^{24} T_w(t)dt = \left[-\frac{T_1}{48}t^2 + T_1 t\right]_0^{24} = 12\ T_1$$

Then by Eq. (7)

$$D_a = -\frac{mc\rho}{h} T_1 + 12T_1 - \frac{80m'\rho}{h} = -\left(\frac{1000d}{h} - 12\right)T_1 - \frac{80d_i\rho}{h} \tag{8}$$

From Table 2 and Eq. (6)

$$\alpha = \frac{Ah}{mc\rho} = 0.031$$

Hence $$h = 0.031\frac{mc\rho}{A}$$

In this case $A = 1\ [m^2]$, $m = Ad = d\ [m^3]$, $\rho = 1000\ [Kg/m^3]$

Hence $$h = 31d \tag{9}$$

For example $h = 3.1$ [Kcal/m^2 hr°C] for $d = 0.1$ (m)

From Eq. (8) and Eq. (9)

$$D_a \fallingdotseq -20\ T_1 - 2580\frac{di}{d} \qquad (10)$$

This relation of D_a and T_1 is shown in Fig. 5 with five plots of incipient ice cover formation data in the Katsurazawa reservoir (1964-1968). $T_1 - D_a$ diagram in Fig. 5 indicates mutual conditions required for a complete ice cover to be formed on the date. T_w of the date when ice cover formation is expected if only D_a becomes lower than some standard value, namely, T_1 may also be estimated by Fig. 5. D_m for T_1, is obtained as D_a for T_w in Fig. 4.

The thickness of ice formation d_i may be estimated as 0.003-0.009 (m) for $d = 0.1$ (m) in the region which covers five observed data. As this thickness means the last part to be closed with ice cover, the thickness of ice cover in most parts of the surface is to be much more.

PROCEDURE OF ESTIMATION

An estimation of incipient ice cover formation in the Katsurazawa reservoir in 1964 is shown for example (see Fig. 2). First, an average decreasing curve for D_a series is drawn by use of moving average of ten days or by a free hand method in order to know D_m value. And D_m becomes lower than -100 [°C hr] on Dec. 7 ($T_1 = 3$°C). Daily D_a after the date in its series is checked and D_a becomes lower then -140 [°C hr] on Dec. 9. And this date is estimated as the incipient formation date. This result is coincided with the observed one. The results of estimation for five years are relatively good with errors within four days.

ACKNOWLEDGMENTS

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REFERENCES

1) J. H. Zumberge, and J. C. Ayers: Hydrology of lakes and swamps, section 23 of the Handbook of applied hydrology, edited by V. T. Chow, Mc-Graw Hill Book Company, 1964.

2) J. G. Knudsen and D. L. Katz: Fluid Dynamics and Heat Transfer, Mc-Graw Hill Book Co., 1958.

Table 1 Observed date on ice cover in two reservoirs
(After the Katsurazawa Dam Control Office,
Hokkaido Development Bureau)

Name	Observed date on ice cover			Significant data for reservoirs
	Year	Formation, no ice-free surface	Breakup, no ice cover	
Katsura-zawa	1958 -1959	Dec. 28	Apr. 8	Drainage area: 299 sq. Km Surface area: 4.99 sq. Km Total capacity: 92.7 million m^3 Planned inflow: (Dec. 1-5) 5.75 m^3/sec Max. water level: 187 m Dam height: 63.6 m (concrete gravity) Dam completed in 1957
	1959 -1960	Dec. 18	Apr. 26	
	1960 -1961	Dec. 18	Apr. 24	
	1961 -1962	Jan. 4	Apr. 16	
	1962 -1963	Dec. 27	Apr. 14	
	1963 -1964	Dec. 28	Apr. 15	Drainage area: 470 sq. Km Surface area: 9.2 sq. Km Total capacity: 150.45 million m^3 Planned inflow: 8.72 m^3/sec (Dec. 1-5) Max. water level: 345 m Dam height: 59.7 m (Concrete hollow gravity) Dam completed in 1967
	1964 -1965	Dec. 9	May 8	
	1965 -1966	Dec. 17	Apr. 26	
	1966 -1967	Dec. 13	Apr. 23	
	1967 -1968	Dec. 13	Apr. 11	
	1968 -1969	Dec. 31	Apr. 24	
	1969 -1970	Dec. 16	May 4	
Kanaya-ma	1968 -1969	Dec. 31		

Table 2

Name	$T_{W_{24}} = \alpha D_a + \beta$ Eq.(6)			
	Used data year	α	β	Correlation coefficient
Katsurazawa reservoir	1964	0.032	5.702	0.970
	1965	0.031	5.874	0.957
	1966	0.029	6.640	0.964
	1967	0.031	6.220	0.967
	1968	0.032	6.140	0.951
	Including 1964-1968	0.031	6.101	0.962
Kanayama reservoir	1968	0.031	7.079	0.938

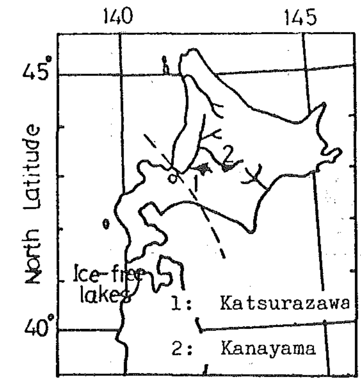

Fig. 1 Map of Hokkaido, Japan showing the observed reservoirs and northern limiting line for non-freezing lakes (After Yoshimura, 1933).

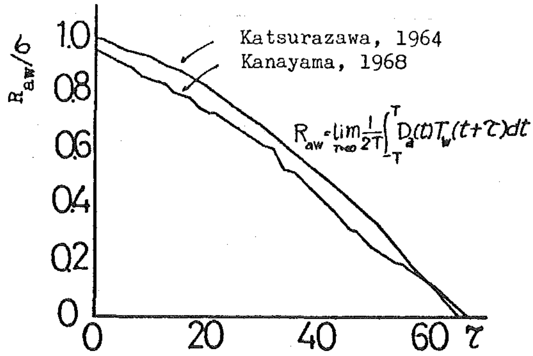

Fig. 3 Cross-correlation of T_w and D_a.

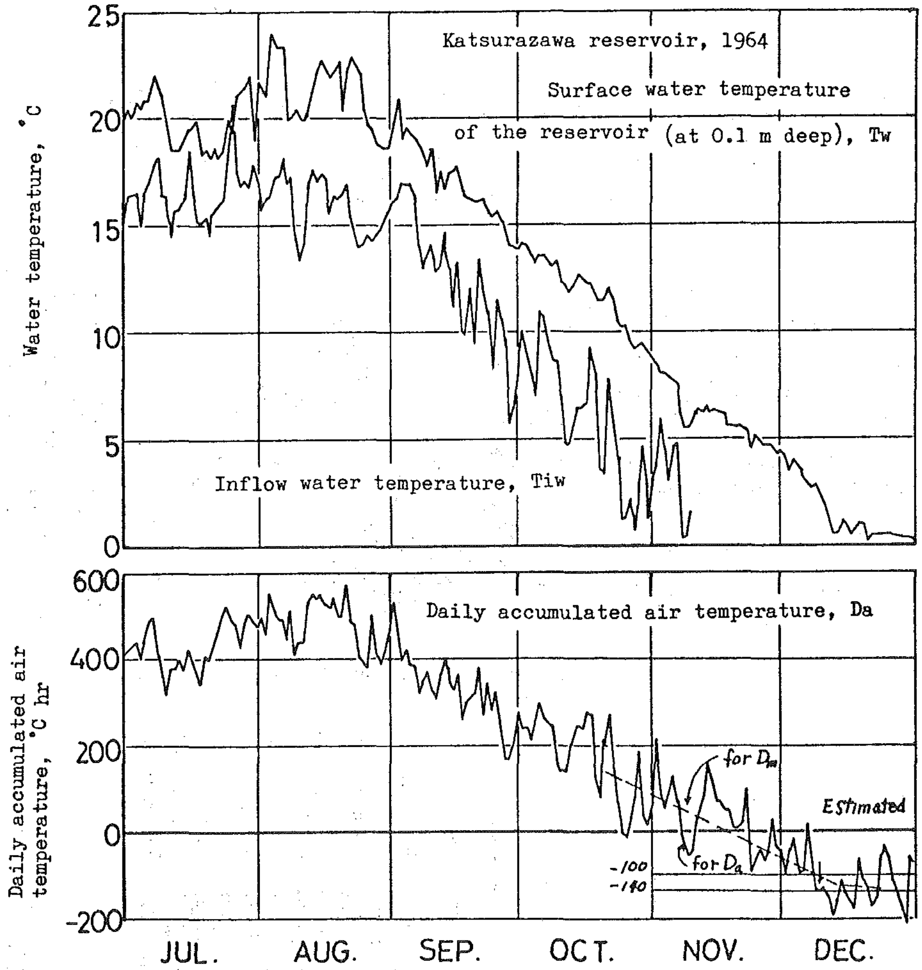

Fig. 2 Water temperature records and a time series of daily accumulated air temperature in 1964 as an example.

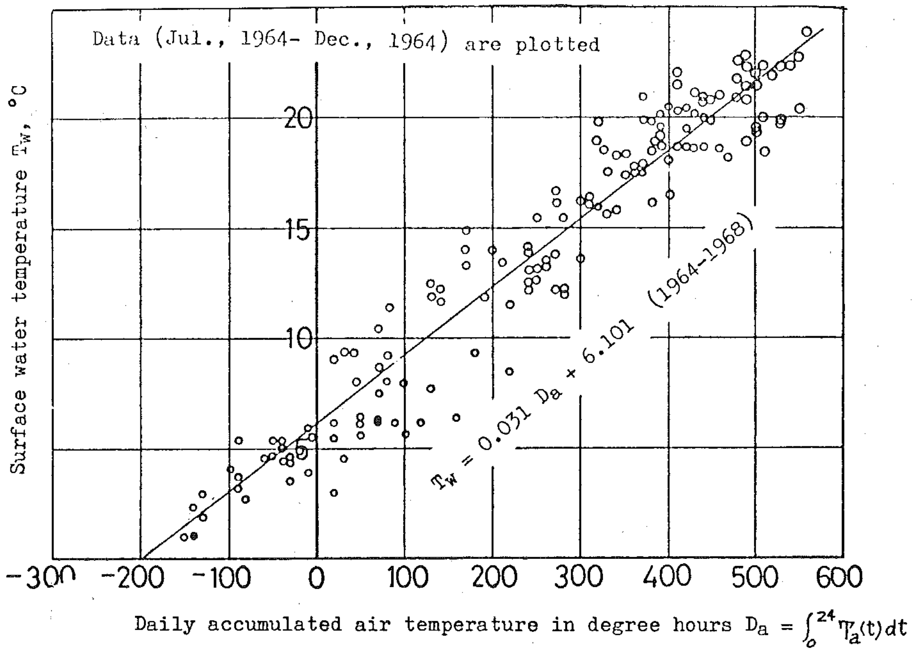

Fig. 4 Relation of surface water temperature to daily accumulated air temperature .

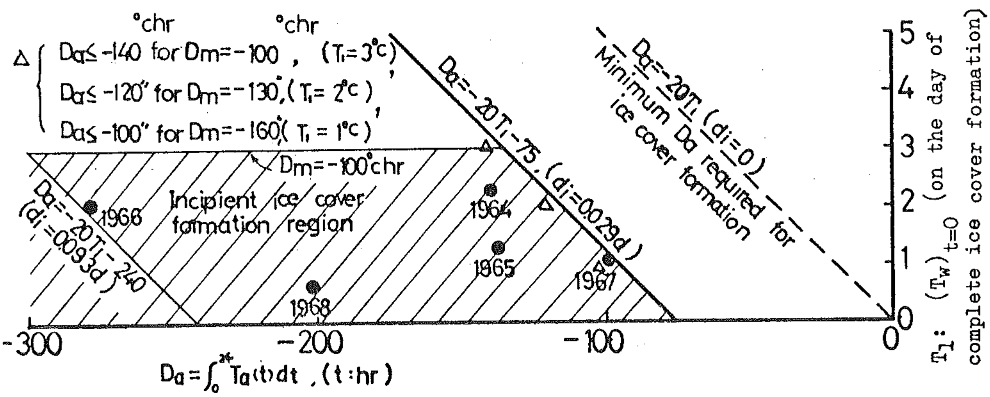

Fig. 5 T_1 - D_a diagram for incipient ice cover formation showing Katsurazawa data (1964-1968).

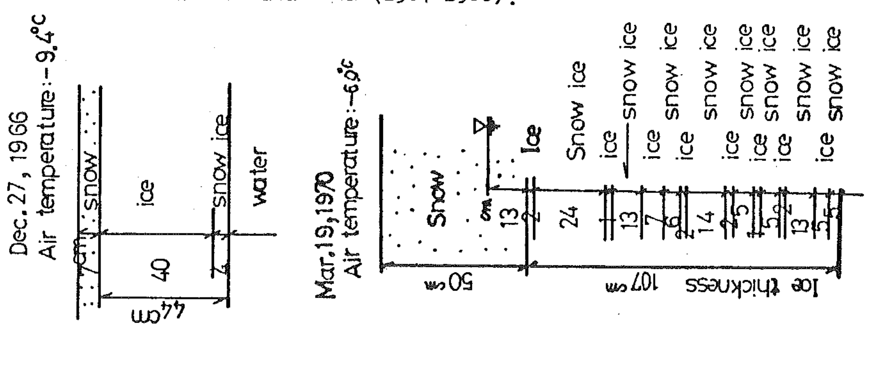

Fig. 6 Developed ice cover profile as examples

On paper by Yamaoka, I. :

DISCUSSION by Tsang, G.

I would like to see some physical argument supporting Prof. Yamaoka's assumption of d, the depth of the affected water layer, equal to 0.1(m). Otherwise the physical and mathematical refinement in the first half of his paper may be tarnished.

DISCUSSION by Yamaoka, I.

I thank Mr. Tsang for his discussion on my paper. My assumption of d to be 0.1(m) as an example is a trial (on the convenience of my having long-term data observed at the depth of 0.1(m) in this case) taking into account that d should be fairly small as I adopt an elementary heat transfer model of ice sheet produced by rapid freezing of the surface-water film.

In order to obtain a further knowledge for assuming appropriate d, some typical water-temperature gradients for surface-temperature variation from T_1 °c to 0 °c were studied by computing the solution of the thermal diffusion equation (or Fourier heat conduction equation) with κ (thermal diffusivity) of 0.000472 (m^2/hr). The convection flow is neglected in this equation. Computed gradients for various values of T_1 show almost linear variations in temperature from the water-surface (0 °c) to depths of d(T_1 °c at the depth lower than d) for the values of d which are less than or approximately equal to 0.1, 0.2, 0.4 and 0.5(m) for T_1 of 0.6, 2, 5, and 10 °c, respectively. In my data, initial water temperatures (at the depth of 0.1(m)) on the beginning date of freeze-up T_1 are scattered between 0.6 and 2 °c as shown in Fig.5. Hence, d should be less than or equal to 0.1-0.2(m). Thus, the assumption d = 0.1(m) may unobjectionably be supported for practical problems. (In my paper the transfer of heat energy required to change the surface water of T_1 °c of the depth d and unit area to the water of 0 °c of the same volume is considered schematically. And the heat energy transferable to the air may be considered the same even in the practical case described in this comments as the surface-water temperature (in Fig.2) has daily periodical variation effected by the air temperature and T_1 may be assumed as the mean value of $2T_1$ and 0 °c for T_1 of low temperature such as 0.6-2 °c.

Namely, $-\frac{1}{2}2T_1 dAc\rho = -mc\rho T_1$)