On the Vibration of Continuous Rectangular Plates

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Synopsis

The author presents the frequency equations of free vibration of such continuous plates as are illustrated in Fig. 1, and gives some numerical illustrations. The present continuous plates are of two types:—one is simply supported by intermediate supports placed at equal intervals and the other is composed of elementary plates joined by means of perfectly smooth hinges. Such two types of plates can be treated in the same manner.

1. Conditions of Continuity

The free vibrations in the portion of r-th span are characterized by the well-known equation

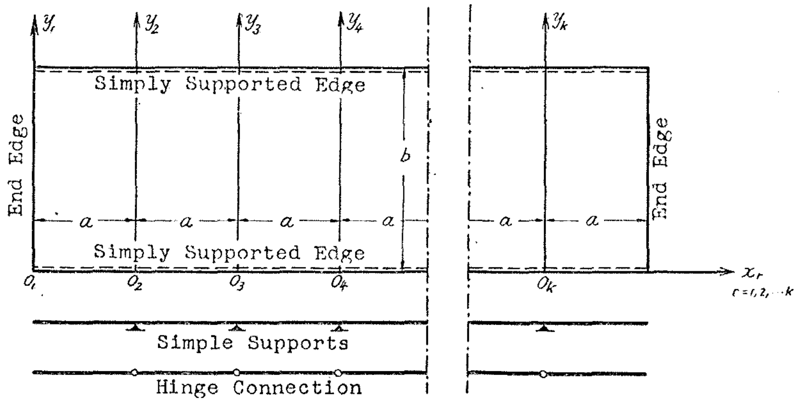

Fig. 1

$$\varDelta\varDelta w_r + \frac{\rho h}{D}\frac{\partial^2 w_r}{\partial t^2} = 0 \ , \tag{1}$$

where w_r=instantaneous deflection in the r-th span, h=thickness of plate, ρ=density of material, D=flexural rigidity of plate$=\frac{Eh^3}{12(1-\nu^2)}$, E=Young's modulus, ν=Poisson's ratio, t=time co-ordinate. Now, the free vibration can be assumed in the following form:

$$w_r = w_r' \cos \omega t \ ,$$

in which w_r' denotes shape function, i.e., normal function, ω the circular frequencies. Then, substitution in Eq. (1) gives

$$\varDelta\varDelta w_r' - \frac{\omega^2 \rho h}{D} w_r' = 0 \ .$$

Such a solution of this equation as satisfies the boundary conditions, i.e., $w_r'=0$, bending moments $=0$, at $y_r=0$ and $y_r=b$, is given by the following expression as well known:

$$\left.\begin{aligned} w_r' &= X_n(\xi_r)\sin n\pi\eta_r\ , \qquad \left(\xi_r=\frac{x_r}{a}\ ,\quad \eta_r=\frac{y_r}{b}\right) \\ X_n(\xi_r) &= K_n\cosh\pi\lambda_n\xi_r + L_n\sinh\pi\lambda_n\xi_r + M_n\cosh\pi\lambda_n'\xi_r + N_n\sinh\pi\lambda_n'\xi_r\ , \\ \lambda_n &= \sqrt{\left(\frac{a}{b}\right)^2 n^2+\mu}\ , \qquad \lambda_n' = \sqrt{\left(\frac{a}{b}\right)^2 n^2-\mu}\ , \\ \mu &= \frac{\omega a^2}{\pi^2}\sqrt{\frac{\rho h}{D}}\ , \qquad \text{(dimensionless quantity)} \end{aligned}\right\} \tag{2}$$

where K_n, L_n, M_n, N_n are integration constants. Next, the conditions of continuity between any two adjacent spans are written respectively for the two types of plates as follows:[1)*]

For the case of the continuous plate simply supported by multiple spans

$$|w_{r-1}'|_{\xi_{r-1}=1}=0\ ,\quad |w_r'|_{\xi_r=0}=0\ ,\quad \left|\frac{\partial w_{r-1}'}{\partial x_{r-1}}\right|_{\xi_{r-1}=1}=\left|\frac{\partial w_r'}{\partial x_r}\right|_{\xi_r=0}\ ,\quad \left|\frac{\partial^2 w_{r-1}'}{\partial x_{r-1}^2}+\nu\frac{\partial^2 w_{r-1}'}{\partial y_{r-1}^2}\right|_{\xi_{r-1}=1}=\left|\frac{\partial^2 w_r'}{\partial x_r^2}+\nu\frac{\partial^2 w_r'}{\partial y_r^2}\right|_{\xi_r=0}.$$

For the case of the plate composed by hinge connection

$$|w_{r-1}'|_{\xi_{r-1}=1}=|w_r'|_{\xi_r=0}\ ,\quad \left|\frac{\partial^2 w_{r-1}}{\partial x_{r-1}^2}+\nu\frac{\partial^2 w_{r-1}'}{\partial y_{r-1}^2}\right|_{\xi_{r-1}=1}=0\ ,\quad \left|\frac{\partial^2 w_r'}{\partial x_r^2}+\nu\frac{\partial^2 w_r'}{\partial y_r^2}\right|_{\xi_r=0}=0\ ,$$

$$\left|\frac{\partial^3 w_{r-1}'}{\partial x_{r-1}^3}+(2-\nu)\frac{\partial^3 w_{r-1}'}{\partial x_{r-1}\,\partial y_{r-1}^2}\right|_{\xi_{r-1}=1}=\left|\frac{\partial^3 w_r'}{\partial x_r^3}+(2-\nu)\frac{\partial^3 w_r'}{\partial x_r\,\partial y_r^2}\right|_{\xi_r=0}.$$

Substituting solution (2) in the above conditions and arranging them, four equations are obtained to represent the relations among integration constants, that is, for the former case

$$\begin{aligned} K_{r-1}\cosh\pi\lambda_1+L_{r-1}\sinh\pi\lambda_1+M_{r-1}\cosh\pi\lambda_1'+N_{r-1}\sinh\pi\lambda_1' &= 0\ , \\ K_r+M_r &= 0\ , \\ K_{r-1}\lambda_1\sinh\pi\lambda_1+L_{r-1}\lambda_1\cosh\pi\lambda_1+M_{r-1}\lambda_1'\sinh\pi\lambda_1'+N_{r-1}\lambda_1'\cosh\pi\lambda_1'-L_r\lambda_1-N_r\lambda_1' &= 0\ , \\ K_{r-1}\beta_1\cosh\pi\lambda_1+L_{r-1}\beta_1\sinh\pi\lambda_1+M_{r-1}\beta_1'\cosh\pi\lambda_1'+N_{r-1}\beta_1'\sinh\pi\lambda_1'-K_r\beta_1-M_r\beta_1' &= 0\ , \end{aligned}$$

where $n=1$ has been taken for the fundamental mode of vibration, and the following notations are used for shortness:

$$\beta_1=\lambda_1^2-\nu\frac{n^2a^2}{b^2}\ ,\qquad \beta_1'=\lambda_1'^2-\nu\frac{n^2a^2}{b^2}\ .$$

Taking $B_r=K_r=-M_r$ by the second equation, the denotation $B_{r-1}=K_{r-1}=-M_{r-1}$ is possible. Then the remaining three equations can be rewritten as follows:

$$\begin{aligned} B_{r-1}(\cosh\pi\lambda_1-\cosh\pi\lambda_1')+L_{r-1}\sinh\pi\lambda_1+N_{r-1}\sinh\pi\lambda_1' &= 0\ , \\ B_{r-1}(\lambda_1\sinh\pi\lambda_1-\lambda_1'\sinh\pi\lambda_1')+L_{r-1}\lambda_1\cosh\pi\lambda_1+N_{r-1}\lambda_1'\cosh\pi\lambda_1'-L_r\lambda_1-N_r\lambda_1' &= 0\ , \\ B_{r-1}(\beta_1\cosh\pi\lambda_1-\beta_1'\cosh\pi\lambda_1')+L_{r-1}\beta_1\sinh\pi\lambda_1+N_{r-1}\beta_1'\sinh\pi\lambda_1'-B_r(\beta_1-\beta_1') &= 0\ . \end{aligned}$$

From the first equation and the third in the above, the following expressions are obtained:

$$\left.\begin{aligned} L_{r-1} &= -B_{r-1}\coth\pi\lambda_1+B_r\,\mathrm{cosech}\,\pi\lambda_1 \\ N_{r-1} &= B_{r-1}\coth\pi\lambda_1'-B_r\,\mathrm{cosech}\,\pi\lambda_1' \end{aligned}\right\} \tag{3}$$

* Numbers in parenthesis refer to the Bibliography at the end of the paper.

By substituting these into the second equation

$$B_{r-1}\, S - B_r\, T + A_r = 0$$

where

$$S = \lambda \operatorname{cosech} \pi\lambda_1 - \lambda_1' \operatorname{cosech} \pi\lambda_1' , \quad T = \lambda_1 \coth \pi\lambda_1 - \lambda_1' \coth \pi\lambda_1' , \quad A_r = L_r\lambda_1 + N_r\lambda_1' .$$

Using the expressions obtained from (3) by substitution of the suffix r instead of $r-1$,

$$A_r = L_r\lambda_1 + N_r\lambda_1' = - B_r T + B_{r+1}\, S$$

is obtained, then the above equation is transformed into

$$B_{r-1} - 2B_r \frac{T}{S} + B_{r+1} = 0 \tag{4}$$

Again, for the latter case, the following equation can be deduced in the same way:

$$B_{r-1} - 2B_r \frac{T'}{S'} + B_{r+1} = 0 \tag{5}$$

where

$$S' = \frac{\gamma_1}{\beta_1} \operatorname{cosech} \pi\lambda_1 - \frac{\gamma_1'}{\beta_1'} \operatorname{cosech} \pi\lambda_1' , \qquad T' = \frac{\gamma_1}{\beta_1} \coth \pi\lambda_1 - \frac{\gamma_1'}{\beta_1'} \coth \pi\lambda_1' ,$$

$$\gamma_1 = \lambda_1 \left\{ \lambda_1^2 - (2-\nu) \frac{n^2a^2}{b^2} \right\} , \qquad \gamma_1' = \lambda_1' \left\{ \lambda_1'^2 - (2-\nu) \frac{n^2a^2}{b^2} \right\} ,$$

The equations above obtained (4), (5) are nothing but the finite differences equations of second order; the solutions, therefore, are obtained at once as follows:

$$\left.\begin{aligned}
&B_r = C_1 \sin r\alpha + C_2 \cos r\alpha \ , \\
&\text{where} \\
&\cos\alpha = \frac{T}{S} \qquad \text{for the former case,} \\
&\cos\alpha = \frac{T'}{S'} \qquad \text{for the latter case,} \\
&C_1 ,\ C_2 = \text{unknown constants.}
\end{aligned}\right\} \tag{6}$$

2. Conditions of the End Edges

Let the number of spans or elementary plates be k. Then with the substitutions $\xi_r = 0$ for $r=1$ and $\xi_r=1$ for $r=k$, the following expressions hold as the conditions of the end edges, viz.,[2]

$$\left.\begin{aligned}
&\text{when the end edges are simply supported,} \\
&w_r' = 0 , \qquad \frac{\partial^2 w_r'}{\partial x_r^2} + \nu \frac{\partial^2 w_r'}{\partial y_r^2} = 0 , \\
&\text{when they are clamped,} \\
&w_r' = 0 , \qquad \frac{\partial w_r'}{\partial x_r} = 0 , \\
&\text{when they are free,} \\
&\frac{\partial^2 w_r'}{\partial x_r^2} + \nu \frac{\partial^2 w_r'}{\partial y_r^2} = 0 , \qquad \frac{\partial^3 w_r'}{\partial x_r^3} + (2-\nu) \frac{\partial^3 w_r'}{\partial x_r\, \partial y_r^2} = 0 ,
\end{aligned}\right\} \tag{7}$$

where $r=1$ or $r=k$. Substitute (2) into these conditions (7), and refer to (3) and (6), then further calculations give the following equations concerning end edges:

For the former case,[3] i.e., the case of the continuous plate simply supported by multiple spans, if the end edges are simply supported,

$$C_1 \sin(k+1)\alpha + C_2 \cos(k+1)\alpha = 0 \qquad \text{for} \quad \xi_k = 1 ,$$
$$C_1 \sin \alpha + C_2 \cos \alpha = 0 \qquad \text{for} \quad \xi_1 = 0 ,$$

when they are clamped,

$$C_1 \cos(k+1)\alpha - C_2 \sin(k+1)\alpha = 0 \qquad \text{for} \quad \xi_k = 1 ,$$
$$C_1 \cos \alpha - C_2 \sin \alpha = 0 \qquad \text{for} \quad \xi_1 = 0 ,$$

when they are free,

$$C_1 \left\{\sin(k+1)\alpha - \frac{\bar{S}S''}{ST'} \sin k\alpha\right\} + C_2 \left\{\cos(k+1)\alpha - \frac{\bar{S}S''}{ST'} \cos k\alpha\right\} = 0 \quad \text{for} \quad \xi_k = 1 ,$$
$$C_1 \left\{\sin \alpha - \frac{\bar{S}S''}{ST'} \sin 2\alpha\right\} + C_2 \left\{\cos \alpha - \frac{\bar{S}S''}{ST'} \cos 2\alpha\right\} = 0 \qquad \text{for} \quad \xi_1 = 0 .$$

For the latter case,[4] i.e., the case of the plate constructed wifh hinge connections, if the end edges are simply supported,

$$C_1 \sin(k+1)\alpha + C_2 \cos(k+1)\alpha = 0 \qquad \text{for} \quad \xi_k = 1 ,$$
$$C_1 \sin \alpha + C_2 \cos \alpha = 0 \qquad \text{for} \quad \xi_1 = 0 ,$$

when they are clamped,

$$C_1 \left\{\sin(k+1)\alpha - \frac{\bar{S}S''}{S'T} \sin k\alpha\right\} + C_2 \left\{\cos(k+1)\alpha - \frac{\bar{S}S''}{S'T} \cos k\alpha\right\} = 0 \quad \text{for} \quad \xi_k = 1 ,$$
$$C_1 \left\{\sin \alpha - \frac{\bar{S}S''}{S'T} \sin 2\alpha\right\} + C_2 \left\{\cos \alpha - \frac{\bar{S}S''}{S'T} \cos 2\alpha\right\} = 0 \qquad \text{for} \quad \xi_1 = 0 ,$$

when they are free,

$$C_1 \cos(k+1)\alpha - C_2 \sin(k+1)\alpha = 0 \qquad \text{for} \quad \xi_k = 1 ,$$
$$C_1 \cos \alpha - C_2 \sin \alpha = 0 \qquad \text{for} \quad \xi_1 = 0 ,$$

where

$$\bar{S} = \frac{\lambda_1}{\beta_1} \operatorname{cosech} \pi\lambda_1 - \frac{\lambda_2}{\beta_2} \operatorname{cosech} \pi\lambda_2 , \qquad S'' = \gamma_1 \operatorname{cosech} \pi\lambda_1 - \gamma_2 \operatorname{cosech} \pi\lambda_2 .$$

3. Frequency Equations

Combining the expressions for $\xi_k=1$ and $\xi_1=0$ which are obtained in the previous section, the frequency equations corresponding to various kinds of end edge conditions can be reduced by eliminating the constants C_1, C_2. Thus, the formulas for each case are tabulated in Table 1.

4. Numerical Examples

Through the following examples, the data $\frac{a}{b}=\frac{1}{2}$ $\nu=0.3$ are assumed and $n=1$ is taken as before stated.

The case of the continuous plate simply supported by multiple spans: 1) The case when both end edges are simply supported. In this case, we can suppose that the portion of each span vibrates as a single plate simply supported along all edges. Then, $\mu=1+\frac{a^2}{b^2}=1.25$[5] is obtained regardless of k.

Table. 1. Frequency Equations

Common part of formulas for both kinds of plates	Complementary part of formulas corresponding to end edge conditions: For the continuous plate simply supported by multiple spans	Complementary part of formulas corresponding to end edge conditions: For the plate constituted with hinge connection
$\cos s\frac{\pi}{k}=\delta$ $s=0, 1, 2, \cdots, 2k-1$	$\delta=\frac{T}{S}$ when both end edges are simply supported	$\delta=\frac{T'}{S'}$ when both end edges are simply supported
$\cos s\frac{\pi}{k}=\delta$ $s=0, 1, 2, \cdots, 2k-1$	$\delta=\frac{T}{S}$ when both end edges are clamped	$\delta=\frac{T'}{S'}$ when both end edges are free
$\cos\left(s+\frac{1}{2}\right)\frac{\pi}{k}=\delta$ $s=0, 1, 2, \cdots, 2k-1$	$\delta=\frac{T}{S}$ when one end edge is simply supported and the other clamped	$\delta=\frac{T'}{S'}$ when one end edge is simply supported and the other free
$\cos\alpha\frac{\sin(k-1)\alpha}{\sin k\alpha}=\frac{TT'}{SS'}$	$\cos\alpha=\frac{T}{S}$ when one end edge is simply supported and the other free	$\cos\alpha=\frac{T'}{S'}$ when one end edge is simply supported and the other clamped
$\cos\alpha\frac{\cos(k-1)\alpha}{\cos k\alpha}=\frac{TT'}{SS'}$	$\cos\alpha=\frac{T}{S}$ when one end edge is clamped and the other free	$\cos\alpha=\frac{T'}{S'}$ when one end edge is free and the other clamped
$\frac{\cos\alpha\sin(k-2)\alpha}{\sin(k-1)\alpha\pm\sin\alpha}=\frac{TT'}{SS'}$	$\cos\alpha=\frac{T}{S}$ when both end edges are free	$\cos\alpha=\frac{T'}{S'}$ when both end edges are clamped

2) The case when both end edges are clamped. When $k=1$ (the case of a single plate), the result $\mu=2.413$[6)] is already known. Next, when $k=\infty$, $\mu=1.25$ can be supposed because of decay of the effects at the end edges. Thus, the inequalities $2.413<\mu<1.25$ can be established when $1<k$. Now, the expression $\frac{T}{S}$ is a monotonic increasing function of μ in the interval above stated (Fig. 2); then the frequency equation gives the least value of μ when $s=k$, but such a value of μ is trivial for the present case because it corresponds to the previous case. The next case $s=k-1$ must, therefore, be taken. From this, the formula is transformed into

$$\cos(k-1)\frac{\pi}{k}=-\cos\frac{\pi}{k}=\frac{T}{S}$$

Whence, by referring to Fig. 2, the results shown in Table 2 are obtained.

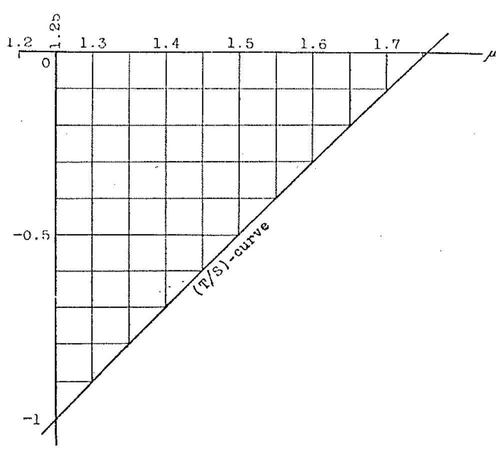

Fig. 2

Table 2

k	$-\cos\frac{\pi}{k}$	μ
2	-0	1.755
3	-0.5	1.498
4	-0.707	1.395
5	-0.809	1.345
6	-0.866	1.317
∞	-1	1.250

3) The case when one end edge is simply supported and the other clamped. It is already known that $\mu=1.756$[7] for $k=1$ and $\mu=1.25$ for $k=\infty$. Then it is coucluded that $1.756<\mu<1.25$ foı $1<k<\infty$. Substituting $s=k-1$ in the formula, the following equation is obtained:

$$\cos\left(s+\frac{1}{2}\right)\frac{\pi}{k}=-\cos\frac{\pi}{2k}=\frac{T}{S}$$

From this equatian, the results shown in Table 3 are obtained by referring to Fig. 2.

Table. 3

k	$-\cos\frac{\pi}{2k}$	μ
2	−0.707	1.395
3	−0.866	1.317
4	−0.923	1.288
5	−0.951	1.275
6	−0.965	1.268
∞	−1	1.250

4) The case when one end edge is simply supported and the other free. The result $\mu=\frac{1}{4}\times 1.6347$[8]$=0.4087$ for $k=1$ is already known. Next, for $k=\infty$, considering that $\alpha=i\alpha'$ (i=imaginary unit) and referring to (6), the following relation is obtained:

$$\lim_{k\to\infty}\cos\alpha\frac{\sin(k-1)\alpha}{\sin k\alpha}=\cosh^2\alpha'-\cosh\alpha'\sinh\alpha'=\left(\frac{T}{S}\right)^2\pm\left(\frac{T}{S}\right)\sqrt{\left(\frac{T}{S}\right)^2-1}\ ,$$

where

$$\cos\alpha=\cosh\alpha'=\frac{T}{S}\ .$$

The frequency equation for this case can, therefore, be rewritten as follows:

$$\frac{TT'}{SS''}=\left(\frac{T}{S}\right)^2\pm\left(\frac{T}{S}\right)\sqrt{\left(\frac{T}{S}\right)^2-1}\ . \tag{8}$$

For $k=2$, the following relation holds:

$$\cos\alpha\frac{\sin(k-1)\alpha}{\sin k\alpha}=0.5$$

Then, the frequency equation becomes in this case

$$0.5=\frac{TT'}{SS''} \tag{9}$$

Using Eqs. (8), (9), the values of μ for $k=\infty$ and $k=2$ can be found by observing the intersection points of the curves which represent respectively the values of each side of those equations. Namely, $\mu=0.492$ by Eq. (8) and $\mu=0.488$ by Eq. (9) as shown in Fig. 3. Thus, it can be supposed that $0.488<\mu<0.492$ if $2<k<\infty$. From this fact, the conclusion follows that μ is almost constant, viz., $\mu\fallingdotseq 0.49$ if $2<k$.

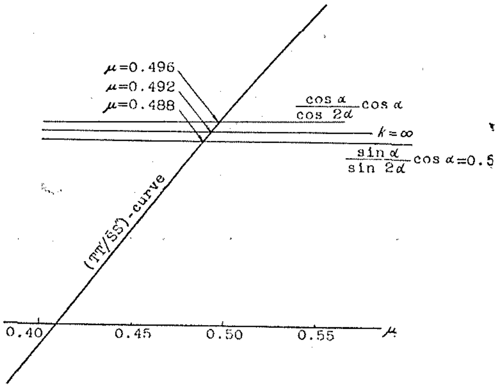

Fig. 3

5) The case when one end edge is clamped and the other free. When $k=\infty$, the following relation holds:

$$\lim_{k\to\infty}\cos\alpha\frac{\cos(k-1)\alpha}{\cos k\alpha}=\left(\frac{T}{S}\right)^2\pm\left(\frac{T}{S}\right)\sqrt{\left(\frac{T}{S}\right)^2-1}\ .$$

so that the frequency equation (8) is applicable to this case and $\mu=0.492$ is obtained. Next, for $k=2$, the following relation holds by referring to (6):

$$\cos\alpha\frac{\cos\alpha}{\cos 2\alpha}=\cos^2\alpha\frac{1}{2\cos^2\alpha-1}=\left(\frac{T}{S}\right)^2\frac{1}{2\left(\frac{T}{S}\right)^2-1}.$$

Using this expression, the left side of the formula for this case can be plotted as the function of μ. Thus $\mu=0.496$ is obtained by observing the intersection point at which the curve above stated and $\frac{TT'}{SS'}$-curve cross. Consequently it can be considered that $0.496>\mu>0.492$ if $2<k<\infty$. Therefore, again, μ is almost constant, i.e., $\mu\fallingdotseq 0.49$ when $2<k$.

6) The case when both end edges are free. If $k=2k'$, the following relations hold:

$$\frac{\cos\alpha\sin(k-2)\alpha}{\sin(k-1)\alpha\pm\sin\alpha}=\begin{cases}\dfrac{\cos\alpha\sin(k'-1)\alpha}{\sin k'\alpha} & \text{for the upper sign in the left member,}\\[2ex] \dfrac{\cos\alpha\cos(k'-1)\alpha}{\cos k'\alpha} & \text{for the lower sign in the left member.}\end{cases}$$

Accordingly, the formula for this case coincides with that for case 4) or case 5), if k is an even number. That is to say, the former relation corresponds to the antisymmetric mode of vibration with respect to the center line perpendicular to x-axis, and the latter relation to the symmetric mode. Further, when $k=\infty$, $\mu=0.492$ is obtained because it can be proved that the present formula coincides with (8). Thus the inequalities $0.488<\mu<0.496$ are concluded if $4<k$. From this, we find that μ is almost constant, i.e., $\mu\fallingdotseq 0.49$ if $4<k$.

The case of the plate composed of elementary plates with hinge connections: 1) The case when both end edges are simply supported. First, supposing the extreme case that $k=\infty$, $\mu=0.241$[9] results because it can be considered that each span comes to vibrate like a single plate which is free along both the end edges. In the vicinity of such a value of μ, the expression $\frac{T'}{S'}$ represents a monotonic decreasing function of μ (Fig. 4). Then, to maximize the left member of the formula is demanded for the least value of μ which corresponds to the fundamental mode; however taking $s=0$ gives a trivial value of μ, viz., 0.241 for this case, because such a value of μ corresponds to the case when both end edges are free. Then it must be taken that $s=1$. Thus the formula, i.e., the frequency equation, becomes

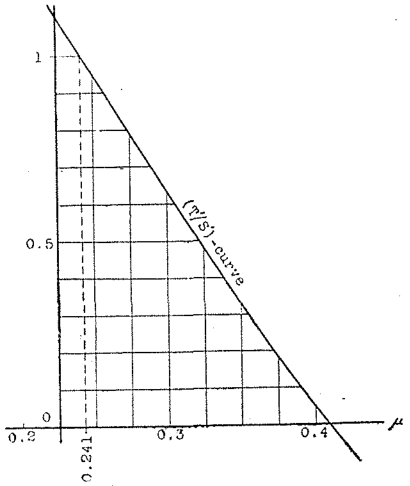

Fig. 4

$$\cos\frac{\pi}{k}=\frac{T'}{S'}$$

The numerical results as shown in Table 4, therefore, are obtained.

2) The case when both end edges are free. Supposing that each span will vibrate as a single plate which is free along both end edges, it can be concluded that $\mu=0.241$.

3) The case when one end edge is simply supported and the other free. Taking $s=0$, the formula is rewritten as $\cos\frac{\pi}{2k}=\frac{T'}{S'}$. Then, by observing $\frac{T'}{S'}$-curve (Fig. 4), the numerical results in Table 5 are obtained.

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k	$\cos\frac{\pi}{k}$	μ
2	0	0.408
3	0.5	0.322
4	0.707	0.288
5	0.809	0.272
6	0.866	0.263
∞	1	0.241

Table 5

k	$\cos\frac{\pi}{2k}$	μ
2	0.707	0.288
3	0.866	0.263
4	0.923	0.254
5	0.951	0.249
6	0.965	0.247
∞	1	0.241

4) The case when one end edge is simply supported and the other clamped. Suppose that $k=2$, then the formula becomes $0.5=TT'/SS''$, which is nothing but Eq. (9). So, $\mu=0.488$ results also in this case. And, since it may be supposed that $\mu=0.241$ when $k=\infty$, the following inequalities can, therefore, be established, if $2<k<\infty$:

$$0.488>\mu>0.241$$

At this interval, the intersection points of two curves which represent respectively each side of the formula must give the values of μ which correspond to such k as is larger than 2.

Concerning the remaining cases: 5) The case when one end edge is free and the other clamped, 6) The case when both end edges are clamped, the procedures can also be carried through in the similar way.

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6) ibid., p. 328.

7) ibid., p. 324.

8) It can be considered that this case of vibration corresponds to the antisymmetric mode of vibration of the plate having two free opposite end edges. See paper by S. Iguchi: "On the Lateral Vibration of a Rectangular Plate with Pairs of Simply Supported and Perfectly Free Two Opposite Edges," Jour. of the Jap. Soc. of Civil Eng., vol. 26, p. 431, Table 1., 1940.

9) ibid., p. 432, Table 2.