A Study of Traffic Assignment to Arterial Street Network

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Abstract

In this paper, two problems were considered.

One is the method of solution for arterial street network capacity and the other is the traffic assignment method that the inter-zone trips are assigned to a network of arterial streets.

We applied the network flow theory and the discrete maximum principle in order to formulate the traffic assignment process.

According to a combination of two methods, we applied them to the traffic assignment of the arterial street network in Sapporo City.

1. Introduction

An important stage of a traffic survey is generally known as traffic assignmet.

Traffic assignment can provide a very reasonable estimate of demand for the usage of service.

Traffic assignment to Street Network serves several useful purposes.

One of the most important of these is in testing a street for its ability to serve the traffic needs of an area.

This test might indicate to a traffic planner that some rearrangements of a street is necessary or new streets should be added.

Therefore, the traffic assignment to Street Network provides the planner with a useful tool which can be used to test the effectiveness of an urban plan from the standpoint of traffic.

2. A synopsis of the model

The following is the features of this assignment procedure.

1) The inter-zone trips are assigned to the network of arterial streets to minimize the total travel time of the system.
2) This method takes into account the effect of traffic in which the running time along each link of the network increases as the flow on the link increases.
3) Also considered herein is the difference of right and left turning time to be assigned.

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4) This network performs even when capacity restraint is placed on any of the links.

This capacity restraint can be calculated from the method of solution for arterial street network capacity.

Such an assignment is known as a capacity restrained assignment.

3. The method of solution for arterial street network capacity

Capacity restraints of an arterial street network with at-grade intersections are on the intersections rather than the links.

In this section, the method that maximizes the steady traffic flow through an arterial street network with signal indicated intersections and link limitation is described.

This method is based on Linear programming and Network flow theory.

(i) Notation

We define the notations as follows.

$P_k(N_s, N_t)$: Path k from starting node N_s to the destination node N_t.
(N_i, N_j): Space from node N_i to N_j.
$P(N_s, N_t) = \{P_k(M_s, N_t)/k = 1, 2, \cdots, n\}$
$C(N_i, N_j)$: Basic capacity restraints on space (N_i, N_j).
$Q_k(N_i)$: Rate of the green time of the intersection N_i on the path $P_k(N_s, N_t)$.

We treat traffic as a countinuous flow with constant arrival rates and departure rates during the green time of a traffic signal.

L_{P_k}; Capacity of path $P_k(N_s, N_t)$

$$L_{P_n} = \min_i C_k(N_i, N_{i+1})$$

We determined the optimal paths that maximize the steady traffic flow of the capacity of network which is saturated.

The problem can be formulated as a Linear programming by enumerating all paths and determining for each path k a column vector P_k whose component is 1, if intersection N_i is passed and 0 is otherwise.

In this way, at each stage of the iteration, we introduce basic vector B, whose order is the number of intermediate nodes of the network.

Then, the initial basic B is the identity matrix and the algorithm proceeds as follows.

1) Calculate P_k and L_{P_k}.
2) Calculate the capacity limitation vector V from equation (1).

$$V = B^{-1} \cdot \alpha \tag{1}$$

$$V = \{V(N_i)/i = 1, 2, \cdots. t\} \quad \alpha = (L_{P_i}/i = 1, 2, \cdots, q)$$

3) Assign the prices of capacity limitation $V(N_i)$ to their corresponding components and call this capacity limitation network M_i.
4) From M_i, find a sequence of minimum price paths network M_i according to the standard shortest route algorithm.
The restricted capacity S_{P_i} is calculated for the following equation (2).

$$S_{P_n} = \underset{P_i \in P - \overset{n-1}{\underset{i}{\cup}} P_i}{\text{Min}} (S_{P_i}) \tag{2}$$

$$S_{P_i} = \sum_i^r V(N_i) \qquad (1 \leqq r \leqq t)$$

5) Find the L_{P_i} for the corresponding path through the original network and dilate from the new capacity limitation network M_{i+1}, all links with capacity limits not exceeding L_{P_i}.
Repeat the process from step 3) to step 5) until M_{i+1} becomes disconnected and no path exists.
6) In step 2, if for any path P_k, we have $S_{P_i} - L_{P_i} \geqq 0$ then, it is added to basic vector B in new component.
7) Repeat from step 1) to step 6) until $S_{P_i} - L_{P_j} \geqq 0$ for all P_i. then, paths $\{P_i\}$ basic vector B is the optimal paths.
After finding the optimal paths $\{P_i\}$, we calculate the optimal traffic flow, taking into account the control of the intersection.
In general

$$L_{P_k} = \underset{P_i \in P - \overset{n-1}{\underset{i}{\subseteq}} P_i}{\text{Max}} (L_{P_i}) \tag{3}$$

$$L_n = \text{Min}\,\{C(N_s,\ N_t)\cdot Q(N_i),\ \cdots,\ (N_j,\ N_t)\}$$

If the space $(N_i,\ N_j)$ as follows,

$$(N_i,\ N_j) \in P_x(N_s,\ N_t) \cap P_y(N_s,\ N_t) \qquad C(N_s,\ N_t)\cdot Q(N_i) > L_x$$

Optimal traffic flow L is as follows

$$L = \text{Min}\,\{C(N_s,\ N_t)\cdot Q(N_i) - L_x,\ C(N_j,\ N_t)\} \tag{4}$$

We repeat this process until the flow becomes disconnected from N_s to N_t.

4. The method of traffic assignment

(i) Formation of the model

We apply the method of the discrete maximum principle to formulate the process. In order to do so, we define notations as follows.

The variable $v_{i,k}^{m,n}$ is the over-all speed of space $(m,\ n)$ in the area k.

$L_{i,k}^{m,n}$ is the over-all distance of space $(m,\ n)$ in the area k.

The variable $F_i^{m,n}$ is the traffic volume to be assigned to the street $(m,\ n)$. Let $i=1,\ 2$ are the directions of horizontal link and vertical link.

$C^{m,n}$ is the restrict capacity of the street $(m,\ n)$.

$V_i^{m,n}$ is the originating traffic volume from the center (m, n).

$K_L^{m,n}$ and $K_R^{m,n}$ are the waiting time for the left-turn and right-turn at node (m, n).

Assuming that the network is held rectangular to keep the notations as simple as possible, however, another is solved by this method.

The common assumption of assignment to a street is based on a combination of distance and speed for arterial streets of an area.

Therefore, in order to formulate the condition in section 2, 2). We introduce the family of the regional speed-volume function $\phi_{i,k}^{m,n}$ satisfying the following equation.

$\phi_{i,k}^{m,n}$ is the function of running time in link (m, n) of each area k (for example, Central Business District, Intermediate Area and Outlying Area).

In general

$$v_{i,k}^{m,n} = c_{i,k}/a_{i,k}(F_i^{m,n}) + b_{i,k} \tag{5}$$

The coefficents $a_{i,k}$, $b_{i,k}$, $c_{i,k}$ are based each area k.

Therefore

$$\phi_{i,k}^{m,n}(F_i^{m,n}) = L_{i,k}^{m,n}\{a_{i,k}(F_i^{m,n}) + b_{i,k}\}/c_{i,k} \tag{6}$$

The process of traffic assignment is considered as the discrete optimal time process.

The restricting conditions are as follows,

$$\sum_{(m,n)\in N} F^{m,n} \leqq \sum_{(m,n)\in N} C^{m,n}, \quad \sum_{(m,n)\in \mathrm{Cut}N} F^{m,n} \leqq \sum_{(m,n)\in \mathrm{Cut}N} C^{m,n}. \tag{7}$$

N is the set of the streets in the network.

Cut N is the set of cuts separeting the network.

In order to minimize the total time of the system, we shall determine the optimal admissible control of traffic assignment $U_{i,j}^{m,n}$.

$$U_{i,j}^{m,n} = \begin{cases} 1 & \text{as traffic is assigned to the street of direction } i. \\ 0 & \text{otherwise} \end{cases}$$

The performance equations are as follows,

$$F_1^{m,n} = U_{1,1}^{m,n}\cdot F_1^{m,n-1} + U_{1,2}^{m,n}\cdot V_1^{m,n} + (1-U_{2,1}^{m,n})\cdot F_2^{m-1,n} + (1-U_{2,2}^{m,n})\cdot V_2^{m,n} \tag{8}$$

$$F_2^{m,n} = (1-U_{1,1}^{m,n})\cdot F_1^{m,n-1} + (1-U_{1,2}^{m,n})\cdot V_1^{m,n} + U_{2,1}^{m,n}\cdot F_2^{m-1,n} + U_{2,2}^{m,n}\cdot V_2^{m,n} \tag{9}$$

$$\begin{aligned} T_1^{m,n} = T_1^{m,n-1} &+ U_{1,1}^{m,n}\cdot\phi_{1,k}^{m,n}(F_1^{m,n-1})\cdot F_1^{m,n-1} + U_{1,2}^{m,n}\cdot\phi_{1,k}^{m,n}(V_1^{m,n})\cdot V_1^{m,n} \\ &+ (1-U_{2,1}^{m,n})\cdot\phi_{1,k}^{m,n}(F_2^{m-1,n})\cdot F_2^{m-1,n} + (1-U_{2,2}^{m,n})\cdot\phi_{1,k}^{m,n}(V_2^{m,n})\cdot V_2^{m,n} \\ &+ K_L^{m,n}(1-U_{2,1}^{m,n})\cdot F_2^{m-1,n} + K_L^{m,n}(1-U_{2,2}^{m,n})\cdot V_2^{m,n} \end{aligned} \tag{10}$$

$$\begin{aligned} T_2^{m,n} = T_2^{m-1,n} &+ U_{2,1}^{m,n}\cdot\phi_{2,l}^{m,n}(F_2^{m-1,n})\cdot F_2^{m-1,n} + U_{2,2}^{m,n}\cdot\phi_{2,l}^{m,n}(V_2^{m,n})\cdot V_2^{m,n} \\ &+ (1-U_{1,1}^{m,n})\cdot\phi_{2,l}^{m,n}(F_1^{m,n-1})\cdot F_1^{m,n-1} + (1-U_{1,2}^{m,n})\cdot\phi_{2,l}^{m,n}(V_1^{m,n})\cdot V_1^{m,n} \\ &+ K_R^{m,n}(1-U_{1,1}^{m,n})\cdot F_1^{m,n-1} + K_R^{m,n}(1-U_{1,2}^{m,n})\cdot V_1^{m,n} \end{aligned} \tag{11}$$

$$T = \sum_{m=1}^{M} T_1^{m,n} + \sum_{n=1}^{N} T_2^{m,n} \tag{12}$$

The procedure for solving such the optimization problem by the discrete maximum principle is to introduce the covariant vector $\psi_i^{m,n}(i=1, 2, \cdots, 4)$ and Hamiltonian function $H^{m,n}$ satisfying.

$$(\psi_3^{m,n}=\psi_4^{m,n}=1)$$

$$\begin{aligned}H^{m,n} = {} & T_1^{m,n-1}+T_2^{m-1,n}+U_{1,1}^{m,n}\cdot F_1^{m,n-1}\{\psi_1^{m,n}+\phi_{1,k}^{m,n}(F_1^{m,n-1})\}+U_{1,2}^{m,n}\cdot V_1^{m,n}\cdot \\ & \{\psi_1^{m,n}+\phi_{1,k}^{m,n}(V_1^{m,n})\}+(1-U_{2,1}^{m,n})\cdot F_2^{m-1,n}\cdot\{\psi_1^{m,n}+\phi_{1,k}^{m,n}(F_2^{m-1,n})+K_L^{m,n}\} \\ & +(1-U_{2,2}^{m,n})\cdot V_2^{m,n}\{\psi_1^{m,n}+\phi_{1,k}^{m,n}(V_2^{m,n})+K_L^{m,n}\}+(1-U_{1,1}^{m,n})\cdot F_1^{m,n-1} \\ & \{\psi_2^{m,n}+\phi_{2,l}^{m,n}(F_1^{m,n-1})+K_R^{m,n}\}+(1-U_{1,2}^{m,n})\cdot V_1^{m,n}\{\psi_2^{m,n}+\phi_{2,l}^{m,n}(V_1^{m,n})+K_R^{m,n}\} \\ & +U_{2,1}^{m,n}\cdot F_2^{m-1,n}\{\psi_2^{m,n}+\phi_{2,l}^{m,n}(F_2^{m-1,n})\}+U_{2,2}^{m,n}\cdot V_2^{m,n}\{\psi_2^{m,n}+\phi_{2,l}^{m,n}(V_2^{m,n})\}\end{aligned} \quad (13)$$

As the admissible control of traffic assignment are given by

$$U_{i,1}^{m,n}=U_{i,2}^{m,n} \qquad (i=1,\ 2)$$

The function $\phi^{m,n}$ is rewritten as

$$\phi^{m,n}(F^{m,n})+\phi^{m,n}(V^{m,n})=\phi^{m,n}(F^{m,n}+V^{m,n}) \quad (14)$$

And, therefore, according to the maximum principle, We put

$$\begin{aligned}\psi_1^{m,n-1} = {} & \partial H^{m,n}/\partial F_1^{m,n-1}=U_{1,1}^{m,n}\{\psi_1^{m,n}+\phi_{1,k}^{m,n}(F_1^{m,n-1})\} \\ & +U_1^{m,n}\cdot F_1^{m,n-1}\cdot\partial\phi_{1,k}^{m,n}(F_1^{m,n-1})/\partial F_1^{m,n-1}+(1-U_{1,1}^{m,n}) \\ & \{\psi_2^{m,n}+\phi_{2,l}^{m,n}(F_1^{m,n-1})+K_R^{m,n}\}+(1-U_{1,1}^{m,n})\cdot F_1^{m,n-1} \\ & \cdot\partial\phi_{1,k}^{m,n}(F_1^{m,n-1})/\partial F_1^{m,n-1}\end{aligned} \quad (15)$$

$$\begin{aligned}\psi_2^{m-1,n} = {} & \partial H^{m,n}/\partial F_2^{m-1,n}=(1-U_{2,1}^{m,n})\ \{\psi_2^{m,n}+\phi_{2,l}^{m,n}(F_2^{m-1,n})+K_L^{m,n}\} \\ & (1-U_{2,1}^{m,n})\cdot F_2^{m-1,n}\cdot\partial\phi_{2,l}^{m,n}(F_2^{m-1,n})/\partial F_2^{m-1,n}+U_{2,1}^{m,n}\cdot \\ & \{\psi_2^{m,n}+\phi_{2,l}^{m,n}(F_2^{m-1,n})\}+U_{2,1}^{m,n}\cdot F_2^{m-1,n}\cdot\partial\phi_{2,l}^{m,n}(F_2^{m-1,n})/F_2^{m-1,n}\end{aligned} \quad (16)$$

(ii) Algorithm for the problem

The computation for this problem can be carried out as follows

1) Assume the admissible control of the traffic assignment at each node.
2) From the starting node, tne traffic assignment volume can be calculated from equations (8), (9).
   In addition, the travel time $\phi^{m,n}$ can be calculated from equation (6).
3) From destination node, in order to test the sequence of the traffic assignment admissible control $\{U_{i,j}^{m,n}\}$.
   We shall determine the new one, so as to minimize Hamiltonian function $H^{m,n}$.
4) If the traffic volume $F^{m,n}$ goes over the capacity of the street (m, n), the sequence of $\{U_{i,j}^{m,n}\}$ is examined by the following condition and returns to 3).

$$U_{i,j}^{m,n}=\begin{cases}1 & F^{m,n}<C^{m,n} \\ 0 & F^{m,n}\geqq C^{m,n}\end{cases}$$

5) The process from 2) to 5) is repeated until the two successive sets of the total travel time are identical.
   This algorithm is the extention of Katz's original treatment, Fan and Wang's the extentional one.

5. Test results

In this chapter, we described the outline of applying the assignment method to the arterial street network in the Sapporo city area.

This assignment is based on the family of the functions that is introduced by the regional speed-volume function.

This family of the regional speed-volume functions is divided into three parts.

These parts are functions in Central Business District, Intermediate Area and Outlying Area.

These functions are shown in Fig. 1, 2, 3.

In order to utilize these functions, the over-all distance of space became necessary to be calculated for the streets.

According to the functions, we can take into account the effect of traffic condition 2) in chapter 2.

Then, we tried to calculate the traffic assignment volume.

Whether the results agree with the actual traffic situation or not, is determined by comparing the result of the traffic flow of the ground count at each point in the arterial streets.

This distribution of the correlation is shown in Fig. 4 and we get the correlation coefficient ; 0.8817.

This high correlation coefficient shows that this method is a useful tool to be assigned to the arterial street network.

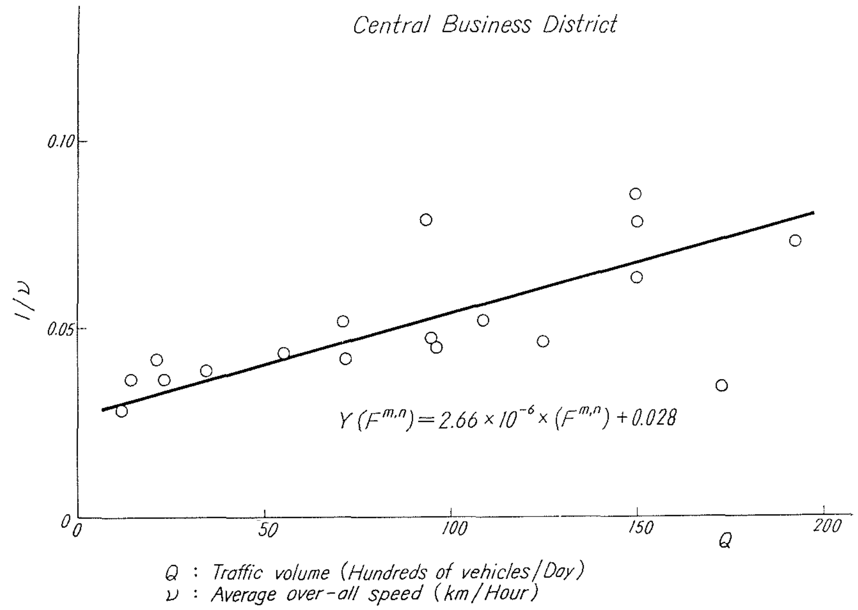
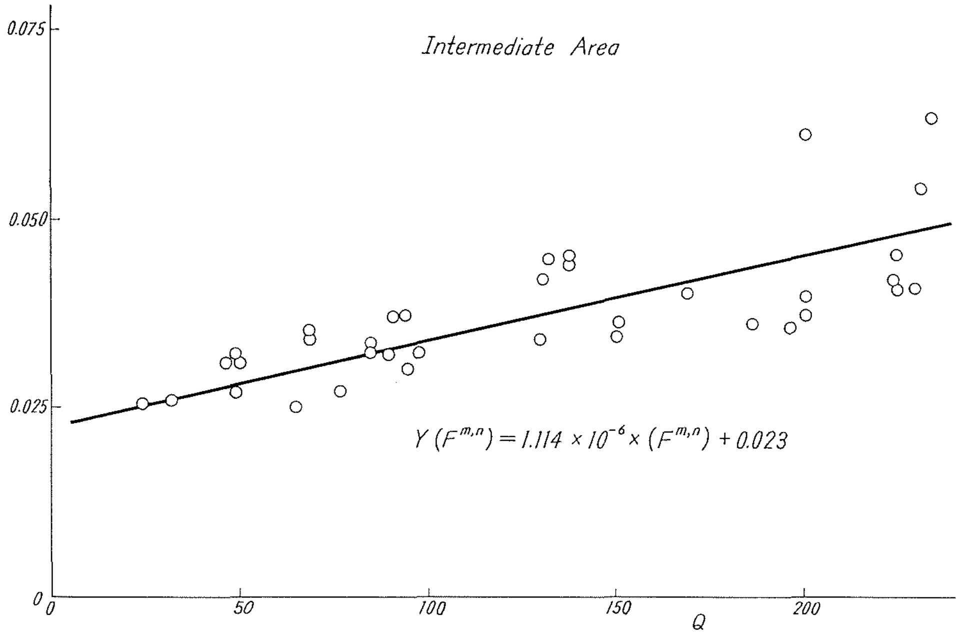
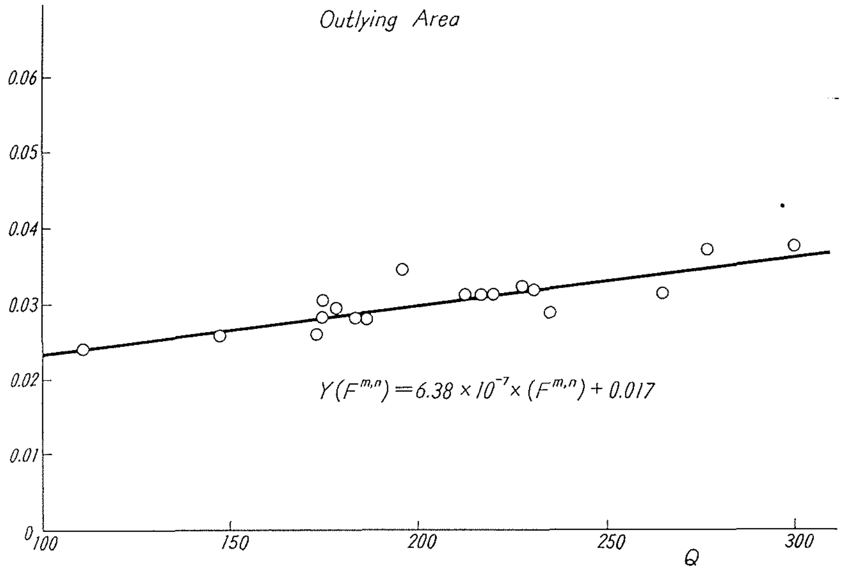


Fig. 1.



Q : Traffic volume (Hundreds of vehicles / Day)
ν : Average over-all speed (km / Hour)

Fig. 2.



Q : Traffic volume (Hundreds of vehicles / Day)
ν : Average over-all speed (km / Hour)

Fig. 3.

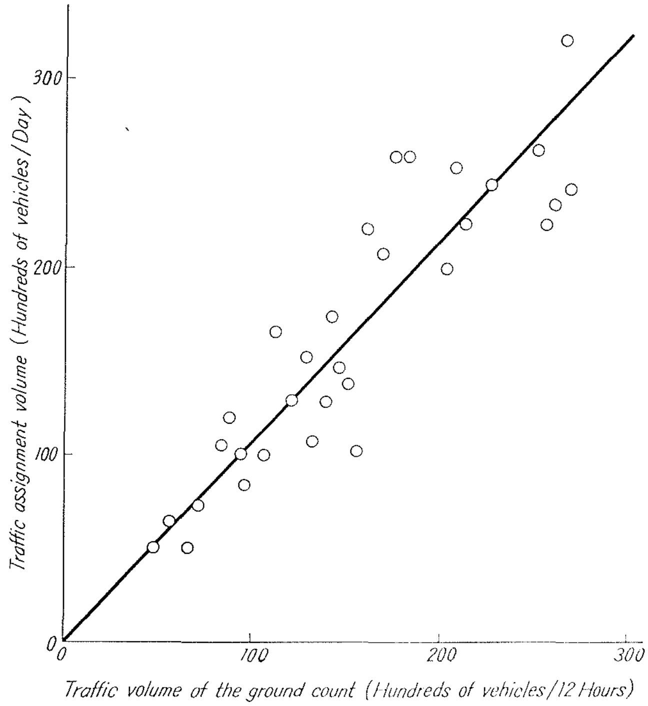


Fig. 4.

6. Conclusion

In this paper, we have mentioned a new approach to a traffic assignment.

It is based on the methods of solution for arterial street network capacity and the traffic assignment by discrete maximum principle.

According to a combination of the two methods, we applied them to the traffic assignment of the arterial street network in Sapporo City.

As a result of this solution, we find the method is effective for the analysis of the traffic assignment to Arterial Street Network.

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