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Longitudinal Distribution of Surface Salinity
in an Estuary

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Abstract

The relation between the behavior of a salt wedge and salinity diffusion at the interface of salt and fresh water have been observed for many years at the mouth of the Ishikari River.

In this paper, the longitudinal distribution of salinity in a fresh water layer and the role of diffusion coefficients of the river flow on salinity distribution are discussed by solving the differential equation of salinity diffusion under adequate boundary conditions. The calculated distribution of surface salinity was compared with the observed results in the Ishikari River.

1. Introduction

The authors and other workers have made observations of a salt wedge by the ultrasonic method[1][2] and also salinity diffusion at the interface of salt and fresh water[3] over a period of many years at the mouth of the Ishikari River in Hokkaido, Japan. The Ishikari River, which has a length of about 300 km, flows through the Ishikari Plains and pours into the Japan Sea (Fig. 1). Because the Japan Sea has only a small tidal range of about

Fig. 1. (a) Hokkaido Territory   (b) The mouth of the Ishikari River

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30 cm at its maximum throughout the year, the behavior of the salt wedge is controlled mainly by the river discharge.

While the salt wedge was observed by echo-sounding along the salt wedge upstream from the river mouth, the surface water was sampled continuously every 30 seconds. The chlorinity concentration of the water samples was measured by chemical analysis to study the salinity diffusion from the salt water layer.

2. Mixing of salt and fresh water

When the river discharge decreases below a critical value of 600 m³/s, the salt wedge begins to invade into the mouth. The front of the salt wedge reaches a point about 15 km upstream from the mouth at a discharge of about 200m³/s, in the low water season in summer.

When the salt wedge stops, the interface of salt and fresh water is very stable. When the discharge increases because of a rainfall, however, the interface changes to an unstable state and the salt wedge begins to decay. In such a case violent mixing of salt and fresh water is found everywhere along the interface.

There is a big river bed projection at a point 3.7 km upstream from the mouth as shown in Fig. 2. Therefore, when the salt wedge existed over that point, a stronger mixing of salt and fresh water was found at this site as compared with all other points.

Fig. 3 shows the vertical distributions of the velocity and salinity at stations A and B in Fig. 1 measured on July 27, 1967. According to the figure, the discontinuity of chlorinity between salt and fresh water is remarkably sharp at station B. At station A immediately downstream from the big projection, however, the mixing of salt and fresh water is so intense that a clear stratification cannot be seen.

Fig. 4 shows the longitudinal distributions of chlorinity of the surface water with a background of 0.01 Cl‰. According to the figure, the chlorinity increases from 0.01 Cl‰ (at the point of the front of the salt wedge) to about 1.0 Cl‰ (at the mouth) in all cases. Therefore, the gradient of chlorinity distribution curve decreases as the salt wedge invades upstream.

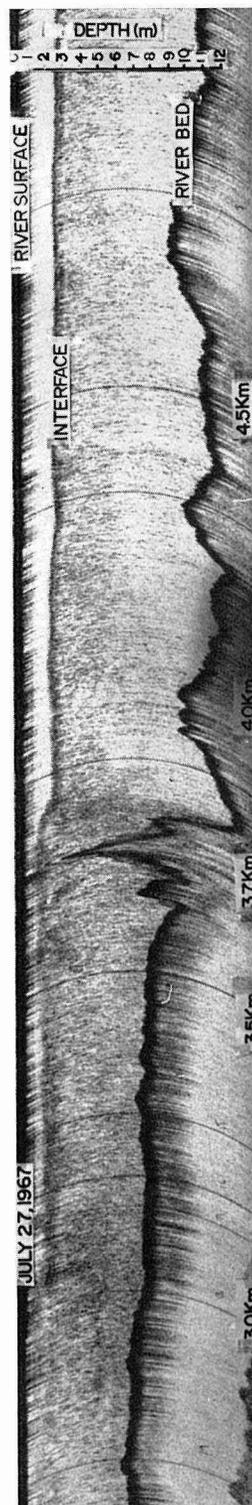In the case of an unstable state at the interface, the

Fig. 2. A longitudinal profile of the salt wedge at the mouth of the Ishikari River (July 27, 1967)
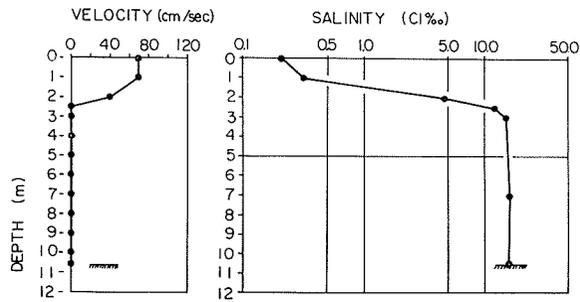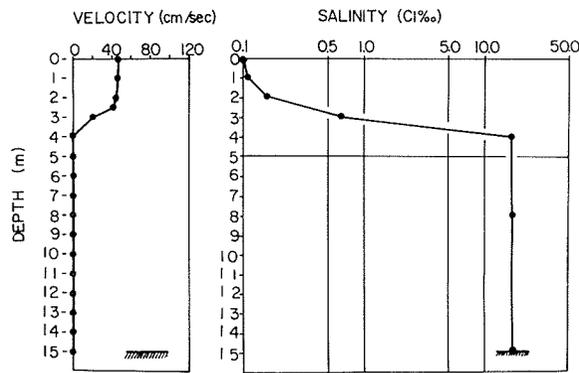
Fig. 3. Vertical distributions of velocity and salnity
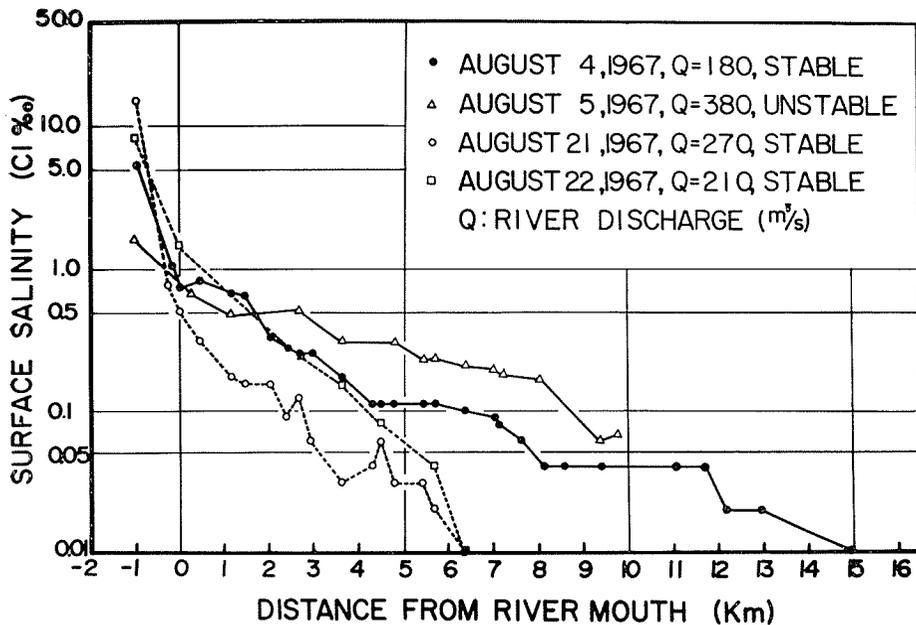(July 27, 1967)

(a) Station A

(b) Station B

Fig. 4. Longitudinal distributions of surface salinity

chlorinity of surface water increases rapidly because the mixing of salt and fresh water becomes highly intense.   On August 22, 1967, the salt wedge had ceased its motion but the salinity distribution was unstable because of mixing by a strong wind with a velocity of 10 m/s.

Fig. 5 shows the relation between the river discharge and the surface chlorinity
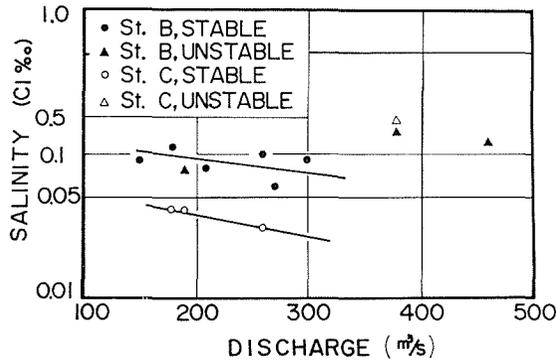
Fig. 5.   Relation between surface salinity and the river discharge

at stations B and C.   As shown in the figure, the diffusion of salinity in the upper layer is highly intense in the case of a retrograding wedge.

3.   Equation of Salinity Diffusion

Taking x in the downstream direction along the river surface and y in a vertically downward direction, the differential equation of salinity diffusion in the fresh water layer is given by

$$U\frac{\partial S}{\partial x} = \eta_x \frac{\partial^2 S}{\partial x^2} + \eta_y \frac{\partial^2 S}{\partial y^2}$$

where $S=$ Cl concentration of the water

$U=$ mean velocity of the fresh water layer

$\eta_x=$ diffusion coefficient of x direction

$\eta_y=$ diffusion coefficient of y direction.

The boundary conditions are expressed by

$$\frac{\partial S}{\partial y} = 0 \quad \text{at} \quad y = 0\,,$$

$$S = S_0 \quad \text{at} \quad y = h_0$$

and $S = f(y)$ at $x = 0\,,$

where h_0 is the depth of the fresh water layer.   The boundary value of Cl concentration $f(y)$ is an arbitrary function of y.

Under these conditions, the equation is solved:

$$S = S_0 - \sum_{n=0}^{\infty} \left\{ \frac{(-1)^n 4S_0}{(2n+1)\pi} - \frac{2}{h_0} \int_0^{h_0} f(\lambda)\cos\frac{(2n+1)\pi\lambda}{2h_0} \, d\lambda \right\} \cdot \cos\frac{(2n+1)\pi y}{2h_0} \cdot$$

$$\exp\left\{ \frac{U}{2\eta_x} - \sqrt{\left(\frac{U}{2\eta_x}\right)^2 + \frac{\pi^2\eta_y}{4h_0^2\eta_x}(2n+1)^2} \right\} x \tag{1}$$

Therefore, the chlorinity of the surface water $(S)_{y=0}$ is given by

$$(S)_{y=0} = S_0 - \sum_{n=0}^{\infty} \left\{ \frac{(-1)^n 4 S_0}{(2n+1)\pi} - \frac{2}{h_0} \int_0^{h_0} f(\lambda) \cos \frac{(2n+1)\pi\lambda}{2h_0} \, d\lambda \right\} \cdot$$

$$\exp \left\{ \frac{U}{2\eta_x} - \sqrt{\left(\frac{U}{2\eta_x}\right)^2 + \frac{\pi^2 \eta_y}{4 h_0^2 \eta_x}(2n+1)^2} \right\} x. \tag{2}$$

When the point of the front of the salt wedge is taken as $x=0$, one of the boundary conditions is represented by $f(y) = 0$, and hence the results can be reduced to

$$S = S_0 - \sum_{n=0}^{\infty} \frac{(-1)^n 4 S_0}{(2n+1)\pi} \cdot \cos \frac{(2n+1)\pi y}{2 h_0} \cdot$$

$$\exp \left\{ \frac{U}{2\eta_x} - \sqrt{\left(\frac{U}{2\eta_x}\right)^2 + \frac{\pi^2 \eta_y}{4 h_0^2 \eta_x}(2n+1)^2} \right\} x \tag{3}$$

and

$$(S)_{y=0} = S_0 - \sum_{n=0}^{\infty} \frac{(-1)^n 4 S_0}{(2n+1)\pi} \cdot \exp \left\{ \frac{U}{2\eta_x} - \sqrt{\left(\frac{U}{2\eta_x}\right)^2 + \frac{\pi^2 \eta_y}{4 h_0^2 \eta_x}(2n+1)^2} \right\} x. \tag{4}$$

These equations give the longitudinal distribution of chlorinity if the values of U, h_0, S_0, η_x, η_y and $f(y)$ are known.

4. Role of diffusion coefficients on salinity distribution

By the use of the above equations, for example, the longitudinal distribution of chlorinity can be obtained with actual values of U, h_0 and S_0 observed at the mouth of the Ishikari River on July 27, 1967, and compared with the observed distribution curve.

Namely, if U, h_0 and S_0 are given as

$U = 41.3$ cm/s
$h_0 = 250$ cm
$S_0 = 0.32$ Cl‰

from the point of the front of the salt wedge (8.4 km upstream from the mouth) to station B (4.5 km), the chlorinity distribution for this section is calculated from equation (3) for various value of diffusion coefficients η_x and η_y. On the other hand, for the section between the big river bed projection (3.7 km) and river mouth, U, h_0, S_0 and boundary condition $f(y)$ are given as

$U = 76.5$ cm/s
$h_0 = 100$ cm
$S_0 = 1.12$ Cl‰
$f(y) = 0.21 + 0.0091\ y$

and thus the chlorinity distribution can be obtained from equation (1).

The vertical distributions of chlorinity at several points calculated taking

$$\eta_x = 5 \times 10^8 \quad \text{C. G. S.}$$
$$\eta_y = 3 \times 10 \quad \text{C. G. S.}$$

for the former section, and

$$\eta_x = 5 \times 10^8 \quad \text{C. G. S.}$$
$$\eta_y = 1 \times 10 \quad \text{C. G. S.}$$

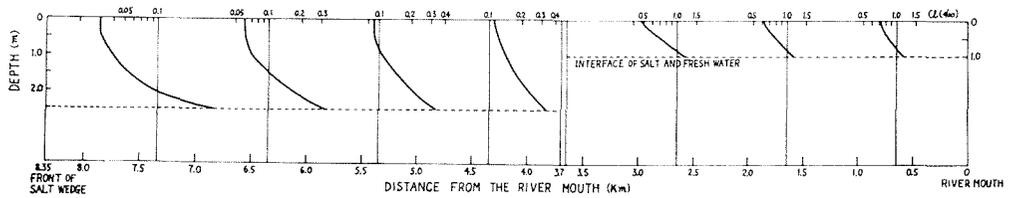for the latter section are shown in Fig. 6.

Fig. 6. Vertical distributions of salinity in the fresh water layer (calculated)

The calculated values of surface salinity for the same values of η_x and η_y as used above are compared with observed values in Fig. 7. The calculated distributions agree fairly well with the observed ones.
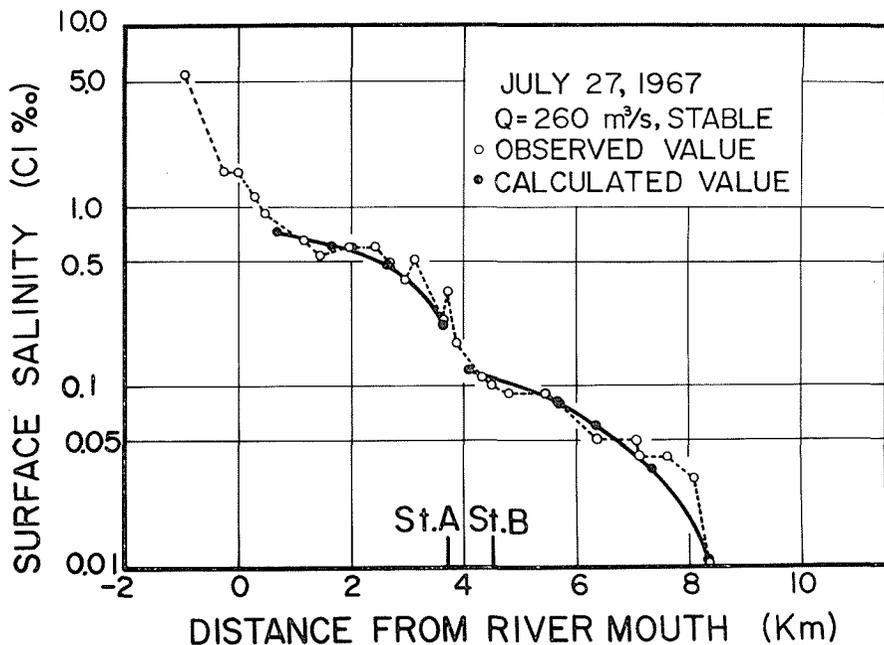

Fig. 7. A longitudinal distribution of surface salinity

Fig. 8 illustrates the role of diffusion coefficients on the surface distribution calculated from equation (4) for a distance of 10 km downstream from the front of the salt wedge. The distribution curve approaches the boundary values S_0 the more rapidly as the value of η_x decreases or η_y increases.
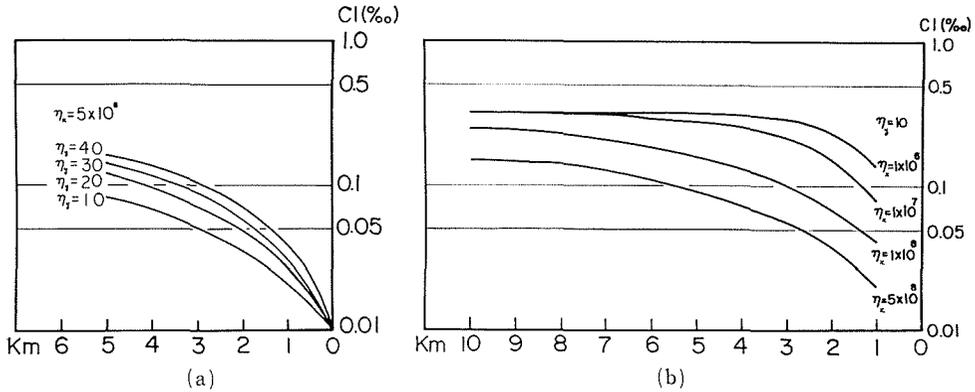
Fig. 8. The role of diffusion coefficients on the longitudinal distribution of surface salinity

5. Conclusion

The longitudinal distribution of surface salinity at a river mouth changes according to the behavior of the salt wedge.

The authors solved a differential equation of salinity diffusion in the fresh water layer. By using the results, the longitudinal distribution of salinity was obtained with actual values of U, h_0, and S_0 observed at the mouth of the Ishikari River.

The calculated salinity distribution of the river surface agreed fairly well with the observed distribution when the values of diffusion coefficients are taken as

$$\eta_x = 5 \times 10^8 \quad \text{C. G. S.}$$
$$\eta_y = 3 \times 10 \quad \text{C. G. S.}$$

for the section between the front and station B, and

$$\eta_x = 5 \times 10^8 \quad \text{C. G. S.}$$
$$\eta_y = 1 \times 10 \quad \text{C. G. S.}$$

downstream from the point of the big river bed projection to the river mouth.

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