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Laminar Wall Jet of an Electrically Conducting Fluid in the Presence of a Transverse Magnetic Field

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Abstract

This note describes a laminar wall jet of an electrically conducting fluid over an isothermal impermeable wall in the presence of a transverse magnetic field of constant strength. The analysis was performed on the basis of the boundary layer theory and the solution was obtained by means of a similarity hypothesis. It was found from the analysis that the magnetic field decreases the skin friction and the heat transfer between the wall and the fluid by a considerable amount.

1. Introduction

Considerable interest has developed in regard to forced-convection heat transfer involving electrically conducting fluids in the presence of magnetic fields. The primary motivation of this interest has been the prospect of reducing aerodynamic heat transfer (Rossow[1], Meyer[2]). In this respect, the effects of transverse magnetic field on the flow characteristics and heat transfer in the boundary layer have been extensively studied from a practical point of view of boundary layer control. However, very little information is available concerning the wall jet of an electrically conducting fluid over a plane surface in a magnetic field.

This note describes a laminar wall jet of an electrically conducting fluid over an isothermal impermeable wall in the presence of a transverse magnetic field of constant strength. Specific considerations are given to the magnitude of its influence on the skin friction and the heat transfer between the wall and the fluid. For simplicity, only a two-dimensional laminar flow is considered.

2. Analysis

The present analysis is based upon the following assumptions: (a) The flow is incompressible, steady and two-dimensional. (b) The properties of the fluid, including electrical conductivity, are constant. (c) Both the induced magnetic field and the charge density are negligible. The wall jet can be treated within the framework of the boundary layer theory (Glauert[3]). According to Rossow[1] the boundary layer equations in accordance with the above assumptions become

$$\frac{\partial u'}{\partial x'}+\frac{\partial v'}{\partial y'}=0 \tag{1 a}$$

$$u'\frac{\partial u'}{\partial x'}+v'\frac{\partial u'}{\partial y'}=\nu\frac{\partial^2 u'}{\partial y'^2}-\frac{\sigma}{\rho}B_0{}^2\,u' \tag{1 b}$$

$$\rho C_p\left(u'\frac{\partial T'}{\partial x'}+v'\frac{\partial T'}{\partial y'}\right)=k\frac{\partial^2 T'}{\partial y'^2} \tag{1 c}$$

where the viscous dissipation and the Joule heating are considered to be negligibly small. In the above equations x' is the streamwise distance measured from an appropriate origin, y' is the distance normal to the wall, u', v' are the corresponding velocity components and T' is the temperature. The fluid properties ρ, ν, C_p, k and σ imply the density, kinematic

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viscosity, specific heat at constant pressure, thermal conductivity and electrical conductivity. B_0 is the magnetic field intensity which is constant. The isothermal wall temperature is denoted by T_w and the temperature of the surrounding fluid by T_∞. If one introduces the non-dimensional variables defined by

$$x = Ux'/\nu,\ y = Uy'/\nu$$

$$u = u'/U,\ v = v'/U,\ \theta = (T' - T_\infty)/(T_w - T_\infty)$$

U being a constant velocity, Eqs. (1 a), (1 b) and (1 c) become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2 a}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - M^2 u \tag{2 b}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\ \frac{\partial^2 \theta}{\partial y^2} \tag{2 c}$$

where P_r is the Prandtl number and M is the Hartmann number defined by $\frac{\nu B_0}{U}\sqrt{\frac{\sigma}{\mu}}$. The boundary conditions are

$$y = 0:\ u = v = 0,\ \theta = 1 \tag{3 a}$$

$$y = \infty:\ u = 0,\ \theta = 0 \tag{3 b}$$

The continuity equation, Eq. (2 a), is automatically satisfied if the stream function Ψ is introduced by the usual definition

$$u = \frac{\partial \Psi}{\partial y},\quad v = -\frac{\partial \Psi}{\partial x}$$

Here one introduces the new variable

$$\eta = y x^{-\frac{3}{4}}$$

and puts the stream function and the temperature into the forms of

$$\Psi\ (x,\ \eta) = 4\,x^{\frac{1}{2}} \sum_{k=0}^{\infty} \varepsilon^k F_k\ (\eta) \tag{4 a}$$

$$\theta\ (x,\ \eta) = \sum_{k=0}^{\infty} \varepsilon^k H_k\ (\eta) \tag{4 b}$$

where

$$\varepsilon = M^2 x^{\frac{3}{2}}$$

Then, the velocity components can be written in terms of ε and η:

$$u = 4\,x^{-\frac{1}{2}} \sum_{k=0}^{\infty} \varepsilon^k F_k'\ (\eta) \tag{5 a}$$

$$v = -x^{-\frac{3}{4}} \sum_{k=0}^{\infty} [(1 + 6\,k)\ \varepsilon^k\ F_k(\eta) - 3\,\eta\ \varepsilon^k\ F_k'(\eta)] \tag{5 b}$$

With Eqs. (4 b), (5 a) and (5 b), the momentum and energy equations can be written in terms of x and η. Then, in each of the resulting equations, the terms are arranged with respect to the powers of ε. In this manner, one obtains a set of ordinary differential equations for the functions F_0, H_0, F_1, H_1, ……,

$$F_0''' + F_0 F_0'' + 2\,(F_0')^2 = 0 \tag{6 a}$$

$$H_0'' + P_r F_0 H_0' = 0 \tag{6 b}$$

$$F_1''' + 7\,F_1 F_0'' + F_0 F_1'' = 2\,F_0' F_1' + F_0' \tag{7 a}$$

$$H_1'' + P_r\,(F_0 H_1' + 7\,F_1 H_0' - 6\,F_0' H_1) = 0 \tag{7 b}$$

$$F_2''' + 13\,F_2F_0'' + 7\,F_1F_1'' + F_0F_2'' = 4\,(F_1')^2 + 8\,F_0'F_2' + F_1' \tag{8 a}$$

$$H_2'' + P_r\,(F_0H_2' + 7\,F_1H_1' + 13\,F_2H_0' - 12\,F_0'H_2 - 6\,F_1'H_1) = 0 \tag{8 b}$$

where the primes denote differentiation with respect to η. It should be noted that Eq. (6 a) coincides with the governing equation for a laminar wall jet in the absence of the magnetic field, the solution of which is plotted by Glauert[3]. Once F_0 is known, it is an easy matter to solve Eq. (6 b) numerically. It thus remains to solve Eqs. (7 a) and (7 b), and their solutions will provide the first order effects of the transverse magnetic field on the wall jet and the heat transfer. Corresponding to the boundary conditions in Eqs. (3 a) and (3 b), the functions F and H take the following boundary values:

$$\begin{array}{rl} \eta = 0\ : & F_0 = F_0' = 0,\ H_0 = 1 \\ & F_1 = F_1' = 0,\ H_1 = 0 \\ & F_2 = F_2' = 0,\ H_2 = 0 \\ \eta = \infty\ : & F_0' = 0,\ H_0 = 0 \\ & F_1' = 0,\ H_1 = 0 \\ & F_2' = 0,\ H_2 = 0 \end{array}$$

Utilizing the Runge-Kutta-Gill method, the solution of Eqs. (7 a) and (7 b) were obtained numerically for $P_r = 0.72$ on HIPAC 103 electronic digital computer installed at the Computer Center of Hokkaido University. The calculated results are tabulated in table 1, together with the values of F_0'' (0) and H_0' (0) that served as the input data for Eqs. (7 a) and (7 b). The information contained in table 1 will be used in the skin friction and heat transfer calculations which follow. Additionally, the functions F_0', F_1' and H_0, H_1, which are respectively related to the velocity and temperature profiles, are plotted in Figure 1. In Figure 2, the velocity and temperature distributions in the wall jet which are computed by means of these functions are shown for three values of the perturbation parameter ε.

Table 1

F_0'' (0)	F_1'' (0)	H_0' (0)	H_1' (0)
0.22222	−0.24822	−0.28676	0.13917

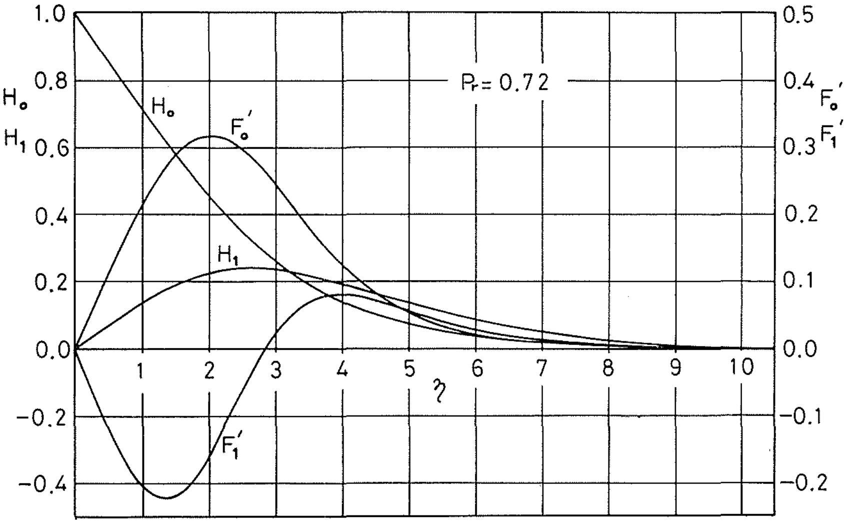
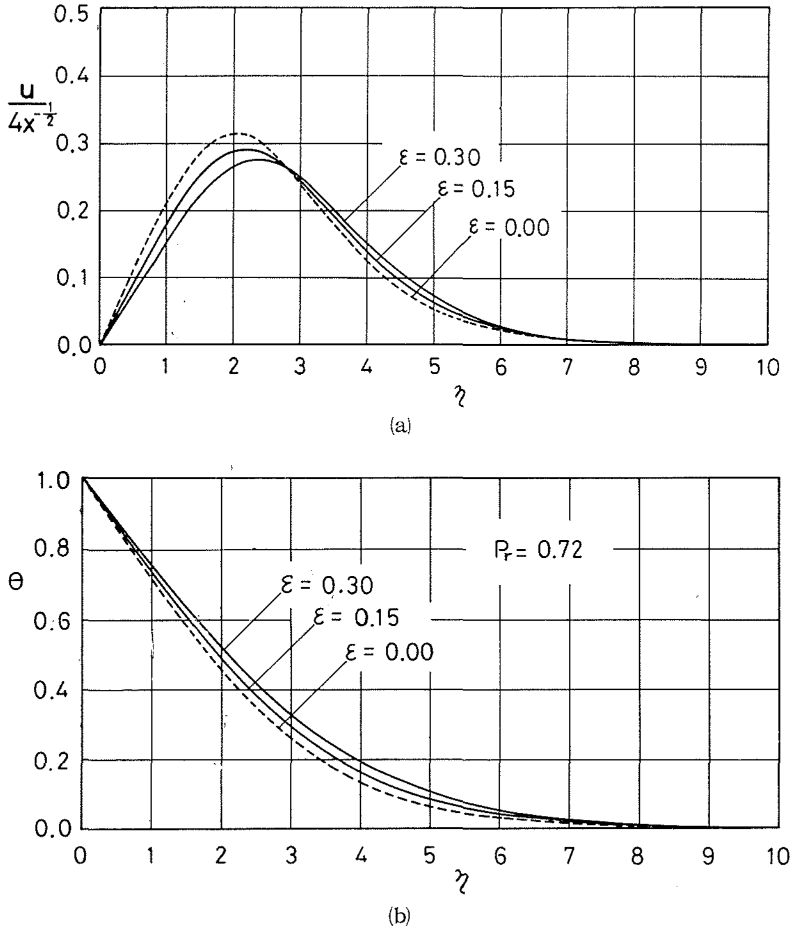


Figure 1 Universal functions.



(a)

(b)

Figure 2 (a), (b) Velocity and temperature distributions.

3. Skin friction and heat transfer

Non-dimensional skin friction τ which is defined as $\tau'/\rho U^2$ may be calculated from $\tau=(\partial u/\partial y)_{y=0}$. Introducing the variable η, the skin friction takes the form

$$\frac{\tau(x)}{\tau_0(x)}=1+\varepsilon\frac{F_1''(o)}{F_0''(o)}+\cdots\cdots$$

$$=1-1.11700\,(M^2x^{\frac{3}{2}})+\cdots\cdots \tag{9}$$

where $\tau_0(x)$ is the non-dimensional skin friction when the magnetic field is absent. Eq. (9) provides the results that the transverse magnetic field decreases the skin friction.

The local heat transfer passing per unit area from the surface to the fluid can be evaluated by Fourier's law $q=-(\partial\theta/\partial y)_{y=0}$ on the non-dimensional basis, q denoting $\nu q'/\{kU(T_w-T_\infty)\}$. In the same way as the skin friction, one obtains the expression

$$\frac{q(x)}{q_0(x)}=1+\varepsilon\frac{H_1'(o)}{H_0'(o)}+\cdots\cdots$$

$$=1-0.48532\,(M^2x^{\frac{3}{2}})+\cdots\cdots \tag{10}$$

in which $q_0(x)$ represents the non-dimensional heat transfer rate for the case of an isothermal impermeable wall in the absence of the magnetic field. As in the skin friction case, the transverse magnetic field decreases the heat transfer. To facilitate interpretation of the results, Eqs. (9) and (10) have been plotted in Figure 3.

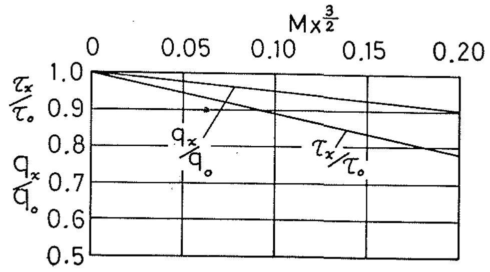


Figure 3 Effects of transverse magnetic field on the skin friction and heat transfer.

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