The Distribution of Effective Quantum Efficiency and Directional Characteristics in Optical Heterodyne Detection*

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Abstract

The effect of the distribution of effective quantum efficiency on directional characteristics in optical heterodyne detection of uniform plane waves was presented.

When the optical wavelengths, the optical input, and the maximum values of effective quantum efficiency are kept constant in the case where the effective quantum efficiencies are uniform, triangular, trapezoidal and Gaussian in distribution, the less angular selectivity, the lower the detection power of precisely parallel and normal incident beams. This is closely related to Siegman's antenna theorem of optical heterodyne detection.

Contents

1. Introduction

The stringent alignment tolerances necessary to keep signal and local-oscillator wavefronts in phase over a photosensitive surface have been pointed out by Siegman, Harris, McMurtry,[1] Corcoran,[2] Stroke[3] and Siegman.[4] The directional characteristic of two-dimensional photo-cathodes has been given by Sakuraba and Tsubo[5] DeLange has shown a curve of effect of beam tilt in Gaussian plane waves on output current.[6] The wavefront curvature effect on detected power output has recently been given by Sakuraba[7] and the fundamental properties of Gaussian plane wave in optical heterodyne detectors have more recently been shown by Sakuraba and Mishima.[8),9] In work done hitherto problems on the distribution of effective quantum efficiency have been mentioned but no quantitative results have been given in detail.[10]

The purpose of this paper is to derive the effect of distribution of effective

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quantum efficiency on the directional characteristics in optical heterodyne detection of uniform plane waves.

2. The Distribution of Effective Quantum Efficiency

The important factor affecting the power available from the photodiode is the effective quantum efficieney, which is defined as the number of electron-hole pairs reaching the active area of the diode per incident photon by Anderson, is less than the intrinsic quantum efficiency because of reflection loss at the interface, recombination in the N-layer and masking of part of the junction area.[11),12)] Experiments using microwave photodiodes have been perfomed in the authors' laboratory. Comparison measurements with various NEC microwave photodiodes showed that the distribution of effective quantum efficiency on the detector surface is not uniform over the photodiode (see Figs. 1 and 2). A study of these results shows that the effective quantum efficiency in one dimension is trapezoidal

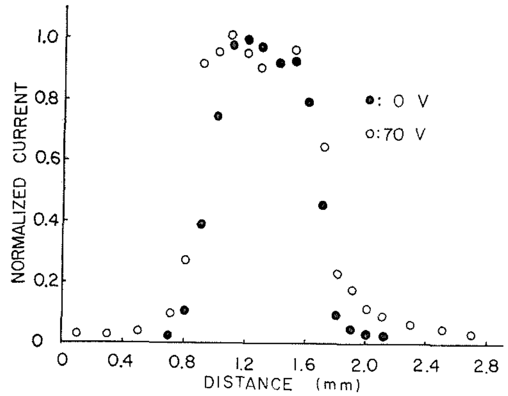

Fig. 1. Normalized current from a photodiode LSD 39B(R4-4) as a function of distance on the photosurface. The data were taken at the reverse voltage shown.

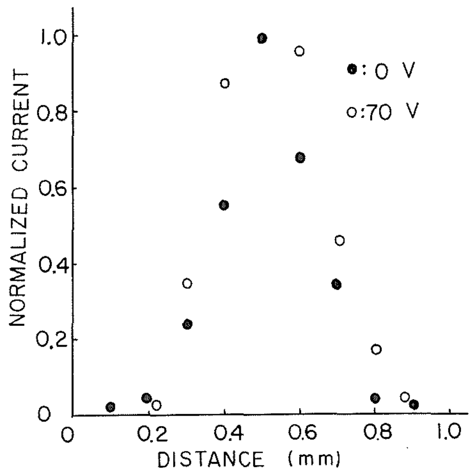

Fig. 2. Normalized current from a photodiode LSD39A(R7-2) as a function of distance on the phtosurface. The data were taken at the reverse voltage shown.

or Gaussian in distribution. It must, therefore, be investigated as to how the distribution of effective quantum efficiency affects the directional characteristics in optical heterodyne detection. In the following section, we will discuss cases where the effective quantum efficiencies are uniform (rectangular), triangular, trapezoidal and Gaussian in distribution.

3. The Effect of Distribution of Effective Quantum Efficiency on Directivity Factors

A schematic representation of the problem of detecting two unfocused light beams is shown in Fig. 3. A local-oscillator wave of complex scalar amplitude A_1 and optical frequency ω_1 emanating from point P_{01} (x_{01}, y_{01}) and a signal wave of complex scalar amplitude A_2 and optical frequency ω_2 emanating from point P_{02} (x_{02}, y_{02}) are superimposed at the square-law detector where is centered at $x = y = 0$ in the $z = 0$ plane. It is assumed that the wavefronts are not too strongly

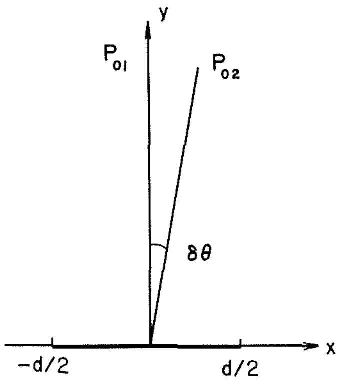

Fig. 3. Schematic diagram showing the illumination of the photosurface.

divergent or convergent on to the active length of the detector and $1/r$ dependence of amplitudes is neglected by the above approximation. It is also assumed that the two beams have the same optical modes and they are identically polarized. The difference-frequency photocurrent is then given by[7),8)]

$$i(t) = e^{-j(\omega_1-\omega_2)t}\int_A \eta(P)E_1(P)E_2^*(P)dA, \quad (1)$$

where

$$E_1(P) = A_1 \exp jk_1r_1, \quad (2)$$
$$E_2(P) = A_2 \exp jk_2r_2, \quad (3)$$

and k_1 and k_2 are the phase constants of the local-oscillator and the signal waves, respectively, $\eta(P)$ is the effective quantum efficiency for light striking the point P and $E_n(P)$ is the field intensity at point P on the photosurface. For simplicity, it is assumed that an equivalent point of origin of the local-oscillator wave is on the line perpendicular to the detector and passes through the detector center and an equivalent point of origin of the signal wave is on the line at the very small angle $\delta\theta$. The photosurface is a strip of width d in the x-direction and uniform in the z-direction. When the quadratic and higher order terms in the expression of r_n expressed by means of the binominal theorem are neglected, the light incident upon the detector from each source is effectively a plane wave. The expression for the output can, therefore, be written as

$$P = \frac{1}{2}R_{eq}|A_1 A_2^*|^2\left|\int_{-d/2}^{d/2} \eta(x)e^{jk2\delta\theta x}\,dx\right|^2, \quad (4)$$

where R_{eq} is the equivalent resistance of the detector.

When the quantum efficiency is uniform over the photosurface, the power output is given by

$$P_1 = P_0 D_1^2, \quad (5)$$
$$P_0 = (R_{eq}/2)|A_1 A_2^*|^2 d^2, \quad (6)$$
$$D_1 = \mathrm{sinc}(\pi d\delta\theta/\lambda), \quad (7)$$

where the notation $\mathrm{sinc}\, x = (\sin x)/x$ is used and λ is the optical wavelength and it was assumed that $\lambda_1 \approx \lambda_2 \approx \lambda$. The quantity P_0 means the power detected in the case of exactly parallel and normal incident beams. The quantity D_1 is the well-known directivity factor of optical heterodyne detection of uniform plane waves. From Eq. (7) the first zero in the directional pattern is seen to occur at

$$\delta\theta_1 = \lambda/d, \quad (8)$$

and the maximum power output becomes

$$(P_1)_{max} = P_0. \quad (9)$$

If the effective quantum efficiency is triangular in distribution,

$$\eta(x) = \begin{cases} 1 + \frac{2}{d}x, & \text{for } -d/2 \leqq x \leqq 0, \\ 1 - \frac{2}{d}x, & \text{for } 0 \leqq x \leqq d/2, \end{cases} \quad (10)$$

the power output becomes

$$P_2 = P_0 D_2^2 \tag{11}$$

$$D_2 = (1/2)\ \mathrm{sinc}^2(\pi d\delta\theta/2\lambda), \tag{12}$$

where D_2 is the directivity factor in the triangular distribution. From Eq. (12) the first zero in the power pattern occurs at

$$\delta\theta_2 \approx 2\lambda/d, \tag{13}$$

and the maximum power is shown by

$$(P_2)_{max} = P_0/4. \tag{14}$$

When the effective quantum efficiency is trapezoidal in distribution,

$$\eta(x) = \begin{cases} (d+2x)/(d-d_1), & \text{for } -\frac{d}{2} \leqq x \leqq \frac{d_1}{2}, \\ 1, & \text{for } -\frac{d_1}{2} \leqq x \leqq \frac{d_1}{2}, \\ (d-2x)/(d-d_1), & \text{for } \frac{d_1}{2} \leqq x \leqq \frac{d}{2}. \end{cases} \tag{15}$$

the power output becomes

$$P_3 = P_0 D_3^2, \tag{16}$$

$$D_3 = [(d+d_1)/2d]\ \mathrm{sin\,c}[\pi\delta\theta(d_1+d)/\lambda]\ \mathrm{sin\,c}[\pi\delta\theta(d_1-d)/\lambda], \tag{17}$$

where d_1 is the width in which the effective quantum efficiency is unity. From Eq. (17) it follows that the first zero in the power pattern occurs at

$$\delta\theta_3 \approx 2\lambda/(d_1+d), \tag{18}$$

and the maximum value is given by

$$(P_3)_{max} = P_0[(d_1+d)^2/4d^2]. \tag{19}$$

The fourth special case of interest is that in which the effective quantum efficiency is Gaussian in distribution,

$$\eta(x) = \exp\,(-x^2/x_0^2), \tag{20}$$

where x_0 is the distribution length of effective quantum efficiency. The detected power output is shown by

$$P_4 = P_0 D_4^2, \tag{21}$$

$$D_4 = (x_0\sqrt{\pi}/d)\exp[-x_0^2\pi^2(\delta\theta)^2/\lambda^2], \tag{22}$$

where it was assumed that $\eta(x)$ is 10^{-3} at $x = \pm d/2$. The condition where $P_4 = 10^{-6}(P)_{max}$ in the directional pattern is seen occurs at

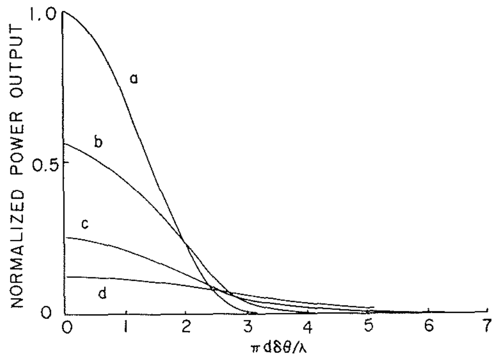

Fig. 4. Normalized power output as a function of $\pi d\delta\theta/\lambda$ in the various distributions of effective quantum efficiency: a) uniform, b) trapezoidal, c) triangular and d) Gaussian.

$$\delta\theta_4 \approx 1.89\ \lambda/\pi x_0 \tag{23}$$

The main lobes for normalized power, P_n/P_o, $n = 1,2,3$ and 4, as a function of $\pi d\delta\theta/\lambda$ are shown in Fig. 4, where it was assumed that $d_1 = d/2$ in the trapezoidal distribution and $x_0 = 0.2\ d$ in the Gaussian distribution. As seen from this figure, the maximum power generally decreases when the directional pattern is less sensitive to angles. It should be noted that the maximum value of effective quantum efficiency is kept constant.

4. Discussion

From the previous equations it follows that

$$(\delta\theta_1)^2(P_1)_{max} = P_0(\lambda/d)^2, \tag{24}$$

$$(\delta\theta_2)^2(P_2)_{max} = P_0(\lambda/d)^2, \tag{25}$$

$$(\delta\theta_3)^2(P_3)_{max} = P_0(\lambda/d)^2, \tag{26}$$

and

$$(\delta\theta_4)^2(P_4)_{max} \approx 1.136 P_0(\lambda/d)^2 \tag{27}$$

The coefficients of $P_0(\lambda/d)^2$ in the right members of Eqs. (24),(25),(26) and (27) are not likely to be far from unity. Therefore, it follows that

$$(\delta\theta_n)^2(P_n)_{max} \approx P_0(\lambda/d)^2, \text{ for } n = 1,\ 2,\ 3 \text{ and } 4. \tag{28}$$

A study of Eq. (28) shows that the quantity $(\delta\theta_n)^2(P_n)_{max}$ for various distributions of effective quantum efficiency is approximately constant when the optical amplitudes, the optical wavelengths, and the maximum values of effective quantum efficiency are kept constant on the photosurface. This means that when the main lobe of the directional power pattern becomes narrowed in the case of the same optical amplitudes and wavelengths, the maximum value of detected power output increases.

From the definition of directivity factors, we obtain

$$(\delta\theta_n)(D_n)_{max}\ d \approx \lambda, \tag{29}$$

this equation means that when the optical wavelength is kept constant and the effective width of detector, $(D_n)_{max}d$, decreases, the directional power pattern is less sensitive to angles between two beams. It should be noted that the maximum value of detected power output decreases in the case of a small effective width.

The effective width for a signal wave parallel to the local-oscillator wave is clearly just $(D_1)_{max}\ d = d$ in the uniform distribution of quantum efficiency. If the phase variation between signal and local-oscillator waves is not to exceed one wavelength across the detector width, the wavefront must not be tilted from parallelism by more than $\delta\theta \approx \lambda/d_e$, where $d_e = (D_n)_{max}d$. Therefore, the incoming signal wave must be confined to a cone of solid angle given by $\Omega_e \approx (\delta\theta_n)^2 \approx \lambda^2/A_e$, where A_e is the effective area of photosurface (diameter d_e). We have, then,

$$(\delta\theta_n)^2 d_e^2 \approx \Omega_e A_e \approx \lambda^2, \tag{30}$$

in agreement with Siegman's antenna theorem of optical heterodyne detection.[4)]

5. Conclusions

The effects of the distribution of effective quantum efficiency on directional characteristics in optical heterodyne detection of uniform plane waves were investigated. The general characteristics may be summarized as follows:

1) When the optical wavelengths, the optical amplitudes and the maximum values of effective quantum efficiency are kept constant, the quantity, $(\delta\theta_n)^2(P_n)_{max}$ is approximately constant in the uniform, triangular, trapezoidal and Gaussian distributions of effective quantum efficiency. The most important point is that the less the critical angular selectivity, the lower the detection power of exactly parallel and normal incident beams.
2) The directional power pattern is sensitive to angles between two beams when the effective width of the detector decreases in the case of constant optical wavelengths and the same maximum values of effective quantum efficiencies.
3) These results are closely related to Siegman's antenna theorem of optical heterodyne detection.

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