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On Hybrid Trees

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Abstract

In this paper *hybrid trees* in a linear graph which consists of two kinds of edges are defined and some of their properties are presented.

The properties are represented by set-theoretical binary operations (e.g. \cup , \cap , \oplus , $-$, $\frac{\partial}{\partial e}$, and $\int de$). And a method of *hybrid tree* generation is also given by using the above. These results are available for topological analysis and synthesis of active networks.

1. Introduction

In the analysis of linear networks, many authors have developed topological methods by using trees or co-trees. And in the topological treatment by tree (co-tree), the linear graph contains only one kind of edge, namely the admittance edge (impedance edge).

In a general network, it may contain any type of dependent sources, gyrators, coupled coils and ideal transformers in addition to L, C, R. In such a case, there are various advantages if each edge of the graph has an admittance dimension or impedance dimension according to the individual element. Thus, we consider a hybrid tree which has properties of both a tree and a co-tree, and we examine the fundamental properties of hybrid trees.

2. Definitions and preliminary considerations

Throughout this paper, we consider only undirected and labeled graph G in which all edges are labeled $e_1, e_2, e_3, \dots, e_m$ and all vertices are labeled $v_1, v_2, v_3, \dots, v_m$. And we can assume that the graph is connected, because similar results are obtained in a non-connected graph.

(1) Operator

g_i is a subgraph of G and e_i is an element (edge) of G . For conveniences sake, each subgraph in G is represented by the "product." Both $A = \{g_i\}$ and $B = \{g_j\}$ are sets of subgraphs in G .

In the following, we define the operators \times , $*$, $\partial/\partial e_i$, and $\int de_i$.

$$A \times B = \{g_i \cup g_j \mid g_i \in A, g_j \in B\} \quad (1)^{*)}$$

$$A * B = \{g_i \oplus g_j \mid g_i \in A, g_j \in B\} \quad (2)$$

$$\frac{\partial g_i}{\partial e_i} = \begin{cases} g_i \oplus e_i, & \text{if } e_i \in g_i \\ \phi, & \text{if } e_i \notin g_i \end{cases} \quad (3)^{*)}$$

$$\int g_i de_i = \begin{cases} g_i \cup e_i, & \text{if } e_i \notin g_i \\ \phi, & \text{if } e_i \in g_i \end{cases} \quad (4)$$

$$\frac{\partial A}{\partial e_i} = \left\{ \frac{\partial g_i}{\partial e_i} \mid g_i \in A, e_i \in g_i \right\} \quad (5)$$

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$$\int Ade_i = \left\{ \int g_i de_i \mid g_i \in A \right\} \quad (6)$$

If $g_i = e_{i_1} e_{i_2} \dots e_{i_m}$, we define

$$\frac{\partial A}{\partial g_i} = \frac{\partial A}{\partial e_{i_1}} \oplus \frac{\partial A}{\partial e_{i_2}} \oplus \dots \oplus \frac{\partial A}{\partial e_{i_m}} \quad (7)$$

$$\int Adg_i = \int Ade_{i_1} \oplus \int Ade_{i_2} \oplus \dots \oplus \int Ade_{i_m}. \quad (8)$$

$\partial^2 A / \partial g_1 \partial g_2$ and $\int dg_1 \int Adg_2$ mean $\partial / \partial g_1 (\partial A / \partial g_2)$ and $\int (\int Adg_2) dg_1$, respectively. And the order of the partial differentiations or integrations is immaterial. Furthermore it can be shown that

$$\frac{\partial(A \oplus B)}{\partial g_i} = \frac{\partial A}{\partial g_i} \oplus \frac{\partial B}{\partial g_i} \quad (9)$$

$$\int (A \oplus B) dg_i = \int Adg_i \oplus \int Bdg_i. \quad (10)$$

If no subgraph in A contains an element (edge) of a subgraph in B ,

$$\frac{\partial A \times B}{\partial g_i} = A \times \frac{\partial B}{\partial g_i} \cup B \times \frac{\partial A}{\partial g_i} \quad (11)$$

$$\frac{\partial A * B}{\partial g_i} = A * \frac{\partial B}{\partial g_i} \cup B * \frac{\partial A}{\partial g_i}. \quad (12)$$

In particular, if no subgraph in A contains an element (edge) in g_i ,

$$\frac{\partial A \times B}{\partial g_i} = A \times \frac{\partial B}{\partial g_i} \quad (11')$$

$$\frac{\partial A * B}{\partial g_i} = A * \frac{\partial B}{\partial g_i} \quad (12')$$

$$\int A \times B dg_i = A \times \int B dg_i \quad (13)$$

$$\int A * B dg_i = A * \int B dg_i. \quad (14)$$

(2) Hybrid tree

A *tree* in a graph G is a connected subgraph of G which contains all the vertices of the graph but does not contain any circuits. The complement of a tree in G is a *co-tree*.

Let E be the set of all edges of G . And let us partition E into two disjoint subsets E_y and E_z . Then we call the graph containing edges of E_y and E_z *hybrid graph* to distinguish it from the original graph. In this paper, however, we use the term "graph G " instead of "hybrid graph" for simplicity.

Hybrid tree: A hybrid tree (*ht*) with respect to E_y in a graph G is a subgraph of G , consisting of edges of a tree (t) contained in E_y and of edges contained in a subgraph E_z-t .

Let HT be the set of all hybrid trees of G . Then

$$HT = \{ht \mid ht = t \oplus E_z, t \in T\}.$$

By definition (2)

$$HT = E_z * T = E_y * CT, \quad (15)$$

where T and CT are the set of all trees and co-trees of G , respectively. In particular, if $E_z = \phi$ ($E_y = \phi$), HT is identical with T (CT).

Furthermore, we define the set of all hybrid k -tree HTv_1, v_2, \dots, v_k of G .

$$HTv_1, v_2, \dots, v_k = E_z * Tv_1, v_2, \dots, v_k \quad (16)$$

where v_1, v_2, \dots, v_k are disjoint subsets of the set of vertices, and Tv_1, v_2, \dots, v_k

is the set of all k -trees of G .

Consider the graph G of Fig. 1. HT is given as follows.

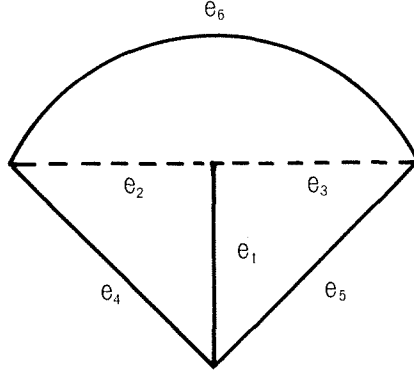


Fig. 1 Hybrid graph

$$E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$$

$$E_z = \{e_2, e_3\}$$

$$E_y = \{e_1, e_4, e_5, e_6\}$$

$$HT = \{e_1, e_4, e_5, e_1e_3e_5, e_1e_3e_6, e_3e_4e_5, e_3e_4e_6, e_3e_5e_6, e_1e_2e_5, e_1e_2e_6, e_2e_4e_5, e_2e_4e_6, e_2e_5e_6, e_1e_2e_3e_4e_5, e_1e_2e_3e_4e_6, e_1e_2e_3e_5e_6\}.$$

3. Some properties of hybrid trees and hybrid k -trees

In this section, we examine the relationships between hybrid trees (hybrid k -trees) and paths (or circuits). Their relationships are represented by operators which are defined in section 2.

Theorem 1

Let P_{zij} be a path between vertices i and j in G and P_{zij} consists of only edges of E_z . Then,

$$HT_{i,j} = \int HT dP_{zij}.$$

Proof: Let $P_{zij} = e_1, e_2, \dots, e_n$, where $e_1 e_2 \dots e_n \subset E_z$.

It is known that

$$T_{i,j} = \partial T / \partial P_{zij}.$$

By the definition (16)

$$\begin{aligned} HT_{i,j} &= E_z * T_{i,j} = E_z * \frac{\partial T}{\partial P_{zij}} \\ &= E_z * \left[\frac{\partial T}{\partial e_1} \oplus \frac{\partial T}{\partial e_2} \oplus \dots \oplus \frac{\partial T}{\partial e_n} \right] \\ &= E_z * \frac{\partial T}{\partial e_1} \oplus E_z * \frac{\partial T}{\partial e_2} \oplus \dots \oplus E_z * \frac{\partial T}{\partial e_n}. \end{aligned}$$

By the hypothesis $e_k \in E_z$

$$E_z * \frac{\partial T}{\partial e_k} = \int E_z * T de_k = \int HT de_k.$$

Therefore

$$\begin{aligned} HT_{i,j} &= \int HT de_1 \oplus \int HT de_2 \oplus \dots \oplus \int HT de_n \\ &= \int HT de_1 e_2 \dots e_n = \int HT dP_{zij}. \end{aligned}$$

Theorem 2

Let P_{ij} be a path between vertices i and j in G . Then

$$HT_{i,j} = \frac{\partial HT}{\partial P_{yij}} \oplus \int HT dP_{zij} = \left(\frac{\partial P_{yij}}{\partial} \oplus \int dP_{zij} \right) HT,$$

where

$$P_{ij} = P_{yij} \cup P_{zij},$$

$$(\text{edges of } P_{yij}) \begin{cases} \in E_y \\ \notin E_z \end{cases} \text{ and } (\text{edges of } P_{zij}) \begin{cases} \in E_z \\ \notin E_y \end{cases}$$

(Herein after, we use suffix y and z to represent the subgraphs)
(consisting of edges of E_y and E_z , respectively.)

Proof: By the definition (16),

$$HT_{i,j} = E_z * T_{i,j} = E_z * \frac{\partial T}{\partial P_{ij}} = E_z * \frac{\partial T}{\partial P_{yij}} \oplus E_z * \frac{\partial T}{\partial P_{zij}}.$$

From the hypothesis and (12)', we may write

$$HT_{i,j} = \frac{\partial E_z * T}{\partial P_{yij}} \oplus E_z * \frac{\partial T}{\partial P_{zij}}.$$

Therefore (by theorem 1)

$$HT_{i,j} = \frac{\partial HT}{\partial P_{yij}} \oplus \int HT dP_{zij} = \left(\frac{\partial}{\partial P_{yij}} \oplus \int dP_{zij} \right) HT.$$

The following theorem is the dual of theorem 2. The proof will not be given here.

Theorem 3

Let q_{ij} be a cutset that separates the vertices i and j in G . Then.

$$HT = \left(\frac{\partial}{\partial q_{zij}} \oplus \int dq_{yij} \right) HT_{i,j}$$

where

$$q_{ij} = q_{zij} \cup q_{yij}.$$

Theorem 4

Let $P_{i_n i_m}$ be a path between vertices i_n and j_m in G . Then,

$$HT_{i_1, i_2, \dots, i_k} = \left(\frac{\partial}{\partial P_{y i_1 i_2}} \oplus \int P_{z i_1 i_2} \right) \left(\frac{\partial}{\partial P_{y i_2 i_3}} \oplus \int dP_{z i_2 i_3} \right) \dots \left(\frac{\partial}{\partial P_{y i_{k-1} i_k}} \oplus \int dP_{z i_{k-1} i_k} \right) HT,$$

where

$$P_{i_n i_m} = P_{y i_n i_m} \cup P_{z i_n i_m}.$$

Proof: If $k = 2$, then the theorem is correct, based on theorem 2. Let us assume that the theorem is correct for $k-1$. $HT_{i_1, i_2, \dots, i_{k-1}}$ is the set of all hybrid trees of graph G' which connects the $k-1$ vertices i_1, i_2, \dots, i_{k-1} together in G . Let HT' be the set of all hybrid trees in G' .

By theorem 2,

$$HT'_{i_{k-1}, i_k} = \left(\frac{\partial}{\partial P_{y i_{k-1} i_k}} \oplus \int dP_{z i_{k-1} i_k} \right) HT'$$

Therefore

$$HT_{i_1, i_2, \dots, i_k} = HT'_{i_{k-1}, i_k} = \left(\frac{\partial}{\partial P_{y i_{k-1} i_k}} \oplus \int dP_{z i_{k-1} i_k} \right) HT_{i_1, i_2, \dots, i_{k-1}}.$$

In the following theorem, we consider fundamental circuits and hybrid trees in G .

Theorem 5

Let $L_{f_1}, L_{f_2}, \dots, L_{f_\mu}$ be fundamental circuits in G .

$$HT = \left(\frac{\partial}{\partial L_{f_{1y}}} \oplus \int dL_{f_{1z}} \right) \left(\frac{\partial}{\partial L_{f_{2y}}} \oplus \int dL_{f_{2z}} \right) \dots \left(\frac{\partial}{\partial L_{f_{\mu y}}} \oplus \int dL_{f_{\mu z}} \right) G',$$

where

$$L_{f_i} = L_{f_{iy}} \cup L_{f_{iz}} \quad (1 \leq i \leq \mu),$$

$$G' = G \oplus E_y$$

and μ is the nullity of G .

Proof: If $\mu = 1$, then

$$L_{f_{1y}} \cup L_{f_{1z}} = L_{f_1} = G$$

and

$$HT = E_z * \left(\frac{\partial G}{\partial L_{f_{1y}}} \oplus \frac{\partial G}{\partial L_{f_{1z}}} \right) = \frac{\partial G'}{\partial L_{f_{1y}}} \oplus \int G' dL_{f_{1z}}.$$

Therefore, the theorem is correct for $\mu = 1$. Assuming that the theorem is correct for $\mu - 1$. Let graph G be a graph which removes a chord $e_k (e_k \in L_{f_\mu})$ with respect to the chosen tree from G . And let HT'' be the set of all hybrid trees in G'' .

By induction hypothesis,

$$HT'' = \left(\frac{\partial}{\partial L_{f_{1y}}} \oplus \int dL_{f_{1z}} \right) \left(\frac{\partial}{\partial L_{f_{2y}}} \oplus \int dL_{f_{2z}} \right) \dots \left(\frac{\partial}{\partial L_{f_{\mu-1y}}} \oplus \int dL_{f_{\mu-1z}} \right) G''',$$

where

$$G''' = G'' \oplus E'_z$$

and

$$E'_z = \begin{cases} E_z \oplus e_k, & \text{if } e_k \in E_z \\ E_z, & \text{if } e_k \notin E_z. \end{cases}$$

G is obtained by the addition of e_k to G'' . And we know,

$$HT = HT_{(1)}^{(e_k)} \cup HT_{(0)}^{(e_k)},$$

where $HT_{(1)}^{(e_k)}$ and $HT_{(0)}^{(e_k)}$ are subsets of HT which contain e_k and which do not contain e_k , respectively. Let e_k be connected between vertices i and j in G . Then, we have two cases, namely $e_k \in E_z$ and $e_k \in E_y$.

Case 1

$$e_k \in E_z$$

Since

$$E_z = E'_z \cup e_k,$$

we have

$$G' = G \oplus E_z = G''',$$

$$HT_{(1)}^{(e_k)} = \int HT'' de_k$$

and

$$HT_{(0)}^{(e_k)} = HT''_{i,j} = \frac{\partial HT''}{\partial L_{f_{\mu y}}} \oplus \int HT'' dL'_{\mu z}$$

where

$$\begin{aligned} P_{ij} &= L_{j_{\mu}y} \cup L'_{j_{\mu}z}, \\ L_{j_{\mu}z} &= L'_{j_{\mu}z} \cup e_k \end{aligned}$$

and P_{ij} is a path between vertices i and j in G .
Therefore,

$$\begin{aligned} HT &= HT_{(1)}^{(e_k)} \cup HT_{(0)}^{(e_k)} = HT_{(1)}^{(e_k)} \oplus HT_{(0)}^{(e_k)} \\ &= \int HT'' de_k \oplus \frac{\partial HT''}{\partial L_{j_{\mu}y}} \oplus \int HT'' dL'_{j_{\mu}z} \\ &= \frac{\partial HT''}{\partial L_{j_{\mu}y}} \oplus \int HT'' dL_{j_{\mu}z} \end{aligned}$$

Since $G' = G''$, the theorem is correct for μ .

Case 2

Since

$$e_k \in E_y,$$

then

$$\begin{aligned} E_z &= E''_z, \\ HT_{(1)}^{(e_k)} &= e_k \times HT''_{1,j} = e_k \times \left(\frac{\partial HT''}{\partial L_{j_{\mu}y}} \oplus \int HT'' dL_{j_{\mu}z} \right) \end{aligned}$$

and

$$HT_{(0)}^{(e_k)} = HT'' = \frac{\partial HT'' \times e_k}{\partial e_k},$$

where

$$\begin{aligned} P_{1j} &= L'_{j_{\mu}y} \cup L_{j_{\mu}z}, \\ L_{j_{\mu}y} &= L'_{j_{\mu}y} \cup e_k \end{aligned}$$

and P_{1j} is a path between vertices i and j in G .
Therefore,

$$\begin{aligned} HT &= HT_{(1)}^{(e_k)} \cup HT_{(0)}^{(e_k)} = HT_{(1)}^{(e_k)} \oplus HT_{(0)}^{(e_k)} \\ &= \int HT'' de_k \oplus \frac{\partial HT''}{\partial L_{j_{\mu}y}} \oplus \int HT'' dL'_{j_{\mu}z} \\ &= \frac{\partial HT''}{\partial L_{j_{\mu}y}} \oplus \int HT'' dL_{j_{\mu}z}. \end{aligned}$$

Since $G' = G'' \times e_k$, the theorem is correct for μ .

In theorem 5, we use fundamental circuits in G . However the property is also correct by using independent circuits in G .

Corollary 1

Let L_1, L_2, \dots, L_μ be independent circuits in G . Then,

$$HT = \left(\frac{\partial}{\partial L_{1y}} \oplus \int dL_{1z} \right) \left(\frac{\partial}{\partial L_{2y}} \oplus \int dL_{2z} \right) \dots \left(\frac{\partial}{\partial L_{\mu y}} \oplus \int L_{\mu z} \right) G',$$

where

$$L_i = L_{iy} \cup L_{iz}, \quad (1 \leq i \leq \mu)$$

and

$$G' = G \oplus E_z.$$

The following theorem is a generalization of theorem 5 to hybrid k -trees and it can be proved as theorem 4 and theorem 5. The proof will be omitted.

Theorem 6

Let G_k be the graph obtained from G by connecting the k vertices i_1, i_2, \dots, i_k together and removing all the self loops consisting of E_y from the resulting graph.

$$HT_{i_1, i_2, i_3, \dots, i_k} = \left(\frac{\partial}{\partial L_{k1y}} \oplus \int dL_{k1z} \right) \left(\frac{\partial}{\partial L_{k2y}} \oplus \int dL_{k2z} \right) \dots \left(\frac{\partial}{\partial L_{kn_y}} \oplus \int dL_{kn_z} \right) G'_k,$$

where

$$G'_k = G_k \oplus E_z \text{ and } L_{ki} = L_{kiy} \cup L_{kiz} \quad (1 \leq i \leq n),$$

in which n and L_{ki} are nullity and an independent circuit of G_k , respectively.

We will illustrate theorem 5 by the following example.

Example 1.

Consider the graph G of Fig.2.

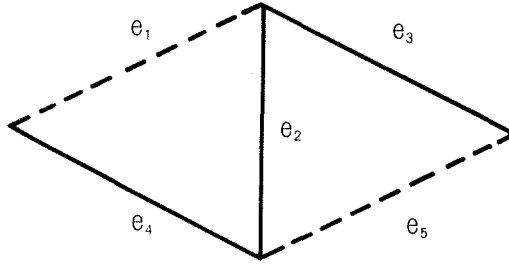


Fig. 2 Generation of hybrid trees

$$\begin{aligned} G &= \{e_1 e_2 e_3 e_4 e_5\} \\ E_z &= \{e_1 e_5\}, \quad E_y = \{e_2 e_3 e_5\} \\ G' &= G \oplus E_z = \{e_2 e_3 e_4\} \\ L_1 &= \{e_1 e_2 e_4\}, : L_{1y} = \{e_2 e_4\}, L_{1z} = \{e_1\} \\ L_2 &= \{e_2 e_3 e_4\}, : L_{2y} = \{e_2 e_3\}, L_{2z} = \{e_5\} \end{aligned}$$

HT is given as follows.

$$\begin{aligned} HT &= \left(\frac{\partial}{\partial L_{1y}} \oplus \int dL_{1z} \right) \left(\frac{\partial G'}{\partial L_{2y}} \oplus \int dL_{2z} \right) \\ &= \frac{\partial \{e_3 e_4, e_2 e_4, e_2 e_3 e_4 e_5\}}{\partial e_2 e_4} \oplus \int \{e_3 e_4, e_2 e_4, e_2 e_3 e_4 e_5\} \alpha_{e_1} \\ &= \{e_2, e_3, e_2 e_3 e_5, e_4, e_3 e_4 e_5, e_3 e_4 e_1, e_2 e_4 e_1, e_1 e_2 e_3 e_4 e_5\}. \end{aligned}$$

4. Generation of hybrid trees from a subset of HT

It can be seen that theorem 5 (theorem 6) is a method of generation of hybrid trees (hybrid k -trees). In this section, we will describe a method of generation of hybrid trees from a subset of HT , paths and cutsets in G .

We give a classification of the set of hybrid tree as follows.

$$HT = \{hT_m, hT_{m-2}, hT_{m-4}, \dots, hT_{m-k}\},$$

where hT_i is the subset of HT and any element of hT_i has the same number of edges i . That is,

$$hT_i = \{h_{zn} \cup h_{ym} \mid (h_{zn} \cup h_{ym}) \in HT, n+m = i\}.$$

If hT_i is given, then we can generate all hybrid trees in G from it. The following theorem is one of the methods.

Theorem 7

Let $H_{zn} = \{h_{zn}\}$ and h_{zn}^c be a element of H_{zn} . Let H_{ym} be the set of h_{ym} with respect to hybrid trees containing h_{zn}^c . Let e_z^c be connected between vertices i

and j .

$$(i) \quad hT_{i-2} = \bigcup_{(1, \dots, n)} \left\{ (e_z^k \oplus h_{zn}^k) \times \frac{\partial H_{ym}}{\partial P_k} \right\},$$

where $e_z^k \in h_{zn}^k$, P_k is a path between vertices i and j in graph G_k and G_k is the graph which is obtained by removing h_{zn}^k from G and by short-circuiting $(E_z - h_{zn}^k)$ in G .

$$(ii) \quad hT_{i+2} = \bigcup_{(1, \dots, n)} \left\{ (e_z^k \cup h_{zn}^k) \times \int H_{ym} dq_k \right\},$$

where $e_z^k \in h_{zn}^k$, q_k is a cutset of G_k separating the vertex i from the vertex j , and G_k is the graph which is obtained by removing h_{zn}^k and e_z^k , and by short-circuiting $(E_z - h_{zn}^k - e_z^k)$ in G .

Proof: By theorem 3, we have

$$HT = \int HT_{i,j} dq_{y^{ij}},$$

where $q_{y^{ij}} \in E_y$, and $q_{y^{ij}}$ is a cutset of G separating the vertex i from the vertex j in G . Using this equation, theorem 2 and *elementary hybrid tree transformations*, we can prove the theorem. Since *elementary hybrid tree transformations*, however, have not been described, we can not explain the details.

The operation (i), (ii) in theorem 7 can be continued until HT is obtained. We will illustrate the above results by the following example.

Example 2.

Consider the graph shown in Fig. 2.

From example 1.,

$$\begin{aligned} hT_5 &= \{e_1 e_2 e_3 e_4 e_5\} \\ h_{22} &= \{e_1 e_5\}, \quad H_{y_3} = \{e_2 e_3 e_4\} \\ hT_3 &= [\{e_5\} \oplus \{e_1 e_5\}] \times \frac{\partial \{e_2 e_3 e_4\}}{\partial e_2 e_3} \cup [\{e_1\} \oplus \{e_1 e_5\}] \times \frac{\partial \{e_2 e_3 e_4\}}{\partial e_2 e_4} \\ &= \{e_1 e_3 e_4, e_1 e_2 e_4, e_3 e_4 e_5, e_2 e_3 e_4\}. \end{aligned}$$

Let

$$h_{21} = \{e_1\}$$

and

$$H_{y_2} = \{e_3 e_4, e_2 e_4\},$$

then,

$$hT_1 = [\{e_1\} \oplus \{e_1\}] \times \frac{\partial \{e_3 e_4, e_2 e_4\}}{\partial e_2 e_4} = \{e_2, e_3, e_4\}.$$

Next, let us obtain hT_5 from

$$\{e_1\} \times \{e_3 e_4, e_2 e_4\}$$

in hT_3 .

$$q_1 = \{e_3\},$$

$$hT_5 = [\{e_5\} \cup \{e_1\}] \times \int \{e_2 e_4, e_3 e_4\} de_3 = \{e_1 e_2 e_3 e_4 e_5\}.$$

5. Conclusion

Hybrid trees are defined and some of their properties are presented. These are an extension of trees and co-trees in G . However, they have some important

properties that neither trees nor co-trees have. And further examination of hybrid trees is required which may give interesting results. For example, a method of hybrid tree generation which is more efficient than tree or co-tree generation may be derived.

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