Second-Order Coherence Effects in Optical Heterodyne Detection*

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Abstract

Effects of the second-order coherence on the signal power and signal-to-noise ratio in optical heterodyne detection were developed for systems in which the distribution of effective responsivity on the photosurface is not uniform over the detector and the local oscillator and normal incident. It is shown that the detection power and signal-to-noise ratio decrease as the local oscillator field coherence and the signal coherence are reduced. It is also shown that the properties of detection are affected by the relation between the Airy disc size and the distribution of effective responsivity.

1. Introduction

Directional characteristics, receiver design and the mutual coherence function of an optical wave in a turbulent atomosphere in optical heterodyne detection have been examined by various authors from different points of view. The effects of coherence functions in local oscillator and signal fields, however, have received little attention[1-4].

The purpose of this paper is to deduce the effects of second-order coherence functions in signal and local oscillator waves on the signal power output and signal-to-noise ratio in optical heterodyne detection when the distribution of effective responsivity on the photosurface is not uniform over the detector.

2. Second-Order Coherence Effects on Signal Power Output

The intensity at a point on the photosurface R is defined in terms of the average power flow per unit time to be[5]

$$I(\bar{R}, t)=\langle\bar{P}(\bar{R}, t)\rangle_{\mathrm{T}}\cdot\bar{n}=\langle\bar{E}^{(r)}(\bar{R}, t)\times\bar{H}^{(r)}(\bar{R}, t)\rangle_{\mathrm{T}}\cdot\bar{n} \tag{1}$$

where $\bar{P}(\bar{R}, t)$ is the Poynting vector at $\bar{R}$ and $\bar{n}$ is a unit vector normal to the photosurface. The brackets $\langle\ \rangle_{\mathrm{T}}$ represent an average over a resolving time T of the photoelectric material which is long compared to a period of signal and local oscillator waves. $\bar{E}^{(r)}(\bar{R}, t)$ is the total electric field resulting from the

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superposition of two linearly polarized waves at the point $\bar{R}$ and the associated magnetic field is $\bar{H}^{(r)}(\bar{R}, t)$. For small angles between wave vector and $\bar{n}$, Eq. (1) can be written as

$$I(\bar{R}, t) \approx \sqrt{\frac{\varepsilon}{\mu}} \langle \bar{E}^{(r)}(\bar{R}, t) \cdot \bar{E}^{(r)}(\bar{R}, t) \rangle_{\mathrm{T}} \tag{2}$$

where μ is the permeability of free space and ε is the permittivity of free space. Using complex representations for the fields, Eq. (2) becomes

$$I(R, t) \approx \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}} \bar{E}(\bar{R}, t) \cdot \bar{E}^*(\bar{R}, t) \tag{3}$$

where $\bar{E}(\bar{R}, t)$ is the complex function associated with $\bar{E}^{(r)}(\bar{R}, t)$ and it was assumed that the difference frequency between two waves is much less than T^{-1}. The incident electric field $\bar{E}$ at the point $\bar{R}$ on the photosurface is given by

$$\bar{E}(\bar{R}, t) = \bar{E}_L(\bar{R}, t) + \bar{E}_S(\bar{R}, t) \tag{4}$$

where $\bar{E}_L(\bar{R}, t)$ and $\bar{E}_S(\bar{R}, t)$ are the local oscillator field and signal field at $\bar{R}$, respectively. when $\bar{E}_L(\bar{R}, t)$ and $\bar{E}_S(\bar{R}, t)$ are quasi-monochromatic. the fields can be shown by

$$\bar{E}_L(\bar{R}, t) = \bar{A}_L(\bar{R}, t) e^{j\omega_L t} \tag{5}$$

$$\bar{E}_S(\bar{R}, t) = \bar{A}_S(\bar{R}, t) e^{j\omega_S t} \tag{6}$$

where $\bar{A}_L(\bar{R}, t)$ and $\bar{A}_S(\bar{R}, t)$, $\bar{\omega}_L$ and $\bar{\omega}_S$ are the complex envelopes and radian center frequencies of the local oscillator and signal waves. If the magnetic field for quasi-monochromatic fields is approximately orthogonally polarized to the electric field and the fields are near normal to the photosurface, the intensity at $\bar{R}$ is

$$I(\bar{R}, t) = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}} \{\bar{E}_L(\bar{R}, t) \cdot \bar{E}_L^*(\bar{R}, t) + \bar{E}_S(\bar{R}, t) \cdot \bar{E}_S^*(\bar{R}, t) + 2\mathrm{Re}[\bar{E}_L(\bar{R}, t) \cdot \bar{E}_S^*(\bar{R}, t)]\}$$

$$= I_L(\bar{R}, t) + I_S(\bar{R}, t) + \sqrt{\frac{\varepsilon}{\mu}} \mathrm{Re}[\bar{A}_L(\bar{R}, t) \cdot \bar{A}_S^*(\bar{R}, t) e^{j\bar{\omega} t}] \tag{7}$$

where $I_L(\bar{R}, t)$ and $I_S(\bar{R}, t)$ are the intensity at $\bar{R}$ due to $\bar{E}_L(\bar{R}, t)$ and $\bar{E}_S(\bar{R}, t)$ alone, respectively and $\bar{\omega} = \bar{\omega}_L - \bar{\omega}_S$. The first two terms are the constant components of the total intensity due to the local oscillator and signal fields individually, while the last term is the fluctuating component varying at the difference frequency $\bar{\omega}$.

The cutput currnt of the photodetector, $i(t)$, is proportional to the average intensity of the fields incident on its photosurface, so that[5)]

$$i(t) = \int_S \rho(\bar{R}) I(\bar{R}, t) dA = \sqrt{\frac{\varepsilon}{\mu}} \int_S \rho(\bar{R}) \langle \bar{E}^{(r)}(\bar{R}, t) \cdot \bar{E}^{(r)}(\bar{R}, t) \rangle_{\mathrm{T}} dA \tag{8}$$

where $I(\bar{R}, t)$ is the intensity at $\bar{R}$ on the photosurface S, which was shown by Eq. (2) and T is the response time of the photodetector (10^{-10} seconds for photoelectric material). The average over T, therefore, eliminates any optical frequency variations in $i(t)$. The quantity $\rho(\bar{R})$ is the effective responsivity of photoelectric material at on the photosurface, which is defined as[4, 6)]

$$\rho(\bar{R}) = Q(\bar{R}) e/hf \tag{9}$$

The quantity $Q(\bar{R})$ is the effective quantum efficiency of the photosurface at $\bar{R}$, e is the magnitude of the electronic charge, h is the Planck's constant and f is the frequency of the field. Using the result of Eq. (3), Eq. (8) can be rewritten as

$$i(t) \approx \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}} \int_S \rho(\bar{R}) \bar{E}(\bar{R}, t) \cdot \bar{E}^*(\bar{R}, t) dA \tag{10}$$

From Eqs. (7) and (10), the output current $i(t)$ becomes

$$i(t)\approx\frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}\left\{\int_S\rho(\bar{R})\bar{E}_L(\bar{R},t)\cdot\bar{E}_L^*(\bar{R},t)dA+\int_S\rho(\bar{R})\bar{E}_S(\bar{R},t)\cdot\bar{E}_S^*(\bar{R},t)dA\right.$$
$$\left.+2\mathrm{Re}\,e^{j\bar{\omega}t}\left[\int_S\rho(\bar{R})\bar{A}_L(\bar{R},t)\cdot\bar{A}_S^*(\bar{R},t)dA\right]\right\}=i_L+i_S+i_S(t) \qquad (11)$$

The terms of i_L+i_S are DC or low frequency components of the photocurrent due to the field intensities of the signal and local oscillator waves individually. The third term, $i_S(t)$, is the heterodyne signal current of the difference frequency $\bar{\omega}$. It should be noted that any amplitude or phase modulation in the signal field results in proportional changes in $\bar{A}_L(\bar{R},t)\cdot\bar{A}_S^*(\bar{R},t)$ and here $i_S(t)$.

Let us consider the case of linearly polarized in the same direction, quasi-monochromatic signal and local-oscillator plane waves superimposed and normally incident on the photosurface. The signal current $i_S(t)$ is given from Eq. (11) by

$$i_S(t)=\sqrt{\frac{\varepsilon}{\mu}}\,\mathrm{Re}\left[\int_S\rho(\bar{R})E_L(\bar{R},t)E_S^*(\bar{R},t)dA\right] \qquad (12)$$

The average signal power delivered by the photodetector to an equivalent resistance R_{eq} is given by

$$P_{Sig}=\langle i_S^2(t)R_{eq}\rangle_T=\left\langle\frac{\varepsilon}{\mu}R_{eq}\left[\mathrm{Re}\int_S\rho(\bar{R})E_L(\bar{R},t)E_S^*(\bar{R},t)dA\right]^2\right\rangle_T \qquad (13)$$

where the equivalent resistance depends on the circuit characteristics of the optical devices and its output connections. The brackets $\langle\ \rangle_T$ indicate an infinite time average. The squared integral can, therefore, be written as

$$P_{Sig}=\frac{\varepsilon}{\mu}R_{eq}\int_{S_1}\int_{S_2}\langle\mathrm{Re}[\rho(\bar{R}_1)E_L(\bar{R}_1,t)E_S^*(\bar{R}_1,t)]\mathrm{Re}[\rho(\bar{R}_2)E_L(\bar{R}_2,t)E_S^*(\bar{R}_2,t)]\rangle_T dA_1\,dA_2$$
$$=\frac{\varepsilon}{2\mu}R_{eq}\int_{S_1}\int_{S_2}\rho(\bar{R}_1)\rho(\bar{R}_2)\mathrm{Re}[\langle E_L(\bar{R}_1,t)E_L^*(\bar{R}_2,t)E_S^*(\bar{R}_1,t)E_S(\bar{R}_2,t)\rangle_T]dA_1dA_2, \qquad (14)$$

where $S_1=S_2=S$. The field processes in optical heterodyne detection can be assumed to be ergodic[7,8], so that Eq. (14) can be rewrtten as

$$P_{Sig}=\frac{\varepsilon}{2\mu}R_{eq}\int_{S_1}\int_{S_2}\rho(\bar{R}_1)\rho(\bar{R}_2)\mathrm{Re}[\langle E_L(\bar{R}_1,t)E_L^*(\bar{R}_2,t)E_S^*(\bar{R}_1,t)E_S(\bar{R}_2,t)\rangle_E]dA_1dA_2 \qquad (15)$$

Also, when the signal and local oscillator fields are statistically independent, Eq. (15) becomes

$$P_{sig}=\frac{\varepsilon}{2\mu}R_{eq}\int_{S_1}\int_{S_2}\rho(\bar{R}_1)\rho(\bar{R}_2)\mathrm{Re}[\langle E_L(\bar{R}_1,t)E_L^*(\bar{R}_2,t)\rangle_E\langle E_S^*(\bar{R}_1,t)E_S(\bar{R}_2,t)\rangle_E]dA_1dA_2$$
$$=\frac{\varepsilon}{2\mu}R_{eq}\int_{S_1}\int_{S_2}\rho(\bar{R}_1)\rho(\bar{R}_2)\mathrm{Re}[\langle E_L(\bar{R}_1,t)E_L^*(\bar{R}_2,t)\rangle_T\langle E_S^*(\bar{R}_1,t)E_S(\bar{R}_2,t)\rangle_T]dA_1dA_2 \qquad (16)$$

It is conventional to consider the coherence effects of the second order for the local oscillator and signal waves[7,8]. The mutual coherence functions are

$$\langle E_L(\bar{R}_1,t)E_L^*(\bar{R}_2,t)\rangle_T=\Gamma_{LT}(\bar{R}_1,\bar{R}_2,0)=\Gamma_{LE}(\bar{R}_1,\bar{R}_2,0)\equiv\Gamma_L(\bar{R}_1,\bar{R}_2,0) \qquad (17)$$
$$\langle E_S(\bar{R}_1,t)E_S^*(\bar{R}_2,t)\rangle_T=\Gamma_{ST}(\bar{R}_1,\bar{R}_2,0)=\Gamma_{SE}(\bar{R}_1,\bar{R}_2,0)\equiv\Gamma_S(\bar{R}_1,\bar{R}_2,0) \qquad (18)$$

where $\Gamma_{LE}(\bar{R}_1,\bar{R}_2,0)$ and $\Gamma_{SE}(\bar{R}_1,\bar{R}_2,0)$ are the mutual functions of the local oscillator and signal fields, respectively, and $\Gamma_{LT}(\bar{R}_1,\bar{R}_2,0)$ and $\Gamma_{ST}(\bar{R}_1,\bar{R}_2,0)$ are the time mutual coherence functions for the local oscillator and signal waves, respectively. Under the conditions of ergodic system the ensemble averages become time-independent and may be replaced by the corresponding time averages[8]. The average signal power, therefore, is

$$P_{sig}=\frac{\varepsilon}{2\mu}R_{eq}\int_{S_1}\int_{S_2}\rho(\bar{R}_1)\rho(\bar{R}_2)\mathrm{Re}[\Gamma_L(\bar{R}_1,\bar{R}_2,0)\overset{*}{\Gamma}_S(\bar{R}_1,\bar{R}_2,0)]dA_1dA_2 \qquad (19)$$

Using terms of the degree of coherence functions, Eq. (19) becomes

$$P_{sig}=2R_{eq}\int_{S_1}\int_{S_2}\rho(\bar{R}_1)\rho(\bar{R}_2)[I_L(\bar{R}_1)I_L(\bar{R}_2)I_S(\bar{R}_1)I_S(\bar{R}_2)]^{1/2}\mathrm{Re}\{\gamma_L(\bar{R}_1,\bar{R}_2,0)\gamma_S^*(\bar{R}_1,\bar{R}_2,0)\}dA_1dA_2 \quad (20)$$

where

$$\gamma_L(\bar{R}_1,\bar{R}_2,0)=\frac{\Gamma_L(\bar{R}_1,\bar{R}_2,0)}{[\Gamma_L(\bar{R}_1,R_1,0)\Gamma_L(\bar{R}_2,\bar{R}_2,0]^{1/2}} \quad (21)$$

$$\gamma_S(\bar{R}_1,\bar{R}_2,0)=\frac{\Gamma_S(\bar{R}_1,\bar{R}_2,0)}{[\Gamma_S(\bar{R}_1,R_1,0)\Gamma_S(\bar{R}_2,\bar{R}_2,0)^{1/2}} \quad (22)$$

$$[\Gamma_L(\bar{R}_1,\bar{R}_1,0)\Gamma_L(\bar{R}_2,\bar{R}_2,0)]^{1/2}=2\sqrt{\mu/\varepsilon}[I_L(\bar{R}_1)I_L(\bar{R}_2)]^{1/2} \quad (23)$$

$$[\Gamma_S(\bar{R}_1,\bar{R}_1,0)\Gamma_S(\bar{R}_2,\bar{R}_2,0)]^{1/2}=2\sqrt{\mu/\varepsilon}[I_S(\bar{R}_1)I_S(\bar{R}_2)]^{1/2} \quad (24)$$

and $\gamma_L(\bar{R}_1, \bar{R}_2, 0)$ and $\gamma_S(\bar{R}_1, \bar{R}_2, 0)$ are the degree of coherence of the local oscillator and signal fields, respectively.

Equation (20) shows that reductions in either the local oscillator coherence or the signal field coherence result in a decrease in the average signal power delivered by the photodetector. It is also concluded that the detected power is affected by the relation between the Airy disc size and the distribution of effective responsivity on the photosurface.

3. Noise Considerations

The principal sources of noise in detectors are thermal noise and shot noise. The thermal noise sources are related to the input termination and the loss of the circuit itself. In general the effects of the gain on these power outputs must be considered, but for small interaction region the gain is negligible and the thermal noise power output P_t is simply

$$P_t=kTB \quad (25)$$

where k is the Boltzmann's constant, T is the temperature of the circuit at the input terminal and B is the equivalent rectangular bandwidth of the detector. The mean square shot noise current, $\langle i_N^2\rangle_T$, at the photosurface is[6]

$$\langle i_N^2\rangle_T=2eB\Big[\int_S\rho(\bar{R})I_L(\bar{R})dA+\int_S\rho(\bar{R})I_S(\bar{R})dA+\int_S\rho(\bar{R})P_B(\bar{R})dA+\int_S\rho(\bar{R})P_{nL}(\bar{R})dA+\int_S\rho(\bar{R})P_D(\bar{R})dA\Big]+\langle i_{\mathrm{mix}}^2\rangle_T \quad (26)$$

where the integral terms of $\int_S\rho(\bar{R})I_L(\bar{R})dA$ and $\int_S\rho(\bar{R})I_S(\bar{R})dA$ and the DC current due to $\bar{E}_L(\bar{R},t)$ and $\bar{E}_S(\bar{R},t)$ alone. P_B is the noncoherent background radiation power acting on the photosurface, P_{nL} is the shot noise power due to the sidebands caused by multimoding or modulation of the local oscillator laser, P_D is the equivalent power of dark current and $\langle i_{\mathrm{max}}^2\rangle_T$ is the mean square current due to the local oscillator self and background beats in the bandwidth B. Eq. (20) shows that the higher the local oscillator power the greater is the average signal level from the detector. However the local oscillator can have sidebands, although very small, which lie at or near the signal carrier frequency and in the frequency bandwidth B of the detector. Thus, as the local oscillator is increased in power, a signal-like noise composed of mixed local oscillator products can override the signal level to be detected. Heterodyning of the local oscillator with background can also produce other extraneous beats, which could limit sensitivity, but these beats will be negligible for noncoherent background. Then the total mean square current due to these mixed products is the sum of the individual mean square values lying in the bandwidth B

$$\langle i_{\mathrm{mix}}^2\rangle_{\mathrm{T}}=\sum_B\langle i_n^2\rangle_{\mathrm{T}} \tag{27}$$

In general the noise output due to velocity fluctuations of electrons is generally much less than the shot noise power. Then the shot noise power P_N is conveniently written as

$$P_N=\langle i_N^2\rangle_{\mathrm{T}}R_{eq}=2eBR_{eq}\left[\int_S\rho(\bar{R})I_L(\bar{R})dA+i_{en}\right]+\sum_B\langle i_n^2\rangle_{\mathrm{T}}R_{eq} \tag{28}$$

where

$$i_{en}=\int_S\rho(\bar{R})I_S(\bar{R})dA+\int_S\rho(\bar{R})P_B(\bar{R})dA+\int_S\rho(\bar{R})P_{nL}(\bar{R})dA+\int_S\rho(\bar{R})P_D(\bar{R})dA \tag{29}$$

Therefore, the overall carrier signal-to-noise ratio becomes

$$\frac{S}{N}=\frac{P_{Sig}}{P_t+P_N}$$
$$=\frac{2R_{eq}\int_{S1}\int_{S2}\rho(\bar{R}_1)\rho(\bar{R}_2)[I_L(\bar{R}_1)I_L(\bar{R}_2)I_S(\bar{R}_1)I_S(\bar{R}_2)]^{1/2}\mathrm{Re}[\gamma_L(\bar{R}_1,\bar{R}_2,0)\gamma_S^*(\bar{R}_1,\bar{R}_2,0)]dA_1dA_2}{kTB+2eBR_{eq}\left[\int_S\rho(\bar{R})I_L(\bar{R})dA+i_{en}\right]+\sum_B\langle i_n^2\rangle_{\mathrm{T}}R_{eq}} \tag{30}$$

If the local oscillator power is large compared to the random noise power and the signal carrier power, the signal-to-noise ratio becomes

$$\frac{S}{N}=\frac{\int_{S1}\int_{S2}\rho(\bar{R}_1)\rho(\bar{R}_2)[I_L(\bar{R}_1)I_L(\bar{R}_2)I_S(\bar{R}_1)I_S(\bar{R}_2)]^{1/2}\mathrm{Re}[\gamma_L(\bar{R}_1,\bar{R}_2,0)\gamma_S^*(\bar{R}_1,\bar{R}_2,0)]dA_1dA_2}{eB\int_S\rho(\bar{R})I_L(\bar{R})dA} \tag{31}$$

Equation (31) means that in the case in which the detection is limited by the local oscillator shot noise a decrease in the coherence either the signal or local oscillator waves causes a reduction in the signal-to-noise ratio of an optical heterodyne detection and the ratio is affected by the relation between the Airy disc size and the distribution of effective responsivity.

4. Conclusions

Effects of the second-order coherence of the signal and local oscillator waves on the signal power output and the signal-to-noise ratio in optical heterodyne detection were analyzed. The results indicate that the properties of detection are influenced by the signal and local oscillator coherence functions and the relation between the Airy disc and the distribution of effective responsivity on the photosurface.

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