An Analysis of Distribution Networks for Leakage Prevention

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Abstract

The method for analyzing the distribution of flow and pressure in a water distribution system is well established and has been in use for a long time. However, this method is of little use in preventing leakage because of the difficulty of separating leakage from demand.

In order to save water lost by leakage, it is necessary to reduce excessive pressure as well as making repairs of water mains and services.

It is the purpose of this paper to develop a method for determining leakage in distribution systems and to demonstrate the application of this method to leakage prevention.

As a result of analyzing a system of pipe network specimen, it was shown that both the nodal residual pressures and the total retes of leakage change almost linearly with the source head.

1. Introduction

Leakage in any water distribution system is always a problem. It is reported in this country that yearly leakage amounts to about 2,000,000,000 m^3 and is about 20% of the total supply[1]. From a view point of water resource economy, prevention of such extensive leakage is one of the most important subjects to be met with.

It is common practice in the presence of leakage in water mains and services to make repairs while conducting leakage survey. On the other hand, it has been suggested that the pressure reduction at the source of the distribution networks is also effective in the prevention of leakage[2]. The present investigation is an exploration of the latter case.

Hitherto, the standard method for analyzing the distribution of flow and pressure in a water distribution system has been the Hardy Cross method. In this method, however, the nodal discharge includes not only the rate of water demand but also the rate of leakage, so that the effect of leakage on the distribution of pressure and the relation of leakage to the source head cannot be clarified.

In this paper, a method for analyzing pipe networks will be developed in such a way that the rate of leakage is separated from the rate of demand. Some illustrative examples will also be presented to show the effectiveness of the method for the prevention of leakage.

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2. Method of Analysis

The Darcy-Weisbach equation states that the relationship between water flow and head loss in a pipe is:

$$H=f\frac{L}{D}\frac{V^2}{2g}=f\frac{8}{\pi^2 g}\frac{Q^2}{D^5} \tag{1}$$

where H is the head loss, in (m); f, dimensionless friction factor; L, length of pipe, in (m); D, diameter of pipe, in (m); V, average velocity of flow, in (m/sec); g, acceleration of gravity, in (m/sec^2); and Q, rate of flow, in (m^3/sec).

Let us designate the pipe connecting the nodes i and j as ij. Then, the relationship between the head loss H_{ij} in the pipe ij and the hydraulic elevations E_i and E_j, in (m), at the nodes i and j may be respectively written as:

$$H_{ij}=E_i-E_j \tag{2}$$

By substituting Eq. (2) into Eq. (1) and giving a solution for Q_{ij},

$$Q_{ij}=\frac{1}{\sqrt{f_{ij}}}\sqrt{\frac{D_{ij}^5}{L_{ij}}\frac{\pi^2 g}{8}}\frac{E_i-E_j}{\sqrt{|E_i-E_j|}}=\frac{M_{ij}}{\sqrt{f_{ij}}}\frac{E_i-E_j}{\sqrt{|E_i-E_j|}} \tag{3}$$

where

$$M_{ij}=\sqrt{\frac{D_{ij}^5}{L_{ij}}\frac{\pi^2 g}{8}} \tag{4}$$

The value of f_{ij} in Eq. (3) may be found by means of the Colebrook-White equation, as follows:

$$\begin{aligned}\frac{1}{\sqrt{f_{ij}}}&=1.14-2\log\left(\frac{k_{ij}}{D_{ij}}+\frac{9.35}{\boldsymbol{R}_{ij}\sqrt{f_{ij}}}\right)\\&=1.14-2\log\left\{\frac{k_{ij}}{D_{ij}}+\frac{9.35\nu}{\sqrt{|E_i-E_j|}}\sqrt{\frac{L_{ij}}{(2gD_{ij})^3}}\right\}\\&=1.14-2\log\left(\frac{k_{ij}}{D_{ij}}+\frac{N_{ij}}{\sqrt{|E_i-E_j|}}\right)\end{aligned} \tag{5}$$

where k_{ij} is the absolute roughness of the pipe ij, in (m); ν, kinematic viscosity of the water, in (m^2/sec); $\boldsymbol{R}_{ij}$, the Reynolds number of the pipe ij, defined as $D_{ij}V_{ij}/\nu$; and

$$N_{ij}=9.35\nu\sqrt{\frac{L_{ij}}{(2gD_{ij})^3}} \tag{6}$$

By substituting Eq. (5) into Eq. (3),

$$Q_{ij}=\frac{M_{ij}(E_i-E_j)}{\sqrt{|E_i-E_j|}}\left\{1.14-2\log\left(\frac{k_{ij}}{D_{ij}}+\frac{N_{ij}}{\sqrt{|E_i-E_j|}}\right)\right\} \tag{7}$$

Eq. (7) is nonlinear with E_i and E_j, so that in order to linearlize it the Newton method[3] is followed.

Let the assumed or approximated values of Q_{ij}, E_i and E_j denote q_{ij}, e_i and e_j respectively, and let the correction factors of Q_{ij}, E_i and E_j be ΔQ_{ij}, ΔE_i and ΔE_i

respectively, then

$$Q_{ij}=q_{ij}+\varDelta Q_{ij}=q_{ij}+\left(\frac{\partial Q_{ij}}{\partial E_i}\right)_{\substack{E_i=e_i\\E_j=e_j}}\varDelta E_i+\left(\frac{\partial Q_{ij}}{\partial E_i}\right)_{\substack{E_i=e_i\\E_j=e_j}}\varDelta E_i \tag{8}$$

$$E_i=e_i+\varDelta E_i \tag{9}$$

and

$$E_j=e_j+\varDelta E_j \tag{10}$$

The two partial derivatives in Eq. (8) are obtainable by partially differentiating Eq. (7) with respect to E_i, and E_i. Thus,

$$\begin{aligned}\frac{\partial Q_{ij}}{\partial E_i}&=\frac{0.5M_{ij}}{\sqrt{|E_i-E_j|}}\left\{1.14-2\log\left(\frac{k_{ij}}{D_{ij}}+\frac{N_{ij}}{\sqrt{|E_i-E_j|}}\right)\right\}\\&\quad+\frac{M_{ij}N_{ij}}{2.303|E_i-E_j|\left(\frac{k_{ij}}{D_{ij}}+\frac{N_{ij}}{\sqrt{|E_i-E_j|}}\right)}\\&=0.5\,Y_{ij}+Z_{ij}\end{aligned} \tag{11}$$

and

$$\frac{\partial Q_{ij}}{\partial E_i}=-0.5\,Y_{ij}-Z_{ij} \tag{12}$$

where

$$Y_{ij}=\frac{N_{ij}}{\sqrt{|E_i-E_j|}}\left\{1.14-2\log\left(\frac{k_{ij}}{D_{ij}}+\frac{N_{ij}}{\sqrt{|E_j-E_i|}}\right)\right\} \tag{13}$$

and

$$Z_{ij}=\frac{M_{ij}N_{ij}}{2.303|E_i-E_j|\left(\frac{k_{ij}}{D_{ij}}+\frac{N_{ij}}{\sqrt{|E_i-E_j|}}\right)} \tag{14}$$

Express Y_{ij} in Eq. (13) and Z_{ij} in Eq. (14) in terms of e_i and e_j, and let them denote y_{ij} and Z_{ij} respectively. Then, substituting Eqs. (11) and (12) into Eq. (8) gives the rate of flow corrected by the Newton method as follows:

$$Q_{ij}=y_{ij}(e_i-e_j)+(0.5\,y_{ij}+z_{ij})\,\varDelta E_i-(0.5\,y_{ij}+z_{ij})\,\varDelta E_i \tag{15}$$

For the first term of the right hand side of Eq. (15), the following relationship is used:

$$q_{ij}=y_{ij}(e_i-e_j) \tag{16}$$

which is obtained by a substitution of Eq. (13) into Eq. (7).

Up to this point the rate of flow in the pipe ij has been derived. Next, the flow balance at the node i will be taken into account.

The discharge at the node i is composed of the rate of water withdrawn by consumers, or demand, p_i and the rate of leakage $\varLambda_i$, both in (m^3/sec), which are assumed to be concentrated at that node. By making use of the experimental results obtained by Sueishi and Ogura[4], the relation of $\varLambda_i$ to the residual pressure (E_i-G_i) at the node i may be written as:

$$\Lambda_i = a_i(E_i - G_i)^b \tag{17}$$

where a_i is a coefficient dependent on the shape and size of leaks, and length of water mains and services connected to the node i; b, an exponent which is found to be about 1.15; and G_i, the topographic elevation at the node i, in (m).

Eq. (17) should also be linearized. Here, if we designate the assumed or approximated values of the rate of leakage as λ_i, and its correction factor as $\Delta\Lambda_i$, then the Newton method gives the following equation:

$$\Lambda_i = \lambda_i + \Delta\Lambda_i = a_i(e_i - G_i)^b + a_i b(e_i - G_i)^{b-1}\Delta E_i \tag{18}$$

Since the algebraic sum of all flows at any nodes together with the rate of demand and the rate of leakage must be zero, the following equation holds at the node i:

$$\left.\begin{aligned} &\sum_j Q_{ij} + p_i + \Lambda_i = 0 \\ &\qquad (i = 2,\ 3,\ \cdots,\ n) \end{aligned}\right\} \tag{19}$$

where j is the adjacent node to the node i; and n, the total number of nodes ($i=1$ refers to the source point). Here, the sign of p_i is positive for withdrawal. By substitution of Eqs. (15) and (17) into Eq. (19) and rearranging,

$$\left.\begin{aligned} &\Delta E_i\left\{\sum_j (0.5\,y_{ij} + z_{ij}) + a_i b(e_i - G_i)|e_i - G_i|^{b-2}\right\} \\ &\quad - \sum_j (0.5\,y_{ij} + z_{ij})\,\Delta E_j = -\left\{\sum_j y_{ij}(e_i - e_j) + p_i + a_i(e_i - G_i)|e_i - G_i|^{b-1}\right\} \\ &\quad (i = 2,\ 3,\ \cdots,\ n) \end{aligned}\right\} \tag{20}$$

This is a linear set of simultaneous equations for $(n-1)$ correction factors of hydraulic elevation.

To deterimine the hydraulic elevations by means of Eq. (20), initial hydraulic elevations at the all nodes in a system are assumed. The assumed hydraulic elevations are then changed by their calculated correction factors, and the process is repeated until a satisfactory balance in Eq. (19) is achieved at each node in the system. Then, the rate of leakage at each node will be evaluated by using Eq. (17).

The method can always become conversible by introducing the improvement for convergence reported in the preceding paper[5] for the case of no leakage.

3. Calculations for a Specimen Network

3.1 Calculation conditions

The street plan of the distribution system used here are shown in Fig. 1. The calculation conditions are as follows;

1) The pipe diameters are as shown in Fig. 1.
2) All the lengths of pipe are 1,000 m.
3) The value of the exponent b in Eq. (17) is taken as 1.1.
4) The absolute roughness of every pipe is 0.001 m.
5) The kinematic viscosity of the water is 1.141×10^{-6} m/sec.
6) The topographic elevation G_i and the maximum rate of demand $p_{i,\max}$ at

each node are as listed in Table 1 for the three cases.

7) The value of the coefficient a_i in Eq. (17) is identical at every node for cases (I) and (II), and different from node to node for case (III) as listed in Table 1.

8) In the lower demand hours in a day, the rate of demand at each node is given by multiplying its maximum value $p_{i,\max}$ by a factor α which is less than unity.

9) The minimum allowable error of Eq. (19) is 0.00005 m³/sec.

10) The minimum required residual pressure at each node is 15 m. It is assumed that there is no suppression of water demand under the residual pressures higher than this limit.

11) The source head or the hydraulic elevation at the location of the source of supply (node 1 in Fig. 1) is maintained at 35 m when the pressure-reducing valve located at this point is not activated.

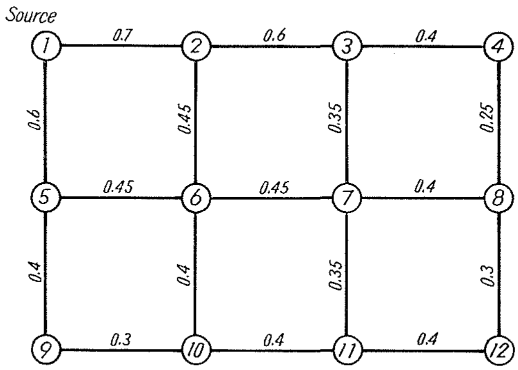

Fig. 1. Street plan of system

Numerals adjacent to the lines represent their diameters in (m). Nodes are designated by circled numerals.

Table 1. Maximum rate of demand and topographic elevation at each node

Node	Case (I)		Case (II)		Case (III)		
(i)	$p_{\max,i}$ (m³/sec)	G_i (m)	$p_{\max,i}$ (m³/sec)	G_i (m)	$p_{\max,i}$ (m³/sec)	G_i (m)	a_i (m-sec unit)
2	0.1	0.0	0.1	8.0	0.05	5.0	0.0004
3	0.1	0.0	0.1	6.0	0.15	5.0	0.0001
4	0.1	0.0	0.1	4.0	0.10	0.0	0.0005
5	0.1	0.0	0.1	8.0	0.15	0.0	0.0005
6	0.1	0.0	0.1	6.0	0.05	5.0	0.0003
7	0.1	0.0	0.1	4.0	0.10	0.0	0.0003
8	0.1	0.0	0.1	2.0	0.15	5.0	0.0002
9	0.1	0.0	0.1	6.0	0.10	10.0	0.0002
10	0.1	0.0	0.1	4.0	0.15	5.0	0.0007
11	0.1	0.0	0.1	2.0	0.05	0.0	0.0005
12	0.1	0.0	0.1	0.0	0.10	0.0	0.0007

3.2 Distribution of resulting pressures

In the conventional method for analyzing the distribution of pressures in a water distribution system, the discharge out of each node includes not only the rate of demand but also the rate of leakage. In the following, a result obtained by the conventional method is compared with the one which is given by using Eq. (20).

For cases (I) and (III), the total discharge, the total rate of leakage, and the

Table 2. Total discharge, total rate of leakage and percentage of leakage

α	Item		Case (I)		Case (III)
			$a=0.0002$	$a=0.0005$	
1.0	Total discharge	(m^3/sec)	1.17007	1.26104	1.25744
	Total rate of leakage	(m^3/sec)	0.07007	0.16104	0.10744
	Percentage of leakage	(%)	6.0	12.8	8.5
0.8	Total discharge	(m^3/sec)	0.96262	1.07158	1.05448
	Total rate of leakage	(m^3/sec)	0.08262	0.19158	0.13448
	Percentage of leakage	(%)	8.6	17.9	12.8
0.6	Total discharge	(m^3/sec)	0.75300	0.87819	0.84783
	Total rate of leakage	(m^3/sec)	0.09300	0.21819	0.15783
	Percentage of leakage	(%)	12.3	24.8	18.6
0.4	Total discharge	(m^3/sec)	0.54105	0.68035	0.63692
	Total rate of leakage	(m^3/sec)	0.10105	0.24035	0.17692
	Percentage of leakage	(%)	18.7	35.3	27.8
0.2	Total discharge	(m^3/sec)	0.32657	0.47750	0.42123
	Total rate of leakage	(m^3/sec)	0.10657	0.25750	0.19123
	Percentage of leakage	(%)	32.6	53.9	45.4

Table 3. Difference of hydraulic elevations in (m) at node 12 calculated by the conventional method and Eq. (20)

α	Method	Case (I)		Case (III)
		$a=0.0002$	$a=0.0005$	
1.0	Conventional	17.683	14.907	15.069
	Eq. (20)	17.894	15.536	15.169
	Difference	−0.211	−0.629	−0.100
0.8	Conventional	23.241	20.457	20.947
	Eq. (20)	23.362	20.853	20.815
	Difference	−0.121	−0.396	0.132
0.6	Conventional	27.769	25.196	25.880
	Eq. (20)	27.828	25.421	25.605
	Difference	−0.059	−0.225	0.285
0.4	Conventional	31.234	29.083	29.820
	Eq. (20)	31.257	29.190	29.490
	Difference	−0·023	−0.107	0.330
0.2	Conventional	33.602	32.055	32.055
	Eq. (20)	33.607	32.094	32.399
	Difference	−0.005	−0.039	0.306

percentage of leakage that is calculated by the following equation:

$$\text{Percentage of leakage} = \frac{\text{Total rate of leakage}}{\text{Total discharge}} \times 100$$

are summarized in Table 2 with respect to different values of α. The hydraulic elevations at node 12 are listed in Table 3, where those in the row designated as "Conventional" are calculated under the conditions that the value of a_i in Eq. (20) is zero at every node and that the discharge at every node is equal to its rate of demand multiplied by the ratio of the total discharge to the total rate of demand.

From the results listed in Table 3, it might be said that the conventional method could show nearly the same pressure distribution as Eq. (20) even if the percentage was considerable as seen in Table 2. It follows that the determination of diameter might be attained as well by using the conventional method as Eq. (20). From a view point of leakage prevention, however, the rate of demand and the rate of leakage must be separated from one another, thus Eq. (20) should be used.

3.3 Change of hydraulic elevation with source head

In Figs. 2, 3 and 4, the hydraulic elevations at nodes of the lowest

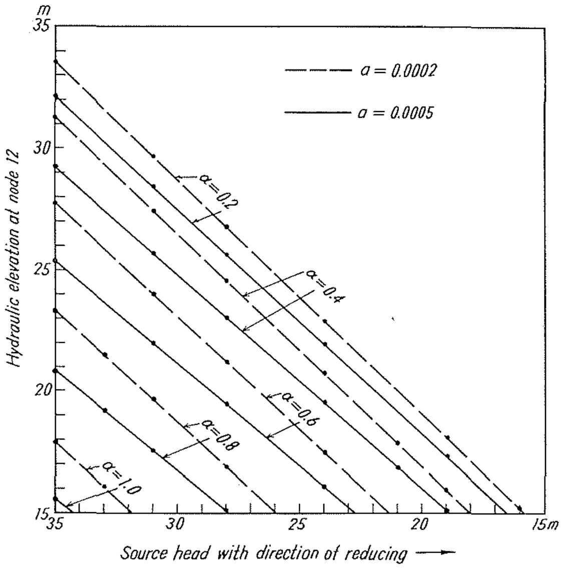

Fig. 2. Hydraulic elevation at node 12 *vs.* source head in case (I)

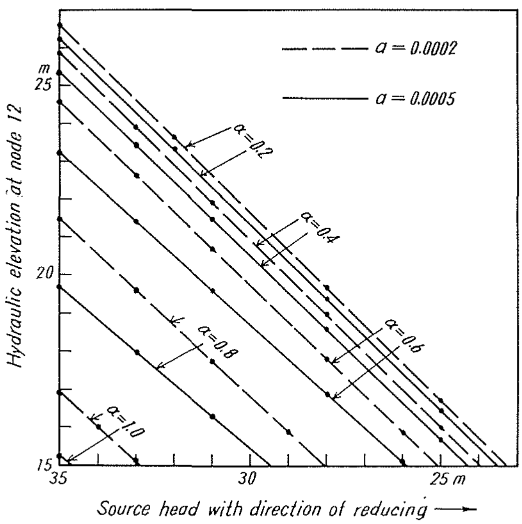

Fig. 3. Hydraulic elevation at node 12 *vs.* source head in case (II)

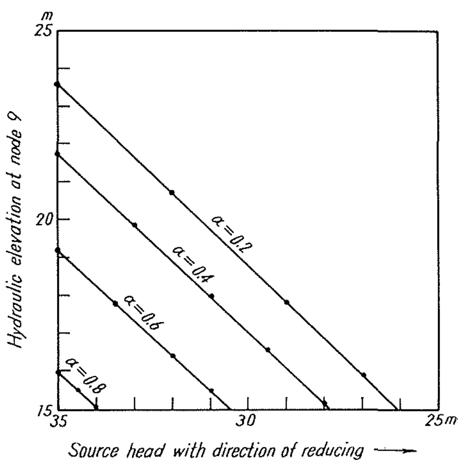

Fig. 4. Hydraulic elevation at node 9 *vs.* source head in case (III)

residual pressure are plotted against the source head. It can be seen that the hydraulic elevations change almost linearly with the source head in all cases.

The gradient appears to be milder than −1. The value of −1 holds if no leakage flow occured and demands were neither encouraged nor suppressed under the residual pressures exceeding 15 m.

3.4 Change of rate of leakage with source head

In Figs. 5, 6, and 7, the total rates of leakage are plotted against the source head. It can be seen in every case that the tatal rate of leakage changes almost linearly with the source head. In these figures, the crossed points correspond to

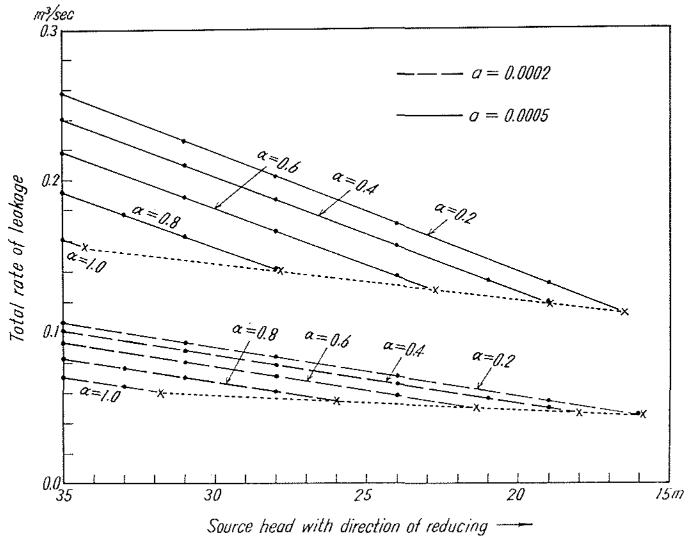

Fig. 5. Total rate of leakage *vs.* source head in case (I)

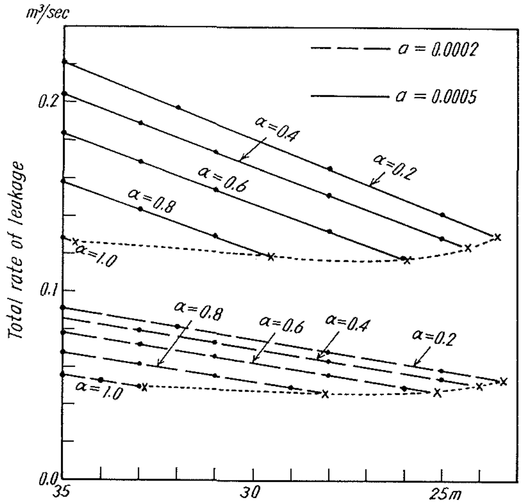

Fig. 6. Total rate of leakage *vs.* source head in case (II)

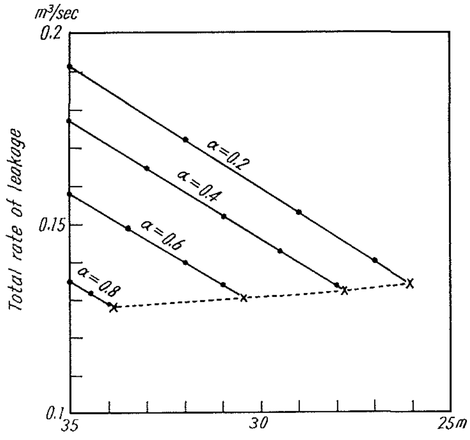

Fig. 7. Total rate of leakage *vs.* source head in case (III)

the limiting values of source head below which the minimum required residual pressure of 15 m cannot be assured.

In these calculations, the source head is maintained at 35 m when the pressure-reducing valve is not activated, so that the difference between the ordinates for this value and the limiting one implies the possible rate of leakage prevented by pressure reduction. Naturally, the lower the rate of demand, the more the amount of pressure reduction at the source; thus the possible amount of leakage to be prevented is increased.

The lines marked by the different values of α in Figs. 5, 6 and 7 are almost in parallel under the same leakage conditions. Hence, it is expected that for a given amount of pressure reduction an identical rate of leakage might be prevented during a day.

Comparing Fig. 5 with Fig. 6, and from Fig. 7, it can be said that the topography scarcely has any effect on the rate of leakage *vs.* source head relationship as described above.

3.5 An example for hourly varition in demand

In general, water demand varies hourly during a day, accompanying hourly variation in rate of leakage. Hence, the possible rate of leakage prevented by reduction of pressure at the source varies from hour to hour. In the following description, for a typical hourly demand variation the total amount of leakage prevented a day will be illustrated. The calculation conditions are as in case (I) and the value of a_i in Eq. (17) is assumed to be 0.0005 (m-sec unit) at every node.

In Fig. 8, the full line and the broken line represent the hourly variations in the total discharge and the total rate of demand respectively. The total discharge implicitly includes the total rate of leakage in addition to that of demand. In this case the source head was kept at 35 m.

When the pressure-reducing valve located at the source point is activated, the variation pattern of the discharge is shifted from that of the full line to that of

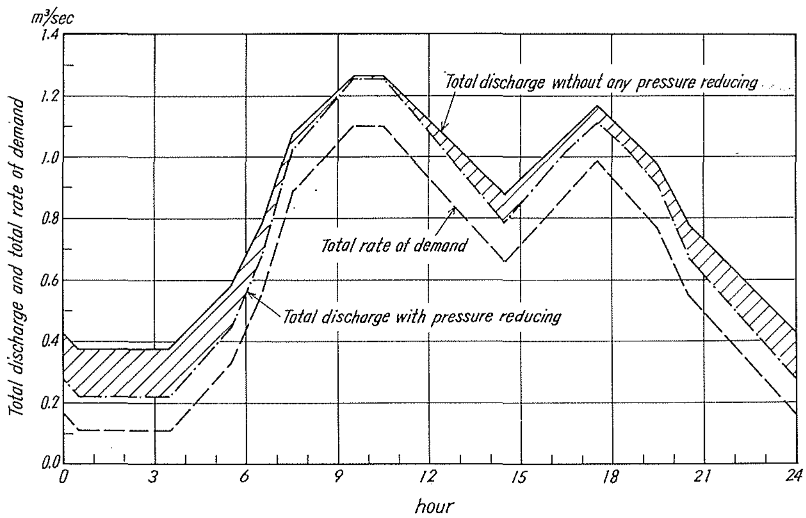

Fig. 8. Hourly variations in total discharge and total rate of demand

Table 4. Summary of the result shown in Fig. 8

	Total amount of dischage a day	Total amount of leakage a day	Percentage of leakage as a whole
without any pressure reducing	71,678 m^3	18,814 m^3	26.2%
with pressure reducing	64,064	11,200	17.4
difference	7,614	7,614	8.8

the chained line. The hatched area represents the amount of leakage prevented a day. Otherwise, the area enclosed by the chained line and the broken line shows the amount of leakage which cannot be controlled by reduction of pressure at the source if the minimum required residual pressure of 15 m is to be kept at every node.

The result is summarized in Table 4. In this example, the percentage of leakage as a whole was reduced from 26.2 to 17.4%, and the total amount of leakage was prevented by 7,614 m^3 or 40.5%.

4. Conclusion

(1) By combining the method of pipe network analysis in which the Colebrook-White equation is applied and the Sueishi-Ogura equation which relates the rate of leakage to the residual pressure, Eq. (20) was derived as a basic equation to discuss the leakage in distribution pipe systems.

(2) For a specimen network, the conventional method, in which the nodal discharge includes both the rates of demand and leakage, gave nearly the same pressure distribution as Eq. (20).

(3) The residual pressure at each node was seen to change almost linearly with the source head even for a high rate of leakage.

(4) The total rate of leakage was found to decrease almost linearly by reduction of pressure at the source and at nearly the same rate for any uniform hourly demand variation.

(5) For a typical hourly demand variation, the effectiveness and limitation of the leakage prevention by means of pressure reducing were demonstrated.

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