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Author(s)	Takai, Nobukatsu
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Experimental Study on the First-Order Probability Density of Speckles

Nobukatsu TAKAI*

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Abstract

The first-order probability density of speckle intensity variations detected with a finite-size circular aperture was experimentally investigated. The approximate probability density, known as a gamma distribution, is applicable for detecting a large number of speckles in excess of approximately ten. When the gamma probability distribution is applicable, the number of the correlation cells in speckles is found by determining the variations. For the detection of a small number of speckles, an exact solution of the probability density is required.

1. Introduction

Numerous studies on the first-order statistics of speckles detected with a finite-size aperture have been performed by various workers¹⁻⁹⁾. This problem is most fundamental and important for revealing the statistical properties of speckles and indicating the usefulness of speckle phenomena in optical measurements^{10, 11)}. It is known that, for a gaussian amplitude light field, the exact solution of the probability density function of the speckle variation detected with a finite aperture is expressed by the integral^{2, 4, 6, 9)}

$$P(I) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-izI}}{\prod_{n=1}^{\infty} (1 - iz\lambda_n)} dz \quad (1)$$

where λ_n indicates mode energies for eigenstates orthonormal over the detecting aperture. Barakat³⁾ presented some numerical evaluations of Eq. (1) for a slit aperture under the assumption of the sinc and gaussian correlations of speckles within the detecting aperture. However, expression (1) is too formalized to be connected with the experimental data. For such a purpose, it is more desirable to find an approximate solution corresponding to Eq. (1) which may be obtained by assuming $\lambda_1 = \lambda_2 = \dots = \lambda_M \equiv a$ (the condition of a short correlation range) in Eq. (1). This assumption produces a gamma probability density function of speckles,

$$P(I) = \frac{I^{M-1}}{a^M \Gamma(M)} e^{-I/a} \quad (2)$$

where $\Gamma(M)$ is a gamma function and M means the number of correlation cells detected¹²⁾, i. e. the number of independent speckles contained within a detecting aperture.

To date, however, the availability of the approximate probability density function of Eq. (2) seems not to be clear because almost all of the past investigations in this field were performed on a theoretical basis rather than an experimental one. In this report, the probability density of speckles is studied experimentally

* Department of Applied Physics, Faculty of Engineering, Hokkaido University, Sapporo, Japan.

by changing the sizes of a detecting circular aperture and speckles with special reference to the approximate probability density function. The limit of the availability of the approximate probability function is also discussed on the basis of experimental results with respect to the values of M .

2. The number of correlation cells

Before dealing with the experiments, a theoretical examination considering an actual situation to be used later was made, and the mean number of speckles produced by a gaussian laser beam and detected with a finite-size aperture was calculated. The amplitude distribution of speckles in the Fraunhofer diffraction field, a distance R away from the scattering object, is expressed by

$$f(\mathbf{x}) = \int A(\mathbf{u}) \exp\{i\theta(\mathbf{u})\} \exp\left(-i\frac{2\pi}{\lambda R}\mathbf{xu}\right) d\mathbf{u} \quad (3)$$

where \mathbf{x} and \mathbf{u} are position vectors at the scattering and detecting planes, λ is the wavelength of light used and $\theta(\mathbf{u})$ and $A(\mathbf{u})$ are a random phase of the scattering object and the amplitude distribution of the gaussian laser-beam illumination. If the random amplitude $f(\mathbf{x})$ of speckles obeys a stochastic gaussian process, the intensity correlation of speckle patterns is characterized by an absolute square of the second-order correlation of $f(\mathbf{x})$:

$$|\langle f(\mathbf{x})f^*(\mathbf{x} + \Delta\mathbf{x}) \rangle|^2 = \left| \iint A(\mathbf{u}_1)A^*(\mathbf{u}_2) \langle e^{i(\theta(\mathbf{u}_1) - \theta(\mathbf{u}_2))} \rangle \exp\left[-i\frac{2\pi}{\lambda R}(\mathbf{xu}_1 - \mathbf{xu}_2 - \Delta\mathbf{xu}_2)\right] d\mathbf{u}_1 d\mathbf{u}_2 \right|^2 \quad (4)$$

where $\langle \dots \rangle$ stands for an ensemble average. Let the phase $\theta(\mathbf{u})$ of the object vary largely with a certain degree so that

$$\langle \exp i[\theta(\mathbf{u}_1) - \theta(\mathbf{u}_2)] \rangle = \delta(\mathbf{u}_1 - \mathbf{u}_2). \quad (5)$$

Then, Eq. (4) becomes

$$|\langle f(\mathbf{x})f^*(\mathbf{x} + \Delta\mathbf{x}) \rangle|^2 = \left| \int |A(\mathbf{u})|^2 \exp\left(-i\frac{2\pi}{\lambda R}\Delta\mathbf{xu}\right) d\mathbf{u} \right|^2. \quad (6)$$

Since a gaussian laser beam is employed for the illumination, $A(\mathbf{u})$ in Eq. (6) is set to be

$$A(\mathbf{u}) = \exp\left(-\frac{|\mathbf{u}|^2}{2w^2}\right) \quad (7)$$

where w is the beam width. Substitution of Eq. (7) into (6) yields

$$|\langle f(\mathbf{x})f^*(\mathbf{x} + \Delta\mathbf{x}) \rangle|^2 = \frac{w^2}{2} \exp\left(-\frac{2\pi^2 w^2}{\lambda^2 R^2} \Delta\mathbf{x}^2\right). \quad (8)$$

The mean radius of correlation cells in speckles is obtained by investigating the extent of this correlation and given by

$$\Delta\mathbf{x}_e = \frac{\lambda R}{\sqrt{2}\pi w} \quad (9)$$

where the extent of $1/e$ of the correlation (8) is taken. Therefore, the number of speckles detected with the circular aperture of radius D is given by

$$M = \frac{2\pi^2 D^2 w^2}{\lambda^2 R^2} \quad (10)$$

which is related only to constants inherent in the experimental condition used.

Consider next another means of obtaining statistically the number M of correlation cells in speckles. When the approximate probability density function of Eq. (2) is applicable, it can be obtained from the experiments in which the probability densities of speckle variations detected with the aperture are analyzed numerically. For this purpose, it is convenient to use the probability density function expressed by

$$P(I') = \frac{M^M I'^{M-1}}{\Gamma(M)} e^{-MI'} \quad (11)$$

where the change of variables by $I' = I/\langle I \rangle$ is taken. Obviously the mean of I' is unity and the variance of speckle intensity variations is given by

$$\langle I'^2 \rangle - \langle I' \rangle^2 = \frac{1}{M} \quad (12)$$

which indicates that the number M of correlation cells is directly related to the variance of speckles. Therefore, by investigating experimentally the probability density in terms of I' and calculating its variance, the values of M are obtained.

3. Experiments and discussion

The experimental arrangement is schematically shown in Fig. 1. The gaussian light from a He-Ne laser ($\lambda = 0.633 \mu\text{m}$) was diverged by an objective lens L and employed for the illumination of the scattering object (ground glass). The extent of the gaussian light on the object was changed by moving the objective lens along the optical axis and, consequently, the sizes of observed speckles were controlled appropriately. The intensity of speckle patterns was detected by the photomultiplier

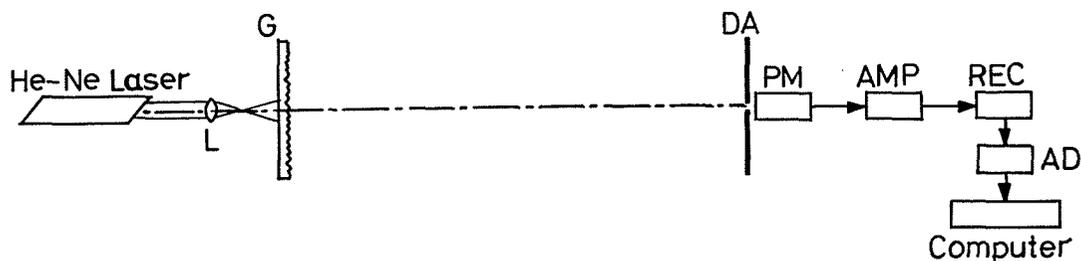


Fig. 1 Experimental arrangement for investigating the probability densities of speckles; L: objective lens, G: ground glass, DA: detecting aperture, PM: photomultiplier, AMP: dc amplifier, REC: data recorder and AD: A-D converter.

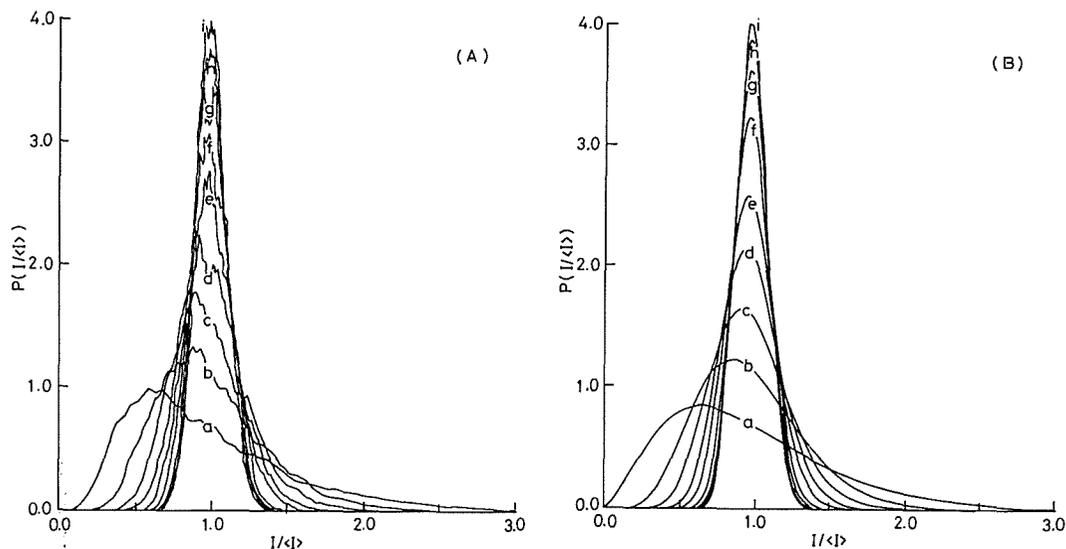


Fig. 2 (A) The probability densities of speckle intensity variations obtained experimentally by various circular apertures. (B) The approximate probability density functions of Eq. (11) calculated using the values of M obtained from results of (A); The values M are (a) 3, (b) 8, (c) 15, (d) 26, (e) 39, (f) 59, (g) 76, (h) 87 and (i) 93.

together with a circular aperture which was placed at the plane DA, 1 m away from the object one. The photocurrent signals from the photomultiplier were obtained as temporal fluctuations by moving the ground glass in a lateral direction and stored once in a data recorder. Then, the probability density functions and their variances were obtained numerically using the computer from experimental data of 4×10^4 provided by employing an A-D converter.

Fig. 2 shows, as an example, the comparison between the probability densities obtained from the experiments and those calculated from Eq. (11) by using the values of M experimentally obtained. The values of M were determined experimentally from the variance of speckle intensity variations by assuming that the approximate probability density function of Eq. (11) holds. In the experiments, the mean size Δx_e of speckle patterns was 0.44 mm which value was known from Eq. (9) and the radius of detecting circular aperture was varied by nine steps from 0.5 to 5.0 mm. Each experimental probability density in this figure is at least qualitatively in agreement with the approximate probability density of the theory.

Furthermore, the probability densities were also obtained from other experiments in which the values of Δx_e took 1.78, 0.88, 0.59 and 0.35 mm. The results in these experiments also almost agree with the theoretical ones,

as is, in the above case. The values of M obtained experimentally are compared in Fig. 3 with those obtained by using the relation of Eq. (10). As may be seen in this figure, the experimental values of M are apparently larger in a region of the small values of M (approximately $M \leq 10$) than the theoretical values from

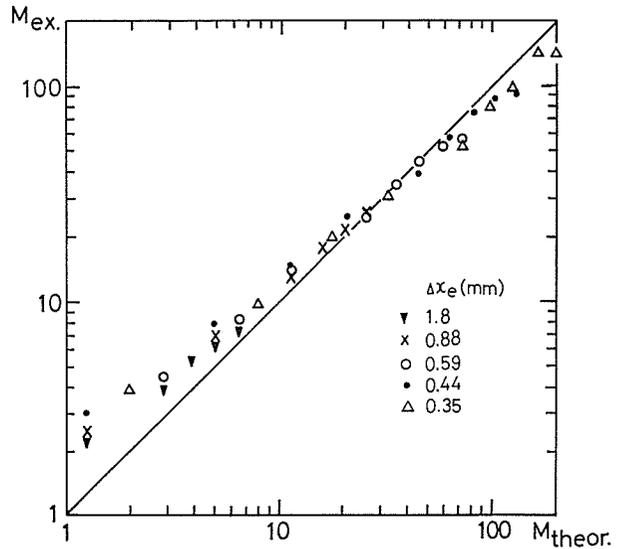


Fig. 3 Comparison between the values of M obtained from the variances of the experimental probability densities (vertical axis) and from Eq. (10) (horizontal axis).

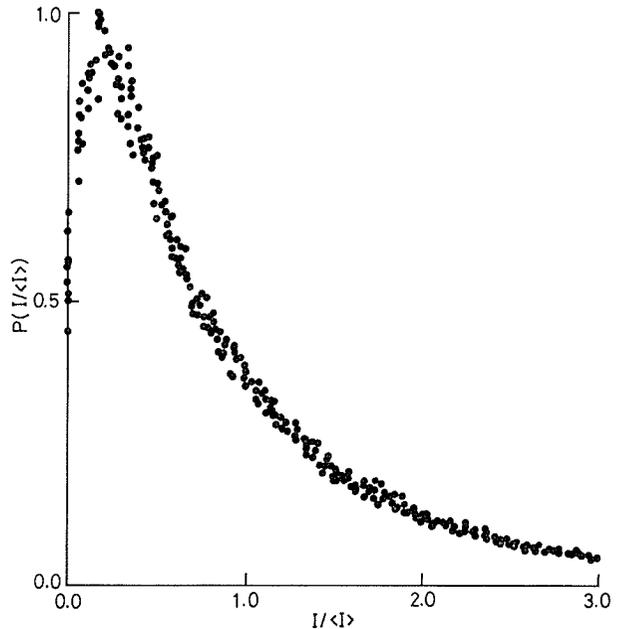


Fig. 4 The probability density obtained in the case of $M < 1$ in which nine cases of $M_{theor.} = 0.13, 0.10, 0.080, 0.062, 0.045, 0.020, 0.011, 0.0050$ and 0.0013 are together shown.

Eq. (10). This fact indicates that, for the case of small values of M , the detected intensity can not be regarded as an ensemble of independent speckles because the size of the aperture used is comparable to that of each speckle. In such a case, therefore, an exact solution of Eq. (1) for the probability density function of speckles is necessarily required by using suitable values of λ_m . On the other hand, the agreement of both values for $M \gtrsim 10$ means that the approximate probability function holds fairly well for a large number of M even though a slight difference between both values is seen. This difference is probably due to the effect of noise (shot noise, light-source fluctuation and etc.) adding to a photocurrent-signal component. The addition of noise leads to an underestimation of the value of M in the calculation using the relation of Eq. (12). This effect may increase as the size of the aperture becomes large.

We finally show an extreme case in which the size of speckles is extremely large in comparison with that of the detecting aperture. The experiments were conducted under the condition that the size of Δx_e was 14.1 mm and radii of the aperture used were varied from 0.5 to 5.0 mm. The value of M corresponding to this condition is between 0.0013 and 0.13. The experimentally observed probability densities were invariable in all the experiments for that value of M . Fig. 4 shows this result in which the data for each experiment are represented by the same symbol of black dots. The behavior of this probability density is almost equal to the negative exponential distribution, except in the vicinity of the origin, which is known as a result of an ideal point detector. Thus, in the case where the size of the detecting aperture is appropriately sufficiently small than that of the correlation cell of speckles, the probability density function converges on the negative exponential density in spite of the value of M defined by Eq. (10).

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