



HOKKAIDO UNIVERSITY

| | |
|------------------|---|
| Title | Internal Laminar Heat Transfer of a Radiating Gas with Gas-Property Variation |
| Author(s) | Seki, Nobuhiro; Fukusako, Shoichiro; Sugawara, Masahiro |
| Citation | 北海道大學工學部研究報告, 82, 1-11 |
| Issue Date | 1976-12-07 |
| Doc URL | https://hdl.handle.net/2115/41391 |
| Type | departmental bulletin paper |
| File Information | 82_1-12.pdf |



Internal Laminar Heat Transfer of a Radiating Gas with Gas-Property Variation

Nobuhiro SEKI* Shoichiro FUKUSAKO*
Masahiro SUGAWARA*

(Received June 30, 1976)

Abstract

The results of a numerical investigation of internal heat transfer to an absorbing and emitting gas with temperature-dependent properties under the condition of uniform wall temperature are reported.

The solution is based on the coupled partial differential equations of continuity, momentum, energy and integral continuity describing the two-dimensional flow of perfect gas between horizontal heated black parallel plates. Numerical examples for CO₂ gas are worked out including (1) simultaneous convection and radiation with constant properties; (2) simultaneous convection and radiation with gas-property variation; and superimposed natural convection with gas-property variation.

Nomenclature

- b distance between parallel plates ;
- C_p specific heat of fluid ;
- C_p dimensionless specific heat of fluid, C'_p/C'_{p0} ;
- $E_n(\tau)$ function of exponential integral, defined by Eq. (6) ;
- g acceleration of gravity
- Gr_0 Grashof number, gb^3/ν_0^2
- Gr_0^* modified Grashof number, Gr_0/Re_0^2 ;
- M_0 Mach number
- N interaction parameter of conduction to radiation, $\lambda'_0/(4\sigma T_0^3)$;
- Nu_x local Nusselt number, defined by Eq. (20) ;
- p' pressure
- P dimensionless pressure defect, $(p'_0 - p')/\rho'_0 u_0'^2$;
- Pr_0 Prandtl number, $\mu'_0 C'_{p0}/\lambda'_0$;
- q_R radiation heat flux ;
- R gas constant ;
- Re Reynolds number, $u'_0 b/\nu'_0$;
- T' temperature of fluid ;
- T dimensionless temperature of fluid, T'/T'_0 ;
- T_m dimensionless mixed mean temperature, defined by Eq. (19) ;
- u' axial velocity ;

* Department of Mechanical Engineering II, Hokkaido University, Sapporo 060, Japan

- u dimensionless axial velocity, u'/u'_0 ;
- u'_0 mean axial velocity at the entrance;
- v' transverse velocity;
- v dimensionless transverse velocity, $(v'Re_0Pr_0)/u'_0$;
- x' axial coordinate;
- x dimensionless axial coordinate, $x'/(bRe_0Pr_0)$;
- y' transverse coordinate;
- y dimensionless transverse coordinate, y'/b ;

Greek symbols

- α_x local heat transfer coefficient;
- κ absorption coefficient of medium;
- λ' thermal conductivity;
- λ dimensionless thermal conductivity, λ'/λ'_0 ;
- μ' viscosity;
- μ dimensionless viscosity, μ'/μ'_0 ;
- ν' kinematic viscosity;
- ρ' density;
- ρ dimensionless density, ρ'/ρ'_0 ;
- τ optical depth;
- τ_w optical thickness;
- Φ azimuthal angle;

Subscripts

- 0 value at the entrance;
- w value at the wall.

1. Introduction

In recent years high temperature heat transfer to gases has attracted a considerable interest, especially in connection with the cooling of nuclear reactors. For the sake of the variation of physical gas properties, that occurs over the cross-section of the duct as well as axially at high heating rates, the classical solutions which are based on an assumption of constant fluid properties may lead to serious errors in their predictions. It is well known that both density and the transport coefficient of a gas are, at least approximately, proportional to the absolute gas temperature raised to some power. By Worsoe-Schmidt and Leppert¹⁾, solutions with constant-properties will give their reasonable results for heat transfer as long as the ratio of absolute wall temperature to absolute gas temperature (for example, the mixed mean temperature) is less than approximately 1.2.

Kotake^{2,3)} discussed the problems on laminar and turbulent flow of gas with a variation of transport properties under uniform wall temperature. In the case of internal flow, Worsoe-Schmidt and Leppert¹⁾ and Swearingen and McEligot⁴⁾ obtained an implicit finite-difference solution for laminar flow of gas without radiation in a heated tube. Recently, Fukusako and Seki⁵⁾ reported a theoretical and experimental investigation of simultaneous heat transfer by radiation and free convection from two-dimensional vertical parallel plates heated under uniform heat flux, they demon-

stated that the experimental data are in good agreement with their predicted results including gas-property variation for a small channel spacing.

In the present study, a theoretical consideration is given to the simultaneous convection and radiation heat transfer with a variation of gas-properties in an absorbing and emitting gas flow between two black horizontal heated parallel flat plates. For the case of a fully developed flow at the starting section of heating, examples are computed under the condition of uniform wall temperature.

2. Basic Equations

Governing differential equations are developed on the basis of the standard boundary layer approximations. This is a common practice for incompressible flow. Wang and Longwell⁶⁾ presented a solution of the complete equations for incompressible flow in the inlet region of a plane duct. Their results indicated that the transverse shear term could be negligible if the Reynolds number, based on a distance from the entrance, was larger than about 200. As for the energy equation, it was assumed that the molecular contribution to axial thermal energy transfer was negligible small. The validity of this assumption has been confirmed by a resulting solution from the energy equation obtained by Singh⁷⁾ and an approximate solution for liquid metal by Schneider⁸⁾. However, the effect of property variation on the magnitude of the neglected terms cannot be estimated *a priori*. Worsoe-Schmidt and Leppert¹⁾ demonstrated that laminar flow heat transfer of gas in a duct, where a large variation of physical gas properties occurred, could be described adequately by the boundary layer equations if the Peclet number, based on a distance from the entrance, was larger than 500.

Now, under the abovementioned conditions, two-dimensional analysis for simultaneous convection and radiation heat transfer is made for a gray medium in a duct formed by two black horizontal parallel flat plates with semi-infinite length and infinite width, as will be shown schematically in Fig. 1.

The resulting basic equations become

$$\frac{\partial}{\partial x'} (\rho' u') + \frac{\partial}{\partial y'} (\rho' v') = 0 \quad (1)$$

$$\rho' u' \frac{\partial u'}{\partial x'} + \rho' v' \frac{\partial u'}{\partial y'} = - \frac{dp'}{dx'} - \frac{\partial}{\partial x'} \int_{y'}^b \rho' g dt' + \frac{\partial}{\partial y'} \left(\mu' \frac{\partial u'}{\partial y'} \right) \quad (2)$$

$$\rho' u' \frac{\partial}{\partial x'} (C_p' T') + \rho' v' \frac{\partial}{\partial y'} (C_p' T') = \frac{\partial}{\partial y'} \left(\lambda' \frac{\partial T'}{\partial y'} \right) + \frac{\mu'}{J} \frac{dp'}{dx'} + \frac{\mu'}{J} \left(\frac{\partial u'}{\partial y'} \right)^2 - \text{div } q_R \quad (3)$$

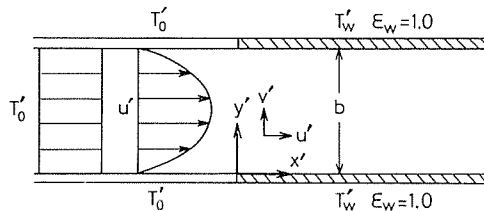


Fig. 1. Definition sketch and coordinate system.

In addition to these equations one needs another equation of state to give the necessary relations between the thermodynamic properties and expressions for the temperature dependence of the transport coefficients (assuming that the latter is independent of pressure). An assumption will be made that the gas obeys the perfect gas law

$$p' = \rho' k T' \quad (4)$$

It is well known that this law holds to a good approximation as long as the gas is not too dense.

If it is assumed that radiant transfer in the longitudinal direction is negligible compared with lateral radiative heat transfer, then

$$\partial q_{Rx'} / \partial x' \ll \partial q_{Ry'} / \partial y'$$

Moreover, if the temperature distribution is assumed to be the same in any other cross section as that in the cross section under consideration, the two-dimensional extinction function of radiation may be replaced by an one-dimensional one. Thus, the divergence of radiative heat flux reduces to

$$-\text{div } q_R = \kappa \left[2 \left\{ \sigma T_w'^4 E_2(\tau) + \sigma T_w'^4 E_2(\tau_w - \tau) + \int_0^{\tau_w} \sigma T'^4 E_1(|\tau - t'|) dt' \right\} - 4\sigma T'^4 \right] \quad (5)$$

where

$$E_n(\tau) = \int_0^1 \mu^{(n-2)} e^{-\tau'/\mu} d\mu, \quad \tau = \int_0^{y'} \kappa dt', \quad (\mu = \cos \Phi) \quad (6), (7)$$

κ is an absorption coefficient of medium and τ_w an optical thickness.

For the diatomic gas, the variation of the specific heat, although less than that of the transport coefficients, is not negligible. For all of three properties the temperature dependence will be taken as power laws which might give a reasonable good approximation to their actual behaviors, as follows

$$C_p / C_{p0} = (T' / T'_0)^a, \quad \mu' / \mu'_0 = (T' / T'_0)^b \quad \text{and} \quad \lambda' / \lambda'_0 = (T' / T'_0)^c \quad (8)$$

At the wall no slip condition, the impermeability of the wall and the imposed thermal condition give the following boundary conditions.

$$u' = 0, \quad v' = 0 \quad \text{and} \quad T' = T'_w \quad (9)$$

For $x'=0$ one has the starting conditions.

$$u' = 6u'_0 \left\{ \left(\frac{y'}{b} \right) - \left(\frac{y'}{b} \right)^2 \right\}, \quad v' = 0, \quad T' = T'_0 \quad \text{and} \quad p' = p'_0 \quad (10)$$

By making the dimensionless variables and parameters substitute, Eqs. (2) and (3) can be placed in the following forms, yielding

$$\rho u \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} \int_0^y \frac{\partial}{\partial x} (\rho u) dy = \frac{dp}{dx} - \frac{Gr_0}{Re_0^2} \frac{\partial}{\partial x} \int_0^y \rho dy + Pr_0 \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) \quad (11)$$

$$\begin{aligned} \rho u \frac{\partial}{\partial x} (C_p T) - \frac{\partial}{\partial y} (C_p T) \int_0^y \frac{\partial}{\partial x} (\rho u) dy &= \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) - (\gamma_0 - 1) M_0^2 \left\{ u \frac{dp}{dx} - Pr_0 \mu \left(\frac{\partial u}{\partial y} \right)^2 \right\} \\ &+ \frac{\tau_w^2}{2N} \left\{ T_w'^4 E_2(\tau) + T_w'^4 E_2(\tau_w - \tau) + \int_0^{\tau_w} T'^4 E_1(|\tau - t|) dt - 2T'^4 \right\} \end{aligned} \quad (12)$$

where the relation

$$v = -\frac{1}{\rho} \int_0^y \frac{\partial}{\partial x} (\rho u) dy$$

derivable from the equation of continuity given by Eq. (1) is substituted.

3. Finite Difference Equations

The finite difference forms of Eqs. (11) and (12) will be now obtained as under-mentioned. The derivatives in the x -direction are replaced by finite differences while the other quantities such as u , $\partial u/\partial y$, $\partial^2 u/\partial y^2$, T , $\partial T/\partial y$, $\partial^2 T/\partial y^2$ etc. are replaced by averages. Assuming that the solutions $u = u_1$, $P = P_1$ and $T = T_1$ at $x = x_1$ are known and solutions $u = u_2$, $P = P_2$ and $T = T_2$ at $x = x_2$ are to be found. After introducing the finite difference approximations in Eqs. (11) and (12), one obtains

$$\rho\phi(\phi - 2u_1) - \phi' \int_0^y (2\rho\phi - \rho_1\phi - 2\rho u_1) dy = 2(P_2 - P_1) - 4Gr_0^* \int_y^1 (\rho - \rho_1) dy + Pr_0 l \left\{ \mu\phi'' + \frac{\partial\mu}{\partial y} \phi' \right\} \quad (13)$$

$$\begin{aligned} \rho\phi \left\{ \frac{1}{2} l\chi \frac{\partial C_p}{\partial x} + C_p(\chi - 2T_1) \right\} - \left(\chi \frac{\partial C_p}{\partial y} + C_p \chi' \right) \int_0^y \left\{ \frac{1}{2} l\phi \frac{\partial \rho}{\partial x} + \rho(\phi - 2u_1) \right\} dy &= l\lambda\chi'' + l \frac{\partial \lambda}{\partial y} \chi' \\ + 4l \frac{\tau_w^2}{N} \left[\int_y^1 \left(\frac{\chi}{2} \right)^3 \frac{\chi'}{2} E_2 \{ \tau_w(t-y) \} dt - \int_0^y \left(\frac{\chi}{2} \right)^3 \frac{\chi'}{2} E_2 \{ \tau_w(y-t) \} dt \right] \\ + (\gamma_0 - 1) M_0^2 \left\{ \phi(P_1 - P_2) + lPr_0 \mu \frac{1}{2} (\phi')^2 \right\} \end{aligned} \quad (14)$$

where a dashed sign denotes the differentiation with respect to y and

$$\phi = u_1 + u_2, \quad \chi = T_1 + T_2 \text{ and } l = x_2 - x_1$$

Moreover, the constancy of flow quantity per unit time through the duct requires that

$$\int_0^1 \rho\phi dy = 2 \int_0^1 \rho_1 u_1 dy \quad (15)$$

Once ϕ and χ are determined as the solutions of Eqs. (13), (14) and (15), one can easily obtain $u_2 = \phi - u_1$ and $T_2 = \chi - T_1$.

Let ϕ_m , $P_{2,m}$ and χ_m define the m -th iterative approximations to the solution of Eqs. (13), (14) and (15) and then $(m+1)$ -th approximations ϕ_{m+1} , $P_{2,m+1}$ and χ_{m+1} are to be found. A rectangular mesh of dimensions (l, h) , as shown in Fig. 2, is now formed by introducing the differences in the y -direction. When the suffix k refers to the k -th mesh point in this direction, Eqs. (13), (14) and (15) are written as

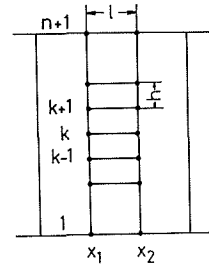


Fig. 2. Mesh size

$$\begin{aligned}
& lPr_0 \mu_{m,k} \phi''_{m+1,k} + \phi'_{m,k} \int_0^{y_k} (2\rho_m - \rho_1) \phi_{m+1} dy - \rho_{m,k} \phi_{m,k} \phi_{m+1,k} + 2P_{2,m+1} \\
& = 2P_1 - 2\rho_{m,k} u_{1,k} \phi_{m,k} + 2\phi'_{m,k} \int_0^{y_k} \rho_m u_1 dy - lPr_0 \left(\frac{\partial \mu}{\partial y} \right)_{m,k} \phi'_{m,k} + 4Gr_0^* \int_{y_k}^1 (\rho_m - \rho_1) dy
\end{aligned} \tag{16}$$

$$\begin{aligned}
& l\lambda_{m,k} \chi''_{m+1,k} + C\rho_{m,k} \chi'_{m+1,k} \left\{ \int_0^{y_k} \phi_m (2\rho_m - \rho_1) dy - \int_0^{y_k} 2\rho_m u_1 dy \right\} - \rho_{m,k} C\rho_{m,k} \phi_{m,k} \chi_{m+1,k} \\
& = \frac{l}{2} \rho_{m,k} \phi_{m,k} \left(\frac{\partial C_p}{\partial x} \right)_{m,k} \chi_{m,k} - 2\rho_{m,k} C\rho_{m,k} \phi_{m,k} T_{1,k} - \left(\frac{\partial C_p}{\partial y} \right)_{m,k} \chi_{m,k} \\
& \times \left\{ \int_0^{y_k} \phi_m (2\rho_m - \rho_1) dy - \int_0^{y_k} 2\rho_m u_1 dy \right\} - l \left(\frac{\partial \lambda}{\partial y} \right)_{m,k} \chi'_{m,k} \\
& - 4l \frac{\tau_w^2}{N} \left[\int_{y_k}^1 \left(\frac{\chi_m}{2} \right)^3 \frac{\chi'_m}{2} E_2 \{ \tau_w (t-y) \} dt - \int_0^{y_k} \left(\frac{\chi_m}{2} \right)^3 \frac{\chi'_m}{2} E_2 \{ \tau_w (y-t) \} dt \right] \\
& - (\gamma_0 - 1) M_0^2 \left\{ \phi_{m,k} (P_1 - P_{2,m}) + lPr_0 \mu_{m,k} \frac{1}{2} (\phi'_{m,k})^2 \right\}
\end{aligned} \tag{17}$$

$$\int_0^1 \rho_m \phi_{m+1} dy = 2 \int_0^1 \rho_1 u_1 dy \tag{18}$$

If the differentiations with respect to y are approximated by the following finite difference ratio

$$\begin{aligned}
\phi''_{m+1,k} &= (\phi_{m+1,k+1} - 2\phi_{m+1,k} + \phi_{m+1,k-1})/h^2 \\
\phi'_{m,k} &= (\phi_{m,k+1} - \phi_{m,k-1})/(2h) \\
(\partial C_p / \partial y)_{m,k} &= (a/4h) (\chi_{m,k}/2)^{a-1} (\chi_{m,k+1} - \chi_{m,k-1}) \\
(\partial C_p / \partial x)_{m,k} &= (a/l) (\chi_{m,k}/2)^{a-1} (\chi_{m,k} - 2T_{1,k}) \text{ etc.}
\end{aligned}$$

and integral by the trapezoidal rule. Eqs. (16), (17) and (18) are now a set of $(2n-1)$ simultaneous linear algebraic equations for the $(2n-1)$ unknown quantities $P_{2,m+1}$, $\phi_{m+1,2}$, $\phi_{m+1,3}$, \dots , $\phi_{m+1,n}$, $\chi_{m+1,2}$, $\chi_{m+1,3}$, \dots , $\chi_{m+1,n}$. These simultaneous linear equations can be written in matrix form:

$$\mathbf{A}_m \mathbf{X}_{m+1} = \mathbf{E}_m$$

where \mathbf{A}_m is a $(2n-1) \times (2n-1)$ matrix, while \mathbf{X}_{m+1} and \mathbf{E}_m are column matrices given by

$$\begin{aligned}
\mathbf{X}_{m+1} &= \{P_{2,m+1}, \phi_{m+1,2}, \phi_{m+1,3}, \dots, \phi_{m+1,n}, \chi_{m+1,2}, \chi_{m+1,3}, \dots, \chi_{m+1,n}\} \\
\mathbf{E}_m &= \{C_{m,2}, C_{m,3}, \dots, C_{m,n}, D_{m,2}, D_{m,3}, \dots, D_{m,n}, E_m\}
\end{aligned}$$

where $C_{m,k}$, $D_{m,k}$ and E_m will be evaluated from the right-hand sides of Eqs. (16), (17) and (18), respectively. The matrix \mathbf{A}_m will be written easily from the left-hand sides of Eqs. (16), (17) and (18). If the inverse matrix of \mathbf{A}_m is denoted by \mathbf{A}_m^{-1} , the solution of these equations is given by

$$\mathbf{X}_{m+1} = \mathbf{A}_m^{-1} \mathbf{E}_m$$

This procedure must be repeated until the following condition is satisfied:

$$\max \left\{ |P_{2,m+1} - P_{2,m}|, |\phi_{m+1,k} - \phi_{m,k}|, |\chi_{m+1,k} - \chi_{m,k}| \right\} \leq \varepsilon$$

where $\max \{ \}$ means the maximum element in $\{ \}$, and the value of ε depends upon an accuracy to be desired. Thus, the values of P_2 , ϕ and χ at $x=x_2$ are determined. The same procedure can then be applied to find the solution at $x=x_2+l$ and so on. By this scheme of calculation, one can obtain the velocity, pressure and temperature profiles at arbitrary temperature developing stage in the thermal entrance region concerned.

4. Numerical Results and Discussions

In the immediate neighbourhood of the entrance section, the mesh size of $n=50$, $l=10^{-4}$ is employed and it is gradually increased in seven steps to the final mesh of $n=25$, $l=5.0 \times 10^{-3}$ in the region of $x \geq 0.1$. The value of ε for convergence is taken to be 5.0×10^{-4} throughout the whole entrance region. The number of iteration of 15 or 16 in the first step is changed to 7 or 8 in the range of $x \geq 10^{-2}$. The computations were performed on FACOM 230-60 Digital Computer at the Compter Center of Hokkaido University, which took about ten minutes to proceed from $x=0.0$ to $x=0.40$. All of the examples are computed with the variability of the properties corresponding to those of carbon-dioxide gas⁹⁾, $Pr_0=0.76$, $\gamma_0=1.259$, $a=0.33$, $b=0.78$ and $c=1.34$.

Figure 3 shows the effect of property variation on developing temperature distributions, where dotted lines correspond to each temperature distribution with constant-properties at several sections. From this figure, it can be seen that the solutions with property variation show the distinguishable deviations from those with constant fluid properties.

The temperature distributions predicted by considering the effect of radiation are shown in Fig. 4. It should be noted that the solution based on the assumption of constant fluid properties may lead to serious errors in the analysis pertaining to

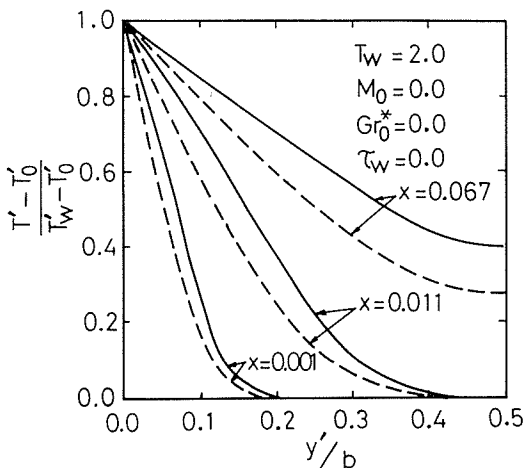


Fig. 3. Temperature profile vs. y
(effect of property variation)

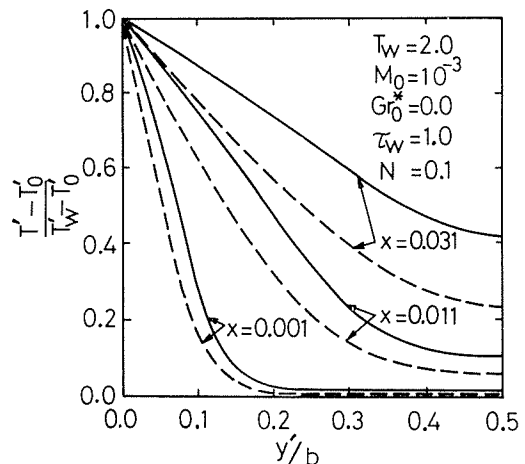


Fig. 4. Temperature profile vs. y
(effect of property variation)

a radiating flow.

In Fig. 5 the development of the axial velocity is shown for $\tau_w=1.0$, $N=0.1$. One notes that the velocity of $y=0.5$ at $x=0.037$ is about 35% larger than that at $x=0.0$.

The local Nusselt numbers in these case are plotted on Fig. 6. It is found, as expected in Figs. 3 and 4, that the heat transfer rate increases due to only the property variation with or without radiation heat transfer, where the mixed-mean temperature and local Nusselt number are defined by the following schemes.

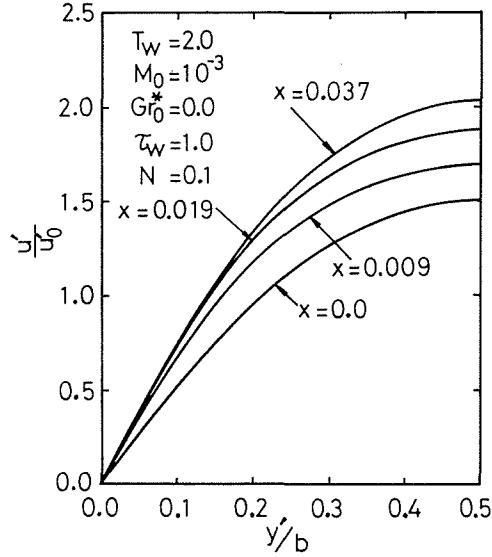


Fig. 5. Velocity profile vs. y (effect of property variation)

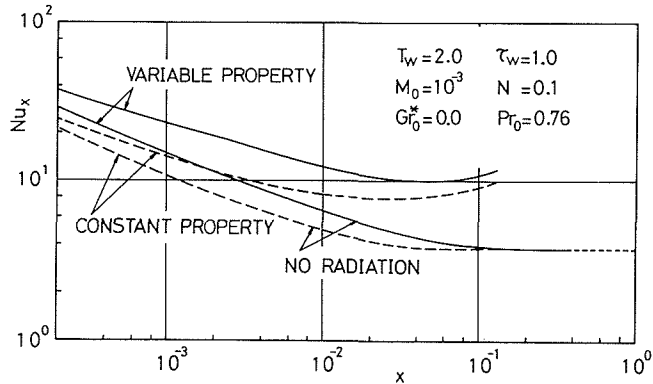


Fig. 6. Local Nusselt number Nu_x vs. x (effect of property variation)

$$T_m = \int_0^1 \rho C_p u T dy / \int_0^1 \rho C_p u dy \quad (19)$$

$$Nu_x = -\frac{1}{T_w - T_m} \frac{\lambda_w}{\lambda_m} \left(\frac{\partial T}{\partial y} \right)_{y=0} + \frac{\tau_w}{2N} \frac{1}{\lambda_m} \left[T_m^4 \{E_3(0) - E_3(\tau_w)\} - \tau_w \int_0^1 T^4 E_2(\tau_w t) dt \right] \quad (20)$$

The effects of τ_w on the temperature profile of a radiating medium with property

variation are shown in Fig. 7 for value of $x=0.009$. The dotted line in this figure shows the temperature profile for a constant property fluid without thermal radiation. One can see that for large optical thickness the radiation heat flux emitted from the plates becomes attenuated so rapidly that the temperature of a medium in a tube rises higher than for small optical thickness. If the optical thickness is the same, the smaller the N is, the more rapid the temperature field tends to develop as will be seen in Fig. 8. From this figure, the medium near the center region of the duct are found to be directly heated by radiation heat flux and the temperature of such a region to rise uniformly.

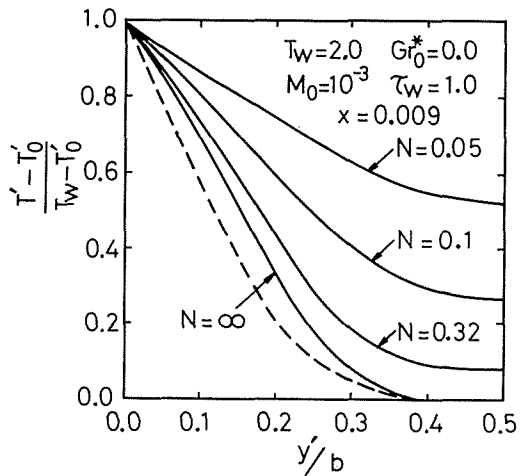
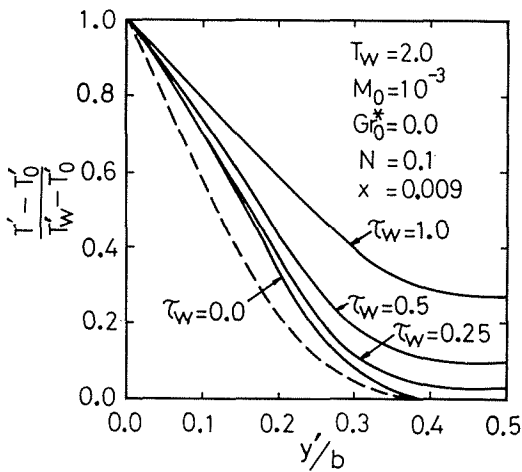


Fig. 7. Temperature profile vs. y (effect of τ_w)

Fig. 8. Temperature profile vs. y (effect of N)

Fig. 9 and 10 demonstrate the relations of the local Nusselt number Nu_x defined by equation (20) vs. x with a parameter of radiation-conduction interaction N and of optical thickness τ_w , respectively. On the first inspection of these figures, it may be seen that the Nu_x has a certain minimum value against x and its characteristic in a medium with property variation shows the same peculiar behavior of combined heat transfer with radiation as in the case of constant property. This fact may be understood as follows; the temperature difference $(T_w - T_m)$ in Eq. (20) decreases

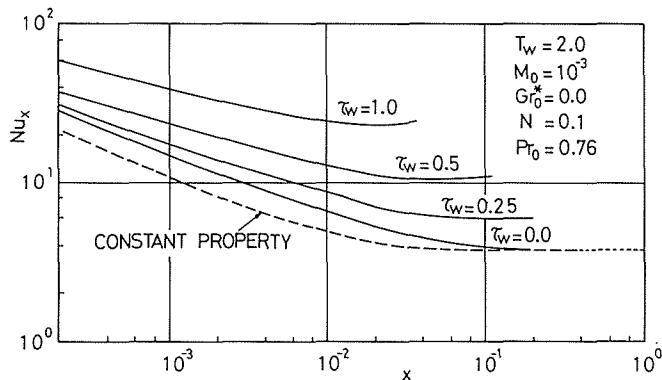


Fig. 9. Local Nusselt number Nu_x vs. x (effect of τ_w)

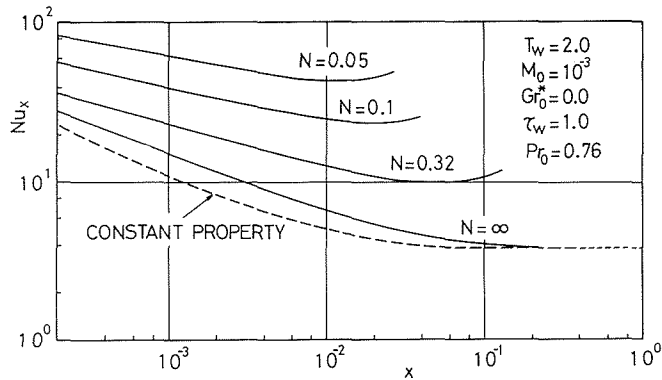


Fig. 10. Local Nusselt number Nu_x vs. x (effect of N)

remarkably due to radiation, because the mixed mean temperature T_m becomes higher than that by only convection heat transfer, as previously shown in Figs. 7 and 8. In Fig. 10 it can be pointed out that for the same optical thickness, the heat transfer rate increase with decrease of N , that is, with increase of contribution of radiation.

In order to further study the effects of superimposed natural convection, computations are carried out for values of $G_{r_0}^*$ of 0.5 and 1.0. A comprehensible conclusion for these examples may be that the velocity and temperature gradients at the lower wall are larger than those at the upper one. The same feature in non-radiating medium was also recognized by previous investigators¹⁰⁾. The asymmetry of the temperature profile with respect to the center line of the duct leads to a different heat transfer rate at the upper and the lower walls of the channel. The variations of the local Nusselt numbers vs. x are tabulated in Table 1. One can see that the local Nusselt number is a little larger on the lower wall than on the upper one. Also interesting is, as expected physically, that the deviation of local Nusselt number between the lower wall and the upper wall is small than that¹⁰⁾ in the case of the constant gas-property without thermal radiation.

Table 1. Local Nusselt number (effect of buoyancy)

$$T_w=2.0 \quad M_0=10^{-3} \quad G_{r_0}^*=1.0 \quad \tau_w=1.0 \quad N=0.32$$

| x | 0.001 | 0.003 | 0.007 | 0.028 |
|------------|-------|-------|-------|-------|
| UPPER WALL | 22.29 | 16.77 | 13.49 | 10.11 |
| LOWER WALL | 23.30 | 17.11 | 13.70 | 10.19 |

5. Conclusions

An analytical study has been performed on the heat transfer by simultaneous radiation and convection in an absorbing and emitting gray gas medium with property variation at high temperature. The physical model employed in the present

investigation is based on laminar flow in horizontal parallel black flat plates and the following conclusions are yielded.

(1) The velocity and temperature profiles predicted with property variation show surprising deviations from those with constant properties.

(2) The development of the temperature field is more rapidly established in a flow direction, if the radiation is taken into account.

(3) The deviation of local Nusselt number between the lower wall and the upper wall resulting from the superimposed natural convection is smaller than that in the case of the constant gas-property without thermal radiation.

References

- 1) Worsoe-Schmidt, P. M. and Leppert, G.: *Int. J. Heat Mass Transfer*, 8 (1965), p. 1281.
- 2) Kotake, S.: *Preprints of JSME-Meeting of Heat Engineering*, No. 720-19 (1972), p. 1.
- 3) Kotake, S.: *Preprints of 10-th Japan Heat Transfer Symposium* (1973), p. 157.
- 4) Swearingen, T. B. and McEligot, D. M.: *Trans. ASME, Ser. C*, 93 (1971), p. 432.
- 5) Fukusako, S. and Seki, N.: *Proceedings of the 5-th. International Heat Transfer Conference*, 1 (1974), p. 113.
- 6) Wang, Y. L. and Longwell, P. A.: *AICHE Journal*, 10 (1964), p. 323.
- 7) Singh, R.: *Applied Scientific Research*, A 10 (1958), p. 325.
- 8) Schneider, P. J.: *Trans. ASME*, 79 (1957), p. 765.
- 9) Fukuda, K., Echigo, R. and Hasegawa, S.: *Trans. JSME*, 38 (1972), p. 2873.
- 10) Kiya, M. Fukusako, S. and Arie, M.: *Bulletin of JSME*, 15 (1972), p. 735.