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Radiation Effect on Developing Free Convection from Two-Dimensional Isothermal Vertical Parallel Plates

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Abstract

Steady laminar convection of a radiating gray gas between vertical parallel plates of isothermal temperature is theoretically investigated. A formulation of the interaction of radiation and free convection in an absorbing and emitting medium was made by a non-linear integro-differential equation. The basic governing continuity, momentum and energy equations, including the temperature dependable transport properties, are expressed in each finite-difference form and solved numerically with a digital computer. Parameter surveys pertaining to the effects of radiation on variations of pressure and temperature profiles throughout a flow field are made. Effects of thermal radiation on heat transfer characteristics of the channel are also discussed in detail.

Nomenclature

d, channel spacing
c_p, dimensionless specific heat of medium
c_p', specific heat of medium
g, acceleration of gravity
Gr_0, modified Grashof number, $gd^3/\nu_0'^2$
L, channel height
N, interaction of conduction to radiation, $\kappa\lambda_0'/(4\sigma T_0'^3)$
Nu_x, local Nusselt number, defined by Eq. (22)
p', pressure
p dimensionless pressure defect, $(p'-p_0')b^2/(\rho_0'\nu_0'^2 Gr_0^2)$
P_r, Prandtl number
q_R, radiation heat flux
Q, dimensionless flow rate, defined by Eq. (14)
R, gas constant
T, dimensionless temperature of medium, T'/T_0'
T', temperature of medium
T_m, dimensionless mixed mean temperature, defined by Eq. (23)
T_0', temperature of medium at entrance of channel
T_w, dimensionless wall temperature, T_w'/T_0'
T_w', wall temperature
u, dimensionless axial velocity, $u'b/(\nu_0' Gr_0)$
u', axial velocity
v, dimensionless transverse velocity, $v'b/\nu_0'$
v', transverse velocity

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x, dimensionless axial coordinate, $x'/(bGr_0)$
x', axial coordinate
y, dimensionless transverse coordinate, y/b
y', transverse coordinate

Greek symbols

κ, absorption coefficient of medium
λ, dimensionless thermal conductivity, λ'/λ'_0
λ', thermal conductivity
μ, dimensionless viscosity, μ'/μ'_0
μ', viscosity
ν', kinematic viscosity
ρ, dimensionless density, ρ'/ρ'_0
ρ', density
τ', optical depth
τ_w, optical thickness

Subscript

0, value at entrance of channel.

1. Introduction

The studies on the laminar free convection between finite vertical parallel plates have been reported by several investigators. Bodoia and Osterle[1] first obtained numerical solutions of this problem and showed that their results were in good agreement with the experimental data of Elenbaas[2]. Aihara[3] and Quintiere and Muller[4] analyzed the same problem by taking the pressure drop resulting from the acceleration of the fluid to the inlet of the channel into account. Aung et al.[5] and Fujii et al.[6] investigated the developing laminar free convection heat transfer in a vertical parallel plate channel with asymmetric thermal boundary conditions. All of the investigators, however, have restricted their considerations to a non-radiating medium under a low heating condition at the walls.

In recent years, the heat transfer problem with thermal radiation under high temperature and high heat flux has become increasingly important in connection with the development of high temperature gas-cooled reactors and nuclear fusion reactors. In this case, due to the variableness of the physical properties of gas under high temperature and high heat flux, it is well known that the classical solutions which are based on the assumption of constant fluid properties may cause considerable errors in the predicted performance. And moreover, the effects of an interaction between radiation and convection on the prediction may be not negligible in a flowing-absorbing emitting medium.

Recently, Fukusako and Seki[7] treated a theoretical and experimental problem of simultaneous heat transfer by radiation and free convection from two-dimensional vertical parallel plates heated at uniform heat flux and concluded that the experimental data are in good agreement with their predicted results for small channel spacing.

The present study concerns a theoretical investigation of the laminar free convection from a finite parallel plate vertical channel of uniform temperature in an absorbing and emitting medium. The calculations are performed by including the temperature dependency of transport properties and density. Parameter surveys pertaining to the effects of thermal radiation on the variableness of pressure and temperature throughout the flow field were carried out. Effects of the thermal radiation on the heat transfer characteristics of the channel are also discussed in detail.

2. Basic Equations

The flow geometry of interest is depicted in Fig. 1. A vertical channel is formed by two parallel black plates of height L and infinite width, separated by a distance b. The chennel walls are maintained at a uniform temperature T'_w which exceeds an ambient fluid temperature T'_0 and as a result of the heat transfer to an absorbing and emitting fluid, the fluid temperature increases. Thus, the fluid which is assumed to enter the channel from the bottom at T'_0 with a parabolic velocity profile rises upwards in the vertical channel by free convection.

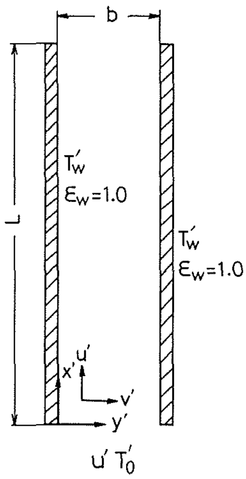

Fig. 1 Definition sketch and coordinate system

Neglecting viscous heat dissipation, the resulting governing equations for a laminar flow of an absorbing and emitting medium become

$$\frac{\partial}{\partial x'}(\rho' u') + \frac{\partial}{\partial y'}(\rho' v') = 0 \tag{1}$$

$$\rho' u' \frac{\partial u'}{\partial x'} + \rho' v' \frac{\partial u'}{\partial y'} = -\frac{dp'}{dx'} + \frac{\partial}{\partial y'}\left(\mu' \frac{\partial u'}{\partial y'}\right) + g(\rho'_0 - \rho') \tag{2}$$

$$\rho' c'_p u' \frac{\partial T'}{\partial x'} + \rho' c'_p v' \frac{\partial T'}{\partial y'} = \frac{\partial}{\partial y'}\left(\lambda' \frac{\partial T'}{\partial y'}\right) - \mathrm{div}\, q_R \tag{3}$$

where q_R denotes the radiation flux vector, and owing to the one dimensional radiative propagation the radiation term in Eq. (3) is frequently approximated in the following form

$$-\mathrm{div}\, q_R = \kappa[2\{\sigma T'^4_w E_2(\tau) + \sigma T'^4_w E_2(\tau_w - \tau) + \int_0^{\tau_w} \sigma T'^4 E_1(|\tau - t'|) dt'\} - 4\sigma T'^4] \tag{4}$$

The optical length τ, the optical depth of a channel τ_w and the exponential integral are defined as follows, respectively

$$\tau = \int_0^{y'} \kappa dt', \quad \tau_w = \int_0^b \kappa dt' \quad \text{and} \quad E_n(\tau) = \int_0^1 s'^{(n-2)} \exp(-\tau/s') ds', \qquad (s' = \cos\phi) \qquad (5), (6)$$

where κ is the absorption coefficient of a medium and ϕ polar angle.

In addition to these equations one needs several more expressions, one describing the necessary relations for thermodynamic equilibrium of gas and the other for the temperature dependability of the transport coefficients. It may be assumed that the gas obeys the perfect gas law

$$p' = \rho' R T' \tag{7}$$

For diatomic and polyatomic gases, the variation of the specific heat, although less than that of the transport coefficients, is not negligible. For the following three properties the dependence will be taken as power laws which respectively give reasonably good approximation to the actual behavior.

$$c'_p/c'_{p0} = (T'/T'_0)^a, \quad \mu'/\mu'_0 = (T'/T'_0)^b \quad \text{and} \quad \lambda'/\lambda'_0 = (T'/T'_0)^c \tag{8}$$

The no-slip condition at the wall, the impermeability of the wall and the imposed thermal condition permit the following boundary conditions.

$$u' = 0, \quad v' = 0 \quad \text{and} \quad T' = T'_w \tag{9}$$

For $x' = 0$, one has the starting conditions.

$$u' = 6u_0'\left\{\left(\frac{y'}{b}\right) - \left(\frac{y'}{b}\right)^2\right\}\ ;\ v'=0\ ;\ T'=T_0'\ ;\ p'=p_0' \tag{10}$$

When the dimensionless variables and parameters are introduced in the foregoing Eqs. (1)~(3), one obtaines the following dimensionless governing equations after some mathematical manipulations,

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{11}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) + (1-\rho) \tag{12}$$

$$\rho c_p u \frac{\partial T}{\partial x} + \rho c_p v \frac{\partial T}{\partial y} = \frac{1}{Pr_0}\frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \frac{2\tau_w}{Pr_0 N}\left[\int_y^1 T^3 \frac{dT}{dt} E_2\{\tau_w(t-y)\}dt - \int_0^y T^3 \frac{dT}{dt} E_2\{\tau_w(y-t)\}dt\right] \tag{13}$$

where the dimensionless flow rate is given by

$$Q = \frac{u_0' b}{\nu_0' Gr_0} = \int_0^1 \rho u dy \tag{14}$$

The boundary conditions in dimensionless form are

$$x>0,\ y=\begin{cases}0\\1\end{cases};\ u=0,\ v=0 \text{ and } T=T_w \tag{15}$$

$$x=0,\ 0 \leqq y \leqq 1\ ;\ u=6Q(y-y^2),\ v=0,\ T=1 \text{ and } p=0 \tag{16}$$

The numerical procedure used in solving these equations which are subject to their boundary conditions is described in the following.

3. Numerical Solution

Equations (11), (12) and (13) constitute simultaneous integro-differential equations with a higher order of non-linearity and it may be very difficult in general to obtain analytical solutions. Therefore, the finite-difference method is applied to this problem in view of the objective to minimize the expected errors. The finite-difference forms of Eqs. (11), (12) and (13) with iterative procedure will now be obtained as follows.

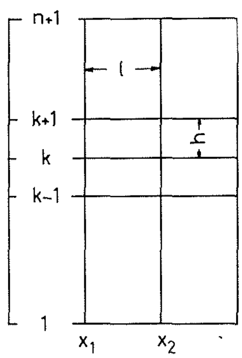

Fig. 2 Mesh points

Assuming that the solutions $u=u_m$, $p=p_m$ and $T=T_m$ at $x=x_m$ are known and the solutions $u=u_{m+1}$, $p=p_{m+1}$ and $T=T_{m+1}$ at $x=x_{m+1}$ are to be found. Applying Eqs. (12) and (13) of finite-difference form to the (m, k) mesh point of a rectangular grid superposed on the channel flow field as shown in Fig. 2, there results

$$p_{m+1} + A_k u_{m+1,k-1} + B_k u_{m+1,k} + C_k u_{m+1,k+1} = F_k \tag{17}$$

$$H_k T_{m+1,k-1} + I_k T_{m+1,k} + J_k T_{m+1,k+1} = W_k \tag{18}$$

where A_k, B_k, ..., W_k have the following expressions, respectively

$$A_k = -\frac{1}{2}\beta h(\rho v)^*_{m+1,i} - \beta \mu^*_{m+1,k} + \frac{1}{2}\beta h\left(\frac{\partial \mu}{\partial y}\right)^*_{m+1,k}$$

$$B_k = (\rho v)^*_{m+1,k} + 2\beta \mu^*_{m+1,k}$$

$$C_k = \frac{1}{2}\beta h(\rho v)^*_{m+1,k} - \beta \mu^*_{m+1,k} - \frac{1}{2}\beta h\left(\frac{\partial \mu}{\partial y}\right)^*_{m+1,k}$$

$$F_k = (\rho u)^*_{m+1,k} u_{m,k} + p_m + l(1-\rho^*_{m+1,k})$$

....

$$W_k = (\rho u)^*_{m+1,k} Cp_{m,k} T_{m,k} + (l\tau_w^2/Pr_0 N)(RAD)^*_{m+1,k}$$

where $B=l/h^2$ and superscript * denotes a value of previous iteration, and

$$(\partial \mu/\partial y)_{m+1,k} = b(T_{m+1,k})^{b-1}(T_{m+1,k+1} - T_{m+1,k-1})/2h$$

$$(\partial\lambda/\partial y)_{m+1,k}=c(T_{m+1,k})^{c-1}(T_{m+1,k+1}-T_{m+1,k-1})/2h$$

$$(RAD)_{m+1,k}=\sum_{i=k+1}^{n}E_2\{\tau_w(y_i-y_k)\}T^3_{m+1,i}(T_{m+1,i+1}-T_{m+1,i-1})$$
$$+E_2\{\tau_w(y_{n+1}-y_k)\}T^3_{m+1,n+1}(T_{m+1,n+1}-T_{m+1,n})$$
$$-\sum_{i=2}^{k-1}E_2\{\tau_w(y_k-y_i)\}T^3_{m+1,i}(T_{m+1,i+1}-T_{m+1,i-1})$$
$$-E_2\{\tau_w(y_k-y_1)\}T^3_{m+1,1}(T_{m+1,2}-T_{m+1,1})$$

Equation (14) may also be rewritten in a finite-difference form by the trapezoidal rule yielding

$$Q=h\sum_{k=2}^{n}\rho_{m+1,k}u_{m+1,k} \tag{19}$$

To evaluate the transverse component v, after the integration of the continuity equation (11), the following form is used:

$$(\rho v)_{m+1,k}=[(\rho v)_{m+1,k-1}-(h/2l)\{(\rho u)_{m+1,k}-(\rho u)_{m,k}+(\rho u)_{m+1,k-1}-(\rho u)_{m,k-1}\}](1-y_k)$$
$$+[(\rho v)_{m+1,k+1}+(h/2l)\{(\rho u)_{m+1,k}-(\rho u)_{m,k}+(\rho u)_{m+1,k+1}-(\rho u)_{m,k+1}\}]y_k \tag{20}$$

Solving the finite-difference equations could be proceeded by selecting first the values for Pr_0, Q and T_w etc. Then, by means of a marching procedure, the u, v, bnd T for each row beginning at row $m+1=2$ are obtained by using the values at the previous row m. Thus applying Eqs. (17), (18) and (19) to the points 2, 3, 4, ..., $n-1$ on row $m+1$, there results $2n-1$ algebraic equations with the $2n-1$ unknowns $u_{m+1,2}$, $u_{m+1,3}$, ..., $u_{m+1,n-1}$, p_{m+1}, $T_{m+1,2}$, ..., $T_{m+1,n-1}$. These equations are then conformably solved by a matrix reduction technique. This procedure is repeated until the following condition is satisfied;

$$\max\{|p_{m+1}-p^*_{m+1}|/|p_{m+1}|,\ |u_{m+1,k}-u^*_{m+1,k}|/|u_{m+1}|,\ |T_{m+1,k}-T^*_{m+1,k}|/|T_{m+1,k}|\}\leqq\varepsilon$$
$$(k=2,\ 3,\ 4,\ ...,\ n-1) \tag{21}$$

where max { } denotes the maximum element induced in { }, and the value of ε depends on the accuracy desired. In the present investigation, the value $\varepsilon=5\times10^{-4}$ is adopted. Thus, an advancement row by row up the channel can be executed. The calculation is continued until the pressure p reaches zero or becomes positive at which point the calculation is terminated. This determines L for input Q. If $p>0$ at termination then L is evaluated at $p=0$ by means of linear interpolation. 41 mesh points are used in the y-direction at the channel entrance and this is reduced to 21 mesh points at the last stage as x increases. A progressively larger step size is also applied in the x-direction, starting with an axial step size of about 0.01 per cent of the channel height. The present calculations are performed on FACOM 230-75 Digital Computer at the Computer Center of Hokkaido University. All of the examples are computed using the properties corresponding to those of carbon dioxide gas[8], $a=0.33$, $b=0.78$ and $c=1.34$.

4. Results and Discussions

4.1 Temprature profile

In Fig. 3 numerically computed axial pressure distributions for several value of N, which indicates the ratio of conduction heat transfer rate to radiation one, are presented for $Q=0.01$, $Gr_0=8.0\times10^6$, $Pr_0=0.77$, $\tau_w=0.36$ and $T_w=2.0$. As will be seen in this figure, the greater the

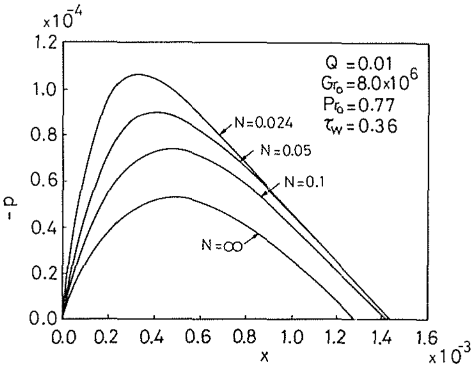

Fig. 3 Pressue variation vs. x (effect of N)

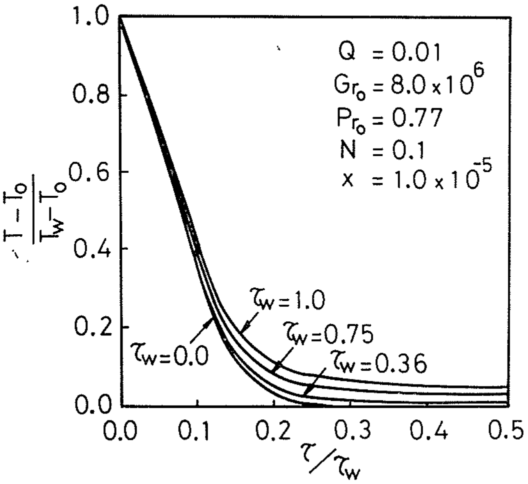

Fig. 4 Temperature profile vs. τ/τ_w (effect of τ_w)

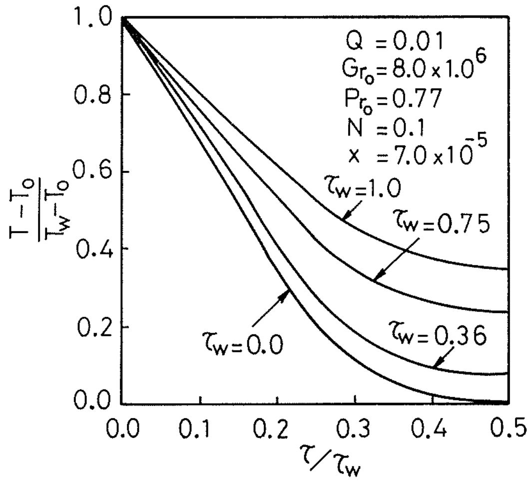

Fig. 5 Temperature profile vs. τ/τ_w (effect of τ_w)

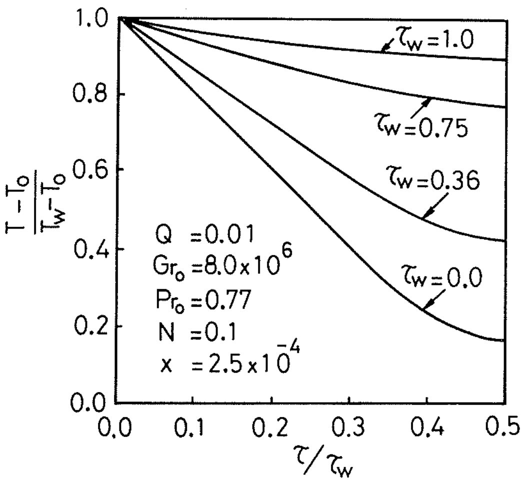

Fig. 6 Temperature profile vs. τ/τ_w (effect of τ_w)

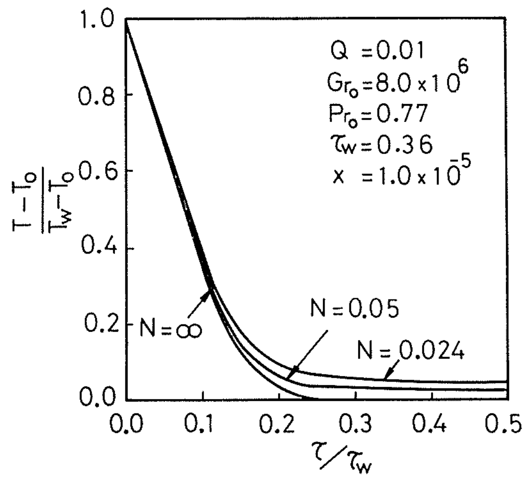

Fig. 7 Temperature profile vs. τ/τ_w (effect of N)

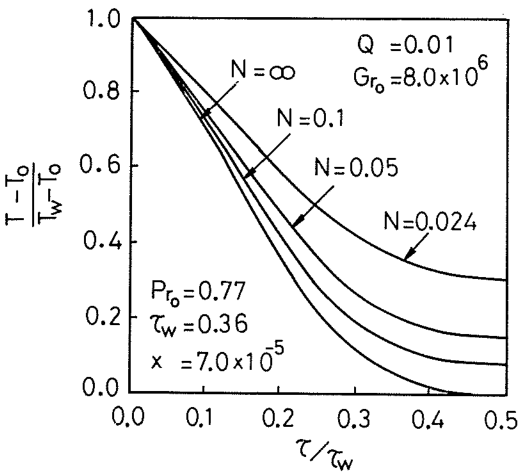

Fig. 8 Temperature profile vs. τ/τ_w (effect of N)

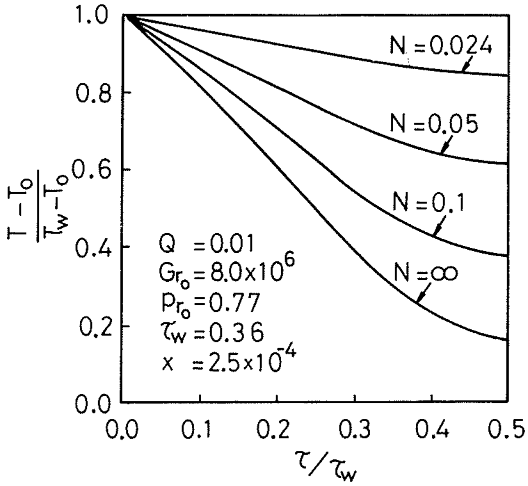

Fig. 9 Temperature profile vs. τ/τ_w (effect of N)

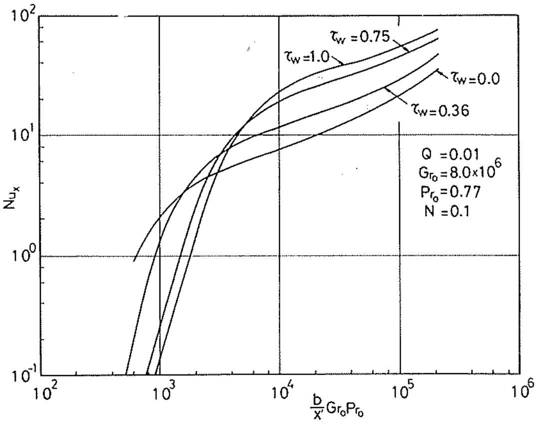

Fig. 10 Local Nusselt number Nu_x (effect of τ_w)

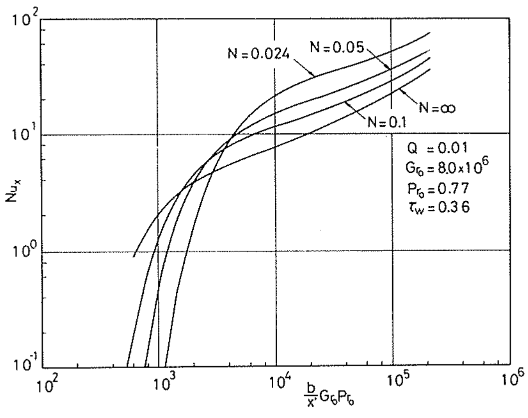

Fig. 11 Local Nusselt number Nu_x (effect of N)

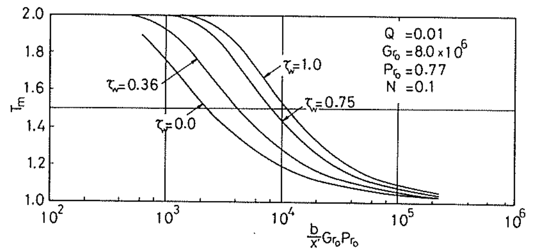

Fig. 12 Mixed mean temperature T_m (effect of τ_w)

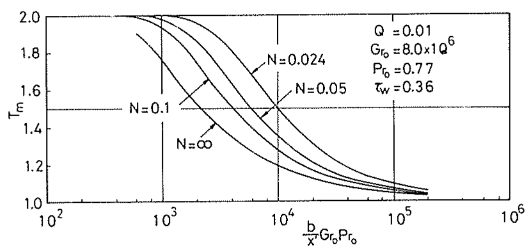

Fig. 13 Mixed mean temperature T_m (effect of N)

effect of radiation (small N) is, tha higher the pressure drop.

Figs. 4, 5 and 6 show the effects of optical thickness τ_w of the channel on developing temperature profile. The curve of $\tau_w=0$ in Figs. 4, 5 and 6 is a hypothetical non-radiating one for carbon dioxide. In Fig. 4, it can be seen that the fluid near the center of the channel is directly heated by the radiation from the walls and the temperature of fluid in the region rises uniformly for large τ_w; that is, for the large absorption coefficient of the medium. Such an effect increases with increasing x, i. e., departing downstream from the starting point of heating section of the region, as shown in Figs. 5 and 6.

The effects of conduction-radiation parameter N on developing temperature profile are indicatad in Figs. 7, 8 and 9 for $\tau_w=0.36$. The curve of $N=\infty$ in these figures corresponds to the case of a hypothetical non-radiating one. From these figures, it is understood that the temperature in the entire region of the channel rises uniformly as N decreases (radiation predominantes over conduction).

4.2 Heat transfer characteristics and mixed mean temperatures

The local Nusselt number and the mixed mean temperature are defined as follows.

$$Nu_x=-\frac{\lambda_w}{T_w-1}\left(\frac{\partial T}{\partial y}\right)_{y=0}+\frac{1}{T_w-1}\frac{\tau_w}{2N}\left[T_w^4E_3(0)-T_w^4E_3(\tau_w)-\tau_w\int_0^1 E_2(\tau_w t)dt\right] \quad (22)$$

$$T_m=\int_0^1 \rho c_p uTdy/\int_0^1 \rho c_p udy \quad (23)$$

Figs. 10 and 11 show the relation of the local Nusselt number Nu_x vs. (b/x) Gr_0Pr_0 for the optical thickness τ_w in the channel and for the conduction-radiation parameter N, respectively. It is found that Nu_x tends to become large with decreasing N or increasing τ_w. But it should be noted that Nu_x at a comparatively large x decreases distinguishably with the increasing τ_w or with the decresing N. This may be attributed to the fact that the medium temperature rises with increasing x and temperature difference between the fluid and the vertical walls decreases over the entire range of the channel, then heat transfer terminates quickly as the effect of radiation increases.

Figs. 12 and 13 illustrate the variation of the dimensionless mixed mean temperature T_m versus x. It can be seen from these figures that for constant N the larger τ_w is, the larger T_m is, while for constant τ_w the smaller N is, the larger T_m is. Such a tendency of T_m indicates that the heat transfer between the walls and the flowing medium in the vertical channel terminates at smaller x as radiation effects increases.

5. Conclusions

An analytical study has been performed on the simultaneous heat transfer by radiation and free convection of an absorbing and emitting gray gas with its property variation. The physical model employed in the present study is based on laminar flow in a isothermal vertical black channel. The effects of thermal radiation on pressure variation, developing temperature profile, the local Nusselt number and mixed mean temperature are discussed in detail.

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