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Scattering of Gaussian Laser Beam by Flowing Particles

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Abstract

The light scattered by the particles flowing with velocity fluctuations shows a random pattern with dynamic motion in space. This phenomenon is dependent on both the mean velocity and the velocity fluctuating around the mean. Such a scattered field produced by the particles under illumination of a gaussian beam is investigated statistically in this paper. The analysis of the scattered field shows that the temporal amplitude-correlation function of the scattered field is characterized by the product of two correlation functions: one is concerned with the mean velocity and the other is the velocity fluctuations. When the mean velocity is much greater than that of the velocity fluctuations, the correlation function is governed mainly by the waist width of the gaussian beam. The experiment was conducted by using an in-line heterodyning system for the amplitude-correlation of the scattered field and confirms the theoretical results on its velocity and beam-waist width dependence.

1. Introduction

A considerable amount of work has been reported on the study of coherent light scattering by random objects in the past years [1, 2, 3, 4]. The randomness of the scattered light field is dependent originally on the stochastic process of scatterers if coherent light is used. It may, therefore, be possible to obtain information about the dynamic behavior of the objects by statistically investigating the scattered light field. The coherent light scattered by small objects moving with velocity fluctuations is detected by a photodetector from which a random signal of the photocurrent is obtained. Analyzing this random signal with photon counting techniques, several workers have already obtained information regarding the velocity of various objects. For example, the motility of microorganisms [5, 6], the velocity of a flow containing the particles [7] and the velocity fluctuations of the Brownian particles [8] have been investigated. On the other hand, the above random signal has been also analyzed from the viewpoint of speckle patterns and various velocimeters for rigid screens have been proposed in this analysis and verified experimentally [9, 10, 11, 12].

A profile of the illuminating light as a probe for investigating the objects plays an important role in the analysis of the scattered light field. Nevertheless, most of the studies seem to have not sufficiently considered the role of the probing light profile.

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The study of speckle pattern, however, has already indicated that the statistical properties of the scattered light field varies remarkably by the effective aperture (the amplitude distribution) of the light illuminating the random objects. Goodman [13] investigated this effect on the statistics of the scattered field and Fujii and Asakura [14] utilized it for measuring the surface roughness. Takai [15] studied the effect of a gaussian beam illuminating a random phase screen on the dynamic speckles, and verified their statistical behavior in relation to the beam width and the surface roughness. Jakeman and McWhirter [16] also investigated the effect of the wavefront curvature of a gaussian beam on the scattered field.

This paper investigates laser light scattered by particles in motion with velocity fluctuations in a flow. The temporal amplitude-correlation function of the scattered field is analyzed in some detail when the velocity of particles consists of the constant mean and radomly fluctuating components. Further analysis is actually conducted for a laser beam with both the gaussian amplitude and the curved wavefront based on the assumption that the velocity fluctuations are given by a gaussian random variable. The result shows that the amplitude-correlation function is determined by the product of two correlation functions related to the mean velocity and the velocity fluctuations. The correlation function associated with the mean velocity is closely connected with the waist of the gaussian laser beam. Experiments based on the theoretical treatment were also performed. The in-line heterodyning technique was used to obtain the amplitude-correlation function from a photocurrent signal detected. Form the experimental results, a theoretical analysis was confirmed with respect to the velocity of the particles and the waist width of the illuminating gaussian beam.

2. Theory

In Fig. 1 the light scattering due to the moving particles and the scattered light field are shown. Scattering particles are distributed in the space of the coordinate (ξ, η, z). A monochromatic light with a wavelength λ, angular frequency ω and an amplitude profile $E_0(\xi, \eta, z)$ incident on the particles is scattered at each of their positions. The resultant scattered field in the plane $(x, y, z=R)$, a distance R away from the particles, is given by the superposition of each diffracted light. If

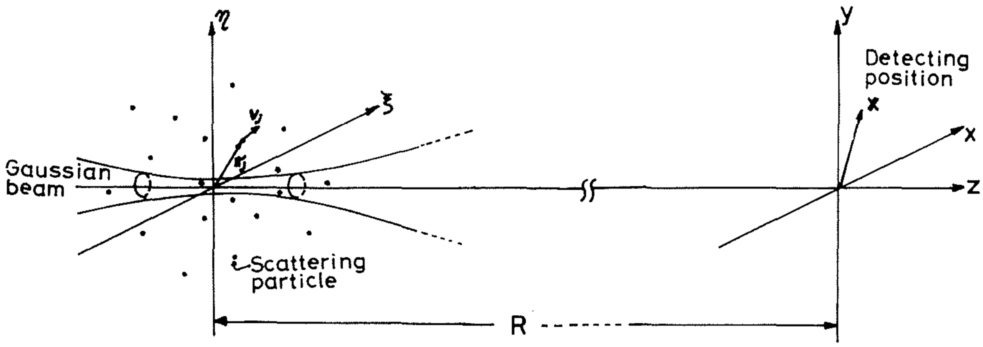

Fig. 1. Coordinate system describing the light scattering from the moving particles.

the particle size is very small and the multiple scattering is negligible, the scattered light field may be written as

$$E_s(\boldsymbol{q}, t) = \sum_{j=1}^{N} \alpha_j E_0\left\{\boldsymbol{r}_j(t)\right\} \exp\left\{i\boldsymbol{q}\cdot\boldsymbol{r}_j(t)\right\} \exp(-i\omega t) \tag{1}$$

were α_j is the scattering coefficient for the j-th particle, N is the total number of the particles contributing to the scattering, and the vectors $\boldsymbol{q}$ and $\boldsymbol{r}_j(t)$ simply indicate the detection point and the position of the j-th particle at time t, i. e. $\boldsymbol{q}=(2\pi x/\lambda R,\ 2\pi y/\lambda R)$ and $\boldsymbol{r}_j(t)=(\xi_j(t),\ \eta_j(t),\ z_j(t))$, respectively.

Consider the case where the particle velocity consists of two components : one is the mean velocity $\boldsymbol{v}_0$ and the other is the time-dependent velocity fluctuation $\boldsymbol{v}_{Bj}(t)$ with respect to the random motion of the particles. Therefore, it is expressed by

$$\boldsymbol{v}_j(t) = \boldsymbol{v}_0 + \boldsymbol{v}_{Bj}(t) \tag{2}$$

where $\langle \boldsymbol{v}_j(t)\rangle = \boldsymbol{v}_0$ ($\langle\cdots\rangle$ means the ensemble average) with the assumption of $\langle \boldsymbol{v}_{Bj}(t)\rangle = 0$. Then, the position of the time-dependent particles is given by

$$\begin{aligned} \boldsymbol{r}_j(t) &= \boldsymbol{r}_j(0) + \int_0^t \boldsymbol{v}_j(t')\,dt' \\ &= \boldsymbol{r}_j(0) + \boldsymbol{v}_0 t + \int_0^t \boldsymbol{v}_{Bj}(t')\,dt' . \end{aligned} \tag{3}$$

Substitution of Eq. (3) into Eq. (1) yields

$$\begin{aligned} E_s(\boldsymbol{q}, t) = \sum_{j=1}^{N} \alpha_j E_0\left\{\boldsymbol{r}_j(0) + \boldsymbol{v}_0 t\right\} \exp\left\{i\boldsymbol{q}\cdot\boldsymbol{r}_j(0)\right\} \exp(i\boldsymbol{q}\boldsymbol{v}_0 t) \\ \times \exp\left\{i\boldsymbol{q}\int_0^t \boldsymbol{v}_{Bj}(t')\,dt'\right\} \exp(-i\omega t) \end{aligned} \tag{4}$$

where the term related to $\boldsymbol{v}_{Bj}(t)$ in the argument of $E_0(\boldsymbol{r})$ was neglected. This approximation is allowable if the values of $E_0(\boldsymbol{r})$ are scarcely varied by the velocity fluctuation of $\boldsymbol{v}_{Bj}(t)$. With the further assumption that the statistical behavior of the scattered field is stationary in time, the temporal-correlation function of Eq. (4) is given by

$$\begin{aligned} \phi(\tau) &= \left\langle E_s(\boldsymbol{q}, 0)\, E_s^*(\boldsymbol{q}, \tau) \right\rangle \\ &= \left\langle \sum_{l=1}^{N} \sum_{m=1}^{N} \alpha_l \alpha_m^* E_0\left\{\boldsymbol{r}_l(0)\right\} E_0^*\left\{\boldsymbol{r}_m(0) + \boldsymbol{v}_0\tau\right\} \exp(-i\boldsymbol{q}\boldsymbol{v}_0\tau) \right. \\ &\quad \left. \times \exp\left[i\boldsymbol{q}\left\{\boldsymbol{r}_l(0) - \boldsymbol{r}_m(0)\right\}\right] \cdot \exp\left\{-i\boldsymbol{q}\int_0^\tau \boldsymbol{v}_{Bm}(t')\,dt'\right\} \right\rangle . \end{aligned} \tag{5}$$

If the total number N of the particles is so large that the contribution of the terms for $l \neq m$ to the scattered field are vanishing in the result of superposition and if the position $\boldsymbol{r}_m$ and velocity $\boldsymbol{v}_m$ of a particle are uncorrelated, the correlation function of Eq. (5) becomes

$$\begin{aligned} \phi(\tau) = \sum_{l=1}^{N} |\alpha_l|^2 \exp(-i\boldsymbol{q}\boldsymbol{v}_0\tau) \left\langle E_0\left\{\boldsymbol{r}_l(0)\right\} E_0^*\left\{\boldsymbol{r}_l(0) + \boldsymbol{v}_0\tau\right\}\right\rangle \\ \times \left\langle \exp\left\{-i\boldsymbol{q}\int_0^\tau \boldsymbol{v}_{Bl}(t')\,dt'\right\}\right\rangle . \end{aligned} \tag{6}$$

When the viewpoint of the statistical identity for the particles is taken, the above expression is rewritten in a form

$$\phi(\tau) = N|\alpha_0|^2 \exp(-i\,\boldsymbol{q}\,\boldsymbol{v}_0\,\tau)\,\phi_{E_0}(\tau)\,\phi_B(\tau) \tag{7}$$

where

$$\phi_{E_0}(\tau) = \left\langle E_0\left\{\boldsymbol{r}(0)\right\} E_0^*\left\{\boldsymbol{r}(0)+\boldsymbol{v}_0\tau\right\}\right\rangle \tag{8}$$

and

$$\phi_B(\tau) = \left\langle \exp\left\{-i\boldsymbol{q}\int_0^\tau \boldsymbol{v}_B(t')\,dt'\right\}\right\rangle. \tag{9}$$

In the above equations, the assumption of $\alpha_1 = \alpha_2 = \alpha_3 = \cdots \equiv \alpha_0$ is taken and the subscripts for random variables $\boldsymbol{r}_l$ and $\boldsymbol{v}_{Bl}$ are dropped since they are not particularly significant. The final equation (7) means that the temporal amplitude-correlation function of the scattered light field is determined by the product of an oscillating term $\exp(-i\,\boldsymbol{q}\,\boldsymbol{v}_0\,\tau)$ and two correlation functions of Eqs. (8) and (9): one correlation function is relevant to both the amplitude distribution of the illuminating light and the mean velocity $\boldsymbol{v}_0$ of the particles, and the other one depends on the random motion of the particles. These temporal-correlation functions will be calculated in the following.

First, we investigate the correlation function of Eq. (8) for the actual case of a gaussian beam employed as the illuminating light. The gaussian beam is usually produced from the laser as a fundamental mode. A three-dimensional gaussian amplitude profile with the waist w_0 is given by [17]

$$E_0(\xi,\eta,z) = \left\{w_0/w(z)\right\}\exp(i\,kz)\exp\left(-\frac{\xi^2+\eta^2}{w^2(z)}\right)$$
$$\times\exp\left(-ik\frac{\xi^2+\eta^2}{2\rho(z)}\right) \tag{10}$$

where k, $w(z)$ and $\rho(z)$ are the wavenumber of the light used, the width and wave-front curvature of the beam. These parameters are given by

$$k = 2\pi/\lambda \tag{11}$$

$$w(z) = w_0(1+z^2/a^2)^{1/2} \tag{12}$$

and

$$\rho(z) = z(1+a^2/z^2) \tag{13}$$

where the replacement of

$$a = \pi w_0^2/\lambda \tag{14}$$

was taken. For simplicity, we consider the case where the velocity has only one component in the ξ direction, i. e. $\boldsymbol{v}_0=(v_0, 0, 0)$. Therefore the correlation function of Eq. (8) becomes

$$\phi_{E_0}(\tau) = \left\langle\left\{w_0^2/w^2(z)\right\}\exp\left\{-\frac{\xi^2+(\xi+v_0\tau)^2+2\eta^2}{w^2(z)}\right\}\exp\left\{ik\frac{v_0^2\tau^2+2v_0\tau\xi}{2\rho(z)}\right\}\right\rangle$$
$$= \left\langle C(z)\exp\left\{-\alpha(\xi+\beta)^2\right\}\exp(-\alpha\eta^2)\exp(-i\gamma\xi)\right\rangle \tag{15}$$

where

$$\alpha = 2/w^2(z)\,, \qquad \beta = v_0\tau/2\,, \qquad \gamma = kv_0\tau/\rho(z) \tag{16}$$

and

$$C(z) = \left\{w_0^2/w^2(z)\right\} \exp\left\{ik \frac{v_0^2\tau^2}{2\rho(z)}\right\} \exp\left\{-\frac{v_0^2\tau^2}{2w_0^2(z)}\right\}. \tag{17}$$

If the position (ξ, η, z) of the particles is regarded as being expressed by a stochastic variable with uniform probability density, the ensemble average of Eq. (15) is replaced by the integral,

$$\phi_{E_0}(\tau) = \int_{z_0-l}^{z_0+l} C(z)\, dz \int_{-\infty}^{\infty} \exp\left\{-\alpha(\xi+\beta)^2\right\} \exp(i\gamma\xi)\, d\xi \int_{-\infty}^{\infty} \exp(-\alpha\eta^2)\, d\eta\,. \tag{18}$$

In this calculation, the regions of the integration over ξ and η are taken from $-\infty$ to $+\infty$ and the region of z is restricted in $z_0-l \leq z \leq z_0+l$ where the scattering particles exist. After some manipulation, we have

$$\phi_{E_0}(\tau) = (w_0^2/4)\int_{z_0-l}^{z_0+l} \exp\left\{-\frac{v_0^2\tau^2}{2w^2(z)}\right\} \exp\left\{-\frac{k^2 v_0^2\tau^2 w^2(z)}{8\rho^2(z)}\right\} dz\,. \tag{19}$$

By means of Eqs. (11)$\sim$(14), the correlation function of Eq. (15) associated with the gaussian beam finally is reduced to

$$\phi_{E_0}(\tau) = (w_0^2 l/2) \exp\left(-\frac{v_0^2\tau^2}{2w_0^2}\right). \tag{20}$$

It should be noted that, while the resultant equation (20) calculated for the illuminating gaussian beam of Eq. (10) is independent of the beam-waist position z_0, it is related to the beam waist w_0 alone. Therefore, the normalized form of this correlation function is always equal to the autocorrelation function of the profile $\exp\{-(\xi^2+\eta^2)/w_0^2\}$ of the illuminating beam at its waist position. The extent of the correlation function is then determined by both the beam-waist width w_0 and the velocity v_0. That is, the correlation length τ_{E_0} of Eq. (20) is given by

$$\tau_{E_0} = \sqrt{2}\, w_0/v_0 \tag{21}$$

where the extent of the correlation to $1/e$ times its central value was taken.

We next investigate the correlation function of Eq. (9) relatied to the random motion of the particles. For a gaussian random variable $x(t)$, there exsists, in general,

$$\left\langle \exp\left\{ic\int_0^t x(t')\, dt'\right\}\right\rangle = \exp\left\{-c^2/2\int_0^t\int_0^t dt_1\, dt_2 \left\langle x(t_1)\, x(t_2)\right\rangle\right\} \tag{22}$$

where c is a certain constant. When the autocorrelation of $x(t)$ is given, with the assumption of a temporal stationarity, by

$$\left\langle x(t_1)\, x(t_2)\right\rangle = \left\langle x^2\right\rangle \phi(\tau)\,, \qquad (\tau = t_2 - t_1) \tag{23}$$

equation (22) becomes

$$\left\langle \exp\left\{ic\int_0^t x(t')\, dt'\right\}\right\rangle = \exp\left\{-\frac{c^2}{2} \langle x^2\rangle \int_0^\tau (\tau-\tau')\, \phi(\tau')\, d\tau'\right\} \tag{24}$$

This relation can directly used to calculate the correlation function of Eq. (9) if the random velocity obeys a gaussian random process. Setting

$$\phi(\tau)=\frac{\langle \boldsymbol{v}_B(0)\,\boldsymbol{v}_B(\tau)\rangle}{\sqrt{\langle|\boldsymbol{v}_B|^2\rangle}}\,, \tag{25}$$

we obtain the correlation function $\phi_B(\tau)$ in the form

$$\phi_B(\tau)=\exp\left\{-\frac{1}{2}\,|\boldsymbol{q}|^2\langle|\boldsymbol{v}_B|^2\rangle\int_0^\tau(\tau-\tau')\,\phi(\tau')\,d\tau'\right\} \tag{26}$$

where the calculation was performed for each component of the velocity $\boldsymbol{v}_B$. When the velocity correlation function of Eq. (25) is further expressed by the following well-known form,

$$\phi(\tau)=\exp\,(-\tau/\tau_c) \tag{27}$$

with the correlation length τ_c for the random velocity $\boldsymbol{v}_B$ of the particles, equation (26) becomes

$$\phi_B(\tau)=\exp\left[-\frac{\langle|\boldsymbol{v}_B|^2\rangle}{2}\,\tau_c^2|\boldsymbol{q}|^2\left\{\exp\,(-\tau/\tau_c)+\tau/\tau_c-1\right\}\right]. \tag{28}$$

The extent of this correlation function seems to be not analytically determinable. If the condition of $\sqrt{\langle \boldsymbol{v}_B|^2\rangle}\;\tau_c\,|\boldsymbol{q}|/\sqrt{2}\ll 1$ is satisfied (this condition holds in many cases of the near-axis detection of $|\boldsymbol{q}|\simeq 0$), however, the correlation length τ_B was found in a heuristic way and approximately given by

$$\tau_B=\frac{2}{\langle|\boldsymbol{v}_B|^2\rangle|\boldsymbol{q}|^2\tau_c}\qquad(\text{for }|\boldsymbol{q}|\approx 0) \tag{29}$$

We now return to the subject of investigating the behavior of the amplitude-correlation function of the scattered light field on the basis of Eqs. (20) and (28). As was mentioned before, the amplitude-correlation function of Eq. (7) is characterized by the product of two correlation functions $\phi_{E_0}(\tau)$ and $\phi_B(\tau)$. If the correlation length of $\phi_B(\tau)$ is much greater than that of $\phi_{E_0}(\tau)$ as is shown in Fig. 2, the resultant correlation function may be governed mainly by the behavior of $\phi_{E_0}(\tau)$ even though there exists the correlation function $\phi_B(\tau)$. In such a case, the actual meaningful region of τ for the amplitude-correlation function is approximately restricted to the region within τ_{E_0}. Althuogh the effect of the oscillating term in Eq. (7) is known to be always negligible in the near-axis detection ($|\boldsymbol{q}|\approx 0$), it also becomes negligible, as is clear from Eq. (21), when a beam with a narrow waist w_0 is used as the illuminating light. Under these circumustances, we may obtain the temporal-correlation function of Eq. (7) in the form,

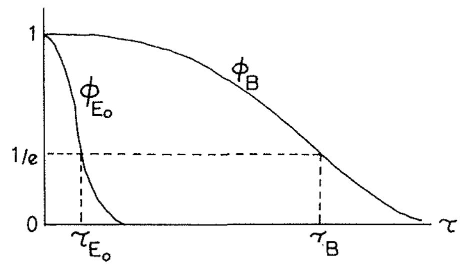

Fig. 2. Correlation functions $\phi_{E_0}(\tau)$ and $\phi_B(\tau)$ forming the amplitude-correlation function of the sattcered light field. In this figure the case of $\tau_{E_0}\ll\tau_B$ is shown.

$$\phi(\tau)\propto\phi_{E_0}(\tau)\propto\exp\left(-\frac{v_0^2\tau^2}{2w_0^2}\right) \tag{30}$$

where the condition of

$$\tau_{E_0} \ll \tau_B \tag{31}$$

has been imposed.

3. Experiments and discussion

In the previous section, we calculated the temporal amplitude-correlation function of the light field scattered by the particles flowing with velocity fluctuations. Before describing the experiments, we present one of the methods to obtain the amplitude-correlation function of the scattered field. It may utilize a heterodyning of the scattered light field situated near the optical axis. The heterodyning is produced by interference between the scattered light $E_{scat}(t)$ from each of the particles and the strong background light E_{uns} which is straightforwardly passing through the particle system without scattering. This circumustance corresponds to a flow having a low density of particles. The photocurrent signal obtained by a photodetector at the heterodyning field is expressed, omitting a constant factor, by

$$\begin{aligned} I(t) &= |E_{scat}(t) + E_{uns}|^2 \\ &= |E_{scat}(t)|^2 + |E_{uns}|^2 + E_{scat}(t)\, E^*_{uns} + E^*_{scat}(t)\, E_{uns} \end{aligned} \tag{32}$$

Since $|E_{uns}| \gg |E_{scat}(t)|$ is established under the above condition of low density of scattering particles, the ac signal obtained through a high-pass filter reaches:

$$I_{ac}(t) = E_{uns}\, E^*_{scat}(t) + E^*_{uns}\, E_{scat}(t)\,. \tag{33}$$

The temporal correlation function of this signal gives

$$\begin{aligned} \left\langle I_{ac}(t)\, I_{ac}(t+\tau) \right\rangle &= 2\,|E_{uns}|^2 Re \left\langle E_{scat}(t)\, E^*_{scat}(t+\tau) \right\rangle \\ &\propto \phi(\tau) \end{aligned} \tag{34}$$

where Re indicates the real part and the very high frequency terms were neglected in the process of the ensemble average. Thus, we can obtain directly the temporal amplitude-correlation function from the ac signal of the photocurrent detected in the neighborhood of the optical axis.

Fig. 3 shows a schematic diagram of the experimental arrangement used. The

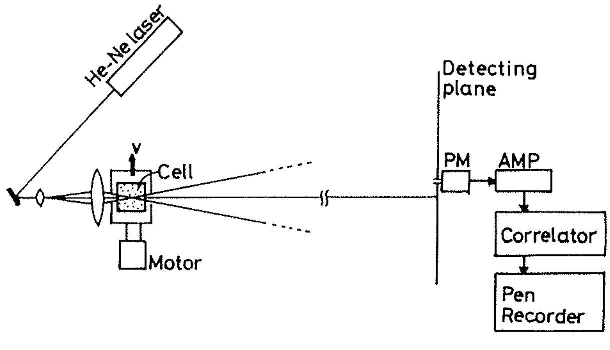

Fig. 3. Experimental arrangement for investigating the amplitude-correlation function of the scattered light field.

single mode He-Ne laser light of a wavelength of 6328 Å was converted by means of lenses L_1 and L_2 to a gaussian beam with the waist width appropriately controlled. The polystyrene spheres of diameter 1.1 μm as scattering particles were suspended in 4% saline solution. The density of the salt water was nearly equal to that of the particles. This density matching eliminates the motion of the particles due to precipitation. The sample water with the particles was placed in a cell of 24 mm thickness. The cell was placed on a stage moving automatically at a given speed by the aid of a motor. In this manner, a constant velocity of the particles was simulated instead of using an actual fiow. A photograph of the scattered light field in the plane, at a distance of $R=50$ cm from the center of the cell, is shown in Fig. 4. In this photograph, the random interference phenomenon between the randomly sacttered light and the background light can be observed in the bright central area. This interference pattern varies in a rather complicated fashion due to the random motion of the particles even when the sample is kept at rest. The exposure time for this photograph was 1/100 sec. Of course, when the constant velocity is added to the natural moiton of the particles, a variation of the pattern rapidly increases corresponding to that velocity.

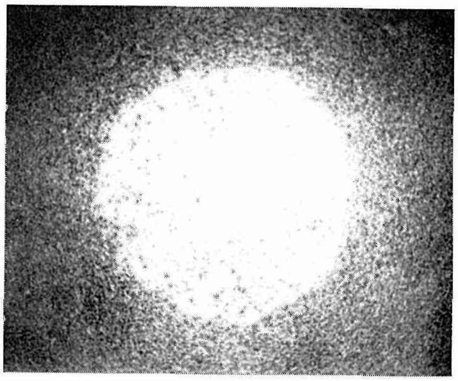

Fig. 4. A photograph of the scattered light field. Interference between the scattered light and the background occurs in the bright region of the center.

A photomultiplier was set at the center in the detecting plane and the photocurrent signal was obtained through a pinhole. After the signal was fed into an RC high-pass filter and then amplified, the autocorrelation function of its ac signal was obtained at real time by using the TEAC correlator C-120.

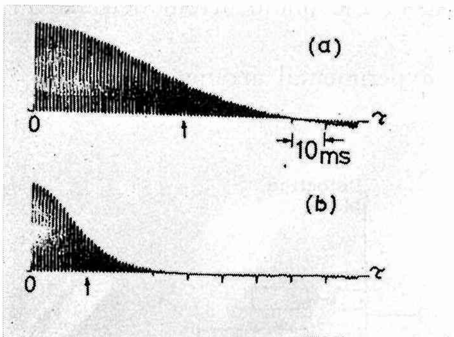

Fig. 5. Dependence of the amplitude-correlation function on the mean velocity: (a) $v_0=133$ μm/sec and (b) $v_0=340$ μm/sec. Measured correlation lengths are shown by an arrow and take 43 msec and 17 msec, respectively.

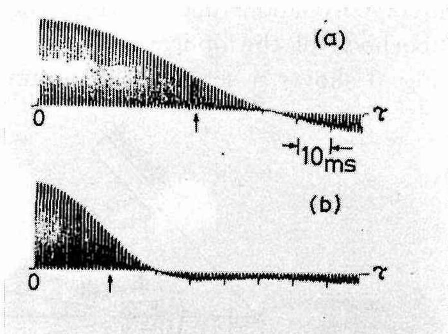

Fig. 6. Dependence of the amplitude-correlation function on the beam-waist width: (a) $w_0=5.1$ μm and (b) $w_0=2.7$ μm. Measured correlation lengths are shown by an arrow and take 44 msec and 22 msec, respectively.

We first invesigate the dependence of the temporal amplitude-correlation function on the constant velocity. Fig. 5 shows the behavior of the correlation functions measured in two cases of $v_0=133$ μm/sec and 340 μm/sec. The values of the measured correlation length were 43 msec and 17 msec and their positions are indicated by an arrow in each figure. When the above experiment obeys the result of Eq. (30), the extent of the correlation function varies inversely proportionally to the mean velocity. The reciprocal ratio of the measured correlation lengths is 2.53 while the ratio of the actual velocities is 2.56. This agreement indicates that the extent of the correlation function is determined by only the constant velocity of the particles, indepently of their random motion.

We next investigate the dependence of the temporal amplitude-correlation function on the beam profile. Fig. 6 shows the two correlation functions measured by using the gaussian beams with waist widths of 5.1 μm and 2.7 μm which were determined by measuring the diverging angles of their intensity distribution. In this case, the waist ratio of two beams is 0.53 which is nearly equal to the ratio 0.50 of the correlation lengths 44 msec and 22 msec measured in respective cases. Consequently, this result verifies the relationship of Eq. (30) between the behavior of the correlation function and the waist of the gaussian beam used. In the experiments, the beam-waist position was chosen at the center of the cell since it was known from the derivation of Eq. (20) that the beam-waist position gives no influence to the correlation function. This fact was actually confirmed by several experiments in which the beam-waist position was varied at several points inside and outside the cell.

4. Conclusion

The temporal amplitude-correlation function of the light field scattered by the particles flowing with velocity fluctuations was closely related to the mean velocity and the velocity fluctuations of a flow. It now becomes clear that the mean flow velocity and illuminating light profile determined the behavior of the temporal amplitude-correlation function when the velocity fluctuations are small. Therefore, the present work may be useful as one of the techniques for measuring the speed of a flow.

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