Bulletin of the Faculty of Engineering,
Hokkaido University, No. 87 (1978)

# Heat Transfer in an Enclosed Rectangular Cavity with a Relatively Small Aspect-Ratio

**Nobuhiro SEKI* Shoichiro FUKUSAKO***
**Hideo INABA***

(Received September 27, 1977)


## Abstract

This paper presents a numerical study on natural convective heat transfer in an enclosed rectangular cavity with a relatively small aspect-ratio $H/W$ with opposing vertical walls having each different temperature.

The present numerical calculations are carried out for an enclosed rectangular cavity having $H/W=0.03\sim1$, $1\sim10^3$ of Prandtl number $Pr$ and $10^2\sim10^6$ of Rayleigh number R$a$. The numerical results obtained indicate that the geometrical aspect-ratio significantly affects the heat transfer through the vertical fluid layer. Moreover, useful correlations of the natural convective heat transfer are derived from the computed results.


## Nomenclature

$c_p$ : specific heat at constant pressure;
$Gr$ : Grashof number, $g\beta\Delta T H^3/\nu^2$;
$H$ : height of rectangular cavity;
$Nu$ : average Nusselt number, $\int_0^1 \frac{\partial T}{\partial Y}|_{X=0,H/W}\,dX$;
$Pr$ : Prandtl number, $\nu/\alpha$;
$Ra$ : Rayleigh number, $Pr\times Gr$;
$T'$, $T$ : temperature, non-dimensional temperature, $(T'-T'_c)/(T'_h-T'_c)$;
$\Delta T$ : temperature difference between hot and cold walls, $(T'_h-T'_c)$;
$U'$, $V'$ : velocity components;
$U$, $V$ : non-dimensional velocity components, $(U', V')\,[H/(\nu\sqrt{Gr})]$;
$X'$, $Y'$ : coordinates;
$X$, $Y$ : non-dimensional coordinates, $(X', Y')/H$;
$W$ : width of rectangular cavity.

Greek symbols

$\alpha$ : thermal diffusivity;
$\beta$ : coefficient of cubical expansion;
$\lambda$ : thermal conductivity;
$\nu$ : kinematic viscosity;
$\rho$ : density;
$\psi$ : stream function;

* Department of Mechanical Engineering II, Hokkaido University, Sapporo 060, Japan.

$\Psi$ : non-dimensional stream function, $\psi/(\nu\sqrt{Gr})$

$\omega$ : vorticity;

$\Omega$ : non-dimensional vorticity, $\omega[H^2/(\nu\sqrt{Gr})]$.

Subscripts

$c$ : refers to cold wall;

$h$ : refers to hot wall.

## 1. Introduction

A numerous multitude of papers are available[1~3] concerned with the problem of natural convective heat transfer in a rectangular cavity having a relatively high aspect-ratio ($H/W>1$), in which opposing vertical walls have each different temperature. However, only a few studies of a rectangular cavity having relatively low aspect-ratio ($H/W<1$), which are of importance in the evaluation of the rate of heat transfer in buildings with a low ratio of height/width, in which one vertical wall is heated by solar heat, or a shallow duct. For example, Hirata et al[4]. made experimental studies on this problem with a low aspect-ratio for $0.42<H/W<1$, but they could not obtain satisfactory results owing to an incompleteness of insulation of both the upper and bottom walls. On the other hand, Cormak et al[5]. made attempts to study analytically the natural convection in a shallow cavity with differentially heated end walls. However, their results were confined to the explanation of the physical behavior of this model and they failed to derive a correlation of heat transfer through the fluid layer.

The purpose of this study is to analytically clarify the effects of the dimension of cavity and the physical properties of fluid on the heat transfer in the rectangular cavity with a relatively small aspect-ratio in a range of $0.03<H/W<1$, in which the opposing vertical walls have each different uniform temperature and while the upper and bottom walls are thermally insulated.

## 2. Description of the Present Mathematical Analysis

The present analysis is carried out by considering an enclosed two-dimensional rectangular cavity with a small aspect-ratio ($H/W<1$), which is filled with fluid. The opposing vertical walls of the cavity are maintained at different uniform temperatures $T'_h$ and $T'_c$, while the upper and the bottom walls are rigid non-slip boundaries which are perfectly insulated. The physical model is shown in Fig. 1. The numerical solutions are obtained by solving the finite-differential approximation of a steady state Boussinesq equations similar to the previous investigation[6]. Introducing stream function $\psi$, vorticity $\omega$ and following non-dimensional variables into appropriate governing equations, namely, the conservation of mass, momentum and energy balance equations:

$$X=\frac{X'}{H},\quad Y=\frac{Y'}{H},\quad U=\frac{U'H}{\nu\sqrt{Gr}},\quad V=\frac{V'H}{\nu\sqrt{Gr}},\quad \Psi=\frac{\psi}{\nu\sqrt{Gr}},\quad \Omega=\frac{\omega H^2}{\nu\sqrt{Gr}}$$

$$T=\frac{T'-T'_c}{T'_h-T'_c},\quad Pr=\nu/\alpha,\quad Gr=\frac{g\beta\Delta TH^3}{\nu^2}.$$

One obtains the following non-dimensional governing equations to be analyzed:

$$\frac{\partial}{\partial X}\left(\frac{\partial \Psi}{\partial X}\right)+\frac{\partial}{\partial Y}\left(\frac{\partial \Psi}{\partial Y}\right)+\Omega=0$$

$$\frac{\partial}{\partial X}\left(\Omega\frac{\partial \Psi}{\partial Y}\right)+\frac{\partial}{\partial Y}\left(\Omega\frac{\partial \Psi}{\partial X}\right)-\frac{1}{\sqrt{Gr}}\left[\frac{\partial}{\partial X}\left(\frac{\partial T}{\partial X}\right)+\frac{\partial}{\partial Y}\left(\frac{\partial T}{\partial Y}\right)\right]+\frac{\partial T}{\partial Y}=0$$

$$Pr\left[\frac{\partial}{\partial X}\left(T\frac{\partial \Psi}{\partial Y}\right)-\frac{\partial}{\partial Y}\left(T\frac{\partial \Psi}{\partial X}\right)\right]-\frac{1}{\sqrt{Gr}}\left[\frac{\partial}{\partial X}\left(\frac{\partial T}{\partial X}\right)+\frac{\partial}{\partial Y}\left(\frac{\partial T}{\partial Y}\right)\right]=0$$

The boundary conditions are as follows:

$$X=0;\quad \frac{\partial \Psi}{\partial X}=\frac{\partial \Psi}{\partial Y}=\Psi=0,\quad \frac{\partial T}{\partial X}=0$$

$$X=1;\quad \frac{\partial \Psi}{\partial X}=\frac{\partial \Psi}{\partial Y}=\Psi=0,\quad \frac{\partial T}{\partial X}=0$$

$$Y=0;\quad \frac{\partial \Psi}{\partial Y}=\frac{\partial \Psi}{\partial X}=\Psi=0,\quad T=1$$

$$Y=W/H;\quad \frac{\partial \Psi}{\partial Y}=\frac{\partial \Psi}{\partial X}=\Psi=0,\quad T=0$$

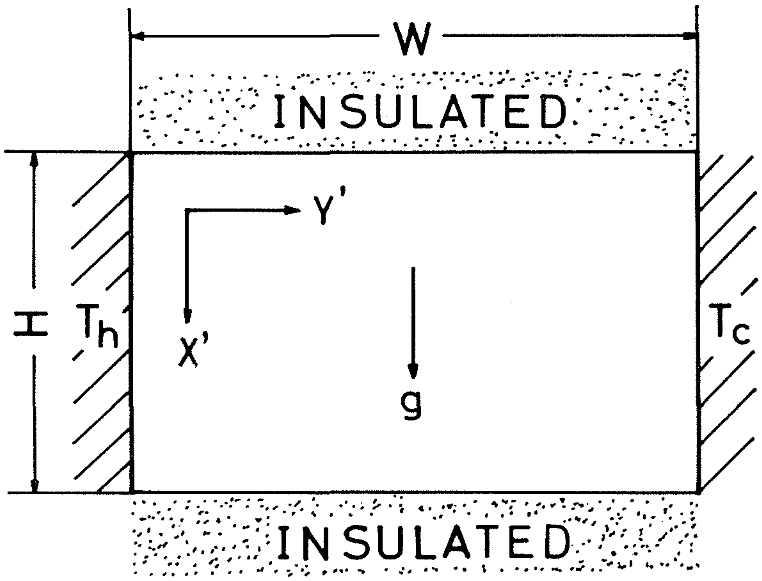


**Fig. 1.** The physical model of the cavity.

In view of the mathematical complexity to solve these equations, the Up-Wind finite-difference technique is applied. The number of grids in the vertical direction [$X$] is 21, while that in the horizontal direction [$Y$] is decided corresponding to the aspect-ratio. The line iterative method is performed until two succesive iteration values of $\Psi$, $\Omega$ and $T$ are attained within 0.1 percent. The computations are performed with a FACOM 230-75 Digital Computer at the Computer Center of Hokkaido University.

## 3. Results and Discussions

A comparison between the present numerical results and the previous ones is shown in Fig. 2 for aspect-ratio $H/W=1$ and Prandtl number $Pr=1$. From this figure, it is clear that the present numerical results in the range of $Ra>5\times10^3$ have approximately the same tendency as those obtained by Batchelor[1] and Emery et al[2].. Where the Nusselt number $Nu$ monotonously increases with increasing Rayleigh number $Ra$. Moreover, it is of interest to note that the numerical results of this study is quite similar to MacGregor's[3] which for Nusselt number do not show a tendency to merge smoothly with the straight line of 1/4 slope in their predicted correlation, but the curve possesses an inflection point in small $Ra$. It might be said that in the present study, an asymptotic flow region, in which the intensity of natural convection is very weak and the conductivity predominates, appears in the range of $Ra<5\times10^3$ for $H/W=1$ and $Pr=1$.

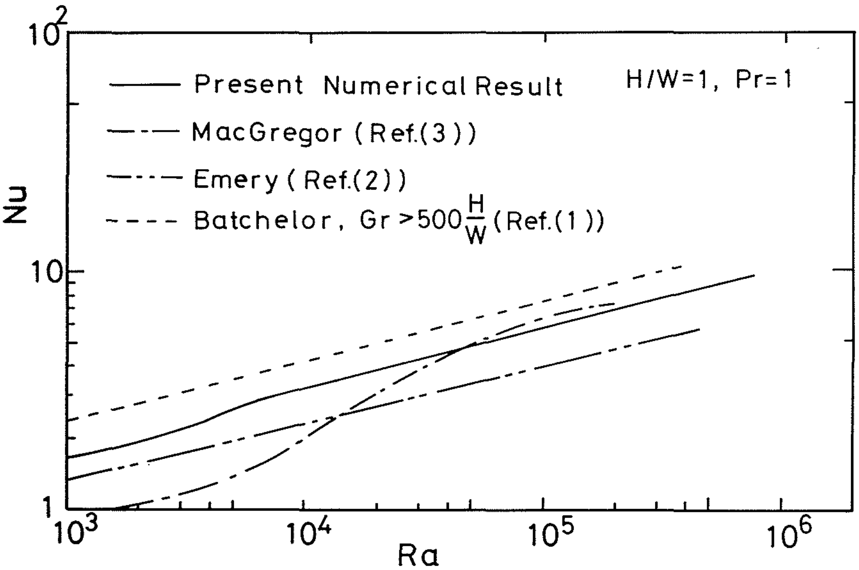


**Fig. 2.** Comparison between the present numerical results and the previous ones for $H/W=1$ and $Pr=1$.

Fig. 3 presents a comparison between the present numerical results and the previous experimental ones reported by Hirata et al[4].. The results shown in the upper figure of Fig. 3 for $H/W=1$ (their experiments were performed for $H/W=0.92$) show a good agreement between the present numerical results and the experimental ones. However, in another figure of Fig. 3 for $H/W=0.5$. their experimental data lie below the solid-line proposed by the present numerical results. These deviations could be caused by not only the difference of aspect-ratio $H/W$ between the present analysis ($H/W=0.5$) and the previous experiment ($H/W=0.42$), but also the difference of the boundary conditions between the analysis and the experiment. In other words, the assumptions in the present analysis in which the upper and the bottom walls are thermally insulated or the surface temperature of the hot and the cold vertical walls are kept uniformly, might not hold in the previous experiments.

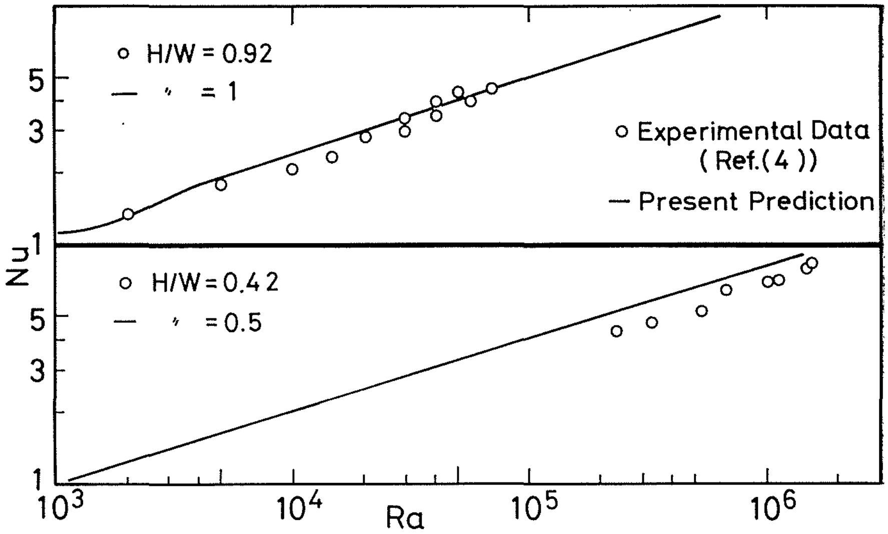


**Fig. 3.** Comparison between the present numerical results and the previous experimental ones.

Fig. 4 shows the relationship between $Nu$ and $Ra$ for a wide range of $H/W$. In this figure, it can be seen that for small $Ra$ the conductive heat transfer which $Nu$ is constant and where $Ra$ is varied predominates in the fluid layer. here the

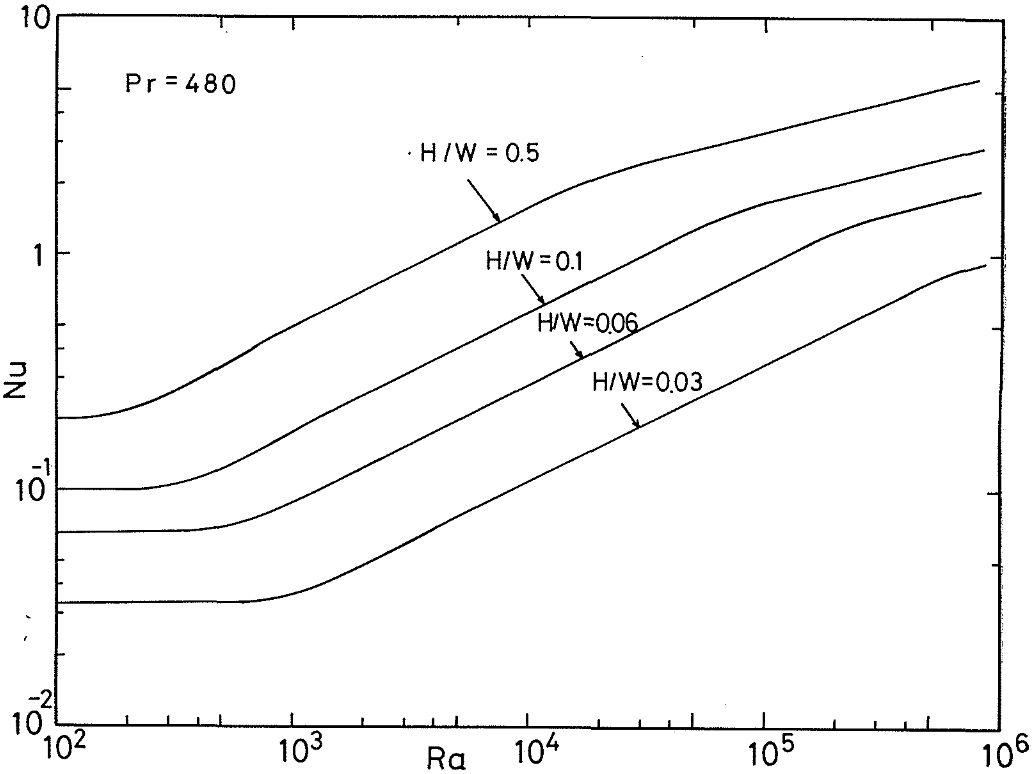


**Fig. 4.** Relationship between $Ra$ and $Nu$.

reader should be kept in mind that the value of $Nu$ is possibly below unity if the $Nu$ is defined by the height of the cavity as a representative length. Moreover, as $Ra$ increases, the mode of heat transfer gradually shifts to the asymptotic heat transfer in which $Nu$ has a slope of 1/2 to $Ra$ on the logarithmic graph, finally the mode becomes a laminar convective heat transfer having a slope of 1/4. One can also see that critical Rayleigh number for the onset of asymptotic heat transfer or the laminar heat transfer increases as $H/W$ decreases. These behaviors might be explained from the fact that as $H/W$ decreases, that is, when the width of the cavity increases under a fixed height, the cumulative effect of locally small viscous effects acting over a sufficient long width of the cavity increases.

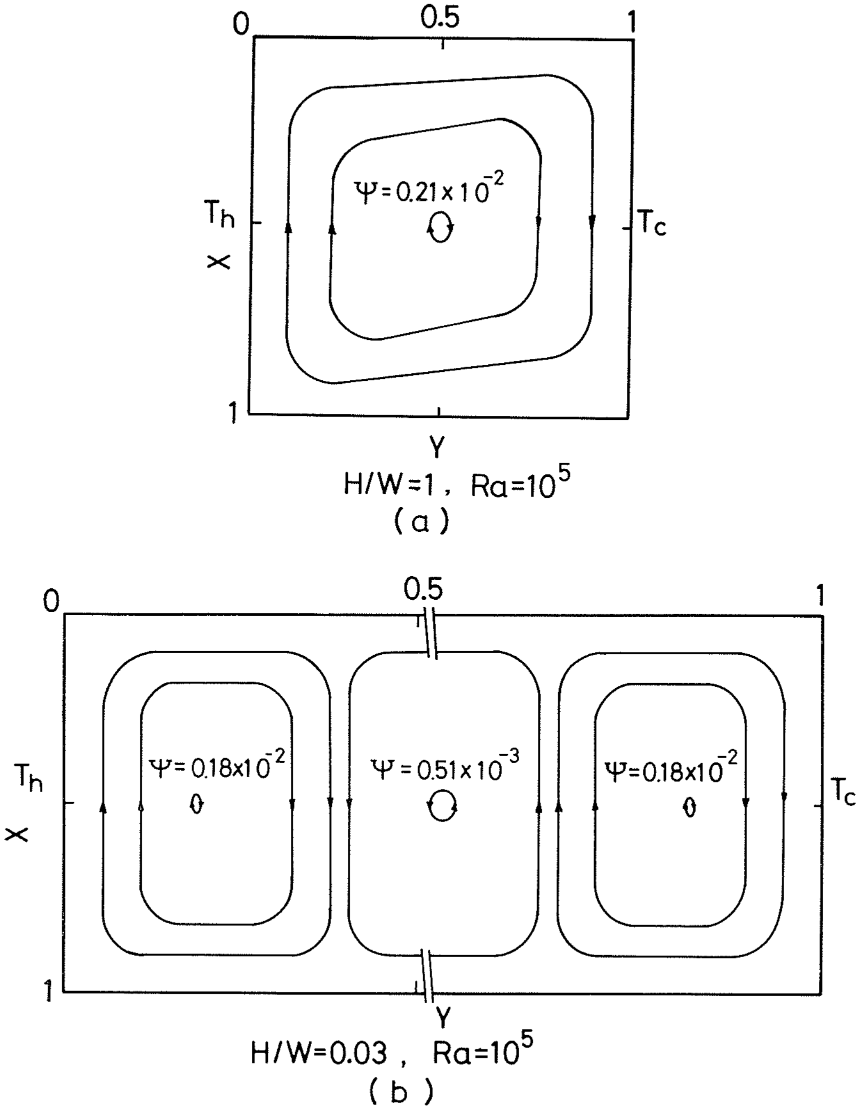


**Fig. 5.** Flow patterns obtained by the present computation, (a) $H/W=1$; (b) $H/W=0.03$.

The typical flow patterns obtained by the present computation are demonstrated in Fig. 5 for $H/W=1$ and 0.03 under $Ra=10^5$. The flow pattern for $H/W=1$ as shown in fig. 5-(a) indicates only one eddy which circulates in a clock-wise direction, while that for $H/W=0.03$ as shown in Fig. 5-(b) indicates a very complicated behavior having three eddies in the fluid layer. From the result of flow patterns obtained, the above-mentioned decrease of $Nu$ with decreasing $H/W$ show that these plural eddies which have a slow speed of circulation, as shown in fig. 5-(b), disturb the heat transfer throught the fluid layer. In the present study, such plural eddies appear in the range of $H/W<0.2$ for $Ra=10^5$. Therefore, it might be concluded that the mechanism of heat transfer is different in the case of $H/W>0.2$ or $H/W<0.2$.

Fig. 6 indicates the non-dimensional velocity distributions in the $X$- direction for non-dimensional distance $Y=0.5$ in order to obtain the intensity of convective flow. One of interesting characteristics in this figure is that the non-dimensional velocity distributions for $H/W=1$ and 0.5 are remarkably different from those for $H/W=0.2$, 0.06 and 0.03, that is, the flow direction of the former is counter to that of the latter. The non-dimensional velocity distributions for $H/W=1$ and 0.5 correspond to the flow pattern show in Fig. 5-(a), while those for $H/W=0.2$, 0.06 and 0.03 correspond to the flow pattern shown in Fig. 5-(b).

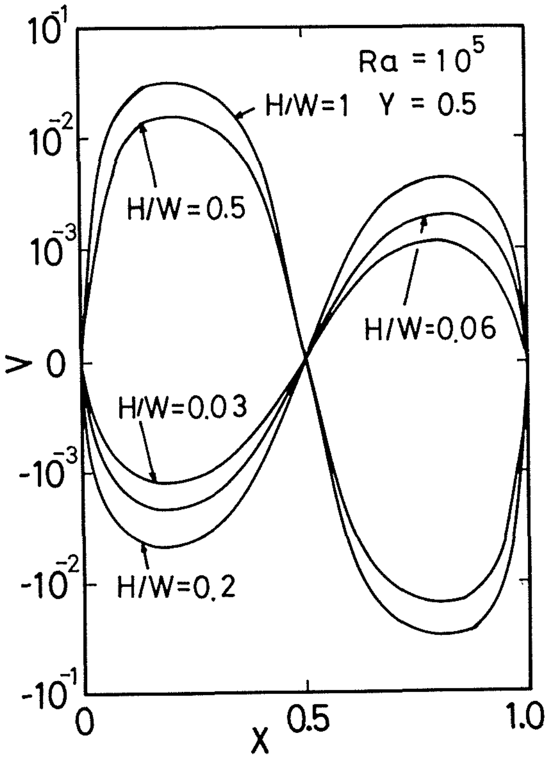


**Fig. 6.** Non-dimensional velocity distributions in the $X$-direction for $Y=0.5$.

Fig. 7 shows the non-dimensional temperature distributions in the $Y$-direction for $X=0.5$ and $Ra=10^5$ in order to check the behavior of heat transfer in the fluid layer. In this figure, it is clear that as $H/W$ decreases, the non-dimensional temperature distribution approaches closely to a line relationship between non-dimensional temperature $T$ and $Y$, which means the conductive heat transfer predominating mainly in the fluid layer. In contrast, as $H/W$ increases, the temperature gradient of the fluid layer in the neighborhood of vertical walls [$Y=0$ or $W/H$] becomes larger and the convective heat transfer predominates mainly in the fluid layer.

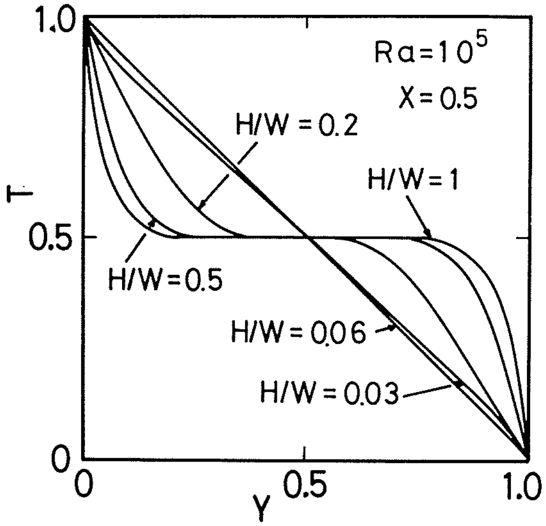


**Fig. 7.** Non-dimensional temperature distributions in the $Y$-direction for $X=0.5$.

The effect of $Pr$ and $Nu$ is presented in Fig. 8 for $H/W=0.2$ and $Ra=10^5$. From this figure, it might be understood that the value of $Nu$ increases linearly as $Pr$ increases on the logarithmic graph. This behavior could be explained by the facts that the increase of $Pr$ (=kinematic viscosity $\nu$/ thermal diffusivity $\alpha$) means the increase of kinematic viscosity due to the thermal diffusivity of a common fluid which has little variation of temperature and the intensity of natural convection is increased with the decreasing viscous resistance. Consequently, $Nu$ representing natural convective heat transfer is increased with the increasing $Pr$. The slope of $Nu$ to $Pr$ obtained in this study is 0.024.

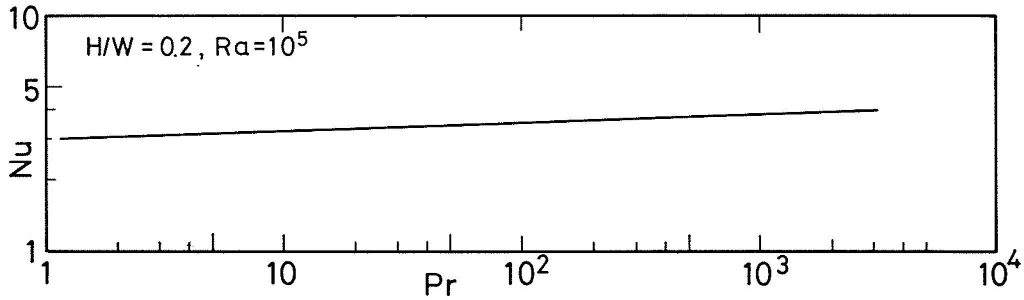


**Fig. 8.** Relationship between $Pr$ and $Nu$ for $H/W=0.2$ and $Ra=10^5$.

Fig. 9 shows the relationship between $Nu$ and $H/W$ together with those for $H/W > 1$ proposed in the previous work[6]. In the laminar region, if $H/W$ approaches 0, the natural convection in the fluid layer becomes quiescent and heat is transferred only by thermal conduction across the fluid layer. On the other hand, if $H/W$ approaches $\infty$, the mechanism of the heat transfer is essentially similar to the natural convective heat transfer between infinite parallel vertical plates and the mode of heat transfer becomes similar to conductive heat transfer. Therefore, it might be considered that the maximum value of $Nu$ exists between $0 < H/W < \infty$. From these results, it could be seen that the maximum value of $Nu$ in the present study exists between $1 < H/W < 1.5$ and also the tendency of $Nu$ related to $H/W$ is different from that the case of $H/W > 0.2$ and $H/W < 0.2$.

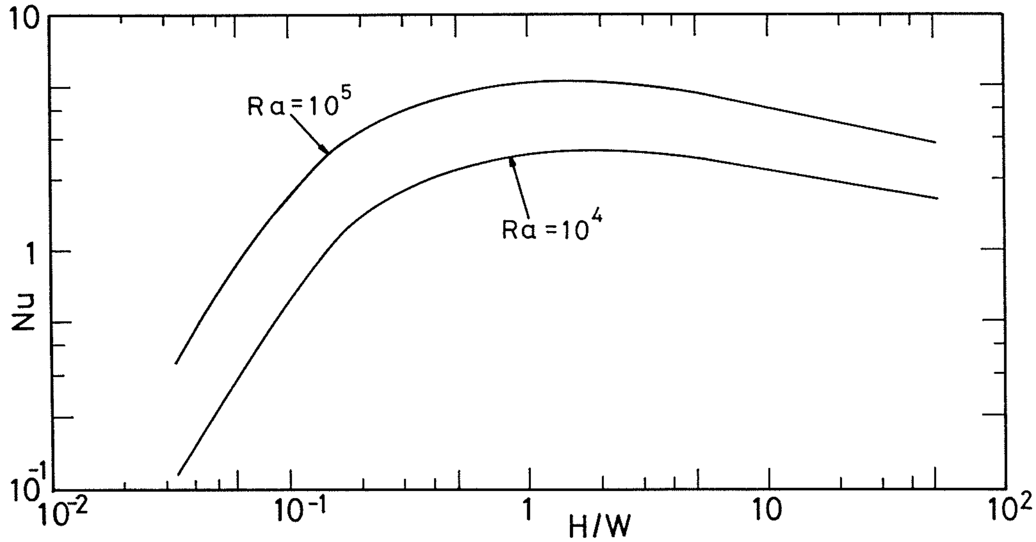


**Fig. 9.** Relationship between $H/W$ and $Nu$ for $Ra=10^4$ and $10^5$.

The following functional relationship given by a product of powers is derived from the result of the present dimensional analysis of heat transfer through a fluid layer :

$$Nu = cPr^{l}(H/W)^{m}\,Ra^{n}\,.$$

Moreover, all of the constant and exponents are determined by numerical computation, in which the deviation is with $\pm 7\%$ in the present study. The correlation derived from analysis is as follows :

In the laminar region ;

$$Nu = 0.223Pr^{-0.024}\,(H/W)^{0.19}\,Ra^{1/4}\,,$$

$$0.2 < H/W < 1\,,\quad 1 < Pr < 10^3\,,\quad Ra > 10^3\,,$$

$$Nu = 0.851Pr^{-0.024}(H/W)^{1.02}\,Ra^{1/4}\,,$$

$$0.03 < H/W < 0.2\,,\quad 1 < Pr < 10^3\,,\quad Ra > 10^5\,;$$

In the asymptotic region ;

$$Nu = 0.280 Pr^{0.024} (H/W)^{1.75} Ra^{1/2},$$
$$0.03 < H/W < 0.2, \quad 1 < Pr < 10^3, \quad 10^2 < Ra < 10^5.$$

## 4. Conclusion

Numerical analyses are carried out on the rectangular cavity having a relatively small aspect-ratio ($0.03 < H/W < 1$), in which the opposing vertical walls have each different temperature, while the upper and bottom walls are insulated. It is clear that aspect- ratio $H/W$ has a significant effect on the heat transfer through the fluid layer. Moreover, the correlation derived from the present analysis is as follows:

In the laminar region;

$$Nu = 0.223 Pr^{0.024} (H/W)^{0.19} Ra^{1/4},$$
$$0.2 < H/W < 1, \quad 1 < Pr < 10^3, \quad Ra > 10^3;$$
$$Nu = 0.851 Pr^{0.024} (H/W)^{1.02} Ra^{1/4},$$
$$0.03 < H/W < 0.2, \quad 1 < Pr < 10^3, \quad Ra > 10^5;$$

In the asymptotic region;

$$Nu = 0.280 Pr^{0.024} (H/W)^{1.75} Ra^{1/2},$$
$$0.03 < H/W < 0.2, \quad 1 < Pr < 10^3, \quad 10^2 < Ra < 10^5.$$

## References

1) Batchelor, G. K.: Quart. Appl. Math., Vol. 12 (1954), p. 209.
2) Emery, A. F. and Chu, N. C.: Trans. ASME, Ser. C, Vol. 87 (1965), p. 110.
3) MacGregor, R. K. and Emery, A. F.: Trans. ASME, Ser. C, Vol. 91 (1969), p. 391.
4) Hirata, M. Nishiwaki, N., Miyashita, H. and Itoh, M.: Semi-International Symposium (Tokyo), JSME (1967), p. 203.
5) Cormack, D. E. and Leal, L. G.: J. Fluid Mech., Vol. 65 (1974), p. 209.
6) Seki, N., Fukusako, S. and Inaba, H.: 13th Japan Heat Transfer Symposium (Kobe), (1976), p. 247.