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A Critical Review of a Matched Filter Receiving Radar Echoes from Extended Targets

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Abstract

This paper presents a critical review of the use of a matched filter in a radar system constructed mainly for probing extended targets. It is shown that the optimality of the matched filter is lost if the objective of the sounding lies in the reconstruction of a continuous scene illuminated by the radar. An inverse filter is shown to be a theoretically natural choice for the task conventionally assigned to the matched filter.

Introduction

The problem of detecting the presence or absence of a signal in the presence of noise is often encountered in the area of signal processing. The detection problem is distinct from other disciplines of signal processing in the point that the signal to be detected has a known waveform ; one is asked to determine whether it is there. An elegant solution to the problelm is the use of a matched filter.[1] The matched filter maximizes the signal to noise ratio of its output for a particular input signal waveform.

A typical detection problem arises in a radar system ; echo response to a point target is a series of pulses, each of which is a replica of the transmitted pulse with a time delay proportional to the round-trip distance to the target. Since the transmitted signal has a known waveform, the presence of each returned pulse can be detected with a filter properly *matched* to the waveform.

Emphasis must be placed on the requirement that the waveform be known for the signal to be detected by the matched filter. The requirement is fulfilled if the object of the sounding consists of a single point or a set of sparsely scattered points, each re-transmitting a separate replica of the original pulse. This may not be the case for a space borne synthetic aperture radar,[2] which continuously scans the surface of the earth.

We review the basic properties of the matched filter so as to obtain an overview

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of the fundamental signal processing technique, which will be referred to in a later section. It is shown that inverse-filtering is a more straightforward solution to the problem of restoring the continuous scene from radar response.

A Review the Matched Filter Principle

In this section the operating principle of the matched filter is briefly reviewed. We make no attempt to develop a comprehensive filtering theory. The reader is referred to reference[1] for in-depth discussion of the subject.

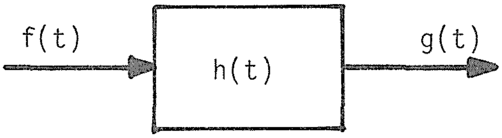

Fig. 1 Shift-invariant system with the impulse response h(t).

Consider the linear shift-invariant system shown in Fig. 1.

The output of the system is given by

$$g(\tau)=\int_{-\infty}^{\infty} f(t)h(\tau-t)dt \tag{1}$$

We can rewrite (1) as

$$g(\tau)=(1/2\pi)\int_{-\infty}^{\infty} \mathbf{f}(\omega)\mathbf{h}(\omega)\exp(j\omega\tau)d\omega \tag{2}$$

where $\mathbf{f}(\omega)$ and $\mathbf{h}(\omega)$ are the Fourier representations of f(t) and h(t), respectively. From (2) and Schwarz's inequality, it follows that

$$|g(\tau)|^2 \leq (1/4\pi^2)\int_{-\infty}^{\infty}|\mathbf{f}(\omega)|^2 d\omega \cdot \int_{-\infty}^{\infty}|\mathbf{h}(\omega)|^2 d\omega = E_f \cdot E_h \tag{3}$$

where

$$E_f=(1/2\pi)\int_{-\infty}^{\infty}|\mathbf{f}(\omega)|^2 d\omega \tag{4}$$

$$E_h=(1/2\pi)\int_{-\infty}^{\infty}|\mathbf{h}(\omega)|^2 d\omega \tag{5}$$

Thus $|g(\tau)|^2$ is bounded by $E_f \cdot E_h$, with both E_f and E_h assumed to be given. The lefthand side of (3) is the maximum of $|g(\tau)|^2$ attained with

$$\tau=0 \tag{6}$$

and

$$\mathbf{h}(\omega)=k\mathbf{f}^*(\omega) \tag{7}$$

The constant k is to be chosen to satisfy (5). But we can set k=1 without severe loss of generality. Hence we have

$$\mathbf{h}(\omega)=\mathbf{f}^*(\omega) \tag{8}$$

as a necessary and sufficient condition for $|g(\tau=0)|^2$ to attain its upper limit. Eq. (8)

is equivalent to

$$h(t)=f^*(-t) \tag{9}$$

as one can readily see.

If and only if f(t) and h(t) satisfy (8), or (9) equivalently, the instantaneous output power of the filter reaches its maximum at $\tau=0$. The filter is termed as the matched filter since it is *matched* to the input signal. Substitution of (9) into (1) yields

$$g(\tau)=\int_{-\infty}^{\infty}f(t)f^*(t-\tau)dt \tag{10}$$

The output $g(\tau)$ of the filter is seen to be the autocorrelation function of the input signal f(t). Therefore, the matched filter is also called a correlation receiver. The output of the correlator reaches its maximum at $\tau=0$ with its associated input function to produce the autocorrelation function, not crosscorrelation function.

With $f(t-t_0)$ a time-shifted version of f(t), the output of the matched filter, a shift invariant system, is equal to $g(t-t_0)$, which attains its maximum magnitude $|g(0)|$ at $t=t_0$. Note that the matched filter can detect the presence of f(t) and a possible shift in time of f(t) as well.

We have presented the fundamental mode of operation of the matched filter in the absence of noise. The results also hold even in the presence of noise uncorrelated with the signal f(t). This will be easily appreciated if we note that matched filter correlates f(t) with its input. A matched filter tuned for a pulse can also be used to detect a train of pulses which are amply separated replicas of the original pulse. This is also a straightforward function of the correlator.

So much for the useful characteristics of the matched filter receiver. We should keep in mind that its optimal operation is based on the perfect knowledge about the waveform of a signal of interest. It merely detects the presence or absence of the known signal. It involves no unknown functions other than the background noise.

Reconstruction of a Scene from Radar Echo Returns

Consider a radar sensor; it transmits a pulse S(t), which is scattered back to the radar by a point target located at range r. The signal received will be S(t-2r/c), where c is the velocity of light. For an extended target with reflectivity w(r), the signal received by the receiver is a linear superposition of echoes:

$$R(t)=\int_{-\infty}^{\infty}w(r)S(t-2r/c)dr \tag{11}$$

where the range illuminated by the radar has been incorporated in w(r). Eq. (11) can be written as

$$\bar{R}(r_t)=\int_{-\infty}^{\infty}w(r)\bar{S}(r_t-r)dr \tag{12}$$

where

$$\bar{R}(r_t) \triangleq R(t) \tag{13}$$
$$\bar{S}(r) \triangleq S(2r/c) \tag{14}$$
$$r_t \triangleq ct/2 \tag{15}$$

Eq. (11) is a convolution integral which we desire to solve for w(r). The deconvolution is most conveniently carried out in the Fourier domain ; taking the Fourier tansforms of both sides of (12), we have

$$\bar{\mathbf{R}}(\omega) = \mathbf{w}(\omega) \cdot \bar{\mathbf{S}}(\omega) \tag{16}$$

where the boldfaced letters indicate the Fourier representations of the respective functions. Assuming that w(r) is band-limited, and $\mathbf{S}(\omega)$ is nonzero in the frequency band of interest, we can solve (16) for $\mathbf{w}(\omega)$:

$$\mathbf{w}(\omega) = \bar{\mathbf{R}}(\omega) \cdot \mathbf{S}^{-1}(\omega) \tag{17}$$

Thus we can accomplish the reconstruction through the inverse filter $\mathbf{S}^{-1}(\omega)$. The band-limitedness of the signal w(r) also helps us combat the presence of noise which has been disregarded in the foregoing arguments. It is possible to design an inverse reconstruction filter that works in a noisy background.

We have seen that the inverse filter enables us to reconstruct the scene. The problem of reconstructing an extended scene is most commonly encountered in SAR (synthetic aperture radar) signal processing. Matched filters have customarily been used to give solutions to the problem[2]. Some explanation is therefore due here to clarify the relation between the two different filters. The matched filter is designed to let us know the presence of the known waveform. It basically has no place in our efforts to estimate the unknown signal w(r). The filter *matched* for the transmitted pulse is given by

$$\mathbf{h}(\omega) = \mathbf{S}^*(\omega) \qquad \text{(See (8))} \tag{18}$$

The matched filter yields at the output port

$$\tilde{\mathbf{w}}(\omega) = \bar{\mathbf{R}}(\omega) \cdot \mathbf{h}(\omega) = \bar{\mathbf{R}}(\omega)\mathbf{S}^*(\omega) \tag{19}$$

with $\bar{\mathbf{R}}(\omega)$ as the input. $\tilde{\mathbf{w}}(\omega)$ in (19) is identical to $\mathbf{w}(\omega)$ in (17) if and only if

$$\mathbf{S}^*(\omega) = \mathbf{S}^{-1}(\omega) \tag{20}$$

from which it immediately follows that

$$|\bar{\mathbf{S}}(\omega)| = 1 \tag{21}$$

Hence, for the matched filter to be equivalent to the inverse filter it must be all-pass. Matched filters found in the SAR literature roughly satisfy the condition (21). They by no means operate according to the matched filter principle.

As pointed out previously, the matched filter is a solution to the maximization problem involving only two functions f(t) and h(t). With an arbitrary filter function $\mathbf{h}(\omega)$, the output is given by

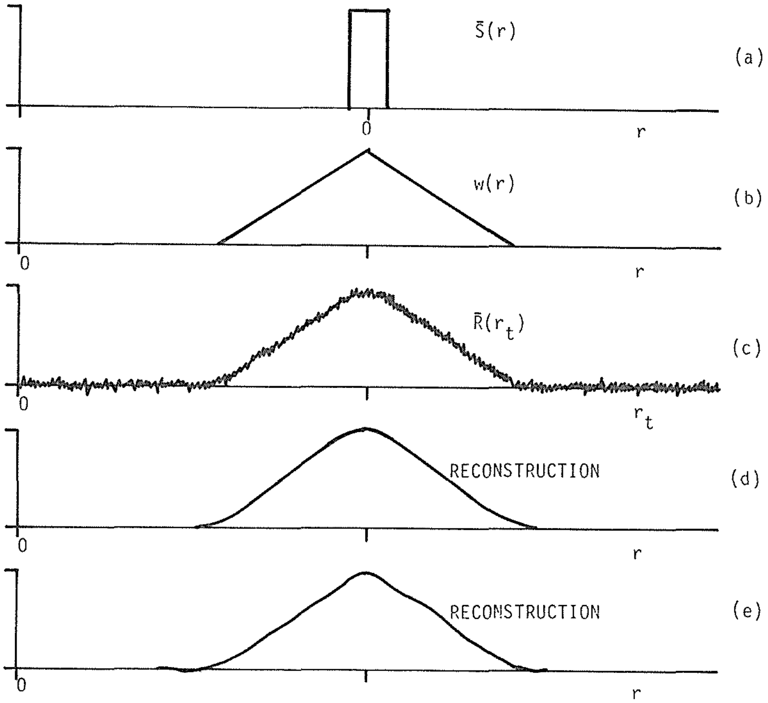

Fig. 2 (a) Transmitted pulse. (b) Extended target. (c) Noisy received signal. (d) Processed by a matched filter. (e) Processed by an inverse filter, SNR=20dB.

$$\mathbf{g}(\omega)=\mathbf{h}(\omega)\cdot\bar{\mathbf{R}}(\omega)=\mathbf{w}(\omega)\bar{\mathbf{S}}(\omega)\mathbf{h}(\omega) \tag{22}$$

where the input is assumed to be the received signal (16). The problem of maximizing $|g(t)|^2$ is exactly the matched filter problem if

$$|\mathbf{w}(\omega)|=\text{constant} \tag{23}$$

This requirement is fulfilled if

$$w(r)=\delta(r-r_1) \tag{24}$$

Eq. (24) represents a point target at range r_1.

Fig. 2 gives an illustrative example of noisy reception processed by a matched filter and an inverse filter. Observe that the inverse filter yields a better reconstruction of the extended target.

Conclusion

We have reviewed the matched filter principle with emphasis on its dependence on the complete knowledge about the signal waveform. The matched filter can be successfully applied to processing echo returns of point targets. Inverse filtering is, however, a straightforward method to use if we desire to restore extended targets. The use of matched filters in SAR signal processing owes its success not to the matched filter principle but to characteristics of the particular inverse filter required; it can be roughly simulated with a conjugate transfer function.

Performance of an inverse filter superior to that of a matched filter has been illustrated by an example. A simple square pulse has been assumed in the example. Investigation of a radar tranasmitting a more sophiscated pulse waveform rendering itself to pulse compression is underway. Results of the study will be reported in a future paper.

References

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2) C. Elachi, T. Bicknell, M. L. Jordan, and C. Wu, Proc. IEEE, **70**, 1174 (1982).