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Author(s)	Maeda, Junji
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Image Restoration by a Constrained Iterative Method

Junji MAEDA

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Abstract

We describe a new digital method for restoring linearly degraded images in the presence of noise. The restoration procedure is an iterative damped least-squares algorithm which has the advantage of the capability of dealing with the wide range of degradations and tolerance to noise. We present some experimental results that demonstrate the effectiveness of the proposed method.

1. Introduction

In this paper we investigate a new digital restoration procedure, which we name the iterative damped least-squares (DLS) method,[1,2] for restoring noisy degraded images. The proposed technique was constructed based on the principle of damped least squares[3]. This method has the advantage of the capability of dealing with a wide range of degradations, including space-invariant (SI) and space-variant (SV) blurs, because our algorithm can be implemented in both spatial and Fourier transform domains.

The iterative DLS method effectively uses additional constraints of non-negativity of the object and adaptive regularization to reduce the effect of noise.[4,5] We also employ median filtering to suppress spiky noise.[6,7]

As a result, the proposed algorithm has another advantage of significant tolerance to noise. After discussing the restoration procedure, we present experimental results which include four different types of degradations.

2. Restoration Procedure

The sampled value of the linearly degraded image can be expressed by a vector-matrix equation:

$$\mathbf{g}=\mathbf{H}\mathbf{f}+\mathbf{n}\ , \tag{1}$$

where $\mathbf{g}$, $\mathbf{f}$, and $\mathbf{n}$ are stacked $N^2\times 1$ vectors containing the $N\times N$ samples of degraded image, original object, and additive noise, respectively. The matrix $\mathbf{H}$ is the $N^2\times N^2$ matrix resulting form the point-spread function (PSF).

We seek the estimate $\hat{\mathbf{f}}$ that minimizes the objective fuction:

$$E(\hat{\mathbf{f}})=\|\mathbf{g}-\mathbf{H}\hat{\mathbf{f}}\|^2+\varepsilon\|\Delta\hat{\mathbf{f}}\|^2\ , \tag{2}$$

* Department of Applied Physics, Faculty of Engineering, Hokkaido University, Sapporo, 060, Japan.

where ε is called the damping factor which is a small positive quantity, and $\Delta\hat{\mathbf{f}}$ is the correction vector.

Differentiating Eq. (2) with respect to $\Delta\hat{\mathbf{f}}$ and setting the result equal to zero yields the damped least-squares solution step as

$$\Delta\hat{\mathbf{f}}=(\mathbf{H}^T\mathbf{H}+\varepsilon\mathbf{I})^{-1}\mathbf{H}^T(\mathbf{g}-\mathbf{H}\hat{\mathbf{f}}_k)\,. \tag{3}$$

Then we get the restored estimate at the $(k+1)$ st iteration by the iterative DLS algorithm as

$$\hat{\mathbf{f}}_{k+1}=\hat{\mathbf{f}}_k+\Delta\hat{\mathbf{f}}_k=\hat{\mathbf{f}}_k+\boldsymbol{w}_k(\mathbf{g}-\mathbf{H}\hat{\mathbf{f}}_k), \tag{4}$$

where we define

$$\boldsymbol{w}_k=(\mathbf{H}^T\mathbf{H}+\varepsilon_k\mathbf{I})^{-1}\mathbf{H}^T \tag{5}$$

as the DLS filter in the spatial domain.

Setting the initial estimate $\hat{\mathbf{f}}_0=0$, we obtain the next guess solution

$$\hat{\mathbf{f}}_1=\boldsymbol{w}_0\mathbf{g}=(\mathbf{H}^T\mathbf{H}+\varepsilon_0\mathbf{I})^{-1}\mathbf{H}^T\mathbf{g}\,. \tag{6}$$

Therefore the estimate at the first iteration in the iterative DLS algorithm is identical with the solution by the conventional (noniterative) regularized least-squares method.[8]

The damping factor ε in each iteration cycle is increased as

$$\varepsilon_k=c\times\varepsilon_{k-1}=c^k\times\varepsilon_0\,, \tag{7}$$

where $c>1$ is the constant which controls the degree of requlalization. We call this procedure as adaptive regularization. As to the additional constraint of the non-negativity of the object, we simply set the negative values of the restored estimate to zero at each cycle of the iteration.

We also employ a median filter to suppress the effect of spiky noise in the restored estimate.

We can also implement the proposed method in the Fourier domain. The Fourier version of the DLS filter is given by

$$W_k(u,v)=H^*(u,v)/(|H(u,v)|^2+\varepsilon_k)\,, \tag{8}$$

where $H(u, v)$ is the discrete Fourier transform (DFT) of the PSF.

If we use $F(u, v)$ and $G(u, v)$ as the DFT of original object and degraded image, respectively, the restored estimate by the iterative DLS method in the Fourier domain can be written from Eq. (4)

$$\hat{F}_{k+1}(u,v)=\hat{F}_k(u,v)+W_k(u,v)[G(u,v)-H(u,v)\hat{F}_k(u,v)]\,, \tag{9}$$

with the initial estimate $\hat{F}_0(u, v)=0$.

3. Experimental Results

In the first three simulations, the restored estimate after the 10th iteration cycle is evaluated by the restoring coefficient R, which is defined as[9]

$$R=\left[1-\frac{\|\mathbf{f}-\hat{\mathbf{f}}_{10}\|^2}{\|\mathbf{f}-\mathbf{g}\|^2}\right]\times 100\,(\%)\ . \tag{10}$$

This R indicates how the restored image approaches the original object in comparison with the degraded image.

The first example is the 1-D restoration with the model object shown in Fig. 1(a). The degraded image by a uniform motion at a SNR of 30 dB is shown in Fig. 1(b).

The restored images obtained by the iterative DLS method without and with median filtering are presented in Figs. 1(c) ($R=95.6\,\%$) and 1(d) ($R=98.0\,\%$), respectively. We used $\varepsilon_0=5\times10^{-3}$ and $c=1.3$.

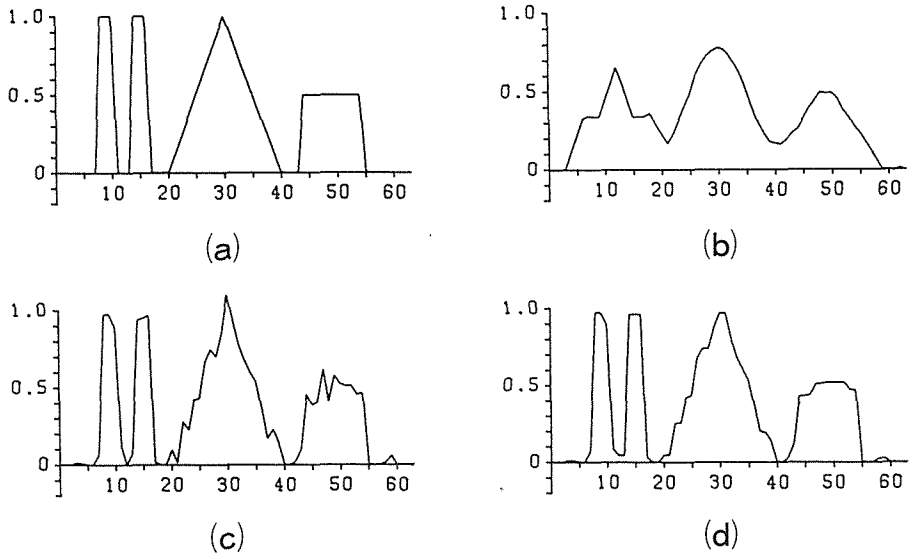


Fig. 1. (a) 1-D model object; (b) noisy degraded image by the SI system (motion blur) at 30-dB SNR; (c) restored image by the iterative DLS method without median filtering; (d) restord image by the iterative DLS method with median filtering.

These results clearly indicate the effectiveness of incorporating the median filter into the iterative DLS algorithm. The median filter was applied in the remaining examples.

Figures 2(a) and 2(b) illustrate the transfer function of the degraded system and the restored transfer function, respectively.

The second example is the 2-D restoration in the SI case. Figures 3(a) and 3(b) show the 2-D model object and the degraded image obtained through the Gaussian PSF system which has 30-dB SNR, respectively. The restored results obtained by the conventional regularized least-squares method and by the proposed method are shown in Figs. 3(c) ($R=$

60.8 %) and 3(d) ($R=82.9$ %), respectively. We used $\varepsilon_0=1\times10^{-3}$ and $c=1.1$.

These figures obviously represent that the iterative DLS method results in a better restoration than the conventional method.

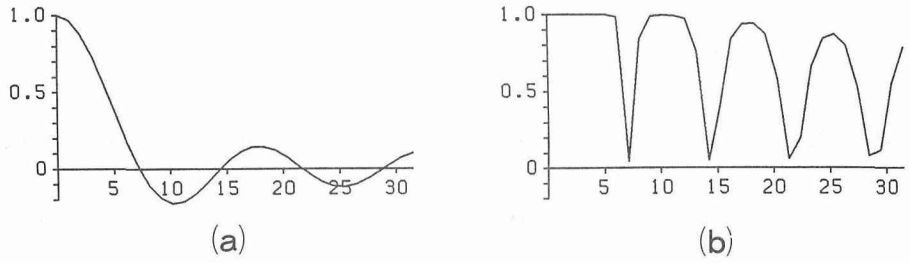


Fig. 2. (a) Transfer function of the degraded system; (b) restored transfer function.

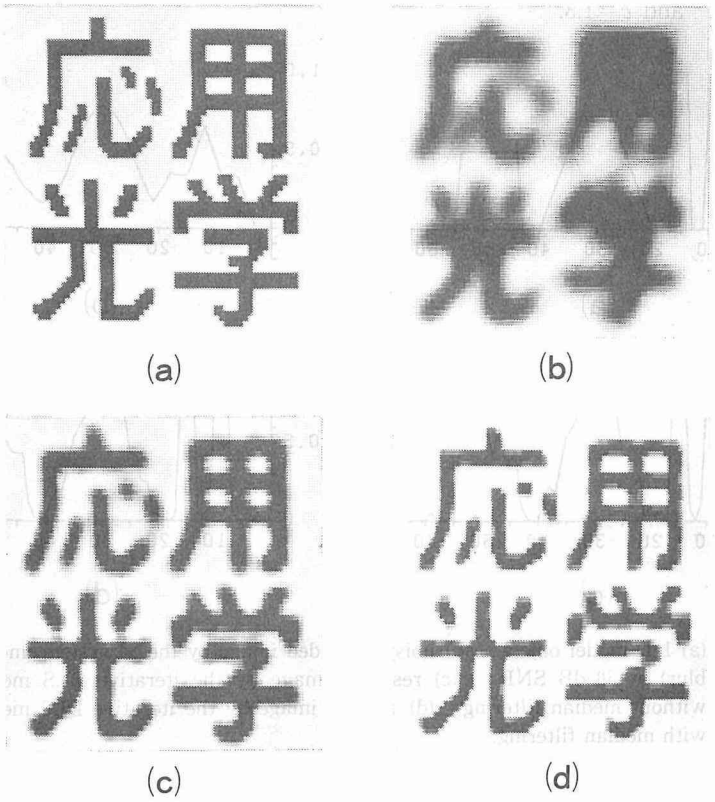


Fig. 3. (a) 2-D model object; (b) noisy degraded image by the SI system at 30-dB SNR; (c) restored image by the conventional regularized least-squares method; (d) restored image by the iterative DLS method.

The third example is related to the SV imaging situation. Figure 4(a) is the degraded image obtained through the SV system which has a better focus at the center and an

increasing blur toward the edges. The restored image obtained by the iterative DLS method is shown in Fig.4 (b) ($R=83.2\,\%$).

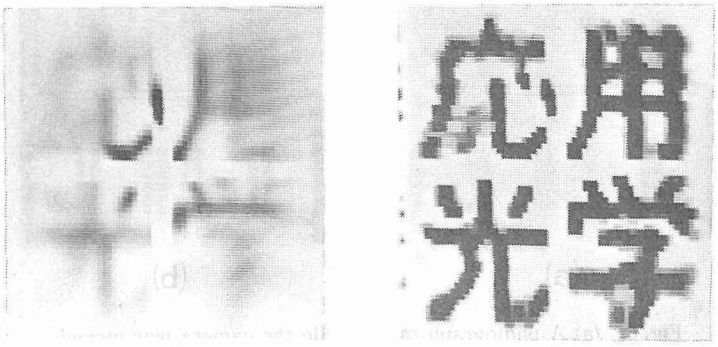


Fig. 4. (a) Noisy degraded image by the SV system at 30-dB SNR; (b) restored image by the iterative DLS method.

The last two examples are restoration with real-world degradations. Figure 5(a) is a photograph of the Furukawa Hall in Hokkaido University taken with the camera defocused.

Figure 6(a) is a photograph taken while the camera was moved horizontally.

The restored pictures obtained by the iterative DLS method are shown in Figs. 5(b) and 6(b), respectively.

These results demonstrate that the proposed method provides successful restoration especially in the part of window sashes.

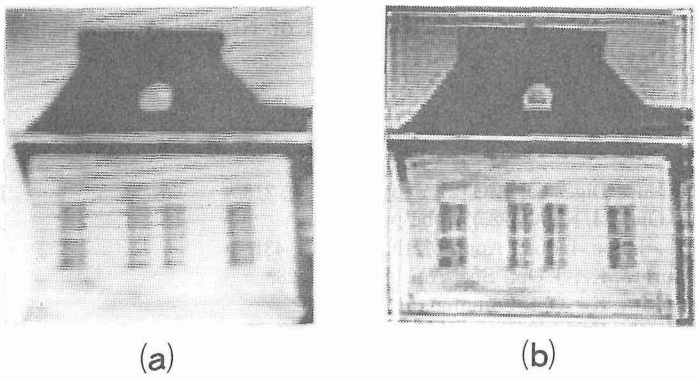


Fig. 5. (a) A photograph taken with the camera defocused; (b) restored image by the iterative DLS method.

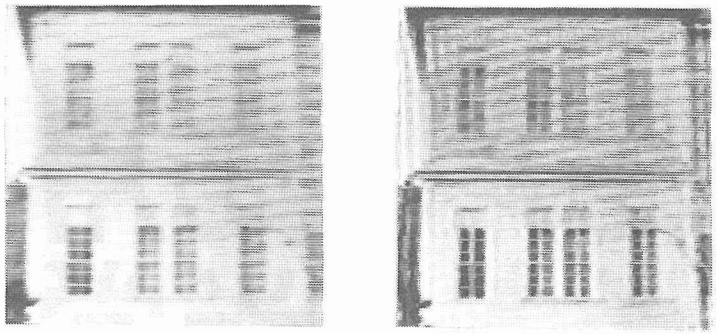


Fig. 6. (a) A photograph taken while the camera was moved horizontally ; (b) restored image by the iterative DLS method.

4. Conclusion

We have described a digital restoration procedure, the iterative DLS method, for restoring degraded images in the presence of noise. This technique has two advantages. One is the capability of processing the wide range of degradations and the other is significant tolerance to noise. The usefulness of the proposed method has been confirmed through computer simulations and restoration of real-world degradations.

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