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Periodicities in Acoustically Modified Optical Coherence Function

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Abstract

Acousto-optic modification of an optical mutual coherence function is described. The resultant optical mutual coherence function formulated by use of acoustically modulated laser light has lateral and longitudinal periodicities in space, as well as temporal periodicity. These periodicities stem from the periodic nature of a progressive acoustic wave itself. In any geometrical plane parallel to the propagation direction of the acoustic wave, the optical mutual intensity, derived from the mutual coherence function obtained, possesses only a lateral periodicity and is independent of the longitudinal coordinate along which non-modulated component of light is propagated.

1. Introduction

In partially coherent optics, the irradiance distribution, observed over any site of optical fields, is primarily dependent upon the illumination condition of partial coherence. In the past there have been a variety of interesting examples[1] such as image resolution of a two-point object, modification of diffraction patterns, and image contrast in a microscope.

Studies on the classical optical coherence theory[2] were extensive hitherto and were almost completed at the end of 1960's, but experimental studies have not been conducted as much as theoretical ones except for those of the coherent and incoherent limits. From a practical point of view, however, Hopkins[3] and Wolf[4] have made an outstanding contribution in the early 1950's to the construction of a bridge from theory to practice in the actual optical system. Hopkins revealed the role of optical coherence in a microscope, whereas Wolf was successful in relating the concept of coherence to the observable, or measurable quantity such as irradiance, and derived a quantitative relationship between the deterioration of spatial coherence and the reduction of visibility of interference fringes.

Most of the traditional experiments for the demonstrations have been performed so far by the use of spatially, partially coherent illumination resulting from an extended thermal source[5] ; a circular opening exposed to such a thermal source plays a role of a secondary incoherent source that yields a Besinc type of optical mutual intensity in the far field, based on the van Cittert-Zernike theorem.[5] However, such an illumination was used for only the demonstration but was not promising for practical use because of some drawbacks such as difficulties in coherence control, tedious analytical treatment, and insufficient irradiance.

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The major objection in promoting the experimental or actual work in partially coherent optics would be reduced to the difficulty in obtaining a continuously coherence-controllable, strong light source.

Meanwhile, the advent of laser in 1960 opened a new era for opto-electronics. Partially coherent optics has also been greatly influenced by this advent ; laser has brought about a new class of spatially, partially coherent fields, produced by degrading its highly coherent state. Coherence deterioration of laser light was successfully achieved by some dynamic scatterers such as a moving diffuser[6] and a liquid crystal[7] under a dc electric field. However, such methods were frequently used for producing a secondary incoherent illumination source, but did not have a definite aim to attempt a quantitative control of coherence. In fact, the scattered laser light may not be suitable for providing a partially coherent field to be controled quantitatively.

In contrast, the present author and his colleagues have made a great effort in the last decade[8–34] to produce an intense, coherence-controllable laser source for illumination with the help of an acousto-optic interaction scheme using a progressive ultrasonic wave. According to their results, the state of spatial coherence follows a periodic variation and is simply modified by the acoustic power level. Over the last several years, a periodic coherence function has been conceptually dealt with by some workers : a two-point or a periodic incoherent source is capable of creating a partially coherent field that results in a periodic coherence function in the far field[35–36]. Quite recently, periodicities in coherent or partially coherent fields have been extensively examined in conjunction with Talbot and Lau effects[37–50].

The major objective of this paper is to reveal what periodicities the optical mutual coherence function has in the Fresnel diffraction zone behind the acoustic cell as a result of acoustic modulation of laser light.

2. Optical Mutual Coherence Function Modified by a Progressive Acoustic Wave

A one-dimensional acousto-optic interaction scheme is sketched in Fig. 1. A plane monochromatic wave of laser light is normally incident on the front plane of an acoustic column, and is modulated in space and time by a progressive acoustic wave propagating in the direction of the x axis in the column. The non-modulated component of the incident light travels behind the acoustic cell in the direction of the z axis whose origin is at the back plane of the acoustic column. Since no acoustic modulation is imposed in the direction of the y axis,

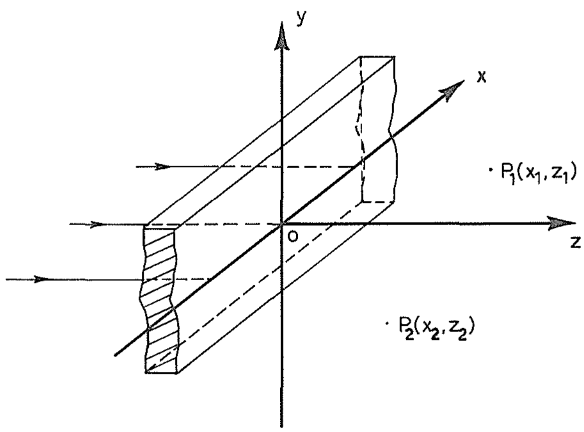

Fig. 1 Acousto-optic interaction scheme for the formulation of an optical mutual coherence function.

it is permissible to deal with the problem with only the x z coordinate.

We are concerned with two points $P_1(x_1, z_1)$ and $P_2(x_2, z_2)$ which specify the optical mutual coherence function in the Fresnel diffraction zone behind the acoustic cell. This function is defined at these points by

$$\Gamma(P_1, P_2, \tau) = \langle V^*(x_1, z_1, t)\, V(x_2, z_2, t+\tau) \rangle, \tag{1}$$

where V is an analytic signal referred to as the complex optical disturbance, V^* is its complex conjugate, τ is the time difference between the stationary optical disturbances arriving at the two points, and angle brackets denote the time average with respect to t. The optical disturbance can be obtained from the solution of the scalar wave equation,

$$\nabla^2 V(x, z, t) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} V(x, z, t), \tag{2}$$

where c is the speed of light outside the acoustic cell. A conventional way to solve Eq. (2) is to expand $V(x, z, t)$ in a Fourier series, taking into account periodic modulation in space and time by the acoustic wave. If a collimated beam of light with a uniform amplitude V_o impinges normally on the acoustic cell, the optical disturbance at a point $P(x, z)$ in the near field behind the acoustic cell and at an instant t can be expressed in a series formula :

$$V(x, z, t) = V_o \exp(i\omega t) \sum_{n=-\infty}^{\infty} U_n(z) \exp\{in(\Omega t - kx)\}, \tag{3}$$

where ω is the angle frequency of the incident monochromatic light, $U_n(z)$ is the expansion coefficient for the nth component dependent on only the z coordinate, and Ω and K are, respectively, the acoustic angle frequency and wave number. Substitution of Eq. (3) into Eq. (2) results in a spatial differential equation with respect to $U_n(z)$:

$$d^2U_n(z)/dz^2 + \{(\omega + n\Omega)^2/c^2 - (nK)^2\}\, U_n(z) = 0, \tag{4}$$

since each of the dindividual differential equations must be zero independently in the summation formula. The solution for $U_n(z)$ is

$$U_n(z) = U_o^n \exp[-i\{K_n^2 - (nK)^2\}^{1/2} z], \tag{5}$$

where $U_n^o = U_n(z=0)$ and $k_n = (\omega + n\Omega)/c$. Note that $\{V_o U_n(z)\}$ expresses the amplitude of frequency component at $(\omega + n\Omega)$, propagating at an angle $\theta_n = \sin^{-1}(nK/k_n)$. We recognize that k_n is much bigger than (nK) in magnitude, so that the square root in the exponent of Eq. (5) can be approximated by expansion to be

$$\{K_n^2 - (nK)^2\} \simeq k\{1 - (1/2)\,(nK/k)^2\}, \tag{6}$$

where $k = \omega/c$ is the wave number of the incident light. Substitution of Eq. (5) with Eq. (6) in Eq. (3) leads to

$$V(x, z, t) = V_o \exp\{i(\omega t - kz)\} \sum_{n=-\infty}^{\infty} U_n^o \exp[i\{n(\Omega t - Kx) + (nK)^2 z/2k\}]. \tag{7}$$

The optical mutual coherence function given by Eq. (1) is now formulated as follows :

$$\Gamma(P_1, P_2, \tau) = |V_o|^2 \exp[i\{\omega\tau + (z_1 - z_2)k\}] \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} U_m^{o*} U_n^o \langle \exp\{i(n-m)\Omega t\}\rangle$$

$$\times \exp[i\{(mx_1 - nx_2)K - (m^2 z_1 - n^2 z_2)K^2/2k + n\Omega\tau\}]. \tag{8}$$

Note that

$$\langle \exp\{i(n-m)\Omega t\}\rangle = \delta_{n,m}, \tag{9}$$

where $\delta_{n,m}$ is Kronecker's delta ; $\delta_{n,m}=1$ or 0 according to $n=m$ or $n \neq m$, and it reduces Eq. (8) to

$$\Gamma(P_1, P_2, \tau) = |V_o|^2 [\exp\{i(\omega\tau + \zeta k)\}] \sum_{n=-\infty}^{\infty} |U_n^o|^2 \exp\{i(n\xi K - n^2 \zeta K^2/2k + n\Omega\tau)\}, \tag{10}$$

where $\xi = (x_1 - x_2)$ and $\zeta = (z_1 - z_2)$ represent, respectively, the x and z component of the two point distance $\overline{P_1P_2}$. Equation (9) has an important physical meaning that any component of the modulated optical waves, shifted in frequency by acoustic frequency times its diffraction order, does not interfere with other components because its frequency differs from the others. This is why if we take any pair of component optical waves they are mutually incoherent in space. In this sense the acoustic wave plays a role of coherence converter from a coherent field to a collection of incoherent fields of light.

A progressive acoustic wave can be treated as a moving lattice with a lattice constant Λ equal to the acoustic wavelength when transmitted by an optical wave. In order to examine what periodicities the optical mutual coherence function of interest follows, we introduce the spatial lateral period Λ, the spatial longitudinal period $z_p = 2\Lambda/\lambda$, and the temporal period $T_p = 1/F$, where λ is the optical wavelength and $F = \Omega/2\pi$ is the acoustic frequency. Since $K = 2\pi/\Lambda$, $k = 2\pi/\lambda$, and $\Omega = 2\pi/T_p$, Eq. (10) is converted into

$$\Gamma(\xi, \zeta, \tau) = |V_o|^2 [\exp\{i(\omega\tau + \zeta k)\}] \sum_{n=-\infty}^{\infty} |U_n^o|^2 \exp\{2\pi i(n\xi/\Lambda - n^2\zeta/z_p + n\tau/T_p)\}. \tag{11}$$

It follows that the optical mutual coherence function obtained has spatial periodicities in both lateral and longitudinal directions in the ξ ζ plane, related to the difference coordinate components of the two points $P_1(x_1, z_1)$, and $P_2(x_2, z_2)$, and also has temporal periodicity. These characteristics originate from the wave nature of a progressive acoustic wave. Except for the temporal periodicity, the spatial periodic characteristics are similar to the statement given by Lohmann and Ojeda-Castaneda[46] and Indebdtouw[48], who discussed spatial periodicities in a partially coherent field in conjunction with Talbot and Lau effects.

3. Periodicities in Optical Mutual Intensity

In the actual optical system it is substantially important to see what formula the illumination condition of partial coherence obeys. To this end the optical mutual intensity is introduced[5], which expresses a functional formula for the illumination condition when any object is set in a spatially, partially coherent field. The optical mutual intensity takes the formula

$$J_{12} = \langle V^*(x_1,\ z_1,\ t)\, V(x_2,\ z_2,\ t)\rangle\ , \tag{12}$$

which results from the mutual coherence function for $\tau=0$. Any two-dimensional object that is concerned is generally arranged in paralell to the x axis along which the acoustic wave propagates (see Fig. 1). Such a situation leads to the mutual intensity at two points on an axis at $z=z_1=z_2$, which is parallel to the x axis. Accordingly, the two conditions $\tau=0$ and $z=z_1=z_2$ reduce Eq. (11) to the mutual intensity of interest :

$$J_{12} = |V_o|^2 \sum_{n=-\infty}^{\infty} |U_n^o|^2 \exp(2\pi i n\xi/\Lambda). \tag{13}$$

This formula is exactly the same expression as the one derived previously from a different point of view.[18]

An emphasis is given to the fact that the optical mutual intensity of Eq. (13) is dependent on only the two-point separation $\xi=(x_1-x_2)$ but is independent of z. It follows that the same formula of the mutual intensity as Eq. (13) is always established across any planes normal to the z axis in the near field zone behind the acoustic column. This gives an advantage to the actual arrangement of an optical system because any object of interest is always subject to the same illumination condition of partial coherence, even though it may be set at any site in that zone.

It has been recognized that the acoustic column, serving as an optical phase grating, forms its self-images due to Talbot effect at distanbes $z=q(2\Lambda^2/\lambda)=qz_p(q=1,\ 2,\ 3...)$ in the Fresnel diffraction zone behind the acoustic column as long as it is coherently illuminated. It is readily understood from this statement that the same optical mutual intensity always results on such any self-imaging plane, but the result obtained here reemphasizes that the same expression for the mutual intensity as Eq. (13) is always established at any plane normal to the z axis, regardless of such self-imaging planes.

As is well known, the acoustic wave works as a pure phase grating in the Raman-Nath acousto-optic interaction scheme[51]. For this scheme, the complex optical disturbance just behind the acoustic column may be written in the form

$$V(x,\ 0,\ t) = V_o\exp[i\{\omega t - v\ sin(\Omega t - Kx)\}], \tag{14}$$

where v is called the Raman-Nath parameter after the names of Raman and Nath[51]. This parameter is controllable in magnitude with the acoustic power level. In terms of the first kind Bessel function $J_n(v)$ of the nth order, Eq. (14) can be expanded into a series :

$$V(x, 0, t) = V_o \exp(i\omega t) \sum_{n=-\infty}^{\infty} J_n(v) \exp\{in(\Omega t - Kx)\}, \tag{15}$$

in which the identity

$$\exp(i v sin\theta) = \sum_{n=-\infty}^{\infty} J_n(v) \exp(in\theta)$$

is used. If we compare Eq. (15) with Eq. (7) at $z=0$, we obtain $U_n^o = J_n(v)$. It follows that Eq. (13) becomes

$$J_{12} = |V_o|^2 \sum_{n=-\infty}^{\infty} J_n^2(v) \exp(2\pi i \xi / \Lambda) = |V_o|^2 J_0[2v sin(\pi \xi / \Lambda)]. \tag{16}$$

The same result as Eq. (16) has already been derived just behind the acoustic column[8].

4. Experimental Verifications

The acoustic cell filled with pure ethylalcohol was driven by an X-cut quartz transducer working at a frequency of 500 kHz ; the acoustic wavelength is $\Lambda = 2.41$ mm at room temperature. The cell was designed in such a way as to serve as a pure phase grating upon which a plane wave of light was normally incident. Care was taken of the fact that all the diffraction components of light by the acoustic wave were still superimposed satisfactorily in the near field behind the cell. For a quantitative discussion it is convenient to introduce the normalized absolute mutual intensity called the degree of spatial coherence :

$$|\mu(\Delta x / \Lambda)| = |J_{12}| / |V_o|^2 = |J_0\{2v sin(\pi \Delta x / \Lambda)\}|, \tag{17}$$

in which $\Delta x = (x_1 - x_2)$ is taken to be the same as $\xi = (x_1 - x_2)$ for the sake of convenience. It should be emphasized that the degree of spatial coherence is simply related to the visibility of interference fringes, created by Young's double-slit experiments, which is a measurable quantity.

Numerically computed and measured results are shown together in Fig. 2 ; dependence on the Raman-Nath parameter v at a distance of 50 mm from the back plane of the acoustic cell is exemplified as a function of $\Delta x / \Lambda$, and what $|\mu|$ is independent of the change in distance from the cell is also given in the figure for $v=1.0$. The degree of coherence varies with a period of Λ and its modification depth is controllable with the Raman-Nath parameter v. As can be seen from all the figures, theoretical predictions are fully verified by the experiments.

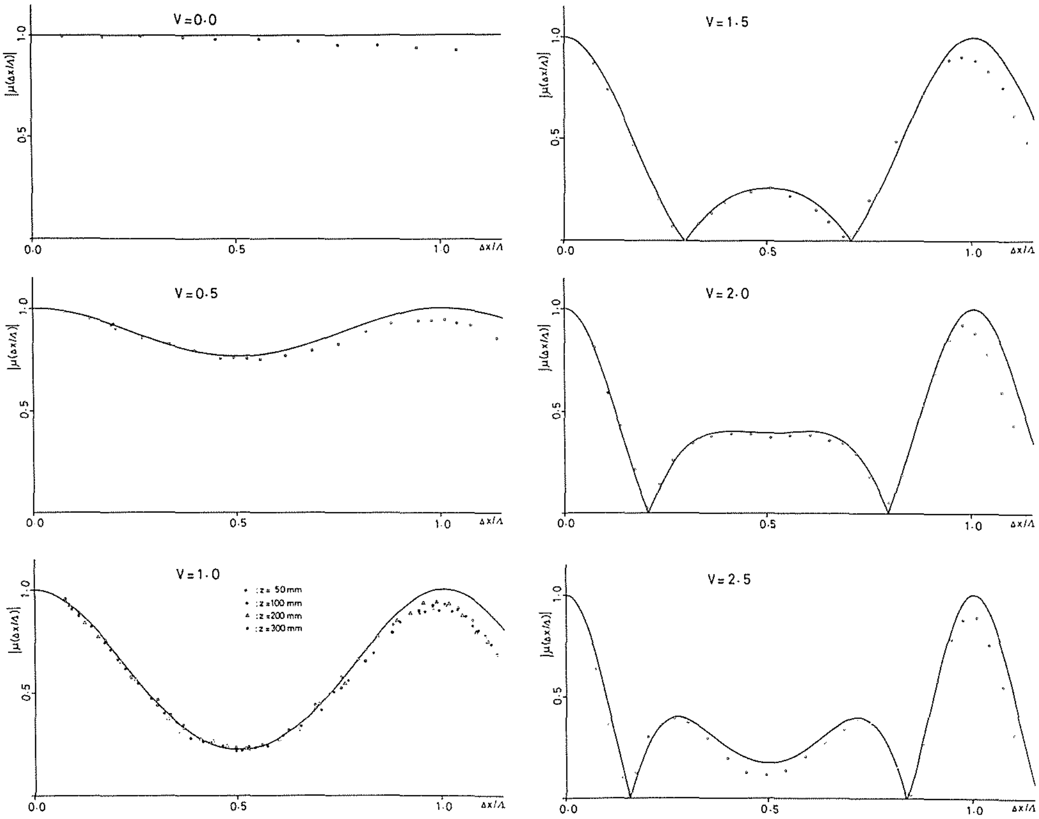

Fig. 2 The degree of spatial coherence as a function of two-point separation normalized by the acoustic wavelength.

5. Conclusions

A complex optical disturbance has been obtained from the solution of a scalar wave equation describing the acousto-optic interaction. The optical mutual coherence function, repesenting a partially coherent field of light, is successfully formulated in the Fresnel diffraction zone behind an acoustic column in which a progressive acoustic wave is present. The resultant mutual coherence function possesses spatial periodicities in both lateral and longitudinal directions, as well as temporal periodicity, which stems from the periodic nature of the acoustic wave. The optical mutual intensity readily results from the optical mutual coherence function obtained. It has only the lateral periodicity if a plane of interest is taken in a site parallel to the acoustic column, and is independent of the longitudinal coordinate along which the non-modulated component of light is propagated.

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