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# Feeding Behavior during Solidification of Binary Alloy Ingots

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**Abstract**

In the solidification process, with the fall in temperature, liquid volume gradually decreases with the increase in solid volume. This results in volumetric shrinkage and necessitates compensation by liquid metal, which is called feeding. Feeding during solidification has been considered to have its limit, because the liquid metal is trapped by the solid at a certain fraction solid due to the growing dendritic network. In this study, Al-3 wt%Si alloy was selected as a representative binary alloy and its feeding behavior during solidification was measured by using an experimental method devised by the authors. The experimental results show that the rate of feeding decreases rapidly and attains a very small value after the fraction solid at the neck of the ingot reaches 0.67. Therefore the limiting fraction solid to feed is determined as 0.67 and the value is in agreement with that of the limit to flow of the interdendritic liquid. Furthermore the density of the ingot is directly related to the limiting fraction solid and the rate of feed.

## 1. Introduction

In the solidification process, with the fall in temperature, liquid volume gradually decreases with the increase in solid volume and the solute in the liquid is enriched. This results in volumetric shrinkage and necessitates compensation by liquid metal, which is called feeding. Lack of feeding during solidification of an alloy results in the deterioration of the soundness of the ingot[1,2,3], due to the formation of pipes, microporosities and cavities. This in turn results in technical and economical problems due to the poor quality of the ingot. Feeding during solidification has been considered to have its limit, because the liquid metal is trapped by the solid at a certain fraction solid due to the growing dendritic network of the fraction solid. This feeding is hereafter called direct feeding. It is important to know the feeding behavior of an alloy and the limiting fraction solid to feed. These results may be useful to determine

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the critical moment for soft reduction in the continuous casting process.

In this study, Al-3 wt%Si alloy was selected as a representative binary alloy and its feeding behavior during solidification was measured. Al-3 wt%Si alloy has a wide range of solidification and therefore it is well suited for the study of the feeding behavior during solidification[4].

The study of feeding behavior was conducted, by measuring the feeder head level during unidirectional and cylindrical solidification, by using an experimental method devised by the authors.

## 2. Experimental method

### 2. 1 Unidirectional solidification

Unidirectional solidification experiments were carried out in graphite molds. The shape and the dimensions of the mold and the placing of the thermocouples are as schematically illustrated in Figure 1.

The mold was cylindrical and consisted of a top mold for the feeder head and a bottom mold for the ingot. The bottom mold had an inner diameter of 60 mm and inner height of 85 mm. The top mold was of 20 mm diameter and 120 mm in length. Six thermocouples were inserted in the bottom mold at 10, 42 and 80 mm heights from the bottom of the mold, along the vertical axis in the center and at 2 mm distance from the wall. One thermocouple was inserted in the feeder head, at 1 mm from its wall and 16 mm above the ingot top. The thermocouples were inserted from the side wall of the mold. In one case, fifteen thermocouples were inserted to obtain more accurate thermal distribution data. The thermocouple sheaths were 2 mm in diameter and occupied 0.13% of the mold volume, in the case of six thermocouples, and 0.33% of the mold volume, in the case of fifteen thermocouples in the mold.

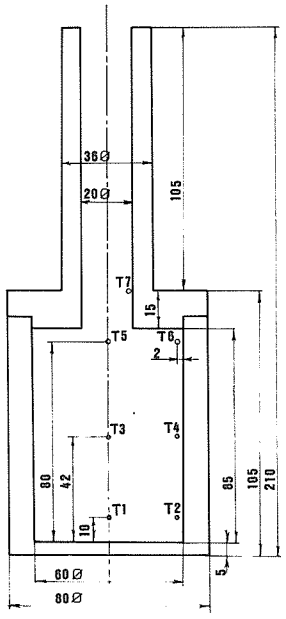


**Fig. 1** Dimensions of graphite mold used for the measurement of feeder head level in unidirectional solidification experiments. Points T1 to T7 indicate the positions of the thermocouples. (unit-mm)

The experimental apparatus is as shown in Figure 2. The mold was mounted on a steel base, which in turn was fixed to a steel stand. The stand had slideable clamps which held i) the furnace to heat the mold, ii) the furnace to heat the feeder head, iii) the tundish for pouring (not shown in the figure, as it was removed after pouring) and iv) the mount for the glass

guiding pipe for the glass rod. The glass rod was connected to the graphite float, resting on the melt surface, in the feeder head. The glass rod was connected at the other end, to one arm of a balance rod, via a thread. The balance rod rested on a pivot point. The other arm was connected to the differential transformer, to measure the movement of the float due to the fall in the feeder head level. This arrangement was designed to enhance the range of the differential transformer by the lever principle and also to keep the float weight, acting on the metal surface, to a minimum.

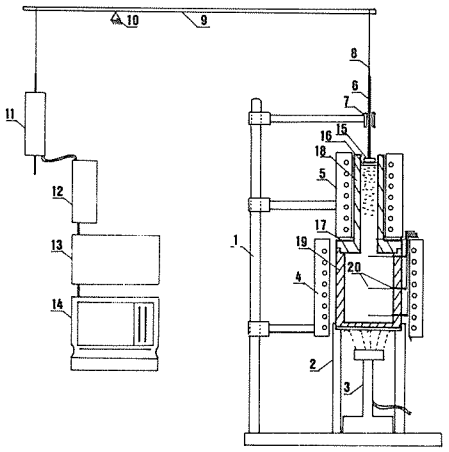


**Fig. 2** Experimental apparatus for measurement of feeder head level during solidification.
1 ; stand, 2 ; steel base, 3 ; shower, 4 ; electrical furnace for the mold, 5 ; electrical furnace for the feeder head, 6 ; silica glass rod, 7 ; guide pipe, 8 ; thread, 9 ; balance rod, 10 ; pivot, 11 ; differential transformer, 12 ; mv converter, 13 ; data lagger, 14 ; computer, 15 ; graphite float, 16 ; salt mixture, 17 ; insulation, 18 ; mold for feeder head, 19 ; mold for ingot, 20 ; thermocouples.

The inserted thermocouples were connected to a data logger. The differential transformer was also connected to the data logger through a millivolt-distance converter. The data logger was in turn connected to a personal computer, for data storage and processing.

The full range of the differential transformer was 20 mm. The error range, given by the manufacturer is $\pm 0.001\%$ of the full range. As the full range of measurement was enhanced to 67 mm, by adjusting the arm lengths of the balance, so the error range in measurement in this experiment was about 0.15 mm, which was negligible.

In all the experiments Al-3 wt%Si alloy was used, which was prepared in an electrical furnace in a alumina crucible, using 99.99% aluminum and Al-24.9% Si alloy, in necessary quantities.

Molten metal was poured into the graphite mold through a tundish at around 800℃, after removing the oxide layer and stirring the melt. The tundish was removed after pouring. The metal level in the feeder head was adjusted to a 10 mm depth from its top, to maintain a constant initial volume. The melt surface was covered with flux, to protect it from oxidation and keep it flat so as to avoid the effect of a curved surface due to the surface tension of the melt in the feeder head. Two grams of powdered flux, consisting of salt mixture ($BaCl_2$, KCl and NaCl in a wt% ratio of 5 : 3 : 2), was sprinkled over the liquid metal surface and it quickly turned to a liquid state. The float was placed on the metal surface in the feeder head. During this preparation the temperature of the liquid metal in the mold was kept at about 800℃.

The mold furnace was then switched off, but kept in place for the duration of the experiment. Data recording was started and water was sprayed on the bottom of the mold for unidirectional cooling. The feeder head level started to fall with the beginning of cooling and was recorded by measuring the movement of the float resting on the melt surface in the feeder head. The alloy in the feeder head was kept in a liquid state to ensure complete feeding conditions to the solidifying ingot. The stoppage of feeding to the solidifying ingot, was judged by monitoring the rate of downward movement of the float, and the cooling water was turned off and the metal in the feeder head was allowed to solidify.

After cooling to room temperature the ingot was cut in half, along the central vertical axis and then polished and etched for observing the macrostructure.

### 2. 2 Cylindrical solidification

Cylindrical solidification was carried out in molds consisting of walls made of a steel pipe, a base made of isolite and the feeder head part made of graphite. The shape and the dimensions of the mold and the placing of the thermocouples are schematically shown in Figure 3. The bottom mold was cylindrical with an inner diameter of 80 mm and inner height of 85 mm. The top mold for the feeder head was of 25 mm in diameter and 120 mm in length. Sixteen thermocouples were inserted in the mold, through the isolite base, at 2, 30, 55 and 80 mm heights from the bottom of the mold, and at 2, 10, 20 and 35 mm distances from the wall. Two thermocouples were inserted in the feeder head at distances of 15 mm and 35 mm from the neck of the ingot. The thermocouple sheaths were 2 mm in diameter and occupied 0.5% of the mold volume.

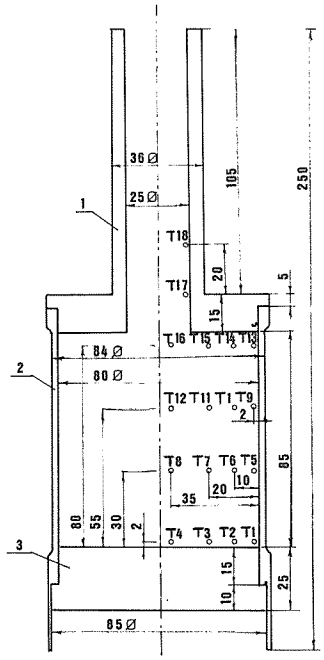


**Fig. 3** Dimensions of the mold used for the measurement of feeder head level in cylindrical solidification experiments. Points T1 to T18 indicate the positions of the thermocouples. (unit-mm)
1 ; graphite top, 2 ; cylindrical steel walls, 3 ; isolite base.

The experimental apparatus was the similar to that shown in figure 2, except that the size and the material of the mold were changed. Furthermore, the full range of the recording was extended to 80 mm.

The experimental procedure was also similar to that of the unidirectional solidification. The mold was fixed in place, and heated, then melt was poured and the float was placed on the melt surface in the feeder head. Then at a given temperature of the alloy the furnace for heating the bottom mold was

lowered and water was sprayed from the side walls of the mold. Solidification proceeded from the circumference of the mold to its center. Cooling was stopped by judging from the rate of the fall of the feeder head level.

After cooling to room temperature the ingot was cut in half along the vertical axis and one half was polished and etched for observing macrostructure.

## 3. Results

### 3. 1 Unidirectionally solidified ingot

#### (1) Macrostructure

The unidirectionally solidified ingots, shown in Figure 4, had cooling rates of (a) 2. 1℃/sec and (b) 3.9℃/sec. The cooling rate in each case was determined as the local cooling rate at 10 mm from the bottom of the ingots, from the beginning of cooling till the start of solidification, and varied from 2℃/sec to 4℃/sec. The bottoms of both the ingots have a fine columnar dendritic structure and along the ingot height the columnar dendrites become coarse. The macrostructures of the unidirectionally solidified ingots were almost similar in the range of cooling rates in these experiments.

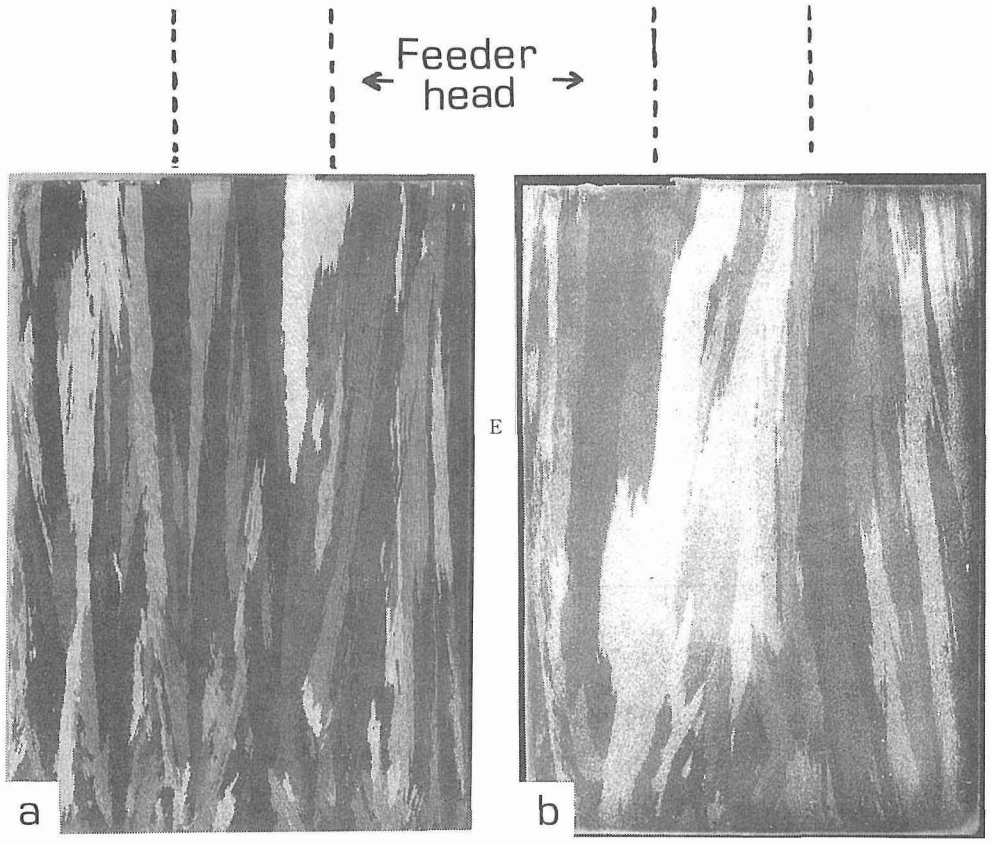


**Fig. 4** Macrostructures of unidirectionally solidified ingots of Al-3 wt%Si alloy having initial cooling rates of (a) 2.1 and (b) 3.9℃/s. (×1.0)

### (2) Feeding and cooling curves

The cooling curves and the feeding measurement curve, for the unidirectionally solidifled ingots with cooling rates of 2.1℃/sec and 3.9℃/sec respectively which are shown in figure 4, are presented in Figures 5 and 6. In both figures the cooling curves are indicated by T1 to T7. T1 and T2 are the cooling curves at the bottom of the ingot, T3 and T4, at the middle of the ingot, and T5 and T6 at the top of the ingot. T7 shows the cooling curve of the feeder head. T1, T3 and T5 are in the center and T2, T4 and T6 are at 2 mm distance from the side wall of the bottom mold.

In both cases solidification started from the bottom at 641℃ and progressed almost paralel to the base of the ingot as there was no great temperature difference between the center and near the inner surface of the ingot. The cooling curve (T7) in the feeder head shows that feeding was guaranteed till the end of solidification in the upper regions of the solidifying ingot, since alloy temperature in the feeder head is still in the liquid region.

The curve F shows the feeding as a fall in the level of molten metal in the feeder head. The rate of feeding suddenly increases with the beginning of solidification, when the temperature near the base of the solidifying ingot passes the liquidus temperature. This is caused by the initiation of the liquid-solid transformation of the alloy. The feeding curve suddenly flattens out when the temperature at the neck of the ingot reaches 607℃, which corresponds to 0.67 fraction solid in the solidification zone. This indicates that the molten metal in the feeder head is unable to feed the solidifying regions at the top of the ingot. This can be seen in all the unidirectionally solidified ingots with different cooling rates.

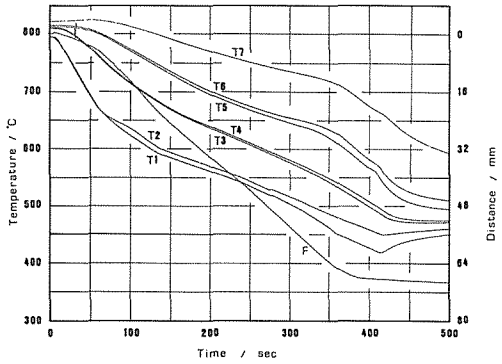


**Fig. 5** Fall in the feeder head level (F) and cooling curves (T1-T7) in the unidirectionally solidified ingot shown in fig. 4(a).

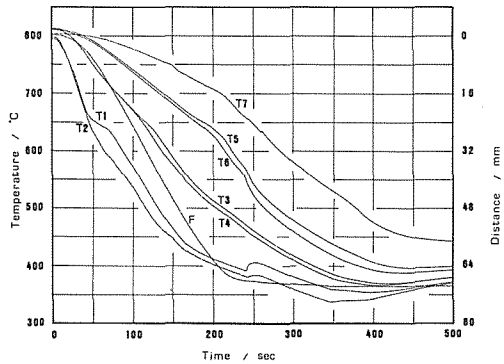


**Fig. 6** Fall in the feeder head level (F) and cooling curves (T1-T7) in the unidirectionally solidified ingot shown in fig. 4(b).

## 3. 2 Cylindrically solidified ingots

### (1) Macrostructure

The macrostructures of the cylindrically solidified ingots, shown in Figure 7, had

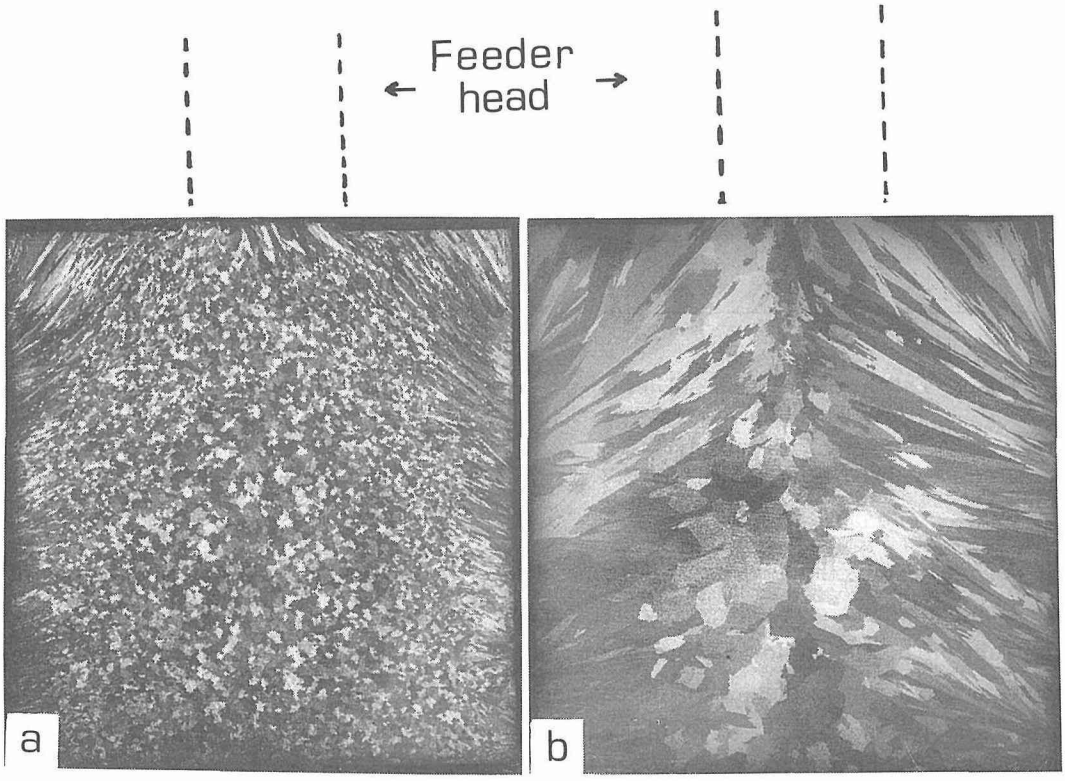


**Fig. 7** Macrostructures of cylindrically solidified ingots of Al-3 wt%Si alloy having initial cooling rates of (a) 14.8 and (b) 19.1℃/s. (×0.9)

cooling rates of (a) 14.8℃/sec and (b) 19.1℃/sec. The cooling rate was taken as the local cooling rate at 2 mm from the side wall, at a height of 55 mm from the bottom of the ingots and varied between 14.8 and 19.1℃/sec. The macrostructure of the ingot (a) shows fine columnar and equiaxed crystals, with a wide region of equiaxed crystals. On the other hand the macrostructure of the ingot (b) shows developed columnar dendrites. The macrostructures showed a tendency to be columnar, with the increase in the cooling rate.

**(2) Feeding and cooling curves**

The cooling curves and feeding measurement curve for the cylindrically solidified ingots with cooling rates of 14.8℃/sec and 19.1℃/sec respectively which are shown in figure 7, are given in Figures 8 and 9 respectively. Cooling curves are marked from T1 to T18 and represent the positions shown in figure 3.

In both cases cooling starts from the side walls of the ingot and the solidification front moves towards the center. Along the vertical axis, the bottom of the ingot solidifies rather earlier than the top, which sufficiently provides opportunity to fully feed the solidifying ingot. The metal in the feeder head remains in the liquid state, till the upper regions of the ingot have solidified, thus guaranteeing complete feeding.

Curve F shows the feeding as a fall in the molten metal level in the feeder head.

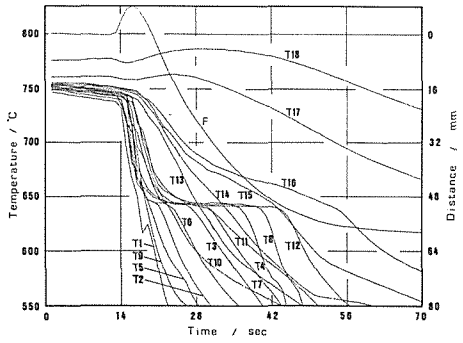


**Fig. 8** Fall in the feeder head level (F) and cooling curves (T1 -T18) in the cylindrically solidified ingot shown in fig 7 (a).

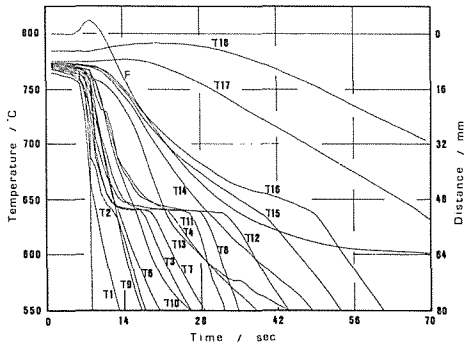


**Fig. 9** Fall in the feeder head level (F) and cooling curves (T1 -T18) in the cylindrically solidified ingot shown in fig. 7 (b).

Unlike the unidirectional experiments, metal level in the feeder head initially rises with the commencement of cooling by water sprays. It seems that initially the volumetric contraction of the mold is more than that of the liquid alloy. The feeding curve shows a decline in feeding with the passage of time because the liquid volume becomes progressively smaller due to cylindrical solidification of the ingot. The progressive decrease in liquid volume does not allow clear observation of the stoppage of feeding, but the feeding becomes negligible after the neck of the ingot has crossed 0.67 fraction solid and the feed curve becomes flatter beyond this stage.

Experiments for cylindrical solidification were conducted with degassed and nondegassed alloys. The feeding curves for degassed ingots were smooth, as shown in figures 8 and 9. On the other hand, the feeding curves of the nondegassed ingots were unpredictable, with haphazard movements of the melt level in the feeder head, hence they could not be recorded.

## 4. Discussion

### 4. 1 Feeding limit

The concept of feed to the solidification zone is not new, but there are different opinions about feeding limit. Some consider that feeding will continue till the end of solidification of the alloy, to guarantee the soundness of the solidifying ingot. Considering the results of the present experiments, it can be said that feeding during solidification does not continue till the end of solidification, and ceases at 0.67 fraction solid.

To have a more clear picture of the solidification zone, it is necessary to consider the relationship between the solidification range of the alloy and the development of the morphology of the growing dendrites[5]. Figure 10 schematically shows the formation of the solidification zone with the temperature profile in it. Lines L and S

represent the liquidus and solidus of the phase diagram. Now, if we consider the average composition of the alloy to be $C_{Lo}$, then $T_L$ and $T_S$ are the liquidus and solidus temperatures of that alloy. By taking the distance along the horizontal axis which is shown at the top of the diagram and by considering the relationship between distance and temperature, the temperature profile $G_A$ can be assumed in the direction of solidification. Point 1 on the temperature profile, which corresponds to the liquidus temperature, is the position of the beginning of the solidification zone. Point 2, which corresponds to the solidus temperature of the alloy, represents the end of the solidification zone. If we consider the conceptual formation of dendritic structure along the solidification range, the distance from point 1 to 2 denotes the solidifying zone Z. This region can be divided, depending upon the state of development of the dendritic structure, into p and q zones. The q zone is further subdivided into $q_1$ and $q_2$ zones.

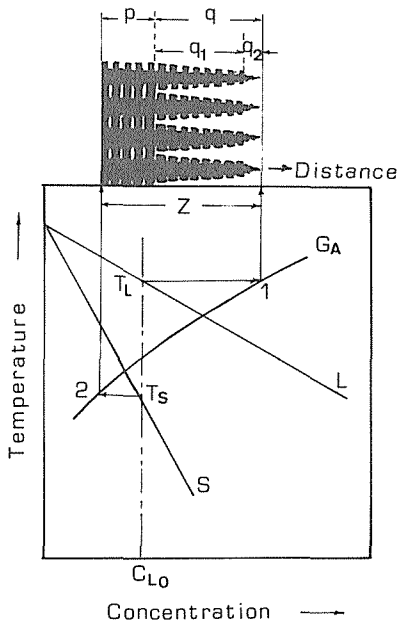


**Fig. 10** Schematic illustration of the formation of solidification zone. $G_A$ ; temperature profile at a certain stage of solidification, Z ; width of the solidification zone, $T_L$ ; temperature at the start of solidification, $T_S$ ; temperature at the end of solidification, $C_{Lo}$ ; average solute concentration of an alloy, L ; liquidus line, S ; solidus line.

The zone adjacent to the liquid metal is termed the $q_2$ zone. In this zone solid particles are not fixed and are in a free floating state in the liquid. The dendrites begin to form a partially intermingled network beyond the $q_2$ zone, with the development of dendrites, along the direction of the solidification. This network gradually develops and the dendrites become fixed, thus forming a solid structure with channels for the flow of the remaining liquid. This is termed the $q_1$ zone. With further development of the dendrites, the channels are closed and the liquid is entrapped by the solid. This is termed as the p zone.

Interdendritic liquid flow can only occur till the boundary of the p and q zones, as the liquid is entrapped by the surrounding solid in the p zone. Thus feeding during solidification is only possible up to the boundary of the p and q zones. Considering in fraction solid terms, this boundary can be termed as the limiting fraction solid to feed.

In this study the limiting fraction solid to direct feeding was established by recording the feeding curve. The feeding curves for the unidirectionally solidified ingots, show a critical point at 0.67 fraction solid as feeding abruptly slows down. This can also be circumstancially observed from the feeding curves of the cylindrically

solidified ingots. This value is the same as that reported for the limit of the interdendritic liquid flow[1].

The presence of the limit of direct feeding during solidification has a very significant importance for the soundness of the castings, ingots and continuous cast semi-products. Determination of the direct feeding limits can effect the installation and weight of risers, installation of external and internal coolers, or even the pouring temperature.

### 4. 2 Relationship between rate of feed and density changes in unidirectionally solidified ingot

Density after solidification, is the direct indicator of the soundness of the ingot because low density means presence of microporosity. Measured density, along the height of the ingot, for the unidirectionally solidified ingot with the cooling rate of 2. 1℃/sec, is given in Figure 11. The room temperature density measurements were carried out by using the Archimedes method. It is interesting to note that the density decreases in the top parts of the ingot in spite of their closeness to the feeder head guaranteeing them complete feeding. This was observed in all the unidirectionally solidified ingots.

In this chapter the effect of the rate of feed to the solidifying region on the density was examined. A comparison of density after solidification with the direct feeding during solidification is a new aspect and is not discussed in the literature.

To make the above mentioned comparison it is necessary to express the density in relation to time since feeding is measured in relation to the passage of time. This can be done by taking a certain effective fraction solid in the solidification zone and expressing the density at a certain level in the ingot through the time at which the effective fraction solid crosses that level. This will be later explained in detail.

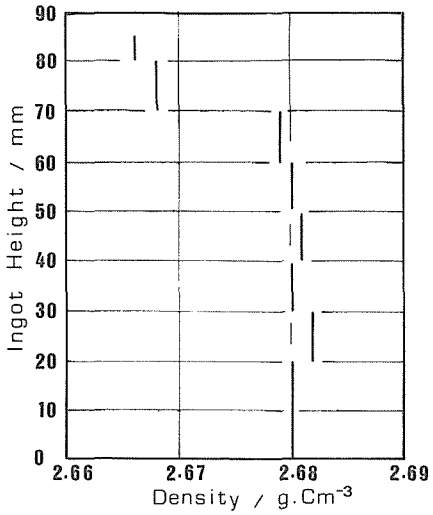


**Fig. 11** Change in solid density along the height of the unidirectionally solidified ingot shown in fig. 4(a).

The first step is to determine the effective fraction solid in the solidification zone. In the q zone there is a direct feeding of liquid alloy to the shrinkage region through interdendritic channels, so the effective fraction solid for comparison, cannot exist in this region, since any pore which may form will have full accessibility to liquid feeding. In the p zone, the remaining liquid is entrapped and it may be said that the future density is already decided at this stage. This leaves only the boundary of the p and q zones, which becomes the limiting fraction solid to direct feed, and lies at 0.67

fraction solid, as was discussed previously.

Figure 12 elaborates the idea of density of an ingot expressed in relation to time through the limiting fraction solid. The left illustration, at the top of the figure, shows the change in density, taken for five segments of a certain width. The densities were measured along the height of the ingot, as in the case of a unidirectional solidification, 0.67 fraction solid moves from the bottom to the top of the ingot. The right illustration, at the top of the figure shows the position of the limiting fraction solid, i.e. 0.67, along the height of the ingot, with the passage of time. The bottom illustration, shows the relationship between density and time, at the position corresponding to 0.67 fraction solid. This takes into account the density of the ingot at the level where the 0.67 fraction solid exists along the ingot height, in relation to the time at which 0.67 fraction solid exists at that level.

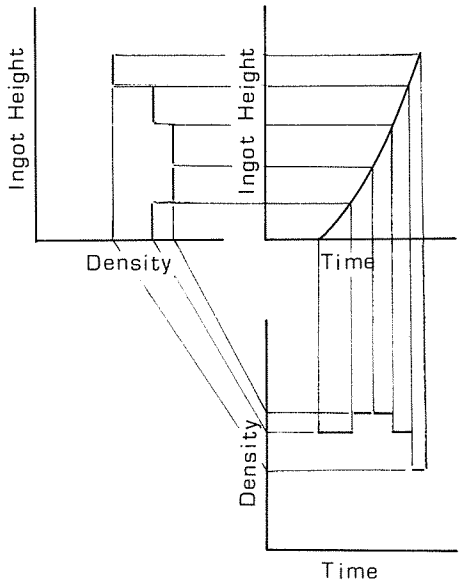


**Fig. 12** Schematic illustration of density as a function of time corresponding to the position of 0.67 fraction solid along the ingot height.

Rate of feed to the ingot was determined, by taking the gradient of the feed curve. The comparison was made between the rate of change of feeding and density both expressed as a function of the passage of time. Comparison was visual, as each curve was made in relation to passage of time.

The results for the unidirectionally solidified ingots, shown in figure 4, are given in Figures 13 and 14. The figures show the observed feed curve as measured by a fall in the metal level in the riser (1), the rate of feed taken for one second interval (2), and the density changes at the position corresponding to the advance of 0.67 fraction solid (3).

In figure 13 the density and the rate of feed are almost constant but both show a decrease during the end of solidification, whereas in figure 14 the density changes are parabolic with the parabolic rate of feed. The change in the rate of feed and the density change curves show good correspondence with each other. Density is constant, when the rate of feed is constant and decreases with decreasing rate of feed. This dependency is also observed for all other unidirectionally solidified ingots.

This lays emphasis on two points. The first is that, 0.67 fraction solid plays a determinant role for the soundness of the ingot, and the second is that, density, controlling the soundness of the ingot, is dependent on the rate of feed.

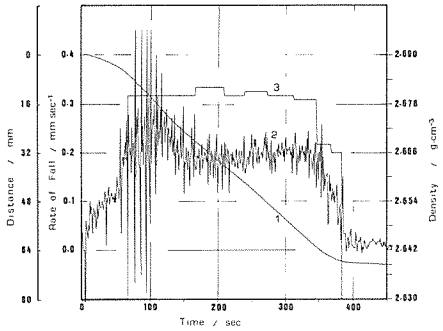


Fig. 13 Relationship between rate of fall of the feeder head level and density corresponding to the advance of the boundary of p and q zones in the solidification zone for unidirectionally solidified Al-3 wt%Si alloy ingot shown in fig. 4(a). 1; feed curve, 2; rate of feed, 3; density changes.

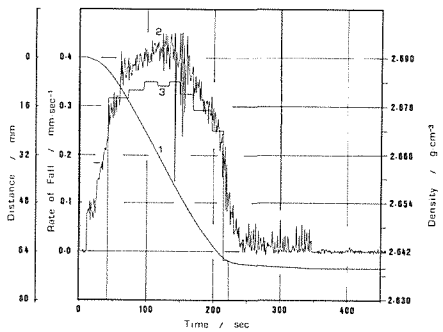


Fig. 14 Relationship between rate of fall of the feeder head level and density corresponding to the advance of the boundary of p and q zones in the solidification zone for unidirectionally solidified Al-3 wt%Si alloy ingot shown in fig. 4(b). 1; feed curve, 2; rate of feed, 3; density changes.

### 4. 3 Effect of solidification factors on negative pressure of pore formation

Pore formation during solidification of an alloy ingot is one of the main factors affecting the soundness of the ingot. This problem has been discussed by many authors[6,7,8], but the stage of formation of the pores, during solidification, is still disputed.

The pressure drop from the position of the beginning of solidification to that at the limiting fraction solid, affects the soundness of the ingot, as contact is broken between the entrapped liquid and the feeding liquid after the limiting fraction solid.

It has been previously reported that pressure drop increases with increasing solidification velocity[9], thus the pressure drop should be dependent on the solidification velocity at 0.67 fraction solid, or in other words, the rate of forward movement of 0.67 fraction solid.

It has also been mentioned previously that an increase in pressure drop is the cause of porosity[9]. Now, as microporosity decreases the density, so the sections of the ingot with low density, must have had a higher pressure drop, during the passage of the limiting fraction solid.

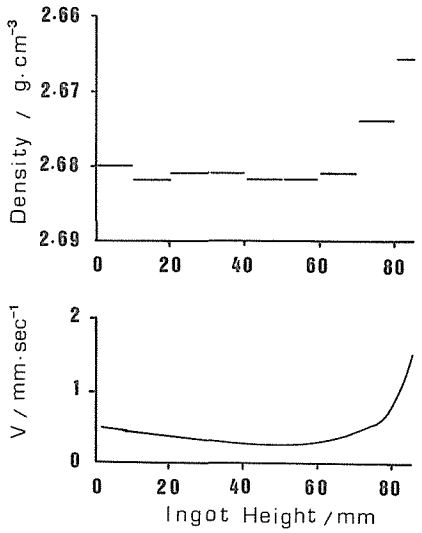


Fig. 15 Solidification velocity of 0.67 fraction solid V and density, along the ingot height for a unidirectionally solidified ingot.

This predicts higher solidification velocity V at the limiting fraction solid 0.67.

Experimental data of this study is in good agreement with the above mentioned factors. Figure 15 shows the relationship between V at 0.67 fraction solid and the density changes, along the direction of solidification, for a unidirectionally solidified ingot. When the rate of movement of 0.67 fraction solid, V, sharply increases, the density simultaneously decreases. This again indicates the importance of the value of 0.67 as the limiting fraction solid to feeding during solidification.

## 5. Conclusions

The conclusions derived from this study are summarized as follow:

1. A new experimental process was developed to measure the feeding behavior during solidification. This process is based on measuring the feeder head level by a light float connected to a differential transformer, through a balancing mechanism. This method ensures perfect recording of minute oscillations of the feeder head level.
2. The experimental results of feeding behavior for unidirectionally and cylindrically solidified ingots show that the rate of feeding becomes faster when the solidification starts. Furthermore, the rate of feeding decreases rapidly and attains a very small value after the fraction solid at the neck of the ingot reaches 0.67. Therefore the limiting fraction solid to feed is determined as 0.67 and the value is in agreement with that of the limit of flow of the interdendritic liquid.
3. The density measurements of the ingots show a decrease in density in the direction of the movement of solidification front.
4. The density of the ingot is directly related to the limiting fraction solid and the rate of feed, since their comparison shows that they are in good agreement.
5. Negative pressure for pore formation during solidification is only effective till the limiting fraction solid.
6. Decrease in density is a direct result of pore formation during solidification. Pore formation depends upon negative pressure, which in turn is related to the permeability of the growing dendrites, the limiting fraction solid to feed, rate of movement of the limiting fraction solid, and the length of the q zone during the liquid-solid transformation region.

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