COMPARISON OF THERMAL MOVEMENTS AND THREE-DIMENSIONAL ELASTIC ANALYTICAL RESULTS OF REINFORCED CONCRETE BUILDINGS

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Abstract

In three actual reinforced concrete (R/C) buildings (1,3 and 6−storied buildings), thermal movements measured until now are very different from analytical results obtained from the plane frame analysis. The present paper is intended to clarify thermal movements analytically, and to find out temperature conditions for analyses. Analysis attempted is three-dimensional elastic analysis considered the effect of slabs and walls, and the validity of analytical results are examined by comparison with the measurements of three buildings. It is shown that results obtained from three-dimensional elastic analysis show a good agreement with measurements, and it is necessary to consider the temperature data corresponding to position of structural members in buildings with heating apparatus.

1 INTRODUCTION

Stresses caused by atmospheric temperature changes under the action of vertical loads occasionally give rise to incremental or cyclic collapse of structures[1]. Thermal movements caused by temperature changes of 1 and 3−storied R/C buildings were measured by K. Mutoh and K.Ohno. According to the literature[2], the conclusions are as follows ;

(1) The thermal movements of the 1−storied building are 0.53−0.64 times as large as the free thermal movements caused by the ambient temperature changes in the same period, and those of the 3−storied building are 0.43−0.72 times.

(2) The relation between atmospheric temperatures and thermal movements is not linear. (This was confirmed in the measurement of the 6−storied building measured by authors.[3])

Meanwhile, thermal movements of the 1−storied building was elastically analyzed as a plane structures without walls. According to comparison with the above−mentioned measurements, the analytical results differed from the measurements, for example, the

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percentage of analytical results against the free thermal movements was more than 85%.[4] This difference seemed to be influenced by the following reason at that time ;

(A) Temperature changes induced thermal movements differs from ambient temperature changes.

(B) Thermal movements are restricted by walls and slabs in buildings.

(C) Thermal movements are influenced by cracks and creep of R/C members.

(D) Thermal movements are influenced by thermal movements of footing, which fact was clarified in the 1−storied building.

The reason (C) or (D) is one of factors in which thermal movements approach the free thermal movements. The reason (A) is investigated now.[5] The reason (B) was previously studied on the basis of the elastic analytical results.

This paper combines the studies on comparison of thermal movements obtained from analyses and the thermal movements of actual buildings.

2 ANALYTICAL METHOD

In this paper, analyses except for preliminary caluculations were made by the following Framework Method. Namely, beams (or columns) were divided into three-dimensional beam-elements. Slabs (or walls) were divided into square rigidly-connected framework elements[6], as shown in Fig. 1. It was assumed that beam-element was connected with framework elements at its axis, as shown in Fig. 2.

In order to investigate the influence of analytical methods and its assumptions, preliminary calculations were made on the model shown in Fig. 3. Here, it was assumed that the temperature change of the analytical model was constant, Poisson's ratio was equal to 1/6, and footings did not move. The calculations were made on the basis of the following four conditions ;

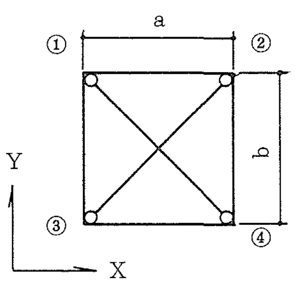


Fig. 1 Framework element

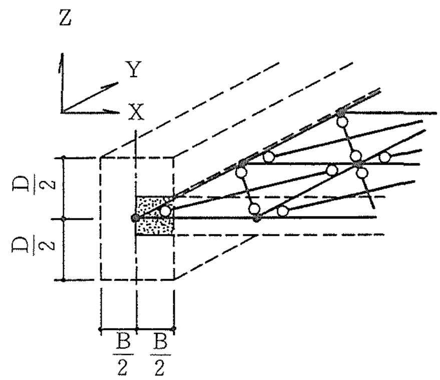


Fig. 2 Connecting relation between framework element and beam-element

(A) three-dimensional FEM in which a plate and beams around the plate were divided into rectangular prisms

(B) two-dimensional FEM in which a plate and beams around the plate divided into quadrilateral elements with two kind of thickness, hence it was assumed that the plate was connected with beams without eccentricity, as shown in Fig. 3 (a).

(C) the case where the model shown in Fig. 3 (a) is calculated by the Framework Method in which it was assumed that node 0 of beam-element was connected with node 1 of the adjacent framework element through a rigid rod, as shown in Fig. 4 (a).

(D) The case where the model shown in Fig. 3 (b) was analyzed on the basis of the Framework method and the assumption shown in Fig. 4 (b).

Although analytical results are generally influenced by an element size, the results under the same size were shown below.[7)]

(1) In analytical results of three-dimensional FEM with the freedom with respect to out-plane direction, partical deformation was induced at the connecting parts of a plate and beams around the plate, which suggests that flexural and axial rigidity in three-dimensional FEM analysis are smaller than those of the other analytical methods.

(2) The analytical results of the Framework Method based on the assumption of Fig. 4 (a) agree well with those of two-dimensional FEM. However, the analytical results of the Framework Method based on the assumption of Fig. 2 differed from the above-mentioned two results. This seems to be influenced by the difference between the assumptions shown in Fig. 5.

(3) The analytical results of three-dimensional FEM differed from those of the Framework Method based on the assumption of Fig. 4.(b). In the Framework Method, it appears to be necessary for the use of framework elements to consider the out-plane direction rigidity. However, it was possible that the outline of stresses and deformations in the case where a plate connected eccentrically with beams, as shown in Fig. 4 (b), was estimated through the above-mentioned Framework Method. The same tendency was confirmed in analytical models continuing in the Y direction.[8)]

In other words, the calculated errors of the Framework Method based on the connec-

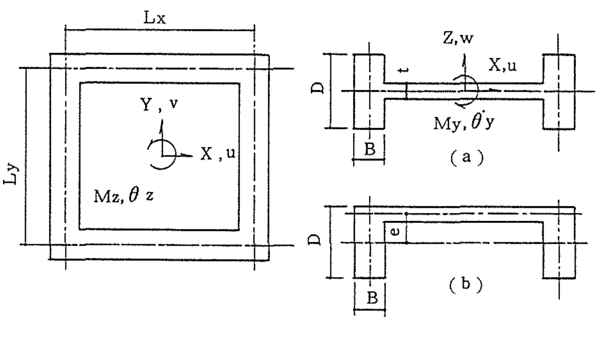


Fig. 3 Analytical model for preliminary calculations

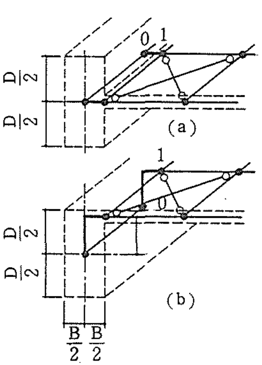


Fig. 4 Non eccentric and eccentric model connected with a rigid rod

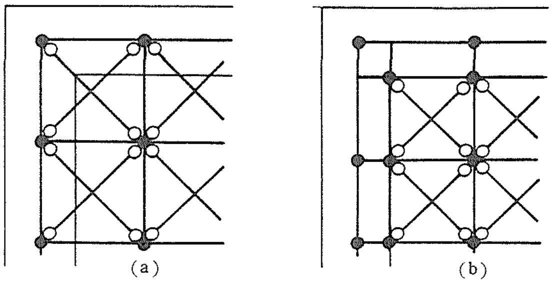


Fig. 5 Direct connection by means of mutual nodes (a) and indirect connection by means of rigid rod (b)

tion shown in Fig. 2 are similar to errors induced owing to ignorance or consideration of rigidity zones in structural designs. Assuming that these errors are ignored, the analytical accuracy of the Framework Method are nearly equal to that of two-dimensional FEM. Also, the Framework Method is more advantageous than other methods since all unknown quantities are relatively limited. From the above-mentioned viewpoints, the Framework Method based on the assumption shown in Fig. 2 was used in the following applications.

3 ON 1−STORIED BUILDING

3.1 Outline of the building and analytical model

Fig. 6 shows the 1−storied building measured by K. Ohno[2]. The only parts enclosed by inner walls and external walls were heated in the winter season.Each bay length of the longitudinal direction and the height of storey were not constant since the building was constructed on sloping ground. As shown in Fig. 6, the inner walls were placed at the left hand side alone of I block, and the opening shape and opening size differed for every span.

However, considering the computer storage, the calculation were made on III block under the following assumptions ;

(a) Although the total span length and the total span number in the longitudinal direction are equal to the actual values, each bay length is constant.

(b) The height of storey is equal to the average one of III block.

(c) Opening size of each span is constant, and that opening shape is similar to the shape of each external wall.

Although the actual average opening ratio was 0.49, calculations were made at every opening ratio 0.2 in order to investigate the influence of the opening ratio on the deformation.

Fig. 7 shows the analytical model considered in the above-mentioned assumption. Each span was divided into ten elements with respect to the longitudinal (Y) direction, 11 elements between Frame Ⓐ and Ⓑ and ten elements between Frame Ⓑ and the symmetrical axis with respect to the transverse (X) direction, ten elements with respect to the height (Z) direction. Here, it was assumed that temperature changes in all members were equal to θ°C.

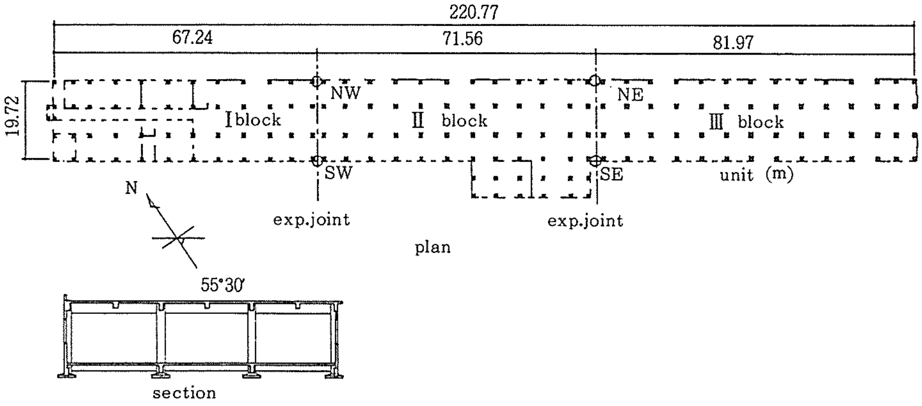


Fig. 6 1-storied building

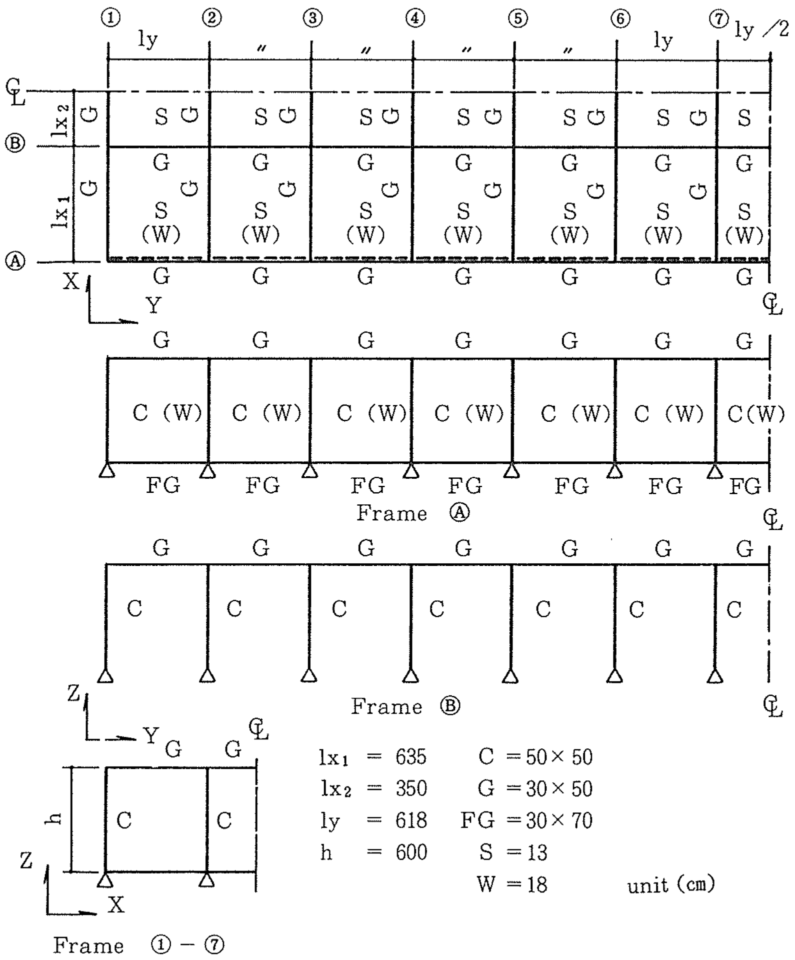


Fig. 7 Analytical model of 1-storied building

3.2 Analytical results and consideration

Column top displacements are shown in Table 1, where the model name (A−n) shows the model with opening ratio n. Fig. 8 shows the relation betweem the opening ratio and the ratio of the outside column top displacement against free thermal movements. With reference to Table 1 and Fig. 8, the following tendency was found in Frame Ⓐ. Namely, Y directional displacements decrease as the opening size becomes small, for example, those of A−1.0 were almost the same as the free thermal movements, but those of A−0 were about 35% as large as the free thermal movements. The increasing rate of Y directional displacements was large within the limits of the opening ratio from 0.4 to 0.8, and that of the other opening ratio was small.

Meanwhile, Y directional displacements at Frame Ⓑ were restricted by the external wall rigidity through the medium of the roof slab. Estimating Y directional displacements of the actual building (n=0.49) from Fig. 8, the values at Frame Ⓐ and Ⓑ were 54%, 63% being as large as the free thermal movement, respectively. These displacements also were similar to the actual measurements (53%−64%).

While, X directional displacements increased as the opening ratio became small, and they were larger than the free thermal movements.

Table 1 Column top displacements

unit ($cm\alpha\theta$)

model name	rigid frame	direction	column position						
			①	②	③	④	⑤	⑥	⑦
A−0	Ⓐ	X	−1309.8	−1035.6	−993.3	−1010.3	−1034.3	−1051.5	−1059.9
		Y	−1309.4	−856.2	−534.8	−338.7	−206.9	−111.4	−35.1
	Ⓑ	X	−512.0	−373.9	−346.7	−352.1	−362.9	−371.0	−375.0
		Y	−1927.1	−1361.7	−917.5	−591.4	−361.4	−194.1	−61.1
A−0.2	Ⓐ	X	−1302.8	−1033.7	−992.8	−1009.6	−1032.9	−1049.5	−1057.6
		Y	−1446.9	−913.2	−584.3	−376.7	−233.1	−126.6	−40.1
	Ⓑ	X	−508.9	−373.1	−346.8	−352.3	−362.7	−370.5	−374.3
		Y	−1968.9	−1407.6	−954.5	−622.4	−384.5	−208.2	−65.8
A−0.4	Ⓐ	X	−1260.0	−1025.3	−991.4	−1006.6	−1026.1	−1039.7	−1046.3
		Y	−1774.0	−1237.7	−866.5	−597.5	−388.5	−218.4	−70.4
	Ⓑ	X	−489.0	−369.5	−347.7	−352.8	−361.4	−367.6	−370.6
		Y	−2216.5	−1647.7	−1173.8	−808.0	−521.4	−292.1	−94.1
A−0.6	Ⓐ	X	−1150.7	−1007.4	−988.0	−998.0	−1009.4	−1017.1	−1020.8
		Y	−2624.5	−2063.4	−1593.7	−1184.5	−816.2	−477.7	−153.7
	Ⓑ	X	−436.7	−361.5	−348.9	−352.3	−357.2	−360.6	−362.2
		Y	−2876.4	−2289.1	−1761.8	−1301.2	−893.7	−522.4	−171.9
A−0.8	Ⓐ	X	−1024.8	−987.1	−982.6	−985.7	−988.8	−990.8	−991.8
		Y	−3634.1	−3031.7	−2454.5	−1892.8	−1342.7	−801.8	−266.6
	Ⓑ	X	−373.4	−352.3	−349.2	−350.4	−351.7	−352.6	−352.9
		Y	−3687.3	−3078.5	−2487.2	−1914.8	−1357.4	−810.4	−269.5
A−1.0	Ⓐ	X	−986.1	−981.0	−980.9	−981.6	−982.0	−982.3	−982.4
		Y	−3949.2	−3336.6	−2726.6	−2118.2	−1511.6	−906.4	−302.0
	Ⓑ	X	−351.7	−349.5	−349.3	−349.6	−349.8	−349.9	−350.0
		Y	−3961.3	−3345.4	−2732.4	−2122.4	−1514.5	−908.1	−302.6

α : thermal expansion coefficient

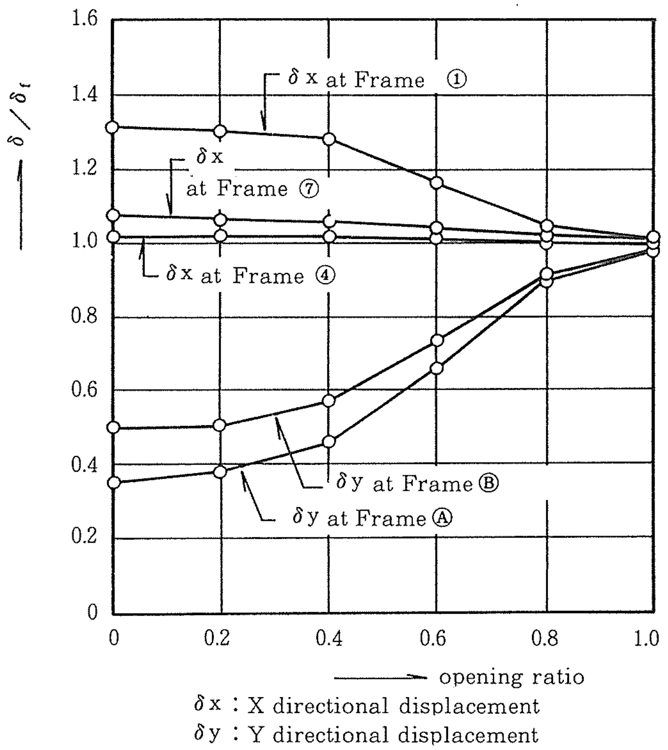


Fig. 8 Relation between opening ratio and ratio of column top displacement (δ) against free thermal displacement (δ_f)

4 ON 3−STORIED BUILDING

4.1 Outline of the building and analytical model

K. Ohno measured the thermal movements of the 3−storied building also[9]. An expansion joint of this building was placed between two columns on the same footing, as shown in Fig. 9. Each bay length in the longitudinal direction, opening shape and opening size of external walls was constant. The calculations were made on I block, where dimensions of structural members for analyses are the same as those of the actual building shown in Table 2. Fig. 10 shows the analytical model. Taking into account the limitation in computer storage, each span was divided into six elements in the longitudinal direction, eight elements between Frame Ⓐ and Ⓑ and six elements between Frame Ⓑ and Ⓒ in the transverse direction, four elements in the height direction. Although these divisions were rather rough, it was confirmed that the analytical results were approximately accurate. Since this building was warmed in winter season, analyses were made under the following three temperature conditions.

(a) the case where temperature changes of all structural members are constant
(b) the cese where temperature changes of beams and slabs are constant and those of other members are equal to zero.

(c) the case where temperature changes of the columns and walls are constant and those of other members are equal to zero.

(d) the combination of (b) and (c)

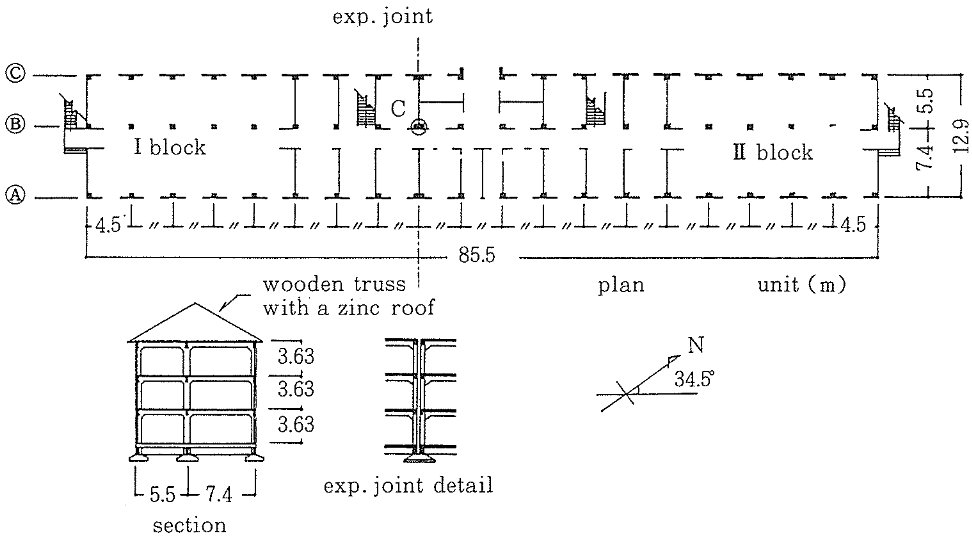


Fig. 9 3-storied building

Table 2 Dimensions of structural members

unit (cm)

Column	50×50
Column at exp. joint	20×50
Beam $_FG_X - {}_RG_X$	30×70
Beam $_3G_y$, $_RG_y$	30×50
$_FG_y$, $_2G_y$	30×60
Slab	12
Wall W15	15
W12	12

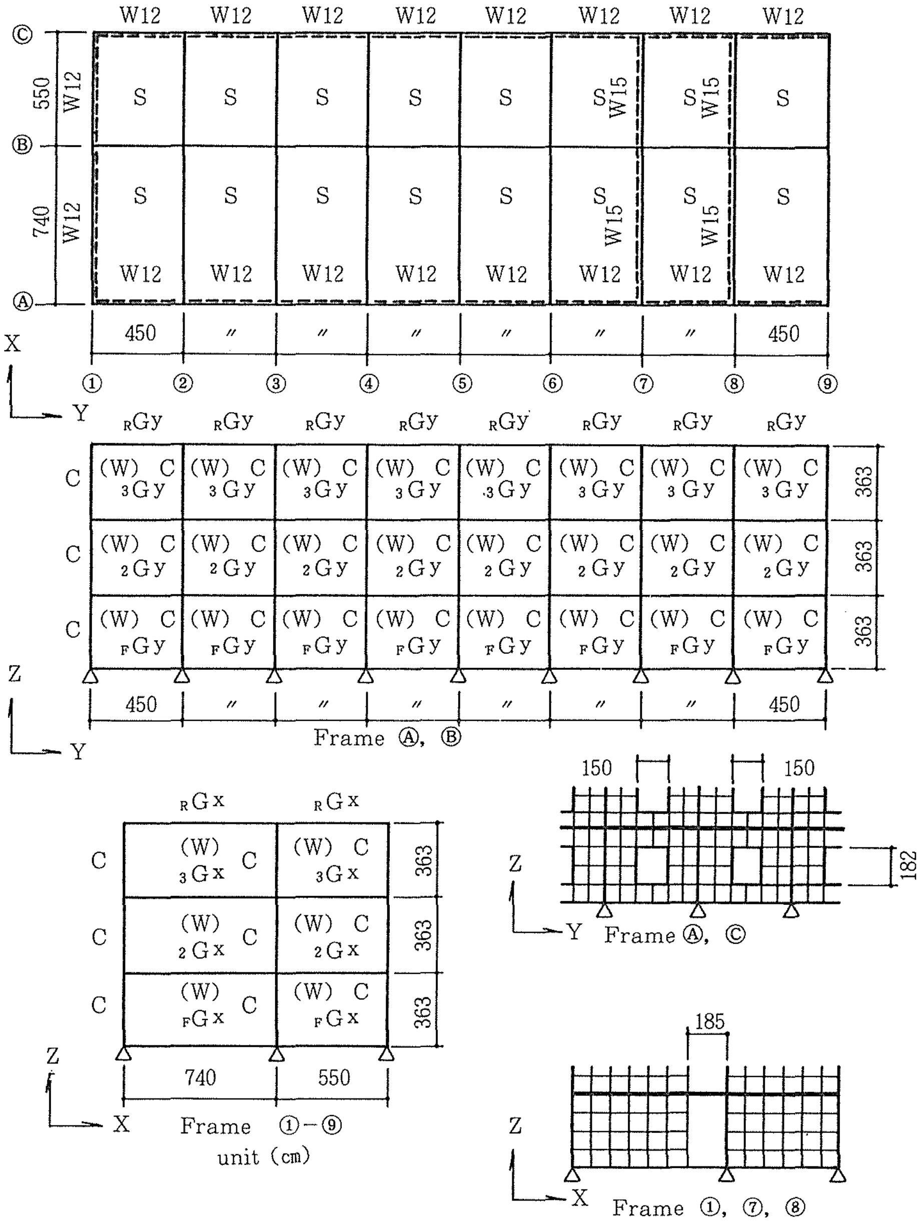


Fig. 10 Analytical model of 3-storied building

4.2 Analytical results and consideration

Table 3. shows the column top displacements obtained under the condition (a) – (d). The displacements at Frame ⑤ was not zero, since the walls were placed at Frame ①, ⑦ and ⑧ in the transverse direction. Table 4, Table 5 show the ratio of analytical results, measured displacements under the condition (a) against the free thermal movement in the longitudinal direction, respectively.

Although the ratio shown in Table 4 is smaller than ratio obtained from the previous plane frame analysis[4)], that of the top storey is almost equal to 1.0, this ratio apparently differs from the ratio showm in Table 5.

The displacements at second and third floor under the condition (b) are smaller than those under the condition (a), but the difference between displacements at third floor and those at the roof slab are large. The displacements under the condition (c) are smaller than those under the condition (b) since the restriction of slab against the movements of wall is larger than that of the opposite case. Also, on the basis of the analytical results under the conditions (b) and (c), a calculation was made under the condition (d) where the temperature changes of beams and slabs, those of columns and walls are equal to θ', θ, respectively. Here, θ' ($=0.51\theta$) was determined in order that the ratio of the outside column top displacement at second floor in Frame Ⓐ against the free thermal movements is equal to the ratio

Table 3 Column top displacements at Frame Ⓐ

unit (10^3 cm$\alpha\theta$)

condition	floor	direction	column position				
			⑤	⑥	⑦	⑧	⑨
(a)	R	X	−0.761	−0.772	−0.799	−0.810	−0.790
		Y	0.007	0.427	0.862	1.320	1.779
		Z	1.202	1.204	1.213	1.153	0.926
	3	X	−0.748	−0.752	−0.752	−0.758	−0.798
		Y	0.006	0.379	0.761	1.162	1.596
		Z	0.831	0.833	0.839	0.784	0.557
	2	X	−0.715	−0.703	−0.635	−0.636	−0.795
		Y	0.005	0.262	0.527	0.817	1.194
		Z	0.438	0.439	0.431	0.395	0.196
(b)	R	X	−0.757	−0.754	−0.668	−0.673	−0.844
		Y	0.010	0.365	0.718	1.063	1.395
		Z	0.037	0.034	0.010	−0.035	−0.231
	3	X	−0.752	−0.738	−0.557	−0.557	−0.872
		Y	0.010	0.316	0.614	0.888	1.135
		Z	0.046	0.043	0.034	−0.010	−0.203
	2	X	−0.725	−0.698	−0.463	−0.459	−0.864
		Y	0.009	0.218	0.421	0.610	0.820
		Z	0.046	0.044	0.026	−0.006	−0.165
(c)	R	X	−0.004	−0.018	−0.131	−0.137	0.054
		Y	−0.003	0.062	0.144	0.257	0.384
		Z	1.165	1.170	1.203	1.188	1.157
	3	X	0.004	−0.014	−0.195	−0.201	0.074
		Y	−0.004	0.063	0.147	0.274	0.461
		Z	0.785	0.790	0.805	0.794	0.760
	2	X	0.010	−0.005	−0.172	−0.177	0.069
		Y	−0.004	0.044	0.106	0.207	0.374
		Z	0.392	0.395	0.405	0.401	0.361
(d)	R	X	−0.390	−0.403	−0.472	−0.481	−0.377
		Y	0.002	−0.248	0.510	0.800	1.097
		Z	1.183	1.187	1.208	1.170	1.039
	3	X	−0.380	−0.391	−1.480	−1.485	−0.371
		Y	0.001	0.224	0.461	0.728	1.041
		Z	0.808	0.812	0.822	0.789	0.656
	2	X	−0.360	−0.361	−0.408	−0.411	−0.372
		Y	0.001	0.155	0.321	0.518	0.793
		Z	0.415	0.417	0.418	0.398	0.277

α : thermal expansion coefficient

(0.44) shown in Table 5. Table 6 shows the ratio of analytical results in the longitudinal direction under the condition (d) against the free thermal movements. According to Table 6, these values agree well with the values shown in Table 5. Hence, it is necessary to consider the temperature changes corresponding to position of structual members in buildings equipped with heating apparatuses.

Table 4 Ratio of column top displacements in the longitudinal direction under the condition (a) against free thermal movements

floor position	2F	3F	RF
Frame ①	0.66	0.89	0.99
Frame ②	0.81	0.94	0.99

Table 5 Ratio of measurements in the longitudinal direction against free thermal movements

floor position	2F	3F	RF
value based on ambient temp.	0.44−0.43	0.56−0.62	0.62−0.72
value based on room temp.	0.52−0.53	0.69−0.80	0.69−0.81

Thermal movements differ owing to the position of Frame (Ⓐ or Ⓑ) ; Ⓐ<Ⓑ.

Table 6 Ratio of column top displacements in the longitudinal direction under the condition (d) against free thermal movements

floor position	2F	3F	RF
Frame Ⓐ	[0.44]	0.58	0.61
Frame Ⓑ	0.44	0.52	0.54

5 ON 6−STORIED BUILDING

5.1 Outline of the building and analytical model

Fig. 11 shows 6−storied building measured by the authors. The only half of III block was analyzed. In order to simplify analyses, the cantilever roof slab was neglected, some

Table 7 Dimensions of column and beam

unit (cm)

floor	C_1	C_2	C_3	C_4	G_1	G_2	B_1
1F (GF)	65× 55	65× 70	60× 75	60× 55	40×120	30× 70	30× 90
2F	65× 55	65× 70	60× 70	60× 55	35× 70	45× 70	30× 70
3F	60× 55	60× 70	55× 70	55× 55	35× 70	40× 70	30× 70
4F	55× 55	55× 70	55× 65	55× 50	35× 70	35× 70	30× 70
5F	55× 55	55× 70	55× 65	55× 50	30× 70	30× 70	30× 70
6F	50× 55	50× 70	50× 60	50× 50	30× 70	30× 70	30× 70
RF	—	—	—	—	25× 65	30× 55	30× 70

Cx × Cy   B × D

walls in the transverse direction were moved to a symmetrical disposition, as shown in Fig. 12. The dimensions of structural members are shown in Table 7. Each bay was divided as shown in Fig. 13, in which the element size only in the height direction was adjusted to the opening level of external wall in the longitudinal direction.

According to the analytical results of the 3—storied building which consists of 2, 4 and 6 floor in the 6—storied building, since the influence of the cantilever roof slab on the deformation was very small except for the top storey, the cantilever roof slab was neglected in the analyses of the 6—storied building. Also, the analytical results of the whole III block differed from those of the half of III block, but its difference were very small.[10)]

On the basis of measurements of atmospheric temperatures, the temperature changes of structural members for analyses were assumed as shown in Table 8. The numerical values shown in Table 8 represent values normalized in terms of the temperature changes of structural members at Frame Ⓓ.

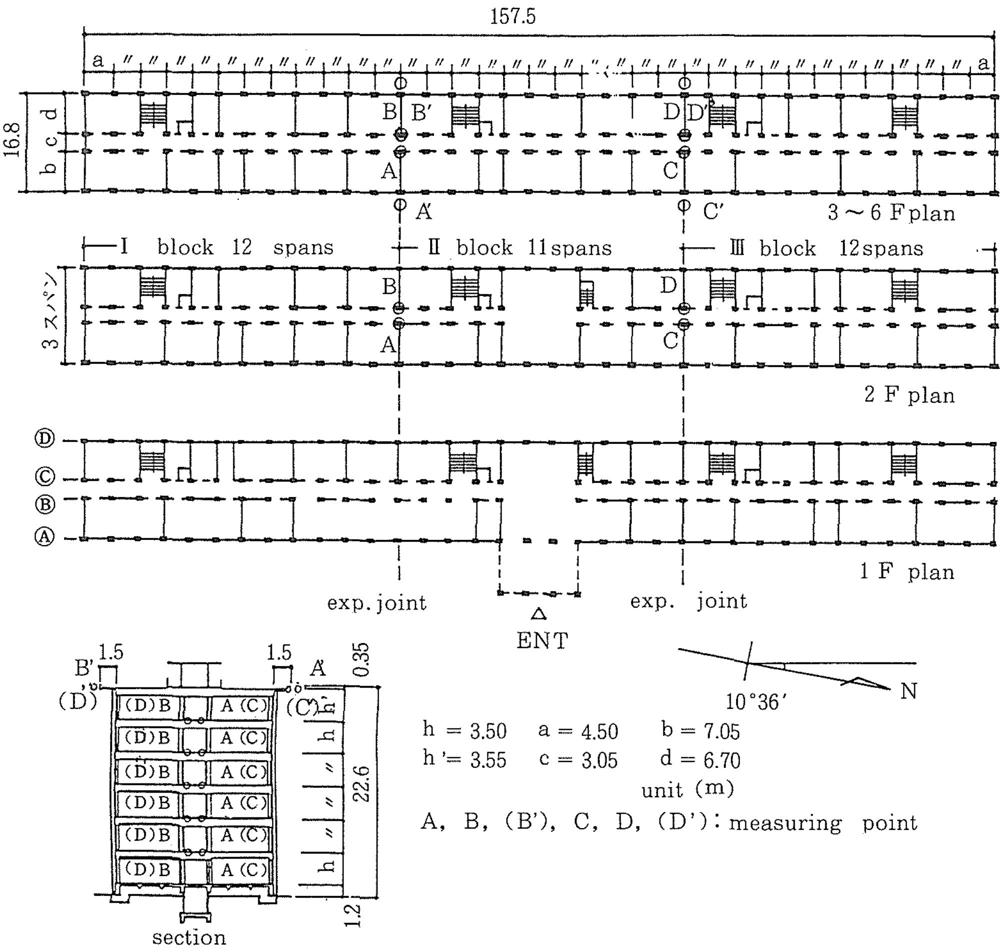


Fig. 11 6-storied building

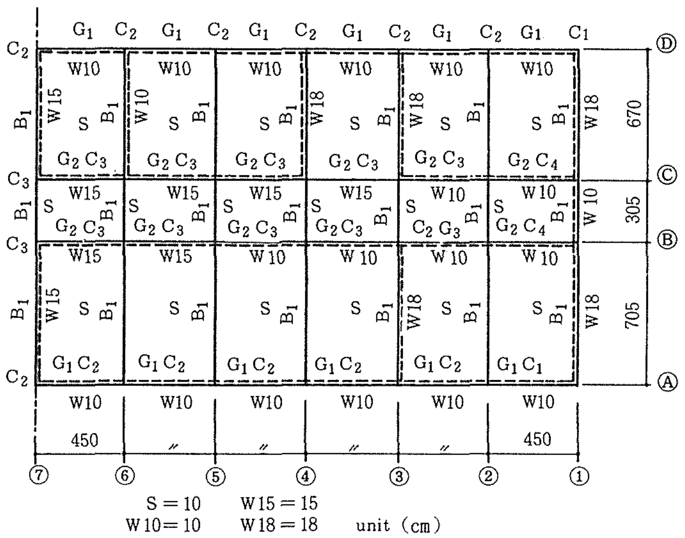
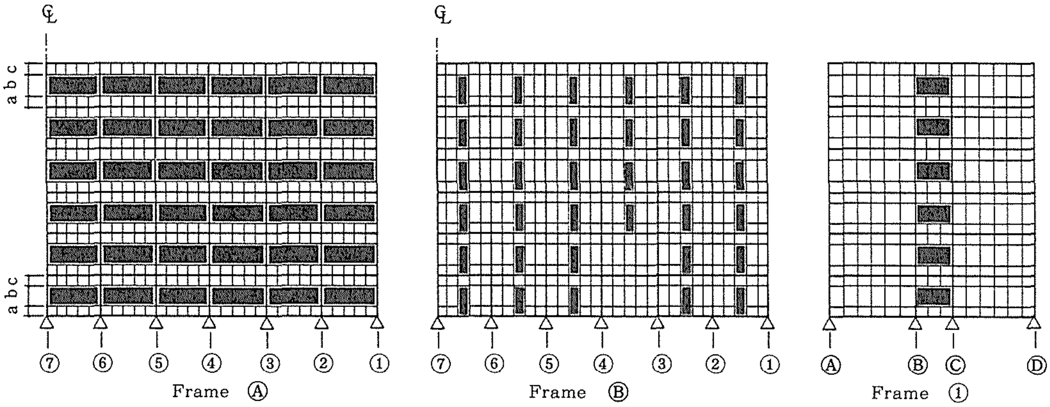


Fig. 12 Analytical model of 3-storied building



the longitudinal direction, the transverse direction : uniform division
the height direction : 1−5F a:b:c= 0.825 :1.85: 0.825,
RF a:b:c= 0.85 :1.85 : 0.85

Fig. 13 Element division

5.2 Analytical results and consideration

Fig. 14 shows the column top displacements of Case A shown in Table 8. The column top displacements at Frame Ⓒ and Ⓓ are similar to those of Frame Ⓑ and Ⓐ,respectively. The displacements at Frame Ⓑ are smaller than those of Frame Ⓐ. This is due to the difference of temperature changes, opening shape and opening size. A comparison of the measurements obtained at the measuring point C or D and the analytical results are shown in Fig. 15, in which each measurement is equal to the gradient of a regression line of the correlation diagram between the ambient temperatures and the thermal movements obtained over a period from May 1, 1984 to August 11, 1984.

Also, Case A—Case D are the analytical results obtained under the assumption that the thermal expansion coefficient α is equal to 10×10^{-6} (1/°C).

According to Fig. 15, the displacements, in particular at the top storey, in the longitudinal direction increase as the temperature changes at the top storey are large. The analytical results agree with the measurements in the case of Case D. This is due to the influence of the

Table 8 Temperature change normalized in terms of temperature changes of structural members in Frame Ⓓ

position		temp. (θ)
external walls, columns, beams in Frame Ⓓ		1.000
external walls, columns, beams in Frame Ⓐ		1.002
external walls, columns, beams in the transverse direction		1.000
roof slab, beams at the top storey		1.000-1.150*
1F-3F	inner walls, columuns, beams	0.706
4F	inner walls, columns, beams	0.655
5F	inner walls, columns, beams	0.670
6F	inner walls, columns, beams	0.652

* Case A : 1.0 Case B : 1.05
Case C : 1.10 Case D : 1.15

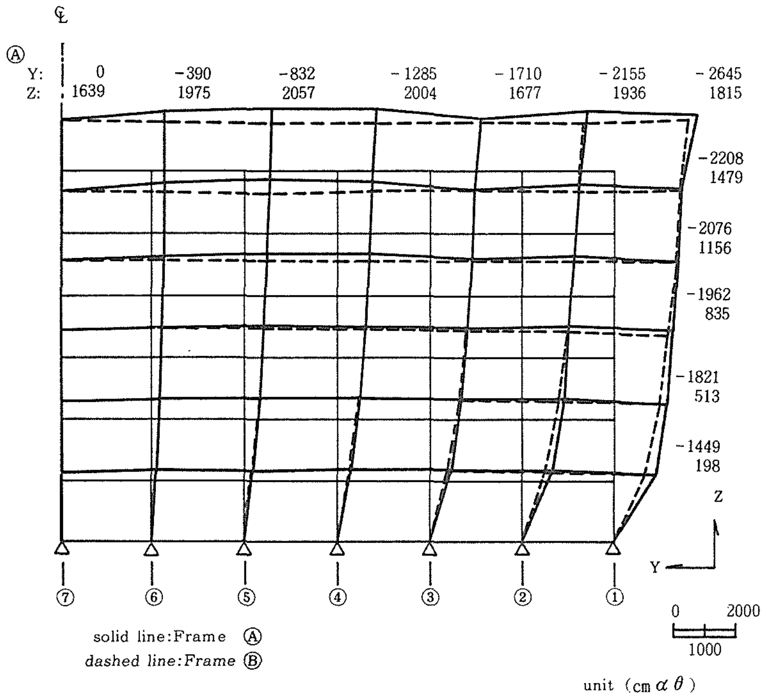


Fig. 14 Column top displacements of Case A

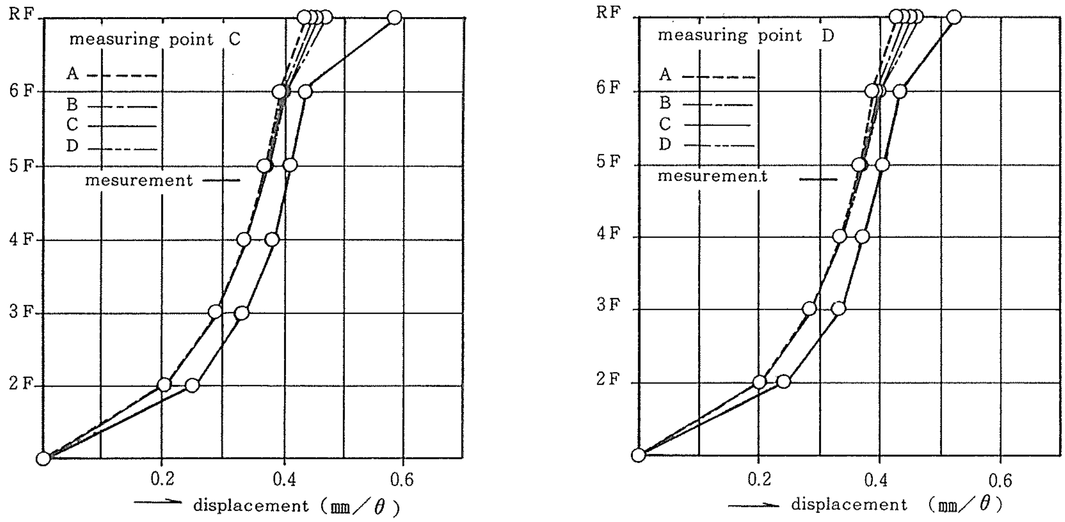


Fig. 15 Outside column top displacements (measurements and analytical results)

large temperature changes induced in the roof slab in terms of solar radiation.

6 CONCLUSION

The thermal movements of three R/C buildings were compared with three-dimensional elastic analytical results obtained from the Framework Method, where beams and columns, slabs and walls were divided into three dimensional beam-elements, framework elements without rigidity with respect to the outplane direction, respectively.

Conclusions are as follows ;

(A) 1–storied building without heating : In the case where the temperature changes of all structural members are constant, the longitudinal directional displacements obtained from the analysis agree well with the measurements. This also was confirmed in Frame Ⓐ with walls and also in Frame Ⓑ without walls.

(B) 3–storied building with heating in winter season : Under the assumption that the temperature changes of all structural members were constant, the results obtained from the three dimensional elastic analysis slightly approached the actual measurements. Under the assumption that the temperature changes of the external walls and column and beams exposed to ambient temperature differed from those of other members, the analytical results further approached the actual measurements.

(C) 6–storied building with heating in winter season : Considering the ratio of ambient temperature changes against room temperature changes, the analytical results agreed well with the actual measuremetns.

Assuming that the more appropriate temperature changes is considered, the accurate thermal movements of R/C buildings will be estimated through the three-dimensional elastic analysis considered walls and slabs.

REFERENCE

1) M. Obata: On Thermal Stresses of the Rectangular Frame Considered Plasticity-part 2, 3, Annual Transac. of AIJ, Hokkaido Branch, No. 25, September 1956, p. 29−36.
2) K. Ohno: The Actual Condition of Thermal Movements of Reinforced Concrete Buildings (In the Case of 1−storied Building), Transac. of AIJ, No. 54, September 1956, p. 233−236.
3) S. Kushiyama, M. Obata: Measurement of Thermal Expansion and Contraction of A−Wing of the Faculty of Engineering Hokkaido University − A Study on Thermal Stresses of Reinforced Concrete Buildings (part 1), J. of Structural and Construction Engineering, Transac. of AIJ, No. 376, July 1987, p. 1−9.
4) M. Obata: On the Thermal Stresses of the Rectangular Frame, and a series of studies, Transac. of AIJ, No. 69, October, p. 297−300.
5) S. Kushiyama, M. Obata: Comparison of Field-measured Surface Temperatures and Analytical Results in Reinforced Concrete Members, Annual Transac. of AIJ, Hokkaido Branch, No. 61, March 1988, p. 41−44.
6) C. W. McCormick: Plane Stress Analysis, ST4, ASCE, Vol. 4, August 1963, P. 37−54.
7) M. Obata et al.: On Thermal Stress Analysis of Plate with Beam Members part 1−3, Annual Transac. of AIJ, Hokkaido Branch, No. 54, 1981, p. 1−12.
8) M. Obata et al.: On Thermal Stress Analysis of Plate with Beam Members (In the Case of Continuous Model), Annual Transac. of AIJ, Hokkaido Branch, No. 55, March 1982, p. 95−102.
9) K. Ohno, K. Mutoh: Measurement of Thermal Movements of Reinforced Concrete Buildings, Transsac. of AIJ, Vol. 27, May 1954, p. 149−150.
10) S. Kushiyama, M. Obata: Thermal Stress Analysis of A−Wing of the Faculty of Engineering Hokkaido University (Comparison of Measuring and Analysis), Annual Transac. of AIJ, Hokkaido Branch, No. 60, March 1987, p. 89−92.