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Properties of Forward Phase Conjugation by Nearly Degenerate Four Wave Mixing with Boxcar Type Phase-Matching

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Abstract

Coupled wave equations are derived and solved for analyzing properties of forward phase conjugation by nearly degenerate four wave mixing with boxcar type phase-matching. The properties are discussed in conjunction with incident angles, polarization directions of the waves and frequency detuning.

1. Introduction

Blaschuk et al.[1] and Prior[2] have proposed a boxcar configuration of wave vectors to realize phase-matching in forward phase conjugation (FPC). Since FPC is considered to have a high speed responsibility, Khyzniak, et al.[3] and Apanasevich, et al.[4] analyzed the transient response of FPC and have shown high speed responsibility. Heer et al.[5], Blaschuk, et al.[1] and Khyzniak et al.[3] have demonstrated FPC. Recently FPC has attracted attention as a squeezed state generator[6], but the main interests have been concentrated into quantum properties of the generated optical field but not into properties of interaction of waves in FPC itself.

The analyses above have assumed the small angles between directions of wave vectors and the demonstrations above have realized under the case of the small angles. The angles must not be zero since those waves are distinguished by their propagation directions in FPC. When the angles become larger the effects of the angles and of polarization directions must be considered in boxcar type phase-matching. Here we discuss the effects of the angles, of the polarization directions and of detuning.

2. Nonlinear polarization

The i-th component of the third order nonlinear polarization P_i is given by for steady state four-wave mixing[7]:

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$$P^{NL}_i = 4 \sum_{jkl} \chi_{ijkl} E_j E_k E_l , \tag{1}$$

where i, j, k and l are one of the Cartesian coordinates, respectively. This expression does not show the wavelength dependence explicitly, but it suffices to consider the effects of phase-mismatching owing to the small detuning of probe wave(hundreds MHz for atomic vapor, for example).

We take spatial symmetry of the nonlinear susceptibility tensor into consideration. For gaseous nonlinear medium inversion symmetry holds and 21 elements of the third-order nonlinear susceptibility are non-zero. Further, only three of these are independent[8]. After some calculation we obtain for the third order nonlinear polarization

$$\begin{aligned} P^{NL} &= P^{NL}{}_x \mathbf{x} + P^{NL}{}_y \mathbf{y} + P^{NL}{}_z \mathbf{z} \\ &= (\chi_{1111}/8)\left[2(\boldsymbol{E}^+ \cdot \boldsymbol{E}^-)\boldsymbol{E}^+ + (\boldsymbol{E}^+ \cdot \boldsymbol{E}^+)\boldsymbol{E}^-\right] + \text{c.c.}, \end{aligned} \tag{2}$$

where

$$\boldsymbol{E} = (1/2)(\boldsymbol{E}^+ + \boldsymbol{E}^-), \tag{3}$$

$$\boldsymbol{E}^+ = (\boldsymbol{E}^-)^*, \tag{4}$$

x, **y**, and **z** are Cartesian unit vectors, and c.c. means a complex conjugate. The nonlinear susceptibility χ_{1111} is assumed to be real and we use χ instead of χ_{1111} hereafter for simplicity.

Each i-th component is considered to be the sum of frequency components:

$$E^+_i = (1/2) \sum_q A_{qi}{}^+ \exp\left[\,\mathrm{i}(\omega_q t - \boldsymbol{k}\cdot\boldsymbol{r})\right], \tag{5}$$

where q=1, 2, p and c indicate two pump waves, signal wave and conjugate wave, $A_{qi}{}^+$ is a slowly varying amplitude. The angular frequency ω_c is

$$\omega_c = \omega_1 + \omega_2 - \omega_p. \tag{6}$$

We assume that the frequencies of the two pump waves are the same ($\omega = \omega_1 = \omega_2$). Besides, each of the pump intensities is assumed to be much larger than probe and conjugate intensity.

The nonlinear polarization also consists of the frequency components:

$$P^{NL}_i = (1/2) \sum_q \left[R_{qi}{}^+ \exp\{\mathrm{i}(\omega_q t - \boldsymbol{k}_q\cdot\boldsymbol{r})\} + \text{c.c.}\right], \qquad \text{i c.c}\ (7)$$

where $R_{qi}{}^+$ is a slowly varying amplitude of the i-th component.

Case A When the polarization of the waves are (see Fig. 1 (a))

$$\boldsymbol{A}_1{}^+ = A_1{}^+ \mathbf{y}, \tag{8}$$

$$\boldsymbol{A}_2{}^+ = A_2{}^+ \mathbf{y}, \tag{9}$$

$$\boldsymbol{A}_p{}^+ = A_p{}^+ \mathbf{x}, \tag{10}$$

and

$$\boldsymbol{A}_c{}^+ = A_c{}^+ \mathbf{x}, \tag{11}$$

each frequency component of nonlinear polarization is reduced to a simple form. Insertion of eq. (4) into eq. (2) and use of eq. (5) lead to

$$\boldsymbol{R}_1^+ = (3/4)\chi(A_1^+A_1^- + 2A_2^+A_2^-)A_1^+\mathbf{y}, \tag{12}$$

$$\boldsymbol{R}_2^+ = (3/4)\chi(A_2^+A_2^- + 2A_1^+A_1^-)A_2^+\mathbf{y}, \tag{13}$$

$$\boldsymbol{R}_p^+ = (1/2)\chi[A_1^+A_2^+A_c^-\exp(-i\Delta\boldsymbol{k}\cdot\boldsymbol{r}) + (A_1^+A_1^- + A_2^+A_2^-)A_p^+]\mathbf{x}, \tag{14}$$

and

$$\boldsymbol{R}_c^+ = (1/2)\chi[(A_1^+A_2^+A_p^-\exp(-i\Delta\boldsymbol{k}\cdot\boldsymbol{r}) + (A_1^+A_1^- + A_2^+A_2^-)A_c^+]\mathbf{x}, \tag{15}$$

where $\Delta\boldsymbol{k}$ is defined by

$$\Delta\boldsymbol{k} \triangleq \boldsymbol{k}_c - (\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{k}_p). \tag{16}$$

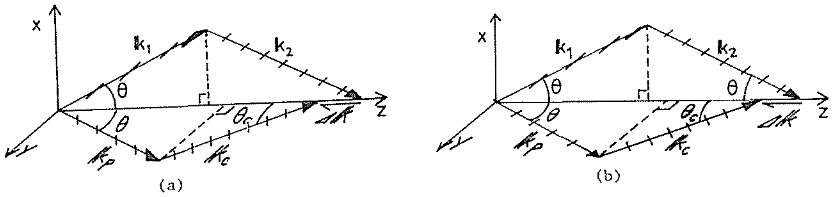

Fig. 1 Polarization of waves and boxcar type phase-matching configuration for FPC. When the detuning δ is equal to zero, the phase-matching condition is assumed to be satisfied. (a) Case A : $\mathbf{e}_1 // \mathbf{e}_2 // \mathbf{y}$ and $\mathbf{e}_p // \mathbf{e}_c // \mathbf{x}$, and (b) Case B : $\mathbf{e}_1 // \mathbf{e}_2 // \mathbf{y}$, $\mathbf{e}_p \perp \mathbf{x}$ and $\mathbf{e}_c \perp \mathbf{x}$, where $\mathbf{e}_q$ is the polarization direction.

Case B When the polarizations are (see Fig. 1 (b))

$$\boldsymbol{A}_1^+ = A_1^+\mathbf{y}, \tag{17}$$

$$\boldsymbol{A}_2^+ = A_2^+\mathbf{y}, \tag{18}$$

$$\boldsymbol{A}_p^+ = A_{py}^+\mathbf{y} + A_{pz}^+\mathbf{z}, \tag{19}$$

and

$$\boldsymbol{A}_c^+ = A_{cy}^+\mathbf{y} + A_{cz}^+\mathbf{z}, \tag{20}$$

we obtain eq. (12) for $\boldsymbol{R}_1$ and eq. (13) for $\boldsymbol{R}_2$ and following for $\boldsymbol{R}_p$ and $\boldsymbol{R}_c$:

$$\boldsymbol{R}_p^+ = (1/2)\chi[3\{A_1^+A_2^+A_{cy}^-\exp(-i\Delta\boldsymbol{k}\cdot\boldsymbol{r}) + (A_1^+A_1^- + A_2^+A_2^-)A_{py}^+\}\mathbf{y} + \{A_1^+A_2^+A_{cz}^-\exp(-i\Delta\boldsymbol{k}\cdot\boldsymbol{r}) + (A_1^+A_1^- + A_2^+A_2^-)A_{pz}^+\}\mathbf{z}], \tag{21}$$

and

$$\boldsymbol{R}_c^+ = (1/2)\chi[3\{A_1^+A_2^+A_{py}^-\exp(-i\Delta\boldsymbol{k}\cdot\boldsymbol{r}) + (A_1^+A_1^- + A_2^+A_2^-)A_{cy}^+\}\mathbf{y} + \{A_1^+A_2^+A_{pz}^-\exp(-i\Delta\boldsymbol{k}\cdot\boldsymbol{r}) + (A_1^+A_1^- + A_2^+A_2^-)A_{cz}^+\}\mathbf{z}]. \tag{22}$$

We note $\boldsymbol{R}_p$ and $\boldsymbol{R}_c$ have z component as well as y component.

3. Boxcar type phase matching and phase mis-matching Δk

It is relatively easy to satisfy phase-matching condition in the case of degenerate interaction, so that we put

$$\omega_p = \omega + \delta \quad \text{and} \quad \omega_c = \omega - \delta, \tag{23}$$

where $>>|\delta|$. When $\delta = 0$ we assume the phase-matching condition is satisfied:

$$\Delta \boldsymbol{k} = 0. \tag{24}$$

For simplicity we assume each angle between z-axis and the propagation directions of the waves is the same, except for conjugate wave:

$$\theta \triangleq \theta_1 = \theta_2 = \theta_p. \tag{25}$$

When $\delta \neq 0$ phase-matching problem arises. The nonlinear medium is assumed to be infinite in x and y directions but finite in z direction (thickness L). In this case phase-matching condition in x and y direction must be strictly satisfied[9]. As a result we obtain

$$\Delta k_z \neq 0 \quad \text{and} \quad \Delta k_x = \Delta k_y = 0. \tag{26}$$

Above equations can be rewritten as

$$\Delta k_z = |\boldsymbol{k}_p|\cos\theta_p + |\boldsymbol{k}_c|\cos\theta_c - |\boldsymbol{k}_1 + \boldsymbol{k}_2|, \tag{27}$$

and

$$|\boldsymbol{k}_p|\sin\theta_p = |\boldsymbol{k}_c|\sin\theta_c. \tag{28}$$

We assume linear refractive index is equal to unity which is adequate for most metal vapors. When $\delta = 2\pi \times 10^9$ rad/s, for example, δ/ω is roughly equal to 0.3×10^{-5}, so that the effect of detuning δ on θ_c can be neglected in most situations. From eq. (27) we obtain

$$\Delta k_z = [\Delta k_z]_{\delta=0} + \left[\frac{\partial(\Delta k_z)}{\partial \delta}\right]_{\delta=0} \delta = -\frac{2\sin\theta\tan\theta}{c_0}\delta. \tag{29}$$

4. Coupled wave equations

By insertion of the optical fields and the nonlinear polarizations into Maxwell equations, and use of slowly varying amplitude approximation we obtain an equation describing the relation between nonlinear polarization and optical field. Separating it into different propagation directions lead to for the case B:

$$\frac{\partial A_1^+}{\partial z_1} = -i\frac{3\chi\omega}{8c_0}(A_1^+A_1^- + 2A_2^+A_2^-)A_1^+, \tag{30}$$

$$\frac{\partial A_2^+}{\partial z_2} = -i\frac{3\chi\omega}{8c_0}(A_2^+A_2^- + 2A_1^+A_1^-)A_2^+, \tag{31}$$

$$\frac{\partial A_{py}^+}{\partial z_p} = -i\frac{3\chi\omega_p}{4c_0}[A_1^+A_2^+A_{cy}^-\exp(-i\Delta k_z z) + (A_1^+A_1^- + A_2^+A_2^-)A_{py}^+], \tag{32}$$

$$\frac{\partial A_{cy}^{+}}{\partial z_c}=-\mathrm{i}\frac{3\chi\omega_c}{4c_0}[A_1^{+}A_2^{+}A_{py}^{-}\exp(-\mathrm{i}\Delta k_z z)+(A_1^{+}A_1^{-}+A_2^{+}A_2^{-})A_{cy}^{+}], \tag{33}$$

$$\frac{\partial A_{pz}^{+}}{\partial z_p}=-\mathrm{i}\frac{\chi\omega_p}{4c_0}[A_1^{+}A_2^{+}A_{cz}^{-}\exp(-\mathrm{i}\Delta k_z z)+(A_1^{+}A_1^{-}+A_2^{+}A_2^{-})A_{pz}^{+}], \tag{34}$$

$$\frac{\partial A_{cz}^{+}}{\partial z_c}=-\mathrm{i}\frac{\chi\omega_c}{4c_0}[A_1^{+}A_2^{+}A_{pz}^{-}\exp(-\mathrm{i}\Delta k_z z)+(A_1^{+}A_1^{-}+A_2^{+}A_2^{-})A_{cz}^{+}], \tag{35}$$

where z_q is the propagation direction of each wave$(\boldsymbol{k}_q/k_q)$.

The nonlinear polarizations $\boldsymbol{R}_p$ and $\boldsymbol{R}_c$ contain z component as well as y component in the case B: Eqs. (34) and (35) are derived from the z component. Eqs. (32) and (34) resemble each other, but the coefficient of eq. (32) is three times larger than that of eq. (34) and so as eqs. (33) and (35).

Because of the lack of the component parallel to pump polarization in the case A we obtain eqs. (30), (31), (34) and (35) where the suffix y and z are omitted. Hereafter, we omit description on the case A.

5. Solutions

We can easily show that $A_1^{+}A_1^{-}$ and $A_2^{+}A_2^{-}$ are constants by using eqs. (30) and (31) and their complex conjugates. The solutions of the pair of eqs. (30) and (31) are

$$A_1^{+}=A_{10}^{+}\exp(-\mathrm{i}k_1' z_1) \tag{36}$$

and

$$A_2^{+}=A_{20}^{+}\exp(-\mathrm{i}k_2' z_2), \tag{37}$$

where A_{10}^{+} and A_{20}^{+} are constants of integration, and

$$k_1' \triangleq \frac{3\chi\omega}{8c_0}(A_1^{+}A_1^{-}+2A_2^{+}A_2^{-}) \tag{38}$$

and

$$k_2' \triangleq \frac{3\chi\omega}{8c_0}(A_2^{+}A_2^{-}+2A_1^{+}A_1^{-}). \tag{39}$$

According to these solutions self focusing or self defocusing will occur if the amplitudes are not uniform. However, we analyze plane wave case, where the magnitudes of the wave vectors change uniformly in space.

Now we assume intensities of the pump waves are equal to each other for making the problem simple. Using the pump wave solutions (eqs. (36) and (37)), the pair of eqs. (32) and (33) can also be solved by transforming the derivatives with respect to z_q into those with respect to x, y and z, and using the technique of separation of variables. The solution of y component of the conjugate wave is

$$A_{cy}^{+}=\frac{-\mathrm{i}3\kappa}{\gamma\cos\theta}A_{py0}^{-}\left(1-\frac{\delta}{\omega}\right)\sinh(\gamma z)$$
$$\times\exp\left[-\mathrm{i}\left\{\frac{9\kappa}{2}\cos\theta+12\kappa\frac{\delta}{\omega}\cos\theta+\frac{\Delta k_z}{2}\right\}z\right], \tag{40}$$

where

$$\gamma \triangleq (2\cos\theta)^{-1}\sqrt{36\kappa^2-[9\kappa\cos^2\theta-12\kappa\{1+(\delta/\omega)\sin^2\theta\}+\Delta k_z\cos\theta]^2} \tag{41}$$

and

$$\kappa \triangleq \frac{\chi\omega}{4c_0}A_{10}{}^{+}A_{10}{}^{-}=\frac{\chi\omega}{4c_0}A_{20}{}^{+}A_{20}{}^{-}. \tag{42}$$

The boundary conditions used for the y component are:

$$A_{cy}{}^{+}(z=0)=0 \quad \text{and} \quad A_{py}{}^{+}(z=0)=A_{py0}{}^{+}. \tag{43}$$

The solution of the z component of the conjugate wave is:

$$A_{cz}{}^{+}=\frac{-i\kappa}{\nu\cos\theta}A_{pz0}{}^{-}(1-\frac{\delta}{\omega})\ \sinh(\nu z)$$
$$\times\exp[-i\{(\frac{9}{2}-2\frac{\delta}{\omega})\kappa\cos\theta+\frac{\Delta k_z}{2}\}z], \tag{44}$$

where

$$\nu \triangleq (2\cos\theta)^{-1}\sqrt{4\kappa^2-[9\kappa\cos^2\theta-4\kappa\{1+(\delta/\omega)\sin^2\theta\}+\Delta k_z\cos\theta]^2}\ . \tag{45}$$

The boundary conditions used for the z component are:

$$A_{cz}{}^{+}(z=0)=0 \quad \text{and} \quad A_{pz}{}^{+}(z=0)=A_{pz0}{}^{+}. \tag{46}$$

We note that the slowly varying amplitudes $A_{cy}{}^{+}$ and $A_{cz}{}^{+}$ do not depend on y and z because of the plane wave input (eqs. (43) and (46)).

6. Increasing/decreasing properties of the solutions

γ and ν could be either real or imaginary, which govern increasing/decreasing property of the solutions. For example, when γ is real $A_{cy}{}^{+}$ increases/decreases monotonously, but when it is imaginary $A_{cy}{}^{+}$ is a periodic function of z. We consider this in the case

$$|\delta/\omega|<<1, \tag{47}$$

using a new parameter ρ, which is the ratio of detuning δ and coupling coefficient κ:

$$\rho \triangleq (2/c_0)(\delta/\kappa). \tag{48}$$

Now, only two parameters, ρ and θ, determine whether γ (or ν) is real or imaginary and ρ-θ plane can be divided into three regions which indicate the increasing/decreasing properties of the solutions (Fig. 2).

In region I, γ and ν are both imaginary, which correspond that $A_{cy}{}^{+}$ and $A_{cz}{}^{+}$ are sine functions of z. In region II, γ is imaginary and ν is real which correspond that $A_{cy}{}^{+}$ is the sine function of z and $A_{cz}{}^{+}$ is hyperbolic function of z. In region III, γ is real and ν is imaginary which correspond to that $A_{cy}{}^{+}$ is hyperbolic function of z and $A_{cz}{}^{+}$ is sine function of z.

In general, γ and ν are not equal to each other in magnitude and there is another factor

"3" in the y component and not in the z component. The polarization direction of $A_c{}^+$ which consists of y and z components is gradually changing with increasing interaction length in the case B. Even when γ and ν are simultaneously equal to zero that occurs at

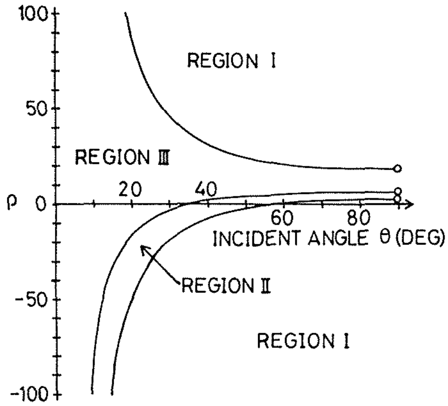

Fig. 2 Regions where γ and ν are real or imaginary, which indicate increasing/decreasing properties of the solutions.
Region I : γ and ν are both imaginary,
Region II : γ is imaginary and ν is real, and
Region III : γ is real and ν is imaginary.

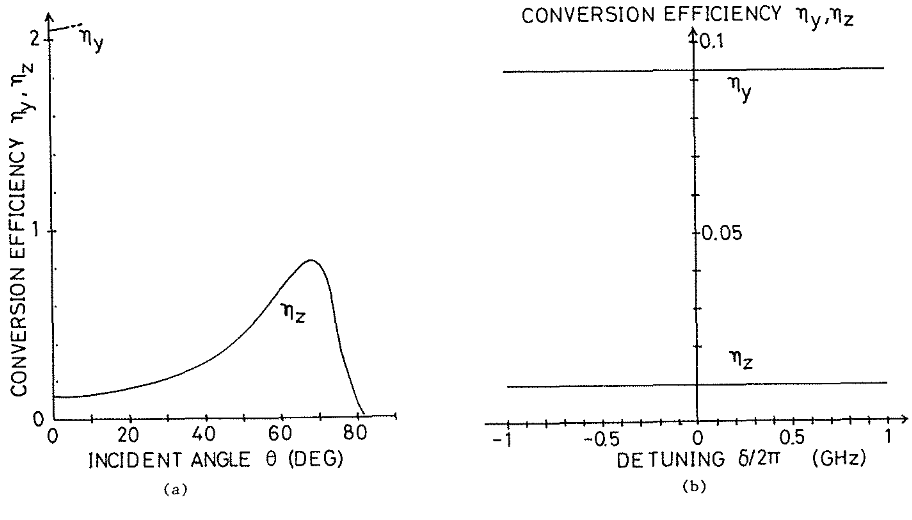

Fig. 3 Examples of conversion efficiency η_y and η_z.
(a) Conversion efficiency η_y and η_z vs. incident angle θ in the case $\lambda=0.85\,\mu$m, $|\kappa L|=0.4$ and $\delta=2\pi\times0.5\times10^9$ rad/s (0.5GHz).
(b) Conversion efficiency η_y and η_z vs. detuning δ in the case $\lambda=0.85\,\mu$m, $|\kappa L|=0.1$ and $\theta=5$ degree.

$$\rho=(6-9\cos^2\theta)/\sin^2\theta, \tag{49}$$

the polarization direction is still changing with z.

In the small coupling coefficient limit ($\kappa\to0$) γ and ν reduce to $i|\Delta k_z/2|$, so that the solution depends on the phase mismatching Δk_z directly. In contrast, in the large coupling coefficient limit ($\kappa\to\infty$ or $\rho\to0$) γ and ν do not depend on the phase mismatching Δk_z.

From the solutions (40) and (44), the conversion efficiencies can be obtained as

$$\eta_y=\left|\frac{3\kappa}{\gamma\cos\theta}(1-\frac{\delta}{\omega})\frac{\exp(\gamma L)-\exp(-\gamma L)}{2}\right|^2 \tag{50}$$

and

$$\eta_z=\left|\frac{\kappa}{\nu\cos\theta}(1-\frac{\delta}{\omega})\frac{\exp(\nu L)-\exp(-\nu L)}{2}\right|^2. \tag{51}$$

Another important feature is that there are nulls in η vs. θ curves when γ (or ν) is a pure imaginary number, because the solution (40) (or (44)) becomes sinc function of γz (or νz).

Some of the numerical calculations are shown in Fig. 3. In these figures η_y is omitted unless $\eta_y/\eta_z\fallingdotseq1$ or $\theta\fallingdotseq0$ where we can not describe the waves under the assumption of constant propagation direction. However, η_z is shown since it is valid in the case A. η_z (and the case A) shows a gentle angle dependence and wide frequency bandwidth but the magnitude of η_z is smaller than that of η_y since the factors in the coupled wave equations in the case of perpendicular polarization are smaller than those in the case of parallel polarization.

7. Conclusions

We have obtained nonlinear polarization and coupled wave equations to analyze the properties of forward phase conjugation by nearly degenerate four wave mixing. Some properties, such as polarization dependence, incident angle dependence, detuning dependence have been discussed. The results show that there are gentle angle dependence and wide frequency bandwidth.

It requires further analysis for the case B because it is found that y and z components show a different behavior and the analysis above is insufficient for describing the case where the polarization direction is changing with the increasing interaction length.

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