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An Uniform Flow Formula for the Partially Full Flow in a Circular Pipe

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Abstract

Sanitary sewage and storm water collection systems are designed and analyzed on the basis of the relation between the water depth and the mean flow velocity of a circular pipe. It is well known that this relation calculated by the Manning formula does not agree with the actual relation. Thus, a new uniform flow formula was developed on the basis of the Ven Te Chow's experimental results.

The new formula has the following advantages over the Manning formula:

(a) it provides a better expression of the flow characteristics in a circular pipe than the Manning formula, and

(b) the applicable pipe roughness range of this formula is wider than the Manning formula.

1. Introduction

Circular pipes are widely used for sanitary sewage and storm water collection systems. Since the flow implies that the pipe is partially full, these systems are designed and analyzed on the basis of the relation between the water depth and the mean velocity (this relation is called the flow characteristics [Ven Te Chow, 1959] and written as FC in this paper) in a circular pipe. This relation has been calculated usually by the Manning formula with a constant roughness coefficient. Manning formula is expressed as follows:

$$V=\frac{1}{n}R^{2/3}I^{1/2} \qquad (1)$$

where V is the mean velocity, n is the Manning roughness coefficient, R is the hydraulic radius and I is the slope of the water surface. But it is well-known that the calculated FC by the Manning formula does not agree with the actual FC as shown in Fig.1. In Fig.1 the actual FC was given by Camp [1946] and V is the mean velocity at a partially full flow of the water depth d and V_0 is the mean velocity at full flow. As to this

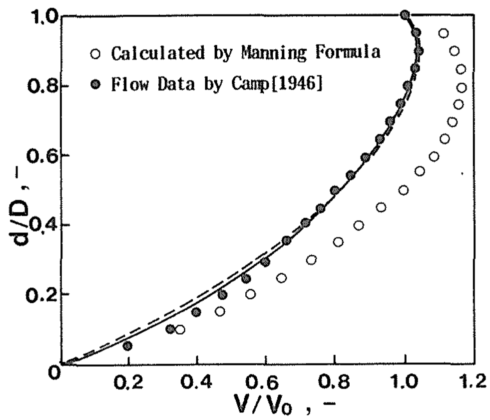

Fig. 1 Flow characteristics of a circular pipe.

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discrepancy one explanation proposed so far is that the Manning roughness coefficient n does not remain constant as the water depth changes [Ven Te Chow, 1959]. There may be another explanation as follows: the representative length of the flow area such as the hydraulic radius R is not adequate for the partially full flow in the closed top channel such as a circular pipe.

In this paper a modified hydraulic radius R' is proposed and a new type of uniform flow formula is developed to predict a more accurate FC.

2. Derivation of A New Uniform Flow Formula

2.1 Assumptions on the Velocity Distribution

Ven Te Chow [1966] measured the velocity distribution of a patially full flow in a circular steel pipe. The diameter D was 0.13 m, the surface roughness was 4.7×10^{-5} m and the slope of water surface was $I=0.002$ in his experiments. He drew the equi-velocity curves in the case where water depths d were $0.33D$, $0.5D$ and $0.68D$ as shown in Fig. 2. He also calculated the shear stress distribution at the pipe wall and showed that the shear stress was uniform except near the water surface. Figure 3 shows the radial velocity distributions drawn by the authors using the experimental results in Fig. 2. From Fig. 3 it is seen that there is no substantial difference between distributions (1) and (2) in Fig. 3 (a), among distributions (3), (4) and (5) in Fig. 3 (b) and among (6), (7) and (8) in Fig. 3 (c).

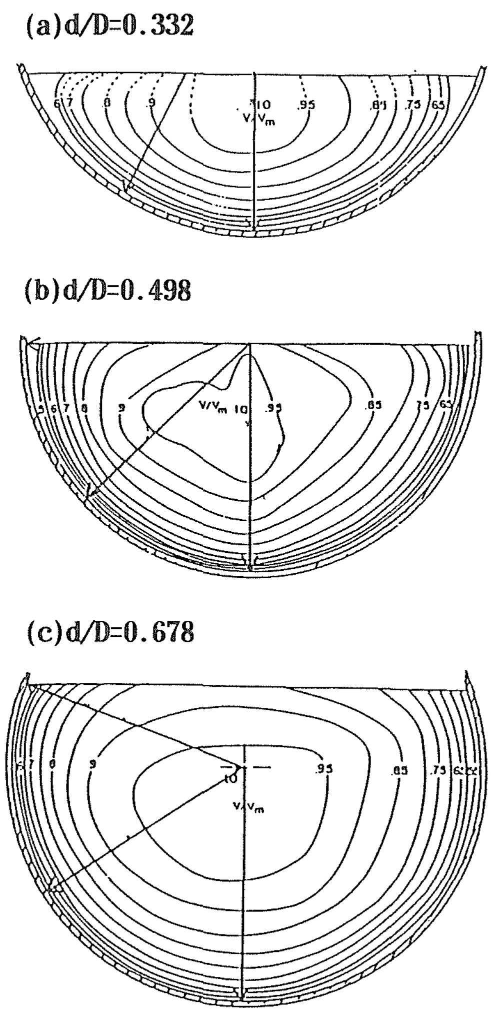

Fig. 2 Equi-velocity curves of the open channel flow in a circular pipe [Ven Te Chow 1966].

Thus in the present work the following assumptions are used for the derivation of a uniform flow formula:

(1) the shape of the equi-velocity curve is a concentric circle, and

(2) the radial velocity distribution can be expressed by the log law for rough surface, namely,

$$\frac{u(y)}{U_*}=8.5+5.75\,log\,\frac{y}{k} \quad (2)$$

where U_* is the shear velocity, y is the distance from pipe wall, $u(y)$ is the velocity at the point y and k is the surface

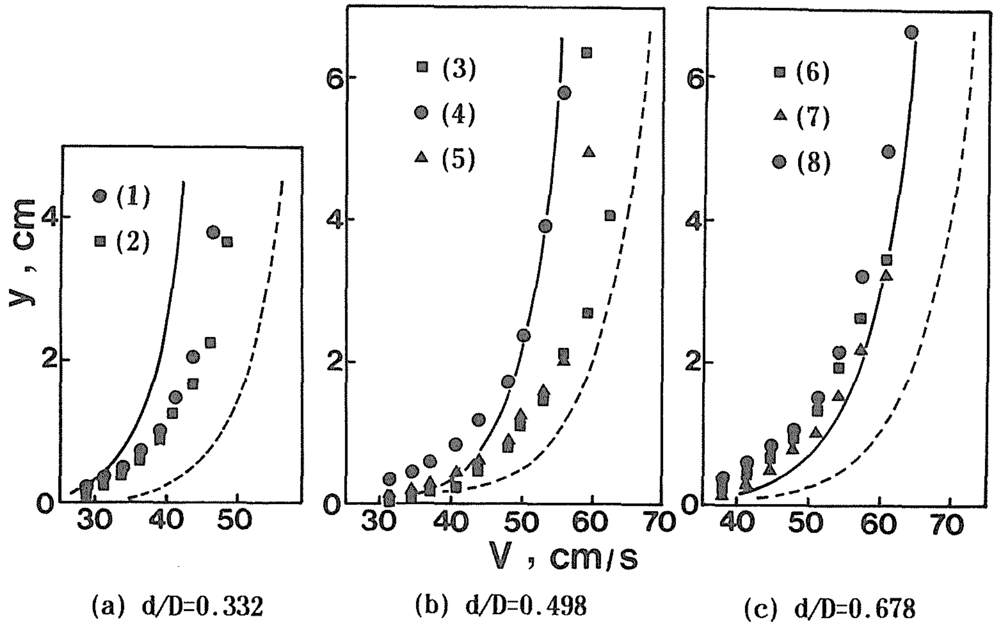

Fig. 3 The radial velocity distribution drawn from the data of Fig. 2. The parenthesized numbers refer to the radial directions drawn in Fig. 2.

roughness.

Integration of Eq.(2) over the flow area leads to the uniform flow formula:

$$V = C \cdot U_* \tag{3}$$

where,

$$C = \frac{1}{A}\int \left[8.5 + 5.75 \log\left(\frac{y}{k}\right)\right] \mathrm{d}A \tag{4}$$

and A is the cross-sectional area of the flow. If the two parameters C and U_* in Eq.(3) are expressed as a function of diameter D, water depth d, surface roughness k and water surface slope I, the mean velocity V can be calculated by Eq.(3).

2.2 Modified Hydraulic Radius R'

The shear velocity U_* of the uniform flow in an open channel is expressed as

$$U_* = \sqrt{gRI} \tag{5}$$

and the hydraulic radius R is

$$R = A/P \tag{6}$$

where P is the wetted perimeter. Radial velocity distributions of the flow in the Ven Te Chow's experiments can be calculated by use of Eqs.(2), (5) and (6) and calculated results are shown in Fig. 3 by the broken lines. It is seen from Fig. 3 that the calculated results (broken lines) are greater than the experimental ones. That is, R value calculated by Eq.(6) may be inadequate for circular pipes.

Since the maximum velocity point is lower than the water surface, it can be assumed that the hydralic radius must express the shear effects of not only the wall but also the water surface. The modified hydraulic radius R' could be in the following form:

$$R' = \frac{A}{P + wT} \quad (7)$$

where T is the width of water surface and w is a parameter. Parameter w may depend on the water depth d and the diameter D. The value of w was determined by the experimental results as follows.

At each d/D value, w was calculated by use of V/V_0 value of Camp's FC and Eqs.(2)-(5) and (7). The calculated results are shown in Fig. 4 and the following relation was obtained by taking into account the usual design water depth of $d/D=0.8$.

$$w = 0.412(d/D)^{-0.86} \quad (8)$$

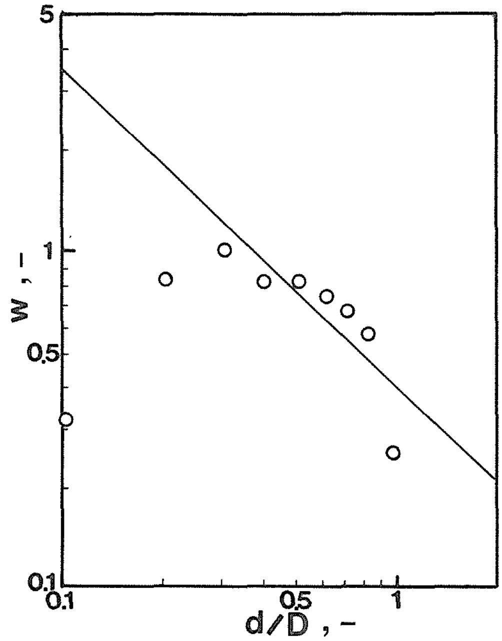

Fig. 4 The relation between w and d/D.

Computational results of the radial velocity distrubutions by Eqs.(7) and (8) are shown in Fig. 3 by the solid lines. The lines in Fig. 3 show a better fit to the experimental results than the broken lines obtained by Eq.(6). In Fig. 1 the broken line is the calculated result of the FC by Eq.(2)-(5), (7) and (8) and shows good agreement with the measured FC.

2.3 Coefficient C in Eq.(3)

In this section the expression of C will be discussed by the numerical calculations. Figure 5 is the graph of C against d/D at various values of the relative roughness k/D. It is seen from Fig. 5 that the value of C changes with d/D. It is desired that the coefficient C in the uniform flow formula does not depend on the water depth but only on the pipe wall roughness. So, the error due to ignoring the water depth dependency of C was examined. The computed FC with the value of C at $d/D=1$ is shown in Fig. 1 by a solid line. The broken line in Fig. 1 is the calculated result of the FC by accounting for the water depth dependency of the C. It is seen from Fig. 1 that both of these two curves provide an equally good fit to the measured FC and it can be assumed that the coefficient C in Eq.(3) depends only on the relative roughness k/D and that the value of C is equal to the value at $d/D=1$, namely, at the full flow condition.

At the full flow condition the modified hydraulic radius R' is $D/4$ and Eq.(3) is transformed by use of Eq.(5) as follows:

$$I = \frac{8}{C^2} \cdot \frac{1}{D} \cdot \frac{V^2}{2g} \quad (9)$$

Comparison between Eq.(9) and Darcy-Weisbach equation gives the relation of C to the friction factor f as follows:

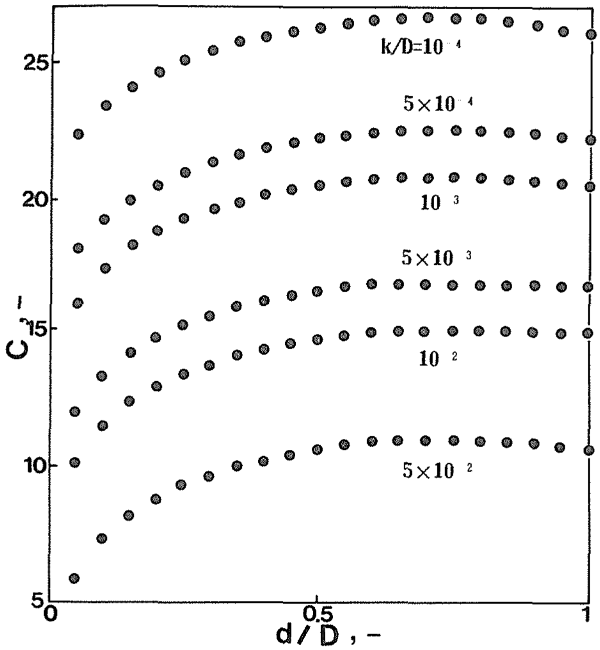

Fig. 5 Effects of d/D on C.

$$C=\sqrt{8/f} \tag{10}$$

Since it has been assumed that the radial velocity distribution is subject to the log law for a rough surface, f can be expressed by the following well-known equation at the full flow condition:

$$\frac{1}{\sqrt{f}}=1.14-2.0\log(k/D) \tag{11}$$

and

$$C=3.224-5.657\log(K/D). \tag{12}$$

3. Discussion

3.1 The Applicability of the New Formula to the Sewer Pipes at a Full Flow Condition

Since the FC calculated by the new uniform flow formula shows good agreement with the measured FC, that is, the value of V/V_0 coincides with the measured one at a partially full condition, here the value of V_0, the mean velocity at the full flow condition, is examined by use of the value of friction factor f.

In the calculation of the friction factor, the flow types are classified into three groups; (a)the rough surface flow, (b)the smooth surface flow and (c)the intermediate flow. It is evident that the applicable range of the new formula is the rough surface flow because the formula has been developed on the basis of the log law velocity distribution of the rough surface flow. The applicability to the intermediate flow range can be checked by comparing the f value by Eq.(11) with the value of f_c for the intermediate range calculated by the Colebrook-White equation:

$$\frac{1}{\sqrt{f_c}} = 1.14 - 2.0 \log\left(\frac{k}{D} + \frac{9.35}{Re\sqrt{f_c}}\right) \quad (13)$$

In the calculation of f and f_c the range of diameter is 0.2 m−5.0m, the mean velocities are 1.0 m/s and 2.0 m/s and the water temperature is 20°C. These conditions are normal in sewer pipe systems. Calculated results are shown in Fig. 6. It is seen from Fig. 6 that the new formula is applicable to the pipes having the roughness of more than 3.0×10^{-4}m, if the acceptable error limit is 5%. Since the roughness of a new concrete pipe is about 3.0×10^{-4} m, it can be said that the new formula is applicable to the whole sewer pipe system.

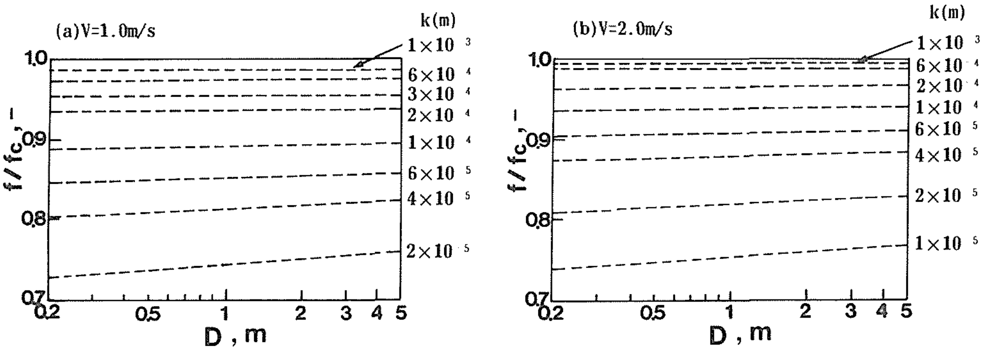

Fig. 6 Comparison between friction factors of Eqs. (11) and (13).

3.2 Comparison of the New Formula with the Manning Formula

In order to compare the new uniform flow formula with the Manning formula the relation between the Manning roughness coefficient n and the surface roughness k is required. To obtain this relation the Manning formula(Eq.(1)) is transformed as follows:

$$I = n^2(D/4)^{-4/3}V^2 \quad (14)$$

Substitution of Eq.(14) into the Darcy-Weisbach equation leads to the friction factor f_M as follows:

$$f_M = 8n^2g/D^{1/3} \quad (15)$$

Since the friction factor f_M has no dimension and n depends only on the roughness k, n must be expressed as

$$n = \alpha k^{1/6}/\sqrt{g} \quad (16)$$

and Eq.(15) is rewritten as

$$f_M = 8\alpha^2(k/D)^{1/3} \quad (17)$$

where α is a constant. The value of α can be calculated by equating Eq.(11) and Eq.(17) and the results are shown in Fig. 7. Figure 7 shows the k/D dependency of α and indicates that the Manning roughness coefficient n does not remain constant over a wide range of diameter even if the surface roghness k is constant. In examining the applicability of the Manning formula to the sewer pipe system, the mean value of α was used. The mean value was calculated at each k over a diameter range of 0.2 m−5.0 m. Figure 8 shows the relation between the mean α and k. The ratios of f_M/f_c computed by Eq.(13) and (17) are shown in

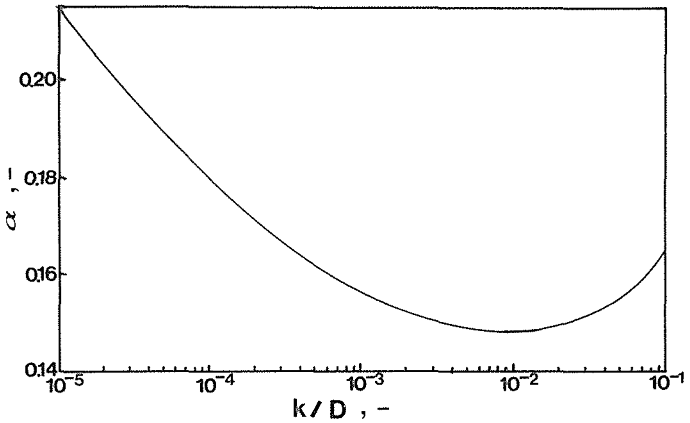

Fig. 7 Variations of α with k/D.

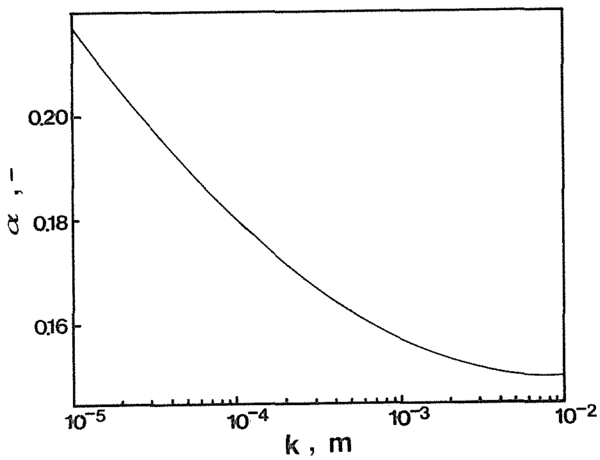

Fig. 8 The mean values of α used for the friction factor calculation.

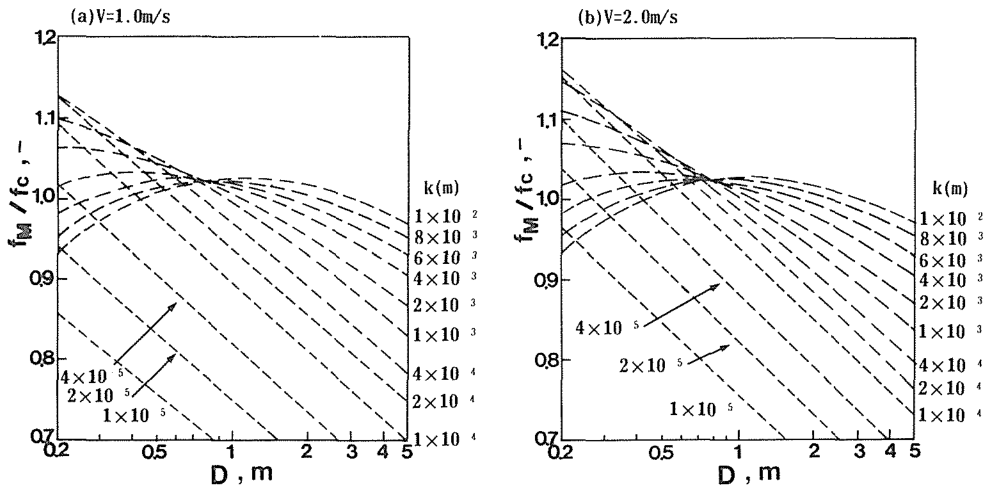

Fig. 9 Comparison between friction factors of Eqs. (13) and (17).

Fig. 9. It may be seen from Fig. 9 that the Manning formula is applicable to the pipes having a roughness of more than 6.0×10^{-3}m. Comparison between Fig. 6 and Fig. 8 shows that the new formula has a wider pipe roughness range than the Manning formula.

4. Conclusions

A new uniform flow formula:

$$V=C\sqrt{gR'I} \tag{18}$$

was developed for the partially full flow in the circular pipe. C is expressed by Eq.(12) and depends on the pipe roughness k and diameter D. R' is the modified hydraulic radius proposed by the authors and expressed by Eq.(7). R' is a function of water depth d and pipe diameter D. This formula has the following advantages over the Manning formula:

(a) it provides better expression of the flow characteristics in a circular pipe than the Manning formula, and

(b) the applicable pipe roughness range of this formula is wider than the Manning formula for the normal sewer system conditions, that is, the range of the mean velocity is 1−2 m/s and the range of the diameter is 0.2−5.0 m.

Acknowlegement

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References

T.R. Camp (1946), Sewage Works J., Vol. 18, pp. 1−16.
V.T. Chow (1959), Open-channel Hydraulics, International student edition, p. 135, McGraw-Hill
V.T. Chow and J.A. Replogle (1966), J. of Hydraulic Div., ASCE, Vol. 92, HY2, pp. 169−191.