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## Strength of Sand as Observed in a newly Developed Direct Shear Box Apparatus

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### Abstract

A direct shear box apparatus (DSBA) has been developed for examining the strength and dilatancy of sand. In this apparatus, the boundary stresses on the bottom and side of the shear box as well as strains of the rectangular-shaped specimen inside the rigid box can be observed throughout testing. This paper first describes features and functions of the DSBA. Second, the results of some pilot tests performed on dry specimens of two kinds of clean sands are presented by which the effects of some factors such as friction between the side wall and the specimen, the size of opening between the upper and lower shear boxes, the specimen height etc. are examined with respect to the peak strength. It is demonstrated that the peak strength of the sands as sheared under a constant vertical stress apparently increased as the size of opening between the rigid boxes, maintained at a constant value in each test, decreased towards zero. The size of opening being equal to, or slightly larger than the width of shear band, which is approximately twenty times the mean diameter of the particles, is recommended to measure the strength free from the boundary constraints. In conclusion, an optimal configuration of the DSBA is proposed so as to yield the strength associated with simple shear conditions.

### 1. Introduction

The direct shear box (DSB) test has been employed most commonly in practice for directly determining a drained Coulomb's strength envelope ( $\tau_f = c' + \sigma'_n \tan \phi_{as}$ ) for use in stability analysis using limit equilibrium methods.

As shown in Fig. 1, interpretation of the DSB test postulates the conditions of 'simple shear' as a reference state, which applies to the element A-E-H-D. The horizontal planes are considered inextensible since the rigid lower and upper halves of the box (i.e., A-B-C-D and E-F-G-H) prevent the development of linear strain in the x-direction (i.e.,  $\epsilon_x = 0$ ). Even though the thickness of the shear zone 't' is unknown, the relation of stress ratio versus

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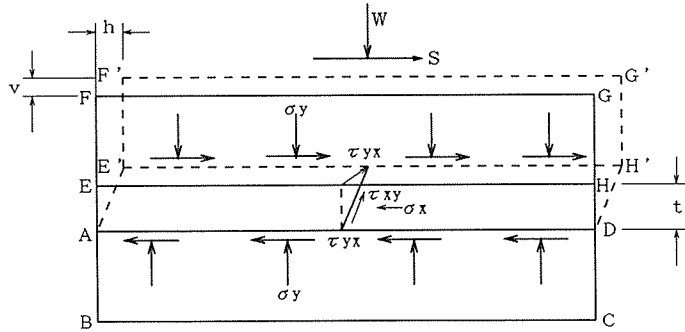


Fig. 1 Mode of simple shear in direct shear box (DSB) test

dilatancy rate in the shear zone (i.e.,  $(\tau_{xy}/\sigma_y)$  versus  $(\delta v/\delta h)$  relation;  $\delta v$ ,  $\delta h$ : the increments of the vertical and horizontal displacements) can be directly measured from the pre-failure to the residual state. Despite criticisms on the non-uniformities in stresses and strains, a recent investigation by means of elasto-plastic FE analysis suggests that the behaviour of the element A-E-H-D may well be interpreted as an ideal simple shear (Potts et al, 1987)<sup>1)</sup>.

However, quite a few unknown effects associated with the conventional DSB test should be properly understood before reaching such a conclusion. These are; a) Is the friction between the shear box and the specimen negligible in the assessment of the mean vertical stress?, b) Does the size of opening between the upper and lower shear box influence the strength of granular materials? (c.f., It is well known that the angle of shearing resistance of coarse gravels when sheared in DSB apparatus exhibits a value far larger than that in the relevant triaxial tests), c) What is the effect of the shape (circular or rectangular) and the size (e.g., the diameter (or length) and its ratio to the height) of the specimen on the strength? and d) What is the effect of the rotation of loading platen? (c.f., refer a hot discussion between Arthur and Jewell in *Geotechnique* in 1987–1988). In this paper, attempts are made to answer some of these questions.

## 2. Testing Procedure in a newly Developed Direct Shear Box Apparatus (DSBA)

Figure 2 shows the configuration of the DSBA developed. Dry sand specimens were prepared by means of the air-pluviation method through multiple sieves (Miura and Toki, 1982<sup>2)</sup>). Very loose specimens were prepared by a similar method without sieves. Note that the preparation was carried out when the lower and upper shear boxes (#19 and #20) were set at the right-hand position in Fig. 2, and the opening between the boxes was maintained at a prescribed value,  $d$ , by thickness gauges (#22) inserted at the four corners. After completing the raining of the dry sand grains, the upper surface of the specimen was flattened, and the initial void ratio,  $e_0$ , was measured. The rectangular specimen had a square shaped cross-section of  $L=15\text{ cm}\times 15\text{ cm}$  with the maximum height of  $H=12\text{ cm}$ .

The shear boxes, together with the specimen, was carefully moved on the cross-roller

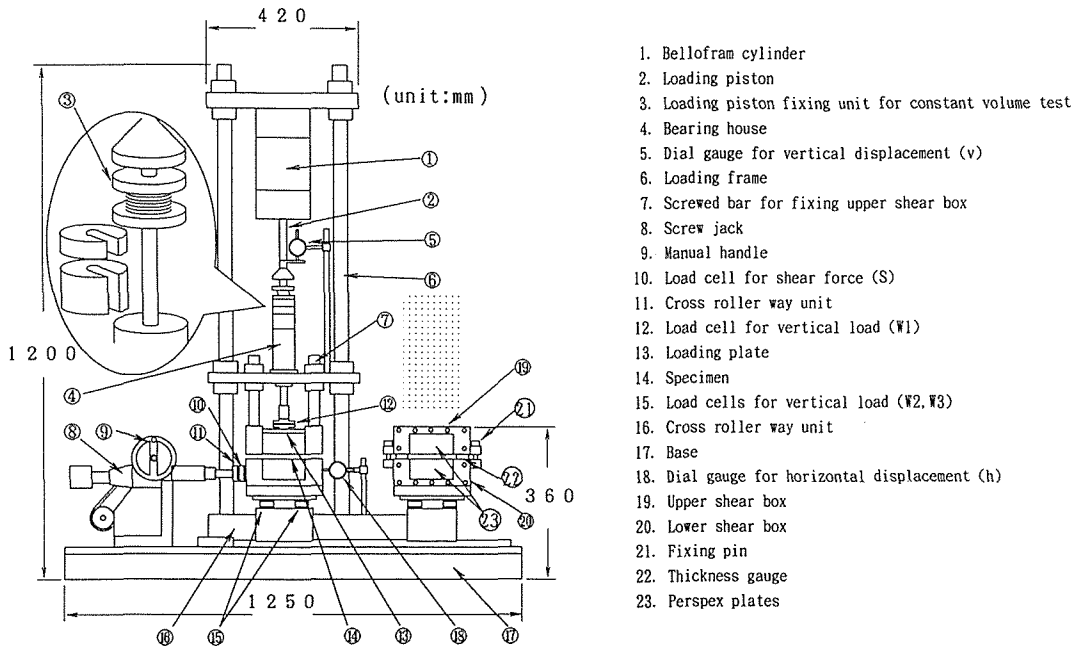


Fig. 2 A newly developed DSB apparatus

way (#16) towards the left-hand position in Fig. 2. The upper shear box was firmly screwed to the cross-head plate by the fixing bars (#7), and the thickness gauges were carefully removed. This configuration ensured the prescribed size of opening being maintained at a constant value throughout the subsequent testing. One-dimensional consolidation took place by gradually increasing the pneumatic pressure in the Bellofram cylinder (#1). After a period of half an hour on reaching the prescribed nominal consolidation pressure,  $\sigma_{vo}$ , the lower shear box was displaced in the horizontal direction at a constant rate of 0.25 mm/min, while the upper box was fixed. Note that the loading plate (#13) was fixed to the piston (#2) for all the tests presented in this paper. During shearing, the vertical and horizontal shear displacements,  $v$  and  $h$ , were measured by dial gauges #5 and #18, respectively.

Two kinds of tests can be performed in this DSBA; i.e., a constant-pressure test and a constant-volume test. The condition of a constant-volume was ensured by that, just before the commencement of shearing, the movement of the loading piston was mechanically constrained. This can be done by first filling the space using spacers and the fine screwed adjuster (#3), and then by applying the supplementary pressure in the Bellofram cylinder to prevent the upward movement in case of the dilation of the specimen. It was found that this technique caused very little change in the vertical stress. It should be mentioned that the system compliance was remarkably small, since high-rigidity load cells (#12, #15) were employed (n.b.; the compliance of each load cell was  $10\mu\text{m}$  for the maximum capacity of 250 kgf). Obviously, this method is much simpler, and more accurate, compared to the conventional technique in which the vertical displacement is controlled to be zero by adjusting the vertical pressure.

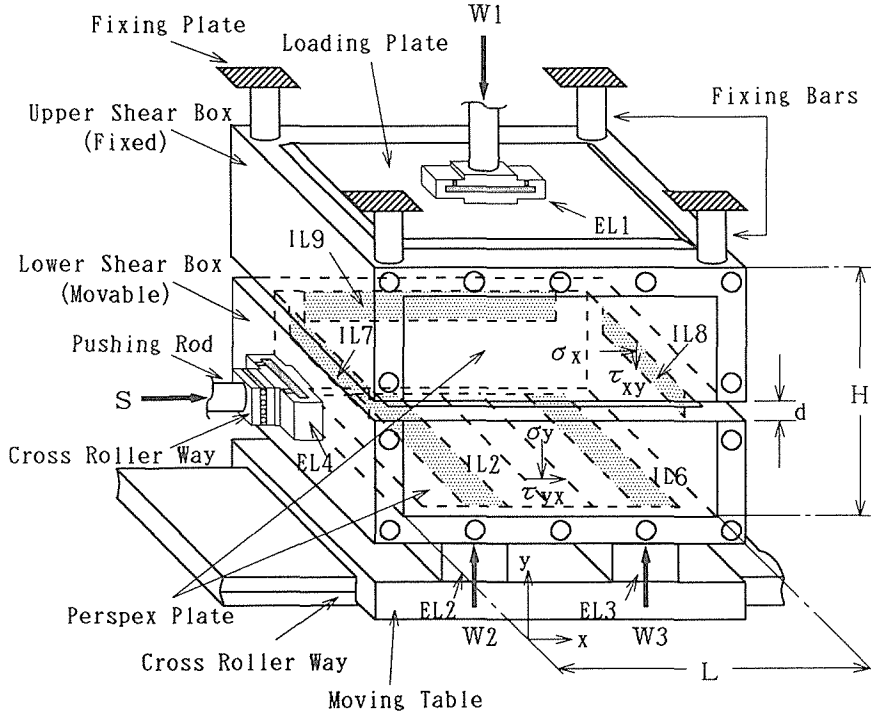


Fig. 3 Details of shear boxes

The image of shearing can be captured in schematic diagrams (Figs. 3 and 4). The mean normal stress on the horizontal shear plane (i.e., vertical stress,  $\sigma_v$ ) can be measured in the following;

$$(\sigma_v)_{upper} = W1/A \quad (1)$$

$$(\sigma_v)_{lower} = (W2 + W3)/A \quad (2)$$

where 'A' stands for cross section area of the specimen. The vertical loads, W1, W2 and W3 are measured using external load cells, EL1, EL2 and EL3, respectively. Similarly, the mean shear stress on the shear plane can be defined as;

$$\tau_h = S/A \quad (3)$$

in which 'S' is the shear force measured by EL4. In view of precisely measuring the vertical stress on the shear plane,  $(\sigma_v)_{upper}$  may involve error due to the friction between the shear box and the specimen. On the other hand, since the opening was kept larger than zero in all the tests, both of  $(\sigma_v)_{lower}$  and  $\tau_h$ , were free from any undesirable soil-to-metal or metal-to-metal frictions.

The boundary stresses in the lower shear box were measured by a total of nine two-way,

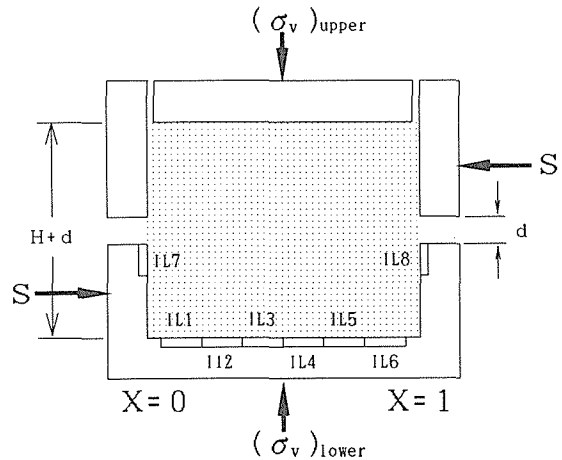


Fig. 4 A sketch of shearing

high-rigidity load cells of IL1 through IL9 (Fig. 3); i.e., the distribution of the vertical stress,  $(\sigma_v)_{\text{inner}}$ , together with the relevant shear stress, can be measured by IL1 through IL6, each of which is equipped with a plate of 2.16 cm in width and 13 cm in length. Similarly, the normal stress on the vertical plane,  $\sigma_h$ , is detected by IL7, 8 and 9, each having the dimension of 1 cm in width and 13 cm in length.

In most of tests, the side walls of the shear box were lubricated using latex membranes. In this case, markers are installed on the membranes in the front. Though the results are not presented in this paper, the local movement of sand inside the boxes can be known by tracing the movement of the markers.

Specimens of two kinds of clean sand; i.e., Toyoura sand ( $D_{50}=0.162$  mm,  $e_{\text{min}}=0.605$ ,  $e_{\text{max}}=0.977$ ) and Sohma sand ( $D_{50}=0.740$  mm,  $e_{\text{min}}=0.530$ ,  $e_{\text{max}}=0.797$ ), were used. The different values of  $e_0$  were obtained by changing the rate of discharge of the sand grains during pluviation.

### 3. Discussions

#### 3.1 $K_o$ -value during one-dimensional consolidation

The ratio of the normal stresses on the vertical plane to the horizontal plane is herein given by

$$K_o = \sigma_h / \sigma_v \quad (4)$$

Typical results of tests on Toyoura and Sohma sands are shown in Figs. 5 and 6, respectively, in which  $\sigma_h$  measured by IL7, 8 and 9 are plotted against  $(\sigma_v)_{\text{lower}}$  (Eq.2). Note that the  $K_o$  value of the dense Toyoura sand and medium-loose Sohma sand was approximately 0.3–0.4 and 0.45–0.5, respectively, which are rather familiar values for normally consolidated quartz sands. Furthermore, in each sand, the size of opening 'd' had little effect on the  $K_o$ .

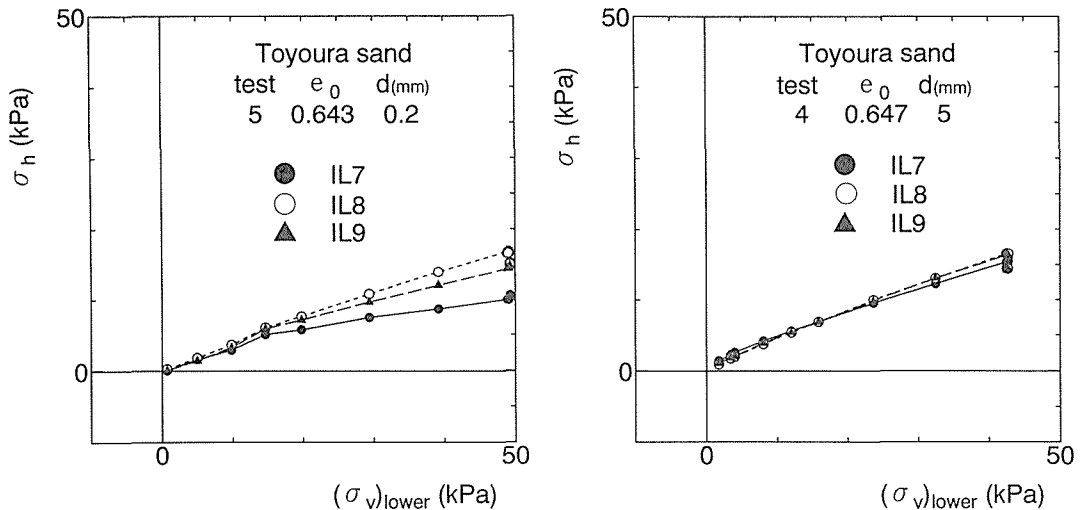


Fig. 5 Normal stresses on vertical and horizontal planes of Toyoura sand specimens during consolidation ( $d=0.2$  mm and  $d=5$  mm)

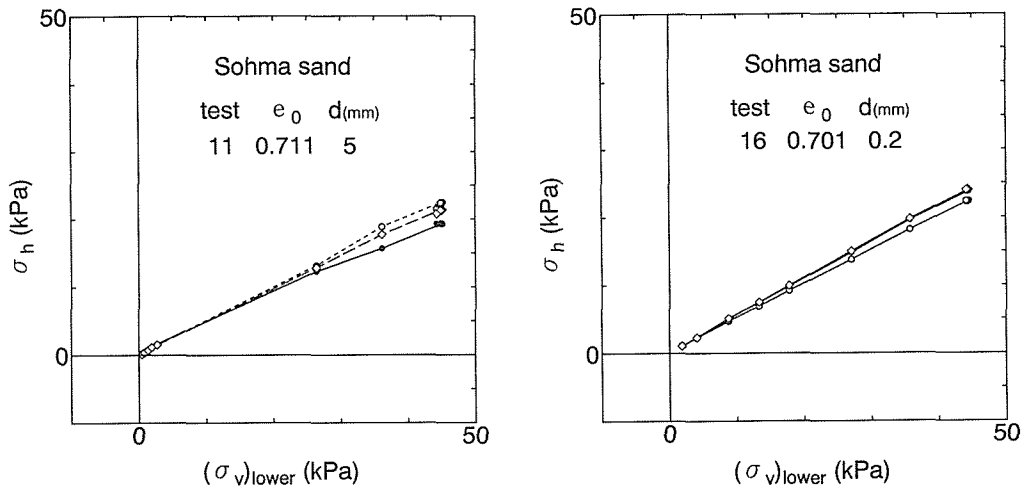


Fig. 6 Normal stresses on vertical and horizontal planes of Sohma sand specimens during consolidation ( $d=0.2$  mm and  $d=5$  mm)

-value. This indicates that the opening of  $d=5$  mm did not cause harmful non-uniformity of stresses even at the edge of the potential shear plane.

### 3.2 Effects of side friction between shear box and specimen

Figure 7 shows the variations of  $\tau_h/(\sigma_v)_{upper}$ ,  $\tau_h/(\sigma_v)_{lower}$  and  $\tau_h$  of test 3, in which a dense Toyoura sand specimen ( $e_0=0.642$ ) was sheared under a constant pressure of  $(\sigma_v)_{upper}=49$  kPa. It should be noted that the mobilization of the peaks amongst these virtually coincided at  $h$  equal to about 2.5 mm. Despite lubrication of the side walls of the shear boxes in this

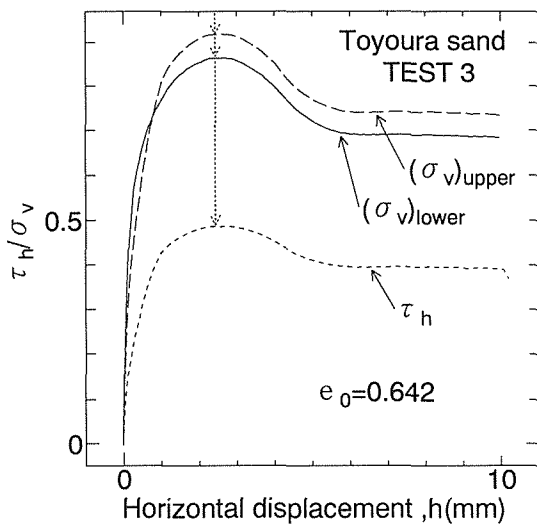


Fig. 7 Variations of stress ratio and shear stress (test 3, lubricated)

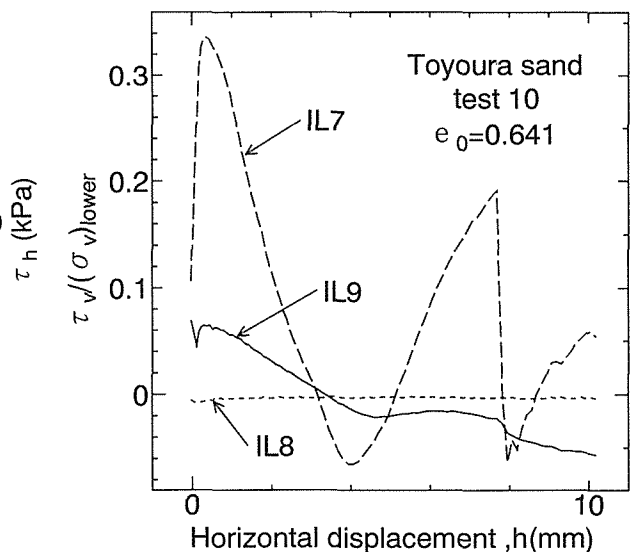


Fig. 8 Variation of shear stress on the vertical plane in the shear box (tests 10, non-lubricated)

test, the peak stress ratio of  $\tau_h/(\sigma_v)_{upper}$  as interpreted in the conventional test was larger than that of  $\tau_h/(\sigma_v)_{lower}$ . Figure 8 shows development of shear stress on the top portion of the lateral surface of the box  $\tau_v (= \tau_{xy}$ , see Fig. 3) in a comparative test 10. In this test, the inner walls of the boxes were not lubricated. It can be seen that a certain amount of shear stress developed at the specimen walls. The discrepancy of the peak strengths may be clearly seen in Fig.9, in which the peak stress ratios of tests performed at  $(\sigma_v)_{upper}=49$  kPa are plotted against  $e_0$ . The following may be noted;

- i) the angle of shearing resistance from  $(\sigma_v)_{upper}$  was higher by about 1-2 degrees than that from  $(\sigma_v)_{lower}$  for the range of  $e_0$  less than about 0.8, whereas the difference was negligible for the looser specimens, and
- ii) the difference was much more significant for test 10 performed under non-lubricated boundary conditions.

It may be mentioned that for a wider range of density, the peak stress ratio cannot be accurately evaluated on the basis of the conventional measurement of the vertical load (i.e.,  $(\sigma_v)_{upper}$ ), especially when the inner walls are not lubricated. In this particular DSBA, this is due to the friction between the upper shear box and the specimen that must be relatively larger for dilative (i.e., dense) specimens.

In this paper, the angle of shearing resistance is therefore given by the following equation, unless otherwise stated:

$$\phi_{ds} = \tan^{-1}(\tau_h/(\sigma_v)_{lower}) \quad (5)$$

Figure 10 shows the variations of  $\sigma_h$  in test 10. Note that  $\sigma_h$  measured by IL7 increased up to about twice than  $(\sigma_v)_{lower}$ , whereas  $\sigma_h$  at the opposite side abruptly reached to zero as soon as shearing took place (refer Fig. 3). This could be an indirect evidence that the sand grains inside the shear box, at least those nearby the shear plane, moved towards the

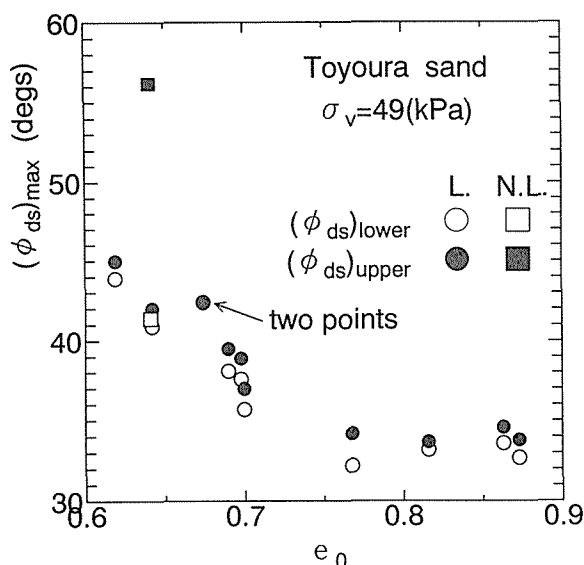


Fig. 9 Maximum angle of shearing resistance versus initial void ratio

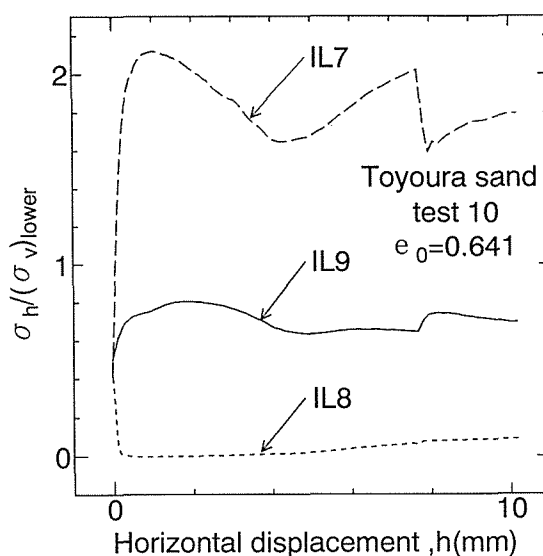


Fig. 10 Variation of normal stress on the vertical plane in the shear box (test 10)

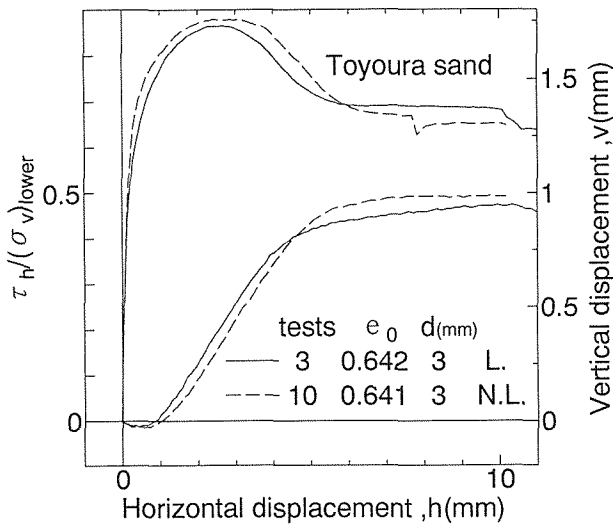


Fig. 11 Variations of stress ratio and vertical displacement in tests 3 and 10

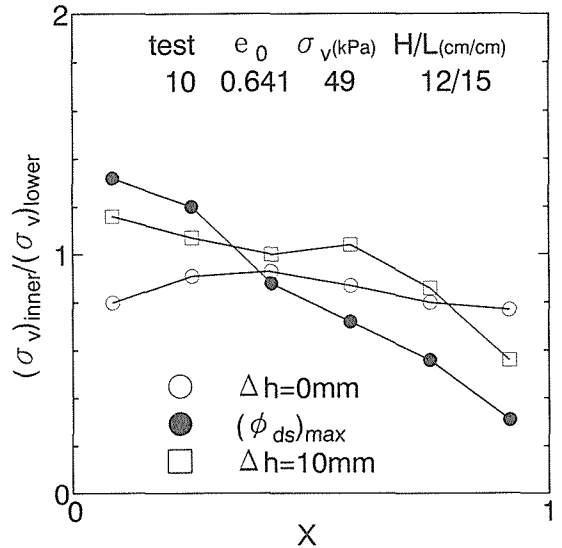


Fig. 12 Distribution of normal stress on the bottom of lower shear box in tests 3.

direction of the applied shear stress,  $\tau_h$ . Accordingly, the horizontal direction may not be the direction of zero-extension (refer Fig. 1). The tendency was more significant for tests having lubricated boundary (i.e., soft boundary) conditions.

The variations of  $\tau_h / (\sigma_v)_{lower}$  in tests 3 and 10 are compared in Fig. 11. Despite a marked difference in the peak strength defined by  $\tau_h / (\sigma_v)_{upper}$  between these tests, the lubrication showed a little influence on the aspect of mobilization of the stress ratio so long as it is examined using  $(\sigma_v)_{lower}$  (see also Fig. 9). In Figs. 12 and 14a, the distribution of  $(\sigma_v)_{inner}$  is examined at different stages of shearing (c.f., the definition of  $x$  in the horizontal axis may be referred to Fig. 4). It may be seen that the distribution of  $(\sigma_v)_{inner}$  at peak was more uniform over the central part in test 3 (i.e., lubricated specimen) than in test 10 (i.e., nonlubricated specimen). However, it should be pointed out that the difference has little to do with the overall relationship between the shear stress ratio and  $h$  (see Fig. 11). Accordingly, in most of tests, the specimen was sheared under lubricated boundary conditions so as to examine the strain distribution inside the shear boxes.

### 3.3 Effects of specimen height

The effects of specimen height are examined in comparative tests 3 and 56 performed on dense Toyoura sand. It is obvious in Fig. 13 that the responses of the mobilized stress ratio and dilatancy are virtually identical between the two specimens which have different heights of  $H = 12$  cm and 5 cm. However, the results shown in Fig. 14 indicate that the distribution of  $(\sigma_v)_{inner}$  is highly nonuniform for both the specimens, and that the aspect of the nonuniformity was quite different between the two. Again, as pointed out in relation to Fig. 12, the boundary stress distribution has little to do with what is going on in the shear zone. Therefore, it may be surmised that the stress/strain distributions inside the shear boxes are

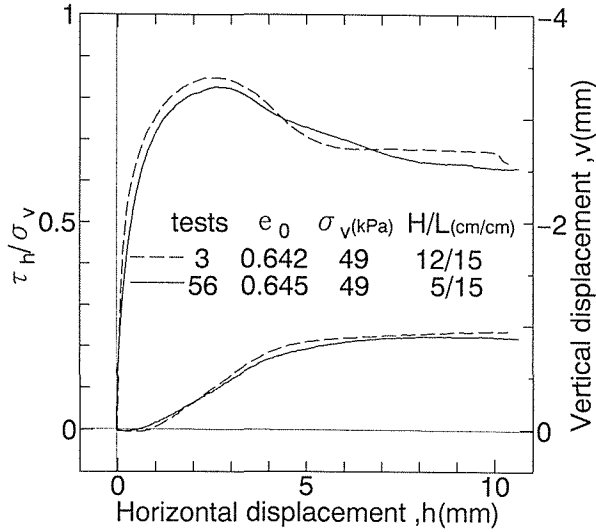


Fig. 13 Effects of specimen height in tests 3 and 56

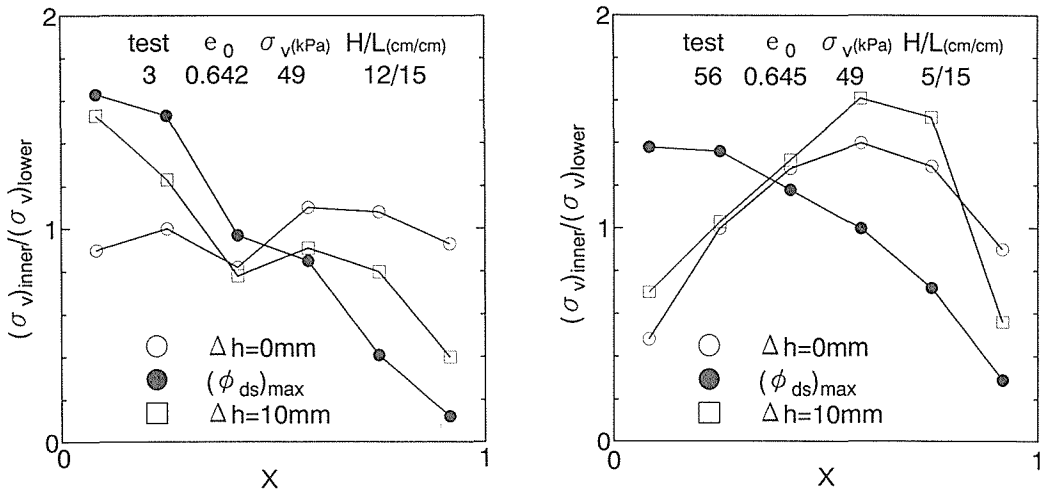


Fig. 14 Distribution of normal stress on the bottom of lower shear box in tests 3 (H=12 cm) and 56 (H=5 cm)

highly non-uniform.

### 3.4 Effects of the size of opening between upper and lower shear boxes

In Fig. 15, the relationship between  $\tan \phi_{ds}$  and  $v$  (n.b., compression is taken as positive) of dense specimens of Toyoura sand and medium-loose specimens of Sohma sand is examined with respect to the horizontal displacement,  $h$ . In each group of tests, the only difference to distinguish the three tests is the size of opening,  $d$ . In Fig. 16, the values of  $\tan \phi_{ds}$  at peak,  $\tan(\phi_{ds})_{max}$ , are plotted against 'd' for each group of tests. The effect of the size of opening was obvious in that  $\tan(\phi_{ds})_{max}$  decreased as the  $d$ -value increased. Note that, in the case of Toyoura sand having  $D_{50}=0.162$  mm, the value of  $\tan(\phi_{ds})_{max}$  reached a stable

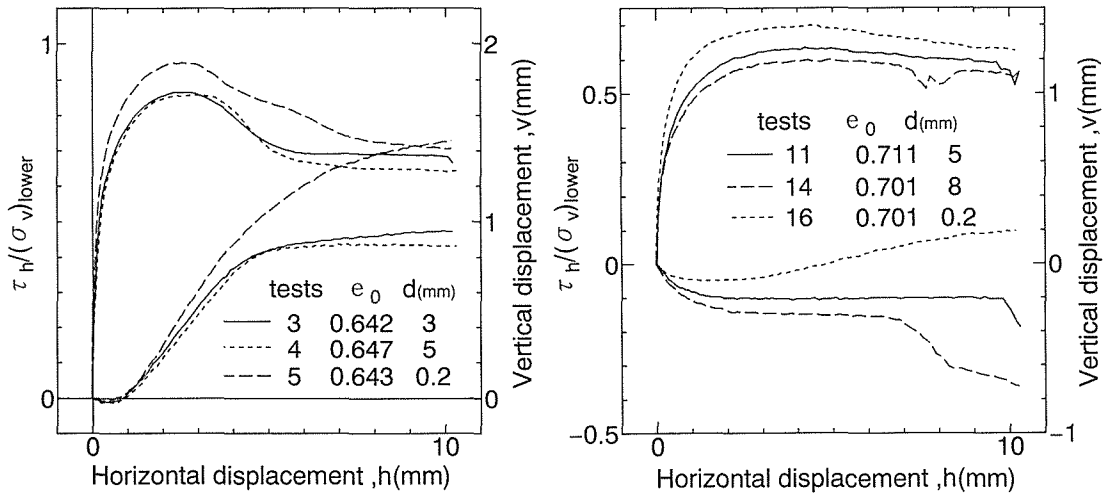


Fig. 15 Effect of size of opening between lower and upper shear boxes on the shear strength; a) Toyoura sand and b) Sohma sand

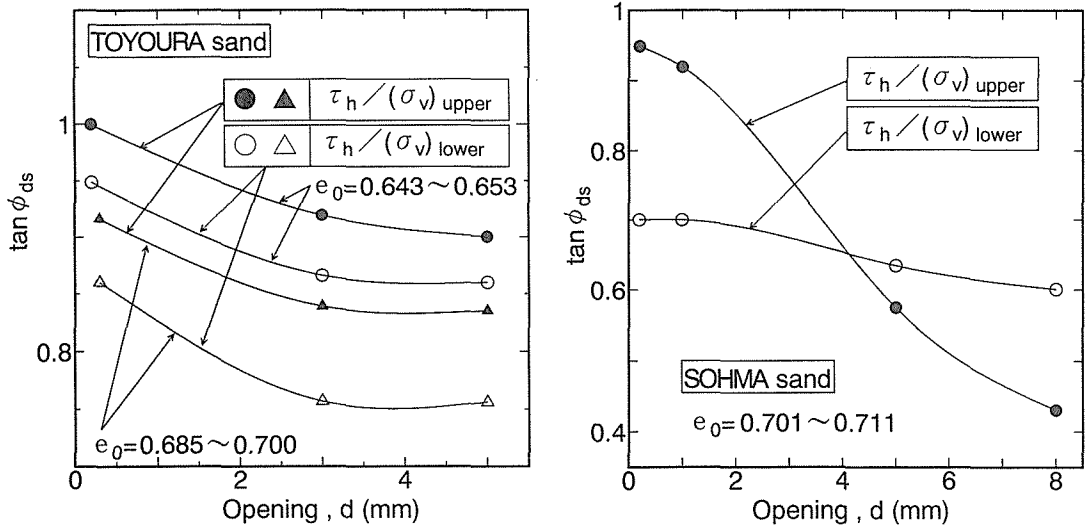


Fig. 16 Effect of size of opening between lower and upper shear boxes on strength; a) Toyoura sand and b) Sohma sand

value at  $d=3.0$  mm. In the case of Sohma sand with  $D_{50}=0.74$  mm, however,  $\tan(\phi_{ds})_{max}$  was still on the way to a continuous, but gradual, decrease even at  $d=8.0$  mm, above which no further increase of the opening was not attempted so as to avoid possible escape of the sand grains from the specimen edges as exclusively observed at  $h=7$  mm in test 14 (see Fig. 15b). The width of shear band,  $t$  (see Fig. 1), is known to be approximately twenty times of  $D_{50}$  (Tatsuoka et al, 1989<sup>3)</sup>), by which those of Toyoura and Sohma sands would be about 3 mm and 15 mm, respectively. Considering the results shown in Figs. 15 and 16, coupled with this observation, the condition of the  $d$ -value equal to, or slightly larger than, the width of shear band could be optimum in the DSB test so as to yield the strength free from the boundary

constraints.

Finally, it should be mentioned that the relationship between  $\tan(\phi_{ds})_{max}$  and the initial density from a series of the DSB tests on Toyoura sand was similar to that from the simple shear tests using a hollow cylindrical specimen.

### Conclusions

In the direct shear box test, the reference state of shearing is the condition of simple shear. To realize this particular mode of shearing, the direct shear box apparatus should satisfy, at least, the following configurations. The measurement of vertical load should exclude the effect of friction on the inner wall of the shear boxes, which was found rather significant even as the wall was well lubricated. Furthermore, the peak strength is affected by the size of opening between the upper and lower shear boxes,  $d$ . From a point of yielding the strength free from the boundary constraints, the optimum  $d$ -value may be equal to, or slightly larger than, the width of shear band which may be about twenty times of the mean diameter of the sand tested.

### Acknowledgement

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