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Simulation of Electromagnetic Fields by Generalized Spatial Network for Vector Potential

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Abstract

In the analysis of electromagnetic fields, the vector potential has important roles especially when sources exist and quantum effects are considered. I have recently proposed a new numerical method for unified analysis of the three-dimensional vector and scalar potential fields on the time domain. The method is based on the Spatial Network Method (SNM) for the Maxwell's equations, in which the use of both equivalent voltage and current variables have the capability of treating the boundary and medium conditions by the equivalent lumped elements at nodes. In the formulation of the vector potential fields, both the magnetic and the electric vector potentials are used to give the fundamental equations to be similar to the Maxwell's equation. Therefore, those advantages in SNM can be inherited. In this paper, this advantages are verified by showing the treatment of typical boundary conditions in a waveguide and the dispersive properties of resonance absorption. It is also shown that the correspondence of the continuity law for the equivalent currents to that for the electric or magnetic flux can simplify the expression of the medium conditions. This property indicates that the spatial network functions as a general network correspondent to each expression of electromagnetic fields.

1. Introduction

In the analysis of electromagnetic fields, the vector potential has important roles especially when current sources exist. But when true charges also exist, the combined analysis considering scalar fields becomes essential. Also, for the Schrödinger equation used in the analysis of quantum effect devices, the Hamiltonian which gives the effects of external electromagnetic fields is described by the vector and scalar potentials. Therefore, the analysis by the vector and scalar potentials can be directly combined with the analysis of those devices.

A new numerical method has been recently developed by the present author for vector analysis of three-dimensional electromagnetic fields on the time domain. This is called as the Spatial Network Method (SNM)[1]-[2]. The use of both voltage and current variables have the capability of applying the method to more general electromagnetic field problems

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including dispersive and anisotropic media[3]-[5]. It has been shown that the SNM is also applied to the vector potential fields with the Coulomb's gauge condition by defining a set of fundamental equations, which are similar to the Maxwell's equations, by using not only the magnetic vector potential but also the electric vector potential[6]-[8]. Still more, the application to the case with Lorentz gauge condition including loss term can be achieved[9]-[13]. This unified time-dependent formulation for both the vector and the scalar potentials is obtained by using the equivalent spatial networks for the scalar field, in which in addition to the conventional electric and magnetic field variables as the current variables, "F" and "F*" functions are newly defined as the voltage variables for electric and magnetic scalar fields, respectively, based on the analogy of the scalar potential field to the velocity potential field in the fluid dynamics. The fundamental characteristics of this formulation in a combined analysis including both the vector and scalar potential fields has been shown in the analysis of a semiconductor device[14] and the perfect diamagnetism[15].

The use of both the equivalent voltage and current variables in the spatial network realizes the treatment of the boundary and medium conditions by lumped elements at each node. The advantage of these treatments as above mentioned can be inherited in the spatial network for the vector potential. In this paper this advantage are verified by showing the treatments of typical boundary conditions in a waveguide partially filled with medium and the dispersive medium condition of resonance absorption[16].

2. Formulation of Vector Potential Field in Spatial Network

It is proved that in the spatial network shown in Fig. 1 the wave field for the vector potentials can be simulated with taking the gauge conditions into consideration. The characteristic equation by the magnetic vector potential "A" and the electric vector potential

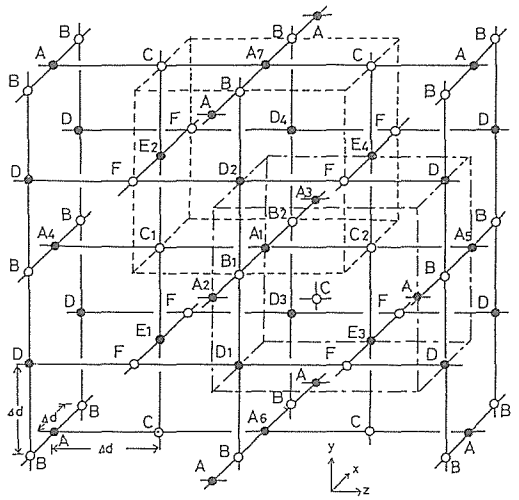

Fig. 1 Three-dimensional lattice network for vector potential field
● : Electric node
○ : Magnetic node
—— : 1-dimensional transmission line

"S" for general medium conditions can be defined as follows. In Table I, the correspodence between variables is shown. In Fig. 2, the fundamental connection in the network is illustrated in detail.

Electric Node			Magnetic Node		
	Constituent Equation	Variables		Constituent Equation	Variables
A_n	$\frac{\partial S_x}{\partial z}-\frac{\partial S_z}{\partial x}=-\varepsilon_*\frac{\partial A_x}{\partial t}$	$V_y=A_y$	F_n	$\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}=\mu_*\frac{\partial S_x}{\partial t}$	$V_y^*=-S_y$
	$-\frac{\partial A_x}{\partial z}=\mu_*\frac{\partial S_x}{\partial t}$	$I_z=S_x$		$-\frac{\partial S_x}{\partial z}=-\varepsilon_*\frac{\partial A_x}{\partial t}$	$I_z^*=A_x$
	$\frac{\partial A_x}{\partial x}=\mu_*\frac{\partial S_z}{\partial t}$	$I_x=-S_z$		$\frac{\partial S_x}{\partial x}=-\varepsilon_*\frac{\partial A_z}{\partial t}$	$I_x^*=-A_z$
D_n	$\frac{\partial S_z}{\partial y}-\frac{\partial S_y}{\partial z}=-\varepsilon_*\frac{\partial A_x}{\partial t}$	$V_x=A_x$	B_n	$\frac{\partial A_x}{\partial x}-\frac{\partial A_x}{\partial y}=\mu_*\frac{\partial S_z}{\partial t}$	$V_z^*=-S_z$
	$\frac{\partial A_x}{\partial z}=\mu_*\frac{\partial S_x}{\partial t}$	$I_z=-S_y$		$\frac{\partial S_z}{\partial y}=-\varepsilon_*\frac{\partial A_x}{\partial t}$	$I_y^*=-A_x$
	$-\frac{\partial A_x}{\partial y}=\mu_*\frac{\partial S_z}{\partial t}$	$I_y=S_z$		$-\frac{\partial S_z}{\partial x}=-\varepsilon_*\frac{\partial A_x}{\partial t}$	$I_x^*=A_y$
E_n	$\frac{\partial S_x}{\partial x}-\frac{\partial S_x}{\partial y}=-\varepsilon_*\frac{\partial A_z}{\partial t}$	$V_z=-A_z$	C_n	$\frac{\partial A_x}{\partial y}-\frac{\partial A_y}{\partial z}=\mu_*\frac{\partial S_x}{\partial t}$	$V_x^*=S_x$
	$\frac{\partial A_z}{\partial y}=\mu_*\frac{\partial S_x}{\partial t}$	$I_y=S_x$		$\frac{\partial S_x}{\partial z}=-\varepsilon_*\frac{\partial A_y}{\partial t}$	$I_z^*=A_y$
	$-\frac{\partial A_z}{\partial x}=\mu_*\frac{\partial S_x}{\partial t}$	$I_x=-S_y$		$-\frac{\partial S_x}{\partial y}=-\varepsilon_*\frac{\partial A_z}{\partial t}$	$I_y^*=-A_z$
Capacitance $C_0=\varepsilon_*/2$			Capacitance $C_0^*=\mu_*/2$		
Inductance $L_0=\mu_*/2$			Inductance $L_0^*=\varepsilon_*/2$		
Polarization $\Delta C=\varepsilon_*\chi_*/2\cdot\Delta d$			Magnetization $\Delta C^*=\mu_*\chi_m/2\cdot\Delta d$		
Conductivity $G=\sigma/2\cdot\Delta d$			Conductivity $G^*=\sigma^*/2\cdot\Delta d$ (For magnetic Current)		

Table Correspondence between vector potential variables and equivalent circuit variables

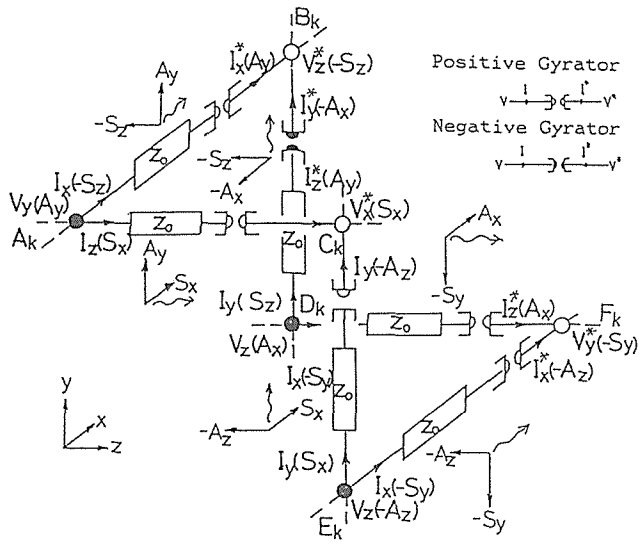

Fig. 2 Fundamental connection in the network

$$\nabla \times A = \sigma^* S + \mu \frac{\partial S}{\partial t} \qquad (\equiv B) \tag{1a}$$

$$\nabla \times S = -\sigma A - \varepsilon \frac{\partial A}{\partial t} \qquad (\equiv D) \tag{1b}$$

Here, σ^* is the conductivity for the hypothetical magnetic current. In each line connected to the node A_1 shown in Fig. 3, the central difference equations for the voltage drop are obtained as follows;

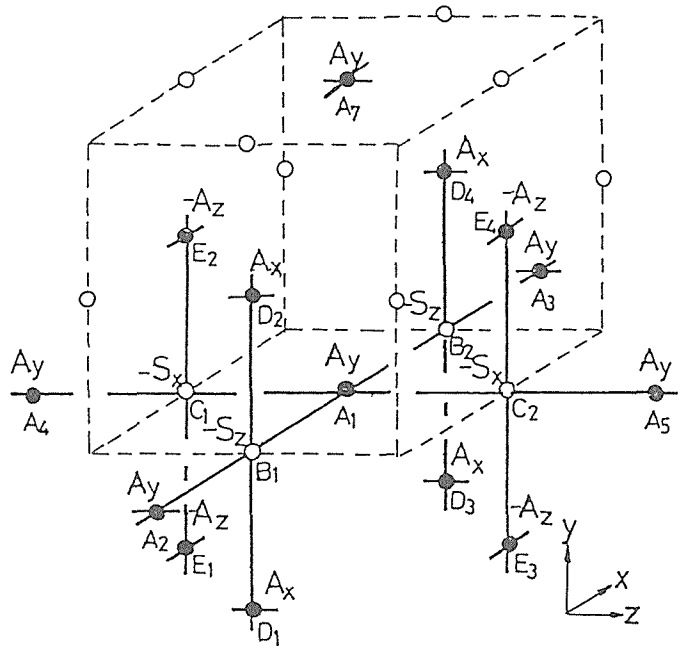

Fig. 3 Arrangement of nodes around node A_1

$$V_y(A_2) - V_y(A_1) = \{2\triangle x(\sigma^*/2) + 2\triangle_x(\mu_0\chi_m/2)\Delta_t + 2\triangle x(\mu_0/2)\Delta_t\}\ V_z^*(B_1) \tag{2a}$$

$$V_x(A_1) - V_y(A_3) = \{2\triangle x(\sigma^*/2) + 2\triangle_x(\mu_0\chi_m/2)\Delta_t + 2\triangle x(\mu_0/2)\Delta_t\}\ V_z^*(B_2) \tag{2b}$$

$$-\{V_x(D_1) - V_x(D_2)\} = \{2\triangle y(\sigma^*/2) + 2\triangle_y(\mu_0\chi_m/2)\Delta_t + 2\triangle y(\mu_0/2)\Delta_t\}\ V_z^*(B_1) \tag{2c}$$

$$-\{V_x(D_3) - V_x(D_4)\} = \{2\triangle y(\sigma^*/2) + 2\triangle_y(\mu_0\chi_m/2)\Delta_t + 2\triangle y(\mu_0/2)\Delta_t\}\ V_z^*(B_2) \tag{2d}$$

$$V_y(A_4) - V_y(A_1) = \{2\triangle z(\sigma^*/2) + 2\triangle_z(\mu_0\chi_m/2)\Delta_t + 2\triangle z(\mu_0/2)\Delta_t\}\ V_x^*(C_1) \tag{2e}$$

$$V_y(A_1) - V_y(A_5) = \{2\triangle z(\sigma^*/2) + 2\triangle_z(\mu_0\chi_m/2)\Delta_t + 2\triangle z(\mu_0/2)\Delta_t\}\ V_x^*(C_2) \tag{2f}$$

$$V_z(E_1) - V_z(E_2) = \{2\triangle y(\sigma^*/2) + 2\triangle_y(\mu_0\chi_m/2)\Delta_t + 2\triangle y(\mu_0/2)\Delta_t\}\ V_x^*(C_1) \tag{2g}$$

$$V_z(E_3) - V_z(E_4) = \{2\triangle y(\sigma^*/2) + 2\triangle_y(\mu_0\chi_m/2)\Delta_t + 2\triangle y(\mu_0/2)\Delta_t\}\ V_x^*(C_2) \tag{2h}$$

Each voltage at node C_1, C_2, B_1, B_2 is expressed by the currents at the central node A_i shown in Fig. 4.

$$\begin{aligned} V_z^*(B_1) &= I_{x1}(A_1) - \triangle x\Delta_x\ V_z^*(B_1) \\ &= I_{x1}(A_1) + (\varepsilon_0/2)\triangle x\Delta_t\ V_y(B_1) \end{aligned} \tag{3a}$$

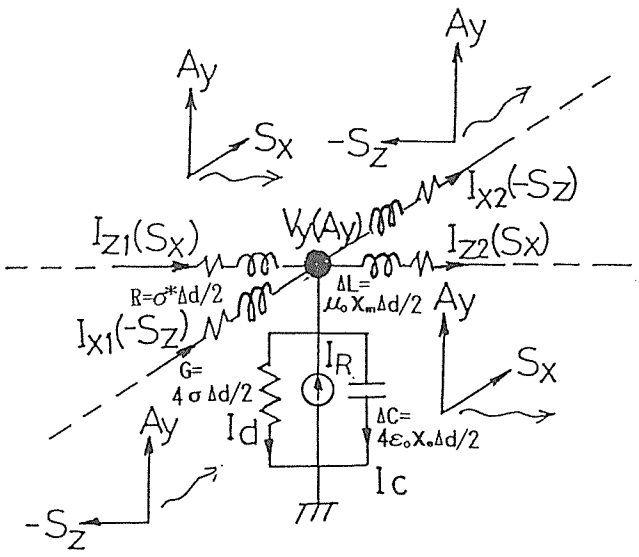

Fig. 4 Definition of voltage and currents at node A_1 with conductance and current source in dielectric and magnetic medium

Similarly

$$V_z^*(B_2) = I_{x2}(A_1) - (\varepsilon_0/2)\,\triangle x \Delta_t\, V_y(B_2) \tag{3b}$$

$$V_x^*(C_1) = I_{z1}(A_1) + (\varepsilon_0/2)\,\triangle z \Delta_t\, V_y(C_1) \tag{3c}$$

$$V_x^*(C_2) = I_{z2}(A_1) - (\varepsilon_0/2)\,\triangle z \Delta_t\, V_y(C_2) \tag{3d}$$

For simplicity, the space discretization to each direction is identical and the following abbreviations are used.

$$\triangle d = \triangle x = \triangle y = \triangle z,\quad \Delta_t = \partial/\partial t,\quad \Delta_t^2 = \partial^2/\partial t^2$$
$$\Delta_x = \partial/\partial_x,\quad \Delta_y = \partial/\partial_y,\quad \Delta_z = \partial/\partial_z$$

Adding Equs. (2) substituted by Equs. (3) side by side, positively for formula with incident currents and negatively for formula with flowing out currents, the following equation can be obtained.

$$\begin{aligned}
&\{V_y(A_2) + V_y(A_3) - 2V_y(A_1)\} \\
&+\{V_y(A_4) + V_y(A_5) - 2V_y(A_1)\} \\
&-[\{V_x(D_4) - V_x(D_2) - V_x(D_3) + V_x(D_1)\} \\
&-\{V_z(E_4) - V_z(E_2) - V_z(E_3) + V_z(E_1)\}] \\
&= 4\triangle d^2(\sigma^*/2)\{I_{z1} - I_{z2} + I_{x1} - I_{x2}\}_{A1}/\triangle d \\
&\quad + 4\triangle d^2(\mu_0(1+\chi_m)/2)\Delta_t\{I_{z1} - I_{z2} + I_{x1} - I_{x2}\}_{A1}/\triangle d \\
&\quad + 4\triangle d^2(\sigma^*\varepsilon_0/4)\Delta_t\{V_y(B_1) + V_y(B_2) + V_y(B_2) + V_y(C_1) + V_y(C_2)\} \\
&\quad + 4\triangle d^2(\varepsilon_0\mu_0(1+\chi_m))\Delta_t^2\{V_y(B_1) + V_y(B_2) + V_y(C_1) + V_y(C_2)\}
\end{aligned} \tag{4}$$

Currents continuity equation (5a) including conductive current (5b), capacity current (5c), and the current source (5d) at node A_1 are given as follows;

$$I_{z1}-I_{z2}+I_{x1}-I_{x2}-I_d-I_c+I_R=0 \tag{5a}$$

$$I_d=4\sigma\left(\frac{\triangle d}{2}\right)V_y \tag{5b}$$

$$I_c=4\varepsilon_0\chi_e\left(\frac{\triangle d}{2}\right)V_y \tag{5c}$$

I_R : Equivalent current correspondent to the polarization (P_R) which derivatives gives the solenoidal current source density (J_R) $\left(J_R=\frac{d}{dt}P_R\right)$ (5d)

After dividing both sides of Equ. (4) by $4\triangle d^2$ and taking the spatial position of each component of the vector potential into consideration, rearrangement of the left hand side of Equ. (4) gives;

$$\frac{V_y(A_2)+V_y(A_3)-2V_y(A_1)}{4\triangle x^2}+\frac{V_y(A_4)+V_y(A_5)-2V_y(A_1)}{4\triangle z^2}$$
$$-\frac{1}{2\triangle y}\left[\left\{\frac{V_x(D_4)-V_x(D_2)}{2\triangle x}\right\}-\left\{\frac{V_x(D_3)-V_x(D_1)}{2\triangle x}\right\}\right.$$
$$\left.-\left\{\left\{\frac{V_z(E_4)-V_z(E_2)}{2\triangle z}\right\}-\left\{\frac{V_z(E_3)-V_z(E_1)}{2\triangle z}\right\}\right\}\right] \tag{6a}$$

Each spatial difference in Equ. (6) is expressed by differential operator when the spatial difference $\triangle d$ becomes sufficiently small and the equivalent voltage variables is rewritten by the correspondent components of the vector potential to clear the physical meaning.

$$=\frac{\partial^2V_y}{\partial x^2}+\frac{\partial^2V_y}{\partial z^2}-\frac{\partial}{\partial y}\left(\frac{\partial V_x}{\partial x}-\frac{\partial V_z}{\partial z}\right)$$
$$=\frac{\partial^2A_y}{\partial x^2}+\frac{\partial^2A_y}{\partial z^2}-\frac{\partial}{\partial y}\left(\frac{\partial A_x}{\partial x}+\frac{\partial A_z}{\partial z}\right) \tag{7}$$

The terms in the parenthesis in equ. (7) are transformed as follows by considerating the gauge condition related to 'div A' from the fundamental cubic shown by dashed lines in Fig. 5 and also in Fig. 1.

$$-\frac{\partial}{\partial y}\left(\frac{\partial A_x}{\partial x}+\frac{\partial A_z}{\partial z}\right)$$
$$=-\frac{\partial}{\partial y}\left(\nabla\cdot A-\frac{\partial A_y}{\partial y}\right)$$
$$=-\frac{\partial}{\partial y}\nabla\cdot A+\frac{\partial^2A_y}{\partial y^2}$$

For the Coulomb's gauge condition 'div A' becomes zero, therefore,

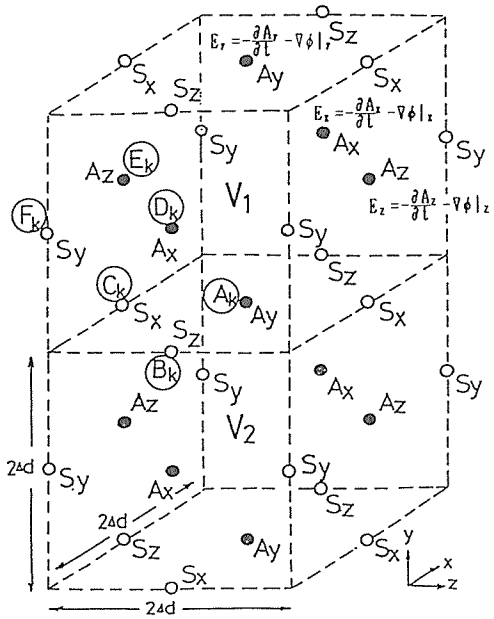

Fig. 5 Fundamental cube for defining "div A"

$$= \frac{\partial^2 A_y}{\partial y^2} \tag{8}$$

Then, the resultant equation of the left hand side of Equ. (4) is given as follows;

$$\therefore \text{Left hand side} = \frac{\partial^2 A_y}{\partial x_y^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z_y^2}$$
$$= \nabla^2 A_y \tag{6b}$$

The treatment for the Lorentz gauge condition is explained in the reference[15].

On the other hand, the currents in the parentheses of the first and the second terms of the right hand side of Equ. (4) is substituted by Equs. (5a)-(5d), and the average of voltages in the third and the fourth terms of the equation converges to the voltage value at the central node A_1 when the spatial difference $\triangle d$ becomes sufficiently small, and the equivalent voltage variable is also rewritten by the correspodent component of the vector potential.

$$\begin{aligned}
\text{Right hand side} &= (\sigma^*/2)\{I_d + I_c - 2\triangle d \cdot I_{Ry}\}_{A1}/\triangle d \\
&\quad + (\mu_o(1+\chi_m)/2)\Delta_t\{I_d + I_c - 2\triangle d \cdot I_{Ry}\}_{A1}/\triangle d \\
&\quad + (\sigma^*\varepsilon_o/4)\Delta_t\{V_y(B_1) + V_y(B_2) + V_y(C_1) + V_y(C_2)\} \\
&\quad + (\varepsilon_o\mu_o(1+\chi_m)/4)\Delta_t^2\{V_y(B_1) + V_y(B_2) + V_y(C_1) + V_y(C_2)\} \\
&= \left(\frac{\sigma^*}{2\triangle d}\right)4\sigma\left(\frac{\triangle d}{2}\right)V_y(A_1) + \left(\frac{\mu_o(1+\chi_m)}{2\triangle d}\right)\frac{\partial}{\partial t}4\sigma\left(\frac{\triangle d}{2}\right)V_y(A_1) \\
&\quad + \varepsilon_o(1+\chi_e)\sigma^*\frac{\partial}{\partial t}V_y(A_1) + \varepsilon_o(1+\chi_e)\mu_o(1+\chi_m)\frac{\partial^2}{\partial t^2}V_y(A_1) \\
&\quad - \mu_o(1+\chi_m)\frac{\partial}{\partial t}I_{Ry}(A_1) - \sigma^* I_{Ry}(A_1)
\end{aligned} \tag{9a}$$

Rewriting this equation by the correspondent components of the vector potential and the current sources to clear the physical meaning gives.

$$= \sigma\sigma^* A_y(A_1) + (\mu\sigma + \varepsilon\sigma^*)\frac{\partial}{\partial t}A_y(A_1) + \varepsilon\mu\frac{\partial^2}{\partial t^2}A_y(A_1) - \mu J_{Ry}(A_1) - \sigma^* P_{Ry}(A_1) \tag{9b}$$

Here, $\varepsilon = \varepsilon_0(1+\chi_e)$, $\mu = \mu_0(1+\chi_m)$ and J_{Ry}, P_{Ry} : y-directed component of J_R, P_R. Resultant equation of Equ. (4) expressed by the correspondent vector potential component A_y is obtained as follows from Equs. (6b) and (9b).

$$\nabla^2 A_y - \varepsilon\mu\frac{\partial^2 A_y}{\partial t^2} - (\mu\sigma + \varepsilon\sigma^*)\frac{\partial A_y}{\partial t} - \sigma\sigma^* A_y = -\mu J_{Ry} - \sigma^* P_{Ry} \tag{10}$$

The vectorial expression for all the components is given asfollows.

$$\nabla^2 A - \varepsilon\mu\frac{\partial^2 A}{\partial t^2} - (\mu\sigma + \varepsilon\sigma^*)\frac{\partial A}{\partial t} - \sigma\sigma^* A = -\mu J_R - \sigma^* P_R \tag{11}$$

The combined vectorial wave equation for both the magnetic and electric vector potentials is given as follows.

$$\left\{\nabla^2 - \varepsilon\mu\frac{\partial^2}{\partial t^2} - (\mu\sigma + \varepsilon\sigma^*)\frac{\partial}{\partial t} - \sigma\sigma^*\right\}\begin{bmatrix} A \\ S \end{bmatrix} = -\begin{bmatrix} \mu J_R + \sigma^* P_R \\ \varepsilon J_R^* + \sigma P_R^* \end{bmatrix}$$

(J_R^* : Magnetic current density
P_R^* : Equivalent magnetic polarization source density) (12)

Conventionally, the second terms in the right hand side of the above equations are omitted because of their hypothetical properties.

3. Analyzed Results

To show the advantage of the spatial network formulation for the vector potential, the treatment of boundary and medium conditions are explained.

3.1 Boundary Condition

To show the advantage of the method in the treatment of boundary condition, the fields in the rectangular waveguide shown in Fig. 6 is simulated. The waveguide has the metal

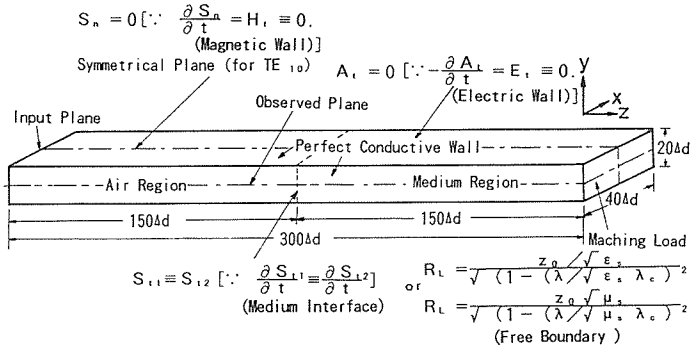

Fig. 6 Definition of typical boundary conditions in each part of rectangular wave guide model, in the rear part of which the medium is partially filled.

wall with the infinite conductivities and its rear portion is filled with dielectric or magnetic medium. Therefore, some kinds of boundary conditions, such as the electric wall, the matching termination, and the air-medium interface, can be treated as shown in the figure. In Fig. 7, an example of the electric field distribution is shown. The standing wave in the fore part caused by the air-medum interface is observed, and the flat distribution in the rear part caused by the matching termination can be also observed. In Fig. 8, the variations of reflective coefficients caused by air-medium boundary are shown as a function of angular frequency normalized by the cut-off angualr frequency ω_C of the waveguide. The good coincidence with the analytical results is shown.

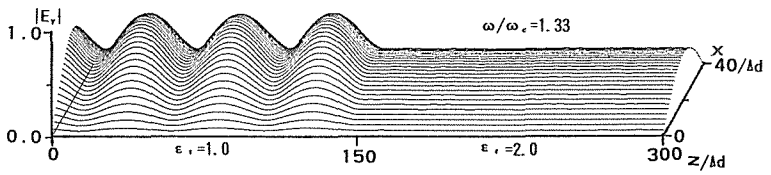

Fig. 7 Electric field distribution in the wave guide ($\omega/\omega_c=1.33$, $\varepsilon_r=2.0$)

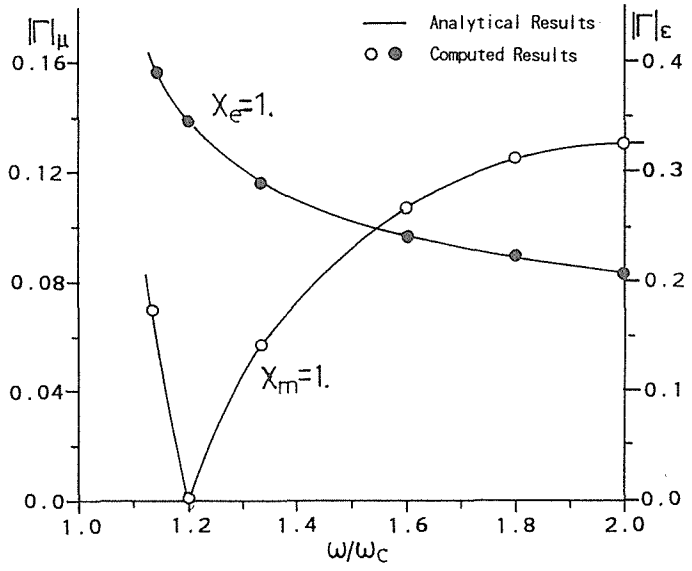

Fig. 8 Variation of reflective coefficients caused by the air-medium boundary in the waveguide as a function of angular frequency normalized by the cut-off angular frequency ω_c.
$|\Gamma|_\varepsilon$: for dielectric medium
$|\Gamma|_\mu$: for magnetic medium

3.2 Dispersive Medium Condition

In this section, it is shown that medium conditions can be treated as lumped elements at each node that decide the voltage-current relation approximated by the trapezoidal formulation of the characteristic equation of the medium including dispersive properties..

The characteristic equation, as an example, for the resonance absorption characteristics is given as follows.

$$\frac{d^2P}{dt^2}+\gamma\frac{dP}{dt}+\omega_0^2P=\varepsilon_0\chi_eE \tag{13}$$

Here,

E : Electric Field
P : Electric Polarization Vector
χ_e : Electric Susceptibility
γ : Attenuation Factor
ω_0 : Resonance Angular Frequency

By using the relation between the electric field and the vector potential, the above equation can be rewritten as follows.

$$\frac{d^2P}{dt^2}+\gamma\frac{dP}{dt}+\omega_0^2P=-\varepsilon_0\chi_e\frac{dA}{dt} \tag{14}$$

Then,

$$\frac{d}{dt}\left(\frac{dP}{dt}+\varepsilon_0\chi_eA+\gamma P\right)=\omega_0^2P \tag{15}$$

Integrating this equation by using the trapezoidal rule gives;

$$\left[\frac{dP}{dt}+\varepsilon_0\chi_eA+\gamma P\right]_t-\left[\frac{dP}{dt}+\varepsilon_0\chi_eA+\gamma P\right]_{t-\triangle t}$$
$$=-\frac{\omega_0^2\triangle t}{2}[P(t)+P(t-\triangle t)] \tag{16}$$

By arranging each term, the following equation is obtained.

$$\frac{dP(t)}{dt}+\varepsilon_0\chi_eA(t)+\left(\gamma+\frac{\omega_0^2\triangle t}{2}\right)P(t)$$
$$=-\frac{\omega_0^2\triangle t}{2}P(t-\triangle t)+K(t-\triangle t) \tag{17a}$$

Here,

$$K(t-\triangle t)=\left[\frac{dP}{dt}+\varepsilon_0\chi_eA+\gamma P\right]_{t-\triangle t} \tag{17b}$$

The second application of the trapezoidal rule to Equ. (17a) gives.

$$P(t)-P(t-\triangle t)+\frac{\varepsilon_0\chi_e\triangle t}{2}(A(t)+A(t-\triangle t))$$
$$+\frac{\triangle t}{2}\left(\gamma+\frac{\omega_0^2\triangle t}{2}\right)[P(t)+P(t-\triangle t)]$$
$$=-\frac{\omega_0^2\triangle t^2}{4}[P(t-\triangle t)+P(t-2\triangle t)]+\frac{\triangle t}{2}[K(t-\triangle t)+K(t-2\triangle t)] \tag{18a}$$

By substituting the last term with the next equation derived from Equ. (17b)

$$K(t-2\triangle t)=K(t-\triangle t)+\frac{\omega_0^2\triangle t}{2}[P(t-\triangle t)+P(t-2\triangle t)] \tag{18b}$$

Equ. (18a) can be arranged as follows.

$$A(t)+\frac{2}{\varepsilon_0\chi_e\triangle t}\left(1+\frac{\gamma\triangle t}{2}+\frac{\omega_0^2\triangle t^2}{4}\right)P(t)$$
$$=-A(t-\triangle t)+\frac{2}{\varepsilon_0\chi_e\triangle t}\left(1-\frac{\gamma\triangle t}{2}-\frac{\omega_0^2\triangle t^2}{4}\right)P(t-\triangle t)+\frac{2}{\varepsilon_0\chi_e}K(t-\triangle t) \tag{19}$$

This resultant equation is simpler than the following equation between the electric field and the displacement current used in the spatial network for the Maxwell's equation. I_P is the displacement current through the polarization.

$$E(t)-\frac{2}{\varepsilon_0\chi_e\triangle t}\left(1+\frac{\gamma\triangle t}{2}+\frac{\omega_0^2\triangle t^2}{4}\right)I_p(t)$$
$$=-E(t-\triangle t)+\frac{2}{\varepsilon_0\chi_e\triangle t}\left(1+\frac{\gamma\triangle t}{2}+\frac{\omega_0^2\triangle t^2}{4}\right)I_p(t-\triangle t)$$
$$+\frac{4}{\varepsilon_0\chi_e\triangle t}\left(\gamma+\frac{\omega_0^2\triangle t}{2}\right)P(t-\triangle t)-\frac{4}{\varepsilon_0\chi_e\triangle t}K(t-\triangle t) \tag{20a}$$

Here,

$$K(t-\triangle t)=-\frac{\omega_0^2\triangle t}{2}[P(t-\triangle t)+P(t-2\triangle t)]$$
$$+\frac{\varepsilon_0\chi_e\triangle t}{2}[E(t-\triangle t)+E(t-2\triangle t)]+K(t-2\triangle t) \tag{20b}$$

This is due to the fact that in the network for the vector potential, the equivalent current corresponds to the electric or magnetic flux density. In Fig. 9, the variation of complex dielectric constant for the dispersive dielectric with resonance absorption is shown as a function of angular frequency normalized by the resonance angular frequency ω_0. The good coincidence with analytical ones is shown.

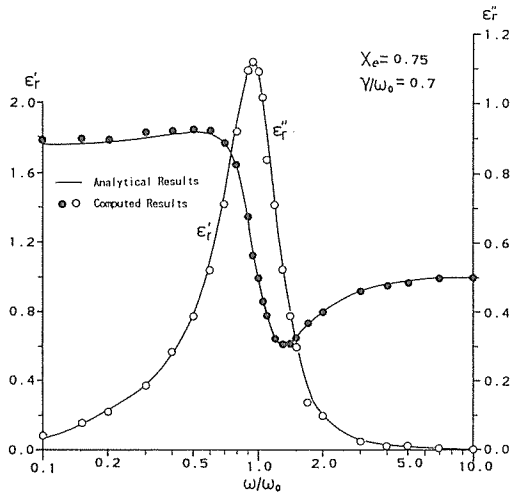

Fig. 9 Variation of complex dielectic constant for the dielectic with resonance absorption as a function of angular frequency normalized by the resonance angular frequency ω_0.

4. Conclusion

1) The spatial network for the vector potential including medium conditions is formulated.
2) In the network, the boundary and medium conditions are treated as the same manner in the spatial network for the electromagnetic field variables.
3) The boundary conditions can be easily formulated by using the correspondence of time derivatives of both the magnetic and electric vector potentials to the electric and magnetic fields, respectively.
4) The formulation of the medium conditions including dispersive properties is based on the fundamental characteristics of the network that in each node the continuity law of the equivalent currents is related physically to that for the electric or magnetic flux. This property can simplify the expression of the medium conditions because of being needless of its time derivative correspondent to electric or magnetic currents. This means also that the vector potential has the intimacy with the mechanism of the medium characteritics.
5) These obtained properties indicates that the spatial network functions as a general network correspondent to each expression of electromagnetic fields. The application of the spatial network to the Hertzian vector also confirms the generality of the spatial network[17]-[18].

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