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| Author(s) | Yoshimura, Yasuo; Nakao, Ikao; Ishibashi, Atsushi |
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# UNIFIED MATHEMATICAL MODEL FOR OCEAN AND HARBOUR MANOEUVRING

**Yasuo Yoshimura (Hokkaido University, Japan)**
**Ikao Nakao (Graduate School of Fisheries Sciences, Hokkaido University, Japan)**
**Atsushi Ishibashi (Tokyo University of Marine Science and Technology, Japan)**

Abstract: Mathematical models for ship manoeuvring have greatly progressed after IMO standard for ship manoeuvrability was settled. These models have been well developed based on experimental and theoretical approaches, particularly for the ocean-going navigation where the forward speed is high. However, for harbour manoeuvring, there are still few works because of the complicated modelling doe to the large drift angle and turning motions that are induced by low forward speed.
In this paper, the authors developed the simple and unified mathematical model that can be available from ocean-going to berthing manoeuvre. The principle of the model is based on linear hydrodynamic force and cross-flow drag theory. Then, the model is applied to the captive model test results of several ship models. From the comparison between measured and calculated by the mathematical model, it is found that the presented model can be well used for the prediction of very wide range of manoeuvring motion. The merit of this model is that the conventional linear hull force derivatives can be applied, and that the coefficients regarding to cross-flow drag are minimized and easily predicted by the regression formulas proposed here. This model is expected to be used for real time simulators as well as the manoeuvring simulation in early design stage.

## 1. INTRODUCTION

The research of a mathematical model and prediction techniques has greatly progressed after IMO standard for ship manoeuvrability was settled. Particularly, for the ocean-going navigation where the forward speed is almost “Full Ahead” or “Navigation Full”, the mathematical model has been developed based on experimental and theoretical approaches including CFD. Then, the manoeuvring ship motion can be well predicted without experimental model tests. This has become possible doe to the development of modular type mathematical model such as MMG, since the force components can be more precisely and reasonably determined based upon theoretical and experimental approaches.
However, for the research on the manoeuvring prediction in harbour navigation, there are still few works, though the prediction in this condition is more important and complicated than in the ocean going. In the harbour manoeuvring, the forward speed is reduced and sway and turning velocities relatively increased doe to the side thrusters or tugboat. Then, these velocity components become almost the same order, this makes it difficult to predict the force components because the drift angle $\beta$ and non-dimensional turning rate $r'$ become extremely larger. Finally, $\beta$ reaches ±90° and $r'$ goes to $\pm\infty$ when the forward speed is zero. In such condition, the conventional expression can not be used.
Oltman [2] presented a mathematical model using cross-flow drag expression in lateral force and yaw moment. Takashina [3] proposed a mathematical model using the harmonic functions with $\beta$ and $r'$. Karasuno [4] introduced a unique expression by means of adopting each hydrodynamic component instead of conventional polynomial terms. However, as these mathematical models are quite different from conventional mathematical models, the mathematical model has to be switched corresponding to the simulated conditions. This causes many difficulties particularly in real-time manoeuvring simulators.
Based on the above background, the authors investigate into the unified mathematical model which can well describe the manoeuvring motion from ocean going to harbour manoeuvring. In this modelling, the characteristic of course stability and initial turning is kept the same as the conventional mathematical model, and it is considered so that the conventional database can be easily applied. Moreover, the coefficients or parameters in the model are also minimized and the ease of presumption in a design phase is secured.

## 2. HULL FORCE MODEL

Co-ordinate system is shown in Fig.1, where $u$ represents forward (surge) velocity, $v$ lateral (sway) velocity and $r$ turning velocity, respectively at mid-ship. $X$ and $Y$ represent hydrodynamic force components at mid-ship in $x$, $y$ axis directions, respectively. $N$ is the force moment around $z$ axis. Since the steady hydrodynamic force is proportional to the square of fluid velocity, the basic structures of hydrodynamic force components of ship hull ($X_H$, $Y_H$ and $N_H$) are basically described as the sum of products of velocity components ($u$, $v$ and $r$).

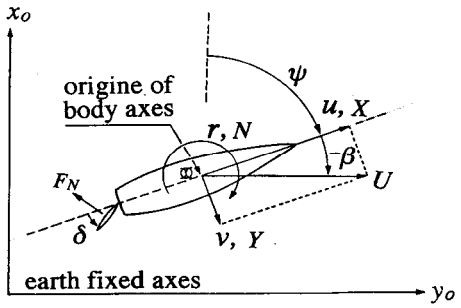



Fig. 1. Co-ordinate system.

As the surge force $X_H$ is an even function about $v$ and $r$, and it is odd function about $u$, the terms of $(vr)$ and $(u|u|)$ or $(uU)$ can be adopted for the expression of $X_H$. The term of $(vr)$ is physically corresponding to the surge component of the centrifugal force of the added mass, and the term of $(u|u|)$ or $(uU)$ hull resistance, respectively. The other term such as $(v|v|)$ or $(r|r|)$ is considered to be one of the force term but not adopted here because of the negligibly small.
As the sway force $Y_H$ and yaw moment $N_H$ are mainly induced by the lifting force, and odd function about $v$ and $r$, they are described by the sum of $(vU)$ and $(rU)$. In the astern condition where $u$ is negative, the term of $(rU)$ in $Y_H$ changes to $(-rU)$, and $(vU)$ in $N_H$ to $(-vU)$, respectively. As the result, $(v|u|)$ and $(ru)$ can be adopted for the expression of $Y_H$, and $(vu)$ and $(r|u|)$ for $N_H$. Considering the above physical structure, the basic mathematical model of hull force can be written as the following.

$$\left.\begin{aligned} X_H &= a_1 uU + a_2 vr \\ Y_H &= b_1 v|u| + b_2 ru + Y_{HN} \\ N_H &= c_1 vu + c_2 r|u| + N_{HN} \end{aligned}\right\} \qquad (1)$$

where, $Y_{HN}$ and $N_{HN}$ represent the non-linear force components induced by lateral drag of hull, for example. The non-dimensional expression of eq.(1) becomes as the following.

$$\left.\begin{aligned} X'_H &= X_H \Big/ \frac{\rho}{2} LdU^2 = a'_1 u' + a'_2 v' r' \\ Y'_H &= Y_H \Big/ \frac{\rho}{2} LdU^2 = b'_1 v'|u'| + b'_2 r' u' + Y'_{HN} \\ N'_H &= N_H \Big/ \frac{\rho}{2} L^2 dU^2 = c'_1 v' u' + c'_2 r'|u'| + N'_{HN} \end{aligned}\right\} \qquad (2)$$

where, $u' = u/U$ , $v' = v/U$ , $r' = r(L/U)$

2.1 Linear hydrodynamic hull force

When the ship is running with large forward speed $u$ and small $v$ and $r$, eq.(2) can be approximated as the following, since $u \cong 1$.

$$\left.\begin{aligned} X'_H &= a'_1 + a'_2 v' r' \\ Y'_H &= b'_1 v' + b'_2 r' + Y'_{HN} \\ N'_H &= c'_1 v' + c'_2 r' + N'_{HN} \end{aligned}\right\} \qquad (3)$$

This equation can be written as the conventional expression as the following.

$$\left.\begin{aligned} X'_H &= X'_0 + (m'_y + X'_{vr}) v' r' \\ Y'_H &= Y'_v v' + (Y'_r - m'_x) r' + Y'_{HN} \\ N'_H &= N'_v v' + N'_r r' + N'_{HN} \end{aligned}\right\} \qquad (4)$$

Accordingly, the coefficients $a_1$, $a_2$, $b_1$, $b_2$, $c_1$, $c_2$ in eq.(1) correspond to the conventional linear hydrodynamic derivatives.

2.2 Non-linear hydrodynamic hull force

Non-linear force components are expressed using cross-flow drag theory. Original cross-flow drag theory is demonstrated in Fig.2. In this figure, the lateral drag force $\Delta Y_H$ and yaw moment $\Delta N_H$ on a segmented hull part whose length is $\Delta x$ and which locates $x$ apart from mid-ship can be expressed as eq.(5) using the geometric lateral velocity in this part.

$$\left.\begin{aligned} \Delta Y_H &= -\frac{\rho}{2} C_D d\Delta x |v + rx|(v + rx) \\ \Delta N_H &= \Delta Y_H \cdot x \end{aligned}\right\} \qquad (5)$$

where, $C_D$ represents the cross-flow drag coefficient. Then, the non-linear force $Y_{HN}$ and moment $N_{HN}$ can be calculated as eq.(6) by means of integrating the above segmented force and moment.

$$\left.\begin{aligned} Y_{HN} &= -\left(\frac{\rho}{2}\right) dC_D \int_{-L/2}^{L/2} |v + rx|(v + rx) \cdot dx \\ N_{HN} &= -\left(\frac{\rho}{2}\right) dC_D \int_{-L/2}^{L/2} |v + rx|(v + rx) \cdot x \cdot dx \end{aligned}\right\} \qquad (6)$$

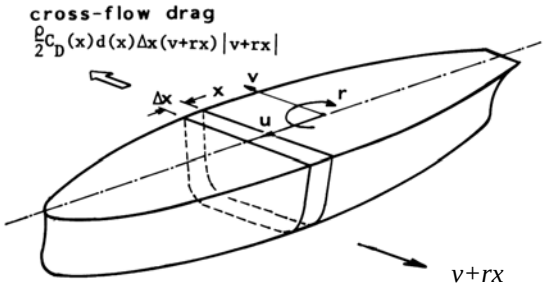



Fig. 2. Schematic view of original cross-flow drag.

This expression is very simple as the non-linear force is described by only one coefficient $C_D$ both for lateral force and yaw moment. However, this simplicity makes the large discrepancies when fitting the measured non-linear hull force and moment. For this solution, Oltman [2] tried to express these force and moment utilizing the high order polynomials for the expression for $C_D$ in stead of constant value. This introduced the other problems and difficulties to the actual applications.

In this paper, correction factors of lateral velocity due to yawing are introduced to the original cross-flow model as the following.

$$\left.\begin{aligned} Y_{HN} &= -\left(\frac{\rho}{2}\right) dC_D \int_{-L/2}^{L/2} |v + C_{rY} rx|(v + C_{rY} rx) dx \\ N_{HN} &= -\left(\frac{\rho}{2}\right) dC_D \int_{-L/2}^{L/2} |v + C_{rY} rx|(v + C_{rY} rx) \cdot x \cdot dx \end{aligned}\right\} \quad (7)$$

where, $C_{rY}$ is a correction factor for lateral force and $C_{rN}$ is for yaw moment, respectively.

Meanwhile, the hull resistance coefficient $X'_0$ in eq.(4) is described as the following, considering the difference of amount between ahead and astern condition.

$$X'_0 = X'_{0(F)} + \left(X'_{0(A)} - X'_{0(F)}\right)\left(|\beta|/\pi\right) \quad (8)$$

where, $X'_{0(F)}$: resistance coefficient of ahead

$X'_{0(A)}$: resistance coefficient of astern

2.3 Total hydrodynamic hull force

Based on the above investigations, the mathematical model of hull force components can be presented as the following formula.

$$\left.\begin{aligned} X_H &= \left(\frac{\rho}{2}\right) Ld \begin{bmatrix} \left\{X'_{0(F)} + \left(X'_{0(A)} - X'_{0(F)}\right)(\beta/\pi)\right\} uU \\ + \left(m'_y + X'_{vr}\right) L \cdot vr \end{bmatrix} \\ Y_H &= \left(\frac{\rho}{2}\right) Ld \begin{bmatrix} Y'_v v|u| + \left(Y'_r - m'_x\right) L \cdot ru \\ -\left(\frac{C_D}{L}\right) \int_{-L/2}^{L/2} |v + C_{rY} rx|(v + C_{rY} rx) dx \end{bmatrix} \\ N_H &= \left(\frac{\rho}{2}\right) L^2 d \begin{bmatrix} N'_v vu + N'_r L \cdot r|u| \\ -\left(\frac{C_D}{L^2}\right) \int_{-L/2}^{L/2} |v + C_{rY} rx|(v + C_{rY} rx) x dx \end{bmatrix} \end{aligned}\right\} \quad (9)$$

The merit of this model is firstly that the course keeping quality of ship can coincide with those of the conventional mathematical model. The linear derivatives also can be used from the conventional database of each organization.

The second merit is that the presented model has a few coefficients or parameters which make it easer for the manoeuvring prediction in a design stage as the coefficients for non-linear force are only $C_D$, $C_{rY}$, $C_{rN}$ as well as resistance coefficient. The third merit is that the model is expected to be used for the simulation from ocean going to harbour manoeuvring even in the forward speed is zero. Actually, substituting $u$=$v$=0 into the eq.(9), the following formula can be obtained for the pure turning ship motion.

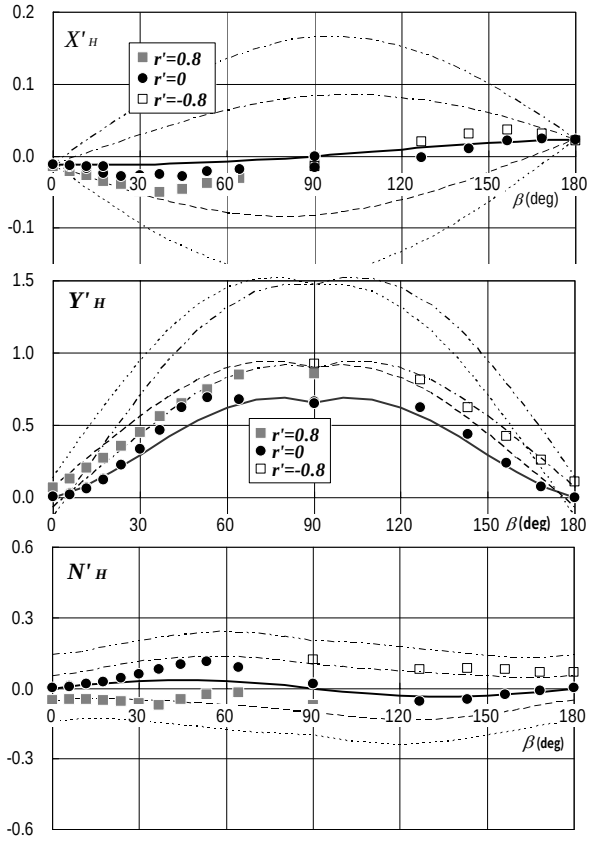
$$\left.\begin{aligned} X_H &= 0, \quad Y_H = 0 \\ N_H &= -\left(\frac{\rho}{2}\right) L^2 d|Lr|(Lr)\left(\frac{C_D C_{rN}^2}{32}\right) \end{aligned}\right\} \quad (10)$$

## 3. APPLICATION TO THE MODEL TESTS

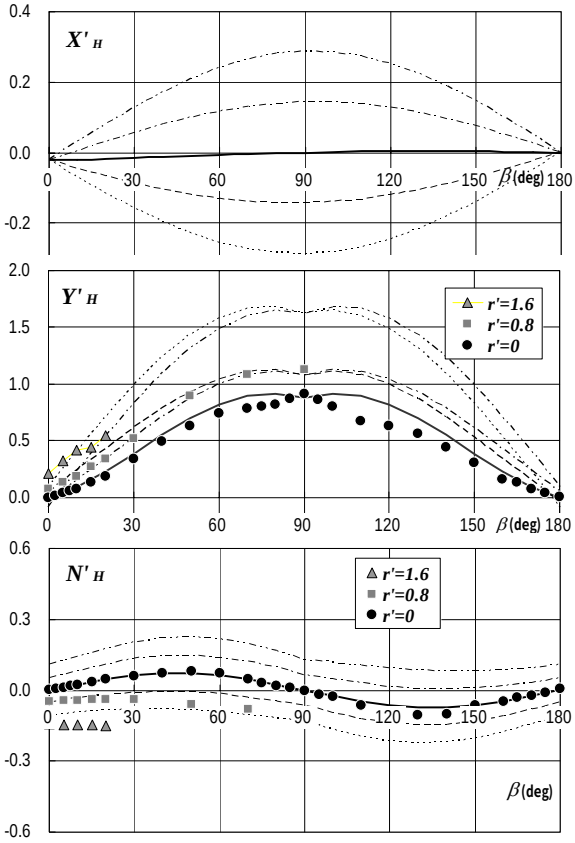
There are not many captive model test data with extremely large drift angle $\beta$ and non-dimensional turning rate $r'$. In 1980's, several experimental data were measured regarding to the Japanese research project whose name is MSS (Manoeuvrability in Shallow and Slow condition). In this project, Nonaka [5] carried out CMT (circular motion test) with 3m model of SR-108 container ship. Yumuro [6] carried out with 4m model of "Esso Osaka" VLCC, and Yoshimura [7] with 3m model of PCC. Recently, the authors also performed almost the same tests with a Fishery Research Ship model [8]. The principal particulars of these ship models are listed in Table 1. Hydrodynamic forces were obtained from CMT in each facility and the maximum range of drift angle $\beta$ and non-dimensional turning rate $r'$ are listed in the table. Conventional captive model tests within the conventional narrow range of $\beta$ and $r'$ were also carried out and linear hydrodynamic derivatives can be obtained from them.

Table 1 Principal Particulars of ship models.

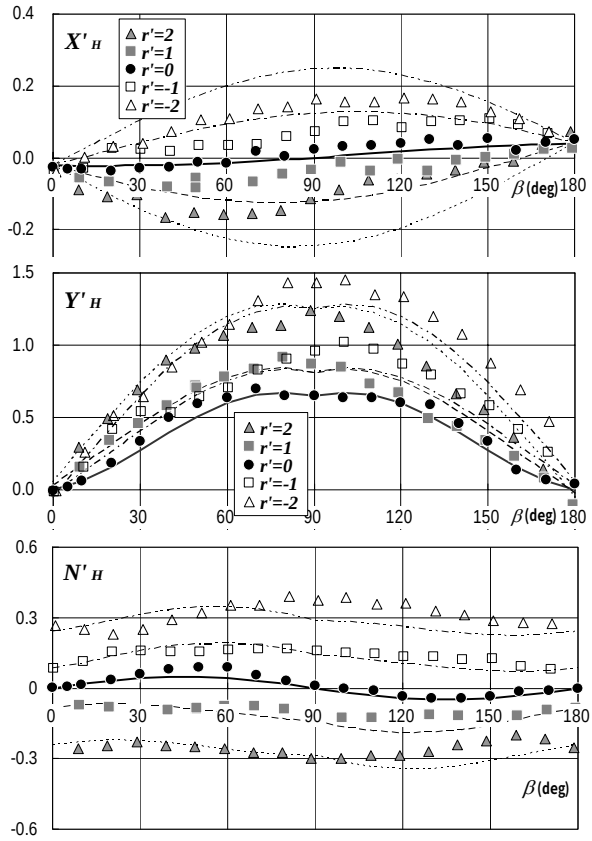
| | SR108 container ship [5] | Esso Osaka VLCC [6] | PCC [7] | Fishery Research ship [8] |
|---|---|---|---|---|
| *Lpp*(=*L*) (m) | 3.000 | 4.000 | 3.0000 | 1.9828 |
| *B* (m) | 0.435 | 0.6523 | 0.5367 | 0.4138 |
| *d* (m) | 0.1629 | 0.2682 | 0.1367 | 0.1517 |
| $x_G$ (=-*lcb*) (m) | | 0.1256 | -0.0422 | -0.0372 |
| $\tau$ (m) | 0 | 0. | 0. | 0. |
| *Cb* | 0.5721 | 0.8303 | 0.547 | 0.5743 |
| *L/B* | 6.897 | 6.132 | 5.590 | 4.792 |
| *L/d* | 18.42 | 14.91 | 21.95 | 13.07 |
| *Cb*/(*L/B*) | 0.0829 | 0.1354 | 0.0980 | 0.1199 |
| 2*d/L*(=*k*) | 0.1086 | 0.1341 | 0.0911 | 0.1530 |
| CMT parameter range | | | | |
| max. $\beta$ (deg.) | 180 | 180 | 180 | 180 |
| max. *r'* | 0.8 | 1.6 | 2.0 | 2.0 |



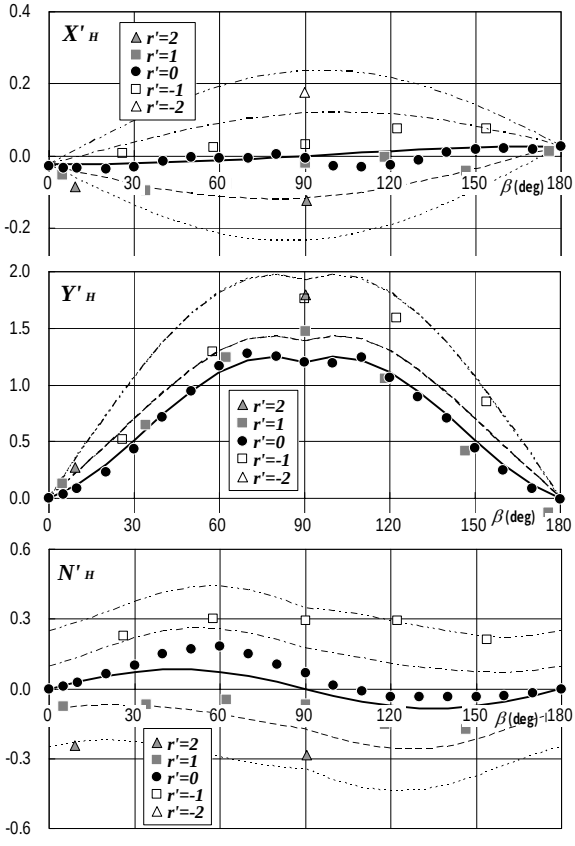

(SR108 Container Ship)




(Esso Osaka VLCC)

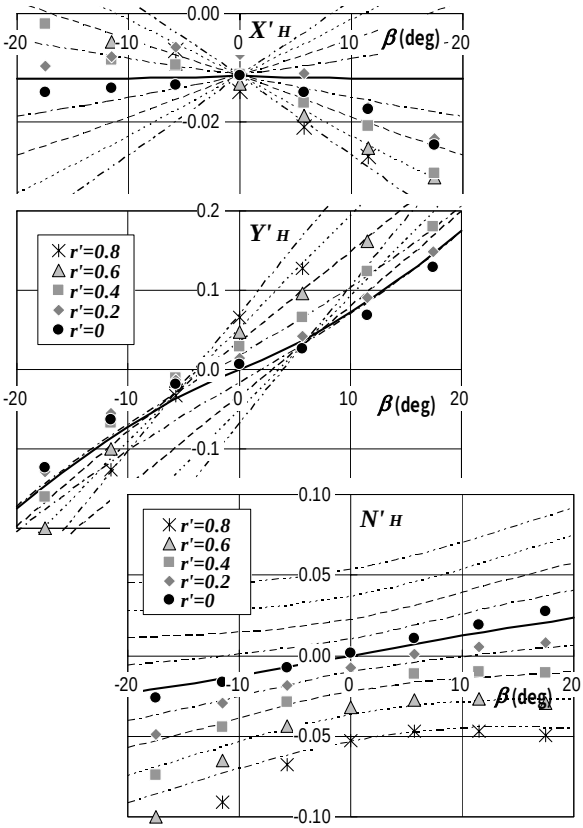



(PCC)

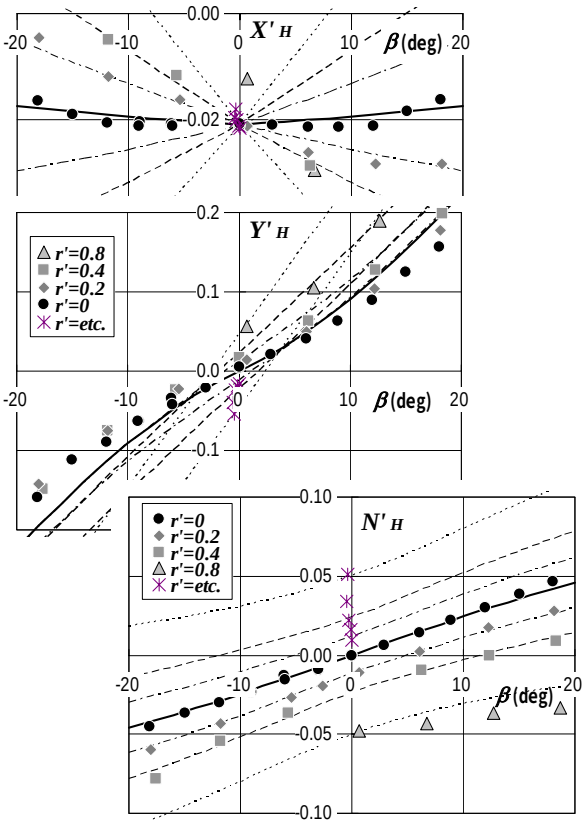



(Fishery Research Ship)

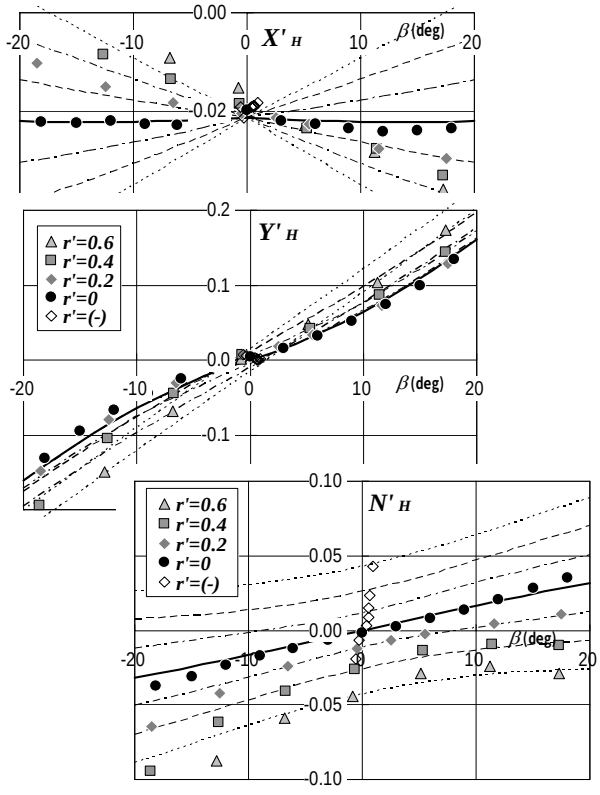
Fig. 3. Comparison of hull force components between measured and approximated.
(wide range of drift angle $\beta$ and non-dimensional turning rate $r'$)



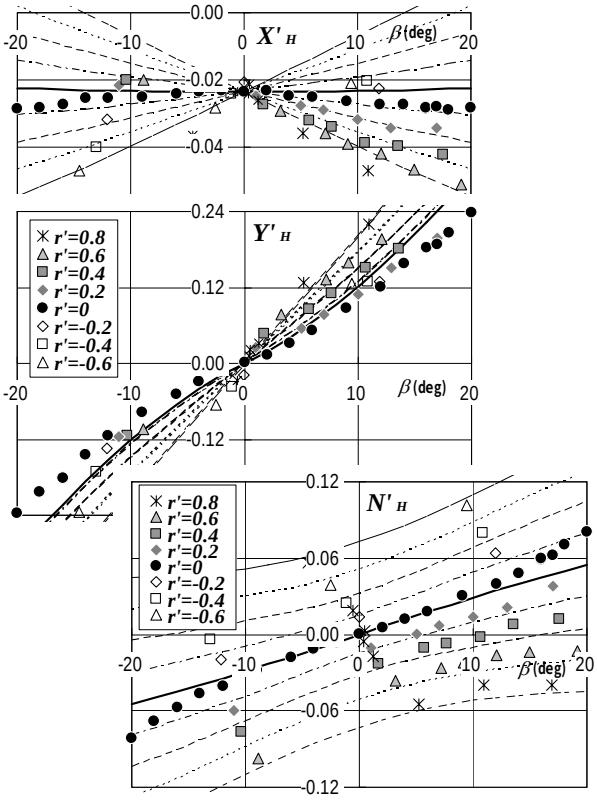

(SR108 Container Ship)




(Esso Osaka VLCC)




(PCC)




(Fishery Research Ship)

Fig. 4. Comparison of hull force components between measured and approximated.
(conventional narrow range of drift angle $\beta$ and non-dimensional turning rate $r'$)

Table 2 Analyzed coefficients in the mathematical model eq.(9).

| | SR108 container ship [5] | Esso Osaka VLCC [6] | PCC [7] | Fishery Research ship [8] |
|---|---|---|---|---|
| hydrodynamic derivatives | | | | |
| $m'_y + X'_{vr}$ | 0.1031 | 0.1795 | 0.1232 | 0.1180 |
| $Y'_v$ | -0.3040 | -0.3777 | -0.2629 | -0.4973 |
| $Y'_r - m'_x$ | 0.0862 | 0.0576 | 0.0262 | 0.0021 |
| $N'_v$ | -0.0726 | -0.1429 | -0.0977 | -0.1696 |
| $N'_r$ | -0.0456 | -0.0562 | -0.0505 | -0.0689 |
| coeff. of cross-flow drag | | | | |
| $C_D$ | 0.659 | 0.879 | 0.648 | 1.201 |
| $C_{rY}$ | 2.590 | 2.058 | 1.735 | 1.355 |
| $C_{rN}$ | 1.124 | 0.543 | 1.309 | 0.858 |

Measured hydrodynamic hull forces for the wide range of drift angle $\beta$ are made non-dimension by ship's speed $U$ and shown in Fig.3. They are plotted by several symbols corresponding to the amount of $r'$ as described in comments in each figure. In this figure $X'_H$ is not measured with "Esso Osaka" VLCC. For the conventional narrow range, measured hydrodynamic hull forces are also made non-dimensional by $U$ and plotted in Fig.4. From these figure, linear hydrodynamic derivatives can be obtained using conventional least square error method with polynomials function of $\beta$ and $r'$. As for the coefficients of cross-flow drag ($C_D$, $C_{rY}$, $C_{rN}$ ), $C_D$ and $C_{rY}$ have been analyzed by the following forms applying to the typical characteristics of $Y'_H$ at $\beta$=90° , and $C_{rN}$ obtained from the characteristics of $N'_H$ at $\beta$=0° in Fig.3.

$$\left.\begin{aligned} C_D &= Y'_{H(\beta=90°,\, r=0)} \\ C_{rY} &= \sqrt{\frac{12}{C_D}\left(\frac{d\left(Y'_{H(\beta=90°)}\right)}{d\left(r'^2\right)}\right)} \\ C_{rN} &= 4\sqrt{\frac{2}{C_D}\left(\frac{d\left(N'_r r' - N'_{H(\beta=0°)}\right)}{d\left(r'^2\right)}\right)} \end{aligned}\right\} \qquad (11)$$

Analyzed coefficients are listed in Table 2 for each ship model. The calculated hull forces by eq.(9) are also made non-dimension as the following,

$$\left.\begin{aligned} X'_H &= X_H \Big/ \left(\frac{\rho}{2}\right) LdU^2 \\ &= \left\{X'_{0(F)} + \left(X'_{0(A)} - X'_{0(F)}\right)(\beta/\pi)\right\}u' + \left(m'_y + X'_{vr}\right)v'r' \\ Y'_H &= Y_H \Big/ \left(\frac{\rho}{2}\right) LdU^2 = Y'_v v'|u'| + \left(Y'_r - m'_x\right)r'u' \\ &\quad - C_D \int_{-1/2}^{1/2} |v' + C_{rY} r'x'|\left(v' + C_{rY} r'x'\right)dx' \\ N'_H &= N_H \Big/ \left(\frac{\rho}{2}\right) L^2 dU^2 = N'_v v'u' + N'_r \cdot r'|u'| \\ &\quad - C_D \int_{-1/2}^{1/2} |v' + C_{rY} r'x'|\left(v' + C_{rY} r'x'\right)x'dx' \end{aligned}\right\} \qquad (12)$$

where, $u' = \cos\beta, \quad v' = -\sin\beta$

And they are compared with the measured data by lines with the parameter of $r'$ in Fig.3 and Fig.4. The bold line shows the pure swaying condition ($r'$=0) in each figure. From the comparisons in Fig.3, the calculated characteristics well expresses in general, though some discrepancies can be seen for the comparison of $X'_H$ and $N'_H$. Particularly, the characteristic of $Y'_H$ that has double peaks around 70° and 110° of $\beta$ instead of single peak at 90° can be well calculated for the fine hull forms such as container ship, PCC and research ship. For the comparisons in Fig.4, the calculated characteristics of hull force components well expresses particularly within 10° of drift angle $\beta$.

As mentioned before, the presented mathematical model eq.(9) can also express the hydrodynamic hull force in pure turning at zero ship's speed. In this condition, $X_H$ and $Y_H$ are zero, and only $N_H$ appears as shown in eq.(10). The measured yaw moment obtained from pure turning test are shown in Fig.5 for the PCC model. The measured yaw moment are made non-dimension by the square of ($Lr$) instead of ship's speed $U$. The non-dimensional yaw moment calculated by the presented model becomes as the following, and it is compared with measured data in Fig.5.

$$N'^*_H = N_H \Big/ \left(\frac{\rho}{2}\right) L^2 d|Lr|(Lr) = -\left(\frac{C_D C_{rN}^2}{32}\right) \qquad (13)$$

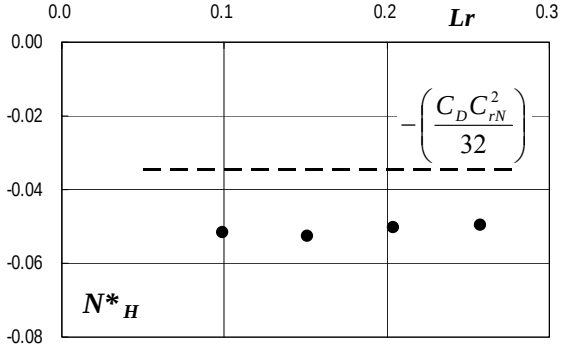



Fig. 5. Comparison of yaw moment at the pure-turning, between measured and approximated. (PCC model)

From the comparison, it is found that the calculated yaw moment is 20~30% smaller than the measured data. This may be introduced by the asymmetric shape of hull-profile under the water that comes from the bulbous bow and the clearance of stern of PCC model. This discrepancy can be improved by shifting the integrating zone of cross-flow drag according to the actual hull-profile under the water, instead of -0.5*L*~0.5*L*.

## 4. SIMULATED RESULTS

Manoeuvring ship motion can be simulated by the following equations of motion.

$$\left.\begin{aligned} m(\dot{u}_G - v_G r) &= X \\ m(\dot{v}_G + u_G r) &= Y \\ I_{zz}\dot{r} &= N - x_G Y \end{aligned}\right\} \tag{14}$$

where, $u_G$, $v_G$ is the velocity components of ship in $x$ and $y$ axis direction at the centre of gravity of ship (C.G). $m$ and $I_{zz}$ are mass and moment of inertia of ship in yaw motion. Since $X$, $Y$, and $N$ are defined at mid-ship as mentioned before, yaw moment is corrected using $x_G$ that is the longitudinal location of (C.G) from the mid-ship. These forces are described separating into the following components from the viewpoint of the physical meaning.

$$\left.\begin{aligned} X &= X_A + X_H + X_R + X_P \\ Y &= Y_A + Y_H + Y_R + Y_P \\ N &= N_A + N_H + N_R + N_P \end{aligned}\right\} \tag{15}$$

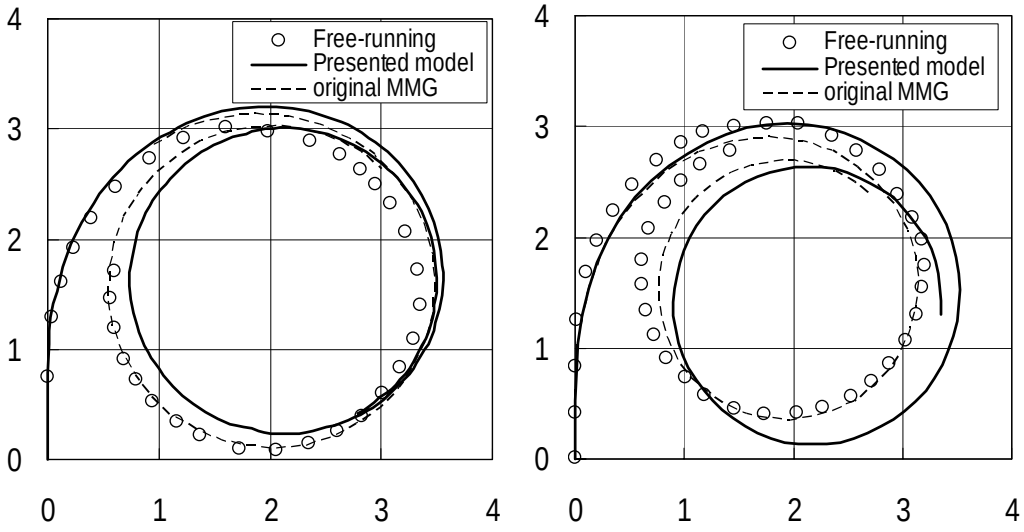
where, the subscripts (*H*), (*P*) and (*R*) refer to hull, propeller and rudder respectively, and the subscript (*A*) to acceleration terms. $X_H$, $Y_H$ and $N_H$ are previously mentioned in detail, and eq.(9) is applied. The acceleration terms are usually described as the following.

$$\left.\begin{aligned} X_A &= -m_x\dot{u} \\ Y_A &= -m_y\dot{v} \\ N_A &= -J_{zz}\dot{r} \end{aligned}\right\} \tag{16}$$

where, $m_x$ and $m_y$ represent added mass on $x$ and $y$ axis direction, and $J_{zz}$ moment of inertia of yawing. Propeller and Rudder forces can be used as following to the previous research [9]. In the harbour manoeuvring where ship's speed is small and the drift angle becomes larger than 30° for example, interaction force coefficients among hull, propeller and rudder can be assumed to be zero.

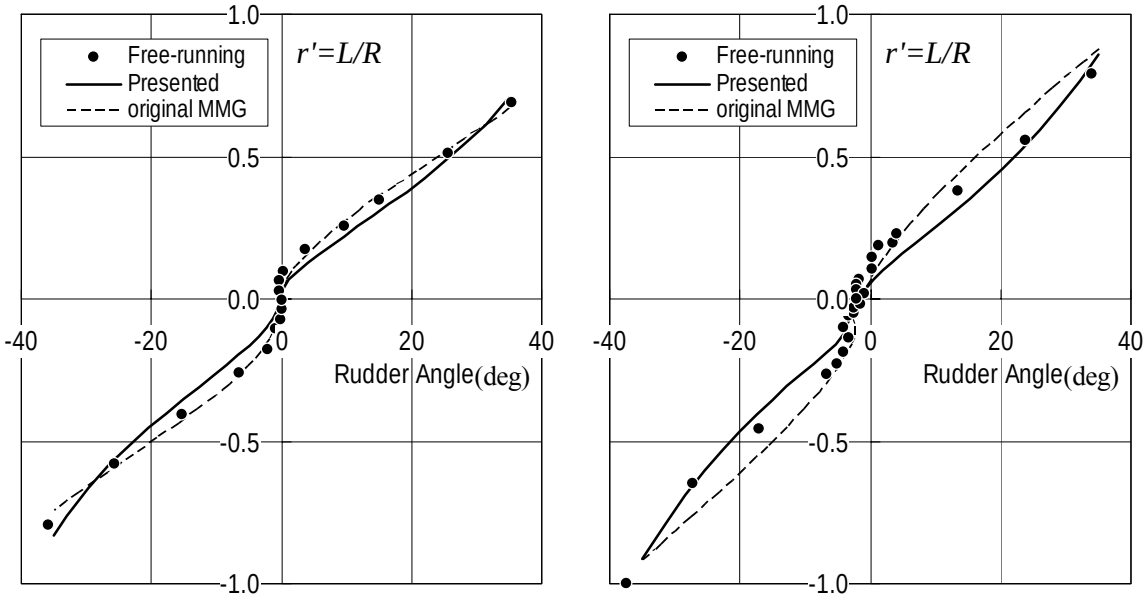
As the example of the simulation, conventional turning test and spiral result are simulated with PCC and "Esso Osaka" VLCC model. Fig.6 shows the simulated conventional turning trajectories with 35° of rudder angle. In this figure, the simulated results by the original MMG model as well as the free-running test result [7] are compared. From the comparison, it can be seen that the presented model can well simulate the turning motion. The simulated spiral curves are shown in Fig.7, where they are also compared with the original MMG model's simulation and free-running test results. It is found that the simulated results by the presented model well predict the actual spiral characteristics particularly for the larger rudder angle, though the predicted results within 10° of rudder angle are slightly smaller than the measured characteristics.

As for the simulation in harbour manoeuvring, it can not be confirmed because of the luck of such free-running model test. However, the presented mathematical model can be expected to be used even in the forward speed is zero, since the hydrodynamic hull forces can be well calculated in such condition.




(PCC model) ("Esso Osaka" VLCC model)

Fig. 6. Comparison of 35° turning trajectory between measured and simulated.




(PCC model) ("Esso Osaka" VLCC model)

Fig. 7. Comparison of spiral curve between measured and simulated.

## 5. PREDICTION OF COEFFICIENTS

When simulating the manoeuvring ship motion, it is convenient that the whole coefficients of the mathematical model can be obtained by some prediction formulas, since the performing of captive model test takes too much cost and time consuming.

### 5.1 Linear derivatives

Linear derivatives in the presented model can be used from the conventional database of each organization as mentioned before. One example is shown in eq.(17) from the reference [10]. These formulas are effective particularly for fine ships whose $C_b$ is less than 0.75.

$$\left.\begin{aligned} m'_y + X'_{vr} &= 2.86\,C_b/(L/B) - 0.21 \\ Y'_v &= Y'_{v0}\left\{1 + 0.54(\tau/d)^2\right\} \\ Y'_r - m'_x &= (Y'_r - m')_0\left\{1 + 1.82(\tau/d)^2\right\} \\ N'_v &= N'_{v0}\left\{1 - 0.51\tau/(dC_b)\right\} \\ N'_r &= N'_{r0}\left\{1 + 1.79(\tau/B)^2\right\} \end{aligned}\right\} \qquad (17)$$

where, $\tau$ : trim by the stern, and

$$\left.\begin{aligned} Y'_{v0} &= -0.5\pi k - 1.4\,C_b/(L/B) \\ (Y'_r - m'_x)_0 &= 0.5\,C_b/(L/B) \\ N'_{v0} &= -k \\ N'_{r0} &= -0.54k + k^2 \end{aligned}\right\} \qquad (18)$$

The above eq.(18) are modified from the original Kijima's model [1].
For the blunt ships such as VLCC and bulk carrier, Kijima's prediction formulas [11] can be applied.

### 5.2 Non-linear coefficients

The resistance coefficient $X'_0$ can be easily predicted by the conventional powering method. The prediction formulas of other non-linear coefficients regarding to the cross-flow drag $C_D$, $C_{rY}$, $C_{rN}$ are proposed here from the regression analysis of the measured data, though the sample is limited at this moment.
As for the cross-flow drag coefficient $C_D$, it is found that the relation between $C_D$ and ($L/d$) is most strong and that $C_D$ becomes small when ($L/d$) increases. The relation is plotted in Fig.8 for several ship models including 4 ship models in this paper. From this figure, the most suitable formula that predicts $C_D$ is obtained as the following.

$$C_D = -0.0591\left(\frac{L}{d}\right) + 1.848 \qquad (19)$$

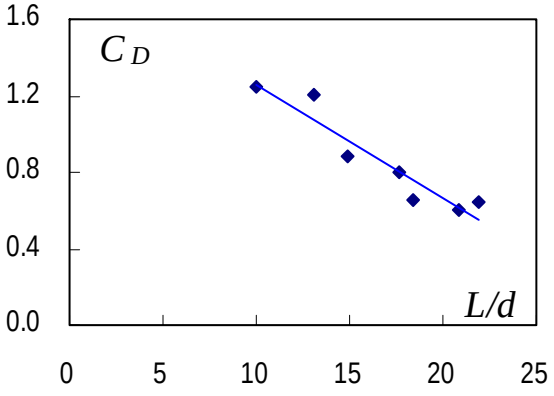



Fig. 8. Relation between cross-flow drag coefficient $C_D$ and ($L/d$)

For the coefficient $C_{rY}$, although it is originally 1.0 in the cross-flow drag theory, it strongly depends on ($L/B$). From the regression analysis, it is found that the relation between $C_{rY}$ and ($L/B$) is best as shown in Fig.9. From this figure, the most suitable formula for predicting $C_{rY}$ is proposed as the following.

$$C_{rY} = 0.520\left(\frac{L}{B}\right) - 1.062 \qquad (20)$$

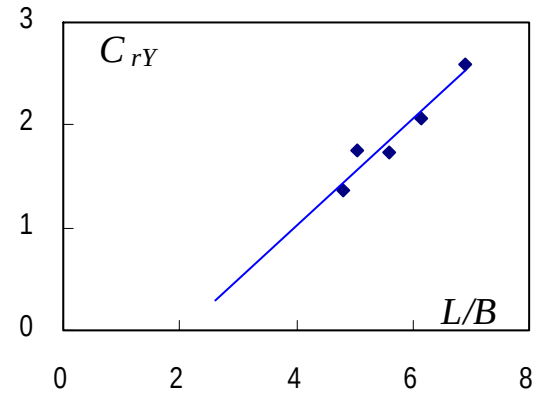



Fig. 9. Relation between $C_{rY}$ and ($L/B$)

Coefficient $C_{rN}$ is also 1.0 in the original cross-flow drag theory. However, it depends on ($L/d$) as shown in Fig.9. From this figure, the predicting formula of $C_{rN}$ is proposed as the following.

$$C_{rN} = 0.0742\left(\frac{L}{d}\right) - 0.297 \qquad (21)$$

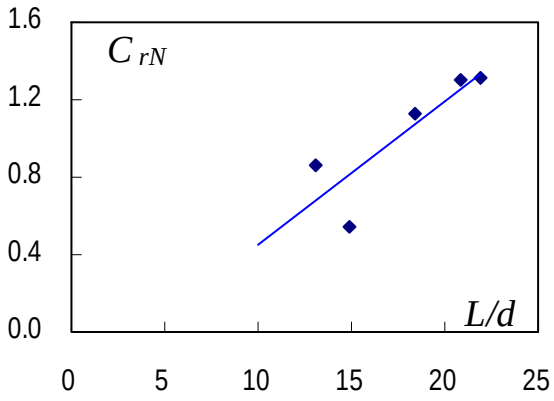



Fig. 10. Relation between $C_{rN}$ and ($L/d$)

## 6. CONCLUSION

The authors try to develop the simple and unified mathematical model that can be available from ocean going to berthing manoeuvre. The principle of the model is based on linear hydrodynamic force and cross-flow drag theory. The concluding remarks are summarized as the following.

1. The presented mathematical model can be well used for the conventional ocean-going manoeuvre. For the harbour manoeuvring, it is expected to be used even in the forward speed is zero, though the simulated results can not be confirmed because of the luck of such free-running model test.
2. The merit of this model is firstly that the course keeping quality of ship can coincide with that of the conventional mathematical model. The linear coefficients also can be applied from the conventional database of each organization.
3. The second merit is that the presented model has a few coefficients which make it easer for the manoeuvring simulation.
4. The regression formulas for predicting the coefficients in the mathematical model are also presented here. This helps for the manoeuvring simulation in early design stage.

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