HOKKAIDO UNIVERSITY

Title	Measurement-Theoretic Foundation of Threshold Utility Maximiser's Preference Logic
Author(s)	Suzuki, Satoru
Description	SOCREAL 2010: 2nd International Workshop on Philosophy and Ethics of Social Reality. Sapporo, Japan, 2010-03-27/28. Session 1: Obligations and Preferences
Citation	SOCREAL 2010: Proceedings of the 2nd International Workshop on Philosophy and Ethics of Social Reality, 83-214
Issue Date	2010
Doc URL	https://hdl.handle.net/2115/43237
Type	conference paper
File Information	Satoru.sli.pdf, Slides

Hokkaido University Collection of Scholarly and Academic Papers : HUSCAP

Measurement-Theoretic Foundation of Threshold Utility Maximiser's Preference Logic

Satoru SUZUKI, PhD.
bxs05253@nifty.com

Faculty of Arts and Sciences, Komazawa University, JAPAN,
Faculty of Political Science and Economics, Meiji University, JAPAN

SOCREAL 2010
27 March 2010, Hokkaido University

Structure of This Talk

1. Introduction
2. Weak Orders as Global Rationality and Semiorders as Bounded Rationality
3. Threshold Utility Maximiser's Preference Logic TUMPL
   - Language of TUMPL
   - Semantics of TUMPL
   - Syntax of TUMPL
   - Metalogic of TUMPL
4. Summary, Further Investigation and Our Related Work

Structure of This Talk

1. Introduction
2. Weak Orders as Global Rationality and Semiorders as Bounded Rationality
3. Threshold Utility Maximiser's Preference Logic TUMPL
   - Language of TUMPL
   - Semantics of TUMPL
   - Syntax of TUMPL
   - Metalogic of TUMPL
4. Summary, Further Investigation and Our Related Work

Counterexample to Transitivity of Indifference (1)

- The economist Armstrong ([Armstrong 1939]) was one of the first to argue that indifference is not always transitive.

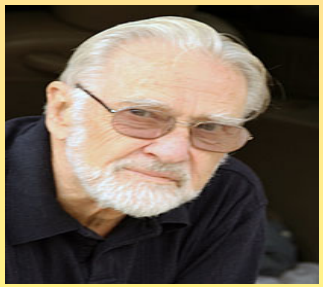
Armstrong, E. W.:
The Determinateness of the Utility Function.
Economic Journal **49** (1939) 453–467.

Counterexample to Transitivity of Indifference (1)

- The economist Armstrong ([Armstrong 1939]) was one of the first to argue that indifference is not always transitive.

Armstrong, E. W.:
The Determinateness of the Utility Function.
Economic Journal **49** (1939) 453–467.

- Luce ([[Luce 1956]: 179]) gave the following counterexample to the transitivity of indifference:



Luce, D.:
Semiorders and a Theory of Utility Discrimination.
Econometrica **24** (1956) 178–191.

Counterexample to Transitivity of Indifference (2)

Example (Avoidance of Sorites Paradox in Preference)

- If indifference were transitive, then he would be unable to detect any weight differences, however great, which is patently false....

Counterexample to Transitivity of Indifference (2)

Example (Avoidance of Sorites Paradox in Preference)

- If indifference were transitive, then he would be unable to detect any weight differences, however great, which is patently false....
- Find a subject who prefers a cup of coffee with one cube of sugar to one with five cubes....

Counterexample to Transitivity of Indifference (2)

Example (Avoidance of Sorites Paradox in Preference)

- If indifference were transitive, then he would be unable to detect any weight differences, however great, which is patently false....
- Find a subject who prefers a cup of coffee with one cube of sugar to one with five cubes....
- Now prepare 401 cups of coffee with $(1 + \frac{i}{100})x$ grams of sugar, $i = 0, 1, \ldots, 400$, where x is the weight of one cube of sugar.

Counterexample to Transitivity of Indifference (2)

Example (Avoidance of Sorites Paradox in Preference)

- If indifference were transitive, then he would be unable to detect any weight differences, however great, which is patently false....
- Find a subject who prefers a cup of coffee with one cube of sugar to one with five cubes....
- Now prepare 401 cups of coffee with $(1 + \frac{i}{100})x$ grams of sugar, $i = 0, 1, \ldots, 400$, where x is the weight of one cube of sugar.
- It is evident that he will be indifferent between cup i and cup $i + 1$, for any i, but by choice he is not indifferent between $i = 0$ and $i = 400$.

■

Counterexample to Transitivity of Indifference (2)

Example (Avoidance of Sorites Paradox in Preference)

- If indifference were transitive, then he would be unable to detect any weight differences, however great, which is patently false....
- Find a subject who prefers a cup of coffee with one cube of sugar to one with five cubes....
- Now prepare 401 cups of coffee with $(1 + \frac{i}{100})x$ grams of sugar, $i = 0, 1, \ldots, 400$, where x is the weight of one cube of sugar.
- It is evident that he will be indifferent between cup i and cup $i + 1$, for any i, but by choice he is not indifferent between $i = 0$ and $i = 400$.

■

This example shows a situation where we would face the Sorites Paradox in preference if indifference were transitive.

Nontransitivity Problem

The first problem now arises:

Nontransitivity Problem

The first problem now arises:

Problem (Nontransitivity Problem)

What kind of preference logic can formalise inferences in which indifference is not transitive? ■

Nontransitivity Problem

The first problem now arises:

Problem (Nontransitivity Problem)

What kind of preference logic can formalise inferences in which indifference is not transitive? ■

We call it the Nontransitivity Problem.

The Aim of This Talk

The aim of this talk is to propose a new version of complete and decidable extrinsic preference logic–threshold utility maximiser's preference logic (TUMPL) that can solve the Nontransitivity Problem.

Fundamental Problem of Intrinsic Preference (1)

- Generally, preference logics are in danger of inviting a serious problem.

Fundamental Problem of Intrinsic Preference (1)

- Generally, preference logics are in danger of inviting a serious problem.
- Von Wright divided preferences into two categories: extrinsic and intrinsic preference.

Von Wright, G. H.:
The Logic of Preference.
Edinburgh UP, Edinburgh (1963).

Fundamental Problem of Intrinsic Preference (2)

- An agent is said to prefer φ extrinsically to ψ if φ is better than ψ in some explicit respect. So we can explain extrinsic preference from some explicit point of view.

Fundamental Problem of Intrinsic Preference (2)

- An agent is said to prefer φ extrinsically to ψ if φ is better than ψ in some explicit respect. So we can explain extrinsic preference from some explicit point of view.
- If we cannot explain preference from any explicit point of view, we call it intrinsic.

Fundamental Problem of Intrinsic Preference (3)

- Von Wright posed the following fundamental problem intrinsic preference logics faced.

Von Wright, G. H.:
The Logic of Preference Reconsidered.
Theory and Decision **3** (1972) 140–169.

Fundamental Problem of Intrinsic Preference (3)

- Von Wright posed the following fundamental problem intrinsic preference logics faced.

Von Wright, G. H.:
The Logic of Preference Reconsidered.
Theory and Decision **3** (1972) 140–169.

Problem (Fundamental Problem of Intrinsic Preference)

The development of a satisfactory logic of preference has turned out to be unexpectedly problematic. The evidence for this lies in the fact that almost every principle which has been proposed as fundamental to one preference logic has been rejected by another one. ■

Fundamental Problem of Intrinsic Preference (3)

- Von Wright posed the following fundamental problem intrinsic preference logics faced.

Von Wright, G. H.:
The Logic of Preference Reconsidered.
Theory and Decision **3** (1972) 140–169.

Problem (Fundamental Problem of Intrinsic Preference)

The development of a satisfactory logic of preference has turned out to be unexpectedly problematic. The evidence for this lies in the fact that almost every principle which has been proposed as fundamental to one preference logic has been rejected by another one. ■

- We call it the Fundamental Problem of Intrinsic Preference.

Fundamental Problem of Intrinsic Preference (4)

Example

	von Wright	Martin	Chisholm and Sosa
Transitivity	+	+	+
Contraposition	−	+	−
Conjunctive Expansion	+	−	−
Disjunctive Distribution	−	−	−
Conjunctive Distribution	+	−	−

Chisholm, R. M. and Sosa, E.:
On the Logic of "Intrinsically Better".
American Philosophical Quarterly **3** (1966) 244–249.

Martin, R. M.:
Intension and Decision.
Prentice-Hall, Inc., Englewood Cliffs (1963).

Mullen's Analysis of Cause of Fundamental Problem (1)

- According to Mullen ([Mullen 1979]), we can analyse the cause of the Fundamental Problem as follows.

Mullen, J. D.:
Does the Logic of Preference Rest on a Mistake?.
Metaphilosophy **10** (1979) 247–255.

Mullen's Analysis of Cause of Fundamental Problem (1)

- According to Mullen ([Mullen 1979]), we can analyse the cause of the Fundamental Problem as follows.

  Mullen, J. D.:
  Does the Logic of Preference Rest on a Mistake?.
  Metaphilosophy **10** (1979) 247–255.

- The adequacy criteria for intrinsic preference principles considered by preference logicians have been whether the principles are consistent with their intuitions of reasonableness.

Mullen's Analysis of Cause of Fundamental Problem (1)

- According to Mullen ([Mullen 1979]), we can analyse the cause of the Fundamental Problem as follows.

  Mullen, J. D.:
  Does the Logic of Preference Rest on a Mistake?.
  Metaphilosophy **10** (1979) 247–255.

- The adequacy criteria for intrinsic preference principles considered by preference logicians have been whether the principles are consistent with their intuitions of reasonableness.
- But each intuitions often disagrees even on the fundamental properties.

Mullen's Analysis of Cause of Fundamental Problem (2)

- Different theories, such as ethics, welfare economics, consumer demand theory, game theory and decision theory make different demands upon the fundamental properties of preference.

Mullen's Analysis of Cause of Fundamental Problem (2)

- Different theories, such as ethics, welfare economics, consumer demand theory, game theory and decision theory make different demands upon the fundamental properties of preference.
- So if we would like to propose preference logic that can avoid the Fundamental Problem of Intrinsic Preference, it should be constructed not from intuition but from a theory or a rule in a theory, that is, it should be extrinsic.

Measurement Theory (1)

- In order to avoid the Fundamental Problem of Intrinsic Preference, we resort to measurement theory.

Measurement Theory (1)

- In order to avoid the Fundamental Problem of Intrinsic Preference, we resort to measurement theory.
- There are two main problems with measurement theory:
  1. the representation problem–justifying the assignment of numbers to objects or propositions,

Measurement Theory (1)

- In order to avoid the Fundamental Problem of Intrinsic Preference, we resort to measurement theory.
- There are two main problems with measurement theory:
  1. the representation problem–justifying the assignment of numbers to objects or propositions,
  2. the uniqueness problem–specifying the transformation up to which this assignment is unique.

Measurement Theory (2)

- A solution to the first problem can be furnished by a representation theorem, which establishes that the chosen numerical system preserves the relations of the relational system.

Measurement Theory (2)

- A solution to the first problem can be furnished by a representation theorem, which establishes that the chosen numerical system preserves the relations of the relational system.
- When we provide TUMPL with a model based on semiorders, by virtue of a corollary of the Scott-Suppes representation theorem, we can adopt threshold utility maximisation as a rule in utility theory that makes demands upon the fundamental properties of preference, which enables TUMPL to avoid the Fundamental Problem of Intrinsic Preference.

Structure of This Talk

1. Introduction
2. Weak Orders as Global Rationality and Semiorders as Bounded Rationality
3. Threshold Utility Maximiser's Preference Logic TUMPL
   - Language of TUMPL
   - Semantics of TUMPL
   - Syntax of TUMPL
   - Metalogic of TUMPL
4. Summary, Further Investigation and Our Related Work

Global Rationality

- The standard model of economics is based on global rationality that requires an optimising behavior.

Global Rationality

- The standard model of economics is based on global rationality that requires an optimising behavior.
- Utility maximisation is a typical example of an optimising behavior.

Representation Theorem for Utility Maximisation

- Cantor ([Cantor 1895]) proved the representation theorem for utility maximisation.

Cantor, G.:
Beiträge zur Begründung der Transfiniten Mengenlehre I.
Mathematische Annalen **46** (1895) 481–512.

Representation Theorem for Utility Maximisation

- Cantor ([Cantor 1895]) proved the representation theorem for utility maximisation.

Cantor, G.:
Beiträge zur Begründung der Transfiniten Mengenlehre I.
Mathematische Annalen **46** (1895) 481–512.

Theorem (Representation for Utility Maximisation, Cantor ([Cantor 1895]))

Suppose $\mathbf{A}$ *is a countable set and* $\succeq$ *is a binary relation on* $\mathbf{A}$. *Then* $\succeq$ *is a weak order (transitive and connected) iff there is a function* $u : \mathbf{A} \to \mathbb{R}$ *such that for any* $x, y \in \mathbf{A}$,

$$x \succeq y \text{ iff } u(x) \geq u(y).$$

■

Bounded Rationality

- But according to Simon ([Simon 1982]), cognitive and information-processing constrains on the capabilities of agents, together with the complexity of their environment, render an optimising behavior an unattainable ideal.

Simon, H. A.:
Models of Bounded Rationality.
MIT Press, Cambridge, Mass. (1982).

Bounded Rationality

- But according to Simon ([Simon 1982]), cognitive and information-processing constrains on the capabilities of agents, together with the complexity of their environment, render an optimising behavior an unattainable ideal.

Simon, H. A.:
Models of Bounded Rationality.
MIT Press, Cambridge, Mass. (1982).

- He dismissed the idea that agents should exhibit global rationality and suggested that they in fact exhibit bounded rationality that allows a satisficing behavior.

Semiorders (1)

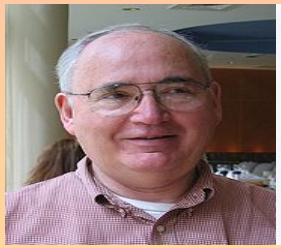
- One explanation for Example 1 is that the nontransitivity of indifference results from the fact that we cannot generally discriminate very close quantities.

Semiorders (1)

- One explanation for Example 1 is that the nontransitivity of indifference results from the fact that we cannot generally discriminate very close quantities.
- The concept of a semiorder was introduced by Luce ([Luce 1956]) to construct a model to interpret situations like Example 1 of nontransitive indifference with a threshold of discrimination.

Semiorders (2)

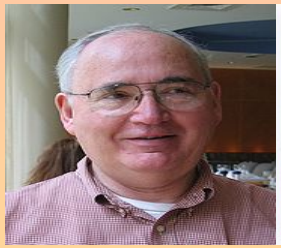
- Scott and Suppes defined ([[Scott and Suppes 1958]: 117]) a semiorder as follows:



Scott, D. and Suppes, P.:
Foundational Aspects of Theories of Measurement.
Journal of Symbolic Logic **3** (1958) 113–128.

Semiorders (2)

- Scott and Suppes defined ([[Scott and Suppes 1958]: 117]) a semiorder as follows:



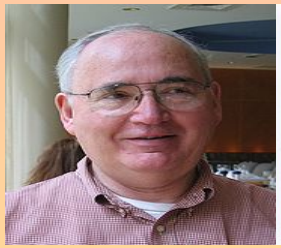
Scott, D. and Suppes, P.:
Foundational Aspects of Theories of Measurement.
Journal of Symbolic Logic **3** (1958) 113–128.

Definition (Semiorder)

$\succ$ on **A** is called a semiorder if, for any $w, x, y, z \in \mathbf{A}$, the following conditions are satisfied:

Semiorders (2)

- Scott and Suppes defined ([[Scott and Suppes 1958]: 117]) a semiorder as follows:



Scott, D. and Suppes, P.:
Foundational Aspects of Theories of Measurement.
Journal of Symbolic Logic **3** (1958) 113–128.

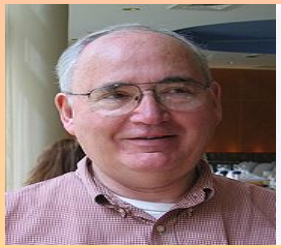
Definition (Semiorder)

$\succ$ on **A** is called a semiorder if, for any $w, x, y, z \in \mathbf{A}$, the following conditions are satisfied:

1. $x \not\succ x$. (Irreflexivity),

Semiorders (2)

- Scott and Suppes defined ([[Scott and Suppes 1958]: 117]) a semiorder as follows:



Scott, D. and Suppes, P.:
Foundational Aspects of Theories of Measurement.
Journal of Symbolic Logic **3** (1958) 113–128.

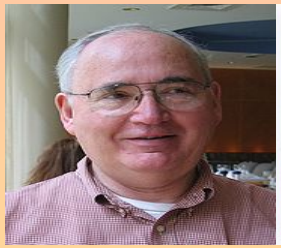
Definition (Semiorder)

$\succ$ on **A** is called a semiorder if, for any $w, x, y, z \in \mathbf{A}$, the following conditions are satisfied:

1. $x \not\succ x$. (Irreflexivity),
2. If $w \succ x$ and $y \succ z$, then $w \succ z$ or $y \succ x$. (Intervality),

Semiorders (2)

- Scott and Suppes defined ([[Scott and Suppes 1958]: 117]) a semiorder as follows:



Scott, D. and Suppes, P.:
Foundational Aspects of Theories of Measurement.
Journal of Symbolic Logic **3** (1958) 113–128.

Definition (Semiorder)

$\succ$ on **A** is called a semiorder if, for any $w, x, y, z \in \mathbf{A}$, the following conditions are satisfied:

1. $x \not\succ x$. (Irreflexivity),
2. If $w \succ x$ and $y \succ z$, then $w \succ z$ or $y \succ x$. (Intervality),
3. If $w \succ x$ and $x \succ y$, then $w \succ z$ or $z \succ y$. (Semitransitivity).

■

Representation Theorem for Threshold Utility Maximisation

- Threshold utility maximisation can be said to be a typical example of a satisficing behavior(bounded rationality).

Representation Theorem for Threshold Utility Maximisation

- Threshold utility maximisation can be said to be a typical example of a satisficing behavior(bounded rationality).
- Scott and Suppes ([Scott and Suppes 1958]) proved a representation theorem for threshold utility maximisation when **A** is finite.

Representation Theorem for Threshold Utility Maximisation

- Threshold utility maximisation can be said to be a typical example of a satisficing behavior(bounded rationality).
- Scott and Suppes ([Scott and Suppes 1958]) proved a representation theorem for threshold utility maximisation when **A** is finite.

Theorem (Representation for Threshold Utility Maximisation, Scott and Suppes ([Scott and Suppes 1958]))

Suppose that $\succ$ is a binary relation on a finite set **A** *and δ is a positive number. Then $\succ$ is a semiorder iff there is a function $u : \mathbf{A} \to \mathbb{R}$ such that for any $x, y \in \mathbf{A}$,*

$$x \succ y \text{ iff } u(x) > u(y) + \delta.$$

■

Structure of This Talk

1. Introduction
2. Weak Orders as Global Rationality and Semiorders as Bounded Rationality
3. Threshold Utility Maximiser's Preference Logic TUMPL
   - Language of TUMPL
   - Semantics of TUMPL
   - Syntax of TUMPL
   - Metalogic of TUMPL
4. Summary, Further Investigation and Our Related Work

Language of TUMPL

We define the language $\mathcal{L}_{\mathsf{TUMPL}}$ of TUMPL.

Language of TUMPL

We define the language $\mathcal{L}_{\mathsf{TUMPL}}$ of TUMPL.

Definition (Language)

- Let **S** denote a set of sentential variables, $\Box$ a necessity operator, **SPR** a strict preference relation symbol.

Language of TUMPL

We define the language $\mathcal{L}_{\mathsf{TUMPL}}$ of TUMPL.

Definition (Language)

- Let **S** denote a set of sentential variables, $\Box$ a necessity operator, **SPR** a strict preference relation symbol.
- The language $\mathcal{L}_{\mathsf{TUMPL}}$ of TUMPL is given by the following rule:

Language of TUMPL

We define the language $\mathcal{L}_{\mathsf{TUMPL}}$ of TUMPL.

Definition (Language)

- Let **S** denote a set of sentential variables, $\Box$ a necessity operator, **SPR** a strict preference relation symbol.
- The language $\mathcal{L}_{\mathsf{TUMPL}}$ of TUMPL is given by the following rule:

$$\varphi ::= s \mid \top \mid \neg\varphi \mid \varphi_1 \& \varphi_2 \mid \Box\varphi \mid \mathbf{SPR}(\varphi_1, \varphi_2),$$

where $s \in \mathbf{S}$, and nestings of **SPR** do not occur.

Language of TUMPL

We define the language $\mathcal{L}_{\mathsf{TUMPL}}$ of TUMPL.

Definition (Language)

- Let **S** denote a set of sentential variables, $\Box$ a necessity operator, **SPR** a strict preference relation symbol.
- The language $\mathcal{L}_{\mathsf{TUMPL}}$ of TUMPL is given by the following rule:

$$\varphi ::= s \mid \top \mid \neg\varphi \mid \varphi_1 \& \varphi_2 \mid \Box\varphi \mid \mathbf{SPR}(\varphi_1, \varphi_2),$$

  where $s \in \mathbf{S}$, and nestings of **SPR** do not occur.
- $\bot, \vee, \rightarrow, \leftrightarrow$ and $\Diamond$ are introduced by the standard definitions.

Language of TUMPL

We define the language $\mathcal{L}_{\mathsf{TUMPL}}$ of TUMPL.

Definition (Language)

- Let **S** denote a set of sentential variables, $\Box$ a necessity operator, **SPR** a strict preference relation symbol.
- The language $\mathcal{L}_{\mathsf{TUMPL}}$ of TUMPL is given by the following rule:

$$\varphi ::= s \mid \top \mid \neg\varphi \mid \varphi_1 \& \varphi_2 \mid \Box\varphi \mid \mathbf{SPR}(\varphi_1, \varphi_2),$$

  where $s \in \mathbf{S}$, and nestings of **SPR** do not occur.
- $\bot, \vee, \rightarrow, \leftrightarrow$ and $\Diamond$ are introduced by the standard definitions.
- We define an indifference relation symbol **IND** and a weak preference relation symbol **WPR** as follows:

Language of TUMPL

We define the language $\mathcal{L}_{\mathsf{TUMPL}}$ of TUMPL.

Definition (Language)

- Let **S** denote a set of sentential variables, $\Box$ a necessity operator, **SPR** a strict preference relation symbol.
- The language $\mathcal{L}_{\mathsf{TUMPL}}$ of TUMPL is given by the following rule:

$$\varphi ::= s \mid \top \mid \neg\varphi \mid \varphi_1 \& \varphi_2 \mid \Box\varphi \mid \mathbf{SPR}(\varphi_1, \varphi_2),$$

  where $s \in \mathbf{S}$, and nestings of **SPR** do not occur.
- $\bot, \vee, \rightarrow, \leftrightarrow$ and $\Diamond$ are introduced by the standard definitions.
- We define an indifference relation symbol **IND** and a weak preference relation symbol **WPR** as follows:

$$\mathbf{IND}(\varphi_1, \varphi_2) := \neg\mathbf{SPR}(\varphi_1, \varphi_2) \& \neg\mathbf{SPR}(\varphi_2, \varphi_1),$$
$$\mathbf{WPR}(\varphi_1, \varphi_2) := \mathbf{SPR}(\varphi_1, \varphi_2) \vee \mathbf{IND}(\varphi_1, \varphi_2).$$

Language of TUMPL

We define the language $\mathcal{L}_{\mathsf{TUMPL}}$ of TUMPL.

Definition (Language)

- Let **S** denote a set of sentential variables, $\Box$ a necessity operator, **SPR** a strict preference relation symbol.
- The language $\mathcal{L}_{\mathsf{TUMPL}}$ of TUMPL is given by the following rule:

$$\varphi ::= s \mid \top \mid \neg\varphi \mid \varphi_1 \& \varphi_2 \mid \Box\varphi \mid \mathbf{SPR}(\varphi_1, \varphi_2),$$

  where $s \in \mathbf{S}$, and nestings of **SPR** do not occur.
- $\bot, \vee, \rightarrow, \leftrightarrow$ and $\Diamond$ are introduced by the standard definitions.
- We define an indifference relation symbol **IND** and a weak preference relation symbol **WPR** as follows:

$$\mathbf{IND}(\varphi_1, \varphi_2) := \neg\mathbf{SPR}(\varphi_1, \varphi_2) \& \neg\mathbf{SPR}(\varphi_2, \varphi_1),$$
$$\mathbf{WPR}(\varphi_1, \varphi_2) := \mathbf{SPR}(\varphi_1, \varphi_2) \vee \mathbf{IND}(\varphi_1, \varphi_2).$$

- The set of all well-formed formulae of $\mathcal{L}_{\mathsf{TUMPL}}$ will be denoted by $\Phi_{\mathcal{L}_{\mathsf{TUMPL}}}$. ■

The Point of Introducing $\Box$

Remark

We introduce $\Box$ to construct a Boolean algebra of subsets of $\mathbf{W}$ that is accessible from $w \in \mathbf{W}$. ■

Model

We define a structured Kripke model $\mathcal{M}$ for TUMPL.

Model

We define a structured Kripke model $\mathcal{M}$ for TUMPL.

Definition (Model)

$\mathcal{M}$ is a quadruple $(\mathbf{W}, R, V, \rho)$, where:

Model

We define a structured Kripke model $\mathcal{M}$ for TUMPL.

Definition (Model)

$\mathcal{M}$ is a quadruple $(\mathbf{W}, R, V, \rho)$, where:

- $\mathbf{W}$ is a nonempty set of possible worlds,

Model

We define a structured Kripke model $\mathcal{M}$ for TUMPL.

Definition (Model)

$\mathcal{M}$ is a quadruple $(\mathbf{W}, R, V, \rho)$, where:

- $\mathbf{W}$ is a nonempty set of possible worlds,
- R is a binary relation on $\mathbf{W}$,

Model

We define a structured Kripke model $\mathcal{M}$ for TUMPL.

Definition (Model)

$\mathcal{M}$ is a quadruple $(\mathbf{W}, R, V, \rho)$, where:

- $\mathbf{W}$ is a nonempty set of possible worlds,
- R is a binary relation on $\mathbf{W}$,
- V is a truth assignment to each $s \in \mathbf{S}$ for each $w \in \mathbf{W}$,

Model

We define a structured Kripke model $\mathcal{M}$ for TUMPL.

Definition (Model)

$\mathcal{M}$ is a quadruple $(\mathbf{W}, R, V, \rho)$, where:

- $\mathbf{W}$ is a nonempty set of possible worlds,
- R is a binary relation on $\mathbf{W}$,
- V is a truth assignment to each $s \in \mathbf{S}$ for each $w \in \mathbf{W}$,
- ρ is a preference space assignment that assigns to each $w \in \mathbf{W}$ a preference space $(\mathcal{F}_w, \succ_w)$ such that $\mathcal{F}_w$ is a Boolean algebra of subsets of $\{w' \in \mathbf{W} : R(w, w')\}$ and $\succ_w$ on $\mathcal{F}_w$ is a semiorder.

■

Representation on Finite Boolean Algebra

Since A is an arbitrary finite set, the next corollary follows directly from Theorem 2.

Representation on Finite Boolean Algebra

Since A is an arbitrary finite set, the next corollary follows directly from Theorem 2.

Corollary (Representation on Finite Boolean Algebra)

Suppose that $\mathbf{W}$ *is a finite set of possible worlds and* $\mathcal{F}$ *is a finite Boolean algebra of subsets of* $\mathbf{W}$ *and* $\succ$ *is a binary relation on* $\mathcal{F}$*, and* δ *is a positive number. Then* $\succ$ *is a semiorder iff there is a function* $u : \mathcal{F} \to \mathbb{R}$ *such that for any* $\alpha, \beta \in \mathcal{F}$*,*

$$\alpha \succ \beta \text{ iff } u(\alpha) > u(\beta) + \delta.$$

■

Representation on Finite Boolean Algebra

Since A is an arbitrary finite set, the next corollary follows directly from Theorem 2.

Corollary (Representation on Finite Boolean Algebra)

Suppose that $\mathbf{W}$ *is a finite set of possible worlds and* $\mathcal{F}$ *is a finite Boolean algebra of subsets of* $\mathbf{W}$ *and* $\succ$ *is a binary relation on* $\mathcal{F}$*, and* δ *is a positive number. Then* $\succ$ *is a semiorder iff there is a function* $u : \mathcal{F} \to \mathbb{R}$ *such that for any* $\alpha, \beta \in \mathcal{F}$*,*

$$\alpha \succ \beta \text{ iff } u(\alpha) > u(\beta) + \delta.$$

■

Remark

This corollary, by virtue of filtration theory, can guarantee that $\succ_w$ *on* $\mathcal{F}_w$ *is a threshold utility maximiser's preference relation.* ■

Truth

We provide TUMPL with the following truth definition relative to $\mathcal{M}$:

Truth

We provide TUMPL with the following truth definition relative to $\mathcal{M}$:

Definition (Truth)

The notion of $\varphi \in \Phi_{\mathcal{L}_{\text{TUMPL}}}$ being true at $w \in W$ in $\mathcal{M}$, in symbols $(\mathcal{M}, w) \models_{\text{TUMPL}} \varphi$ is inductively defined as follows:

Truth

We provide TUMPL with the following truth definition relative to $\mathcal{M}$:

Definition (Truth)

The notion of $\varphi \in \Phi_{\mathcal{L}_{\text{TUMPL}}}$ being true at $w \in W$ in $\mathcal{M}$, in symbols $(\mathcal{M}, w) \models_{\text{TUMPL}} \varphi$ is inductively defined as follows:

- $(\mathcal{M}, w) \models_{\text{TUMPL}} s$ iff $V(w)(s) = \textbf{true}$,
- $(\mathcal{M}, w) \models_{\text{TUMPL}} \top$,
- $(\mathcal{M}, w) \models_{\text{TUMPL}} \varphi_1 \& \varphi_2$
  iff $(\mathcal{M}, w) \models_{\text{TUMPL}} \varphi_1$ and $(\mathcal{M}, w) \models_{\text{TUMPL}} \varphi_2$,
- $(\mathcal{M}, w) \models_{\text{TUMPL}} \neg\varphi$ iff $(\mathcal{M}, w) \not\models_{\text{TUMPL}} \varphi$,
- $(\mathcal{M}, w) \models_{\text{TUMPL}} \Box\varphi$
  iff, for any w' such that $R(w, w')$, $(\mathcal{M}, w') \models_{\text{TUMPL}} \varphi$,

Truth

We provide TUMPL with the following truth definition relative to $\mathcal{M}$:

Definition (Truth)

The notion of $\varphi \in \Phi_{\mathcal{L}_{\text{TUMPL}}}$ being true at $w \in W$ in $\mathcal{M}$, in symbols $(\mathcal{M}, w) \models_{\text{TUMPL}} \varphi$ is inductively defined as follows:

- $(\mathcal{M}, w) \models_{\text{TUMPL}} s$ iff $V(w)(s) = \textbf{true}$,
- $(\mathcal{M}, w) \models_{\text{TUMPL}} \top$,
- $(\mathcal{M}, w) \models_{\text{TUMPL}} \varphi_1 \& \varphi_2$
  iff $(\mathcal{M}, w) \models_{\text{TUMPL}} \varphi_1$ and $(\mathcal{M}, w) \models_{\text{TUMPL}} \varphi_2$,
- $(\mathcal{M}, w) \models_{\text{TUMPL}} \neg\varphi$ iff $(\mathcal{M}, w) \not\models_{\text{TUMPL}} \varphi$,
- $(\mathcal{M}, w) \models_{\text{TUMPL}} \Box\varphi$
  iff, for any w' such that $R(w, w')$, $(\mathcal{M}, w') \models_{\text{TUMPL}} \varphi$,
- $(\mathcal{M}, w) \models_{\text{TUMPL}} \textbf{SPR}(\varphi_1, \varphi_2)$ iff $[\![\varphi_1]\!]^{\mathcal{M}}_w \succ_w [\![\varphi_2]\!]^{\mathcal{M}}_w$,

where $[\![\varphi]\!]^{\mathcal{M}}_w := \{w' \in \mathbf{W} : R(w, w') \text{ and } (\mathcal{M}, w') \models_{\text{TUMPL}} \varphi\}$. ■

Validity

Definition (Validity)

- If $(\mathcal{M}, w) \models_{\mathsf{TUMPL}} \varphi$ for all $w \in \mathbf{W}$, we write $\mathcal{M} \models_{\mathsf{TUMPL}} \varphi$ and say that φ is valid in $\mathcal{M}$.

Validity

Definition (Validity)

- If $(\mathcal{M}, w) \models_{\mathsf{TUMPL}} \varphi$ for all $w \in \mathbf{W}$, we write $\mathcal{M} \models_{\mathsf{TUMPL}} \varphi$ and say that φ is valid in $\mathcal{M}$.
- If φ is valid in all structured Kripke models for TUMPL, we write $\models_{\mathsf{TUMPL}} \varphi$ and say that φ is valid.

■

Counter-Model of Transitivity of Indifference (1)

- We would like to provide a counter-model of the transitivity of indifference.

Counter-Model of Transitivity of Indifference (1)

- We would like to provide a counter-model of the transitivity of indifference.
- We now return to Example 1.

Counter-Model of Transitivity of Indifference (1)

- We would like to provide a counter-model of the transitivity of indifference.
- We now return to Example 1.
- Assume that $\mathcal{U} := (\mathbf{W}, R, V, \rho)$ is given, where:

Counter-Model of Transitivity of Indifference (1)

- We would like to provide a counter-model of the transitivity of indifference.
- We now return to Example 1.
- Assume that $\mathcal{U} := (\mathbf{W}, R, V, \rho)$ is given, where:
  - $\mathbf{W} := \{w_0, \ldots, w_{400}\}$, where w_i is a possible world in which you try a cup of coffee with $(1 + \frac{i}{100})x$ grams of sugar, for any i $(0 \leq i \leq 400)$,

Counter-Model of Transitivity of Indifference (1)

- We would like to provide a counter-model of the transitivity of indifference.
- We now return to Example 1.
- Assume that $\mathcal{U} := (\mathbf{W}, R, V, \rho)$ is given, where:
  - $\mathbf{W} := \{w_0, \ldots, w_{400}\}$, where w_i is a possible world in which you try a cup of coffee with $(1 + \frac{i}{100})x$ grams of sugar, for any i $(0 \leq i \leq 400)$,
  - R is a binary relation on $\mathbf{W}$,

Counter-Model of Transitivity of Indifference (1)

- We would like to provide a counter-model of the transitivity of indifference.
- We now return to Example 1.
- Assume that $\mathcal{U} := (\mathbf{W}, R, V, \rho)$ is given, where:
  - $\mathbf{W} := \{w_0, \ldots, w_{400}\}$, where w_i is a possible world in which you try a cup of coffee with $(1 + \frac{i}{100})x$ grams of sugar, for any i $(0 \leq i \leq 400)$,
  - R is a binary relation on $\mathbf{W}$,
  - V is a truth assignment to each $s \in \mathbf{S}$ for each $w_i \in \mathbf{W}$,

Counter-Model of Transitivity of Indifference (1)

- We would like to provide a counter-model of the transitivity of indifference.
- We now return to Example 1.
- Assume that $\mathcal{U} := (\mathbf{W}, R, V, \rho)$ is given, where:
  - $\mathbf{W} := \{w_0, \ldots, w_{400}\}$, where w_i is a possible world in which you try a cup of coffee with $(1 + \frac{i}{100})x$ grams of sugar, for any i $(0 \leq i \leq 400)$,
  - R is a binary relation on $\mathbf{W}$,
  - V is a truth assignment to each $s \in \mathbf{S}$ for each $w_i \in \mathbf{W}$,
  - ρ is a preference space assignment that assigns to each $w_i \in \mathbf{W}$ a preference space $(\mathcal{F}_{w_i}, \succ_{w_i})$, where:

Counter-Model of Transitivity of Indifference (1)

- We would like to provide a counter-model of the transitivity of indifference.
- We now return to Example 1.
- Assume that $\mathcal{U} := (\mathbf{W}, R, V, \rho)$ is given, where:
  - $\mathbf{W} := \{w_0, \ldots, w_{400}\}$, where w_i is a possible world in which you try a cup of coffee with $(1 + \frac{i}{100})x$ grams of sugar, for any i $(0 \leq i \leq 400)$,
  - R is a binary relation on $\mathbf{W}$,
  - V is a truth assignment to each $s \in \mathbf{S}$ for each $w_i \in \mathbf{W}$,
  - ρ is a preference space assignment that assigns to each $w_i \in \mathbf{W}$ a preference space $(\mathcal{F}_{w_i}, \succ_{w_i})$, where:
    - $\mathcal{F}_{w_i}$ is a Boolean algebra of subsets of $\{w_i' \in \mathbf{W} : R(w_i, w_i')\}$,

Counter-Model of Transitivity of Indifference (1)

- We would like to provide a counter-model of the transitivity of indifference.
- We now return to Example 1.
- Assume that $\mathcal{U} := (\mathbf{W}, R, V, \rho)$ is given, where:
  - $\mathbf{W} := \{w_0, \ldots, w_{400}\}$, where w_i is a possible world in which you try a cup of coffee with $(1 + \frac{i}{100})x$ grams of sugar, for any i $(0 \leq i \leq 400)$,
  - R is a binary relation on $\mathbf{W}$,
  - V is a truth assignment to each $s \in \mathbf{S}$ for each $w_i \in \mathbf{W}$,
  - ρ is a preference space assignment that assigns to each $w_i \in \mathbf{W}$ a preference space $(\mathcal{F}_{w_i}, \succ_{w_i})$, where:
    - $\mathcal{F}_{w_i}$ is a Boolean algebra of subsets of $\{w_i' \in \mathbf{W} : R(w_i, w_i')\}$,
    - $\succ_{w_i}$ on $\mathcal{F}_{w_i}$ is a semiorder,

Counter-Model of Transitivity of Indifference (1)

- We would like to provide a counter-model of the transitivity of indifference.
- We now return to Example 1.
- Assume that $\mathcal{U} := (\mathbf{W}, R, V, \rho)$ is given, where:
  - $\mathbf{W} := \{w_0, \ldots, w_{400}\}$, where w_i is a possible world in which you try a cup of coffee with $(1 + \frac{i}{100})x$ grams of sugar, for any i $(0 \leq i \leq 400)$,
  - R is a binary relation on $\mathbf{W}$,
  - V is a truth assignment to each $s \in \mathbf{S}$ for each $w_i \in \mathbf{W}$,
  - ρ is a preference space assignment that assigns to each $w_i \in \mathbf{W}$ a preference space $(\mathcal{F}_{w_i}, \succ_{w_i})$, where:
    - $\mathcal{F}_{w_i}$ is a Boolean algebra of subsets of $\{w_i' \in \mathbf{W} : R(w_i, w_i')\}$,
    - $\succ_{w_i}$ on $\mathcal{F}_{w_i}$ is a semiorder,
    - $\{w_j\} \sim_{w_i} \{w_{j+1}\}$, for any j $(0 \leq j \leq 400)$,

Counter-Model of Transitivity of Indifference (1)

- We would like to provide a counter-model of the transitivity of indifference.
- We now return to Example 1.
- Assume that $\mathcal{U} := (\mathbf{W}, R, V, \rho)$ is given, where:
  - $\mathbf{W} := \{w_0, \ldots, w_{400}\}$, where w_i is a possible world in which you try a cup of coffee with $(1 + \frac{i}{100})x$ grams of sugar, for any i $(0 \leq i \leq 400)$,
  - R is a binary relation on $\mathbf{W}$,
  - V is a truth assignment to each $s \in \mathbf{S}$ for each $w_i \in \mathbf{W}$,
  - ρ is a preference space assignment that assigns to each $w_i \in \mathbf{W}$ a preference space $(\mathcal{F}_{w_i}, \succ_{w_i})$, where:
    - $\mathcal{F}_{w_i}$ is a Boolean algebra of subsets of $\{w_i' \in \mathbf{W} : R(w_i, w_i')\}$,
    - $\succ_{w_i}$ on $\mathcal{F}_{w_i}$ is a semiorder,
    - $\{w_j\} \sim_{w_i} \{w_{j+1}\}$, for any j $(0 \leq j \leq 400)$,
    - $\{w_0\} \nsucc_{w_i} \{w_{400}\}$.

Counter-Model of Transitivity of Indifference (2)

- Let φ_i denote the sentence "You try a cup of coffee with $(1 + \frac{i}{100})x$ grams of sugar", for any i $(0 \leq i \leq 400)$.

Counter-Model of Transitivity of Indifference (2)

- Let φ_i denote the sentence "You try a cup of coffee with $(1 + \frac{i}{100})x$ grams of sugar", for any i $(0 \leq i \leq 400)$.
- Then we have, for any i $(0 \leq i \leq 400)$,

Counter-Model of Transitivity of Indifference (2)

- Let φ_i denote the sentence "You try a cup of coffee with $(1 + \frac{i}{100})x$ grams of sugar", for any i $(0 \leq i \leq 400)$.
- Then we have, for any i $(0 \leq i \leq 400)$,

$$[\![\varphi_j]\!]^{\mathcal{U}}_{w_i} \sim_{w_i} [\![\varphi_{j+1}]\!]^{\mathcal{U}}_{w_i}, \text{ for any } j \ (0 \leq j \leq 400),$$

Counter-Model of Transitivity of Indifference (2)

- Let φ_i denote the sentence "You try a cup of coffee with $(1 + \frac{i}{100})x$ grams of sugar", for any i $(0 \leq i \leq 400)$.
- Then we have, for any i $(0 \leq i \leq 400)$,

$$\begin{array}{l} [\![\varphi_j]\!]^{\mathcal{U}}_{w_i} \sim_{w_i} [\![\varphi_{j+1}]\!]^{\mathcal{U}}_{w_i}, \text{ for any } j \ (0 \leq j \leq 400), \\ [\![\varphi_0]\!]^{\mathcal{U}}_{w_i} \nsim_{w_i} [\![\varphi_{400}]\!]^{\mathcal{U}}_{w_i}, \end{array}$$

Counter-Model of Transitivity of Indifference (2)

- Let φ_i denote the sentence "You try a cup of coffee with $(1 + \frac{i}{100})x$ grams of sugar", for any i $(0 \leq i \leq 400)$.
- Then we have, for any i $(0 \leq i \leq 400)$,

$$\begin{array}{l} [\![\varphi_j]\!]^{\mathcal{U}}_{w_i} \sim_{w_i} [\![\varphi_{j+1}]\!]^{\mathcal{U}}_{w_i}, \text{ for any } j \ (0 \leq j \leq 400), \\ [\![\varphi_0]\!]^{\mathcal{U}}_{w_i} \nsim_{w_i} [\![\varphi_{400}]\!]^{\mathcal{U}}_{w_i}, \end{array}$$

for $[\![\varphi_j]\!]^{\mathcal{U}}_{w_i} = \{w_j\}$ holds for any j $(0 \leq j \leq 400)$.

Counter-Model of Transitivity of Indifference (2)

- Let φ_i denote the sentence "You try a cup of coffee with $(1+\frac{i}{100})x$ grams of sugar", for any i $(0 \leq i \leq 400)$.
- Then we have, for any i $(0 \leq i \leq 400)$,

$$\begin{array}{l} [\![\varphi_j]\!]^{\mathcal{U}}_{w_i} \sim_{w_i} [\![\varphi_{j+1}]\!]^{\mathcal{U}}_{w_i}, \text{ for any } j \ (0 \leq j \leq 400), \\ [\![\varphi_0]\!]^{\mathcal{U}}_{w_i} \nsim_{w_i} [\![\varphi_{400}]\!]^{\mathcal{U}}_{w_i}, \end{array}$$

  for $[\![\varphi_j]\!]^{\mathcal{U}}_{w_i} = \{w_j\}$ holds for any j $(0 \leq j \leq 400)$.
- It must be noted that, for any i $(0 \leq i \leq 400)$, because $\succ_{w_i}$ on $\mathcal{F}_{w_i}$ is a semiorder, $[\![\varphi_j]\!]^{\mathcal{U}}_{w_i} \sim_{w_i} [\![\varphi_{j+1}]\!]^{\mathcal{U}}_{w_i}$ for any j $(0 \leq j \leq 400)$ does not imply $[\![\varphi_0]\!]^{\mathcal{U}}_{w_i} \sim_{w_i} [\![\varphi_{400}]\!]^{\mathcal{U}}_{w_i}$.

Counter-Model of Transitivity of Indifference (3)

- So we have, or any i $(0 \leq i \leq 400)$,

Counter-Model of Transitivity of Indifference (3)

- So we have, or any i $(0 \leq i \leq 400)$,

$$(\mathcal{U}, w_i) \not\models_{\mathsf{TUMPL}} (\mathbf{IND}(\varphi_0, \varphi_1) \& \cdots \& \mathbf{IND}(\varphi_{399}, \varphi_{400})) \rightarrow \mathbf{IND}(\varphi_0, \varphi_{400}).$$

Counter-Model of Transitivity of Indifference (3)

- So we have, or any i $(0 \leq i \leq 400)$,

  $$(\mathcal{U}, w_i) \not\models_{\mathsf{TUMPL}} (\mathbf{IND}(\varphi_0, \varphi_1) \& \cdots \& \mathbf{IND}(\varphi_{399}, \varphi_{400})) \rightarrow \mathbf{IND}(\varphi_0, \varphi_{400}).$$

- Therefore we obtain the following proposition.

Counter-Model of Transitivity of Indifference (3)

- So we have, or any i $(0 \leq i \leq 400)$,

$$(\mathcal{U}, w_i) \not\models_{\mathsf{TUMPL}} (\mathbf{IND}(\varphi_0, \varphi_1) \& \cdots \& \mathbf{IND}(\varphi_{399}, \varphi_{400})) \rightarrow \mathbf{IND}(\varphi_0, \varphi_{400}).$$

- Therefore we obtain the following proposition.

Proposition (Nontransitivity of Indifference)

$$\not\models_{\mathsf{TUMPL}} (\mathbf{IND}(\varphi_0, \varphi_1) \& \cdots \& \mathbf{IND}(\varphi_{399}, \varphi_{400})) \rightarrow \mathbf{IND}(\varphi_0, \varphi_{400}).$$

■

Proof System

We provide TUMPL with a proof system.

Definition (Proof System)

The proof system of TUMPL consists of the following:

Proof System

We provide TUMPL with a proof system.

Definition (Proof System)

The proof system of TUMPL consists of the following:

1. all tautologies of classical sentential logic,

Proof System

We provide TUMPL with a proof system.

Definition (Proof System)

The proof system of TUMPL consists of the following:

1. all tautologies of classical sentential logic,
2. $\Box(\varphi_1 \rightarrow \varphi_2) \rightarrow (\Box\varphi_1 \rightarrow \Box\varphi_2) \qquad (K)$,

Proof System

We provide TUMPL with a proof system.

Definition (Proof System)

The proof system of TUMPL consists of the following:

1. all tautologies of classical sentential logic,
2. $\Box(\varphi_1 \to \varphi_2) \to (\Box\varphi_1 \to \Box\varphi_2) \qquad (K)$,
3. $\Box(\varphi_1 \leftrightarrow \varphi_2)\&\Box(\psi_1 \leftrightarrow \psi_2) \to (\mathbf{SPR}(\varphi_1, \psi_1) \leftrightarrow \mathbf{SPR}(\varphi_2, \psi_2))$ (Replacement of Necessary Equivalents),

Proof System

We provide TUMPL with a proof system.

Definition (Proof System)

The proof system of TUMPL consists of the following:

1. all tautologies of classical sentential logic,
2. $\Box(\varphi_1 \rightarrow \varphi_2) \rightarrow (\Box\varphi_1 \rightarrow \Box\varphi_2) \qquad (K)$,
3. $\Box(\varphi_1 \leftrightarrow \varphi_2)\&\Box(\psi_1 \leftrightarrow \psi_2) \rightarrow (\mathbf{SPR}(\varphi_1, \psi_1) \leftrightarrow \mathbf{SPR}(\varphi_2, \psi_2))$
   (Replacement of Necessary Equivalents),
4. $\neg\mathbf{SPR}(\varphi, \varphi)$
   (Syntactic Counterpart of Irreflexivity),

Proof System

We provide TUMPL with a proof system.

Definition (Proof System)

The proof system of TUMPL consists of the following:

1. all tautologies of classical sentential logic,
2. $\Box(\varphi_1 \to \varphi_2) \to (\Box\varphi_1 \to \Box\varphi_2) \qquad (K)$,
3. $\Box(\varphi_1 \leftrightarrow \varphi_2)\&\Box(\psi_1 \leftrightarrow \psi_2) \to (\mathbf{SPR}(\varphi_1, \psi_1) \leftrightarrow \mathbf{SPR}(\varphi_2, \psi_2))$
   (Replacement of Necessary Equivalents),
4. $\neg\mathbf{SPR}(\varphi, \varphi)$
   (Syntactic Counterpart of Irreflexivity),
5. $(\mathbf{SPR}(\varphi_1, \varphi_2) \wedge \mathbf{SPR}(\varphi_3, \varphi_4)) \to$
   $(\mathbf{SPR}(\varphi_1, \varphi_4) \vee \mathbf{SPR}(\varphi_3, \varphi_2))$
   (Syntactic Counterpart of Intervality),

Proof System

We provide TUMPL with a proof system.

Definition (Proof System)

The proof system of TUMPL consists of the following:

1. all tautologies of classical sentential logic,
2. $\Box(\varphi_1 \to \varphi_2) \to (\Box\varphi_1 \to \Box\varphi_2) \qquad (K)$,
3. $\Box(\varphi_1 \leftrightarrow \varphi_2)\&\Box(\psi_1 \leftrightarrow \psi_2) \to (\mathbf{SPR}(\varphi_1, \psi_1) \leftrightarrow \mathbf{SPR}(\varphi_2, \psi_2))$
   (Replacement of Necessary Equivalents),
4. $\neg\mathbf{SPR}(\varphi, \varphi)$
   (Syntactic Counterpart of Irreflexivity),
5. $(\mathbf{SPR}(\varphi_1, \varphi_2) \wedge \mathbf{SPR}(\varphi_3, \varphi_4)) \to$
   $(\mathbf{SPR}(\varphi_1, \varphi_4) \vee \mathbf{SPR}(\varphi_3, \varphi_2))$
   (Syntactic Counterpart of Intervality),
6. $(\mathbf{SPR}(\varphi_1, \varphi_2) \wedge \mathbf{SPR}(\varphi_2, \varphi_3)) \to$
   $(\mathbf{SPR}(\varphi_1, \varphi_4) \vee \mathbf{SPR}(\varphi_4, \varphi_3))$
   (Syntactic Counterpart of Semitransitivity),

Proof System

We provide TUMPL with a proof system.

Definition (Proof System)

The proof system of TUMPL consists of the following:

1. all tautologies of classical sentential logic,
2. $\Box(\varphi_1 \to \varphi_2) \to (\Box\varphi_1 \to \Box\varphi_2) \qquad (K)$,
3. $\Box(\varphi_1 \leftrightarrow \varphi_2)\&\Box(\psi_1 \leftrightarrow \psi_2) \to (\mathbf{SPR}(\varphi_1, \psi_1) \leftrightarrow \mathbf{SPR}(\varphi_2, \psi_2))$
   (Replacement of Necessary Equivalents),
4. $\neg\mathbf{SPR}(\varphi, \varphi)$
   (Syntactic Counterpart of Irreflexivity),
5. $(\mathbf{SPR}(\varphi_1, \varphi_2) \wedge \mathbf{SPR}(\varphi_3, \varphi_4)) \to$
   $(\mathbf{SPR}(\varphi_1, \varphi_4) \vee \mathbf{SPR}(\varphi_3, \varphi_2))$
   (Syntactic Counterpart of Intervality),
6. $(\mathbf{SPR}(\varphi_1, \varphi_2) \wedge \mathbf{SPR}(\varphi_2, \varphi_3)) \to$
   $(\mathbf{SPR}(\varphi_1, \varphi_4) \vee \mathbf{SPR}(\varphi_4, \varphi_3))$
   (Syntactic Counterpart of Semitransitivity),
7. Modus Ponens,

Proof System

We provide TUMPL with a proof system.

Definition (Proof System)

The proof system of TUMPL consists of the following:

1. all tautologies of classical sentential logic,
2. $\Box(\varphi_1 \to \varphi_2) \to (\Box\varphi_1 \to \Box\varphi_2) \qquad (K)$,
3. $\Box(\varphi_1 \leftrightarrow \varphi_2)\&\Box(\psi_1 \leftrightarrow \psi_2) \to (\mathbf{SPR}(\varphi_1, \psi_1) \leftrightarrow \mathbf{SPR}(\varphi_2, \psi_2))$ (Replacement of Necessary Equivalents),
4. $\neg\mathbf{SPR}(\varphi, \varphi)$ (Syntactic Counterpart of Irreflexivity),
5. $(\mathbf{SPR}(\varphi_1, \varphi_2) \wedge \mathbf{SPR}(\varphi_3, \varphi_4)) \to (\mathbf{SPR}(\varphi_1, \varphi_4) \vee \mathbf{SPR}(\varphi_3, \varphi_2))$ (Syntactic Counterpart of Intervality),
6. $(\mathbf{SPR}(\varphi_1, \varphi_2) \wedge \mathbf{SPR}(\varphi_2, \varphi_3)) \to (\mathbf{SPR}(\varphi_1, \varphi_4) \vee \mathbf{SPR}(\varphi_4, \varphi_3))$ (Syntactic Counterpart of Semitransitivity),
7. Modus Ponens,
8. Necessitation. ■

Provability

Definition (Provability)

- A proof of $\varphi \in \Phi_{\mathsf{TUMPL}}$ is a finite sequence of $\mathcal{L}_{\mathsf{TUMPL}}$-formulae having φ as the last formula such that either each formula is an instance of an axiom, or it can be obtained from formulae that appear earlier in the sequence by applying an inference rule.

Provability

Definition (Provability)

- A proof of $\varphi \in \Phi_{\mathsf{TUMPL}}$ is a finite sequence of $\mathcal{L}_{\mathsf{TUMPL}}$-formulae having φ as the last formula such that either each formula is an instance of an axiom, or it can be obtained from formulae that appear earlier in the sequence by applying an inference rule.
- If there is a proof of φ, we write $\vdash_{\mathsf{TUMPL}} \varphi$. ■

Soundness

- We prove the metatheorems of TUMPL.

Soundness

- We prove the metatheorems of TUMPL.
- It is easy to prove the soundness of TUMPL.

Soundness

- We prove the metatheorems of TUMPL.
- It is easy to prove the soundness of TUMPL.

Theorem (Soundness)

For any $\varphi \in \Phi_{\mathcal{L}_{\text{TUMPL}}}$, *if* $\vdash_{\text{TUMPL}} \varphi$, *then* $\models_{\text{TUMPL}} \varphi$. ■

Completeness

- We now turn to the task of proving the completeness of TUMPL.

Completeness

- We now turn to the task of proving the completeness of TUMPL.
- We prove it by developing the idea of Segerberg ([Segerberg 1971]) that we modify filtration theory in such a way that completeness can be established by Corollary(Representation on Finite Boolean Algebra).

Segerberg, K.:
Qualitative Probability in a Modal Setting. in Fenstad, J. E. (ed.):
Proceedings of the Second Scandinavian Logic Symposium. North-Holland, Amsterdam (1971) 341–352.

Completeness

- We now turn to the task of proving the completeness of TUMPL.
- We prove it by developing the idea of Segerberg ([Segerberg 1971]) that we modify filtration theory in such a way that completeness can be established by Corollary(Representation on Finite Boolean Algebra).

Segerberg, K.:
Qualitative Probability in a Modal Setting. in Fenstad, J. E. (ed.):
Proceedings of the Second Scandinavian Logic Symposium. North-Holland, Amsterdam (1971) 341–352.

Theorem (Completeness)

For any $\varphi \in \Phi_{\mathcal{L}_{\mathsf{TUMPL}}}$, *if* $\models_{\mathsf{TUMPL}} \varphi$, *then* $\vdash_{\mathsf{TUMPL}} \varphi$. ■

Decidability

- TUMPL has the finite model property that every non-theorem of TUMPL fails in a structured Kripke model for preference with only finitely many elements.

Decidability

- TUMPL has the finite model property that every non-theorem of TUMPL fails in a structured Kripke model for preference with only finitely many elements.
- So we can prove the decidability of TUMPL.

Decidability

- TUMPL has the finite model property that every non-theorem of TUMPL fails in a structured Kripke model for preference with only finitely many elements.
- So we can prove the decidability of TUMPL.

Theorem (Decidability)

TUMPL *is decidable.* ■

Structure of This Talk

1. Introduction
2. Weak Orders as Global Rationality and Semiorders as Bounded Rationality
3. Threshold Utility Maximiser's Preference Logic TUMPL
   - Language of TUMPL
   - Semantics of TUMPL
   - Syntax of TUMPL
   - Metalogic of TUMPL
4. Summary, Further Investigation and Our Related Work

Summary and Further Investigation

- SUMMARY: In this talk we have proposed a new version of complete and decidable extrinsic preference logic–threshold utility maximiser's preference logic (TUMPL) that can solve the Nontransitivity Problem and avoid the Fundamental Problem of Intrinsic Preference.

Summary and Further Investigation

- SUMMARY: In this talk we have proposed a new version of complete and decidable extrinsic preference logic–threshold utility maximiser's preference logic (TUMPL) that can solve the Nontransitivity Problem and avoid the Fundamental Problem of Intrinsic Preference.
- FURTHER INVESTIGATION: This talk is only a part of a larger measurement-theoretic study. We are now trying to construct such logics as dyadic deontic logic, logic for goodness and badness, and logic of questions and answers by means of measurement theory.

Our Related Work

Satoru Suzuki:
Preference Logic and Its Measurement-Theoretic Semantics.
In: Accepted Papers of 8th Conference on Logic and the Foundations of Game and Decision Theory (LOFT 2008), Universiteit van Amsterdam (2008).

Satoru Suzuki:
Prolegomena to Dynamic Epistemic Preference Logic.
In: Hattori, H. et al. (eds.), New Frontiers in Artificial Intelligence, LNAI 5447, Springer-Verlag, Berlin (2009) 177–192.

Satoru Suzuki:
Measurement-Theoretic Foundation of Preference-Based Dyadic Deontic Logic.
In: He, X. et al. (eds.), Proceedings of the Second International Workshop on Logic, Rationality, and Interaction (LORI-II), LNAI 5834, Springer-Verlag, Berlin, (2009) 278–291.

Satoru Suzuki:
Measurement-Theoretic Foundation of Logic for Goodness and Badness.
In: Bekki, D. (ed.), Proceedings of the Sixth Workshop on Logic and Engineering of Natural Language Semantics (LENLS 2009), JSAI, (2009) 1–14.

Structure of This Talk

1. Introduction
2. Weak Orders as Global Rationality and Semiorders as Bounded Rationality
3. Threshold Utility Maximiser's Preference Logic TUMPL
   - Language of TUMPL
   - Semantics of TUMPL
   - Syntax of TUMPL
   - Metalogic of TUMPL
4. Summary, Further Investigation and Our Related Work

Thank You for Your Attention!