A Logical Typology of Normative Systems

(and its relation to deontic logic)

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The theoretical context

This talk is a continuation of the theoretical approach given in the following works:

Carlos E. Alchourrón and Eugenio Bulygin.
The expressive conception of norms.
In R. Hilpinen (ed.)*New Studies in Deontic Logic*, pp. 95–125, D. Reidel Publishing Company, Dordrecht, 1981.

John Broome.
Requirements.
In T. Rønnow-Rasmussen, B. Petersson, J. Josefsson, and D. Egonsson, editors,*Homage a Wlodek: Philosophical Papers Dedicated to Wlodek Rabinowicz*, pages 1–41. Lunds universitet, Lund, 2007.
http://www.fil.lu.se/hommageawlodek.

Georg Henrik von Wright.
Deontic logic: a personal view.
Ratio Juris, **12**: 26–38, 1999.

Set theoretic approach in the logical theory of normative systems

Set theoretical approach

1. introduced by Alchourrón and Bulygin (1981): the "force" of norm is represented by the membership of its norm-content in the set (normative system);
2. discussed as a possible interpretation of deontic logic by von Wright (1999);
3. generalized by treating the sets of norm-contents as values of code functions by Broome (2007).

Quote (C. Alchourrón and E. Bulygin, 1981)

We ... define the concept of a normative system as the set of all the propositions that are consequences of the explicitly commanded propositions.

Quote (G. H. von Wright, 2007)

... classic deontic logic, on the descriptive interpretation of its formulas, pictures a gapless and contradiction-free system of norms.

Generalization of the set-theoretic approach

Quote

We must allow for the possibility that the requirements you are under depend on your circumstances. . . . There is a set of worlds, at each of which propositions have a truth value. The values of all propositions at a particular world conform to the axioms of propositional calculus. For each source of requirements s, each person *i* and each world *w*, there is a set of propositions $\mathrm{k_s}(i, w)$, which is to be interpreted as the set of things that s requires of *i* at *w*. Each proposition in the set is a required proposition. The function $\mathrm{k_s}$ from *i* and *w* to $\mathrm{k_s}(i, w)$ I shall call s's *code* of requirements".
Broome [2007] p. 14

Code function

- Code is ternary function:

$$k_{\boxed{1}}\left(\boxed{2},\boxed{3}\right)=\boxed{4}$$

- Input:
  1. A normative source[1]
  2. An agent
  3. A world
- Output
  4. A set of sentences.

[1] Broome does not explicate the notion of different normative sources but introduces the notion by the way of examples ("survival," "prudence" and "rationality"). I will not give explication for the notion of normative sources either, but I will give a sketch of the notion that was implicit in my thoughts. Normative sources are: formal and material. Formal normative sources regulate relations between intentional states either within one category (e.g. theoretical rationality) or between categories (e.g. practical rationality). Material normative sources are those that require a specific content to be present in an intentional state. I posit theoretical type of normative source as requiring certain beliefs, and practical type of normative source as requiring certain desires and decisions (intentions).

Preliminary steps

- Metanormative theory speaks about the language $\mathcal{L}_{\mathrm{n}}$, the language in which the norm-contents are expressed.
- $\mathcal{L}_{\mathrm{n}}$ is a language of propositional logic whose formative syntax also allows modalities: B_i for '*i* believes that', D_i for '*i* desires that', I_i for '*i* intends that'.

Definition

The normative language $\mathcal{L}_{\mathrm{n}}$ is built over the base language of propositional logic $\mathcal{L}_{PL}$:

$$\text{Sentences of } \mathcal{L}_{PL} ::= \text{propositional letters} \mid \neg\varphi \mid (\varphi \wedge \psi)$$

Let $i \in A$, $\mathrm{X} = \mathrm{B}, \mathrm{D}, \mathrm{I}$, and $p \in \mathcal{L}_{PL}$

$$\text{Sentences of } \mathcal{L}_{\mathrm{n}} ::= p \mid [\mathrm{X}_i]\varphi \mid \neg\varphi \mid (\varphi \wedge \psi)$$

The definitions of truth-functional connectives are standard.

Quasi-literals

Remark

The sentences of $\mathcal{L}_\mathrm{n}$ whose main operator is $[\mathrm{B}_i]$, $[\mathrm{D}_i]$, or $[\mathrm{I}_i]$ will be termed 'modals'.

Definition

The set $\mathrm{lit}(\mathcal{L}_\mathrm{n})$ of quasi-literals with respect to propositional logic is the smallest subset of $\mathcal{L}_\mathrm{n}$ containing the set of propositional letters and their negations, and the set of modals and their negations.

Considered in isolation, the language $\mathcal{L}_\mathrm{n}$ is not committed to any particular logic. Still, if a subset of $\mathcal{L}_\mathrm{n}$ has a logical property definable within some particular logic $< l >$, than that property will be noted as $< l >$-property.

Extension to infinitary language

For theoretical purposes the language $\mathcal{L}_{\mathrm{n}}$ will be extended to the language $\mathcal{L}_{\mathrm{n}(\omega_1)}$ of a variant of infinitary logic which has the same symbols as $\mathcal{L}_{\mathrm{n}}$, but in $\mathcal{L}_{\mathrm{n}(\omega_1)}$ the conjunction symbol $\bigwedge$ may be applied to subsets of the set of literals $\mathrm{lit}(\mathcal{L}_{\mathrm{n}})$.

Definition

Let $p \in \mathcal{L}_{\mathrm{n}}$ and let $x \subseteq \mathrm{lit}(\mathcal{L}_{\mathrm{n}})$ be countably infinite.

$$\text{Sentences of } \mathcal{L}_{\mathrm{n}(\omega_1)} ::= p \mid \bigwedge x \mid \neg\varphi \mid (\varphi \wedge \psi)$$

Extension of deduction rules and valuation function to infinitary propositional logic

For theoretical purposes the deductive system $\vdash_{\mathrm{pl}}$ (e.g. natural deduction) of propositional logic will be extended to an *ad hoc* variant of infinitary propositional logic $\vdash_{\mathrm{pl}(\omega_1)}$ containing the rules of $\vdash_{\mathrm{pl}}$ and the additional rules for the countably infinite conjunctions of literals.

According to the grammar of $\mathcal{L}_{\mathrm{n}}$ the introduction and elimination rules for $\bigwedge$ are applicable to the sets of literals only:

for $x \subseteq lit(\mathcal{L}_n)$,

1. $\Gamma, \bigwedge x \vdash_{\mathrm{pl}(\omega_1)} p$ for all $p \in x$, and
2. if $\Gamma \vdash_{\mathrm{pl}(\omega_1)} p$ for all $p \in x$, then $\Gamma \vdash_{\mathrm{pl}(\omega_1)} \bigwedge x$

On the side of semantics, the definition of the truth assignment $\hat{h}$ is extended in an obvious way: $\hat{h}(\bigwedge x) = \mathrm{t}$ iff $\hat{h}(p) = \mathrm{t}$ for all $p \in x$.

Conservative extension

The *ad hoc* system $\vdash_{\mathrm{pl}(\omega_1)}$ is a conservative extension of $\vdash_{\mathrm{pl}}$.

Proposition

For $x \cup \{p\} \subseteq \mathcal{L}_{\mathrm{n}}$, if $x \vdash_{\mathrm{pl}(\omega_1)} p$, then $x \vdash_{\mathrm{pl}} p$.

Proof.

The proof will be sketched. Assume $x \vdash_{\mathrm{pl}(\omega_1)} p$. The deductive system $\vdash_{\mathrm{pl}(\omega_1)}$ is sound, as can be easily checked. Therefore, $x \models_{\mathrm{pl}(\omega_1)} p$. Then also $x \models_{\mathrm{pl}} p$ thanks to coincidence of the semantic definitions for sentences in $\mathcal{L}_{\mathrm{n}}$. Finally, $x \vdash_{\mathrm{pl}} p$ by the completeness of the propositional logic. □

Multi-sorted first order language $\mathcal{L}_{\mathrm{meta}}$

In order to achieve technical clarity we define a first-order metanormative language $\mathcal{L}_{\mathrm{meta}}$ in which variables of different sorts range over different objects in the domain with the following **extralogical vocabulary**:

- individual constants for normative sources, agents and worlds: $\mathrm{s}, \mathrm{s}_1, \ldots,$ $\mathrm{a}, \mathrm{a}_1, \ldots, \mathrm{v}, \mathrm{v}_1, \ldots;$
- function symbols for code of requirement, propositional logic consequence, and "axiomatic basis (of a modal logic)" function: k^3, Cn^1, l^1;
- function symbols for generating sentential forms occurring in the object language: neg^1, conj^2, and a set of symbols $\mathrm{mod}^1_{\mathrm{B}i}$, $\mathrm{mod}^1_{\mathrm{D}i}$, $\mathrm{mod}^1_{\mathrm{I}i}$ for each $i = \mathrm{a}, \mathrm{a}_1, \ldots,$ and $\mathrm{infconj}^2$;
- function symbols for extraction of "quasi-literals" from a given set: lt^1;
- dyadic predicate symbol for relation of membership: $\in^2$;
- and additionally we may introduce a dispensable part of vocabulary containing monadic predicate symbols expressing properties of being a normative source, an agent, a sentence in $\mathcal{L}_{\mathrm{n}}$, a possible world: $\mathrm{Source}^1, Ag^1, \mathrm{Sen}^1, W^1$.

Sorts of variables and shortcut notations

Variables comprise:

- "general variables" ranging over everything: $x, x_1, \ldots, y, y_1, \ldots$;
- sorts of variables:
  - $i, i_1, \ldots$ ranging over $\{x \in \mathrm{D} \mid Ag(x)\}$;
  - $p, p_1, \ldots, q, q_1, \ldots$ ranging over $\{x \in \mathrm{D} \mid \mathrm{Sent}(x)\}$;
  - $w, w_1, \ldots$ ranging over $\{x \in \mathrm{D} \mid W(x)\}$.

Notation

The shorthand notations for $\mathrm{neg}(p)$, $\mathrm{conj}(p, q)$, $\mathrm{mod}_{\mathrm{B}i}(p)$, $\mathrm{mod}_{\mathrm{D}i}(p)$, $\mathrm{mod}_{\mathrm{I}i}(p)$, $\mathrm{infconj}(x)$ are $\ulcorner \neg p \urcorner$, $\ulcorner p \wedge q \urcorner$, $\ulcorner [\mathrm{B}_i] p \urcorner$, $\ulcorner [\mathrm{D}_i] p \urcorner$, $\ulcorner [\mathrm{I}_i] p \urcorner$, $\ulcorner \bigwedge x \urcorner$. For the ease of reading "Quine quotes" will be used also for the standardly defined connectives, e.g. $\ulcorner p \rightarrow q \urcorner$ for $\mathrm{neg}(\mathrm{conj}(p, \mathrm{neg}(q)))$.
The sole variable written between "Quine quotes" is the same as the variable itself. Sometimes this redundant notation will be (ab)used in order to emphasize sentence variables and sentence functions within a formula.

Standard definitions of terms, formulas and sentences in $\mathcal{L}_{\mathrm{meta}}$

Definitions

Let c stand for individual constant, v for any variable, f for function symbol and P^n for predicate symbol.

$$\begin{array}{rcl} \text{terms } (t) & ::= & \mathrm{c} \mid \mathrm{v} \mid f(t_1, ..., t_n) \\ \text{atomic formulas } (p) & ::= & P^n(t_1, ..., t_n) \end{array}$$

Let p be an atomic formula

$$\text{Formulas of } \mathcal{L}_{\mathrm{meta}} ::= p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid \forall \mathrm{v}\ \varphi$$

Sentences of $\mathcal{L}_{\mathrm{meta}}$ are formulas of $\mathcal{L}_{\mathrm{meta}}$ with all variables bound.

Objects in the domain

The purpose of the metanormative language is to enable talking:

- about the syntax of sentences in $\mathcal{L}_{\mathrm{n}(\omega_1)}$,
- about the properties of sentences and their sets that they could have in different logics (most notably "world logic" and "intentionality logics"),
- about the semantics of sentences in $\mathcal{L}_{\mathrm{n}(\omega_1)}$, i.e. about sentence-world relation.

- The basic ontology for the code functions requires: normative sources, agents, worlds and sets of sentences.
- Some objects will be constructed using $\mathcal{L}_{\mathrm{n}(\omega_1)}$ sentences:
  - the worlds, which are theoretically identified with proposition-logically maximal consistent sets of $\mathcal{L}_{\mathrm{n}(\omega_1)}$,
  - code values, which are "logic free" sets of sentences,
  - axiomatic bases of logics, which are sets of substitutional instances of the sentences in a given set,
  - sentences, which **are** sentences.
  -

Worlds free of modal logic

Definition

$\text{MaxCon}(\mathcal{L}_{\text{n}(\omega_1)}) = \{x \subseteq \mathcal{L}_{\text{n}(\omega_1)} \mid x \nvdash_{\text{pl}(\omega_1)} \bot, \forall y \in \mathcal{L}_{\text{n}(\omega_1)}(y \notin x \rightarrow x \cup \{y\} \vdash_{\text{pl}(\omega_1)} \bot)\}$ is the set of possible worlds.

No modal axiom for belief, desire or intention does hold in all the possible worlds. Therefore, any kind and any measure of violations of logics of intentionality may occur.

Unbounded irrationality

Quote

What sets a limit to the amount of irrationality we can make psychological sense of is a purely conceptual or theoretical matter—the fact that mental states and events are the states and events they are by their location in a logical space.
Donald Davidson (2004) *Problems of Rationality*. Clarendon Press, Oxford, p. 183

In the modeling the worlds characterized by an extreme "amount of irrationality" on the side of an agent *i* are admitted in the modeling. This fact should not be interpreted as an violation of Davidson's thesis, rather it should be understood as an unrealistic but harmless and dispensable theoretical possibility.

No place for axiom T

- The T axiom ($\Box p \rightarrow p$) poses a serious threat to the modeling that keeps modality and world apart.
- If modalities obeying "reflexive" axiom T are allowed, then possible worlds, *i.e.* maximal consistent sets in propositional logic, would become intuitively impossible.
- For example, although $\{p, [\mathrm{K}]_i \neg p\}$ is pl-consistent set, we do not want to have it included in any world since no false proposition may be known as a true proposition.
- T axioms constitute an important part of the meaning of verbs of knowledge and of action. So, epistemic and praxeological modalities must be excluded from the language of norms $\mathcal{L}_{\mathrm{n}}$.
- The exclusion strategy may seem drastic. The forthcoming analysis does not depend on the inclusion of "T modalities", so this strategy may be adopted as a provisional method.

Content of the norm

- Von Wright (1963, Norm and Action : A Logical Enquiry) defined 'content of a norm' as "that which ought to or may or must not be or be done".
- The normative language $\mathcal{L}_{\mathrm{n}(\omega_1)}$ departs from von Wright's definition by taking norm-content to be *the intentional state or relation of intentional states that ought to or may or must not be present in the mind of the norm addressee on a particular occasion*.
- The reduction and the switch may seem drastic but there is a rationale for it.
  - The requirement that agent i knows that p could be replaced by $p \rightarrow [\mathrm{B}_i]p$; a required action to see to it that p could be replaced by the required intention, i.e. $[\mathrm{I}_i]p$.
  - Actions if successful require "cooperation of the world," and "the world" is not a norm addressee.

Sets of requirements as logic free theories

- Broome claims that code values are closed under pl-equivalence and he seems to tacitly hold that this congruence[2] property constitutes the whole of the logic of "source requirements". A recent proponent is Loui Goble (2009, Normative Conflicts and The Logic of Ought, *Noûs*:**43**).
- Broome bases the acceptability of the congruence principle on the argument from the absence of contrary evidence, while Goble takes it for granted since "[it] seems [to be] a minimum requirement for a logic of ought," (p. 483).
- On the other hand, Alchourrón and Bulygin [1] propose an approach that is both more restrictive and more permissive. First, contrary to Broome's weak, "congruence logic", Alchourrón and Bulygin argue that there is *no logic of norms* since the existence of a norm depends on the empirical fact of promulgation. Second, they claim that *there is a logic of normative systems* since the set of norm-contents is deductively closed.
- It seems that there is no *consensus* and no conclusive reason for presupposing the existence of any particular logical property of code values. On the other hand, a "logic free" conception of code values operates at a higher level of generality.

[2] If p and q are equivalent in propositional logic, then p is a member of a code just in case q

Axiomatic bases for logics of intentionality

If the normative character the intentionality realm consists in its subjection to requirements of different normative sources and if rationality is a normative source, then some logic for rational relations between intentional states will be needed. On the other hand, a code function may deliver sets having logical properties other from those definable within propositional logic.

Sets of substitutional instances

The uniform substitutions are restricted to the finitary, i.e. $\mathcal{L}_{\mathrm{n}}$ part of the metanormative language since infintary sentences are not allowed to embed within each other.

Definitions

Any function g from $\mathcal{L}_{\mathrm{n}}$ to $\mathcal{L}_{\mathrm{n}(\omega_1)}$ is a substitution function iff (i) $g(p) \in \mathcal{L}_{\mathrm{n}(\omega_1)}$ if p is a propositional letter, (ii) $g(\neg p) = \neg g(p)$, (iii) $g(p \wedge q) = g(p) \wedge g(q)$, (iv) $g([\mathrm{X}_i]p) = [\mathrm{X}_i]g(p)$ for $\mathrm{X} = \mathrm{B}, \mathrm{D}, \mathrm{I},\ i \in \mathrm{A}$. The set Sb is the set of all substitution functions. The set of all substitutional instances of the sentences in a given set $x \subseteq \mathcal{L}_{\mathrm{n}}$ is the set $\mathrm{l}(x) = \{q \mid \exists p \exists f(p \in x \wedge f \in \mathrm{Sb} \wedge f(p) = q)\}$.

Definition

The set $\mathrm{Cn}(\mathrm{l}(x)) = \{p \mid \exists y(y \subseteq \mathrm{l}(x) \wedge y \vdash_{\mathrm{pl}(\omega_1)} p\}$ is the logic for axiomatic basis x.

Digression: normal logics

Definition

Let $\top_{[\mathrm{X}_i]}$ and $\mathrm{K}_{[\mathrm{X}_i]}$ denote axiom schemata $((p \vee \neg p) \leftrightarrow q) \rightarrow [\mathrm{X}_i]q$ and $[\mathrm{X}_i](p \rightarrow q) \rightarrow ([\mathrm{X}_i]p \rightarrow [\mathrm{X}_i]q)$ respectively. A set $\mathrm{Cn}(\mathrm{l}(x))$ is a normal logic for a set of modal operators $\mathrm{o}/x \subseteq \{[\mathrm{X}_i] \mid \mathrm{X} = \mathrm{B}, \mathrm{D}, \mathrm{I},\ i \in \mathrm{A},\ [\mathrm{X}_i]$ occurs in some $p \in x\}$ iff

$$\mathrm{Cn}(\mathrm{l}(\{\top_x \mid x \in \mathrm{o}/x\} \cup \{\mathrm{K}_x \mid x \in \mathrm{o}/x\})) \subseteq \mathrm{Cn}(\mathrm{l}(x))$$

The domain

- The domain for metanormative language $\mathcal{L}_{\text{meta}}$ comprises the following objects:
  - normative sources, $x \in \mathrm{S}$
  - agents, $x \in \mathrm{A}$
  - sentences, $x \in \mathcal{L}_{\mathrm{n}(\omega_1)}$,
  - sets of sentences (code values, and axiomatic bases for logics), $x \in \wp\mathcal{L}_{\mathrm{n}(\omega_1)}$
  - worlds, $x \in \mathrm{MaxCon}(\mathcal{L}_{\mathrm{n}(\omega_1)}) \subseteq \wp\mathcal{L}_{\mathrm{n}(\omega_1)}$

Definition

$\mathrm{D} = \mathrm{S} \cup \mathrm{A} \cup \mathcal{L}_{\mathrm{n}(\omega_1)} \cup \wp\mathcal{L}_{\mathrm{n}(\omega_1)}$
where $\mathrm{S} \neq \emptyset$, $\mathrm{A} \neq \emptyset$, $\mathrm{O} \cap \mathrm{A} = \emptyset$.

Interpretation function (1st part)

Interpretation function $\mathcal{I}$ for $\mathcal{L}_{\text{meta}}$:

- (Names of sources) $\mathcal{I}(\mathrm{s}_i) \in \mathrm{S}$,
- (Code function) $\mathcal{I}(\mathrm{k})$ is a function: $\mathrm{S} \times \mathrm{A} \times \mathrm{MaxCon}(\mathcal{L}_{\mathrm{n}(\omega_1)}) \to \wp\mathcal{L}_{\mathrm{n}(\omega_1)}$,
- (Axiomatic basis function) $\mathcal{I}(\mathrm{l})$ is a function: $\wp\mathcal{L}_{\mathrm{n}} \to \wp\mathcal{L}_{\mathrm{n}(\omega_1)}$, such that for any $x \subseteq \mathcal{L}_{\mathrm{n}}$, $\mathrm{l}(x) = \{f(p) \mid p \in x \wedge f \in \mathrm{Sb}\}$
- (pl-consequence function) $\mathcal{I}(\mathrm{Cn})$ is a function: $\wp\mathcal{L}_{\mathrm{n}(\omega_1)} \to \wp\mathcal{L}_{\mathrm{n}(\omega_1)}$, such that for any $x \subseteq \mathcal{L}_{\mathrm{n}(\omega_1)}$, $\mathrm{Cn}(x) = \{y \in \mathcal{L}_{\mathrm{n}(\omega_1)} \mid x \vdash_{\mathrm{pl}(\omega_1} y\}$.

Interpretation function (2nd part)

Sentence forms

- $\mathcal{I}(\mathrm{neg})$, $\mathcal{I}(\mathrm{conj})$, $\mathcal{I}(\mathrm{mod}_{\mathrm{B}i})$, $\mathcal{I}(\mathrm{mod}_{\mathrm{D}i})$, $\mathcal{I}(\mathrm{mod}_{\mathrm{I}i})$, $\mathcal{I}(\mathrm{infconj})$ are functions: $\mathcal{L}_{\mathrm{n}(\omega_1)} \to \mathcal{L}_{\mathrm{n}(\omega_1)}$, such that

$$
\begin{array}{rcl}
\mathcal{I}(\mathrm{neg}) & = & \{\langle x, y\rangle \mid y = \neg \frown x\} \\
\mathcal{I}(\mathrm{conj}) & = & \{\langle x, y, z\rangle \mid z = x \frown \wedge \frown y\} \\
\mathcal{I}(\mathrm{mod}_{\mathrm{B}i}) & = & \{\langle x, y\rangle \mid y = \mathrm{B}_i \frown x\} \\
\mathcal{I}(\mathrm{mod}_{\mathrm{D}i}) & = & \{\langle x, y\rangle \mid y = \mathrm{D}_i \frown x\} \\
\mathcal{I}(\mathrm{mod}_{\mathrm{I}i}) & = & \{\langle x, y\rangle \mid y = \mathrm{I}_i \frown x\} \\
\mathcal{I}(\mathrm{infconj}) & = & \left\{ \langle x, y\rangle \mid \begin{array}{c} x \subseteq \mathrm{lt}(\mathcal{L}_{\mathrm{n}}) \wedge \\ y = seq(x)(1) \frown \wedge \frown \\ \ldots \frown \wedge \frown seq(x)(n) \frown \wedge \frown \ldots) \end{array} \right\}
\end{array}
$$

where $seq(x) \in \prod_{i,j \in \mathbb{N}}^{funct} x$

and where $\prod_{i \in \mathbb{N}}^{funct} x = \{f : \mathbb{N} \to x | \forall i \forall j \; f(i) \neq f(j)\}$.

Interpretation function (3rd part)

- (extraction of quasi-literals) lt is the function: $\wp\mathcal{L}_{\mathrm{n}(\omega_1)} \to \wp\mathcal{L}_{\mathrm{n}(\omega_1)}$ such that for any $x \subseteq \mathcal{L}_{\mathrm{n}(\omega_1)}$, $\mathrm{lt}(x) = \{y \in \mathcal{L}_{\mathrm{n}} \mid y \in x \wedge y \in \mathrm{lit}(\mathcal{L}_{\mathrm{n}})\}$,
- (superfluous predicates)
    - (Source predicate) $\mathcal{I}(\mathrm{Source}) = \mathrm{S}$,
    - (Agent predicate) $\mathcal{I}(A) = \mathrm{A}$,
    - (Sentence predicate) $\mathcal{I}(\mathrm{Sen}) = \mathcal{L}_{\mathrm{n}}$,
    - (World predicate) $\mathcal{I}(\mathrm{W}) = \mathrm{MaxCon}(\mathcal{L}_{\mathrm{n}})$,
- (Relation of having a property corresponding to a source s in a world) $\mathcal{I}(\mathrm{K}_s) \subseteq \mathrm{O} \times \mathrm{MaxCon}(\mathcal{L}_{\mathrm{n}(\omega_1)})$,
- (Membership relation) $\mathcal{I}(\in) \subseteq \mathcal{L}_{\mathrm{n}(\omega_1)} \times \wp\mathcal{L}_{\mathrm{n}(\omega_1)} \cup \mathrm{A} \times \wp\mathrm{A} \cup \mathrm{S} \times \wp\mathrm{S}$.

Variable assignment

Definition

$\mathfrak{M}_{\mathrm{mn}} = \langle \mathrm{D}, \mathcal{I} \rangle$

Definition

Variable assignment g in $\mathfrak{M}_{\mathrm{mn}} = \langle \mathrm{D}, \mathcal{I} \rangle$ is possibly partial function g such that for any variable v

$$g(v) \in \mathrm{D} \text{ if } v \in domain(g)$$

For sorts of variables: (world variables) $g(v) \in \mathrm{MaxCon}(\mathcal{L}_{\mathrm{n}})$ if $v = w, w_1, \ldots$; (sentence variables) $g(v) \in \mathcal{L}_{\mathrm{n}}$ if $v = p, p_1, \ldots, q, q_1, \ldots$, (agent variables) $g(v) \in \mathrm{A}$.

Definition

The variable assignment g is appropriate for formula p iff all free variables in p are in the domain of g .

Singular terms

Notation

By $g_{[x/d]}$ we denote the variable assignment that differs from g on at most x:

$$g_{[x/d]}(v) = \begin{cases} g(v), \text{ if } x \neq v \\ d, \text{ otherwise.} \end{cases}$$

The special case of the empty variable assignment $g_{\emptyset}$:

$$range(g_{\emptyset}) = \emptyset$$

Definitions

$$\mathcal{I}(f)(x_1, .., x_n) = \begin{cases} y, \text{ if } \langle x_1, .., x_n, y\rangle \in \mathcal{I}(f) \\ \text{undefined, otherwise.} \end{cases}$$

$$[\![t]\!]_g^{\mathfrak{M}_{mn}} = \begin{cases} \mathcal{I}(t), \text{ if } t \text{ is an individual constant} \\ g(t), \text{ if } t \text{ is an individual variable} \\ \mathcal{I}(f)([\![t_1]\!]_g^{\mathfrak{M}_{mn}}, \ldots, [\![t_n]\!]_g^{\mathfrak{M}_{mn}}), \text{ if } t \text{ is } f(t_1, ..., t_n) \end{cases}$$

Satisfaction and truth in $\mathfrak{M}_{mn}$

Definition

(Satisfaction) Let g be an assignment in $\mathfrak{M}_{\mathrm{mn}}$ that is appropriate for the formulas being evaluated.

$$\begin{array}{lll}
\mathfrak{M}_{mn} & \models & P(t_1, ..., t_n)\ [g] \text{ iff } \langle [\![t_1]\!]_g^{\mathfrak{M}_{mn}}, \ldots, [\![t_n]\!]_g^{\mathfrak{M}_{mn}} \rangle \in \mathcal{I}(P) \\
\mathfrak{M}_{mn} & \models & \neg\varphi\ [g] \text{ iff not } \mathfrak{M}_{mn} \models \varphi\ [g] \\
\mathfrak{M}_{mn} & \models & (\varphi \wedge \psi)\ [g] \text{ iff } \mathfrak{M}_{mn} \models \varphi\ [g] \text{ and } \mathfrak{M}_{mn} \models \psi\ [g] \\
\mathfrak{M}_{mn} & \models & \forall v\, \varphi\ [g] \text{ iff for all } \mathrm{d} \in \mathrm{D}, \mathfrak{M}_{mn} \models \varphi\ [g_{[v/d]}]
\end{array}$$

Definition (Truth in a metanormative model)

Formula φ is true in $\mathfrak{M}_{mn}$ iff $g_\emptyset$ satisfies φ in $\mathfrak{M}_{mn}$: $\mathfrak{M}_{mn} \models \varphi$ iff $\mathfrak{M}_{mn} \models \varphi\, [g_\emptyset]$

Properties of a code within "world logic"

Quantifications over different argument positions in the code function enable a number of interesting type distinctions, some of which will be introduced below using a $\mathcal{L}_{\mathrm{meta}}$ formula in the *definiens*.
Some pl logical properties of codes:

- $\mathrm{k_s}$ is a pl-*congruent* code iff

$$\ulcorner p \leftrightarrow q \urcorner \in \mathrm{Cn}(\varnothing) \rightarrow (p \in \mathrm{k_s}(i, w) \leftrightarrow q \in \mathrm{k_s}(i, w))$$

- $\mathrm{k_s}$ is a pl-*consistent* code iff

$$\exists w_2 \, \mathrm{k_s}(i, w_1) \subseteq w_2$$

- $\mathrm{k_s}$ is a pl-*deductively closed* iff

$$\mathrm{k_s}(i, w) = \mathrm{Cn}(\mathrm{k_s}(i, w))$$

Codes and the world

Some world related code properties:

- $\mathrm{k_s}$ code is *futile* iff

$$\forall i \forall w \; \mathrm{k_s}(i, w) \subseteq w$$

- $\mathrm{k_s}$ is an *achievable* code iff

$$\exists w \, \mathrm{k_s}(i, w) \subseteq w$$

- $\mathrm{k_s}$ is a *relativistic* code iff

$$\exists i \exists w_1 \exists w_2 \, \mathrm{k_s}(i, w_1) \neq \mathrm{k_s}(i, w_2)$$

- a code is *absolute* iff it is not relativistic

Social aspects of codes

Some social code properties:

- $\mathrm{k_s}$ is a *socially consistent* code iff

$$\exists w_2 \, \mathrm{k_s}(i_1, w_1) \cup \mathrm{k_s}(i_2, w_1) \subseteq w_2$$

- $\mathrm{k_s}$ is an *socially achievable* code (for group G) iff

$$\exists w \, \forall i (i \in G \rightarrow \mathrm{k_s}(i, w) \subseteq w)$$

Relations between normative sources

Some relations between normative sources:

- codes $\mathrm{k}_{\mathsf{s}_m}$ and $\mathrm{k}_{\mathsf{s}_n}$ are *realization-equivalent* iff

$$\mathrm{k}_{\mathsf{s}_m}(i, w) \subseteq w \leftrightarrow \mathrm{k}_{\mathsf{s}_n}(i, w) \subseteq w$$

- codes $\mathrm{k}_{\mathsf{s}_m}$ and $\mathrm{k}_{\mathsf{s}_n}$ are *compatible* iff

$$\exists w_1 \exists w_2\, \mathrm{k}_{\mathsf{s}_m}(i, w_1) \cup \mathrm{k}_{\mathsf{x}_n}(i, w_1) \subseteq w_2$$

- code $\mathrm{k}_{\mathsf{s}_m}$ is *maximally compatible* iff

$$\forall x \exists w_1 \exists w_2\, \mathrm{k}_{\mathsf{s}_m}(i, w_1) \cup \mathrm{k}_{\mathsf{x}}(i, w_1) \subseteq w_2$$

Codes and logics

Some $\mathrm{l}(x)$ logical properties of codes:

- code is *consistent with respect to* $\mathrm{l}(x)$ iff

$$\exists w_2\ \mathrm{Cn}(\mathrm{l}(x) \cup \mathrm{k_s}(i, w_1)) \subseteq w_2$$

- code is *logic* iff

$$\exists x\ \mathrm{k_s}(i, w) = \mathrm{Cn}(\mathrm{l}(x))$$

- code is *deductively closed with respect to logic* $\mathrm{Cn}(\mathrm{l}(x))$ ("more than a logic") iff

$$\exists x \exists y (\neg y \subseteq \mathrm{Cn}(\mathrm{l}(x)) \wedge \mathrm{k_s}(i, w) = \mathrm{Cn}(\mathrm{l}(x) \cup y))$$

- code is *less than a logic* iff

$$\exists x \exists y (\neg y \subseteq \mathrm{Cn}(\mathrm{l}(x)) \wedge \mathrm{k_s}(i, w) = \mathrm{Cn}(\mathrm{l}(x) \cup y) - \mathrm{Cn}(\mathrm{l}(x))).$$

KD deontic logic without iterated modalities

Definition

Let $p \in \mathcal{L}_{PL}$ be a formula of propositional logic:

$$\text{Formulas of } \mathcal{L}^{\mathrm{O}}_{\mathrm{KD}} ::= p \mid \mathrm{O}p \mid \mathrm{P}p \mid \neg\varphi \mid (\varphi \wedge \psi)$$

Let us introduce the translation τ^1 from the restricted language $\mathcal{L}^{\mathrm{O}}_{\mathrm{KD}}$ to the metanormative language $\mathcal{L}_{\mathrm{meta}}$, where $\mathrm{O}\,p$ and $\mathrm{P}\,p$ will be translated as '*a* in v has *s*-obligation (*s*-permission) to *p*'.

Two-step translation: typo

Remark

Typo in the booklet: instead in τ "Quine quotes" should have been introduced in τ^1.

Two-step translation

Definitions

Function τ maps sentences from the fragment $\mathcal{L}^{\mathrm{O}}_{\mathrm{KD}} \cap \mathcal{L}_{PL}$ to the set of sentential variables and sentential function terms of $\mathcal{L}_{\mathrm{meta}}$:

$$\begin{array}{rcll} \tau(l) & \in & \{p, p_1, \ldots, q, q_1, \ldots\} & \text{for propositional letters } l \in \mathcal{L}_{PL} \\ \tau(\neg\varphi) & = & \neg\tau(\varphi) & \\ \tau(\varphi \wedge \psi) & = & (\tau(\varphi) \wedge \tau(\psi)) & \end{array}$$

Translation $\tau^1 : \mathcal{L}^{\mathrm{O}}_{\mathrm{KD}} \to \mathcal{L}_{\mathrm{meta}}$

$$\begin{array}{rcll} \tau^1(p) & = & \ulcorner\tau(p)\urcorner \in \mathrm{v} & \text{if } p \in \mathcal{L}_{PL} \\ \tau^1(\mathrm{O}\varphi) & = & \ulcorner\tau(\varphi)\urcorner \in \mathrm{k_s}(\mathrm{a}, \mathrm{v}) & \\ {}^1(\mathrm{P}\varphi) & = & \ulcorner\tau(\neg\varphi)\urcorner \notin \mathrm{k_s}(\mathrm{a}, \mathrm{v}) & \\ \tau^1(\neg\varphi) & = & \neg\tau^1(\varphi) & \\ \tau^1(\varphi \wedge \psi) & = & (\tau^1(\varphi) \wedge \tau^1(\psi)) & \end{array}$$

An example

Example

$$
\begin{aligned}
\tau^1(\mathrm{O}\neg p \rightarrow \mathrm{O}\neg(p \wedge q)) \quad &\Leftrightarrow \quad \tau^1(\mathrm{O}\neg p) \rightarrow \tau^1(\mathrm{O}\neg(p \wedge q)) \\
&\Leftrightarrow \quad \ulcorner \tau(\neg p) \urcorner \in \mathrm{k_s}(\mathrm{a}, \mathrm{v}) \rightarrow \ulcorner \tau(\neg(p \wedge q)) \urcorner \in \mathrm{k_s}(\mathrm{a}, \mathrm{v}) \\
&\Leftrightarrow \quad \ulcorner \neg\tau(p) \urcorner \in \mathrm{k_s}(\mathrm{a}, \mathrm{v}) \rightarrow \ulcorner \neg\tau(p \wedge q) \urcorner \in \mathrm{k_s}(\mathrm{a}, \mathrm{v}) \\
&\Leftrightarrow \quad \ulcorner p \urcorner \in \mathrm{k_s}(\mathrm{a}, \mathrm{v}) \rightarrow \ulcorner \neg(\tau(p) \wedge \tau(q)) \urcorner \in \mathrm{k_s}(\mathrm{a}, \mathrm{v}) \\
&\Leftrightarrow \quad \ulcorner p \urcorner \in \mathrm{k_s}(\mathrm{a}, \mathrm{v}) \rightarrow \ulcorner \neg(p \wedge q) \urcorner \in \mathrm{k_s}(\mathrm{a}, \mathrm{v})
\end{aligned}
$$

Narrow and wide scope reading of conditional obligation

Example

There are two interpretations of conditional obligation in standard deontic logic: (N-scope) "narrow scope interpretation" ("if p is the case, then q ought to be the case"): $p \rightarrow \mathrm{O}q$, 2. (W-scope) "wide scope interpretation" ("it ought to be the case that: if p is the case, then q is the case") or $\mathrm{O}(p \rightarrow q)$. Narrow scope formula, i.e. $p \rightarrow \mathrm{O}q$ is translated as $p \in \mathrm{v} \rightarrow q \in \mathrm{k_s}(\mathrm{a}, \mathrm{v})$. Wide scope formula, i.e. $\mathrm{O}(p \rightarrow q)$ is translated as $\ulcorner p \rightarrow q \urcorner \in \mathrm{k_s}(\mathrm{a}, \mathrm{v})$.
There is a tendency between natural language speakers to consider (N-scope) and (W-scope) expressions as equivalent.

Wide and narrow scope reading *vs.* absolute and relativistic codes

The impression of equivalence in meaning is justified by two theoretically derived facts.

1. A code $\mathrm{k_s}(\mathrm{a}, \mathrm{v})$ has its conditionalized variant $\mathrm{k_s}(\mathrm{a}, \mathrm{v})$ (Proposition below)

$$\underbrace{(\ulcorner p \urcorner \in w \rightarrow \ulcorner q \urcorner \in \mathrm{k_s}(\mathrm{a}, w))}_{\text{narrow scope}}$$
$$\leftrightarrow$$
$$\underbrace{(\ulcorner p \urcorner \in \mathrm{Cn}(\{\ulcorner \bigwedge \mathrm{lt}(w) \urcorner\}) \rightarrow \ulcorner \bigwedge \mathrm{lt}(w) \rightarrow q \urcorner \in \mathrm{k}_{\mathrm{s}}^{cond}(\mathrm{a}, w))}_{\text{wide scope (generalized)}}$$

(right side translation: in all *p*-worlds there is a norm content stating that in a such a world *q* is a case).

2. A code and its conditionalized variant are realization equivalent. Therefore, from the behavioristic of view there is no difference between them.

KD as a descriptive theory of consistent and deductively closed code values

Quote

. . . classical deontic logic, on the descriptive interpretation of its formulas, pictures a gapless and contradiction-free system of norms.
Von Wright [1999] p. 32

According to our translation scheme von Wright's claim should be appended: classical deontic logic "pictures a system of norms" that is deductively closed too.

The translation of the axioms

- "gaplessness" condition $\mathrm{P}p \vee \mathrm{O}\neg p$ translates to $\ulcorner\neg p\urcorner \notin \mathrm{k_s}(\mathrm{a},\mathrm{v}) \vee \ulcorner\neg p\urcorner \in \mathrm{k_s}(\mathrm{a},\mathrm{v})$ and that property obviously holds for any set of requirements whatsoever;
- K axiom becomes $\ulcorner p \to q\urcorner \in \mathrm{k_s}(\mathrm{a},\mathrm{v}) \to (p \in \mathrm{k_s}(\mathrm{a},\mathrm{v}) \to q \in \mathrm{k_s}(\mathrm{a},\mathrm{v}))$ and that property holds for any pl-deductively closed set;
- D axiom becomes $p \in \mathrm{k_s}(\mathrm{a},\mathrm{v}) \to \ulcorner\neg p\urcorner \notin \mathrm{k_s}(\mathrm{a},\mathrm{v})$ and that is just another way of stating pl-consistency;
- mutual definability, $\mathrm{P}_1 p \leftrightarrow \neg\mathrm{O}\neg p$ holds if the set of requirements is congruent.

Are there any problems?

One may ask whether these properties provide "an adequate" description of a formally sound set of requirements.

Example

For example, τ^1 translation for the D does not allow

$$[\mathrm{B}_i]p \wedge \neg[\mathrm{B}_i]p$$

to enter the set of requirements, but it does allow

$$[\mathrm{B}_i]p \wedge [\mathrm{B}_i]\neg p$$

So the question arises whether consistency property of a set of requirements is — a property that is connected to the logic of reality, or rather — a property that a set inherits when it obeys the logic of its contents (i.e. Logic of intentionality)?

Iterated operators

Although iterated deontic operators receive no translation in the scheme proposed above, one may extend the line of thought by giving additional translation rules for language of standard deontic $\mathcal{L}_{\mathrm{KD}}^{\mathrm{OO}}$ restricted to the maximum of two iterations of deontic operators, treating iterated deontic modalities as a sequence of heterogenous operators and introducing the distinction into the syntax:

$$\mathcal{L}_{\mathrm{KD}}^{\mathrm{O_2O}} ::= p \in \mathcal{L}_{\mathrm{KD}}^{\mathrm{O}} \mid \mathrm{O}_2 p \mid \mathrm{P}_2 p \mid \neg\varphi \mid (\varphi \wedge \psi)$$

Definition

Let $Sub(\varphi)_{[\frac{c_1}{x_1} \cdots \frac{c_n}{x_n}]}$ denote substitutional instance of $\varphi \in \mathcal{L}_{\mathrm{meta}}$ in which constants $c_1, \ldots, c_n$ are replaced by variables $x_1, \ldots, x_n$. Translation $\tau^2 : \mathcal{L}_{\mathrm{KD}}^{\mathrm{O_2O}} \to \mathcal{L}_{\mathrm{meta}}$

$$\begin{aligned}
\tau^2(\mathrm{O}_2 p) &= \forall i \forall w\ Sub(\tau^1(p))_{[\frac{\mathrm{a}}{i}\frac{\mathrm{v}}{w}]} && \text{for } p \in \mathcal{L}_{\mathrm{KD}}^{\mathrm{O}} \\
\tau^2(\mathrm{P}_2 p) &= \exists i \exists w\ Sub(\tau^1(p))_{[\frac{\mathrm{a}}{i}\frac{\mathrm{v}}{w}]} && \text{for } p \in \mathcal{L}_{\mathrm{KD}}^{\mathrm{O}} \\
\tau^2(\neg\varphi) &= \neg\tau^2(\varphi) \\
\tau^2(\varphi \wedge \psi) &= (\tau^2(\varphi) \wedge \tau^2(\psi))
\end{aligned}$$

Generalization over agents and worlds

Such an approach to iterated deontic modalities departs from von Wright's [3] "second order descriptive interpretation" where e.g. O_2 would stand for existence of "normative demands on normative systems" ("norms for the norm givers"). The "first order" translation τ^1 as well as the "second order" translation τ^2 give us statements in metanormative language $\mathcal{L}_{\mathrm{meta}}$ both of which may "picture" some type of "normative system". The difference lies in the fact that τ^1 gives a local picture of a set of requirements (for a particular source, agent and world) while τ^2 gives a more global picture of a code function. In the second case the properties depicted are the properties of a code function for a particular source with respect to any agent and any world.

Are iterations significant under translation?

Let us consider KD45 deontic logic! The τ^2 translations of reinterpreted axioms 4, $\mathrm{O}_1 p \to \mathrm{O}_2\mathrm{O}_1 p$ and 5, $\mathrm{P}_1 p \to \mathrm{O}_2\mathrm{P}_1 p$ amount to stating that any s-obligation and any s-permission holds universally. So, the reinterpreted axioms will hold only if s-code is absolute.

Definition

An agent i at world w has an "all-or-nothing" normative property K_s that corresponds to the source s iff the set of requirements $\mathrm{k}_\mathsf{s}(i, w)$ is satisfied in w, i.e. $\mathrm{K}_\mathsf{s}(i, w) \leftrightarrow \mathrm{k}_\mathsf{s}(i, w) \subseteq w$.

If the only way to satisfy some relativistic code and some absolute code is to satisfy them simultaneously, then these codes define the same normative property. The question arises as to whether (non)absoluteness of a code function introduces a difference with respect to normative properties. The next theorem provides a negative answer.

Conditionalization of codes

There is a number of ways to define a conditionalized variant of a code. Below we introduce one of the variants using an infinite conjunction of literals to single out a world, and assigning a conditional for each requirement.

Definition

A code k_s^{cond} is the conditionalized variant of a code k_s iff

$$\forall p \forall w_1 (p \in \mathrm{k}_s^{cond}(i, w_1) \leftrightarrow \exists q \exists w_2 (q \in \mathrm{k}_s(i, w_2) \wedge p = \ulcorner \bigwedge \mathrm{lt}(w_2) \rightarrow q \urcorner))$$

*Quasi*literals determine the world

Lemma

For all $p \in \mathcal{L}_{\mathrm{n}(\omega_1)}$, $\ulcorner p \urcorner \in \mathrm{Cn}(\mathrm{lt}(w))$ or $\ulcorner \neg p \urcorner \in \mathrm{Cn}(\mathrm{lt}(w))$.

Proof.

We use transfinite induction on the pl-complexity of formulas[a] We will consider the cases of limit ordinals. (0) The lemma holds for propositional letters and modal formulas in virtue of pl-maximality of w. (ω) Suppose p is $\bigwedge x$. According to the definition, any $p_i \in x$ is a quasi-literal, and by inductive hypothesis the lemma holds for each p_i. Either all quasi-literals in x are consequences of $\mathrm{lt}(w)$, and therefore $\ulcorner \bigwedge x \urcorner \in \mathrm{Cn}(\mathrm{lt}(w))$, or some of quasi-literals are not consequences of $\mathrm{lt}(w)$, and therefore $\ulcorner \neg \bigwedge x \urcorner \in \mathrm{Cn}(\mathrm{lt}(w))$. □

[a] We define the complexity of modal formulas and propositional letters to be 0; the complexity of $\neg p$ to be one greater than complexity of p; the complexity of $(p \wedge q)$ to be one greater than the maximum of that of p and q, the complexity of $\bigwedge x$ to be ω.

Proposition

$\mathrm{Cn}(\mathrm{lt}(w)) = w$

Proof.

First, suppose $p \in \mathrm{Cn}(\mathrm{lt}(w))$. Then, $p \in w$ since w is deductively closed. Second, suppose $p \in w$. By lemma, $\ulcorner p \urcorner \in \mathrm{Cn}(\mathrm{lt}(w)) \vee \ulcorner \neg p \urcorner \in \mathrm{Cn}(\mathrm{lt}(w))$, and so $\ulcorner p \urcorner \in \mathrm{Cn}(\mathrm{lt}(w))$ since w is consistent. □

Definition

A code k_s^{cond} is the conditionalized variant of a code k_s iff

$$\forall p \forall w_1 (p \in \mathrm{k}_s^{cond}(i, w_1) \leftrightarrow \exists q \exists w_2 (q \in \mathrm{k}_s(i, w_2) \wedge p = \ulcorner \bigwedge \mathrm{lt}(w_2) \rightarrow q \urcorner))$$

Lemma

Any conditionalized code is absolute.

Proof.

Let w_1 and w_2 be arbitrary worlds. Assume $p \in \mathrm{k}_\mathsf{s}^{cond}(i, w_1)$. Then, by definition of conditionalization, $\exists q \exists w_3 (q \in \mathrm{k}_\mathsf{s}(i, w_3) \wedge p = \ulcorner \bigwedge \mathrm{lt}(w_2) \rightarrow q \urcorner)$. Then, by (universal instantiation of) the same definition, $p \in \mathrm{k}_\mathsf{s}^{cond}(i, w_2)$. Obviously the same holds in the opposite direction. □

Theorem

For each relativistic code there is a realization equivalent absolute code.

Proof.

Each relativistic code has its conditionalized counterpart. By lemma, each conditionalized code is absolute. It remains to prove that:

$$\mathrm{k}_s(i, w) \subseteq w \leftrightarrow k_s^{cond}(i, w) \subseteq w$$

□

L-R, 1st pt.

1		$\mathrm{k_s}(i, w_1) \subseteq w_1$	
2	p	$p \in \mathrm{k}_s^{cond}(i, w)$	
3		$\exists q \exists w_2 (q \in \mathrm{k_s}(i, w_2) \wedge p = \ulcorner \bigwedge \mathrm{lt}(w_2) \rightarrow q \urcorner)$	2/ def. of cond. code
4	q w_2	$q \in \mathrm{k_s}(i, w_2) \wedge p = \ulcorner \bigwedge \mathrm{lt}(w_2) \rightarrow q \urcorner$	
5		$p = \ulcorner \bigwedge \mathrm{lt}(w_2) \rightarrow q \urcorner$	4/ $\wedge$ Elim
6		$\mathrm{Cn}(\mathrm{lt}(w_2)) = w_1 \vee \mathrm{Cn}(\mathrm{lt}(w_2)) \neq w_1$	Taut.
7		$\mathrm{Cn}(\mathrm{lt}(w_2)) = w_1$	
8		$q \in \mathrm{k_s}(i, w_1)$	4, 7/ =Elim; lemma
9		$q \in w_1$	1, 8/ *FO*con
10		$\ulcorner \bigwedge \mathrm{lt}(w_2) \rightarrow q \urcorner \in w_1$	9/ *w* closure
11		$p \in w_1$	5, 10/ =Elim

L-R, 2nd pt.

12	$\mathrm{Cn}(\mathrm{lt}(w_2)) \neq w_1$	
13	$\ulcorner \bigwedge \mathrm{lt}(w_2) \urcorner \notin w_1$	12/ lemma, w closure
14	$\ulcorner \neg \bigwedge \mathrm{lt}(w_2) \urcorner \in w_1$	13/ w completeness
15	$\ulcorner \bigwedge \mathrm{lt}(w_2) \to q \urcorner \in w_2$	14/ w closure
16	$p \in w_1$	5, 15/ =Elim
17	$p \in w_1$	6, 7–11, 12–16/ $\vee$ Elim
18	$p \in w_1$	3, 4–17/ $\exists$ Elim
19	$\mathrm{k}_s^{cond}(i, w_1) \subseteq w_1$	2–18/ $\forall$Intro

□

R-L.

1	$\mathrm{k}_s^{cond}(i,w)) \subseteq w$	
2	$p \quad p \in \mathrm{k}_s(i,w))$	
3	$\ulcorner \bigwedge \mathrm{lt}(w) \to p \urcorner \in \mathrm{k}_s^{cond}(i,w)$	2/ def. cond. code.
4	$\ulcorner \bigwedge \mathrm{lt}(w) \to p \urcorner \in w$	1, 3/ *FO* con.
5	$\ulcorner \bigwedge \mathrm{lt}(w) \urcorner \in w$	lemma
6	$p \in w$	4, 5/ w closure
7	$\mathrm{k}_s(i,w)) \subseteq w$	2–6/ ∀Intro

□

It seems that generalized set theoretic approach opens up a number of interesting topics:

- historical,
- theoretical and philosophical,
- ethical.

Historical topic : Leibniz and the relation between the normative properties and requirements

Gottfried Wilhelm Leibniz.
Leibniz an Antoine Arnauld [Anfang November 1671].
Saemtliche Schriften Und Briefe. Zweite Reihe: Philosophischer Briefwechsel. Erster Band 1663-1685, 274–286, Akademie Verlag, 2006, Berlin

Quote

Licitum enim est, quod viro bono possibile est. Debitum sit, quod viro bono necessarium est.[a]

[a] That is permitted which a good man possibly is. That is obligatory which a good man necessary is.

Theoretical and philosophical topics

- Develop a typology of normative properties.
- Determine the deontic logic that describes the structure of the "property requirements".
- Use the "code approach" to explicate the notion of the "normativity of the mental" (e.g. Zangwill's thesis that it is of essence of the mental to be *subject* to the norms of rationality, not to conform to them).
    - If rationality is a normative source, what is its logical phenomenology (maximal compatibility?, formality?)?
    - What kind of property the rationality is?

Ethical topics

- Determination of the logical type of a "valid" code and of configurations of codes taking into account distinctions such as "code compatibility", "social consistency", "achievability", "logicality" etc.