Folded Edge of Turbulence in a Pipe

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(Received 24 May 2010; revised manuscript received 2 September 2010; published 22 October 2010)

The results of an experimental investigation into the threshold boundary between laminar and disordered pipe flow are presented. Complex features have been uncovered using a highly refined experimental approach where an intermediate periodic state forms an integral part of the transition sequence. In accord with the suggestions produced by a numerical investigation, the boundary is found to be folded with a complicated structure. This raises important questions about accepted definitions of threshold amplitudes in this long-standing problem.

DOI: 10.1103/PhysRevLett.105.174502 PACS numbers: 47.27.nf, 47.20.−k, 47.60.−i

The transition to turbulence in pipe flow remains an outstanding challenge to hydrodynamic stability theory although there have been significant advances in recent years [1–3]. The central issue is that all numerical and theoretical results indicate that the flow is linearly stable and yet pipe flow is typically turbulent at modest values of the Reynolds number Re. (Here $\mathrm{Re} = UD/\nu$ where U is the mean speed, D is the diameter of the pipe and ν is the kinematic viscosity of the fluid.) It has been recognized since Reynolds' original investigations [4] that finite amplitude disturbances are responsible for transition in practice. More recently, progress has been made in providing experimental estimates for the stability boundary between laminar and turbulent flow [5,6] and various scalings of the amplitude of disturbance required to cause transition have been found [7,8] some of which are in accord with theoretical predictions [9,10]. A reasonable question to ask is whether the boundary is sharp or, as in examples from low-dimensional dynamical systems it is folded or interleaved? In this Letter, the results of an experimental investigation into this issue are reported and the outcomes are interpreted with the aid of numerical results.

The possibility that the boundary basin of Poiseuille flow is folded has been investigated numerically where the lifetimes of perturbations are used to determine the boundary between laminar and disordered flow [11]. A dynamical systems approach is taken where the calculation domain is $5D$ long. Support for the folded stability boundary found in the numerical results lies in experimental observations [5] where a degree of spottiness is found in the transition probability plotted as a function of disturbance amplitude and Re. Precisely mapping out the stability boundary and elucidating any features in an experiment requires the use of a highly reproducible disturbance whose amplitude can be adjusted without changing its form. The experiments [5] were performed using short duration pulses of relatively large amplitude and unknown spatial structure and direct connection with the numerical work is unclear. In general, it is difficult to specify the precise form of a disturbance in an experiment where fluid is either injected and/or withdrawn through small holes [7,8]. Despite this, repeatability of the estimates of thresholds and scaling laws for the dependency of the transition amplitude on Re are found. However, the absolute amplitude level of the scaling laws can differ by an order of magnitude and making connections with available numerical work [12,13] remains an outstanding challenge.

Here the technique used to perturb the flow is sufficiently refined to provide the control and reproducibility required to enable the detection of subtle features which lie within the transition threshold. Initially, the sensitivity of the perturbation was increased using a push-pull disturbance through two adjacent small holes [8]. More detailed investigations revealed that the zero net mass flux aspect of the disturbance is not of primary importance in our system but it is the size of the holes through which the perturbation fluid is injected which is the determining factor. Hence, the simple methodology of injecting through a fine hole [8] surprisingly increases the sensitivity of the flow to the disturbance such that it is now possible to promote transition using very small injected amounts of fluid which are typically 0.1%–0.3% of the mean flux along the pipe to be compared with the 1% to 10% used previously at these values of Re. Moreover, a definite intermediate step in transition emerges and this gives optimism for the prospect of forming a firm link between theory and experiment.

The experiments were performed using a constant mass flux pipe flow apparatus and details can be found elsewhere [7,8]. In summary, it consisted of a tube of diameter $D = 20 \pm 0.01$ mm which had a total length of 15.7 m ($785D$). A 26 cm diameter aluminum-PTFE composite piston which was 14 cm long was mounted on piston rings and guides inside a ground 110 cm long steel barrel. A feedback controlled motor drove the piston which pulled the water at a set rate along the pipe from a reservoir through a smooth trumpet shaped inlet. When a small amount of fluid was injected into the flow as a disturbance or if the fluid in the pipe became turbulent, the net mass flux pulled through the pipe was unaffected so that Re remained constant. Temperature variations along the length of the pipe were measured to be at most 0.2 °C and this control together with the accurate construction of the apparatus enabled us to maintain an accuracy of better than 1% in Re. The facility enabled laminar flow to be achieved up to values of Re = 20 000.

The flow was perturbed by injecting small amounts of fluid through an 0.3 mm hole using a specially developed pump which produced a short duration boxcar disturbance [7]. This created a small jet which was orthogonal to the main flow and its duration was set to influence $10D$ of the flow for the Re range investigated. Amplitudes of the disturbances were typically in the range 0.1% to 0.3% of the mass flux through the pipe and they will be denoted by $\Phi_{\rm inj}/\Phi_{\rm pipe}$. They were injected $220D$ from the inlet to ensure that fully developed Hagen-Poiseuille flow was the initial state for all experiments. The flow was monitored using both dye and Pearlessence flow visualization where both 100 and 25 frames-per-second video images were recorded. Pressure measurements were also used to monitor both the disturbance and the flow field using a standard transducer. Primarily, the flow state was observed at $250D$ from the injection point and at the end of the pipe but other tests carried out to monitor the spatial development of the flow. Data from a minimum of 20 rehearsals of the experiment were used for "data points" to enable good statistical estimates of the thresholds. Further checks were performed by doubling this number and 24 runs were used in the spatial development tests reported in Fig. 2. Each run of the experiment took ~30 mins to perform [14].

A significant feature of transition with this small amplitude perturbation is the production of hairpin vortices or waves [8] and an image of a typical set of waves is shown in Fig. 1(a). The waves are reminiscent of hairpin vortices [12] generated by jets in a cross-flow [15] and,

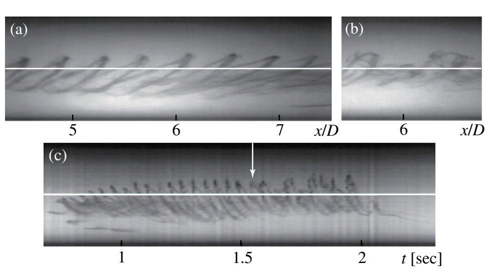

FIG. 1 (color online). Images of waves formed at Re = 1900 when Poiseuille flow is disturbed by a small jet. (a) Dye visualization with $\Phi_{\rm inj}/\Phi_{\rm pipe} = 0.17\%$. The scale is indicated in units of distance D from the injection point. Image taken at 1.3 sec after the injection. (b) Closeup of breakdown at ~1.6 sec. (c) A "timeline" evolution of a set of waves towards a puff. The location of the vertical video line was fixed at $6D$. (Arrow indicates approximate location of breakdown.)

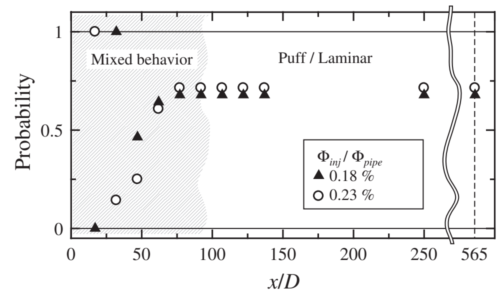

FIG. 2. Probability of observation of a puff as a function of distance x/D from the disturbance injection point at Re = 1900. Measurements were made every $15D$ from $17D$ after the injection point up to $137D$ and further observations were made at $250D$ and $565D$, the end of the pipe. Each data point was obtained from 24 runs of the experiment.

interestingly, have similar wavelengths to "edge states" in pipe flow [16]. Here, the waves are primarily on the injector side of the pipe and their "heads" travel at $(0.8–0.9)U_{\rm max} = (1.6–1.8)U_{\rm mean}$; i.e., they travel at the local flow speed and hence are distorted by the parabolic profile as can be seen in Fig. 1(a). When the amplitude of the disturbance is such that the waves cross the center line of the pipe, they tend to break down [detail in Fig. 1(b)] to form patches of turbulence normally termed "puffs" for Re ≲ 3 000 and "slugs" above this value [17]. An example of this is shown as a timeline image in Fig. 1(c) which was constructed from a sequence of vertical video lines located at the fixed location $6D$ from the injection point. A clear indication of an intermediate step transition process [8] can be seen with the creation of distinct waves between ~1 and ~1.5 sec. and breakdown towards a patch beginning at ~1.8 sec. after injection. This can be contrasted with the majority of previous experimental investigations dating back to Reynolds where the onset of turbulence is found to be catastrophic.

The probability of observing a laminar flow or puff was measured at various downstream locations at Re = 1900 and the results are presented in Fig. 2. Data were taken every $15D$ from $17D$ after the injection point up to $137D$ with further checks at the main measuring stations of $250D$ and the end of the pipe $565D$. There is initially a complicated transient phase which is labeled "mixed behavior" in Fig. 2 where switching between laminar and turbulent flow took place. This involved an interplay between waves which traveled at the local flow speed with turbulent patches which generally travel more slowly at $0.9U_{\rm mean}$ [17]. The transient behavior was followed by consistent outcomes after ~$100D$ with transition probabilities that do not depend on the position of observation. These results are consistent with previous observations [5,8] that transient effects typically persist for ~$100D$ in this constant mass flux system.

Once it was established that transient behavior was restricted to ≲ $100D$ all observations of transition thresholds were made at $x = 250D$ from the disturbance injection point and at the end of the pipe. The probability for emergence of a turbulent puff or slug was investigated as a function of the amplitude of the applied perturbation, $\Phi_{\rm inj}/\Phi_{\rm pipe}$, at three values of Re, Re = 1900, 2200, and 3000 and the results are shown in Fig. 3. The vertical dark strip in each of the plots shown in Fig. 3 are estimates of the critical amplitude above which waves appear. They are $\Phi_{\rm inj}/\Phi_{\rm pipe} = 0.046 \pm 0.002\%$, $0.033 \pm 0.002\%$, and $0.020 \pm 0.002\%$ for Re = 1900, 2200, and 3000, respectively. For amplitudes below these values, the injected disturbance produced indistinct features which decayed within $30D$. Above these critical amplitudes and below those required for the creation of puffs or slugs, patches of up to ten waves formed transiently and they decayed within $\sim 50D$ as they propagated downstream. They remained on the same side of the pipe as the injection hole and it appears that crossing of the centerline is an essential feature of the breakdown to disordered motion. An example of a periodic state is shown in Fig. 1.

The simplest threshold to consider is shown in Fig. 3(c), which was obtained at Re = 3000. All disturbances with

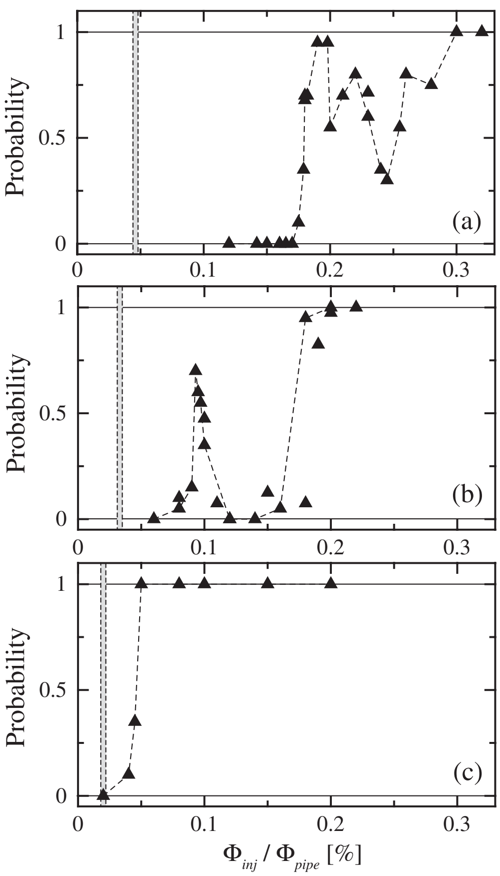

FIG. 3. Probability of observation of waves or puffs or slugs as a function of $\Phi_{\rm inj}/\Phi_{\rm pipe}$ at (a) Re = 1900 and (b) Re = 2200 (c) Re = 3000. The vertical strip indicates the threshold for the appearance of waves (Fig. 1). Observations made at $250D$ and $565D$.

an amplitude greater than 0.05% gave rise to turbulent slugs whereas those with amplitudes $\simeq 0.02\%$ all decayed. Hence there is a relatively sharp threshold boundary between disturbances which create slugs and those which decay at this value of Re. These results have the same qualitative "*S*-shaped" form as found for larger disturbances amplitudes [7,8,18]. The transition is sharp and details of the processes involved are difficult to distinguish but note that the lower end of the threshold is close to the critical amplitude for the onset of the hair-pin vortices. The other data sets shown in Figs. 3(a) and 3(b) at Re = 1900 and Re = 2200, respectively, are more interesting and show clear evidence for the existence of complicated structure in the threshold. There are definite peaks in the probability of transition so that the dependence of this measure on the disturbance amplitude is nonmonotonic. The total widths of the transition regions in terms of $\Phi_{\rm inj}/\Phi_{\rm pipe}$ are 0.12% at Re = 1900 and 0.1% at Re = 2200 so that ascribing a definite value to the threshold with a suitable error bar is not particularly useful.

Two computational domains were used in the numerical investigations. In one, the flow in a pipe segment of length $L = 5D$ was used while this was increased to $L = 15D$ in another. Details of the numerical methods are given elsewhere [11]. Re was held fixed and the evolution of the flow was followed for the initial perturbation with various amplitudes A_0 and fixed spatial structure so that $\vec{u}_0 = \vec{u}_{\rm lam} + A_0\vec{v}$. Here the strength of the initial perturbation is characterized by its kinetic energy E_0 with respect to that of the laminar flow generating the same flow rate, so that $A_0 = \sqrt{E_0/E_{\rm lam}}$ if the fixed field $\vec{v}$ is normalized to one. A pair of counterrotating downstream vortices that have been slightly twisted to break axial invariance is used for $\vec{v}$. In order to detect if a perturbation induced turbulence, the kinetic energy of the flow was monitored after time $T = 150D/U_{\rm mean}$ chosen to be longer than the decay time of nonturbulent transients [19] so that as in the experiment the dynamics is no longer affected by the initial transient behavior (cf. Figure 2). This was used to determine if turbulence was induced for a pair (Re, A_0) as in [5]. Transition probabilities were estimated by ensemble averaging over ~ 10 realizations where the set of realizations was constructed by allowing small variations in A_0 and Re.

Transition probabilities for parameters where folds in the stability boundary were found are shown in Fig. 4(a), where probability estimates both from binning ~ 10 nearby data points into statistically independent nonoverlapping bins (boxes) and from using overlapping bins sliding along the amplitude axis (solid line) are given. The clear nonmonotonic variation of the transition probability which can be directly linked to folds in the stability boundary [11] is in excellent qualitative agreement with the experimental findings. Since the periodic domain in the numerical study cannot capture the spatial structure of a puff, the details of the stability boundary will differ between the experimental

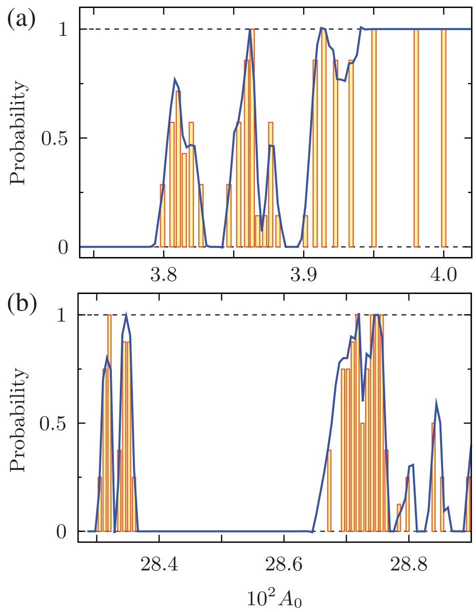

FIG. 4 (color online). Nonmonotonic probability of inducing turbulence in the simulations plotted as a function of perturbation amplitude A_0. Probabilities are estimated from relative frequencies in overlapping "sliding" bins (blue curve) and in nonoverlapping statistically independent bins (boxes). (a) Data for a periodically continued domain of length $L = 5D$ at $\mathrm{Re} = 3878 \pm 8$ (raw data from [11]). (b) Data for a longer domain with $L = 15D$ at $\mathrm{Re} = 1,900$ and perturbations of different spatial structure (raw data from [19]).

and numerical system so that quantitative agreement is not expected. In order to demonstrate the robustness of the nonmonotonicity the analysis has been repeated for data from [19] for a domain with $L = 15D$ at $\mathrm{Re} = 1900$ and a snapshot from a turbulent run was used to define the spatial structure of the initial condition. Again a nonmonotonic probability variation is recovered as shown in Fig. 4(b). Interestingly, it was not found in the experiment for $\mathrm{Re} \gtrsim 3000$. However, folds in the boundary are also not present for all values of Re in the numerical results [11].

A convincing experimental demonstration of a nonmonotonic probability of transition in pipe flow has been provided using a very accurate control of initial perturbations both in terms of the reproducibility of its spatial structure and in terms of amplitude resolution. The qualitative agreement with the numerical work indicates that close to the "critical" perturbation amplitude, folds in the stability boundary of pipe flow are generic features of transition, at least over a range of Re. From a dynamical systems point of view folds and thus nonmonotonic transition probabilities can be observed if the stability boundary (considered as a codimension one object in phase space) is folded in a region probed by the chosen initial conditions and if it is crossed in a direction (defined by the spatial structure of the chosen perturbation) which permits details of the folds to be revealed. Since the structure of the stability boundary varies with Re we expect to observe the nonmonotonic behavior of the probability for some Re but not for others [20].

These results bring into question the notion of the requirement of a "critical amplitude" for transition. It has been demonstrated that if the structure of the initial perturbation can be controlled sufficiently, fine details of the processes involved can be revealed. Coarse graining and hence washing out features of the stability boundary as a result of inherent noise permits the notion of a critical perturbation in a statistical sense. However, refined control and the creation of a well-defined intermediate step has enabled the uncovering of fine structure in the transition boundary which is consistent with ideas from low-dimensional dynamical systems and numerical results.

The visit of Y. T. to Manchester was funded by MEXT-Japan and T. M. S. acknowledges the support by the German Research Foundation under Grant No. SCHN 1167/1, the Kavli Institute for Bionano Science and Technology at Harvard, and Harvard MRSEC.

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[1] R. R. Kerswell, Nonlinearity **18**, R17 (2005).
[2] B. Eckhardt *et al.*, Annu. Rev. Fluid Mech. **39**, 447 (2007).
[3] T. Mullin, Annu. Rev. Fluid Mech. (to be published), http://www.annualreviews.org/doi/abs/10.1146/annurev-fluid-122109-160652.
[4] O. Reynolds, Proc. R. Soc. A **34**, 84 (1883).
[5] A. G. Darbyshire and T. Mullin, J. Fluid Mech. **289**, 83 (1995).
[6] A. A. Draad *et al.*, J. Fluid Mech. **377**, 267 (1998).
[7] B. Hof, A. Juel, and T. Mullin, Phys. Rev. Lett. **91**, 244502 (2003).
[8] J. Peixinho and T. Mullin, J. Fluid Mech. **582**, 169 (2007).
[9] M. I. Gavarini *et al.*, J. Fluid Mech. **517**, 131 (2004).
[10] S. J. Chapman, J. Fluid Mech. **451**, 35 (2002).
[11] T. M. Schneider, B. Eckhardt, and J. A. Yorke, Phys. Rev. Lett. **99**, 034502 (2007).
[12] P. O. Åsén *et al.*, Comput. Fluids **39**, 926 (2010).
[13] F. Mellibovsky and A. Meseguer, Phys. Fluids **19**, 044102 (2007).
[14] In a previous publication [8] it was stated that forty runs were performed for all experiments whereas the average number was actually twenty five.
[15] T. H. New *et al.*, Exp. Fluids **40**, 859 (2006).
[16] T. M. Schneider and B. Eckhardt, Phil. Trans. R. Soc. A **367**, 577 (2009).
[17] I. J. Wygnanski and F. H. Champagne, J. Fluid Mech. **59**, 281 (1973).
[18] S. Williams Ph.D. thesis, University of Manchester, 2001.
[19] T. M. Schneider and B. Eckhardt, Phys. Rev. E **78**, 046310 (2008).
[20] J. Vollmer and T. M. Schneider, and B. Eckhardt, New J. Phys. **11**, 013040 (2009).