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Buckling of Bridging Fibers in Composites*

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In brittle materials such as concrete and ceramics, fiber reinforcement has been widely accepted as an effective way of improving their strength and toughness. In addition, a notable pseudo strain-hardening phenomenon can contribute to a significantly enhanced ductility of the composite when adequately designed fiber system is used. This condition was first proposed by Aveston et al. [1], and later extended by Marshall et al. [2] for continuous aligned fiber reinforced brittle matrix composites. More recently, further extension to randomly oriented discontinuous fiber reinforced composites have been presented [3,4]. Upon satisfying these conditions described in the above mentioned micromechanical models, the ultimate tensile strains of the composites are usually improved by two order of magnitude (e.g. see [5], [6], [7]). The total fracture energy reaching 35 kJ/m² was also reported for a 2% polyethylene fiber reinforced cement paste [8]. This kind of ductile fracture resembles metal instead of brittle materials.

The pseudo strain-hardening behavior of fiber reinforced brittle matrix composites is associated with multiple cracking, and results from adequate stress transfer capability of bridging fibers. Studies are typically conducted under monotonic tensile loading only. In reality, composites are usually subject to cyclic loads. As a

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preliminary report of an ongoing research on the cyclic behavior of pseudo strain-hardening cementitious composites, we present an initial finding on buckling of bridging nylon fibers across fracture planes in a cement composite after complete unloading in tension. An analytic model is also proposed to account for this buckling phenomenon.

Type I Portland cement, silica fume and superplasticizer were used to form the cement paste with water /cementitious ratio of 0.27. Discontinuous nylon fibers ($L_f=21$ mm, $d_f=25$ μ m, and $E_f=5.2$ GPa) were used to reinforce the paste at a volume fraction of 2 %. Tensile coupon specimens of size 304.8 x 76.2 x 12.7 mm were prepared and tested under direct tension in a servo hydraulic tester. Detailed mix proportions and testing procedure can be found elsewhere [9]. Tensile stress-strain curves were recorded. Optical microscope with 50 times magnification was used to examine the bridging fibers after the specimen was completely unloaded.

The stress-strain curve is shown in Fig. 1 where four peaks are identified, corresponding to four multiple cracks occurred within a length of 200 mm. After reaching the ultimate load, the main crack opened up continuously with descending load carrying capacity while other cracks were unloaded. Examination of these closing cracks using a microscope reveals both buckled fibers as well as straight fibers across the crack surfaces. This is shown in Fig. 2. It is clearly demonstrated that a portion of the fibers underwent buckling. In the following, an analysis is made to examine this buckling phenomenon.

It is possible to explain why some of the nylon fibers buckle between the closing crack surfaces by examining whether the critical buckling load of a fiber is exceeded or not. The critical buckling load, P_c , for a circular column with both ends fixed is given by

$$P_c = \frac{\pi^2 EI}{(0.5L)^2} \quad (1)$$

where E =Young's modulus of the column, $I = \pi r^4/4$, r =radius of the column, and L =length of the column [10]. Assuming that the critical buckling load given by the above equation is applicable to a single fiber under compression imposed by the closing crack surfaces, eq. (1) becomes

$$P_c = \frac{\pi^2 E_f I}{(0.5\delta)^2} \quad (2)$$

where E_f =fiber modulus, $I = \pi(d_f/2)^4/4$, d_f =fiber diameter, and δ =crack opening displacement (cod). This equation yields the relation between critical buckling load, P_c , and cod, δ , with two fiber parameters included. For a given cod and fiber parameters, P_c can be calculated and compared with axial load applied at the ends of the single fiber. When the axial load exceeds P_c , the fiber is expected to buckle.

The relation between applied load, P , and cod, δ , (P - δ relation) of a single discontinuous fiber bridging a matrix crack with weak (friction controlled) interface has been derived by Li [11]. The derived P - δ relation consists of ascending and descending portion, corresponding to debonding and sliding process respectively. In the ascending portion, as P increases, debonding region extends towards the embedded end of the fiber. During this debonding process the ascending portion of P - δ relation is expressed by

$$P = \frac{\pi}{2} \sqrt{E_f d_f^3 \tau \delta} \quad \text{for } \delta \leq \delta_0 \quad (3)$$

where τ =constant frictional stress and $\delta_0 = 4\tau l^2/E_f d_f$ corresponds to the cod at which frictional debonding is completed where l =embedment length of the fiber. After debonding reaches the embedded end of the fiber, cod is attributed entirely to sliding out of the fiber. In this process P decreases with cod and is given by

$$P = \pi \tau d_f l \left(1 - \frac{\delta - \delta_0}{l} \right) \quad \text{for } \delta_0 < \delta \leq l. \quad (4)$$

Applied axial load, P , imposes a push-back force on the fiber following unloading in tension. Hence P - δ relation (originally accounted for tensile load only) needs to be extended to cover compressive load as well. This applied push-back force (in compression) will then be compared with the critical buckling load, P_c , to determine the possibility of fiber buckling. During crack closure, if the applied push back force P exceeds the critical buckling load, then the fiber will buckle instead of sliding back into the matrix. Derivation of this additional relation (ΔP - $\Delta \delta$ relation) is described as follows.

Because of random distribution of fiber embedment length at a designated fracture plane in the matrix, fibers can be divided into two groups, i.e. some fibers undergo debonding process (for long embedment length) and others sliding (for short embedment length). When unloading occurs ($\Delta P = P_{\max} - P_{\min}$), ΔP - $\Delta \delta$ relation can be derived by the following equation

$$\Delta \delta = \int_0^{L_f} (\varepsilon_{\max}(x) - \varepsilon_{\min}(x)) dx \quad (5)$$

where $\Delta \delta$ =cod change, L_f =fiber length, $\varepsilon_{\max}(x)$ =strain in the fiber at $P=P_{\max}$, $\varepsilon_{\min}(x)$ =strain in the fiber at $P=P_{\min}$, and x is measured from the shorter embedded end of the fiber (see Fig. 3). The change of cod, $\Delta \delta$, between the two crack surfaces result from the difference of axial strain in the fiber and requires summation over entire fiber length, L_f . Strain in the fiber, $\varepsilon(x)$, can be given by

$$\varepsilon(x) = F(x) / \left(\pi \left(\frac{d_f}{2} \right)^2 E_f \right) \quad (6)$$

where $F(x)$ =force in the fiber. Assuming that interfacial frictional stress, τ , is constant in resisting pullout and that unloading creates a zone where τ acts in the reversed direction to resist unstretching or push-back of the fiber, $F(x)$ becomes a piecewise linear function and so does $\varepsilon(x)$ (movement of the crack surfaces relative to the fiber is assumed to have little effect on the cod change). Figs. 3 and 4 show the distribution of axial strain of the fiber before and after unloading for fibers in debonding and sliding process respectively. As seen in Figs. 3 and 4, the axial strain arises within the debonded region and the exposed portion of the fiber between the crack surfaces, and equals to zero in the undebonded region. Thus the integration in eq. (5) can be carried out only for the debonded region and the exposed portion of the fiber, and is equivalent to computing the shaded area. The shaded area can yield ΔP - $\Delta\delta$ relation for fibers in debonding and sliding separately.

For fibers in debonding

$$\Delta\delta = \frac{2(\Delta P)^2}{\pi^2 \tau d_f^3 E_f} \quad (7)$$

and for fibers in sliding

$$\Delta\delta = \frac{1}{\pi^2 \tau d_f^3 E_f} \left(2(\Delta P)^2 + 6\Delta P(P_0 - P_{\max}) \right) \quad (8a)$$

where P_0 =pullout load at which debonding is completed.

These equations hold also for compressive loading. However, for fibers in sliding stage, $\Delta\delta$ obtained by unstretching or contracting fiber is insufficient to attain full crack closure. At $P_{\min} = -P_{\max}$, the debonded region where interfacial frictional stress acts in the reversed direction reaches the embedded end of the fiber. Further compressive loading beyond $-P_{\max}$ is then required to push the fiber back into the matrix. Therefore eq. (8a) is followed by

$$\Delta\delta = \frac{\Delta P - 2P_{\max}}{\pi\tau d_f} + \frac{1}{\pi^2\tau d_f^3 E_f} (12P_{\max}P_0 - 4P_{\max}^2) \quad (8b)$$

for $-P_0 \leq P_{\min} \leq -P_{\max}$. During push-back of the fiber, $\Delta\delta$ is attributed mainly to sliding distance, which is represented by the first term of eq. (8b), under constant interfacial frictional stress.

Using eq. (3), (4), (7), and (8), P- δ curves including loading and unloading can be constructed, as shown in Fig. 5. For the nylon fiber composite, P- δ curves are shown in Fig. 6 for various fiber embedment length, l , and for fibers all perpendicular to the fracture surface. Load reversal point is set at cod (505 μm in this case) which corresponds to the peak load of a fiber with its embedded length equal to $L_f/2$. This corresponds also to the maximum composite strength [3,11]. Material parameters of the composite used are listed above and $\tau=0.15$ MPa is estimated from the maximum bridging stress. As revealed in Fig. 6, it is possible for fibers with $1.9 \leq l \leq 5.7$ mm to buckle, since the critical buckling load is reached for these fibers. It should be noted that this range of fiber embedded lengths for undergoing buckling depends on the location of the load reversal point. In our experiment, unloading occurred following the peak of the composite strength. Hence the load reversal point is chosen as above. Full crack closure can be obtained for fibers with embedded length above 5.7 mm, since the critical buckling load is higher than the necessary compressive load required to push the fibers back into the matrix due to shorter exposed fiber lengths (at the beginning of sliding). On the other hand, maximum contraction load ($P_{\min}$) achievable is too small to cause fiber buckling due to shorter embedded length for those fibers with initial embedded length below 1.9 mm. Furthermore there is no chance for the sliding curve to intersect the critical buckling load curve, since the gradient of P_c is much steeper. It should be noted that complete pullout occurs for fibers with initial embedded length less than 505 μm .

In addition to the random distribution of fiber embedment length, angle of a fiber to the crack surface is also distributed randomly in a real composite. It is found that pullout force increases with increasing inclination angle of the fiber to the loading axis, ϕ , and can be accounted for by the snubbing effect [12,13]. Under assumption that the snubbing effect applies to ΔP - $\Delta\delta$ relation,

$$\Delta P(\Delta\delta, \phi) = \Delta P(\Delta\delta, \phi = 0) e^{f\phi} \quad (9)$$

where f =snubbing coefficient ($f=0.994$ for nylon fiber [13]). Figs. 7 and 8 have been drawn for $\phi=30$ and 60 degree. Since additional force due to snubbing effect is required for the same amount of cod change and critical buckling load remains unchanged, more fibers tend to undergo buckling as shown in Figs. 7 and 8. The possible range of fiber embedment length undergoing buckling is 1.3 to 7.4 mm and 0.9 to 8.7 mm respectively for $\phi=30$ and 60 degree.

It is shown that a portion of the bridging nylon fibers in a cement paste undergo buckling when the specimen is unloaded. This can be accounted for by combining simple buckling theory and fiber debonding and sliding mechanisms in a load-unload cycle. Furthermore, the snubbing effect is shown to enhance fiber buckling.

Acknowledgment

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Captions

Figure 1: Tensile stress-strain curve of nylon fiber reinforced cement paste ($V_f=2\%$).

Figure 2: Portion of bridging fibers undergo buckling (computer scanned image).

Figure 3: Strain distribution in the debonding fiber with shorter embedment length, l , before and after unloading. The shaded area shows $\Delta\delta$.

Figure 4: Strain distribution in the sliding fiber with shorter embedment length, l , before and after unloading. The shaded area shows $\Delta\delta$.

Figure 5: Schematic representation of loading-unloading curves with corresponding governing equations. Insert shows complete curve.

Figure 6: $P-\delta$ curves and critical buckling load for nylon fiber perpendicular to the crack plane ($\phi=0^\circ$).

Figure 7: $P-\delta$ curves and critical buckling load for nylon fiber inclined at $\phi=30^\circ$ to the crack plane.

Figure 8: $P-\delta$ curves and critical buckling load for nylon fiber inclined at $\phi=60^\circ$ to the crack plane.

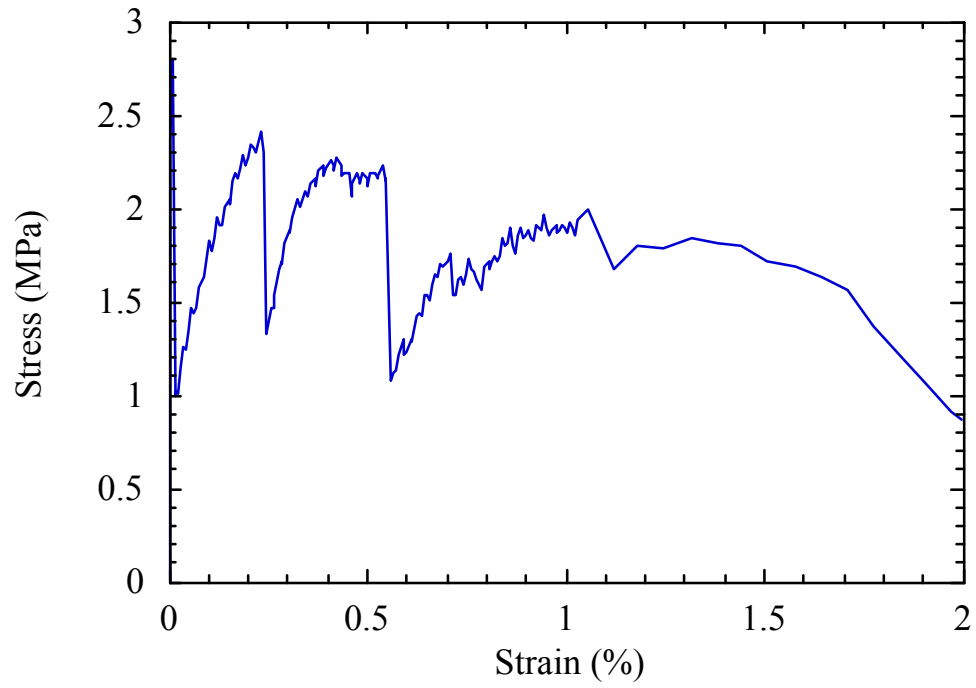


Figure 1: Tensile stress-strain curve of nylon fiber reinforced cement paste ($V_f=2\%$).

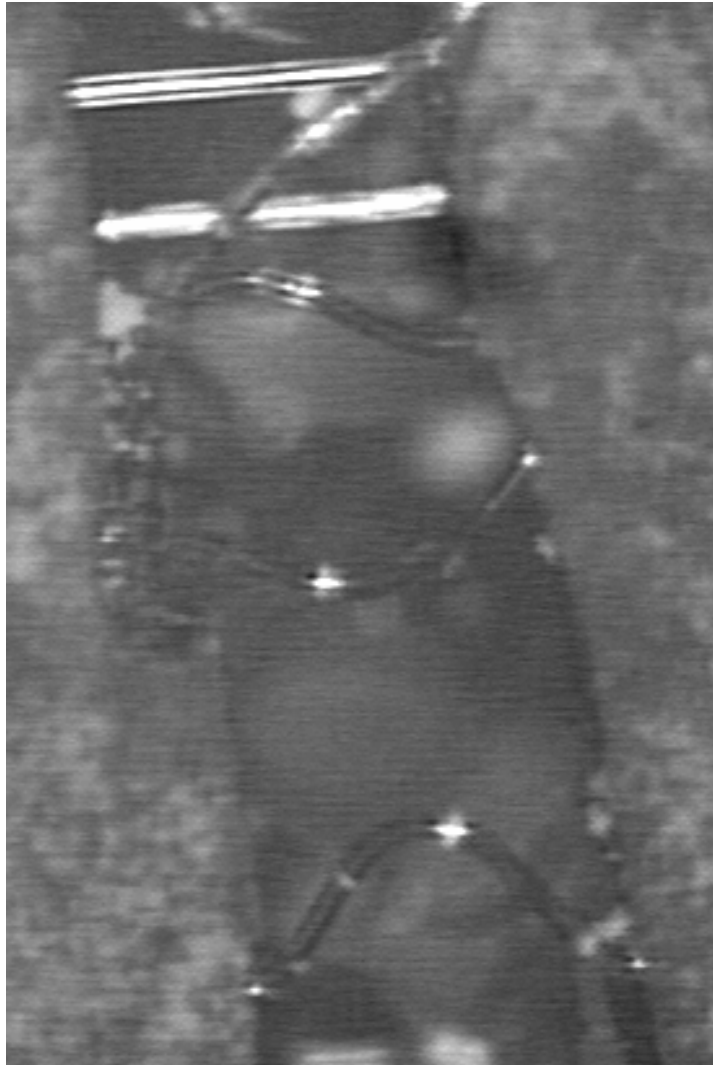


Figure 2: Portion of bridging fibers undergo buckling (computer scanned image).

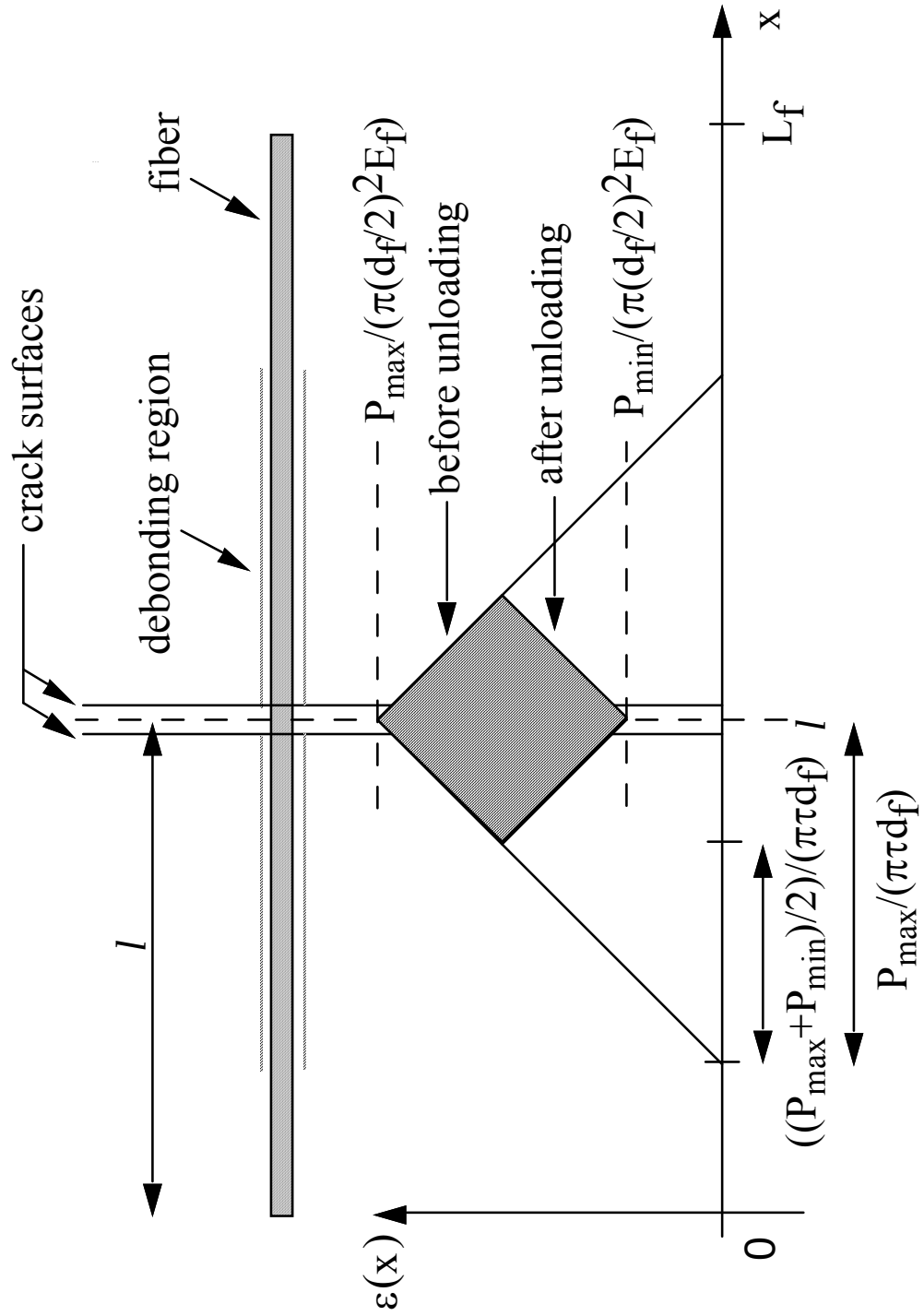


Figure 3: Strain distribution in the debonding fiber with shorter embedment length, l , before and after unloading. The shaded area shows $\Delta\delta$.

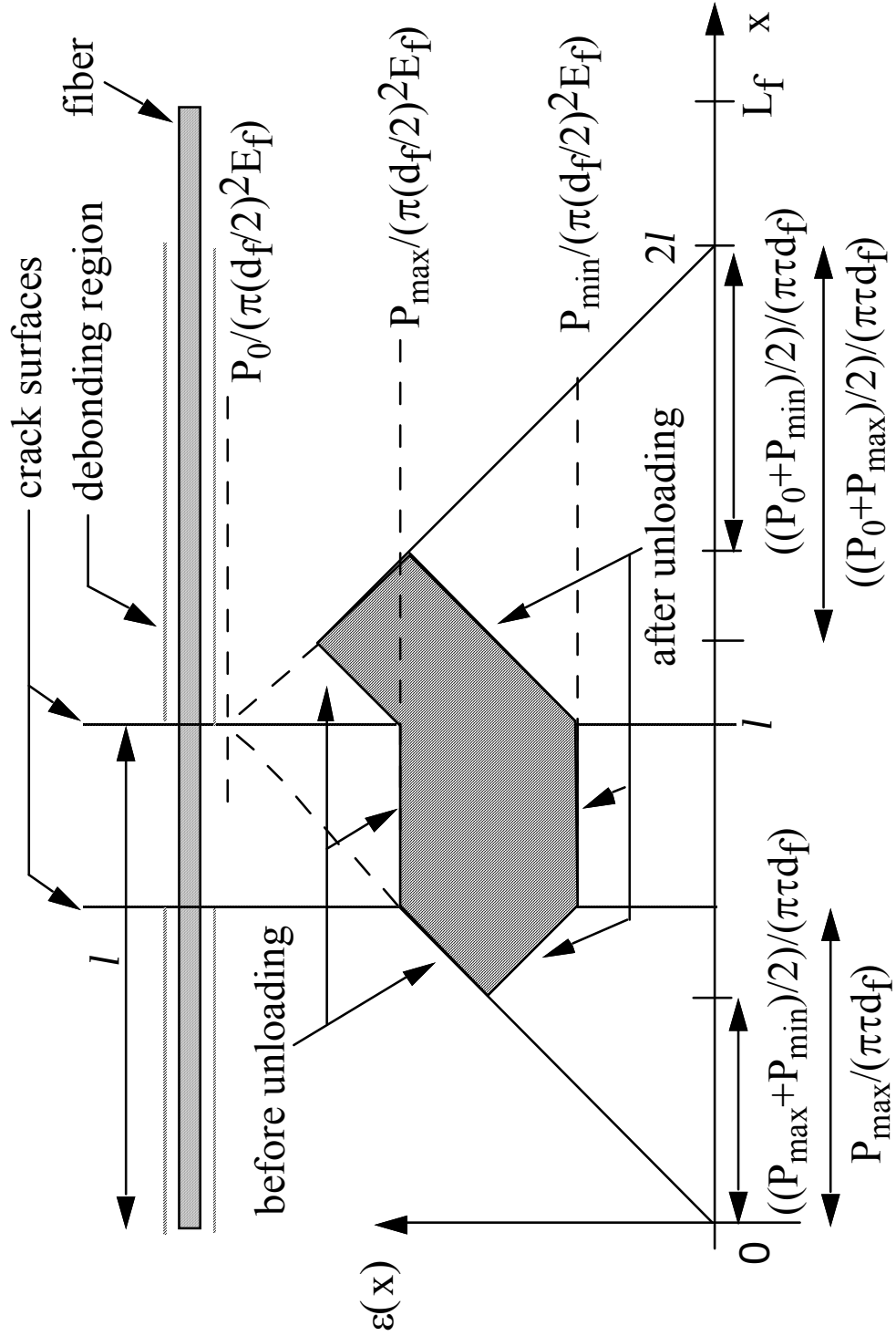


Figure 4: Strain distribution in the sliding fiber with shorter embedment length, l , before and after unloading. The shaded area shows $\Delta\delta$.

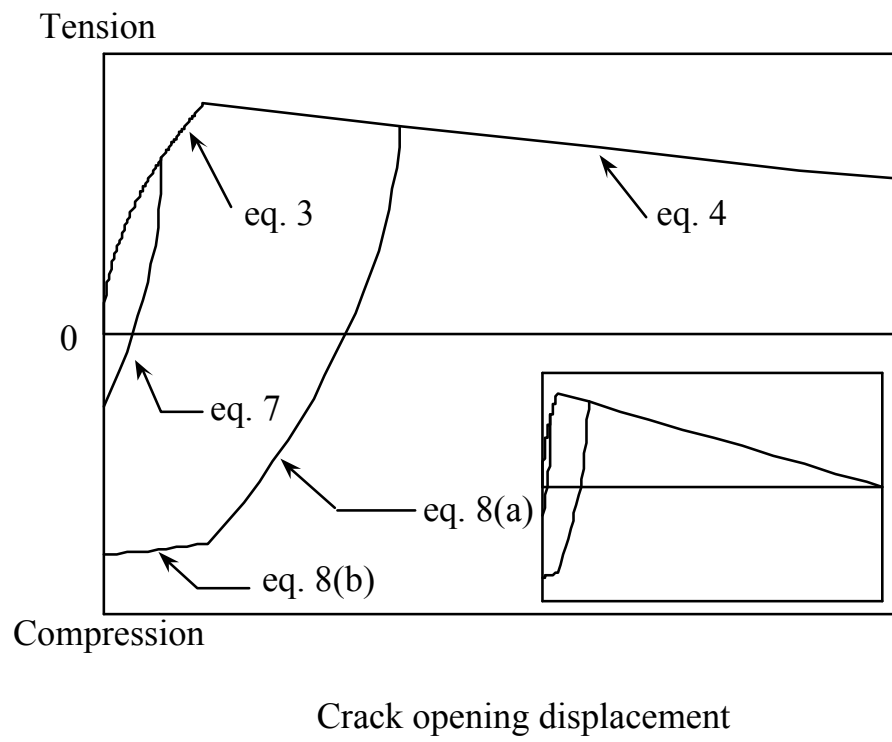


Figure 5: Schematic representation of loading-unloading curves with corresponding governing equations. Insert shows complete curve.

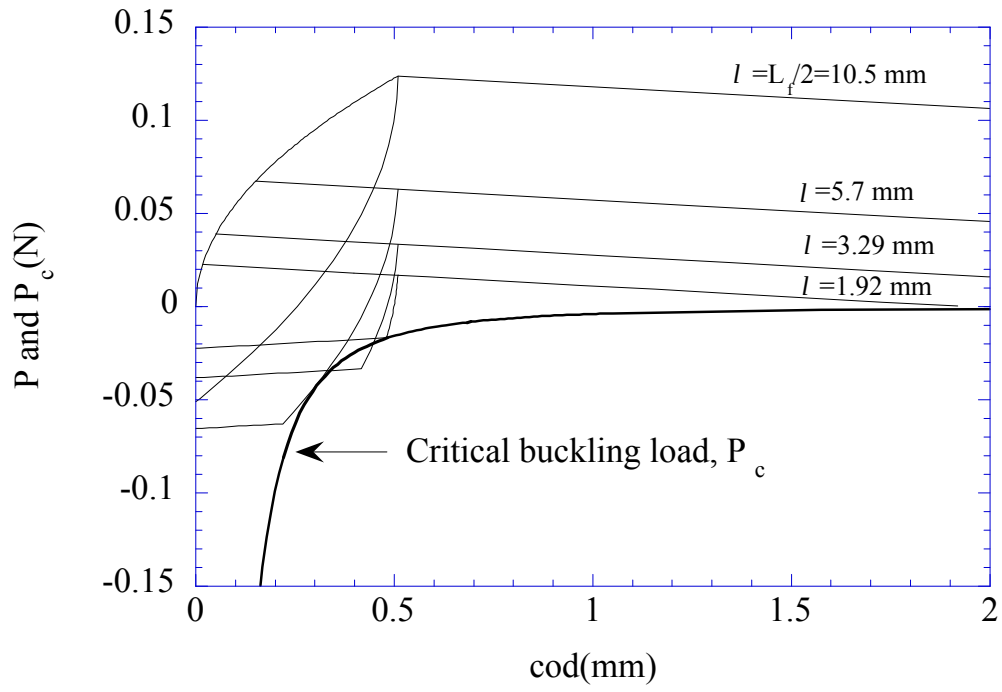


Figure 6: P- δ curves and critical buckling load for nylon fiber perpendicular to the crack plane ($\phi=0^\circ$).

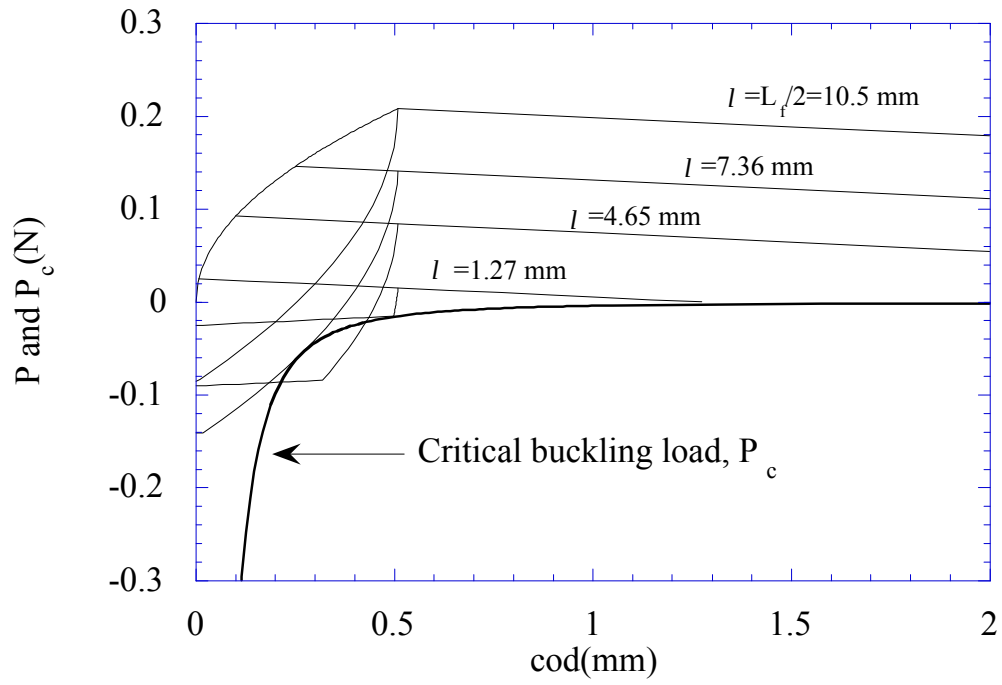


Figure 7: P- δ curves and critical buckling load for nylon fiber inclined at $\phi=30^\circ$ to the crack plane.

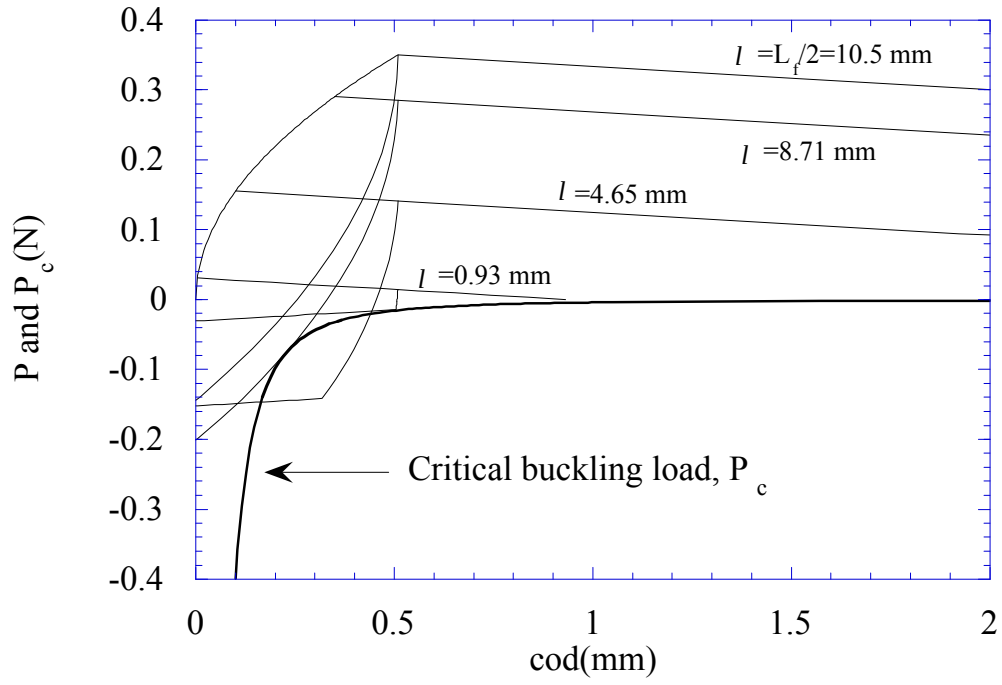


Figure 8: P - δ curves and critical buckling load for nylon fiber inclined at $\phi=60^\circ$ to the crack plane.