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Author(s)	Akera, H.
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Crossover of the Hall-voltage distribution in AC quantum Hall effect

H. Akera

Division of Applied Physics, Faculty of Engineering, Hokkaido University, Sapporo, Hokkaido, 060-8628, Japan

Abstract

The distribution of the Hall voltage induced by low-frequency AC current is studied theoretically in the incoherent linear transport of quantum Hall systems. It is shown that the Hall-voltage distribution makes a crossover from the uniform distribution to a concentrated-near-edges distribution as the frequency is increased or the diagonal conductivity is decreased. This crossover is also reflected in the frequency dependence of AC magnetoresistance.

Keywords: quantum Hall effect, AC transport, current distribution, magnetoresistance

1. Introduction

In the quantum Hall effect [1, 2] observed in two-dimensional electron systems (2DES) in strong magnetic fields, the Hall voltage V_H divided by the current I is quantized as

$$\frac{V_H}{I} = \frac{h}{ie^2} \quad (1)$$

with i an integer. The distribution of this quantized Hall voltage along the width of the 2DES has been studied theoretically and experimentally, but has a problem that remains to be solved.

MacDonald, Rice and Brinkman [3] have studied theoretically the Hall-voltage distribution in the ideal 2DES with no disorder for integer values of the Landau-level filling factor ν at absolute zero. They have considered an infinitely-long sample with width W in the xy plane in the magnetic field B along the z direction ($B > 0$). The 2DES considered has a macroscopic size: W is much larger than the magnetic length $l = (\hbar c/eB)^{1/2}$ ($e > 0$). In this paper we choose the x axis along the current and the y axis along the width (the 2DES is in $-W/2 < y < W/2$). In the ideal 2DES with a constant current, the electric field along the current E_x is zero and the dissipation is absent. The Hall field $E_y(y)$ induces a shift of each wave function by $\Delta y = -eE_y/m\omega_c^2$ with $\omega_c = eB/mc$ (m : the effective mass), and the resulting polarization gives the Hall charge density $\rho_{\text{polar}}(y)$. MacDonald et al. [3] have obtained the formula for $\rho_{\text{polar}}(y)$ by making the summation of contributions from each of these shifted wave functions. The same formula is obtained by starting with the polarization or the dipole moment per area which is given by

$$P_y = -e\Delta y\nu/2\pi l^2 = \chi_{yy}^0 E_y, \quad (2)$$

where χ_{yy}^0 is the DC dielectric susceptibility (the superscript 0 means DC) given by

$$\chi_{yy}^0 = e^2\nu/h\omega_c. \quad (3)$$

With use of $\rho_{\text{polar}} = -\nabla_y P_y$ ($\nabla_y = \partial/\partial y$), we obtain

$$\rho_{\text{polar}} = -\chi_{yy}^0 \nabla_y E_y = \chi_{yy}^0 \nabla_y^2 \phi. \quad (4)$$

The electrostatic potential $\phi(y)$ ($E_y = -\nabla_y \phi$) in this equation is given in terms of $\rho_{\text{polar}}(y)$ by

$$\phi(y) = -\frac{2}{\varepsilon} \int_{-W/2}^{W/2} dy' \ln|y - y'| \rho_{\text{polar}}(y') \quad (5)$$

where ε is the dielectric constant of the intrinsic semiconductor. Equations (4) and (5) give the Hall potential $\phi(y)$ as a function of y . The calculated result [3] shows that *the Hall voltage is concentrated near edges*.

In the presence of dissipation the Hall-voltage distribution changes drastically [4]. Here we assume that the transport current densities j_x and j_y are related to E_x and E_y by the local DC conductivity tensor $\sigma_{xx}^0 = \sigma_{yy}^0$, $\sigma_{xy}^0 = -\sigma_{yx}^0$, which has no spatial dependence, that is,

$$j_x = \sigma_{xx}^0 E_x + \sigma_{xy}^0 E_y, \quad j_y = \sigma_{yx}^0 E_x + \sigma_{yy}^0 E_y. \quad (6)$$

The density of the charge accumulated due to the transport, ρ_{trans} , evolves according to the equation of charge conservation:

$$\frac{\partial \rho_{\text{trans}}}{\partial t} = -\nabla \cdot \mathbf{j} = -\sigma_{yy}^0 \nabla \cdot \mathbf{E}. \quad (7)$$

If we consider a state which is steady and uniform along x , the above equation shows that *the Hall field is uniform along the width*. The uniform Hall field means a uniform current density, which is along the x direction. Such distributions of the Hall voltage and the current are those minimizing the total entropy production, which is in accordance with the theorem of the minimum entropy production [5].

Many other theoretical works have been performed on the Hall-voltage and current distributions both in the absence and in the presence of dissipation. In the dissipationless case, quantum wires with width comparable to l have been studied by calculating the wave function numerically and taking into account ρ_{polar} in this way [6, 7, 8]. In several papers [9, 8, 10] the edge charge due to electrons added to (and subtracted from) edge states was considered. Such edge charge can be described as the

charge due to the polarization in eq.(4) since adding and subtracting electrons in this way is equivalent to shifting the whole electrons by the appropriate distance. The theory has also been extended to the fractional quantum Hall states [11]. In the dissipative case, the theory has been extended to a state with compressible and incompressible strips in a slowly-varying confining potential [12, 13, 14].

Fontein et al. [15, 16] have measured the Hall potential in a 2-mm-wide 2DES formed in a GaAs/AlGaAs heterostructure using the linear electro-optic effect. They have observed a crossover of the Hall-voltage distribution from the *concentrated-near-edges* to the *uniform* distribution by increasing the temperature or the current. This observation suggests that the crossover occurs with increasing the dissipation σ_{xx}^0 . At first glance this seems to contradict the expectation from the above theories: any real systems of macroscopic size should have nonzero dissipation and should show the uniform distribution. A possible reason for the contradiction may be the difference in angular frequency ω of current: the theories assumed $\omega = 0$ (the steady state), while the experiment applied AC current of $\omega/2\pi = 235\text{Hz}$ to employ the lock-in technique. The theorem of the minimum entropy production [5], which leads to the uniform distribution, is applicable only to the case of $\omega = 0$.

In this paper we study theoretically the Hall-voltage distribution in the case of $\omega \neq 0$. The 2DES we consider here is uniform except at sharp edges, while a 2DES with a slowly-varying confining potential will be studied elsewhere. The value of the filling factor in the uniform bulk region is not restricted to integers, but we neglect the electron correlation such as in the fractional quantum Hall effect by considering a relatively high temperature. In this paper we study only the incoherent linear transport by employing the local conductivity tensor. A crossover between coherent and incoherent regimes has been studied theoretically [17, 18] for the voltage distribution in the 2DES with source and drain contacts in strong magnetic fields.

The organization of the paper is as follows. In §2, we introduce a model and derive an equation for the Hall potential as a function of y . In §3, we present an analytical solution for the Hall potential when complex conductivities are constant and the interaction is short-ranged. In §4, we study numerically the Hall potential in the long-range interaction as well as in the short-range interaction. We present our model for the complex conductivities in the edge region, our method of numerical calculation, and calculated results. In §5, conclusions and discussion are given. In Appendix we estimate the value of the complex conductivities.

2. Model and Equations

2.1. Current Density and Complex Conductivity

We assume that the response of the current to the electric field is local. That is, the current density $j_\alpha(\mathbf{r}, t) = j_\alpha(\mathbf{r}, \omega)e^{i\omega t}$ at the position $\mathbf{r} = (x, y)$ is determined only by the electric field at the same position $E_\beta(\mathbf{r}, \omega)e^{i\omega t}$ ($\alpha, \beta = x, y$):

$$j_\alpha(\mathbf{r}, \omega) = \sum_\beta \sigma_{\alpha\beta}(\mathbf{r}, \omega) E_\beta(\mathbf{r}, \omega). \quad (8)$$

This local relation may be applicable to macroscopic samples where $W \gg l_\phi$ with l_ϕ the phase coherence length, because in this case the length scale of variations of the electric field, which is of the order of W , is much larger than l_ϕ . The conductivity $\sigma_{\alpha\beta}(\mathbf{r}, \omega)$ in this case can be determined by calculating the uniform-current response to the uniform electric field and taking the average over the random potential with the length scale l_{ran} since we assume here that $l_{\text{ran}} < l_\phi$. This averaging procedure makes the 2DES isotropic in the xy plane so that we have $\sigma_{xx}(\mathbf{r}, \omega) = \sigma_{yy}(\mathbf{r}, \omega)$ and $\sigma_{xy}(\mathbf{r}, \omega) = -\sigma_{yx}(\mathbf{r}, \omega)$. The spatial dependence of $\sigma_{\alpha\beta}(\mathbf{r}, \omega)$ in this paper is due to the decrease of the local electron density as approaching a boundary of the 2DES. When $\omega \neq 0$, the above relation eq.(8) can be rewritten, in terms of the polarization $P_\alpha(\mathbf{r}, \omega) = j_\alpha(\mathbf{r}, \omega)/i\omega$ and the dielectric susceptibility $\chi_{\alpha\beta}(\mathbf{r}, \omega) = \sigma_{\alpha\beta}(\mathbf{r}, \omega)/i\omega$, as $P_\alpha(\mathbf{r}, \omega) = \sum_\beta \chi_{\alpha\beta}(\mathbf{r}, \omega) E_\beta(\mathbf{r}, \omega)$.

Now we restrict our discussion to the low-frequency region. The relevant energy scales in the response to the AC electric field are $\hbar\omega_c$ and the Landau-level broadening due to the random potential. By assuming that $\hbar\omega$ is much smaller than such energy scales, we expand $\sigma_{\alpha\beta}(\omega)$ in a power series of ω and retain terms up to the first order of ω . Since the real and imaginary parts of $\sigma_{\alpha\beta}(\omega) = \sigma'_{\alpha\beta}(\omega) + i\sigma''_{\alpha\beta}(\omega)$ satisfy the following relation: $\sigma'_{\alpha\beta}(-\omega) = \sigma'_{\alpha\beta}(\omega)$ and $\sigma''_{\alpha\beta}(-\omega) = -\sigma''_{\alpha\beta}(\omega)$, we can write $\sigma_{\alpha\beta}(\omega)$ as

$$\sigma_{\alpha\beta}(\omega) = \sigma_{\alpha\beta}^0 + i\omega\chi_{\alpha\beta}^0, \quad (9)$$

where $\sigma_{\alpha\beta}^0$ is the DC conductivity and $\chi_{\alpha\beta}^0$ is the DC susceptibility.

2.2. Hall Charge Density

In this paper we consider a 2DES with two boundaries which are both parallel to the x axis. We assume that $\sigma_{\alpha\beta}$ and E_β are uniform along x . In the vicinity of the boundaries, $\sigma_{\alpha\beta}$ has a y dependence, which will be specified in §4.1. From the equation of charge conservation, the Hall charge density ρ is given by

$$i\omega\rho(y) = -\nabla_y j_y(y). \quad (10)$$

In this equation and in the following, the ω dependence of the variables and coefficients will not be shown explicitly. From eq.(8), we have

$$j_y(y) = \sigma_{yx}(y)E_x + \sigma_{yy}(y)E_y(y). \quad (11)$$

Here we have also assumed that E_x has no dependence on y since $\nabla_y E_x - \nabla_x E_y \approx 0$ when ω is small. The above equations show that, when $E_x \neq 0$, the y dependence of σ_{yx} gives the Hall charge density ρ and the Hall field E_y .

2.3. Current due to the Chemical-Potential Gradient

Corresponding to the two terms of $\sigma_{\alpha\beta}$ in eq.(9), $\rho(y)$ has two components: $\rho(y) = \rho_{\text{trans}}(y) + \rho_{\text{polar}}(y)$, where ρ_{trans} is the transport charge density defined by

$$i\omega\rho_{\text{trans}}(y) = -\nabla_y [\sigma_{yx}^0(y)E_x + \sigma_{yy}^0(y)E_y(y)]. \quad (12)$$

and ρ_{polar} is the polarization charge density defined by

$$\rho_{\text{polar}}(y) = -\nabla_y [\chi_{yx}^0(y)E_x + \chi_{yy}^0(y)E_y(y)]. \quad (13)$$

The transport charge density $\rho_{\text{trans}}(y)$ gives a deviation of the chemical potential μ from its equilibrium value μ_{eq} . The deviation $\Delta\mu = \mu - \mu_{\text{eq}}$ is given by

$$\Delta\mu(y) = \rho_{\text{trans}}(y)/(-eD_T), \quad (14)$$

where $D_T = \partial n / \partial \mu$ (n : electron density per unit area) is the thermodynamic density of states. The gradient of $\Delta\mu$ induces the current and the total current density is given in terms of the gradient of the electrochemical potential (μ_{ec}), $\nabla_y \mu_{\text{ec}} = eE_y + \nabla_y \Delta\mu$, as

$$j_y(y) = \sigma_{yx}(y)E_x + \sigma_{yy}^0(y)e^{-1}\nabla_y \mu_{\text{ec}} + i\omega\chi_{yy}^0(y)E_y(y). \quad (15)$$

Note that $\nabla_x \Delta\mu = 0$ and the polarization current is induced only by the electric field. We have calculated numerically the value of $\nabla_y \Delta\mu$ and have obtained $|\nabla_y \Delta\mu| \ll e|E_y|$, which is also supported by an analytical result below in eq.(29). Therefore we will neglect the term proportional to $\nabla_y \Delta\mu$ in the following.

2.4. Hall Potential

The electrostatic potential due to the Hall charge, $\phi(y)$, in a 2DES uniform along x is given by

$$\phi(y) = \int_{-\infty}^{\infty} dy' K(y - y') \rho(y'). \quad (16)$$

where $K(y - y')$ is the potential due to the unit line charge at a distance $|y - y'|$. We consider the two models of the electrostatic interaction. One is the long-range interaction with

$$K(y - y') = -\frac{2}{\varepsilon} \ln |y - y'|. \quad (17)$$

This is the potential in a dielectric material with the dielectric constant ε and has been used in the previous work by MacDonald et al. [3].

The other model is the short-range interaction with

$$K(y - y') = r_K \delta(y - y'). \quad (18)$$

In this model $\phi(y) = r_K \rho(y)$. This model is valid when the range of $K(y - y')$ is much shorter than the length scale of variation of $\rho(y)$, L_ρ . If we consider a 2DES with a parallel gate electrode at distance d , this condition becomes $d \ll L_\rho$. In such system

$$r_K = 4\pi d / \varepsilon. \quad (19)$$

3. Short-Range Interaction and Constant Conductivity

We first consider the simpler case of the short-range interaction. If the interaction is short-ranged, electrostatics, in addition to transport, becomes local and the Hall potential $\phi(y)$ is described by a differential equation. In this section we consider the bulk uniform region, as the simplest case, where complex conductivities σ_{yx} and σ_{yy} have no spatial dependence and

$\sigma_{yx} = \sigma_{yx}^{0b} + i\omega\chi_{yx}^{0b}$, $\sigma_{yy} = \sigma_{yy}^{0b} + i\omega\chi_{yy}^{0b}$. In this case $\phi(y)$ is described by a differential equation with constant coefficients:

$$i\omega\phi(y) = D(1 + i\tilde{\omega})\nabla_y^2 \phi(y), \quad (20)$$

where D is a diffusion constant given by

$$D = r_K \sigma_{yy}^{0b}, \quad (21)$$

and $\tilde{\omega}$ is a normalized angular frequency defined by

$$\tilde{\omega} = \omega\chi_{yy}^{0b} / \sigma_{yy}^{0b}. \quad (22)$$

When $\tilde{\omega} \ll 1$, the equation for $\phi(y)$ becomes

$$i\omega\phi(y) = D\nabla_y^2 \phi(y). \quad (23)$$

Then $\phi(y)$ is given by

$$\phi(y) = \phi_0 \exp[-(1 + i)y / \lambda(\tilde{\omega})], \quad (24)$$

and is decaying and oscillating with y . We also have a solution: $\phi(y) = \phi_0 \exp[(1 + i)y / \lambda]$. Here $\lambda(\tilde{\omega})$ is the decay length given by

$$\lambda(\tilde{\omega}) = \sqrt{2D/\omega}, \quad (25)$$

and is equal to the diffusion length in the time interval $1/\omega$. In this section we consider the change of λ when either ω or σ_{yy}^{0b} is changed. Since $\lambda(\tilde{\omega}) = \sqrt{2r_K\chi_{yy}^{0b}/\tilde{\omega}}$, $\lambda(\tilde{\omega})$ decreases with the increase of $\tilde{\omega}$, that is $\lambda(\tilde{\omega})$ decreases when ω is increased or σ_{yy}^{0b} is decreased.

When $\tilde{\omega} \gg 1$, the equation for $\phi(y)$ becomes

$$\phi(y) = r_K \chi_{yy}^{0b} \nabla_y^2 \phi(y). \quad (26)$$

The decaying solution in this case is

$$\phi(y) = \phi_0 \exp(-y/\lambda_\infty), \quad (27)$$

and the decay length is

$$\lambda_\infty = \sqrt{r_K \chi_{yy}^{0b}}. \quad (28)$$

If we employ eq.(19) and an estimate using eq.(A.12), $\chi_{yy}^{0b} = e^2 \nu_b / (h\omega_c)$ with ν_b the bulk filling factor, we have $\lambda_\infty = \sqrt{2\nu_b l^2 d / a_B^*}$ where $a_B^* = \hbar^2 \varepsilon / me^2$ is the effective Bohr radius. If we use the value of m and ε of GaAs, $\nu_b = 4$, $d = 0.1 \mu\text{m}$ and $B = 5\text{T}$, we obtain $\lambda_\infty \sim d$ which means that the spatial variation of ϕ and ρ in this case is too steep to satisfy the condition ($\lambda_\infty \gg d$) for the short-range model. In the lower-magnetic-field region, however, $\lambda_\infty (\propto B^{-1})$ becomes larger and the condition becomes satisfied.

Starting from $\tilde{\omega} = 0$, we increase $\tilde{\omega}$. Then we first encounter a crossover around the point satisfying $\lambda(\tilde{\omega}) = W$ when W is large enough. In this crossover the Hall-voltage distribution changes from *uniform* to *concentrated-near-edges* profile. If we increase $\tilde{\omega}$ further, we come to another crossover around the point satisfying $\tilde{\omega} = 1$, where the dominant response changes from *transport current* to *polarization current*. In the second

crossover ($\tilde{\omega} = 1$) the decay length becomes of the order of λ_∞ . To distinguish the second crossover from the first one, $\lambda_\infty \ll W$ must be satisfied, in addition to the condition for the short-range model of $\lambda_\infty \gg d$.

We examine the validity of the approximation to neglect the current due to the chemical-potential gradient in the present uniform case in the short-range model by showing $|\nabla_y \Delta \mu| \ll e|E_y|$. Using eqs.(14), (12), (20) and (21), we have

$$\frac{|\nabla_y \Delta \mu|}{|eE_y|} = (e^2 D_T r_K \sqrt{1 + \tilde{\omega}^2})^{-1} \sim \frac{l}{d}, \quad (29)$$

where we have used $D_T \sim 1/(2\pi l^2 \hbar \omega_c)$ as well as eq.(19) and $\tilde{\omega} \sim 1$. For $d/l = 10$, $|\nabla_y \Delta \mu|/|eE_y| \sim 1/10$ and we can neglect $\nabla_y \Delta \mu$.

4. Numerical Calculation

4.1. Model for σ_{yx} and σ_{yy} in the edge region

In the long-range interaction, the relation between the Hall potential $\phi(y)$ and the Hall charge density $\rho(y)$ is nonlocal. Therefore, the value of $\phi(y)$ in the bulk region is influenced by that of $\rho(y)$ in the edge region, which is in turn determined by eqs.(10)(11) with $\sigma_{yx}(y)$ and $\sigma_{yy}(y)$ in the edge region. Here we introduce a model of $\sigma_{yx}(y) = \sigma_{yx}^0(y) + i\omega \chi_{yx}^0(y)$ and $\sigma_{yy}(y) = \sigma_{yy}^0(y) + i\omega \chi_{yy}^0(y)$ in the edge region.

In the edge region the electron density and the equilibrium chemical potential $\mu_{eq}(y)$ decrease as approaching the boundary from the bulk region. We describe this y dependence by a simple function:

$$\begin{aligned} \mu_{eq}(y) &= \mu_b, \quad (0 < y < W/2) \\ \mu_{eq}(y) &= \mu_b \left[1 - (y - W/2)^2 / W_e^2 \right], \quad (y > W/2) \end{aligned} \quad (30)$$

and $\mu_{eq}(-y) = \mu_{eq}(y)$. The parameter W_e represents the width of the edge region since $\mu_{eq}(W/2 + W_e) = 0$.

We assume that the y dependence of $\sigma_{yx}(y)$ and $\sigma_{yy}(y)$ originates from the y dependence of $\mu_{eq}(y)$, that is $\sigma_{yx}(y) = \sigma_{yx}(\mu_{eq}(y))$ and $\sigma_{yy}(y) = \sigma_{yy}(\mu_{eq}(y))$. As for the μ_{eq} dependence of σ_{yx}^0 and σ_{yy}^0 , we employ a model [19] which retains the observed features of $\sigma_{yx}^0(B)$ and $\sigma_{yy}^0(B)$:

$$\sigma_{yx}^0(\mu_{eq}) = \frac{2e^2}{h} \sum_N f_N, \quad (31)$$

$$\sigma_{yy}^0(\mu_{eq}) = \frac{2e^2 D_0}{k_B T} \sum_N (2N + 1) f_N (1 - f_N), \quad (32)$$

$$f_N = \left\{ 1 + \exp \left[(\varepsilon_N - \mu_{eq}) / k_B T \right] \right\}^{-1} \quad (33)$$

where $\varepsilon_N = \hbar \omega_c (N + 1/2)$, $N = 0, 1, 2, \dots$, T is the temperature and D_0 is a constant. As for $\chi_{yx}^0(\mu_{eq})$ and $\chi_{yy}^0(\mu_{eq})$ we use a simple formula

$$\chi_{yx}^0(\mu_{eq}) = 0, \quad \chi_{yy}^0(\mu_{eq}) = e^2 \nu / (\hbar \omega_c), \quad (34)$$

which is derived in Appendix. The derivation assumes that Landau-level mixings are negligible and also that Γ is negligible compared to $\hbar \omega_c$ with Γ the Landau-level broadening. Since such assumptions are not always satisfied in the cases we consider below, the above formula itself should be considered an assumption. Note that the conclusion of this paper does not change even when $\sigma_{yx}^0(\mu_{eq})$, $\sigma_{yy}^0(\mu_{eq})$ and $\chi_{yy}^0(\mu_{eq})$ change substantially, as will be shown below. In calculating the filling factor ν at a given μ_{eq} and T , we use the following density of states:

$$\begin{aligned} D(\varepsilon) &= 1/(2\pi l^2 \Gamma), \quad (|\varepsilon - \varepsilon_N| < \Gamma) \\ D(\varepsilon) &= 0, \quad (\text{otherwise}) \end{aligned} \quad (35)$$

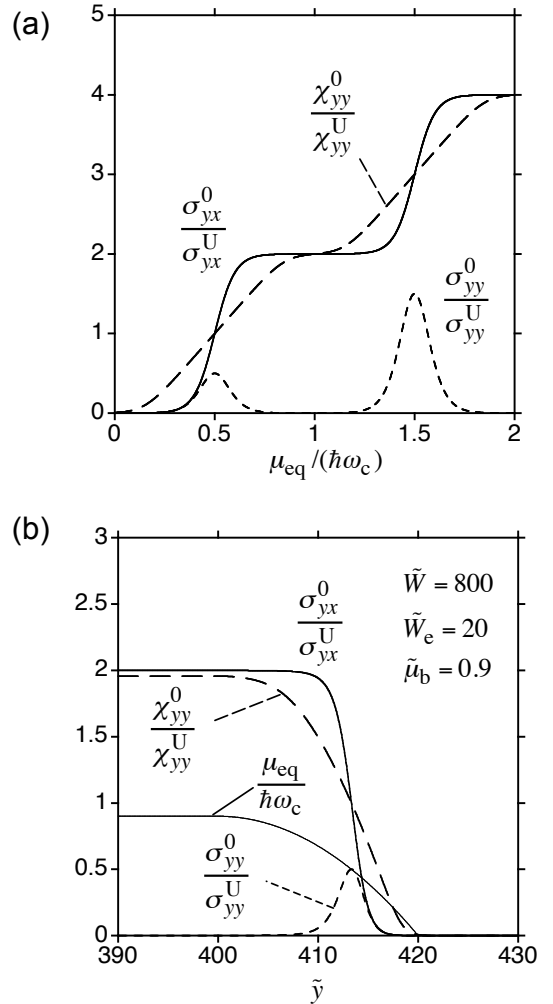


Figure 1: (a) Dependence of σ_{yx}^0 , σ_{yy}^0 and χ_{yy}^0 on the equilibrium chemical potential, μ_{eq} . $\sigma_{yx}^U = e^2/h$, $\sigma_{yy}^U = e^2 D_0 / (k_B T)$ and $\chi_{yy}^U = e^2 / (\hbar \omega_c)$. (b) y dependence of σ_{yx}^0 , σ_{yy}^0 and χ_{yy}^0 as well as μ_{eq} . $\tilde{y} = y/l_U$, $\tilde{W} = W/l_U$, $\tilde{W}_e = W_e/l_U$ and $\tilde{\mu}_b = \mu_b / (\hbar \omega_c)$ with l_U defined in eq.(39).

Figure 1(a) presents the μ_{eq} dependences of σ_{yx}^0 , σ_{yy}^0 and χ_{yy}^0 , while Fig.1(b) shows an example of the y dependences of σ_{yx}^0 , σ_{yy}^0 and χ_{yy}^0 as well as that of μ_{eq} . In the numerical calculation the values of Γ and T are fixed as $\Gamma = 0.35 \hbar \omega_c$ and $k_B T = 0.05 \hbar \omega_c$.

4.2. Method of Numerical Calculation

We calculate the Hall potential $\phi(y)$ and the Hall charge density $\rho(y)$ by solving eqs.(10) and (16) with eq.(17). We consider a periodic array of infinitely-long 2DES strips. The n th strip is in $-W/2 + nW_p < y < W/2 + nW_p$ where n is the integer and W_p is the periodicity. The Hall potential satisfies

$$\phi(-y) = -\phi(y), \quad \phi(y + W_p) = \phi(y), \quad (36)$$

which leads to $\phi(y) = 0$ at $y = \pm W_p/2$. The same is the case for $\rho(y)$. Therefore we expand $\phi(y)$ and $\rho(y)$ in the Fourier series as

$$\phi(y) = \sum_{k=1}^{k_{\max}} \phi_k \sin\left(\frac{2\pi k}{W_p} y\right), \quad \rho(y) = \sum_{k=1}^{k_{\max}} \rho_k \sin\left(\frac{2\pi k}{W_p} y\right). \quad (37)$$

Then eqs.(10) and (16) become a system of linear equations for ϕ_k and ρ_k with a nonhomogeneous term proportional to E_x . By solving this numerically, we obtain ϕ_k and ρ_k . We have confirmed that $\phi(y)$ within the 2DES has little dependence on W_p if the gap between 2DES strips is wide enough. In the following we present results for $W_p = 2W$.

4.3. Calculated Results

We use the following dimensionless variable:

$$\tilde{y} = y/l_U, \quad (38)$$

with a unit

$$l_U = 2\chi_{yy}^{0b}/\varepsilon = \nu_b l^2/(\pi a_B^*), \quad (39)$$

where an estimate using eq.(A.12) is substituted for χ_{yy}^{0b} . When we use $B = 5T$, $\nu_b = 4$ and the value of m and ε of GaAs, we have $l_U \sim l$, while $l_U(\propto B^{-2})$ becomes larger at smaller B . We introduce the normalized Hall field and potential as

$$\tilde{E}_y = E_y \sigma_{yy}^{0b}/(E_x \sigma_{yx}^{0b}), \quad \tilde{\phi} = \phi \sigma_{yy}^{0b}/(E_x \sigma_{yx}^{0b} l_U). \quad (40)$$

From this definition $\tilde{E}_y = -1$ within the uniform bulk region in the steady state since $j_y = \sigma_{yx}^{0b} E_x + \sigma_{yy}^{0b} E_y = 0$. From eqs.(10), (16) and (17) we can show that $\tilde{\phi}$ as a function of $\tilde{y}$ is determined only by $\tilde{\omega}$ and $\tilde{W} = W/l_U$ if the edge region is negligible and the gap between 2DES strips is wide enough.

Figure 2(a) presents the absolute value of $\tilde{\phi}$ in the long-range interaction as a function of $\tilde{y}$ for several values of $\tilde{\omega}$ when $\tilde{W} = 800$ and the y dependence of σ_{yx}^0 , σ_{yy}^0 and χ_{yy}^0 is given as in Fig.1(b). Its spatial dependence within the 2DES ($-W/2 < y < W/2$) demonstrates a crossover from a slope with a constant angle (uniform Hall field) to that with a larger angle at both edges compared to the center (concentrated Hall voltage) with increasing $\tilde{\omega}$. This crossover in the long-range interaction is essentially the same as that obtained in the short-range interaction in §3. The argument of $\phi(y)$ shown in Fig.2(b) exhibits a delay relative to that of E_x . The phase delay is absent at $\tilde{\omega} = 0$, increases with increasing $\tilde{\omega}$, and approaches $\pi/2$ at $\tilde{\omega} \rightarrow \infty$. It is shown from eqs.(10), (16) and (17) that, as $\tilde{\omega} \rightarrow \infty$, $\phi(y)i\omega/E_x$ approaches a real value independent of ω .

Figure 3 shows calculated results focused on the crossover by plotting a normalized Hall potential $\phi(y)/\phi(y_B)$ with $y_B =$

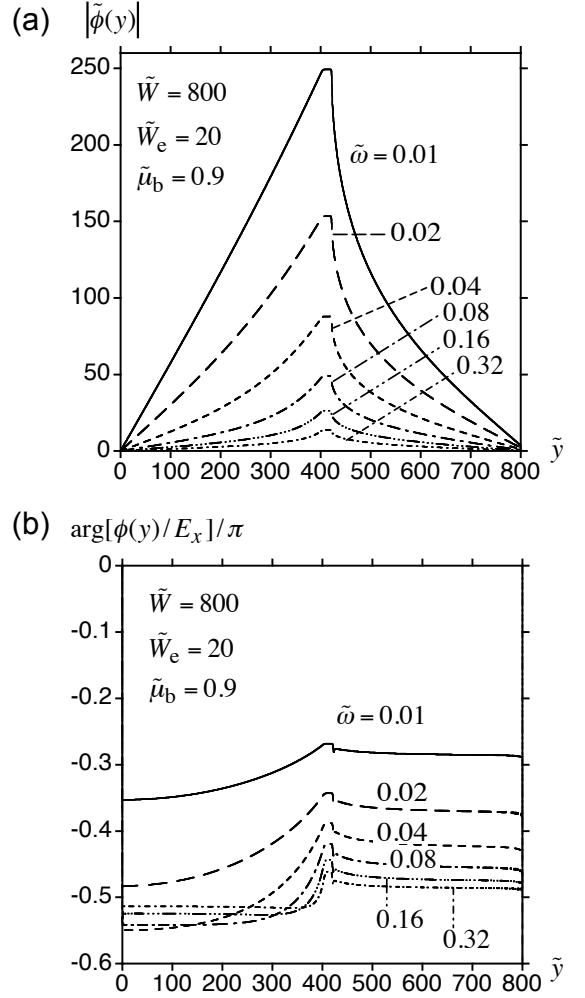


Figure 2: Spatial profile of the Hall potential $\phi(y)$ in the long-range interaction eq.(17) at several values of $\tilde{\omega}$ defined by eq.(22).

$W/2$ in Fig.3(a) and a normalized current density $\tilde{j}_x(y) = j_x(y)\rho_{xx}^{0b}/E_x$ with $\rho_{xx}^{0b} = \sigma_{yy}^{0b}/(\sigma_{xx}^{0b}\sigma_{yy}^{0b} - \sigma_{xy}^{0b}\sigma_{yx}^{0b})$ in Fig.3(b). The normalized current density $\tilde{j}_x(y)$ depends on $\sigma_{yy}^{0b}/\sigma_{yx}^{0b}$. In Fig.3(b) the value of $\sigma_{yy}^{0b}/\sigma_{yx}^{0b} = 0.01$ is used. However, $\tilde{j}_x(y)$ at such a small value of $\sigma_{yy}^{0b}/\sigma_{yx}^{0b}$ is approximately the same in the bulk uniform region as $\tilde{j}_x(y)$ at $\sigma_{yy}^{0b}/\sigma_{yx}^{0b} = 0$, which is equal to $-\tilde{E}_y$ in the bulk uniform region where σ_{yx}^0 takes a constant value σ_{yx}^{0b} . Figure 3(b) demonstrates the crossover in the y dependence of $|j_x|$ and $|E_y|$. Although $|j_x|$ and $|E_y|$ decrease with increasing $\tilde{\omega}$ also at the boundary $y_B = W/2$, the decrease is faster in the central region than at the boundary. Also note that $|j_x|$ and $|E_y|$ at higher $\tilde{\omega}$ in the long-range interaction have a longer tail into the bulk region compared to those in the short-range interaction which show an exponential decay as derived in §3. Such a longer tail is understood from the nonlocal relation between $\phi(y)$ and $\rho(y')$ in eq.(16) in the long-range interaction.

Figure 4(a) presents the absolute value of a normalized current $\tilde{I}_x = I_x \rho_{xx}^{0b}/(E_x W)$ as a function of $\tilde{\omega}$ where I_x is the current

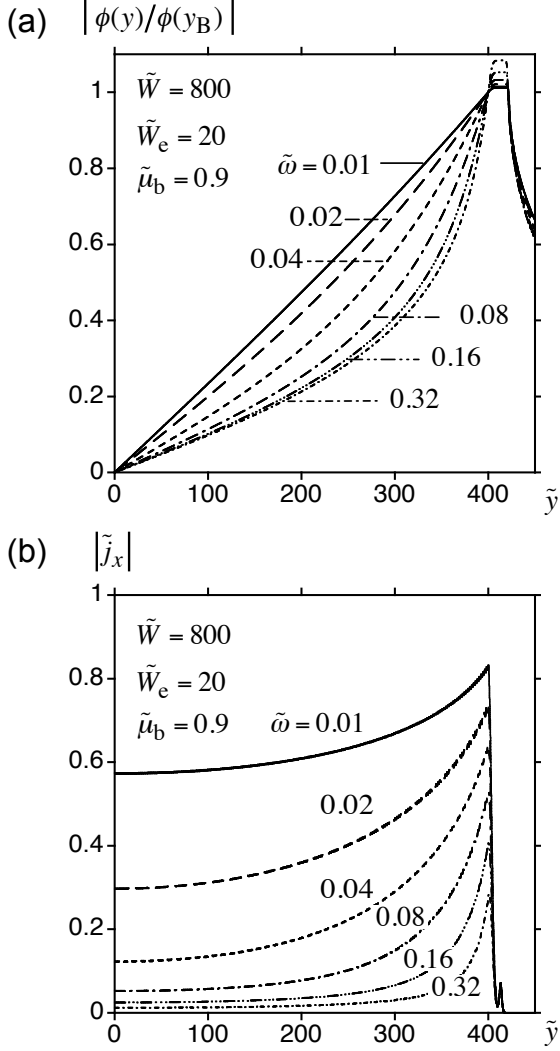


Figure 3: Crossover in the long-range interaction eq.(17), (a) in the Hall potential $\phi(y)/\phi(y_B)$ with $y_B = W/2$ and (b) in the current density $j_x(y)$.

per strip defined by

$$I_x = \int_{-W_p/2}^{W_p/2} dy j_x. \quad (41)$$

Note that $I_x/(E_x W)$ is equal to the inverse of the AC magnetoresistance $R_{xx} = V_x/I_x$ when the distance between the voltage probes is W . The absolute value of I_x exhibits a drop with increasing $\tilde{\omega}$. The value of $\tilde{\omega}$ at the drop coincides roughly with that at the crossover from the uniform to the concentrated distribution, $\tilde{\omega}_{\text{cross}}$, while the drop of I_x is not only by the reduction of $|E_y(y)|$ in the central region of the strip but also by the reduction around $y = \pm y_B$.

Figure 4(a) shows that $\tilde{\omega}_{\text{cross}}$ decreases with increasing W as

$$\log \tilde{\omega}_{\text{cross}} \approx -\log W + \text{const.} \quad (42)$$

This W dependence of $\tilde{\omega}_{\text{cross}}$ in the long-range interaction is different from that in the short-range interaction shown in Fig.4(b):

$$\log \tilde{\omega}_{\text{cross}} \approx -2 \log W + \text{const.} \quad (43)$$

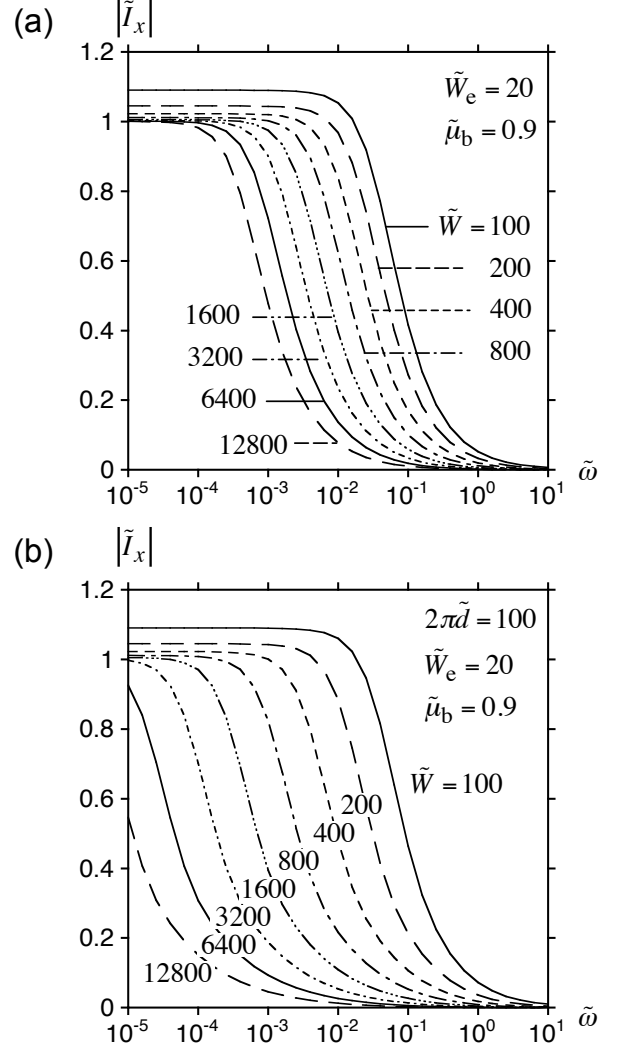


Figure 4: Current I_x as a function of $\tilde{\omega}$ (a) in the long-range interaction eq.(17) and (b) in the short-range interaction eqs.(18)(19).

The analytical expression of $\lambda(\tilde{\omega})$ in the short-range interaction given by eq.(25) leads to $\tilde{\omega}_{\text{cross}} \propto W^{-2}$ if we use $\lambda(\tilde{\omega}_{\text{cross}}) = W$, in agreement with the numerical result eq.(43).

Finally we show that the crossover from the uniform to the concentrated distribution in the bulk uniform region does not change, at least qualitatively, when the values of $\sigma_{yx}^0(y)$, $\sigma_{yy}^0(y)$ and $\chi_{yy}^0(y)$ in the edge region are changed, even in the long-range interaction. We introduce a variation of $\sigma_{yx}^0(y)$, $\sigma_{yy}^0(y)$ and $\chi_{yy}^0(y)$ by changing the equilibrium chemical potential in the bulk region as $\tilde{\mu}_b = 0.5$ (Fig.5(a)), $\tilde{\mu}_b = 0.9$ (Fig.1(b)) and $\tilde{\mu}_b = 1.4$ (Fig.5(b)). Such a change in $\tilde{\mu}_b$ gives a large change in the normalized coefficients $\sigma_{yx}^0(y)/\sigma_{yx}^{\text{ob}}$, $\sigma_{yy}^0(y)/\sigma_{yy}^{\text{ob}}$ and $\chi_{yy}^0(y)/\chi_{yy}^{\text{ob}}$ in the edge region. Figure 6 shows that the large differences in the normalized coefficients in the edge region give only small differences in $|\phi(y)/\phi(y_B)|$ in the bulk region. Note that $\phi(y)/\phi(y_B)$ depends only on $\tilde{y}$, $\tilde{\omega}$, and the normalized coefficients. In addition, Fig.6 shows that, such differences in $|\phi(y)/\phi(y_B)|$ decrease with decreasing the width of the edge region $\tilde{W}_e$.

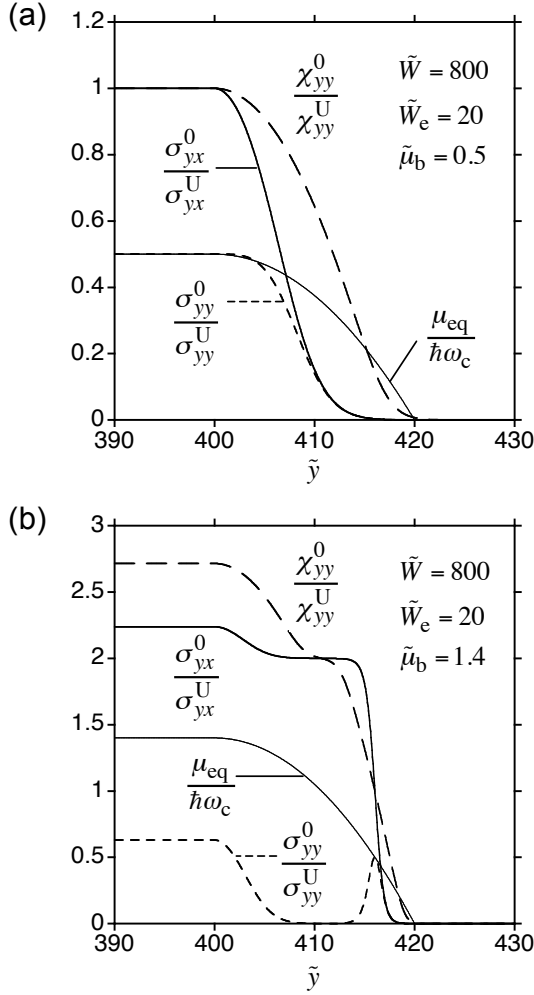


Figure 5: Spatial profiles of σ_{yx}^0 , σ_{yy}^0 and χ_{yy}^0 as well as μ_{eq} for different equilibrium chemical potentials in the bulk region: (a) $\tilde{\mu}_b = 0.5$ and (b) $\tilde{\mu}_b = 1.4$.

In this paper we have chosen a quite simple model for the μ_{eq} dependence of σ_{yx}^0 , σ_{yy}^0 and χ_{yy}^0 . However, differences in $\sigma_{yx}^0(\mu_{eq})$, $\sigma_{yy}^0(\mu_{eq})$ and $\chi_{yy}^0(\mu_{eq})$ between this model and the more accurate model affect little the crossover, since we have shown above that $|\phi(y)/\phi(y_B)|$ in the bulk region is quite insensitive to $\sigma_{yx}^0(y)$, $\sigma_{yy}^0(y)$ and $\chi_{yy}^0(y)$ in the edge region.

5. Conclusions and Discussion

We have studied the Hall potential $\phi(y)$ and the Hall field $E_y(y)$ as a function of y (in the width direction) in quantum Hall systems with width W in the case of low- ω AC current in the incoherent linear transport. The dynamics of the local Hall-charge density in the uniform bulk region is determined by the complex diagonal conductivity $\sigma_{yy} = \sigma_{yy}^{ob} + i\omega\chi_{yy}^{ob}$ where σ_{yy}^{ob} and χ_{yy}^{ob} are the DC conductivity and the DC dielectric susceptibility, respectively, in the bulk. We have made calculations in the long-range interaction as well as in the short-range interaction, and have obtained the following conclusions common to both interactions. In the lower- ω region of $\tilde{\omega} = \omega\chi_{yy}^{ob}/\sigma_{yy}^{ob} \ll 1$

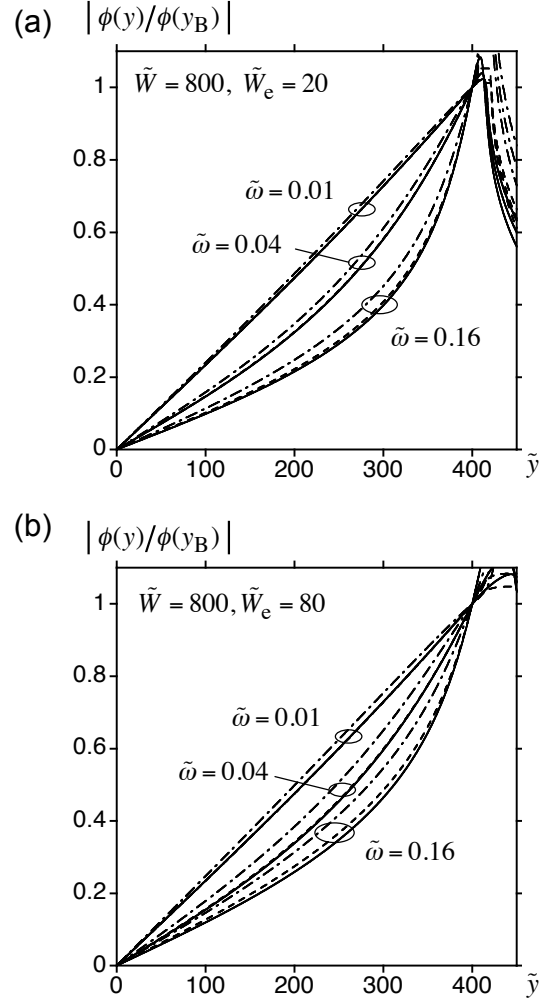


Figure 6: Hall potential $\phi(y)$ in the long-range interaction eq.(17) for different equilibrium chemical potentials in the bulk region, $\tilde{\mu}_b = 0.5$ (—), $\tilde{\mu}_b = 0.9$ (---) and $\tilde{\mu}_b = 1.4$ (- · - · -). (a) $\tilde{W}_e = 20$, (b) $\tilde{W}_e = 80$.

the transport component is dominant and the decay length λ of $E_y(y)$ decreases with increasing ω . When the decay length becomes comparable to W ($\omega = \omega_{cross}$), $E_y(y)$ makes a crossover from *uniform* to *concentrated-near-edges* profile [20]. In the higher- ω region of $\tilde{\omega} \gg 1$, on the other hand, the polarization component is dominant and the decay length approaches a constant value. The crossover of the Hall-voltage distribution at $\omega = \omega_{cross}$ is reflected in the frequency dependence of the magnetoresistance $R_{xx}(\omega)$. With increasing ω around ω_{cross} , $|R_{xx}|$ rises and the delay appears in the phase of the current relative to the voltage. Note that such a crossover also occurs when σ_{yy}^{ob} is decreased at a fixed ω .

The Hall-voltage distribution depends on ω and σ_{yy}^{ob} mainly through $\tilde{\omega} = \omega\chi_{yy}^{ob}/\sigma_{yy}^{ob}$. In the vicinity of $(\omega, \sigma_{yy}^{ob}) = (0, 0)$ we therefore obtain different distributions depending on the order of taking the limit of $\omega \rightarrow 0$ and that of $\sigma_{yy}^{ob} \rightarrow 0$. The decay length λ approaches a constant value when the limit of $\sigma_{yy}^{ob} \rightarrow 0$ is taken first ($\tilde{\omega} \rightarrow \infty$), while λ becomes infinity when that of $\omega \rightarrow 0$ is taken first ($\tilde{\omega} \rightarrow 0$). The theory in the ideal 2DES

by MacDonald et al. [3] corresponds to the case of $\tilde{\omega} \rightarrow \infty$ since $\sigma_{yy} = i\omega\chi_{yy}^0$ in their theory. In fact this theory obtains the concentrated-near-edges distribution, which we have reproduced in the case of $\tilde{\omega} \rightarrow \infty$. On the other hand, the dissipative DC transport giving the uniform distribution corresponds to the case of $\tilde{\omega} \rightarrow 0$, in which we have reproduced the uniform Hall field.

Low-frequency admittance has been theoretically studied in the edge-channel picture of quantum Hall conductors [21], in which the electrochemical potential of an edge channel is equal to that of a contact connected to the channel. Since the current through the channel is determined by the distant contact, the transport is nonlocal. On the other hand, this paper is based on the incoherent bulk picture in which the electrochemical potential of an edge state is considered to be equal to that of the neighboring bulk region. In this picture the transport is assumed to be local as in eq.(8).

In the experiment by Fontein et al. [16] the Hall-voltage distribution has been measured at a fixed frequency of $\omega/2\pi = 235\text{Hz}$ for two sets of temperature and current values: (A) $T = 1.5\text{K}$, $I = 5\mu\text{A}$ and (B) $T = 55\text{K}$, $I = 20\mu\text{A}$. In (A) the Hall voltage is concentrated near edges, while in (B) it is uniformly distributed. The value of σ_{yy}^{ob} is much larger in (B). If we apply the present theory to interpret this experiment, $\tilde{\omega}$ is decreased with the increase of σ_{yy}^{ob} and therefore the crossover has occurred from concentrated-near-edges to uniform distribution. In this interpretation, from the condition that the experimental value of $\tilde{\omega}$ coincides with its theoretical value, we can obtain an estimate of σ_{yy}^{ob} at the crossover in the experiment, which should be between those in (A) and (B).

Here we make such estimation of σ_{yy}^{ob} at the crossover from $\tilde{\omega}_{\text{cross}} = \omega\chi_{yy}^{\text{ob}}/\sigma_{yy}^{\text{ob}}$ where we use the theoretical value for $\tilde{\omega}_{\text{cross}}$ and the experimental value for $\omega/2\pi = 235\text{Hz}$. The sample width $W = 2\text{mm}$ and $l_U \sim l \sim 0.01\mu\text{m}$ give $\tilde{W} = W/l_U \sim 10^5$. The largest $\tilde{W}$ at which the crossover has been demonstrated in Fig.4(a) is $\tilde{W} = 12800$, for which we have obtained $\tilde{\omega}_{\text{cross}} \sim 10^{-3}$. By extrapolating the relation $\tilde{\omega}_{\text{cross}} \propto \tilde{W}^{-1}$ in eq.(42), we have $\tilde{\omega}_{\text{cross}} \sim 10^{-4}$ at $\tilde{W} = 10^5$. We use $\chi_{yy}^{\text{ob}} = e^2\nu_b/(\hbar\omega_c)$ from eq.(A.12), $\nu_b = 4$, $B = 5\text{T}$ and the effective mass of GaAs. Then we obtain an estimate of $\sigma_{yy}^{\text{ob}} = 10^{-10}\Omega^{-1}$. It may not be unrealistic that this value of σ_{yy}^{ob} is between the values of σ_{yy}^{ob} in (A) and (B).

Time scales longer than 0.01s have been observed in various experiments in quantum Hall systems [22, 23, 24, 25] and some of them have already been attributed to small values of σ_{yy}^{ob} at the time of publication [22, 25]. A theory based on small values of σ_{yy}^{ob} has also been proposed [26] to explain the experiments in the vicinity of the breakdown of the quantum Hall effect [23, 24].

For a contactless 2DES, the response to the AC electric field has been studied with use of capacitively-coupled electrodes in strong magnetic fields and a sharp drop of the response with increasing frequency has been observed in the MHz region [27]. To explain this drop, a theory for a contactless 2DES has been developed which assumes the short-range interaction as in eq.(18) and takes into account only the trans-

port component [27, 28]. This theory has derived the length scale of charge accumulation l_E , which is essentially the same as eq.(25). By comparing with the theory, the observed drop has been attributed to a crossover from bulk to edge response which occurs when l_E becomes smaller than the sample size.

Finally we note that the decay length of the Hall electric field (eq.(25)) and the length scale of charge accumulation [27, 28] are the penetration depth in the AC diffusion problem. The same penetration depth is encountered in the velocity distribution in fluid dynamics (the Stokes layer) [29] and in the temperature distribution in the AC calorimetry [30].

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Appendix A.

In this Appendix we estimate the value of χ_{yy}^0 and χ_{yx}^0 in eq.(9) by neglecting the Landau-level mixings and by neglecting the Landau-level broadening compared to $\hbar\omega_c$.

The conductivity $\sigma_{\alpha\beta}$, which corresponds to the uniform current density in the α direction induced by a uniform electric field with angular frequency ω applied along β ($\alpha, \beta = x, y$), is expressed by the Kubo formula [31, 32]:

$$\sigma_{\alpha\beta}(\omega) = \frac{1}{S} \int_0^\infty dt e^{-i\omega t - \varepsilon t} \int_0^\beta d\lambda \langle \hat{j}_\beta(-i\hbar\lambda) \hat{j}_\alpha(t) \rangle, \quad (\text{A.1})$$

where S is the area of the 2DES, ε the positive infinitesimal, and $\beta = (k_B T)^{-1}$ with k_B the Boltzmann constant and T the temperature. The current operator $\hat{j}_\alpha(t)$ in the above equation is given by $\hat{j}_\alpha(t) = e^{i\hat{H}t/\hbar} \hat{j}_\alpha e^{-i\hat{H}t/\hbar}$, where $\hat{H}$ is the Hamiltonian and $\hat{j}_\alpha$ is given by

$$\hat{j}_\alpha = \sum_\sigma \int \psi_\sigma^\dagger(\mathbf{r})(-ev_\alpha)\psi_\sigma(\mathbf{r})d\mathbf{r}, \quad (\text{A.2})$$

where v_α is the velocity operator and $\psi_\sigma(\mathbf{r})$ is the quantized wave function for spin σ ($\sigma = \uparrow, \downarrow$). The bracket $\langle \cdots \rangle$ in eq.(A.1) means that, for an operator $\hat{A}$, $\langle \hat{A} \rangle = \text{tr}(\hat{\rho}_{\text{eq}}\hat{A})$ with the equilibrium density matrix $\hat{\rho}_{\text{eq}} = e^{-\beta\hat{H}}/\text{tr}(e^{-\beta\hat{H}})$.

We employ the one-electron approximation in which

$$\hat{H} = \sum_\sigma \int \psi_\sigma^\dagger(\mathbf{r})H_\sigma\psi_\sigma(\mathbf{r})d\mathbf{r}. \quad (\text{A.3})$$

The one-electron operator H_σ has the eigenfunction $\varphi_p(\mathbf{r})$ with p a set of quantum numbers and the eigenvalue $\varepsilon_{p\sigma}$ which satisfy $H_\sigma\varphi_p(\mathbf{r}) = \varepsilon_{p\sigma}\varphi_p(\mathbf{r})$. We expand $\psi_\sigma(\mathbf{r})$ in terms of the eigenfunctions $\varphi_p(\mathbf{r})$:

$$\psi_\sigma(\mathbf{r}) = \sum_p c_{p\sigma}\varphi_p(\mathbf{r}), \quad (\text{A.4})$$

and then obtain

$$\hat{H} = \sum_{p\sigma} \varepsilon_{p\sigma} c_{p\sigma}^\dagger c_{p\sigma}. \quad (\text{A.5})$$

The current operator $\hat{j}_\alpha$ is also expressed as

$$\hat{j}_\alpha = \sum_{pp'\sigma} j_\alpha^{pp'} c_{p'\sigma}^\dagger c_{p\sigma}, \quad (\text{A.6})$$

with

$$j_\alpha^{pp'} = \int \varphi_{p'}^*(\mathbf{r})(-ev_\alpha)\varphi_p(\mathbf{r})d\mathbf{r}. \quad (\text{A.7})$$

In such one-electron approximation, we obtain

$$\sigma_{\alpha\beta}(\omega) = \frac{1}{S} \sum_{pp'\sigma} j_\beta^{pp'} j_\alpha^{p'p} T_{p'p\sigma} g(E_{p'p\sigma} - \hbar\omega), \quad (\text{A.8})$$

with

$$\begin{aligned} T_{p'p\sigma} &= \hbar(f_{p\sigma} - f_{p'\sigma})/E_{p'p\sigma} \quad (p \neq p'), \\ &= \hbar\beta f_{p\sigma}(1 - f_{p\sigma}) \quad (p = p'), \end{aligned} \quad (\text{A.9})$$

where $E_{p'p\sigma} = \varepsilon_{p'\sigma} - \varepsilon_{p\sigma}$ and $f_{p\sigma} = 1/[\exp[\beta(\varepsilon_{p\sigma} - \mu_{\text{eq}})] + 1]$ with μ_{eq} the equilibrium chemical potential, and

$$g(E) = i\frac{\mathcal{P}}{E} + \pi\delta(E). \quad (\text{A.10})$$

First we consider the ideal 2DES where the random potential $V_{\text{ran}} = 0$. In the ideal 2DES the eigenfunction is labeled by N and k where $N = 0, 1, \dots$ is the Landau index and k is the momentum along x . In this case, $j_\alpha^{N'k, Nk}$ is diagonal in k and

$$j_\alpha^{N'k, Nk} = 0, \quad \text{except } N' = N \pm 1, \quad (\text{A.11})$$

and we obtain

$$\begin{aligned} \sigma_{yy}^0 &= 0, \quad \chi_{yy}^0 = e^2\nu/(\hbar\omega_c), \\ \sigma_{yx}^0 &= e^2\nu/h, \quad \chi_{yx}^0 = 0. \end{aligned} \quad (\text{A.12})$$

Next we consider the 2DES where $V_{\text{ran}} \neq 0$. We neglect the Landau-level mixings induced by V_{ran} for simplicity. Then the eigenfunction is written as

$$\phi_{N\gamma}(\mathbf{r}) = \sum_k a_{\gamma k}^N \varphi_{Nk}(\mathbf{r}), \quad (\text{A.13})$$

which leads to

$$j_\alpha^{N'\gamma', N\gamma} = 0, \quad \text{except } N' = N \pm 1. \quad (\text{A.14})$$

In addition we assume that $\Gamma \ll \hbar\omega_c$ where Γ is the Landau-level broadening due to V_{ran} . Then we obtain the same formulas for σ_{yy}^0 , χ_{yy}^0 , σ_{yx}^0 and χ_{yx}^0 as those in the absence of V_{ran} , eq.(A.12), except that ν is the spatial average of the filling factor in the presence of V_{ran} . When Landau-level mixings are taken into account, σ_{yy}^0 and χ_{yx}^0 become nonzero.

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