Measuring Inequality of Subjective Well-Being: A Bayesian Approach*

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Abstract

This article proposes a new measure for inequality of subjective well-being and shows the empirical results using a Bayesian ordered probit model. We introduce a new concept called "regret" as a measure for inequality of subjective well-being. Regret is the probability with which a respondent who chooses an option in a multiple-choice question pertaining to subjective well-being does not choose any other option indicative of better well-being. Regret is estimated in connection with demographic factors using the Markov chain Monte Carlo (MCMC) method and data of the World Values Survey. Furthermore, the relationships between regret and GDP per capita and the changes therein are shown to investigate those between inequality of subjective well-being and economic conditions.

Keywords: Bayesian ordered probit model, Markov chain Monte Carlo (MCMC), regretted subjective well-being

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1 Introduction

There has been considerable research on happiness (see, e.g. Frank 1999, Frey and Stutzer 2002, and Layard 2005). In particular, Frey and Stutzer (2002) conducted a detailed literature survey and carried out an empirical analysis on happiness. In the empirical analyses presented by Frey and Stutzer (2002, Chapter 3) and Blanchflower and Oswald (2004), happiness is regarded as life satisfaction. For example, the research by Blanchflower and Oswald (2004) deals with the degree of happiness in the United States, while their empirical analyses deal with life satisfaction in the United Kingdom.

The comments of Becker and Rayo (2008) and Krueger (2008) on Stevenson and Wolfers (2008a) are useful in understanding the difference between happiness and utility. Stevenson and Wolfers (2008a) show a positive relationship between GDP and subjective well-being across countries. However, Becker and Rayo (2008) note that the differences between utility and happiness are paid scant attention to in Stevenson and Wolfers (2008a) and that happiness is not a measure of utility but a commodity like other commodities usually used in the utility function.[1] Krueger (2008) also makes a similar comment on the relationship between subjective well-being and utility.[2]

This article primarily aims to propose a new measure for subjective well-being and to show the empirical analyses using a Bayesian ordered probit model. In a questionnaire survey concerning subjective outcomes such as subjective well-being, options are often ordinally arranged. An ordered probit model can be used to statistically analyze the data on such ordinal options. Kalmijn and Veenhoven (2005) state the possibility such that "an egalitarian policy aimed at reducing differences in happiness would differ from a utilitarian policy aimed at producing a higher average level of happiness."[3] That is, a society with the lowest level of happiness for all individuals is equal but unhappy, whereas a society in which a few people have the lowest level of happiness and most have the highest level of happiness is a happier one. Thus, it is necessary to consider an index that identifies an individual's situation at each level of happiness, or each ordinal option.

Subsequent to Albert and Chib's (1993) seminal work, which utilizes latent variable representation, the Bayesian analysis using the Markov chain Monte Carlo (MCMC) method has become popular for the estimation of the ordered probit model. [4] One merit of the Bayesian analysis is that we can obtain the values of latent variables directly from the posterior results.[5] In this article,

[1] See Becker and Rayo (2008, pp.88–89).

[2] "At best, subjective well-being captures a component of utility." See Krueger (2008, p.100).

[3] "The main reason for looking at inequality of happiness in nations is in the possibility that this may reveal differences across nations other than those observed for the level of happiness. If so, this would mean that an egalitarian policy aimed at reducing differences in happiness would differ from a utilitarian policy aimed at producing a higher average level of happiness. To check this possibility we need measures for the general happiness level of a nation or nations and for inequality of the same, which are mutually independent, at least ideally." (Kalmijn and Veenhoven 2005, p.359)

[4] From the frequentist viewpoint, an ordered probit model can be estimated by using the maximum likelihood method. See, for example, Greene (2008, Chapter 23).

[5] Although the latent variables are unknown, their full conditional distributions (FCD) follow a truncated normal distribution. This makes the estimation of the ordered probit model very tractable in Bayesian analysis. See Albert and Chib (1993).

using the probabilities associated with the values of the latent variables, we propose a new concept called "regret" to measure subjective well-being. Subjective well-being is often investigated using surveys that are responded to by choosing an option representing its degree. Regret is the probability with which a respondent who chooses an option in a multiple-choice question pertaining to subjective well-being does not choose any other option indicative of better well-being. Thus, it enables us to capture the degree of unhappiness of the respondents.

Further, how to measure inequality of happiness is a major research issue. Kalmijn and Veenhoven (2005) consider nine statistics of inequality of happiness and conclude that four measures, including the standard deviation and the mean absolute difference, are suitable statistics to measure inequality, but the other five measures, including the Gini coefficient and Theil's entropy measure, are not suitable for both theoretical and empirical reasons.[6] Stevenson and Wolfers (2008b) estimate the mean and variance of happiness with the ordered probit model using the data of the General Social Survey (GSS) and show that inequality of happiness has declined in the United States since the 1970s. They use the variance as a convenient measure of inequality of happiness and decompose it to "within-group" variance and "between-group" variance to analyze the influence of change in the average level of happiness.[7] In these previous studies, inequality of happiness is computed using numerical values to represent the rating of happiness, which are measured with ladder scale and verbal rating scale. They measure inequality of happiness using ordinal variables. Thus, differences of happiness between people are not measurable and ignored as pointed out in Kalmijn and Veenhoven (2005).[8]

Regret is computed using a Bayesian ordered probit model in this article. It is possible to compute the individual regret and the average regret of a country. It is also possible to analyze inequality of happiness using distributions of regret and some inequality measures of regret. Thus, regret can denote an index of subjective well-being of a country and be used for time-series and cross-national comparisons of subjective well-being. These are the advantages in regret.

The empirical analyses on subjective well-being across countries are conducted using the data of the World Values Survey (WVS). Regret is estimated in connection with the demographic factors. The relationships between regret and GDP per capita are shown to investigate those between subjective well-being and economic conditions. For some countries, regret and GDP per capita were found to change in the same direction, which means that their subjective well-being decreases despite economic growth, and vice versa.[9]

The article proceeds as follows. In Section 2, we describe the Bayesian ordered probit model and its estimation procedure by using the algorithm in Nandram and Chen (1996) and Chen and Dey (2000). One merit of Bayesian analysis is that we can obtain the values of latent variables directly from the posterior results. In Section 3, using the values of latent variables, we propose a new inequality measure of subjective well-being. In Section 4, we present the posterior results for this measure using WVS data. Section 5 provides the

[6] See Kalmijn and Veenhoven (2005, pp.389–390).

[7] See Stevenson and Wolfers (2008b, S43, S70–S71).

[8] See Kalmijn and Veenhoven (2005, p.361).

[9] It seems that cases similar to the well-known Easterlin paradox are observed. See Easterlin (1974).

concluding remarks.

2 Bayesian Ordered Probit Model

Let y_i denote the ordinal discrete response of individual i for $i = 1, \cdots, n$; that is, $y_i = c$ for $c = 1, \cdots, C$. Further, let z_i denote the latent variable of individual i such that

$$y_i = c \ \text{ if } \ z_i \in (\gamma_{c-1}, \gamma_c], \ i = 1, \cdots, n, \ c = 1, \cdots, C, \tag{1}$$

where γ_c is a cutoff point of ordinal response. We specify that

$$-\infty = \gamma_0 < \gamma_1 = 0 < \gamma_2 < \cdots < \gamma_{C-1} < \gamma_C = \infty,$$

where the condition $\gamma_1 = 0$ is required to establish the identifiability of the cutoff parameters.[10] The latent variable z_i is assumed to be determined by the following linear model:

$$z_i = \boldsymbol{x}_i'\boldsymbol{\beta} + u_i, \ i = 1, \cdots, n,$$

where $\boldsymbol{x}_i = (x_{i1}, \cdots, x_{ik})'$ and $\boldsymbol{\beta} = (\beta_1, \cdots, \beta_k)'$. Defining

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}, \ \boldsymbol{z} = \begin{pmatrix} z_1 \\ \vdots \\ z_n \end{pmatrix}, \ \boldsymbol{X} = \begin{pmatrix} \boldsymbol{x}_1' \\ \vdots \\ \boldsymbol{x}_n' \end{pmatrix}, \ \boldsymbol{u} = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix},$$

the linear model for the latent variables is rewritten as

$$\boldsymbol{z} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{u}.$$

Now, we assume that $\boldsymbol{u} \sim \mathrm{N}(\boldsymbol{0}, \sigma^2\boldsymbol{I}_n)$, that is,

$$\boldsymbol{z}|\boldsymbol{\beta}, \sigma^2, \boldsymbol{X} \sim \mathrm{N}(\boldsymbol{X}\boldsymbol{\beta}, \sigma^2\boldsymbol{I}_n). \tag{2}$$

Furthermore, following Nandram and Chen (1996) and Chen and Dey (2000), we assume that $\gamma_{C-1} = 1$; this is in addition to $\gamma_1 = 0$.[11] Since $\boldsymbol{\gamma} = (\gamma_2, \cdots, \gamma_{C-2})'$ is restricted as $0 < \gamma_2 < \cdots < \gamma_{C-2} < 1$, Chen and Dey (2000) propose the following transformation of cutoff points:

$$\delta_c = \log\left(\frac{\gamma_c - \gamma_{c-1}}{1 - \gamma_c}\right), \ c = 2, \cdots, C-2, \tag{3}$$

where $\boldsymbol{\delta} = (\delta_2, \cdots, \delta_{C-2})'$ is unrestricted. The prior distributions are specified as follows:

$$p(\boldsymbol{\beta}, \sigma^2, \boldsymbol{\gamma}) = p(\boldsymbol{\beta})p(\sigma^2)p(\boldsymbol{\gamma}),$$

[10] See, for example, Albert and Chib (1993, p.673) and Johnson and Albert (1999, p.131).

[11] Although the unit variance of disturbance, that is, $\mathrm{var}(u_{it}) = 1$, is the standard identification restriction, there are other ways of identifying the ordered probit model. Nandram and Chen (1996) and Chen and Dey (2000) provide an identification restriction based on the reparameterization of the cutoff points, that is, $\gamma_{C-1} = 1$. See also Jeliazkov et al. (2009, Section 2.2).

where the prior distribution of $\boldsymbol{\gamma}$, $p(\boldsymbol{\gamma}) = p(\boldsymbol{\delta}(\boldsymbol{\gamma}))$, is based on the transformation (3), and

$$\boldsymbol{\beta} \sim \mathrm{N}(\boldsymbol{\beta}_0, \boldsymbol{B}_0),\ \sigma^{-2} \sim \mathrm{Gam}(a_0, b_0),\ \boldsymbol{\delta}(\boldsymbol{\gamma}) \sim \mathrm{N}(\boldsymbol{\delta}_0, \boldsymbol{D}_0).$$

Here, $\mathrm{Gam}(a_0, b_0)$ denotes a gamma distribution with location parameter a_0 and scale parameter b_0. The algorithm drawing the parameters in the ordered probit model is based on the algorithms of Nandram and Chen (1996) and Chen and Dey (2000) and is as follows.

Algorithm:

1. Sample $\boldsymbol{\beta}$ and σ^2 from their full conditional distributions (FCDs):

$$\begin{aligned}
&\boldsymbol{\beta}|\cdots \sim \mathrm{N}(\tilde{\boldsymbol{\beta}}, \tilde{\boldsymbol{B}}) \\
&\quad \tilde{\boldsymbol{B}} = \left(\boldsymbol{B}_0^{-1} + \frac{1}{\sigma^2}\boldsymbol{X}'\boldsymbol{X}\right)^{-1},\ \tilde{\boldsymbol{\beta}} = \tilde{\boldsymbol{B}}\left(\boldsymbol{B}_0^{-1}\boldsymbol{\beta}_0 + \frac{1}{\sigma^2}\boldsymbol{X}'\boldsymbol{z}\right) \\
&\sigma^{-2}|\cdots \sim \mathrm{Gam}(\tilde{a}, \tilde{b}) \\
&\quad \tilde{a} = a_0 + \frac{n}{2},\ \tilde{b} = b_0 + \frac{1}{2}(\boldsymbol{z} - \boldsymbol{X}\boldsymbol{\beta})'(\boldsymbol{z} - \boldsymbol{X}\boldsymbol{\beta}),
\end{aligned}$$

   where "$|\cdots$" denotes conditioning on the values of all other parameters and data.

2. If $y_i = c$, sample z_i from

$$z_i|\cdots \sim \mathrm{N}(\boldsymbol{x}_i'\boldsymbol{\beta}, \sigma^2)1_{(z_i \in (\gamma_{c-1}, \gamma_c])}, \tag{4}$$

   where $1_{(\cdot)}$ is an indicator function.[12]

3. Sample $\boldsymbol{\delta}$ from the Metropolis-Hastings (M-H) algorithm and calculate $\gamma_c = \dfrac{\gamma_{c-1} + \exp\delta_c}{1 + \exp\delta_c}$, $c = 2, \cdots, C-2$.[13]

3 New Measure of Subjective Well-Being

As described in Section 1, Kalmijn and Veenhoven (2005) state the possibility such that "an egalitarian policy aimed at reducing differences in happiness would differ from a utilitarian policy aimed at producing a higher average level of happiness." That is, a society with $y_i = 1$ for all individual i is equal but unhappy whereas a society in which a few people have $y_i = 1$ and most have $y_i = C$ is a happier one. Thus, it is necessary to consider an index that identifies an individual's situation in each category c.

In this section, we propose a new measure of subjective well-being. We calculate the following probability $r(z_i|\gamma_c, y_i)$, which denotes the difference between

[12] We can utilize the method proposed by Damien and Walker (2001) for sampling truncated normal variables.

[13] The details of this sampling algorithm are provided in Appendix A.

the cutoff points γ_c and z_i:

$$
\begin{aligned}
r(z_i|\gamma_c, y_i) &= \Pr(z_i < z < \gamma_c|y_i) \\
&= \Pr\left(\frac{z_i - \boldsymbol{x}_i'\boldsymbol{\beta}}{\sigma} < \frac{z - \boldsymbol{x}_i'\boldsymbol{\beta}}{\sigma} < \frac{\gamma_c - \boldsymbol{x}_i'\boldsymbol{\beta}}{\sigma} \;\middle|\; y_i \right) \\
&= \begin{cases} \Phi\left(\dfrac{\gamma_c - \boldsymbol{x}_i'\boldsymbol{\beta}}{\sigma}\right) - \Phi\left(\dfrac{z_i - \boldsymbol{x}_i'\boldsymbol{\beta}}{\sigma}\right) & \text{if } z_i < \gamma_c \\ 0 & \text{if } z_i \geq \gamma_c, \end{cases} \\
& \quad i = 1, \cdots, n, \;\; c = 1, \cdots, C,
\end{aligned} \tag{5}
$$

where $\Phi(\cdot)$ is a distribution function of the standard normal distribution.[14] We call $r(z_i|\gamma_c, y_i)$ the regretted subjective well-being of individual i in category c. In the case of $c = C$, (5) becomes

$$
r\left(z_i|\gamma_C, y_i\right) = 1 - \Phi\left(\frac{z_i - \boldsymbol{x}_i'\boldsymbol{\beta}}{\sigma}\right) = 1 - \Phi\left(\frac{u_i}{\sigma}\right).
$$

Since $\boldsymbol{u} \sim \mathrm{N}(0, \sigma^2 \boldsymbol{I}_n)$, the average value of $r\left(z_i|\gamma_C, y_i\right)$ becomes 0.5.[15]

4 Analysis of Subjective Well-Being Using the WVS Data

4.1 Data and Bayesian Estimation of Probit Model

We use WVS data for the empirical analysis of subjective well-being. In WVS data, there are five waves of surveys on the perception of life, attitudes toward the environment, job aspects, economical situation, and so on, from 1981 to 2007. The data of income level and subjective well-being are chosen for our analysis from four waves of the surveys: wave 1 (1981–1984), wave 2 (1989–1993), wave 3 (1994–1998), and wave 4 (1999–2004).

Data of income and subjective well-being are used in the following probit model. WVS only provides household income band as income data. Therefore, following Layard et al. (2008, p.1850), we constructed the numerical values of income from the income bands.[16] Subjective well-being is investigated in WVS using multiple-choices questionnaires. There are four options: "very happy," "quite happy," "not very happy," and "not at all happy." In our study, integer values 1 to 4 are allocated to these options, with 4 denoting "very happy."

[14] If the loss function is defined as $L(z_i, \gamma_c) = 1_{\left(z \in (z_i, \gamma_c)\right)}$, the posterior risk function can be written as

$$
r\left(z_i|\gamma_c, y_i\right) = \mathrm{E}\left[L(z_i, \gamma_c)|\cdots\right] = \int 1_{\left(z \in (z_i, \gamma_c)\right)} p(z_i|\cdots) dz_i,
$$

which is the regret function defined in (5). For the Bayesian decision theory, see, for example, Press (2003, Chapter 11).

[15] One of the referees pointed this out.

[16] Layard et al. (2008, p.1850) constructed the income data as follows: "In the cross-section surveys only income bands are available, and these we converted into numerical values using the midpoint of each band. For respondents in the lowest income band we assumed an income of two thirds of the upper limit of the band, and for respondents in the highest income band we assumed an income of 1.5 of the lower income limit of the band."

The probit model for happiness is as follows:

$$\begin{aligned} y_i &= c \ \text{ if } \ z_i \in (\gamma_{c-1}, \gamma_c], \ c = 1, \cdots, 4, \ i = 1, \cdots, n \\ z_i &= \beta_1 + \beta_2 \, \texttt{income}_i + \beta_3 \, \texttt{married}_i + \beta_4 \, \texttt{male}_i + \beta_5 \, \texttt{work}_i \\ &\quad + \beta_6 \, \texttt{age}_i + u_i, \ i = 1, \cdots, n, \end{aligned} \tag{6}$$

where `income` is the logarithm of total household income; `married`, `male`, and `work` are dummy variables that indicate marital status, gender, and labor force participation, respectively; and `age` denotes the respondent's age. The MCMC simulation was run for 20,000 iterations and the first 5,000 samples were discarded as the burn-in period. The posterior results obtained thereafter were generated using Ox version 5.10 (Doornik, 2007). We set the prior distributions as follows:

$$\boldsymbol{\beta} \sim \mathrm{N}(\mathbf{0}, 100 \times \boldsymbol{I}), \ \sigma^{-2} \sim \mathrm{Gam}(5, 0.05), \ \boldsymbol{\delta}(\boldsymbol{\gamma}) \sim \mathrm{N}(\mathbf{0}, 100 \times \boldsymbol{I}).$$

The posterior results of 22 countries are available in both waves 2 and 4; these are listed in Table 1. We also include the countries used in Deaton (2008) and Easterlin (2009), since the WVS data they use overlap the data we use. Deaton (2008) uses the WVS data for 1996 and Easterlin (2009) uses the data for the period from the end of the 1980s to the end of the 1990s.[17]

The following two points are observed from the posterior results of the countries given in Table 1 on the estimated coefficients of explanatory variables.[18]

Among the explanatory variables, the 95% credible intervals of `married` (β_3) include the least number of zeros and are followed by those of `income` (β_2) and `age` (β_6). Most posterior means of `married` and `income` are positive and those of `age` are negative. Therefore, marital status and income have a positive effect on the degree of subjective well-being, while aging has a negative effect on it. In contrast, most credible intervals of `work` (β_5) include zero.

4.2 Regret and GDP Per Capita

We obtained the posterior means of regretted subjective well-being of the individual respondents measured by (5) from the posterior results. We use these posterior means of the individual regretted subjective well-being in the following analyses and simply call it "regret." Figures 1 and 2 show the relationships between regret and GDP per capita in waves 2 and 4, respectively. The regret and GDP per capita are available for 26 and 44 countries in waves 2 and 4 respectively; these are listed in Table 2. In the figures, we use the data of GDP per capita in 1990 International Geary-Khamis dollars, whose source is *Statistics on World Population, GDP and Per Capita GDP, 1-2006 AD* by Maddison, A.[19] Figures of waves 1 and 3 are not shown since the number of countries is not sufficient in these waves. There are eight countries in wave 1 and eleven in wave 3. In these figures, `regret1` to `regret4` denote the regretted subjective

[17]See Deaton (2008, p.57) and Easterlin (2009, p.143).

[18]We estimated (6) by using maximum likelihood estimation, but we obtained results similar to those obtained by using the Bayesian method. Further, we estimated an ordered probit model that includes variables on education in consideration of the impact of omitted variables. However, we also obtained similar results on regrets. All the posterior results of the probit models for the countries are available upon request.

[19]See http://www.ggdc.net/maddison/.

well-being for categories $c = 1$ to $c = 4$, respectively. The graphs in Figure 1 show the relationships between `regret1` to `regret4` and GDP per capita in wave 2. In the graphs, the perpendicular lines passing through the dots indicate "the sample mean ± the sample standard deviation" of the individual posterior regretted subjective well-being. The sample mean and the sample standard deviation of the individual posterior regretted subjective well-being are defined as follows: for $c = 1, \cdots, 4$,

$$\texttt{regret}c: \ \bar{r}_c = \frac{1}{n}\sum_{i=1}^{n} r_{post}\left(z_i|\gamma_c, y_i\right),$$

$$s_c = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(r_{post}\left(z_i|\gamma_c, y_i\right) - \bar{r}_c\right)^2},$$

where $r_{post}\left(z_i|\gamma_c, y_i\right)$ denotes the posterior mean of the regretted subjective well-being of individual i. The dotted lines represent the fitted lines between regret and GDP per capita. From Figure 1, the following observations can be made.

- The fitted lines are downward sloping in the graphs of `regret2` and `regret3`. This means that the higher the GDP per capita, the lower the regret. It is difficult to observe the relationships between regret and GDP per capita in the graphs of `regret1` and `regret4` because the regret of every country is approximately zero in `regret1` and 0.5 in `regret4`.[20]

- In the graph of `regret2`, the regret of countries with lower GDP per capita – India (IND), South Africa (ZAF), Brazil (BRZ), and Turkey (TRK) – is below the fitted line, while that of the former communist countries – Russia (RUS), Lithuania (LTU), Latvia (LVA), Estonia (EST), and Slovakia (SVK) – which have a higher GDP per capita above the fitted line.

- In the graph of `regret3`, a similar pattern is observed more clearly. The regret of India, South Africa, Brazil, Mexico (MEX), Turkey, and Chile (CHL) is below the fitted line, while that of the former communist countries like Hungary (HUN), Czech Republic (CZE), and ex-Soviet republics is above the fitted line.

- In the graphs of `regret2` and `regret3`, among the countries with high GDP per capita, the regret of Ireland (IRL), Belgium (BEL), and Netherlands (NLD) is below the fitted line.

[20] We estimate fitted lines in Figures 1 and 2 by OLS, where regret is an explained variable and per-capita GDP is an explanatory variable. The following table shows the results concerning the explanatory variable and the adjusted coefficient of determination.

	Figure 1 (wave 2)			Figure 2 (wave 4)		
	estimate	std. error	$\bar{R}^2$	estimate	std. error	$\bar{R}^2$
`regret1`	-8.191×10^{-8}	2.940×10^{-8}	0.2129	-1.400×10^{-7}	5.113×10^{-8}	0.1312
`regret2`	-3.345×10^{-6}	1.097×10^{-6}	0.2493	-2.812×10^{-6}	7.214×10^{-7}	0.2481
`regret3`	-6.447×10^{-6}	3.328×10^{-6}	0.09915	-6.730×10^{-6}	1.890×10^{-6}	0.2135
`regret4`	-7.385×10^{-8}	3.749×10^{-8}	0.1033	-3.018×10^{-8}	1.500×10^{-8}	0.06614

Figure 2 shows similar graphs of wave 4. From Figure 2, the following observations can be made.

- The fitted lines in the graphs of `regret2` and `regret3` are also downward sloping.
- In these graphs, it is less clearer that the regret of countries whose GDP per capita is low is below the fitted line. However, the regret of former communist countries such as Russia, Bulgaria (BGR), Moldova (MDA), Albania (ALB), Latvia, and Estonia is above the fitted line, as in wave 2.
- In the graph of `regret3`, the regret of Lithuania, Croatia (HRV), Slovakia, Czech Republic, and Slovenia (SVA) along with that of former communist countries mentioned in the figure of `regret2` is above the fitted line.
- The regret of Latin American countries such as Mexico, Venezuela (VEN), Argentina (ARG), and Chile is below the fitted line in the graphs of `regret2` and `regret3` and the same is true for Peru (PER) in the graphs of `regret3`. Therefore, all Latin American countries in the graph of `regret3` appear below the fitted line.

As observed in Figures 1 and 2, the sample means of the regret of former communist countries are higher than the values on the fitted line for their GDP per capita. In contrast, the regret of Latin American countries is lower than the values on the fitted line for their GDP per capita. In the study on the relationship between GDP per capita and life satisfaction using WVS data by Deaton (2008), among countries with low GDP per capita, the degrees of life satisfaction of Eastern European countries and ex-Soviet republics are low, while those of countries like Mexico and Brazil are high, as in our findings.[21]

4.2.1 Change in the relation between regret and GDP per capita

Figure 3 shows the changes in the relationship between regret and GDP per capita from waves 2 to 4. The figure illustrates 22 countries for which data of both waves 2 and 4 can be used. The origin of an arrow indicates regret and GDP per capita in wave 2, and its endpoint indicates the same in wave 4. The names of countries are not denoted in the graphs of `regret1` and `regret4` because the change in the regret of most countries is too small. Table 3 complements Figure 3 and shows the ratio of the rate of change of regret to that of GDP per capita from waves 2 to 4. The values in the table represent the direction and magnitude of the change of regret.

The countries cannot be commonly grouped through all figures, but it is possible to do so according to the graph of `regret3`.

Group A: Chile, India, Mexico, South Africa, Turkey

Group B: Czech, Estonia, Latvia, Lithuania, Russia, Slovakia

Group C: Germany, Italy, Japan, Portugal, Spain

Group D: Austria, Belgium, Canada, Ireland, Netherlands, United States

[21] See Deaton (2008, p.59, Figure 3).

Each group has the following characteristics. GDP per capita in the countries of groups A and B is lower than that in the countries of groups C and D. The regret of group A countries is lower than that of group B countries. The regret of group D countries is lower than that of group C countries. These are summarized as follows. Group A consists of countries with lower GDP per capita and lower regret; group B consists of countries with lower GDP per capita and higher regret; group C consists of countries with higher GDP per capita and higher regret; and group D consists of countries with higher GDP per capita and lower regret.

The following findings can be pointed out from the figures of `regret2` and `regret3` wherein the changes are perceptible.

- The regret of most group A countries, like Mexico and South Africa, changes significantly as compared to the changes in the GDP per capita. This is especially distinguished in the graph of `regret3`. It seems plausible that regret decreases when GDP per capita increases. The case of Turkey in the graph of `regret2` does not follow the expected results. As Turkey's GDP per capita increases, its `regret2` increases, although it is obvious from Table 3 that `regret2` and `regret3` of most countries decrease.

- In the former communist countries that comprise group B, GDP per capita of four countries, the Baltic states of Estonia, Latvia, and Lithuania, and Russia, decreases from waves 2 to 4 according to the graph of `regret2`. In spite of the decrease in GDP per capita, `regret2` of Estonia, Latvia, and Lithuania decreases. In particular, a large decrease in `regret2` is seen in Lithuania. Table 3 also shows that the ratio of the rate of change of `regret2` to that of GDP per capita of Estonia is remarkably high, but that of Latvia is small. Of this group, the direction of the changes of `regret3` and GDP per capita is the same as that of `regret2` in all countries, and the ratio of the rate of change of `regret3` to that of GDP per capita is smaller than that of `regret2` in most countries.

- In the group C countries whose regret is relatively high among the developed countries, `regret3` changes more than `regret2` does. The change of `regret3` in Japan and Portugal is larger than that in Germany and Italy, when GDP per capita increases. However, `regret2` of Germany and Italy and `regret3` of Spain increases with GDP per capita.

- The regret of group D countries does not change much. However, it is observed that the increase in `regret2` of Austria and Belgium and in `regret3` of Ireland, Belgium, Netherlands, and the United States are accompanied by an increase in GDP per capita.

According to Easterlin (2009), life satisfaction and real GDP in the Baltic states decreased between 1990 and 1999.[22] In the same period, `regret2` and `regret3` decreased in our study. It seems that this results in greater subjective well-being in these countries. Therefore, our findings on the Baltic states are different from those in Easterlin (2009), although the problems on comparability remain because the life satisfaction in his article does not always correspond with regret in our study.

[22]See Easterlin (2009, p.134).

4.2.2 Change in the distribution

We rank countries in order of Gini coefficient computed using regret. Regret denotes the probability with which a respondent who chooses an option in a multiple-choice question pertaining to subjective well-being does not choose any other option indicative of better well-being. Thus, the distribution of the probability in a country that has relatively high Gini coefficient is more biased than that in other countries. This means that the Gini coefficient using regret can play the role of an index for comparing inequality of subjective well-being.

Tables 4a and 4b show the results of the countries in Figure 3. They are separated into four groups in 4.2.1. The parenthetic group name from A to D is attached after the country name in both tables. Overall, the tables provide the following findings.

The Gini coefficients of group A countries are low for `regret1`. They increase as number of the `regret` increases from `regret1` to `regret4`. In contrast, the Gini coefficients of group C countries are high for `regret1`. They decrease as the number of `regret` increases. These two patterns are observed more clearly in wave 2 than in wave 4. The Gini coefficients of Group B countries are low and those of Group D countries are high for `regret1`, `regret2` and `regret3`.[23]

5 Concluding Remarks

In this article, we estimated the ordered probit model for well-being function by using the Bayesian method and proposed a new inequality measure represented as regret for subjective well-being. We used WVS data for the estimation. Regret enables us to investigate the state of the subjective well-being of the respondents who choose a certain option in a multiple-choice questionnaire survey like WVS. Using regret, we can analyze subjective well-being by measuring the unhappiness with which an individual is unable to attain more happiness. Our method offers a way to analyze subjective well-being in more detail than earlier models do.

The main findings of this article are as follows.

- Being married and having a higher income have a positive effect on subjective well-being. The effect of aging is negative.
- The higher a country's per-capita GDP is, the lower the regret across countries at a point in time becomes. This indicates that the relationship between regret and per-capita GDP is negative across countries. This finding is observed especially for regret 2 and regret 3 in Figures 1 and 2. However, within countries across time, there are some countries whose regret changes in the same direction as their per-capita GDP change in Figure 3.
- The regret of former communist countries is higher than the values on the fitted line for their GDP per capita. The regret of Latin American countries is lower than the values on the fitted line for their GDP per capita in Figures 1 and 2.

[23] Although the Gini coefficient using regret is an index of inequality of subjective well-being, it does not provide information on shapes of the distributions of regret in each country. Boxplots in the electoronic supplemetary material provide this.

- In the Baltic states, `regret2` and `regret3` decrease with a decrease in the GDP per capita. Such changes where `regret2` and `regret3` move with GDP per capita in the same direction are also observed in some other countries in Figure 3.

- According to the Gini coefficients computed using regret, those of group A countries are low for `regret1`. They increase as number of the `regret` increases from `regret1` to `regret4`. In contrast, the opposite is observed for group C countries.

Appendix

A Sampling Algorithm for γ

The FCD of $\boldsymbol{\gamma}$ is

$$\begin{aligned} p(\boldsymbol{\gamma}|\cdots) &\propto p(\boldsymbol{\delta}(\boldsymbol{\gamma})) \prod_{i:y_i=2} \left[\Phi\left(\frac{\gamma_2 - \boldsymbol{x}_i'\boldsymbol{\beta}}{\sigma}\right) - \Phi\left(-\frac{\boldsymbol{x}_i'\boldsymbol{\beta}}{\sigma}\right) \right] \\ &\times \prod_{i:y_i=3} \left[\Phi\left(\frac{\gamma_3 - \boldsymbol{x}_i'\boldsymbol{\beta}}{\sigma}\right) - \Phi\left(\frac{\gamma_2 - \boldsymbol{x}_i'\boldsymbol{\beta}}{\sigma}\right) \right] \\ &\times \cdots \times \prod_{i:y_i=C-1} \left[\Phi\left(\frac{1 - \boldsymbol{x}_i'\boldsymbol{\beta}}{\sigma}\right) - \Phi\left(\frac{\gamma_{C-2} - \boldsymbol{x}_i'\boldsymbol{\beta}}{\sigma}\right) \right]. \end{aligned}$$

Since the Jacobian of the transformation of $\boldsymbol{\gamma}$ to $\boldsymbol{\zeta}$ is $\displaystyle\prod_{c=2}^{C-2} \frac{(1-\gamma_{c-1})\exp\delta_c}{(1+\exp\delta_c)^2}$, we have

$$p(\boldsymbol{\delta}|\cdots) = p(\boldsymbol{\gamma}|\cdots) \prod_{c=2}^{C-2} \frac{(1-\gamma_{c-1})\exp\delta_c}{(1+\exp\delta_c)^2}$$

We use the following Metropolis-Hastings (M-H) algorithm for generating $\boldsymbol{\zeta}$. We use a multivariate t distribution, $\mathrm{Mt}(\boldsymbol{\delta}|\tilde{\boldsymbol{\delta}}, \tilde{\boldsymbol{\Sigma}}_{\boldsymbol{\delta}}, \nu)$, as a proposal distribution for generating $\boldsymbol{\delta}$, where $\tilde{\boldsymbol{\delta}}$ is the mode of $p(\boldsymbol{\delta}|\cdots)$,

$$\tilde{\boldsymbol{\Sigma}}_{\boldsymbol{\delta}} = \left\{ \left[-\frac{\partial \log p(\boldsymbol{\delta}|\cdots)}{\partial\boldsymbol{\delta}\partial\boldsymbol{\delta}'} \right]_{\boldsymbol{\delta}=\tilde{\boldsymbol{\delta}}} \right\}^{-1}$$

and ν is the degrees of freedom. The M-H algorithm for generating $\boldsymbol{\delta}$ is as follows:

1. Let $\boldsymbol{\delta}^{(t)}$ denote the value of $\boldsymbol{\delta}$ at the tth iteration.

2. At the $(t+1)$th iteration, sample $\boldsymbol{\delta}^p$ from $\mathrm{Mt}(\boldsymbol{\delta}|\tilde{\boldsymbol{\delta}}, \tilde{\boldsymbol{\Sigma}}_{\boldsymbol{\delta}}, \nu)$.

3. The transition probability from $\boldsymbol{\delta}^{(t)}$ to $\boldsymbol{\delta}^p$ is

$$\alpha = \min\left\{ \frac{p(\boldsymbol{\delta}^p|\cdots)\,\mathrm{Mt}(\boldsymbol{\delta}^{(t)}|\tilde{\boldsymbol{\delta}}, \tilde{\boldsymbol{\Sigma}}_{\boldsymbol{\delta}}, \nu)}{p(\boldsymbol{\delta}^{(t)}|\cdots)\,\mathrm{Mt}(\boldsymbol{\delta}^p|\tilde{\boldsymbol{\delta}}, \tilde{\boldsymbol{\Sigma}}_{\boldsymbol{\delta}}, \nu)}, 1 \right\}$$

4. Generate $u \sim \mathrm{U}(0, 1)$, the uniform distribution on $(0, 1)$, and take

$$\boldsymbol{\delta}^{(t+1)} = \begin{cases} \boldsymbol{\delta}^p & \text{if } \ u < \alpha \\ \boldsymbol{\delta}^{(t)} & \text{otherwise.} \end{cases}$$

$\boldsymbol{\gamma}$ is derived from $\boldsymbol{\delta}$ such as $\gamma_c = \dfrac{\gamma_{c-1} + \exp \delta_c}{1 + \exp \delta_c}, \ \ c = 2, \cdots, C-2.$

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Table 1: Countries available in both wave 2 and wave 4

Austria	Belgium	Canada	Chile
Czech	Estonia	Germany	India
Ireland	Italy	Japan	Latvia
Lithuania	Mexico	Netherlands	Portugal
Russia	Slovakia	South Africa	Spain
Turkey	United States		

Table 2: Countries in Figures 1, 2 and 3[a]

(wave2)			
Austria	Belgium	Brazil	Canada
Chile	Czech	Estonia	Finland
Germany	Great Britain	Hungary	India
Ireland	Italy	Japan	Latvia
Lithuania	Mexico	Netherlands	Portugal
Russia	Slovakia	South Africa	Spain
Turkey	Unites States		
(wave4)			
Albania	Algeria	Argentina	**Austria**
Belgium	Bosnia Herzegovina	Bulgaria	**Canada**
Chile	China	Croatia	**Czech**
Denmark	Egypt	**Estonia**	France
Germany	Greece	**India**	**Ireland**
Israel	**Italy**	**Japan**	Jordan
Latvia	**Lithuania**	Macedonia	**Mexico**
Morocco	**Netherlands**	Peru	**Portugal**
Rep. Moldova	**Russia**	**Slovakia**	Slovenia
South Africa	South Korea	**Spain**	Sweden
Turkey	**United States**	Venezuela	Zimbabwe

a: Countries in boldface are listed in both wave 2 and wave 4.

Table 3: The ratio of change rate of regret to that of GDP per capita

country	year	regret2	regret3
Austria	$90 - 99$	0.983	-0.946
Belgium	$90 - 99$	1.769	0.120
Canada	$90 - 00$	-5.181	-1.998
Chile	$90 - 00$	-0.804	-0.136
Czech	$91 - 99$	-4.125	-0.542
Estonia	$90 - 99$	71.957	21.782
Germany	$90 - 99$	8.448	-0.221
India	$90 - 00$	-0.363	-0.121
Ireland	$90 - 99$	-0.961	0.129
Italy	$90 - 99$	7.585	-0.496
Japan	$90 - 00$	-5.750	-2.454
Latvia	$90 - 99$	0.233	0.360
Lithuania	$90 - 99$	2.479	0.068
Mexico	$90 - 00$	-4.826	-3.490
Netherlands	$90 - 99$	-0.068	0.298
Portugal	$90 - 99$	-1.779	-0.423
Russia	$90 - 99$	-0.846	-0.012
Slovakia	$91 - 99$	-1.636	-0.074
South Africa	$90 - 01$	-7.835	-6.121
Spain	$90 - 99$	-0.522	0.129
Turkey	$90 - 01$	9.722	-0.248
USA	$90 - 99$	-3.32	0.039

Table 4a : Gini coefficients of regret in wave 2

regret 1		regret 2	
country	Gini	country	Gini
South Africa (A)	0.951	Latvia (B)	0.605
Estonia (B)	0.955	Russia (B)	0.606
Canada (D)	0.966	Lithuania (B)	0.613
Slovakia (B)	0.971	Slovakia (B)	0.689
India (A)	0.973	Estonia (B)	0.693
Portugal (C)	0.974	Mexico (A)	0.751
Russia (B)	0.975	India (A)	0.775
Latvia (B)	0.976	Chile (A)	0.783
Lithuania (B)	0.977	Portugal (C)	0.797
Turkey (A)	0.979	South Africa (A)	0.831
Chile (A)	0.982	Czech (B)	0.835
Mexico (A)	0.983	Canada (D)	0.837
Czech (B)	0.987	Turkey (A)	0.864
Italy (C)	0.987	Japan (C)	0.871
Belgium (D)	0.990	Spain (C)	0.891
Ireland (D)	0.991	Italy (C)	0.895
United States (D)	0.991	Germany (C)	0.918
Austria (D)	0.992	United States (D)	0.920
Spain (C)	0.992	Austria (D)	0.942
Netherlands (D)	0.994	Belgium (D)	0.950
Germany (C)	0.995	Ireland (D)	0.955
Japan (C)	0.996	Netherlands (D)	0.966
regret 3		regret 4	
country	Gini	country	Gini
Czech (B)	0.247	Czech (B)	0.230
Germany (C)	0.272	Germany (C)	0.235
Latvia (B)	0.278	Italy (C)	0.240
Italy (C)	0.284	Japan (C)	0.261
Estonia (B)	0.288	Spain (C)	0.261
Lithuania (B)	0.298	Latvia (B)	0.271
Slovakia (B)	0.315	Austria (D)	0.274
Russia (B)	0.315	Estonia (B)	0.275
Japan (C)	0.320	Portugal (C)	0.278
Portugal (C)	0.332	Netherlands (D)	0.282
Spain (C)	0.334	Ireland (D)	0.282
Austria (D)	0.409	Lithuania (B)	0.282
India (A)	0.424	Slovakia (B)	0.283
Mexico (A)	0.448	Belgium (D)	0.285
Turkey (A)	0.453	United States (D)	0.286
Canada (D)	0.456	Russia (B)	0.288
South Africa (A)	0.465	Canada (D)	0.296
United States (D)	0.499	Turkey (A)	0.297
Belgium (D)	0.515	India (A)	0.301
Chile (A)	0.515	South Africa (A)	0.302
Ireland (D)	0.530	Mexico (A)	0.306
Netherlands (D)	0.568	Chile (A)	0.309

Table 4b : Gini coefficients of regret in wave 4

regret 1		regret 2	
country	Gini	country	Gini
Turkey (A)	0.909	Russia (B)	0.607
Russia (B)	0.928	Latvia (B)	0.669
India (A)	0.965	Estonia (B)	0.729
Latvia (B)	0.966	Slovakia (B)	0.780
Slovakia (B)	0.967	India (A)	0.797
Estonia (B)	0.968	Turkey (A)	0.802
South Africa (A)	0.977	Lithuania (B)	0.827
Italy (C)	0.980	Chile (A)	0.837
Lithuania (B)	0.984	South Africa (A)	0.864
Belgium (D)	0.985	Germany (C)	0.865
Germany (C)	0.987	Italy (C)	0.871
Portugal (C)	0.988	Portugal (C)	0.883
Czech (B)	0.991	Spain (C)	0.898
Spain (C)	0.991	Czech (B)	0.901
Netherlands (D)	0.994	Austria (D)	0.930
Austria (D)	0.995	Japan (C)	0.933
Ireland (D)	0.995	Belgium (D)	0.945
Mexico (A)	0.995	Mexico (A)	0.947
Chile (A)	0.995	United States (D)	0.957
Japan (C)	0.995	Netherlands (D)	0.967
United States (D)	0.996	Canada (D)	0.973
Canada (D)	0.997	Ireland (D)	0.973
regret 3		regret 4	
country	Gini	country	Gini
Czech (B)	0.235	Czech (B)	0.220
Lithuania (B)	0.240	Lithuania (B)	0.232
Slovakia (B)	0.299	Spain (C)	0.253
Estonia (B)	0.300	Germany (C)	0.267
Spain (C)	0.317	Mexico (A)	0.268
Latvia (B)	0.321	Portugal (C)	0.269
Germany (C)	0.327	Slovakia (B)	0.269
Russia (B)	0.332	Italy (C)	0.270
Portugal (C)	0.341	Estonia (B)	0.273
Italy (C)	0.342	Ireland (D)	0.274
Japan (C)	0.405	Japan (C)	0.274
India (A)	0.436	United States (D)	0.276
United States (D)	0.472	Canada (D)	0.276
Austria (D)	0.482	Netherlands (D)	0.284
Turkey (A)	0.490	Austria (D)	0.287
Ireland (D)	0.494	Latvia (B)	0.289
Chile (A)	0.513	Belgium (D)	0.290
Canada (D)	0.521	Russia (B)	0.299
Belgium (D)	0.532	India (A)	0.299
Netherlands (D)	0.554	Chile (A)	0.302
South Africa (A)	0.580	South Africa (A)	0.304
Mexico (A)	0.669	Turkey (A)	0.311

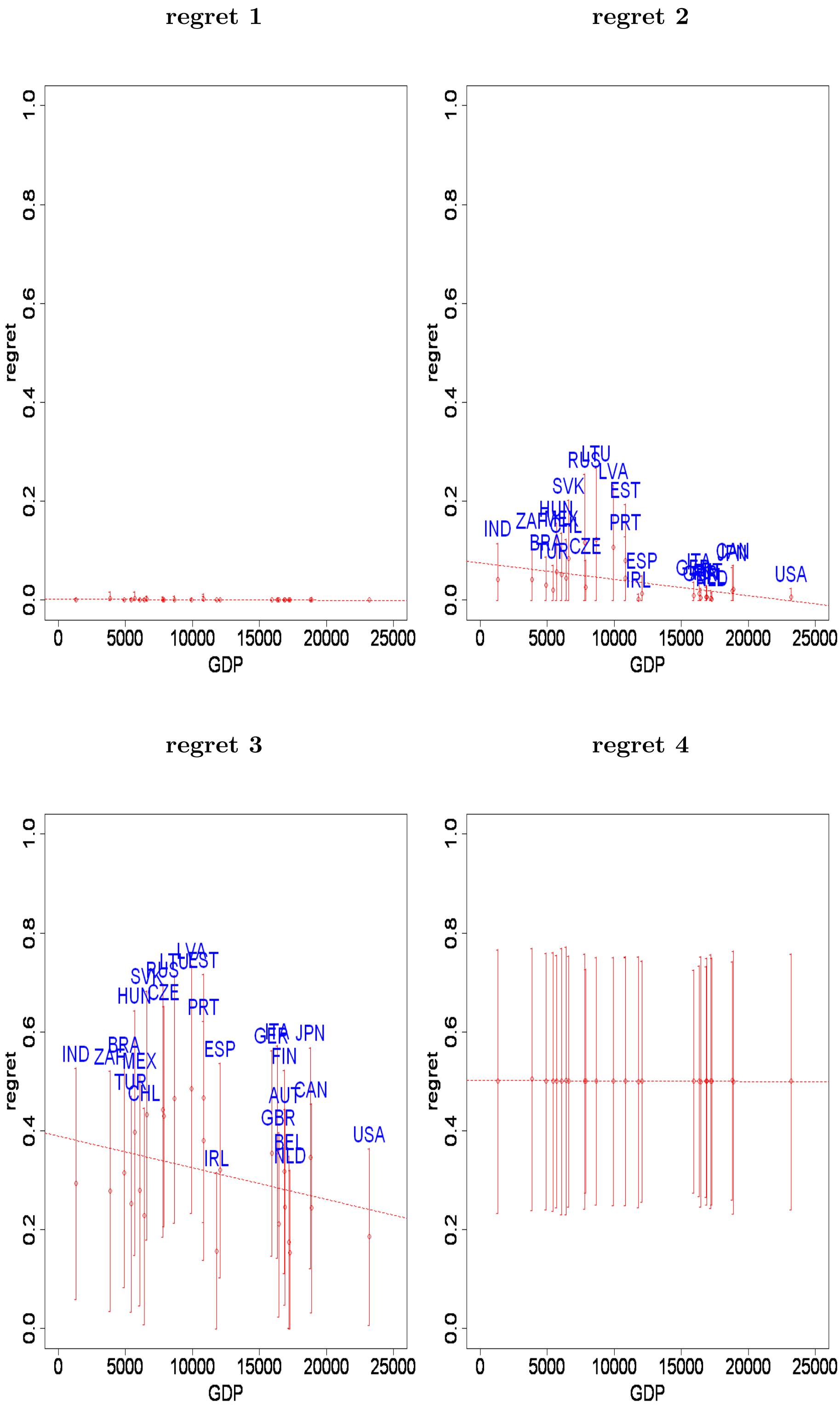


Figure 1: Regret and GDP per capita (wave=2)

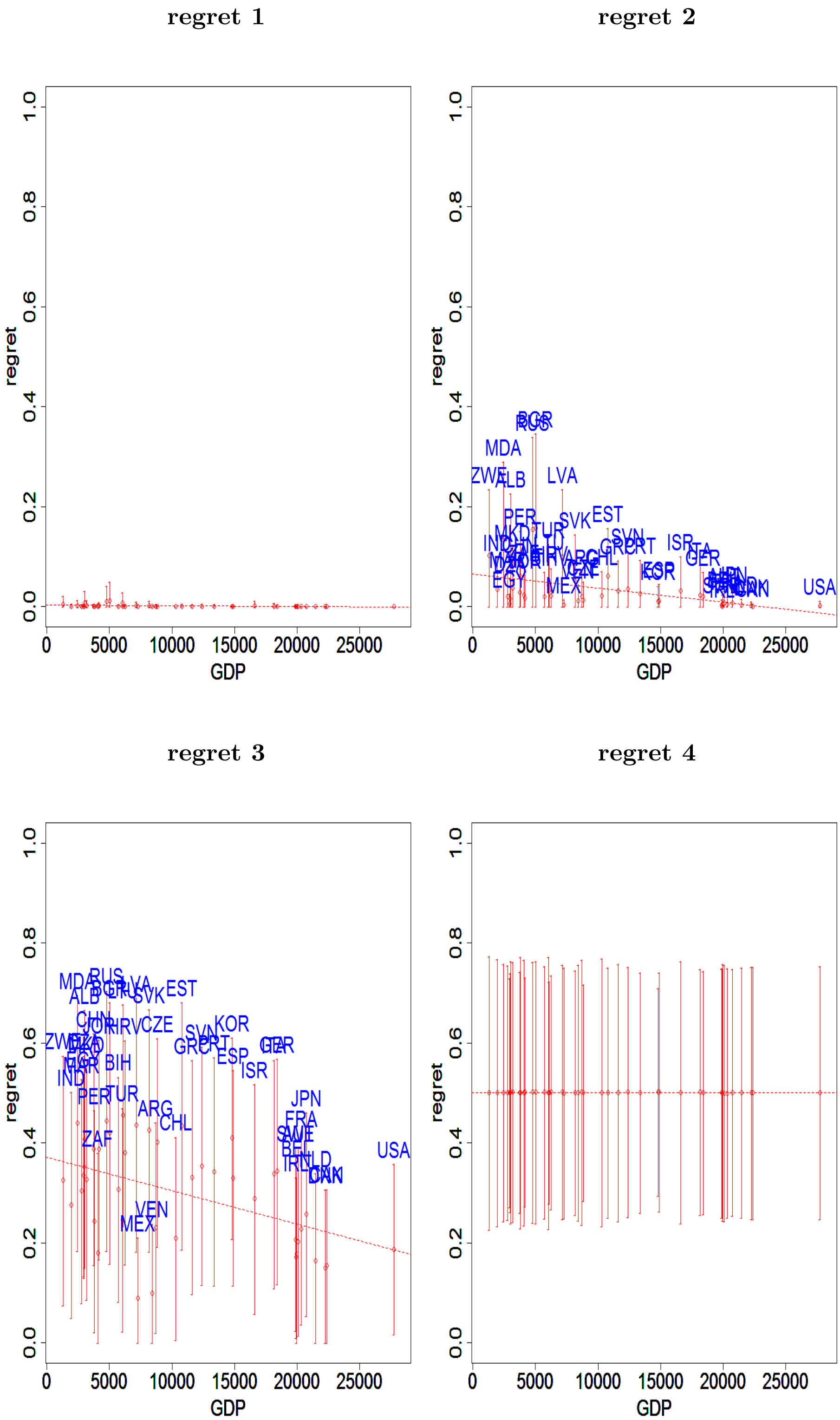


Figure 2: Regret and GDP per capita (wave=4)

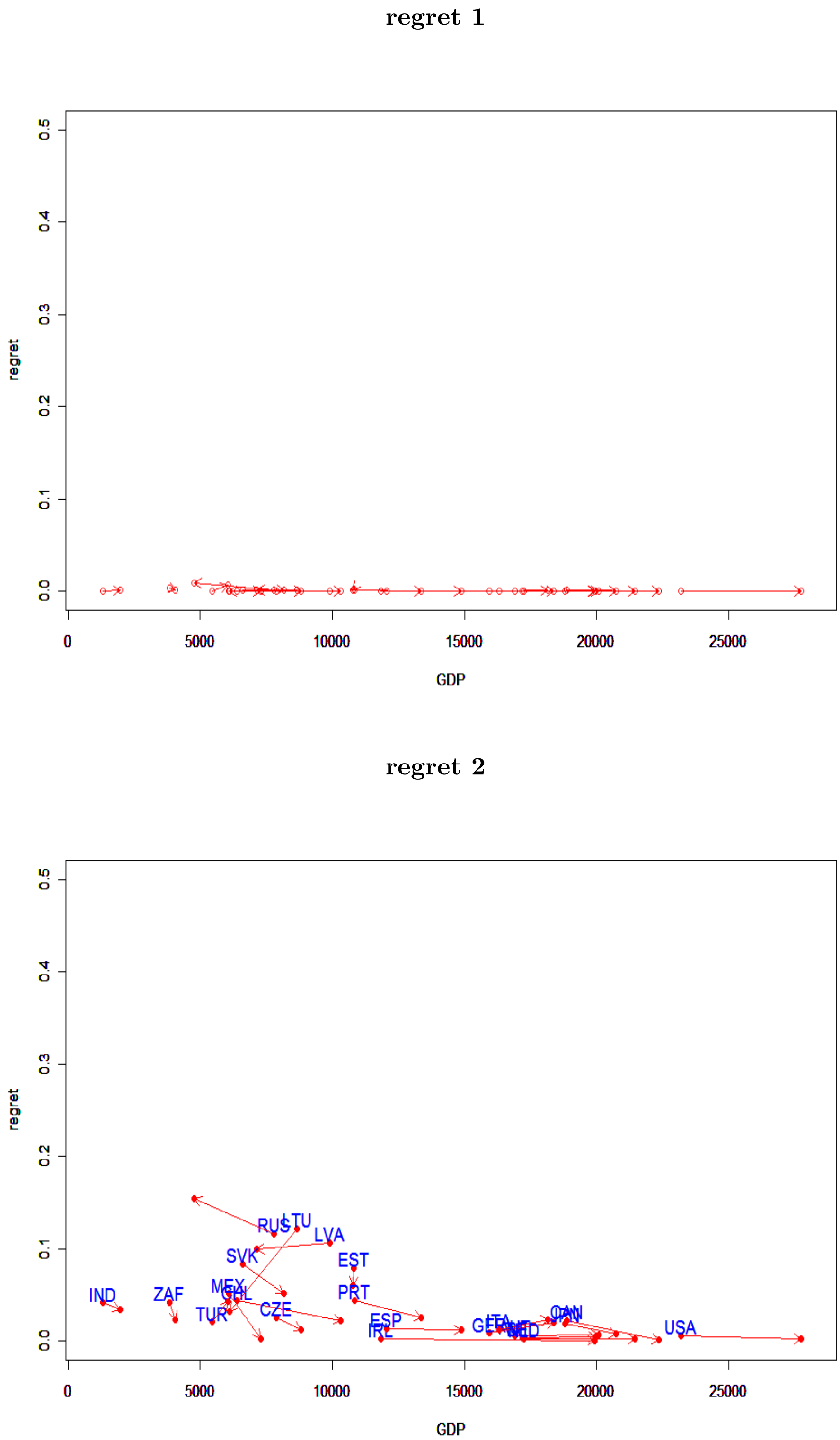
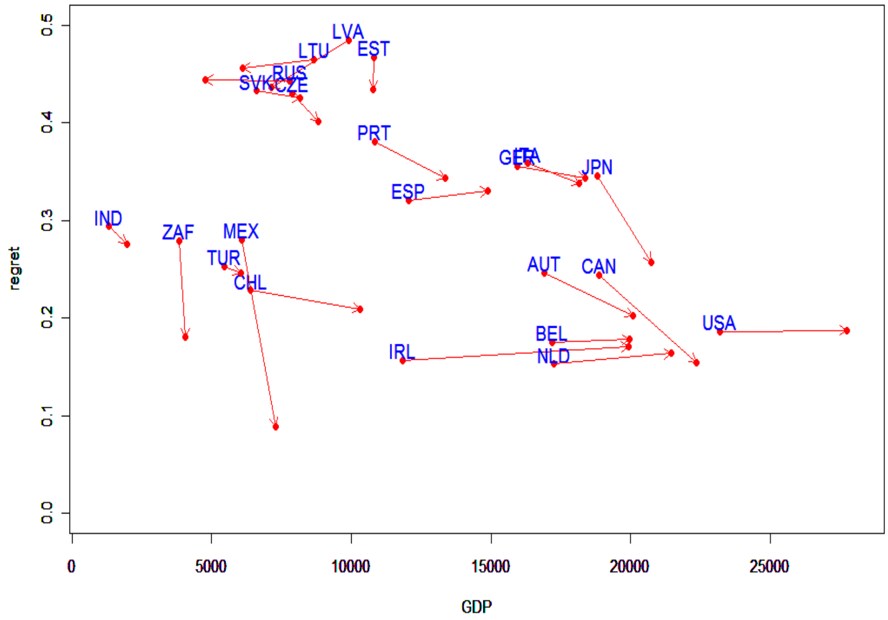


Figure 3: Change in regret and GDP per capita

regret 3



regret 4

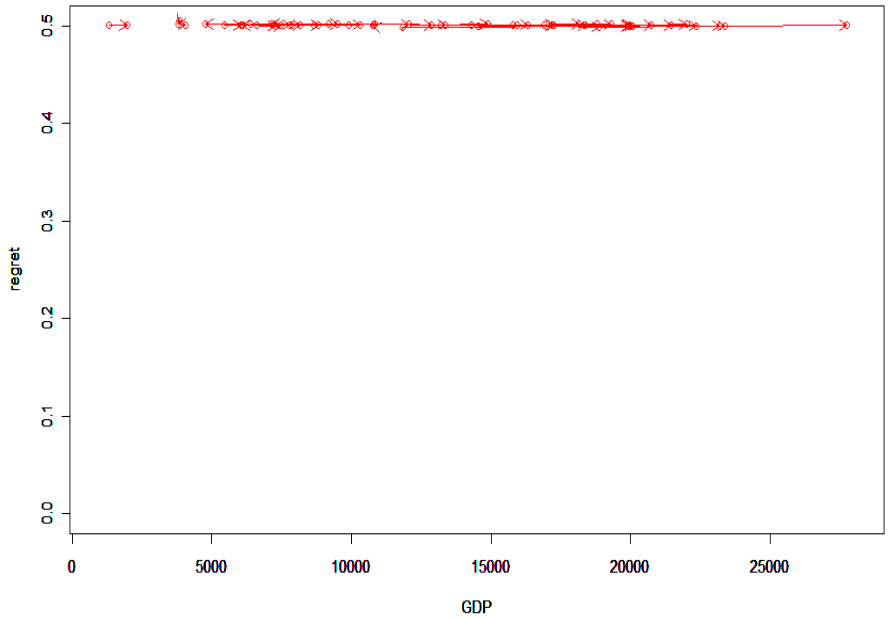


Figure 3: Continued