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Author(s)	山下, 茂
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二分決定グラフによる量子回路設計

立命館大・情報理工・山下茂

- 量子回路-2^nx2^nの複素行列
- ある種の量子回路の場合２分決定グラフで効率的に表現可能
- そして，ほぼストレートフォワードに量子回路設計可能
- 解きたい問題

(古典なら2^nの0/1)

Shigeru Yamashita
Shin-ichi Minato
D. Michael Miller

1993 京都大学工学部情報工学科卒業
1995 京都大学大学院工学研究科情報工学専攻修了
1995 日本電信電話株式会社　ＮＴＴコミュニケーショ ン科学基礎研究所 研究員
2000 科学技術振興事業団 ERATO 今井 量子計算機構プロジェクト研究員（兼務）
2003 奈良先端科学技術大学院大学 情報科学研究科 助教授

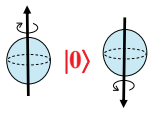
- 論理合成，量子アルゴリズム，対故障回路設計，GPU，量子回路設計

What is Q.C. in short?

To perform computation by using "quantum states," controlling by quantum gates

Quantum States – 2 level physical system

- light phase（0° and 90°）
- spins of electron（↑ and ↓）
- energy level of quantum dots (ground and exited)
- NMR(average spin of 6x10^{23} molecules)



Quantum Gate

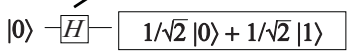
We can control quantum states by adding magnetic fields, irradiate an electromagnetic waves, ...

Quantum Bit $\sqrt{0.7}\,|0\rangle + \sqrt{0.3}\,|1\rangle$

Any superposition of two distinguishable quantum states

Quantum Bit & Quantum Gate

Quantum Gate = Certain Physical Operation



Superposition of |0⟩ and |1⟩ w.p. ½ each

$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix} \quad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix} \quad H = 1/\sqrt{2}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

$$H\begin{pmatrix}1\\0\end{pmatrix} = 1/\sqrt{2}\begin{pmatrix}1\\0\end{pmatrix} + 1/\sqrt{2}\begin{pmatrix}0\\1\end{pmatrix}$$

- Quantum Bit: 1x2 column vector
- Quantum Gate: 2x2 Matrix

Operations in Q. C.

$$R(\theta) = \begin{pmatrix}\cos\frac{\theta}{2} & -i\sin\frac{\theta}{2}\\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{pmatrix} \qquad NOT = iR(\pi) = i\begin{pmatrix}\cos\frac{\pi}{2} & -i\sin\frac{\pi}{2}\\ -i\sin\frac{\pi}{2} & \cos\frac{\pi}{2}\end{pmatrix} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

Classical Operation : NOT

$$|0\rangle \underset{NOT}{\Leftrightarrow} |1\rangle \qquad \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix} \qquad \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$$

- Quantum Bit: 1x2 column vector
- Quantum Gate: 2x2 Matrix

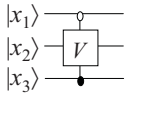
Quantum Operation

$$R(\theta) = \begin{pmatrix}\cos\frac{\theta}{2} & -i\sin\frac{\theta}{2}\\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{pmatrix} \qquad \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix} \qquad \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$$

Representation for General Q. C.

- Quantum Bit: 1x2 column vector
- Quantum Gate: 2x2 Matrix

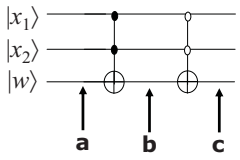
- n Quantum Bits: 1x2^n column vector
- n Quantum Gate: 2^nx2^n Matrix



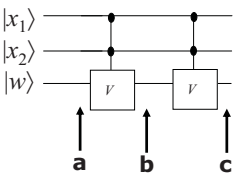
$$V = \frac{1+i}{2}\begin{pmatrix}1 & -i\\ -i & 1\end{pmatrix}$$

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \frac{1+i}{2} & \frac{1-i}{2} & 0 & 0 & 0 & 0\\
0 & 0 & \frac{1-i}{2} & \frac{1+i}{2} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

SCQCs and Matrix Functions

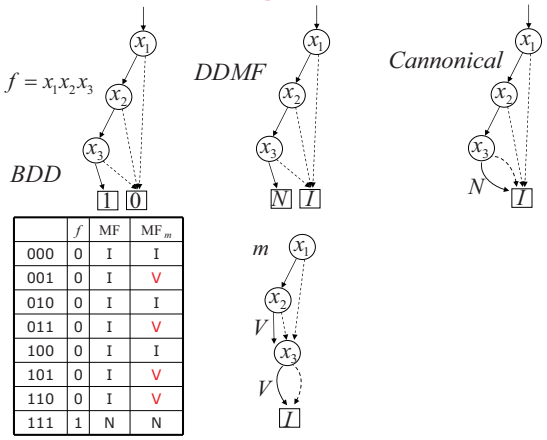


	a	b	c	f
00	I	I	N	1
01	I	I	I	0
10	I	I	I	0
11	I	N	N	1



	a	b	c	f
00	I	I	I	0
01	I	I	I	0
10	I	I	I	0
11	I	V	N	1

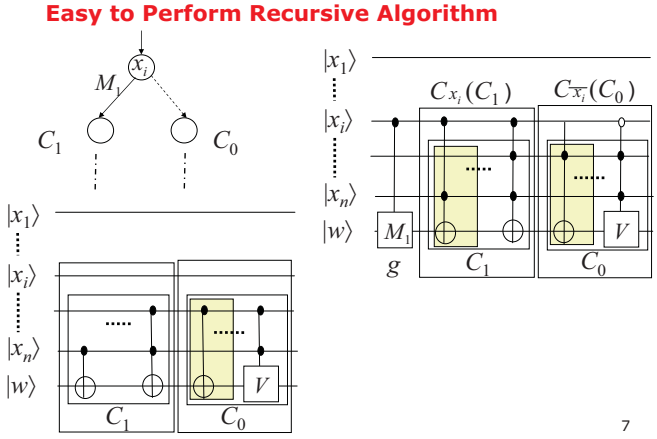
DDMF: Decision Diagram for Matrix Function



	f	MF	MF_m
000	0	I	I
001	0	I	V
010	0	I	I
011	0	I	V
100	0	I	I
101	0	I	V
110	0	I	V
111	1	N	N

DDMF-> Quantum Circuit

Easy to Perform Recursive Algorithm



Problem Formulation

1. **How about variable ordering?**
2. **How to divide the target func. into several MFs?**

$x_1x_2x_3$	X_1	X_2	X_3	R	Target Func.
000	I	I	I	I	I
001	I	I	V	V	N
010	I	V	I	V	N
011	I	V	V	I	N
100	V^{+1}	I	I	V	I
101	V^{+1}	I	V	I	I
110	V^{+1}	V	I	I	I
111	V^{+1}	V	V	V	N

Easy ?

